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*National Aerospace University named after N. Zhukovsky (KhAI), Kharkov, Ukraine***ANALYTICAL AND NUMERICAL LINEARIZATION OF THE FLIGHT VEHICLE
NONLINEAR DYNAMIC MODEL IN THE PRESENCE OF WIND**

In this paper, nonlinear mathematical dynamic model for standard unmanned aerial vehicle, Aerosonde was developed at low velocities in the presence of wind and then its linearized model was derived using numerical and analytical-numerical methods. These linearized dynamic models and nonlinear one were simulated and compared with each other. It was shown that analytical-numerical and numerical linear models with nonlinear dynamic model confirm each other adequately. It must be mentioned that analytical-numerical linear model more accurate and closer to nonlinear model than numerical linear model, because of errors in calculating numerical linear model due to hard nonlinearities. Achieved linear models can be used for control system design purposes. Worst-case linear model was obtained for aerodynamic coefficients uncertainties. It means that these linearized models can be used for obtaining required accurate nominal linear models and uncertainties to design robust control system

Keywords: *unmanned aerial vehicle, nonlinear and linear dynamic model, analytical and numerical linearization, nominal model, control system design, aerodynamic coefficient uncertainty*

Introduction

The field of unmanned aerial vehicle (UAV) is rapidly growing due to its wide range of potential applications. Low operating cost and risk makes UAV a substitution for many airborne applications, which involves higher expenses and greater risks. Nowadays, technological advances in wireless networks and micro electromechanical systems (MEMS) make it possible to use inexpensive micro autopilots on small UAVs [1]. In [2] UAVs generally classified based on the 16 fundamental features such as type of control system, flight rules, wing types, types of landing and take-off, engine type, weight and maximum range of flight, flight altitude, functions and so on. The payload weight may reach up to three kg, for example, gyro stabilized platforms are used by the micro and mini UAVs developed in Russia such as ZALA 421-16 [3].

Due to the high nonlinearities, time varying and uncertainties of the mini UAV dynamics, a lot of classical and advanced control techniques, such as PID control, neural network, fuzzy logic, sliding mode control, gain scheduled, adaptive and robust control have been used in autopilot systems to guarantee a smooth desirable trajectory navigation. A common drawback of both classical and modern control techniques is that they are essentially based on certain models, and therefore, the resulted controllers may work accurately only on the select operating point. However, the UAVs dynamics varies significantly. Hence designing work must be repeated at large quantities of operating points that cover

the whole flight envelop, and gain-schedule mechanism is indispensable to account for the transition between these points. One of the attractive features of the adaptive controllers is that the control implementation does not require a priori knowledge of unknown constant parameters. Two disadvantages of the adaptive controllers are that large amounts of on-line calculation are required, and the lack of robustness to additive bounded disturbances. Therefore, it is difficult to implement adaptive robust control in UAVs due to the large amount of on-line calculation [4, 5]. Attractive features of the robust controllers are that on-line computation kept at a minimum and their inherent robustness to additive bounded disturbances. It must be considered that robust controllers require a priori known bounds on the uncertainty.

Here mentioned fixed-wing UAVs. UAVs are not like commercial airplanes because they have fixed aerodynamic configuration. This is due to their simplicity and low cost. They are expendable, easy to be built and operated. Most of them can be operated by one to two people, or even be hand-carried and hand launched. UAVs usually flies at low speeds to perform their missions. There are some reasons that robust control systems are essential for them to successfully perform their missions:

1. It is difficult to measure aerodynamic velocity at low speeds because of sensor failure and accuracy (The air data system measures the dynamic pressure; it is necessary to measure static pressure, dynamic pressure, and static temperature for calculating true air-

speed) [6].

2. Blowing wind influences as strong external disturbance and also because of that it is impossible to use velocity measured with GPS as airspeed.

3. Fixed aerodynamic configuration prevent to compensate aerodynamically low velocities with change of wing areas, sweeps and so on.

4. There is difficult to capture UAVs in taking off and recover them in landing because of very low velocities.

5. Nonlinear equations of motions and time varying mass, moments of inertia and center of mass change during the flight.

6. It is very difficult to accurately predict and calculate aerodynamic coefficients.

Then it is necessary to model nonlinear equations of motions and linearized at suitable operating points and trim conditions for the purposes of robust control design. The goal of uncertainty modelling is to improve robust performance while maintaining the validity of the model provides a mechanism for achieving robust stability and performance of multiple-input multiple-output (MIMO) systems, and based on a generalized system description that separates the nominal and uncertain system components [7]. In [8] State-space model of UAV from basic aerodynamic equations and by using DATCOM and Simulink derived.

Software simulation helps to minimize design development time and cost for the overall design. The simulation environment that will use for development is an aircraft dynamics simulator built by Unmanned Dynamics called AeroSim® toolbox. So MATLAB® standard configuration environment and the AeroSim® Aeronautical Simulation Block Set utilized for simulation studies [9]. UAVs such as the Aerosonde operated by the Australian meteorological office used to sample the atmosphere over wide areas [10].

The trim settings required to maintain the flight condition are consisted of the airspeed, altitude, bank angle and fuel mass.

The paper was organized in the following way. In section II, the problem was defined and nonlinear equations of UAVs were described using the basic force, moment and kinematic equations. In section III, trimming method is discussed and then analytical and numerical methods for linearization were discussed, in section IV, responses of two the calculated linearized models were compared with each other's and also with nonlinear model responses. It can be seen that they confirm each other's precisely. Then for variation of aerodynamic coefficients in given intervals, distribution of short and long period poles of longitudinal motion calculated. On the other hand, it developed linear model verified and can be used for studying and obtaining nominal model and bounds of uncertainties.

Problem statement

The *goal* is to develop the nonlinear mathematical dynamic model for the flight vehicle and then to linearize the standard UAV, Aerosonde nonlinear dynamic model at low velocities in the presence of wind using numerical and analytical-numerical methods.

Four axis systems are used in the dynamic modelling of UAV: Body frame, Inertial or Earth frame, Aerodynamic frame and Flight path or Velocity frame.

The total velocity vector, V does not in general lie in Earth or Body frame but its orientation defined by the two angles of attack, α and sideslip, β with using linear velocities defined in Body frame. u, v, w were given here:

$$\alpha = \text{tg}^{-1}\left(\frac{w}{u}\right), \beta = \sin^{-1}\left(\frac{v}{V}\right), \quad (1)$$

$$V = \sqrt{u^2 + v^2 + w^2}.$$

The dynamic equations of the UAV were written in Body axis here. It was supposed that mass, m , moments of inertia, I_B and center of gravity (CG) of UAV are changing slowly during the flight. Therefore, the nonlinear translational and rotational dynamical equations of motion for UAV given as:

$$\begin{aligned} \vec{F}_B &= \begin{bmatrix} F_X \\ F_Y \\ F_Z \end{bmatrix} = m \left(\begin{matrix} \dot{\vec{V}}_E + \vec{\omega}_{B/E} \times \vec{V}_E \end{matrix} \right), \\ \begin{matrix} \dot{\vec{V}}_E \\ \dot{\vec{\omega}}_{B/E} \end{matrix} &= \begin{pmatrix} \frac{1}{m} \begin{bmatrix} X + X_T - mg \sin \theta + X_d \\ Y + Y_T + mg \cos \theta \sin \phi + Y_d \\ Z + Z_T + mg \cos \theta \cos \phi + Z_d \end{bmatrix} \\ \begin{bmatrix} qw - rv \\ ru - pw \\ pv - qu \end{bmatrix} \end{pmatrix}, \quad (2) \\ \vec{M}_B &= \begin{bmatrix} M_X \\ M_Y \\ M_Z \end{bmatrix} = I_B \begin{pmatrix} \dot{\vec{\omega}}_{B/E} + \vec{\omega}_{B/E} \times I_B \vec{\omega}_{B/E} \\ \dot{\vec{\omega}}_{B/E} = I_B^{-1} \begin{bmatrix} L + L_T + L_d \\ M + M_T + M_d \\ N + N_T + N_d \end{bmatrix} - \vec{\omega}_{B/E} \times I_B \vec{\omega}_{B/E} \end{pmatrix}, \end{aligned}$$

where \vec{F}_B , \vec{M}_B are vectors of total forces and moments acting on the UAV, X, Y, Z, L, M, N and X_T to Z_T are the scalar form of aerodynamic and engine's forces and moments, X_d to N_d are disturbances, \vec{V}_E , $\vec{\omega}_{B/E}$ are linear and angular velocities u, v, w, p, q, r , respectively.

Aerodynamic coefficients are normally modelled as the sum of several effects, each of which may vary

with flight conditions. For example, the pitching-moment coefficient is a function of center-of-mass location, Mach number, angle of attack, pitch rate, angle of attack rate, and elevator deflection. Aerodynamic and thrust forces act on points that are different from the center of mass. Effect of changing in the center of mass must be considered in calculating of aerodynamic and thrust moments.

Aerodynamic forces and moments usually calculated in the aerodynamic axis frame, they presented as functions of angles of attack and sideslip, Mach number, deflections of control surfaces and angular velocities. Forces and moments transformed from the aerodynamic to the Body frame for calculation of forces and moments in Body frame.

The orientation of the airplane is given by a series of three consecutive rotations: roll, pitch and yaw called Euler angles, whose order is important.

Aerodynamic forces and moments depend on the air-relative velocity. Consider how wind effects are treated in an inertial reference frame. Given an air-mass velocity V_w with respect to the Inertial frame and an inertial aircraft velocity, V viewed within the Inertial frame, the air-relative velocity vector of the aircraft is:

$$V_a = V_E - V_w = \begin{bmatrix} u_E \\ v_E \\ w_E \end{bmatrix} - \begin{bmatrix} u_w \\ v_w \\ w_w \end{bmatrix}. \quad (3)$$

Headwind increases the air-relative velocity of a forward-moving aircraft. In the Body frame of reference, the air-relative velocity vector is:

$${}^B \vec{V}_a = \begin{bmatrix} u_a \\ v_a \\ w_a \end{bmatrix} = C_{BI} V_E - C_{BI} V_w = {}^B V_E - C_{BI} V_w, \quad (4)$$

where ${}^B V_E$ represents the inertial velocity viewed in the Body frame. Then air-relative velocity magnitude or airspeed and the angles of attack and sideslip are:

$$\begin{aligned} \alpha_a &= \text{tg}^{-1} \left(\frac{w_a}{u_a} \right) \approx \frac{w - w_w}{u - u_w} = \alpha - \alpha_w, \\ \beta_a &= \text{sin}^{-1} \left(\frac{v_a}{V_a} \right) \approx \frac{v - v_w}{V_a} = \beta - \beta_w, \\ V_a &= \sqrt{u_a^2 + v_a^2 + w_a^2}. \end{aligned} \quad (5)$$

These definitions are used to compute aerodynamic forces and moments, and the equations of motion. It must be considered that an aircraft cruising with a fixed power setting reaches equilibrium at a given airspeed rather than a given inertial speed. If the wind field is changing in time either explicitly or implicitly, then this must be taken into account in the dynamic equation:

$$\frac{d\vec{V}_a}{dt} = \frac{d\vec{V}_E}{dt} - \frac{d\vec{V}_w}{dt}, \quad (6)$$

where $\frac{d\vec{V}_w}{dt}$ produces specific force (i.e., force per unit mass).

Trimming and linearization

Suppose a nonlinear dynamical system described as below:

$$\begin{aligned} \dot{X}(t) &= f(X(t), u(t), w(t), t), \\ Y(t) &= g(X(t), u(t), w(t), t), \end{aligned} \quad (7)$$

where $X_{n \times 1}$, $Y_{r \times 1}$, $u_{m \times 1}$, $w_{p \times 1}$ and t are state vector, output vector, input disturbance and time consequently, f and g are general nonlinear functions. Any nominal or reference trajectory satisfies the equation. Actual dynamics can be expressed as the sum of the nominal dynamics plus perturbation effects. With cancelation of nominal terms from the both sides of equation, the following linear model derived:

$$\begin{aligned} \Delta \dot{X}(t) &= \left. \frac{\partial f}{\partial X} \right|_N \Delta X(t) + \left. \frac{\partial f}{\partial u} \right|_N \Delta u(t) + \left. \frac{\partial f}{\partial w} \right|_N \Delta w(t) = \\ &= A_{n \times n} \Delta X(t) + B_{n \times m} \Delta u(t) + E_{n \times p} \Delta w(t), \\ \Delta Y(t) &= \left. \frac{\partial g}{\partial X} \right|_N \Delta X(t) + \left. \frac{\partial g}{\partial u} \right|_N \Delta u(t) + \left. \frac{\partial g}{\partial w} \right|_N \Delta w(t) = \\ &= C_{r \times n} \Delta X(t) + D_{r \times m} \Delta u(t) + F_{r \times p} \Delta w(t), \end{aligned} \quad (8)$$

where A , B and E are matrixes with the appropriate dimensions can be calculated numerically.

It is convenient to separate the linearized equations into longitudinal equations and lateral-directional equations. This is possible because the two sets of perturbation motions are uncoupled for a symmetric aircraft in steady cruise, climbing or descending flight. For example, the purpose of trimming in the longitudinal channel is to determine the angle of attack and elevator angle required for a given flight condition in a steady cruise, climb, or descent. For steady flight, in the longitudinal channel assumed that \dot{V} , \dot{q} , $\dot{\theta}$ and $\dot{\alpha}$ are zero.

The longitudinal state, control, and disturbance vectors are:

$$\begin{aligned} \Delta X_{Lon} &= [\Delta u \quad \Delta w \quad \Delta q \quad \Delta x \quad \Delta z \quad \Delta \theta]^T, \\ \Delta U_{Lon} &= \begin{bmatrix} \Delta \delta_E \\ \Delta \delta_r \end{bmatrix}, \Delta W_{Lon} = \begin{bmatrix} \Delta u_w \\ \Delta w_w \end{bmatrix}. \end{aligned} \quad (9)$$

The linearized longitudinal equations of motion take the general form:

$$\Delta \dot{X}_{Lon} = A_{lon} \Delta X + B_{Lon} \Delta U + E_{Lon} \Delta W, \quad (10)$$

where

$$A_{Lon} = \begin{bmatrix} X_u & X_w & a_{13} & 0 & X_z & a_{16} \\ a_{21} & a_{22} & a_{23} & 0 & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & 0 & 0 & a_{36} \\ a_{41} & a_{42} & 0 & 0 & 0 & b \\ a_{51} & a_{52} & 0 & 0 & 0 & c \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$B_{Lon} = \begin{bmatrix} X_{\delta_E} & X_{\delta_T} \\ \frac{Z_{\delta_E}}{1-Z_{\dot{w}}} & \frac{Z_{\delta_T}}{1-Z_{\dot{w}}} \\ M_{\delta_E} + aZ_{\delta_E} & M_{\delta_T} + aZ_{\delta_T} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, E_{Lon} = \begin{bmatrix} -a_{11} & -a_{12} \\ -a_{21} & -a_{22} \\ -a_{31} & -a_{32} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

and

$$a_{13} = X_q - w_0, a_{16} = -g \cos(\theta_0)$$

$$a_{31} = M_u + aZ_u, a_{32} = M_w + aZ_w$$

$$a_{26} = \frac{-g \sin(\theta_0)}{1-Z_{\dot{w}}}, a = \frac{M_{\dot{w}}}{1-Z_{\dot{w}}}$$

$$a_{36} = M_\theta - ag \sin(\theta_0),$$

$$a_{21} = \frac{Z_u}{1-Z_{\dot{w}}}, a_{22} = \frac{Z_w}{1-Z_{\dot{w}}},$$

$$a_{23} = \frac{Z_q + u_0}{1-Z_{\dot{w}}}, a_{25} = \frac{Z_z}{1-Z_{\dot{w}}}.$$

$$a_{41} = \cos(\theta_0), a_{42} = -\sin(\theta_0),$$

$$a_{51} = \sin(\theta_0), a_{52} = \cos(\theta_0),$$

$$b = -u_0 \sin(\theta_0) + w_0 \cos(\theta_0),$$

$$c = -u_0 \cos(\theta_0) - w_0 \sin(\theta_0),$$

$$a_{33} = M_q + a(Z_q + u_0).$$

Wind is disturbance input. Wind disturbances have the same aerodynamic effects as airspeed variations but with opposite sign. Note that state matrix A_{Lon} contains Coriolis or non-aerodynamic components, while E_{Lon} does not [11]. Wind affects the dynamics of the UAV through the aerodynamic model of the forces and moments and direct accelerations due to the wind can be supposed equal to zero in the presence of slowly changing wind.

It must be considered that some aerodynamic coefficients are provided in look up tables, therefore, not only analytical but also numerical linearization are needed for this case.

Models Comparison

In this section, simulation results were compared in nonlinear and linearized models for longitudinal channel. Simulation results for three models were pre-

sented at a low-velocity operating point to prove similarity of responses:

$$\begin{aligned} u_{Trim} &= 20 \text{ m/s}, & \delta_{ETrim} &= -0.2070 \text{ rad}, \\ w_{Trim} &= 1.81 \text{ m/s}, & \delta_{TTrim} &= 0.3253 \text{ rad}, \\ \theta_{Trim} &= 0.0725 \text{ rad/s}, & \text{mass} &= 9.5 \text{ kg}, \\ w_{Trim} &= 1.81 \text{ m/s}, & \omega_{EngineTrim} &= 445 \text{ rev/min}, \end{aligned} \quad (11)$$

where δ_{ETrim} and δ_{TTrim} are consequently, trimmed elevator and throttle control inputs, mass and $\omega_{EngineTrim}$ are consequently, trimmed mass of UAV and its engine speed.

Three dynamic systems at given low velocity operating point were simulated for periodic square form signal with frequency equal 0.2 Hz as a persistent excitation for elevator deflection control input: original nonlinear, numerical and analytical-numerical linear models were simulated and results were compared in Fig. 1 and Fig. 2. It can be seen that their responses are very similar to each other's adequately. But it must be mentioned that analytical-numerical linear model more accurate and closer to nonlinear model than the numerical linear model, because of errors in calculating numerical linear model due to hard nonlinearities. It means that analytical linear model verified and can be used to study parametric uncertainties.

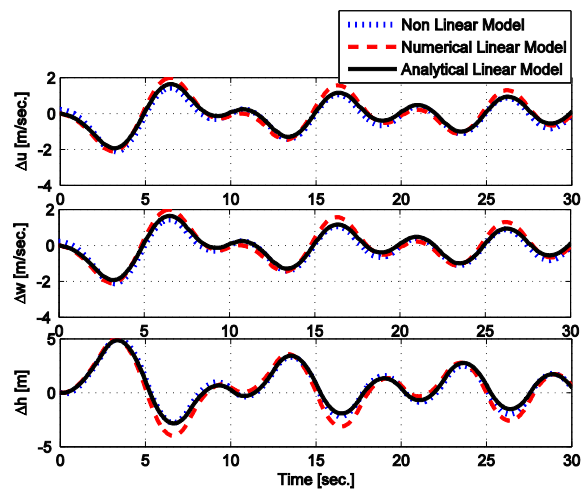


Fig. 1. Linear velocities and height in nonlinear and linear models

The direction of the wind can be supposed constant or slowly varying in the Inertial frame. Wind affects the dynamics of the UAV through the aerodynamic model of forces and moments and direct accelerations due to the wind can be supposed equal to zero. The effect of the wind has been studied in linear and nonlinear models of the Aerosonde in Body frame for the constant wind vector, $[4 \ 2 \ 0]^T$ in Inertial frame. Wind effect in the linear and nonlinear models was compared in Fig. 3 and Fig. 4. It can be seen that wind effect can not be

neglected and in comparison with the nonlinear model the linear model with acceptable accuracy can show the effect of the wind.

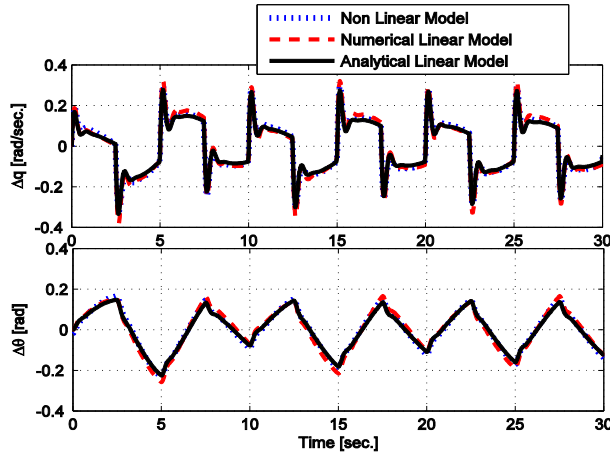


Fig. 2. Pitch angular rate and pitch angle in nonlinear and linear models

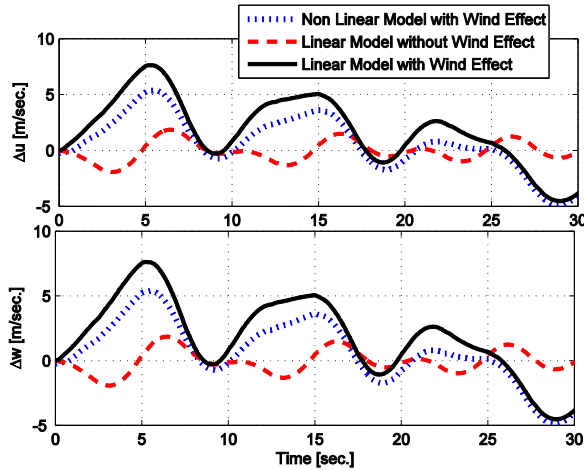


Fig. 3. Wind effect on linear velocities in nonlinear and linear models in Body frame

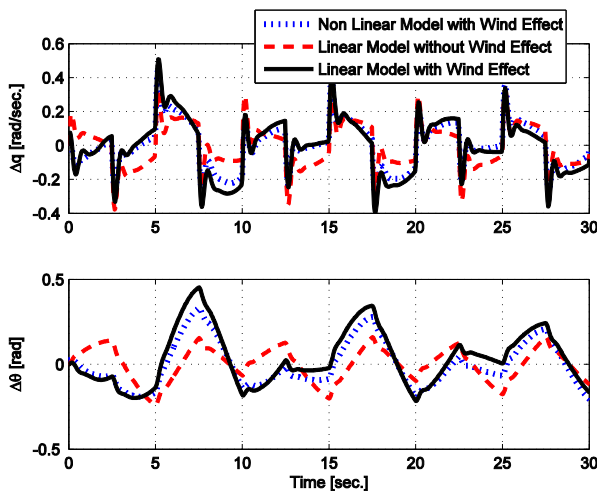


Fig. 4. Wind effect on pitch angular rate and pitch angle in nonlinear and linear models

Aerodynamic Coefficients Uncertainties

Aerodynamic model for UAVs cannot be predicted or calculated exactly specially at low velocities. Here set of linear models of the mini UAV were calculated for the given intervals of aerodynamic coefficients uncertainties in the longitudinal channel:

$$\begin{aligned} C_{L_{Min}} &= C_{L_{MinN}} \pm 20\%, & C_{m\alpha} &= C_{m\alpha N} \pm 50\%, \\ C_{m0} &= C_{m0N} \pm 30\%, & C_{L_{\delta_E}} &= C_{L_{\delta_EN}} \pm 30\%, \\ C_{L_\alpha} &= C_{L_\alpha N} \pm 15\%, & C_{m_{\delta_E}} &= C_{m_{\delta_EN}} - 90\% + 100\%, \\ C_{L_{\dot{\alpha}}} &= C_{L_{\dot{\alpha}N}} \pm 100\%, & C_{m\dot{\alpha}} &= C_{m\dot{\alpha}N} - 100\% + 200\%, \\ C_{L_q} &= C_{L_qN} \pm 100\%, & C_{m_q} &= C_{m_qN} \pm 100\%, \end{aligned} \quad (12)$$

where index N was used for normal values. Eight aerodynamic coefficients affect the positions of poles of MIMO system. $C_{L_{\delta_E}}$ and $C_{m_{\delta_E}}$ do not affect poles of the system. For various combinations of aerodynamic coefficients that means for $38=6561$ cases, poles of the multi inputs multi outputs system of longitudinal channel were calculated. Poles or eigenvalues of the MIMO system for 6561 cases were shown in Fig. 5 and Fig. 6. It must be noticed that state vector of the system combined of seven variables:

$$[u \ w \ q \ x \ z \ \theta \ \omega_r]^T. \quad (13)$$

Real and imaginary part of complex poles of the system in short period mode change about 4.71 and 2.31 times consequently. Real and imaginary part of complex poles of the system change in long period mode about 1.6 and 1.87 times consequently due to aerodynamic coefficients variations. Minimum and maximum real part of poles for short-period mode told above were happened when aerodynamic coefficients were in the opposite extreme sides of changing intervals.

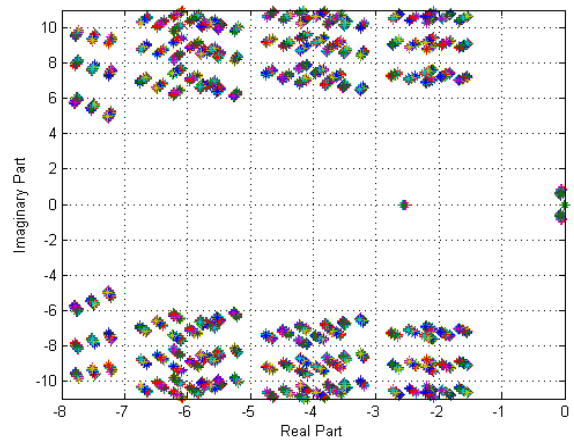


Fig. 5. All of the poles of longitudinal channel for various combination of aerodynamic coefficients uncertainties

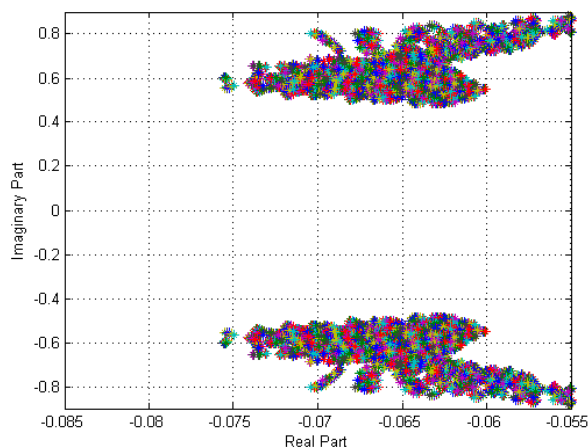


Fig. 6. Long period poles of longitudinal channel for various combination of aerodynamic coefficients uncertainties

Various techniques of system identification exist that provide a nominal model and an uncertainty bound. An important question is what the implications are for the particular choice of the structure in which the uncertainty is described when dealing with robust stability/performance analysis of a given controller and when dealing with robust synthesis [12]. Nominal model can be found with different method. If the goals of a nominal model are reduction or simplification for control system design, then these methods can be used: comparison principles, singular perturbation, weighted functions, matrix inequality, and approximation or reduction method [13].

Conclusions

Nonlinear and linear dynamic model of a mini UAV at low velocities were presented. Linear model of the mini UAV for longitudinal motions was calculated using numerical and analytical methods in the presence of the wind. Numerical and analytical-numerical linear model responses were compared and verified with nonlinear model responses. Achieved verified linear model can be used for studying and modelling of uncertainties and finding nominal model. For aerodynamic coefficients uncertainties at low velocity, a set of linear models was calculated and studied. Maximum range of changes for long and short-period poles were calculated. For practical control problems, verified uncertainty modelling, which accurately characterizes realistic system uncertainties is very important; because the robustness results obtained using these algorithms depend directly on the uncertainty model used for the analysis and design.

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АНАЛИТИЧЕСКАЯ И ЧИСЛЕННАЯ ЛИНЕАРИЗАЦИЯ НЕЛИНЕЙНОЙ ДИНАМИЧЕСКОЙ МОДЕЛИ ЛЕТАТЕЛЬНОГО АППАРАТА ПРИ НАЛИЧИИ ВЕТРА

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В данной работе использована нелинейная математическая модель беспилотного летательного аппарата типа Aerosonde и разработана линейная при малых скоростях полета и при наличии ветра, а затем линеаризованная модель была получена с помощью численного и численно – аналитического метода. Эти линеаризованные и нелинейная динамические модели сравнены при различных режимах. Путем имитационного моделирования показано, что численно-аналитическая и численная линейная модели соответствуют нелинейной динамической. Следует отметить, что численно-аналитическая линейная модель более точна и ближе к нелинейной модели, чем численная линейная модель, из-за ошибок при вычислении численной линейной модели. Полученные линейные модели могут быть использованы для проектирования системы управления летательного аппарата. Неопределенная линейная модель была получена при задании неопределенности аэродинамических коэффициентов. Это означает, что эти линеаризованные модели могут быть использованы для получения номинальных линейных моделей и диапазонов неопределенности при разработке робастной системы управления.

Ключевые слова: беспилотный летательный аппарат, нелинейная и линейная динамические модели, аналитическая и численная линеаризации, номинальная модель, система управления, неопределенность аэродинамических коэффициентов.

**АНАЛІТИЧНА ТА ЧИСЕЛЬНА ЛІНЕАРИЗАЦІЯ НЕЛІНІЙНОЇ ДИНАМІЧНОЇ
МОДЕЛІ ЛІТАЛЬНОГО АПАРАТУ ЗА НАЯВНОСТІ ВІТРУ*****Р. М. Фархаді, В. І. Кортунів***

В даній роботі використано нелінійну математичну модель безпілотного літального апарату типу Aerosonde та розроблено лінійну при малих швидкостях польоту і при наявності вітру, а далі лінеаризовану модель було отримано за допомогою чисельного та чисельно – аналітичного методу. Ці лінеаризовані і нелінійна динамічні моделі порівняно при різних режимах. Шляхом імітаційного моделювання показано, що чисельно-аналітична і чисельна лінійні моделі відповідають нелінійній динамічній. Слід зазначити, що чисельно-аналітична лінійна модель більш точна і ближче до нелінійної моделі, ніж чисельна лінійна модель, через помилки при обчисленні чисельної лінійної моделі. Отримані лінійні моделі можуть бути використані для проектування системи керування літального апарату. Невизначена лінійна модель була отримана при завданні невизначеності аеродинамічних коефіцієнтів. Це означає, що ці лінеаризовані моделі можуть бути використані для отримання номінальних лінійних моделей і діапазонів невизначеності при розробці робастної системи управління.

Ключові слова: безпілотний літальний апарат, нелінійна та лінійна динамічні моделі, аналітична і чисельна лінеаризації, номінальна модель, система управління, невизначеність аеродинамічних коефіцієнтів.

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