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MATHEMATICAL MODEL OF A VERTICAL-AXIAL WIND MOTOR IN A VISCOUS GAS FLOW

The development of vertical axial wind turbines in Ukraine is in its infancy for many reasons: the lack of systematic theoretical and experimental studies of the aerodynamic characteristics of various schemes of wind turbines, the lack of an appropriate experimental base in technical universities, design organizations, insufficient number of available publications in foreign literature due to high competition between by monopoly firms. At present, various numerical methods are widely used to solve urgent problems of aero hydrodynamics, which are used for the approximate solution of boundary value problems in the form of differential forms of mathematical models. Their common disadvantages are the particularity and laboriousness of solutions, high requirements for computing resources, and, as a consequence, the complexity of solving optimization problems and economic feasibility. These problems can be avoided by using exact or approximate analytical dependences, which allow solving some urgent problems of studying the interaction of a viscous gas with the bearing elements of both aircraft and engineering structures. The existing methods for calculating the aerodynamic characteristics, based on the ideology of the mathematical model of the motion of an ideal medium without viscous interaction, do not correspond to the real processes and demands of practice. The article presents the ideology of determining the aerodynamic characteristics of the interacting system of solid profiles in the configuration of a vertical-axial wind turbine in a viscous gas flow. Based on generalized vector-tensor analysis, contour integral representations of solutions to the main problem of fluid and gas mechanics related to the determination of kinematic and dynamic characteristics of interaction have been constructed. In addition, the existence of a vector potential of the tensor of stresses and deformation velocities has been proved, reducing, in the simplest cases, the process of determining characteristics to integration. The limit values of these integral representations are a system of boundary integral equations, allow for elementary algorithmization, and lead to a system of linear algebraic equations having a single solution.

Keywords: *viscous gas; conservation laws; boundary integral equations; system of solid profiles; wind turbine; aerodynamic characteristics.*

Introduction

Wind energy is a field of alternative energy that specializes in converting the kinetic energy of wind into electrical energy.

For the best use of wind energy, it is important to study in detail the daily and seasonal changes in wind flows, changes in wind speed depending on the height above the earth's surface, the number of wind gusts in short periods of time, as well as statistical data for at least the last 20 years.

Humankind has used wind energy for a long time. One of the first devices to use wind energy was a windmill built somewhere in the fifth millennium BC. In the first century BC, the ancient Greek scientist Heron of Alexandria invented a windmill that controlled an organ.

Windmills for processing grain, were invented in the middle Ages. It believed that the first windmills, were built in Sistani, somewhere between present-day Iran and Afghanistan, between the ninth and seventh centuries BC. They had a vertical axis of rotation, from six to twelve wings made of linen or poles, and were used as mills and water pumps.

In recent years, wind energy has been increasingly used to generate electricity. High-capacity windmills are created and installed in areas where frequent and strong winds blow. The quantity and quality of such engines is increasing every year, their serial production has been established.

Energy production from renewable (or alternative) sources around the globe is growing at a rapid pace. One of such inexhaustible sources of energy on Earth is the wind.

However, the most common wind turbines today with a horizontal axis of rotation of the wind turbine cannot yet exceed the power figure of 5–7 MW, which, in turn, limits the possibility of reducing the cost of a kWh to a competitive value. For example, the most powerful wind turbine of this type put into operation today with a capacity of five MW was created in

Germany. It is installed in the North Sea at a depth of 40 m, the blade length of its wind turbine is 61 m, and the height of the tower is 120 m [1].

At the same time, the power of a wind turbine with a vertical axis of rotation (VAR) of a wind rotor (of the Darrius rotor type, but with straight blades) (Fig. 1) can reach, according to experts, 10 - 30 MW.

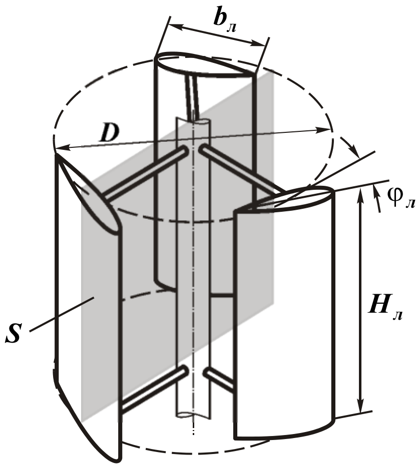

Fig. 1. Three-blade wind turbine

It is possible to enumerate such advantages of these wind turbines as the independence of operation from the direction of the wind flow, the possibility of switching from a cantilever mounting of the wind rotor axis to a two-support one, the possibility of placing an energy consumer (electric generator, pump) at the base of the wind turbine (reducing the requirements for height, strength and rigidity of the support), simplifying the design of the blades and reducing their material consumption (and hence cost), reducing the noise level of wind turbines and the area of land for its placement, etc. [1]. In addition, the significant advantages of vertical-axial layout include:

– the ability to work with very small gusts of wind from 0.17 m/s;

– noiseless operation and absence of vibration, which allows you to install the windmill in the immediate vicinity of the house, or even on the roof;

– system operation does not depend on wind direction;

– possibility of capacity increase without disassembly of the whole system;

– resistance to strong, hurricane gusts of wind;

– reliability and durability of the installation.

The ever-increasing interest of scientists and designers around the world in wind turbines of this type can be illustrated by the fact that at the 8th World Conference on Wind Energy held in Canada in June 2008 in the section "Design of wind turbines" all reports (from the USA, Canada, etc.), were devoted to wind turbines with WWII with straight blades. Intensive research continues on promising layout schemes for vertical-axis wind turbines [2].

The process of creating the Ukrainian wind power industry began in 1996, when the Novoazovsk wind farm was designed with a design capacity of 50 MW. In 1997, the Truskavets wind farm was put into operation. In 2000, 134 turbines were already operating in Ukraine and about 100 foundations were laid for turbines with a capacity of 100 kW. In 1998-1999, three more new wind farms began to generate energy.

A significant expansion of the construction of wind farms has been observed since 2009, after the introduction of the "Green Tariff" by the Government of Ukraine.

1. Statement of the problem

Nonlinear aerodynamic characteristics of a vertical axis engine are determined (Fig. 1). It is assumed that when flowing around this turbine, aerodynamic forces arise, leading it to intensive rotation around the vertical axis. In addition, such an engine design with rectangular bearing elements has a number of technical and economic advantages over propeller-type engines. The rotation of the turbine at a constant speed ω occurs in a stationary flow of a viscous incompressible fluid inside the control region (Σ), the choice of the size of which guarantees the attenuation of the arising disturbances of the medium (Fig. 2).

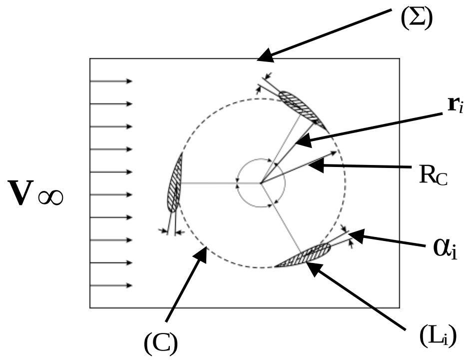

Fig. 2. Plane section of a three-blade wind turbine in the control area

The fundamental feature of the differential forms of mathematical models of gas-dynamic processes, which creates significant difficulties for theoretical studies, is their non-linearity. That is why, for the majority of practically important gas-dynamic problems, existence, uniqueness, and stability theorems for solutions have not yet been proven. Therefore, the numerical implementation of computational algorithms that are widely used now is not provided with an appropriate correct justification, which, even with a

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large expenditure of resources, does not lead to reliable results. The theory of flows of viscous compressible media seems to be one of the most important for practice and the most interesting branch of continuum mechanics for mathematical research.

It is no coincidence that J. Leray and J. Schauder took the first steps in applying the methods of functional analysis in the problems of viscous flow dynamics, and recently the Navier-Stokes equations have become one of the first objects of application of numerical methods. The theoretical foundations of modern technologies for studying the flows of viscous continuums were laid back in the middle of the last century by O. A. Ladyzhenskaya, J.-L. Lions, R. Temam, O. M. Belotserkovsky, K. I. Babenko and many others [3, 4], who also carried out numerical simulation of the motion of bodies in various media. To date, many scientists have conducted numerous studies and obtained many results that are relevant to this work.

Unfortunately, the theoretical research methods used today and the corresponding application software packages based on finite-difference approaches are far from perfect [5], and the results of the fundamental works of our predecessors (N. E. Kochin, I. I. Lyashko, B. E. Pobedrya, P. K. Rashevsky, E. Cartan, V. D. Kupradze and many others) are waiting for their demand. However, interest in this ideology has recently been growing (Vector, tensor and the basic equations of fluid mechanics. R. Aris. University of Minnesota; publications in Pergamon Press, Springer, Philosophical Transactions of the Royal Society of London Series A, Mathematical and Physical Sciences, The Journal of Fluid Mechanics, AIAA Paper, Acta Mechanica Sinica, International Journal of Mechanical Engineering and Automation, Journal of aircraft, NASA cooperative agreement, NASA Langley research center, etc.).

The most promising method for solving initial-boundary value problems based on the system of Navier-Stokes equations or correct linearization is the method of boundary integral equations [8]. Moreover, this method is often used to solve a wide range of applied problems [9, 11]. Reducing the boundary value problem to a boundary integral equation or to an adequate system of boundary integral equations allows:

- reduce the dimension of the problem and consider more complex classes of problems than those that can solved by other methods;
- immediately determine unknown quantities at the boundaries, without calculating them in the entire space of motion; solutions at interior points of the domain found by integration;
- non-linear problems for differential equations or their systems lead to a system of linear boundary integral equations with respect to unknown boundary values of the sought parameters of the problem or functions from them;
- to set and solve problems of optimizing gas-dynamic characteristics by studying their extreme values.

1.1. Mathematical model

1.1.1. Conservative form of the laws of conservation of fluid and gas mechanics

The mathematical model of the dynamics of a viscous gas is the initial-boundary value problem for the system of conservation laws of continuum mechanics in differential form [4, 10]. Here we consider a two-dimensional problem of the motion of a non-heat-conducting viscous incompressible fluid.

The introduction of dimensionless coordinates with a characteristic size *L* and parameters related to the characteristics of the undisturbed flow, allows us to represent the laws of conservation of aerohydrodynamics in a conservative form (we restrict ourselves to studying the stationary case, when $\frac{\partial}{\partial t} \equiv 0$ and in the absence of external influences):

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0\,; \tag{1.1}$$

$$\begin{cases} \dfrac{\partial}{\partial x}\left(\rho u^2 + p - \tau_{xx}\right) + \dfrac{\partial\left(\rho uv - \tau_{xy}\right)}{\partial y} = 0; \\ \dfrac{\partial\left(\rho uv - \tau_{xy}\right)}{\partial x} + \dfrac{\partial}{\partial y}\left(\rho v^2 + p - \tau_{yy}\right) = 0. \end{cases} \tag{1.2}$$

Here, in the system of differential equations of the momentum conservation law (1.2), the components of the strain velocity tensor are calculated by the formulas [4, 5, 10]:

$$\tau_{xx} = \frac{2}{3}\frac{\mu}{Re}\left(2\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) = 2\frac{\mu}{Re}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\mu}{Re}\left(\nabla, \mathbf{V}\right);$$

$$\tau_{xy} = \frac{\mu}{Re}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \tau_{yx}\,; \tag{1.3}$$

$$\tau_{yy} = \frac{2}{3}\frac{\mu}{Re}\left(2\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right) = 2\frac{\mu}{Re}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\mu}{Re}\left(\nabla, \mathbf{V}\right)$$

at the Reynolds number $Re = \dfrac{\rho_\infty V_\infty L}{\mu_\infty}$, and u, v are the components of the velocity vector **V** in the Cartesian coordinate system with divergence at a constant density of the medium - ρ = const. in (1.1) and the absence of external sources of mass, μ is the coefficient of dynamic viscosity, depending only on the absolute temperature T, which in the main problems of aerodynamics can be considered a constant value.

Here, it is especially necessary to emphasize the fact that the conservation laws (1.1), (1.2) are the fundamental physical conservation laws of continuum mechanics and in the process of their derivation, no assumptions about the kinematic characteristics of the medium were involved, which, along with the dynamic characteristics, are determined in the process of solving the initial -boundary value problems for this system of equations.

In practical stationary problems of aerogasdynamics, it is sometimes possible to assume the absence of both heat and mass transfer and the presence of mass and surface forces (although this does not fundamentally affect the approach developed in the future). In this case, the conservation equations (1.1), (1.2) have a conservative form [4, 8, 10], which is conveniently written in tensor form

$$\nabla,\mathbf{P} = 0\,, \tag{1.4}$$

where the tensor P has the form

$$\mathbf{P} = \begin{Bmatrix} \rho u & \rho v \\ \rho u^2 + p - \tau_{xx} & \rho uv - \tau_{xy} \\ \rho uv - \tau_{xy} & \rho v^2 + p - \tau_{yy} \end{Bmatrix}. \tag{1.5}$$

In what follows, it is expedient to represent the components of the strain rate tensor in the form of standard vector analysis operators. Then the expressions in (1.3) can give the form:

$$\tau_{xy} = \tau_{yx} = \frac{\mu}{\mathrm{Re}}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \\ = \frac{\mu}{\mathrm{Re}}\left(2\frac{\partial u}{\partial y} + \Omega_z\right) = \frac{\mu}{\mathrm{Re}}\left(2\frac{\partial v}{\partial x} - \Omega_z\right); \tag{1.6}$$

where in the plane case $\Omega_Z = (\mathbf{k},\mathbf{\Omega}) = \left(\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}\right)$ is the only component of the vorticity vector orthogonal to the flow plane.

The system of differential equations of the momentum conservation law (1.2) in the stationary case of flow in the absence of body forces has a conservative form:

$$\begin{cases} \dfrac{\partial}{\partial x}\left(\rho u^2 + p - 2\dfrac{\mu}{\mathrm{Re}}\dfrac{\partial u}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(\rho uv - \dfrac{\mu}{\mathrm{Re}}\left(2\dfrac{\partial u}{\partial y} + \Omega_z\right)\right) = 0; \\ \dfrac{\partial}{\partial x}\left(\rho uv - \dfrac{\mu}{\mathrm{Re}}\left(2\dfrac{\partial v}{\partial x} - \Omega_z\right)\right) + \dfrac{\partial}{\partial y}\left(\rho v^2 + p - 2\dfrac{\mu}{\mathrm{Re}}\dfrac{\partial v}{\partial y}\right) = 0, \end{cases}$$

which can be associated with the tensor Π, which can be expressed in multiple forms:

$$\mathbf{\Pi} = \mathbf{i}\left\{\mathbf{i}\left(\rho u^2 + p - 2\frac{\mu}{\mathrm{Re}}\frac{\partial u}{\partial x}\right) + \mathbf{j}\left(\rho uv - \frac{\mu}{\mathrm{Re}}\left(2\frac{\partial u}{\partial y} + \Omega_z\right)\right)\right\} + \\ + \mathbf{j}\left\{\mathbf{i}\left(\rho uv - \frac{\mu}{\mathrm{Re}}\left(2\frac{\partial v}{\partial x} - \Omega_z\right)\right) + \mathbf{j}\left(\rho v^2 + p - 2\frac{\mu}{\mathrm{Re}}\frac{\partial v}{\partial y}\right)\right\} = \\ = \begin{vmatrix} \rho u^2 + p - 2\dfrac{\mu}{\mathrm{Re}}\dfrac{\partial u}{\partial x} & \rho uv - \dfrac{\mu}{\mathrm{Re}}\left(2\dfrac{\partial u}{\partial y} + \Omega_z\right) \\ \rho uv - \dfrac{\mu}{\mathrm{Re}}\left(2\dfrac{\partial v}{\partial x} - \Omega_z\right) & \rho v^2 + p - 2\dfrac{\mu}{\mathrm{Re}}\dfrac{\partial v}{\partial y} \end{vmatrix} \tag{1.7}$$

and which, by virtue of (1.2), (1.3), (1.6), is symmetric and conservative, since

$$\nabla,\mathbf{\Pi} = \mathbf{i}\left\{\frac{\partial}{\partial x}\left(\rho u^2 + p - 2\frac{\mu}{\mathrm{Re}}\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho uv - \frac{\mu}{\mathrm{Re}}\left(2\frac{\partial v}{\partial x} - \Omega_z\right)\right)\right\} + \\ + \mathbf{j}\left\{\frac{\partial}{\partial x}\left(\rho uv - \frac{\mu}{\mathrm{Re}}\left(2\frac{\partial u}{\partial y} + \Omega_z\right)\right) + \frac{\partial}{\partial y}\left(\rho v^2 + p - 2\frac{\mu}{\mathrm{Re}}\frac{\partial v}{\partial y}\right)\right\} = 0. \tag{1.8}$$

The coordinate representations of this tensor $\mathbf{\Pi} = \mathbf{i\Pi}_x + \mathbf{j\Pi}_y$ fully correspond to the system of equations of the momentum conservation law (1.2).

Indeed, the vector components of the tensor (1.7) have the form:

$$\mathbf{\Pi}_x = \mathbf{i}\ \rho u^2 + p\ + \mathbf{j}\ \rho uv\ - 2\frac{\mu}{\mathrm{Re}}\nabla u + \frac{\mu}{\mathrm{Re}}\ \mathbf{i},\mathbf{\Omega}\ ; \\ \mathbf{\Pi}_y = \mathbf{i}\ \rho uv\ + \mathbf{j}\ \rho v^2 + p\ - 2\frac{\mu}{\mathrm{Re}}\nabla v + \frac{\mu}{\mathrm{Re}}\ \mathbf{j},\mathbf{\Omega}\ ; \tag{1.9}$$

and are conservative in full agreement with equations (1.8):

$$\begin{cases} \nabla,\mathbf{\Pi}_x = \dfrac{\partial}{\partial x}\ \rho u^2 + p\ + \dfrac{\partial}{\partial y}\ \rho uv\ - \\ \left. -2\dfrac{\mu}{\mathrm{Re}}\Delta u - \dfrac{\mu}{\mathrm{Re}}\dfrac{\partial \Omega_z}{\partial y}\right\} = 0; \\ \nabla,\mathbf{\Pi}_y = \dfrac{\partial}{\partial x}\ \rho uv\ + \dfrac{\partial}{\partial y}\ \rho v^2 + p\ - \\ \left. -2\dfrac{\mu}{\mathrm{Re}}\Delta v + \dfrac{\mu}{\mathrm{Re}}\dfrac{\partial \Omega_z}{\partial x}\right\} = 0; \end{cases} \tag{1.10}$$

Conservative forms of conservation laws (1.8), (1.10) are convenient to represent as second-order differential equations [4, 5]:

$$\nabla,\mathbf{V} = 0 \Rightarrow \nabla\ \nabla,\mathbf{V} = 0\,, \\ \nabla(\nabla,\mathbf{\Pi}) = 0 \Rightarrow \begin{cases} \nabla(\nabla,\mathbf{\Pi}_x) = 0; \\ \nabla(\nabla,\mathbf{\Pi}_y) = 0, \end{cases} \tag{1.11}$$

for the solution of which the necessary mathematical apparatus of generalized vector-tensor analysis is fully developed [8].

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1.1.2. Theory of generalized hydrodynamic potentials

Potential theory is a branch of mathematics that has developed in close connection with the theory of classical boundary value problems of mathematical physics (Laplace's equations, heat conduction, wave equations, and a number of others). Apparently, the first significant step was associated with the study of potential flows of an ideal incompressible fluid. The number of such flows turned out to be quite extensive, and the available mathematical methods for their study were almost perfect. However, all attempts to eliminate the well-known paradoxes within the framework of the theory of an ideal incompressible fluid were in vain, which indicated the imperfection of this theory.

Conservatism (1.8) of the tensor (1.7) allows for the existence of a vector potential $\mathbf{\Psi}$:

$$\begin{aligned} &\mathbf{\Pi} = \mathbf{i}\ \ \mathbf{i}\ \ \rho u^2 + p\ \ + \mathbf{j}\ \ \rho uv\ \ + \\ &+\mathbf{j}\ \ \mathbf{i}\ \ \rho uv\ \ + \mathbf{j}\ \ \rho v^2 + p\ \ - 2\frac{\mu}{Re}\nabla^{*}\mathbf{V} + \\ &+\frac{\mu}{Re}\ \mathbf{I},\mathbf{\Omega}\ = \left[\nabla,\ \mathbf{kk},\mathbf{\Psi}\ \right] = \nabla^{*}\mathbf{\Psi} - \mathbf{I}\ \ \nabla,\mathbf{\Psi}\ \ , \end{aligned} \qquad (1.12)$$

where $\mathbf{I} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ - unit tensor and tensor

$$\nabla^{*}\mathbf{\Psi} = \mathbf{i}\nabla\Psi_x + \mathbf{j}\nabla\Psi_y = \begin{vmatrix} \frac{\partial\Psi_x}{\partial x} & \frac{\partial\Psi_x}{\partial y} \\ \frac{\partial\Psi_y}{\partial x} & \frac{\partial\Psi_y}{\partial y} \end{vmatrix} =$$

$$= \mathbf{i}\left(\mathbf{i}\frac{\partial\Psi_x}{\partial x} + \mathbf{j}\frac{\partial\Psi_x}{\partial y}\right) + \mathbf{j}\left(\mathbf{i}\frac{\partial\Psi_y}{\partial x} + \mathbf{j}\frac{\partial\Psi_y}{\partial y}\right)$$

is conjugate to a tensor

$$\nabla\mathbf{\Psi} = \mathbf{i}\frac{\partial\mathbf{\Psi}}{\partial x} + \mathbf{j}\frac{\partial\mathbf{\Psi}}{\partial y} = \begin{vmatrix} \frac{\partial\Psi_x}{\partial x} & \frac{\partial\Psi_y}{\partial x} \\ \frac{\partial\Psi_x}{\partial y} & \frac{\partial\Psi_y}{\partial y} \end{vmatrix} =$$

$$= \mathbf{i}\left(\mathbf{i}\frac{\partial\Psi_x}{\partial x} + \mathbf{j}\frac{\partial\Psi_y}{\partial x}\right) + \mathbf{j}\left(\mathbf{i}\frac{\partial\Psi_x}{\partial y} + \mathbf{j}\frac{\partial\Psi_y}{\partial y}\right).$$

Here, the vector components of tensor $\mathbf{\Pi}$ (1.9) are of the form:

$$\mathbf{\Pi}_x = \nabla\Psi_x - \mathbf{i}\ \ \nabla,\mathbf{\Psi}\ \ ; \qquad (1.13)$$

$$\mathbf{\Pi}_y = \nabla\Psi_y - \mathbf{j}\ \ \nabla,\mathbf{\Psi}\ \ . \qquad (1.14)$$

From the expressions (1.9), (1.13), (1.14) follow the following differential properties of the vector potential $\mathbf{\Psi}$:

$$\mathbf{i},\mathbf{\Pi}_x\ + \ \mathbf{j},\mathbf{\Pi}_y\ = \rho u^2 + p - 2\frac{\mu}{Re}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \rho v^2 + p =$$

$$= \frac{\partial\Psi_x}{\partial x} - \left(\frac{\partial\Psi_x}{\partial x} + \frac{\partial\Psi_y}{\partial y}\right) + \frac{\partial\Psi_y}{\partial y} - \left(\frac{\partial\Psi_x}{\partial x} + \frac{\partial\Psi_y}{\partial y}\right) = -\left(\frac{\partial\Psi_x}{\partial x} + \frac{\partial\Psi_y}{\partial y}\right).$$

Thus,

$$\nabla,\mathbf{\Psi}\ = \frac{\partial\Psi_x}{\partial x} + \frac{\partial\Psi_y}{\partial y} = -\rho V^2 - 2p, \qquad (1.15)$$

where $V^2 = u^2 + v^2$.

Further:

$$\mathbf{i},\mathbf{\Pi}_x\ = \mathbf{k}\left(\rho uv - 2\frac{\mu}{Re}\frac{\partial u}{\partial y} - \frac{\mu}{Re}\Omega_z\right) = \mathbf{k}\frac{\partial\Psi_x}{\partial y};$$

$$\left[\mathbf{j},\mathbf{\Pi}_y\right] = -\mathbf{k}\left(\rho uv - 2\frac{\mu}{Re}\frac{\partial v}{\partial x} + \frac{\mu}{Re}\Omega_z\right) = -\mathbf{k}\frac{\partial\Psi_y}{\partial x}.$$

Therefore

$$\left[\mathbf{i},\mathbf{\Pi}_x\right] + \left[\mathbf{j},\mathbf{\Pi}_y\right] = 2\mathbf{k}\frac{\mu}{Re}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - 2\mathbf{k}\frac{\mu}{Re}\Omega_z =$$

$$= -\mathbf{k}\left(\frac{\partial\Psi_y}{\partial x} - \frac{\partial\Psi_x}{\partial y}\right)$$

and $$\nabla,\mathbf{\Psi}\ = \mathbf{k}\left(\frac{\partial\Psi_y}{\partial x} - \frac{\partial\Psi_x}{\partial y}\right) = 0. \qquad (1.16)$$

Then from the conservatism of the tensor $\mathbf{\Pi}$ (1.12), taking into account the potentiality of the vector $\mathbf{\Psi}$ (1.16), should

$$\begin{aligned} &\nabla,\mathbf{\Pi}\ = 0 \Rightarrow\ \nabla,\nabla\mathbf{\Psi}\ - \nabla\ \ \nabla,\mathbf{\Psi}\ = \\ &= -\left[\nabla,\ \nabla,\mathbf{\Psi}\ \right] = 0. \end{aligned} \qquad (1.17)$$

Let us move on to calculating the vortex of the tensor $\mathbf{\Pi}$:

$$\nabla,\mathbf{\Pi}\ = \mathbf{k}\left(\frac{\partial\mathbf{\Pi}_y}{\partial x} - \frac{\partial\mathbf{\Pi}_x}{\partial y}\right) = -\left[\mathbf{I},\nabla\ \ \nabla,\mathbf{\Psi}\ \right], \qquad (1.18)$$

since $\left[\nabla,\ \nabla\mathbf{\Psi}\ \right] \equiv 0,$ and for vectors from (1.13 – 1.14) have

$$\nabla,\mathbf{\Pi}_x \;=\mathbf{k}\left(\frac{\partial\Pi_{xy}}{\partial x}-\frac{\partial\Pi_{xx}}{\partial y}\right)=$$
$$=-\left[\nabla,\ \mathbf{i}\ \nabla,\mathbf{\Psi}\ \ \right]=\left[\mathbf{i},\nabla\ \ \nabla,\mathbf{\Psi}\ \right];$$
$$\left[\nabla,\mathbf{\Pi}_y\right]=\mathbf{k}\left(\frac{\partial\Pi_{yy}}{\partial x}-\frac{\partial\Pi_{yx}}{\partial y}\right)=$$
$$=-\left[\nabla,\ \mathbf{j}\ \nabla,\mathbf{\Psi}\ \ \right]=\left[\mathbf{j},\nabla\ \ \nabla,\mathbf{\Psi}\ \right],$$

that's why $\mathbf{i},\ \nabla,\mathbf{\Pi}_x\ \ =0;\quad \mathbf{j},\left[\nabla,\mathbf{\Pi}_y\right]\ =0$.

It makes sense to represent tensor (1.7)

$$\mathbf{\Pi}=\mathbf{i}\left\{\mathbf{i}\ \ \rho u^2+p\ \ +\mathbf{j}\ \ \rho uv\ \ -2\frac{\mu}{Re}\nabla u-\frac{2}{3}\frac{\mu}{Re}\mathbf{i}\ \ \nabla,\mathbf{V}\ \ +\right.$$
$$\left.+\frac{\mu}{Re}\ \mathbf{i},\mathbf{\Omega}\ \right\}+\mathbf{j}\ \ \mathbf{i}\ \ \rho uv\ \ +\mathbf{j}\ \ \rho v^2+p\ \ -$$
$$-2\frac{\mu}{Re}\nabla v-\frac{2}{3}\frac{\mu}{Re}\mathbf{j}\ \ \nabla,\mathbf{V}\ \ +\frac{\mu}{Re}\ \ \mathbf{j},\mathbf{\Omega}\ \Big\},$$

it is advisable to submit records in invariant form

$$\mathbf{\Pi}=\rho\mathbf{VV}+\mathbf{I}\left\{p-\frac{2}{3}\frac{\mu}{Re}\ \ \nabla,\mathbf{V}\ \ \right\}-2\frac{\mu}{Re}\nabla^*\mathbf{V}+$$
$$+\frac{\mu}{Re}\ \mathbf{I},\mathbf{\Omega}\ \ =\rho\mathbf{VV}+\mathbf{I}\left\{p-\frac{2}{3}\frac{\mu}{Re}\ \ \nabla,\mathbf{V}\ \ \right\}-$$
$$-2\frac{\mu}{Re}\nabla\mathbf{V}-\frac{\mu}{Re}\ \mathbf{I},\mathbf{\Omega}\ , \qquad (1.19)$$

where, unlike the previous one, in a curvilinear coordinate system, the unit tensor $\mathbf{I}$ has the form $\mathbf{I}=\mathbf{ss}+\mathbf{nn}$.

Then the vector components of the tensor P can be written as

$$\begin{cases}\mathbf{\Pi}_s=\rho v_s\mathbf{V}+\mathbf{s}p-2\dfrac{\mu}{Re}\dfrac{\partial\mathbf{V}}{\partial s}-\dfrac{\mu}{Re}\ \ \mathbf{s},\mathbf{\Omega}\ ;\\ \mathbf{\Pi}_n=\rho v_n\mathbf{V}+\mathbf{n}p-2\dfrac{\mu}{Re}\dfrac{\partial\mathbf{V}}{\partial n}-\dfrac{\mu}{Re}\ \ \mathbf{n},\mathbf{\Omega}\ .\end{cases} \qquad (1.20)$$

1.2. Fundamental solutions of differential operators

1.2.1. Generalized differential operations of vector-tensor analysis

It should be noted that here of particular interest are generalized representations of solutions to the main problems of vector analysis [6], which are also associated with differential operators of the form (1.11). For the first time, a fundamental solution was constructed as a singular solution to Laplace's equation [7]. By definition, a singular solution is a generalized function, i.e. a functional defined on finite functions. However, in the present case, the singular solution is locally summable, which significantly expands the scope of its application and allows you to get many important results.

For the further development of the method of boundary integral equations, the solution of boundary problems of viscous gas dynamics takes place proved in [8], the following generalized differential operations of vector-tensor analysis in the case when $\varphi=\varphi(x,y)$ and $\mathbf{G}=\mathbf{i}G_x(x,y)+\mathbf{j}G_y(x,y)$ have the necessary differential properties:

Table 1

Basic differential operation of vector-tensor analiz

$[\nabla,\mathbf{kk}\varphi]=[\nabla\varphi,\mathbf{kk}]=[\mathbf{I},\nabla\varphi];$	$\left(\nabla,[\mathbf{I},\mathbf{a}]\right)=[\nabla,\mathbf{a}];$
$[\nabla,[\mathbf{kk},\mathbf{a}]]=[\nabla,[\mathbf{a},\mathbf{I}]]=$ $=\nabla^*\mathbf{a}-\mathbf{I}(\nabla,\mathbf{a});$	$[\mathbf{I},[\nabla,\mathbf{a}]]=\nabla^*\mathbf{a}-\nabla\mathbf{a};$
$[\nabla,[\mathbf{I},\mathbf{a}]]=-\mathbf{kk}(\nabla,\mathbf{a});$	$\nabla(\nabla,\mathbf{a})=\left(\nabla,\nabla^*\mathbf{a}\right)=$ $=\Delta\mathbf{a}+[\nabla,[\nabla,\mathbf{a}]]$

Of considerable interest are also generalized algebraic and differential operations with combinations of vectors:

$$[\mathbf{\Psi},[\mathbf{I},\mathbf{G}]]=\left[\left(\mathbf{i}\Psi_x+\mathbf{j}\Psi_y\right),\left[\mathbf{kk},\left(\mathbf{i}G_x+\mathbf{j}G_y\right)\right]\right]=$$
$$=\left[\left(\mathbf{i}\Psi_x+\mathbf{j}\Psi_y\right),\mathbf{k}\left(\mathbf{j}G_x-\mathbf{i}G_y\right)\right]=\left(\mathbf{i}G_x+\mathbf{j}G_y\right)\left(\mathbf{i}\Psi_x+\mathbf{j}\Psi_y\right)-$$
$$-\mathbf{I}\left(G_x\Psi_x+G_y\Psi_y\right)=\mathbf{G\Psi}-\mathbf{I}(\mathbf{G},\mathbf{\Psi}),$$

where do we have symmetric expressions:

$$[\mathbf{\Psi},[\mathbf{I},\mathbf{G}]]=\mathbf{G\Psi}-\mathbf{I}(\mathbf{G},\mathbf{\Psi}),$$
$$[\mathbf{G},[\mathbf{I},\mathbf{\Psi}]]=\mathbf{\Psi G}-\mathbf{I}(\mathbf{\Psi},\mathbf{G}). \qquad (1.21)$$

Generalizing the well-known formula of vector analysis for arbitrary vector functions $\mathbf{a}$ and $\mathbf{b}$ $\left[\nabla,\ \mathbf{a},\mathbf{b}\ \right]=\ \mathbf{b},\nabla\ \mathbf{a}\ -\ \mathbf{a},\nabla\ \ \mathbf{b}-\mathbf{b}\ \ \nabla,\mathbf{a}\ \ +\mathbf{a}\ \ \nabla,\mathbf{b}\ $, onto the functions of tensor nature, we get

$$\left[\nabla,\ \mathbf{\Psi},\mathbf{\Gamma}\ \right]=\left[\nabla,\mathbf{k}\ \ \Psi_x\Gamma_y-\Psi_y\Gamma_x\ \ \right]=$$
$$=\ \nabla^*\mathbf{\Psi},\mathbf{\Gamma}\ \ -\mathbf{\Gamma}\ \ \nabla,\mathbf{\Psi}\ \ -\ \ \mathbf{\Psi},\nabla\mathbf{\Gamma}\ \ +\mathbf{\Psi}\ \ \nabla,\mathbf{\Gamma}\ \ =$$
$$=-\left[\mathbf{\Psi},\ \nabla,\mathbf{\Gamma}\ \right]+\left[\ \nabla,\mathbf{\Psi}\ ,\mathbf{\Gamma}\right]+\ \nabla\mathbf{\Psi},\mathbf{\Gamma}\ \ -$$
$$-\ \mathbf{\Psi},\nabla\mathbf{\Gamma}\ \ +\mathbf{\Psi}\ \ \nabla,\mathbf{\Gamma}\ \ -\mathbf{\Gamma}\ \ \nabla,\mathbf{\Psi}\ \ ;$$

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and finally,

$$\left[\nabla,\ \mathbf{\Psi},\mathbf{\Gamma}\ \right] = \ \nabla^{*}\mathbf{\Psi},\mathbf{\Gamma}\ - \mathbf{\Gamma}\ \nabla,\mathbf{\Psi}\ - \\ -\ \mathbf{\Psi},\nabla\mathbf{\Gamma}\ + \mathbf{\Psi}\ \nabla,\mathbf{\Gamma}\ . \quad (1.22)$$

In addition, for (1.21) we have

$$\begin{aligned}&\left[\nabla,\left[\mathbf{G},\left[\mathbf{I},\mathbf{\Psi}\right]\right]\right] = \left[\nabla,\left(\mathbf{\Psi G} - \mathbf{I}\left(\mathbf{\Psi},\mathbf{G}\right)\right)\right] = \\ &= \left[\nabla,\mathbf{\Psi}\right]\mathbf{G} - \left[\mathbf{\Psi},\nabla\mathbf{G}\right] - \\ &- \left[\mathbf{I},\nabla\left(\mathbf{\Psi},\mathbf{G}\right)\right] = \left[\nabla,\mathbf{\Psi}\right]\mathbf{G} - \left[\mathbf{\Psi},\nabla\mathbf{G}\right] - \\ &- \left[\mathbf{I},\left\{\left(\mathbf{G},\nabla^{*}\right)\mathbf{\Psi} + \left(\mathbf{\Psi},\nabla^{*}\mathbf{G}\right)\right\}\right].\end{aligned} \quad (1.23)$$

1.2.2. Vector potential of the tensor of the momentum conservation law

It was shown above that the vector potential Ψ is a solution to the vector equation (1.17)

$$\left[\nabla,\left[\nabla,\mathbf{\Psi}\right]\right] = 0\,. \quad (1.24)$$

This linear differential operator is self-adjoin, and its fundamental solution is the solution of the operator equation

$$\left[\nabla,\left[\nabla,\mathbf{\Gamma}\right]\right] = 0 \quad (1.25)$$

has the form

$$\mathbf{\Gamma} = \mathbf{I}\varphi - \left[\mathbf{I},\mathbf{k}\psi\right], \quad (1.26)$$

where $\varphi = \dfrac{\ln\left|\mathbf{r} - \mathbf{r}_0\right|}{2\pi}$ - the fundamental solution of the Laplace equation, and the function $\psi = \dfrac{1}{2\pi}\mathrm{arctg}\dfrac{x - x_0}{y - y_0}$ - conjugate φ solution of laplace's equation $\Delta\psi = 0$.

Due to Cauchy-Riemann terms:

$$\begin{cases}\dfrac{\partial\varphi}{\partial x} = \dfrac{\partial\psi}{\partial y}; \\ \dfrac{\partial\varphi}{\partial y} = -\dfrac{\partial\psi}{\partial x};\end{cases} \Leftrightarrow \begin{cases}\nabla\varphi = \left[\nabla,\mathbf{k}\psi\right]; \\ \nabla\psi = -\left[\nabla,\mathbf{k}\varphi\right],\end{cases} \quad (1.27)$$

tensor **Γ** (1.26) is conservative

$$\left(\nabla,\mathbf{\Gamma}\right) = 0 \Leftrightarrow \nabla\varphi = \left[\nabla,\mathbf{k}\psi\right], \quad (1.28)$$

and due to the same Cauchy-Riemann conditions (1.27) also potential

$$\begin{aligned}&\left[\nabla,\mathbf{\Gamma}\right] = \left[\nabla,\left(\mathbf{I}\varphi\right)\right] - \left[\nabla,\left[\mathbf{I},\mathbf{k}\psi\right]\right] = \\ &= \left[\mathbf{I},\left(\nabla\varphi\right)\right] - \left[\mathbf{I},\left(\mathbf{i}\frac{\partial\psi}{\partial y} - \mathbf{j}\frac{\partial\psi}{\partial x}\right)\right] = \left[\mathbf{I},\left(\nabla\varphi\right)\right] - \\ &- \left[\mathbf{I},\left(\mathbf{i}\frac{\partial\varphi}{\partial x} + \mathbf{j}\frac{\partial\varphi}{\partial y}\right)\right] = \left[\mathbf{I},\left(\nabla\varphi\right)\right] - \left[\mathbf{I},\left(\nabla\varphi\right)\right] = 0.\end{aligned} \quad (1.29)$$

1.2.3. Boundary problem of streaming the system of body profiles VAWT stream of viscous gas

In the stationary case, the law of conservation of mass, vorticity, momentum and its components (1.11) have the form:

$$\nabla\ \nabla,\mathbf{V}\ = 0,\quad \nabla\ \nabla,\mathbf{\Omega}\ = 0,\quad \nabla\ \nabla,\mathbf{\Pi}_i\ = 0\,. \quad (1.30)$$

It is especially important to highlight the fact that the tensor **Γ** (1.26) is a fundamental solution of the differential operator of the second order of equations (1.30):

$$\nabla\left(\nabla,\mathbf{\Gamma}\right) = \Delta\mathbf{\Gamma} + \left[\nabla,\left[\nabla,\mathbf{\Gamma}\right]\right] = \mathbf{I}\Delta\varphi\,. \quad (1.31)$$

The differential conservation laws of kind (1.11) belong to the same class and, by virtue of (1.30), have a fundamental solution of kind (1.28).

Thus, in order to determine the aerodynamic characteristics of a certain engine as a whole and each of its load-bearing elements separately, it is necessary not just for the laws of conservation of dynamics of a viscous incompressible fluid (1.30), but also for the vector potential Ψ of (1.12)

$$\left[\nabla,\left[\nabla,\mathbf{\Psi}\right]\right] = 0 \quad (1.32)$$

solve the boundary problem:

$$\begin{cases}\mathbf{V}_i\big|_{(L_i)} = \Omega\ \mathbf{k},\mathbf{r}_i\ = \Omega\ \mathbf{j}x_i - \mathbf{i}y_i\ ,\ \mathbf{r}_i = \mathbf{i}x_i + \mathbf{j}y_i\ ; \\ \mathbf{V}\big|_{(\Sigma)} = \mathbf{V}_\infty = \mathrm{const.}; \\ \mathbf{\Omega}_i\big|_{(L_i)} = 0;\quad \mathbf{\Omega}\big|_{(\Sigma)} = 0; \\ p\big|_{(\Sigma)} = p_\infty\end{cases} \quad (1.33)$$

with respect to the profiles of scalar pressure p distributed along the boundaries, and vortex - $\mathbf{\Omega} = \mathbf{k}\left(\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}\right)$, which determine the force interaction of the turbine with the operating environment. Here, in the flat case, the velocity vector $\mathbf{V}\ x,y\ = \mathbf{i}u + \mathbf{j}v$.

1.3. Generalized integral theorems of vector-tensor analysis

The basis of the method of boundary integral equations – it is integral representations of the solution of corresponding boundary value problems, since the boundary integral equations are the limiting values of such representations.

The application of integral theorems of vector-tensor analysis is based on the existence of fundamental solutions of the corresponding differential operators of the mathematical model. Fundamental solutions are well known for second-order differential operators with constant coefficients, such as Laplace, Cauchy-Riemann, wave, transfer [6]. In the present case, such operators are expressions from (1.25) and (1.31):

$$\begin{cases}\left[\nabla,\ \nabla,\boldsymbol{\Gamma}\ \right]=0;\\ \ \nabla\ \ \nabla,\boldsymbol{\Gamma}\ \ \ =0.\end{cases} \qquad (1.34)$$

1.3.1. Green's formula for the differential operator of the vector potential

Performing standard integration by parts of a combination of operators from (1.25), (1.32) [6, 8] in a closed domain E:

$$\iint_E \ \left[\nabla,\ \nabla,\boldsymbol{\Psi}\ \right],\boldsymbol{\Gamma}\ -\ \boldsymbol{\Psi},\left[\nabla,\ \nabla,\boldsymbol{\Gamma}\ \right]\ dE\ \ =0\Rightarrow$$

$$\Rightarrow\iint_E\ \ \left[\nabla,\ \nabla,\boldsymbol{\Psi}\ \right],\boldsymbol{\Gamma}\ -\ \ \nabla,\boldsymbol{\Psi}\ ,\ \nabla,\boldsymbol{\Gamma}\ \ +$$

$$+\ \ \nabla,\boldsymbol{\Psi}\ ,\ \nabla,\boldsymbol{\Gamma}\ \ -\ \boldsymbol{\Psi},\left[\nabla,\ \nabla,\boldsymbol{\Gamma}\ \right]\ \ dE=$$

$$=\iint_E\ \ \nabla,\ \left[\ \nabla,\boldsymbol{\Psi}\ ,\boldsymbol{\Gamma}\right]+\left[\boldsymbol{\Psi},\ \nabla,\boldsymbol{\Gamma}\ \right]\ \ \ dE=$$

$$=\oint_L\ \ \mathbf{n},\ \left[\ \nabla,\boldsymbol{\Psi}\ ,\boldsymbol{\Gamma}\right]+\left[\boldsymbol{\Psi},\ \nabla,\boldsymbol{\Gamma}\ \right]\ \ \ dl=0$$

we get Green's generalized formula:

$$\oint_L\ \ \mathbf{n},\left[\ \nabla,\boldsymbol{\Psi}\ ,\boldsymbol{\Gamma}\right]\ -\ \boldsymbol{\Psi},\left[\mathbf{n},\ \nabla,\boldsymbol{\Gamma}\ \right]\ \ dl=0, \quad (1.35)$$

where L ≡ ∂E is the closed boundary of domain E.

Using the differential expressions from Table 1. $\left(\left[\nabla,\boldsymbol{\Psi}\right]=0,\left[\nabla,\boldsymbol{\Gamma}\right]=0\right)$, we have:

$$\mathbf{n},\left[\nabla,\ \boldsymbol{\Psi},\boldsymbol{\Gamma}\ \right]\ =\ \mathbf{n},\boldsymbol{\Psi}\ \ \ \nabla,\boldsymbol{\Gamma}\ -\left(\boldsymbol{\Psi},\frac{\partial\boldsymbol{\Gamma}}{\partial n}\right)-$$

$$-\ \mathbf{n},\boldsymbol{\Gamma}\ \ \ \nabla,\boldsymbol{\Psi}\ +\left(\frac{\partial\boldsymbol{\Psi}}{\partial n},\boldsymbol{\Gamma}\right). \qquad (1.36)$$

And since

$$\oint_L\ \mathbf{n},\left[\nabla,\ \boldsymbol{\Psi},\boldsymbol{\Gamma}\ \right]\ dl=\oint_L\left\{\ \mathbf{n},\boldsymbol{\Psi}\ \ \ \nabla,\boldsymbol{\Gamma}\ -\left(\boldsymbol{\Psi},\frac{\partial\boldsymbol{\Gamma}}{\partial n}\right)-\right.$$

$$\left.-\ \mathbf{n},\boldsymbol{\Gamma}\ \ \ \nabla,\boldsymbol{\Psi}\ +\left(\frac{\partial\boldsymbol{\Psi}}{\partial n},\boldsymbol{\Gamma}\right)\right\}dl=0.$$

Then

$$\oint_L\ \ \left[\mathbf{n},\ \nabla,\boldsymbol{\Psi}\ \right],\boldsymbol{\Gamma}\ -\ \boldsymbol{\Psi},\left[\mathbf{n},\ \nabla,\boldsymbol{\Gamma}\ \right]\ \ dl=$$

$$=\oint_L\left\{\left(\left[\frac{\partial^*\boldsymbol{\Psi}}{\partial n}-\frac{\partial\boldsymbol{\Psi}}{\partial n}\right],\boldsymbol{\Gamma}\right)-\left(\boldsymbol{\Psi},\left[\frac{\partial^*\boldsymbol{\Gamma}}{\partial n}-\frac{\partial\boldsymbol{\Gamma}}{\partial n}\right]\right)\right\}dl=$$

$$=\oint_L\left\{\left(\left[\frac{\partial^*\boldsymbol{\Psi}}{\partial n}-\mathbf{n}\ \ \nabla,\boldsymbol{\Psi}\ -\frac{\partial\boldsymbol{\Psi}}{\partial n}+\mathbf{n}\ \ \nabla,\boldsymbol{\Psi}\ \right],\boldsymbol{\Gamma}\right)-\right.$$

$$\left.-\oint_L\left\{\left(\boldsymbol{\Psi},\left[\frac{\partial^*\boldsymbol{\Gamma}}{\partial n}-\mathbf{n}\ \ \nabla,\boldsymbol{\Gamma}\ -\frac{\partial\boldsymbol{\Gamma}}{\partial n}+\mathbf{n}\ \ \nabla,\boldsymbol{\Gamma}\ \right]\right)\right\}.$$

But (see (1.22))

$$\oint_L\ \mathbf{n},\left[\nabla,\ \boldsymbol{\Psi},\boldsymbol{\Gamma}\ \right]\ dl=\oint_L\left\{\left(\left[\frac{\partial^*\boldsymbol{\Psi}}{\partial n}-\mathbf{n}\ \ \nabla,\boldsymbol{\Psi}\ \right],\boldsymbol{\Gamma}\right)-\right.$$

$$\left.-\left(\boldsymbol{\Psi},\left[\frac{\partial^*\boldsymbol{\Gamma}}{\partial n}-\mathbf{n}\ \ \nabla,\boldsymbol{\Gamma}\ \right]\right)\right\}dl=0, \qquad (1.37)$$

and $\left[\mathbf{n},\ \nabla,\boldsymbol{\Psi}\ \right]=\frac{\partial^*\boldsymbol{\Psi}}{\partial n}-\frac{\partial\boldsymbol{\Psi}}{\partial n}=0.$ (see (1.16) and then $\frac{\partial\boldsymbol{\Psi}}{\partial n}-\mathbf{n}\ \ \nabla,\boldsymbol{\Psi}\ \ =\ \mathbf{n},\ \mathbf{kk},\boldsymbol{\Psi}\ \ =\ \mathbf{n},\boldsymbol{\Pi}$ (see (1.12), $\nabla,\boldsymbol{\Gamma}\ \ =0,\quad \nabla,\boldsymbol{\Psi}\ \ =-\rho V^2-2p.$ Therefore, Green's formula (1.37) for operator b (1.34) takes the final form:

$$\oint_L\ \ \mathbf{n},\left[\ \nabla,\boldsymbol{\Psi}\ ,\boldsymbol{\Gamma}\right]\ -\ \boldsymbol{\Psi},\left[\mathbf{n},\ \nabla,\boldsymbol{\Gamma}\ \right]\ \ dl=$$

$$=\oint_L\left\{\ \mathbf{n},\boldsymbol{\Pi},\boldsymbol{\Gamma}\ -\left(\boldsymbol{\Psi},\frac{\partial\boldsymbol{\Gamma}}{\partial n}\right)\right\}dl=0. \qquad (1.38)$$

1.3.2. Generalized Green's formula for the conservation differential operator

Obviously, the basic conservation laws, written in a conservative form (1.30), can be represented in vector form of second-order differential equations:

$$\nabla\left(\nabla,\mathbf{a}\right)=0;\quad \nabla\left(\nabla,\mathbf{B}\right)=0, \qquad (1.39)$$

where vector **a** **a**nd tensor **B** are related to the study of a specific law of conservation of continuum mechanics.

Then

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$$\left(\nabla,\left((\nabla,\mathbf{a})\mathbf{B}\right)\right)-(\nabla,\mathbf{a})(\nabla,\mathbf{B})=\left(\nabla(\nabla,\mathbf{a}),\mathbf{B}\right),$$

Similarly,

$$\left(\nabla,\left(\mathbf{a}(\nabla,\mathbf{B})\right)\right)-(\nabla,\mathbf{a})(\nabla,\mathbf{B})=$$
$$=(\nabla,\mathbf{B})\frac{\partial a_x}{\partial x}+\frac{\partial(\nabla,\mathbf{B})}{\partial x}a_x+(\nabla,\mathbf{B})\frac{\partial a_y}{\partial y}+\frac{\partial(\nabla,\mathbf{B})}{\partial y}a_y-$$
$$-(\nabla,\mathbf{a})(\nabla,\mathbf{B})=\left(\mathbf{a},\left(\nabla(\nabla,\mathbf{B})\right)\right)$$

and

$$\left(\mathbf{a},\left(\nabla(\nabla,\mathbf{B}.)\right)\right)=\left(\nabla,\left(\mathbf{a}(\nabla,\mathbf{B})\right)\right)-(\nabla,\mathbf{a})(\nabla,\mathbf{B}).$$

From here we have

$$\left(\mathbf{a},\left(\nabla(\nabla,\mathbf{B}.)\right)\right)-\left(\left(\nabla(\nabla,\mathbf{a})\right),\mathbf{B}\right)=\left(\nabla,\left(\mathbf{a}(\nabla,\mathbf{B})\right)\right)-$$
$$-(\nabla,\mathbf{a})(\nabla,\mathbf{B})-\left(\nabla,\left((\nabla,\mathbf{a})\mathbf{B}\right)\right)+(\nabla,\mathbf{a})(\nabla,\mathbf{B})=$$
$$=\left(\nabla,\left(\mathbf{a}(\nabla,\mathbf{B})\right)\right)-\left(\nabla,\left((\nabla,\mathbf{a})\mathbf{B}\right)\right).$$

Integrating this expression in the bounded domain E, we obtain a second generalized Greene formula for operators in (1.39):

$$\iint_E \nabla,\ \nabla,\mathbf{a}\ \mathbf{B}\ -\ \nabla,\ \mathbf{a}\ \nabla,\mathbf{B}\quad dE=$$
$$=\oint_L \nabla,\mathbf{a}\ \ \mathbf{n},\mathbf{B}\ -\ \mathbf{n},\mathbf{a}\ \ \nabla,\mathbf{B}\ \ dl. \tag{1.40}$$

And returning to natural quantities (**a**≡ **Π**, **B** ≡ **Γ**), we have:

$$\oint_L \nabla,\mathbf{a}\ \ \mathbf{n},\mathbf{B}\ -\ \mathbf{n},\mathbf{a}\ \ \nabla,\mathbf{B}\ \ dl. \tag{1.41}$$
$$\oint_L \nabla,\mathbf{\Pi}\ \ \mathbf{n},\mathbf{\Gamma}\ -\ \mathbf{n},\mathbf{\Pi}\ \ \nabla,\mathbf{\Gamma}\ \ dl=0.$$

In the case of a vector $\mathbf{a}\equiv\begin{cases}\mathbf{V}\\ \mathbf{\Omega}\\ \mathbf{\Pi}_i\end{cases}$, for further application, this formula can be converted using the integral formula (1.40) to Green's formula for the differential operator in (1.39):

$$\oint_{(L)} \mathbf{n},\mathbf{a}\ \ \nabla,\mathbf{\Gamma}\ -\ \mathbf{n},\mathbf{\Gamma}\ \ \nabla,\mathbf{a}\ \ dl=\oint_{(L)} \left[\mathbf{n},\ \nabla,\mathbf{a}\ \right],\mathbf{\Gamma}\ -$$
$$-\ \mathbf{a},\left[\mathbf{n},\ \nabla,\mathbf{\Gamma}\ \right]-\left(\frac{\partial\mathbf{a}}{\partial n},\mathbf{\Gamma}\right)+\left(\mathbf{a},\frac{\partial\mathbf{\Gamma}}{\partial n}\right)\Bigg\}dl, \tag{1.42}$$

where $[\nabla,\mathbf{\Gamma}]=0$ (see (1.29)).

2. Integral representations of solutions of a boundary value problem of viscous gas flow around a system of thick airfoils VAWE

To the original system of differential conservation laws

$$\begin{cases}\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0;\\[2mm] \dfrac{\partial}{\partial x}\left(\rho u^2+p-2\dfrac{\mu}{Re}\dfrac{\partial u}{\partial x}\right)+\dfrac{\partial}{\partial y}\left(\rho uv-\dfrac{\mu}{Re}\left(2\dfrac{\partial u}{\partial y}+\Omega_z\right)\right)=0;\\[2mm] \dfrac{\partial}{\partial x}\left(\rho uv-\dfrac{\mu}{Re}\left(2\dfrac{\partial v}{\partial x}-\Omega_z\right)\right)+\dfrac{\partial}{\partial y}\left(\rho v^2+p-2\dfrac{\mu}{Re}\dfrac{\partial v}{\partial y}\right)=0\end{cases} \tag{2.1}$$

it is also necessary to attach the vector potential **Ψ** tensor of stresses and velocities of deformations (1.7), taking into account the law of conservation of mass:

$$\nabla,\mathbf{V}\ \equiv\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$$

$$\mathbf{\Pi}=\rho\mathbf{V}\mathbf{V}+\mathbf{I}p-2\frac{\mu}{Re}\nabla^*\mathbf{V}+\frac{\mu}{Re}\ \mathbf{I},\mathbf{\Omega}\ =$$
$$=\left[\nabla,\ \mathbf{kk},\mathbf{\Psi}\ \right]=\nabla^*\mathbf{\Psi}-\mathbf{I}\ \ \nabla,\mathbf{\Psi}\ . \tag{2.2}$$

The values sought here are scalar characteristics: pressure – p and swirling – Ω_Z = ω, as well as vector potential **Ψ**.

Green's generalized formula (1.38) $\oint_{(L)}\left\{\left((\mathbf{n},\mathbf{\Pi}),\mathbf{\Gamma}\right)-\left(\mathbf{\Psi},\frac{\partial\mathbf{\Gamma}}{\partial n}\right)\right\}dl=0$, where $\left(\mathbf{\Psi},\frac{\partial\mathbf{\Gamma}}{\partial n}\right)=$ $=\mathbf{\Psi}\frac{\partial\varphi}{\partial n}-[\mathbf{\Psi},\mathbf{k}]\frac{\partial\psi}{\partial n}$, due to the known properties of the double layer potential $\frac{\partial\varphi}{\partial n}$ [6, 7], leads to an integral representation of the solution of the equation (1.24)

$$\mathbf{\Psi}=\oint_{(L)}\left\{\left((\mathbf{n},\mathbf{\Pi}),\mathbf{\Gamma}\right)-\left(\mathbf{\Psi},\frac{\partial\mathbf{\Gamma}}{\partial n}\right)\right\}dl\,. \tag{2.3}$$

As for the conservation laws (2.1), then the justification formula Grin (1.42) $\oint_{(L)}\left\{\left(\frac{\partial^*\mathbf{a}}{\partial n},\mathbf{\Gamma}\right)+\left(\mathbf{a},\frac{\partial\mathbf{\Gamma}}{\partial n}\right)\right\}dl=0$, leads to an integral representation of the solution:

$$\mathbf{a}=-\oint_{(L)}\left\{\ \left[\mathbf{n},\ \nabla,\mathbf{a}\ \right],\mathbf{\Gamma}\ +\left(\frac{\partial\mathbf{a}}{\partial n},\mathbf{\Gamma}\right)+\left(\mathbf{a},\frac{\partial\mathbf{\Gamma}}{\partial n}\right)\right\}dl, \tag{2.4}$$

which for specific physical vectors takes forms:

$$\mathbf{V}=-\oint_{(L)}\left\{\ \left[\mathbf{n},\ \nabla,\mathbf{V}\ \right],\mathbf{\Gamma}\ +\left(\frac{\partial\mathbf{V}}{\partial n},\mathbf{\Gamma}\right)+\left(\mathbf{V},\frac{\partial\mathbf{\Gamma}}{\partial n}\right)\right\}dl; \tag{2.5}$$

$$\mathbf{\Omega}=-\oint_{(L)}\left\{\ \left[\mathbf{n},\ \nabla,\mathbf{\Omega}\ \right],\mathbf{\Gamma}\ +\left(\frac{\partial\mathbf{\Omega}}{\partial n},\mathbf{\Gamma}\right)+\left(\mathbf{\Omega},\frac{\partial\mathbf{\Gamma}}{\partial n}\right)\right\}dl; \tag{2.6}$$

$$\mathbf{\Pi}_i=-\oint_{(L)}\left\{\ \left[\mathbf{n},\ \nabla,\mathbf{\Pi}_i\ \right],\mathbf{\Gamma}\ +\left(\frac{\partial\mathbf{\Pi}_i}{\partial n},\mathbf{\Gamma}\right)+\left(\mathbf{\Pi}_i,\frac{\partial\mathbf{\Gamma}}{\partial n}\right)\right\}dl, \tag{2.7}$$

where in (2.5), in accordance with the expression (2.2),

$$\begin{aligned}
&\mathbf{\Pi} = \rho\mathbf{VV} + \mathbf{I}p - \frac{\mu}{Re}(\mathbf{I},\mathbf{\Omega}) - 2\frac{\mu}{Re}\nabla\mathbf{V} = \\
&= [\nabla, (\mathbf{kk},\mathbf{\Psi})] = \nabla^{*}\mathbf{\Psi} - \mathbf{I}(\nabla,\mathbf{\Psi}); \\
&(\mathbf{n},\mathbf{\Omega}) = [\mathbf{n}, [\nabla,\mathbf{V}]] = \\
&= \frac{Re}{\mu}\left(\rho v_n\mathbf{V} + \mathbf{n}p\right) - 2\frac{\partial\mathbf{V}}{\partial n} - \frac{Re}{\mu}\left(\mathbf{n},[\nabla, (\mathbf{kk},\mathbf{\Psi})]\right),
\end{aligned} \tag{2.8}$$

which allows the last term, containing $\mathbf{\Psi}$, integrated into parts. Indeed,

$$\begin{aligned}
&\oint_{(L)} \left(\left(\mathbf{n},[\nabla, (\mathbf{kk},\mathbf{\Psi})]\right),\mathbf{\Gamma}\right) dl = \\
&= \oint_{(L)} \left(\left(\mathbf{n},[\nabla, (\mathbf{kk},\mathbf{\Psi})]\right)\varphi - \left[\left(\mathbf{n},[\nabla, (\mathbf{kk},\mathbf{\Psi})]\right),\mathbf{k}\psi\right]\right) dl = \\
&= \oint_{(L)} \left(-[(\mathbf{n},\nabla\varphi),\mathbf{\Psi}] + ((\mathbf{n},\nabla\psi),\mathbf{k\Psi})\right) dl,
\end{aligned}$$

where the curvilinear integral with the kernel as a tangent derivative of a simple layer $[\mathbf{n},\nabla\varphi]$ is a continuous function.

In representation (2.6) summand $[\mathbf{n}, [\nabla,\mathbf{\Omega}]]$ in agreement with the momentum conservation law (1.8), (1.10)

$$\left[\mathbf{n},\nabla\left(\rho\frac{v^2}{2} + p\right)\right] - \rho[\mathbf{n}, [\mathbf{V},\mathbf{\Omega}]] + \frac{\mu}{Re}[\mathbf{n}, [\nabla,\mathbf{\Omega}]] = 0,$$

since $(\mathbf{n},\mathbf{\Omega}) = (\mathbf{n},\mathbf{k}\omega) = 0$, has the form

$$[\mathbf{n}, [\nabla,\mathbf{\Omega}]] = -\frac{Re}{\mu}\left[\mathbf{n},\nabla\left(\rho\frac{v^2}{2} + p\right)\right] - \frac{Re}{\nu}\mathbf{\Omega}V_n, \tag{2.9}$$

where the first term admits integration by parts. Really,

$$\oint_{(L)} ([\mathbf{n}, [\nabla,\mathbf{\Omega}]],\mathbf{\Gamma})\, dl = -Re\oint_{(L)}\left(\left\{\frac{1}{\mu}\left[\mathbf{n},\nabla\left(\rho\frac{v^2}{2} + p\right)\right] + \frac{\mathbf{\Omega}V_n}{\nu}\right\},\mathbf{\Gamma}\right) dl.$$

It follows

$$\begin{aligned}
&\oint_{(L)}\left(\left[\mathbf{n},\nabla\left(\rho\frac{v^2}{2} + p\right)\right],\mathbf{\Gamma}\right) dl = \\
&= \oint_{(L)}\left(\left[\mathbf{n},\nabla\left(\rho\frac{v^2}{2} + p\right)\right], (\mathbf{I}\varphi - [\mathbf{I},\mathbf{k}\psi])\right) dl = \\
&= \oint_{(L)}\left[\mathbf{n},\nabla\left(\rho\frac{v^2}{2} + p\right)\right]\varphi dl - \oint_{(L)}\left[\left[\mathbf{n},\nabla\left(\rho\frac{v^2}{2} + p\right)\right],\mathbf{k}\psi\right] dl.
\end{aligned}$$

Here $\left[\mathbf{n},\nabla\left(\rho\frac{v^2}{2} + p\right)\right]\varphi =$

$$= \left[\mathbf{n},\nabla\left\{\left(\rho\frac{v^2}{2} + p\right)\varphi\right\}\right] - (\mathbf{n},\nabla\varphi)\left(\rho\frac{v^2}{2} + p\right)$$

and

$$\left[\left[\mathbf{n},\nabla\left(\rho\frac{v^2}{2} + p\right)\right],\mathbf{k}\psi\right] =$$

$$= \left[\left\{\mathbf{k}\left(n_x\frac{\partial}{\partial y}\left(\rho\frac{v^2}{2} + p\right) - n_y\frac{\partial}{\partial x}\left(\rho\frac{v^2}{2} + p\right)\right)\right\},\mathbf{k}\psi\right] = 0.$$

In this way $\oint_{(L)}\left(\left[\mathbf{n},\nabla\left(\rho\frac{v^2}{2} + p\right)\right],\mathbf{\Gamma}\right) dl =$ $= -\oint_{(L)}(\mathbf{n},\nabla\varphi)\left(\rho\frac{v^2}{2} + p\right) dl$ and integral representations of solutions have the final form:

$$\begin{aligned}
&\mathbf{V} = -\oint_{(L)}\left\{\left(\left\{\frac{Re}{\mu}(\rho v_n\mathbf{V} + \mathbf{n}p)\right\},\mathbf{\Gamma}\right) + \frac{Re}{\mu}[(\mathbf{n},\nabla\varphi),\mathbf{\Psi}] - \right. \\
&\left. - ((\mathbf{n},\nabla\psi),\mathbf{k\Psi}) - \left(\frac{\partial\mathbf{V}}{\partial n},\mathbf{\Gamma}\right) + \left(\mathbf{V},\frac{\partial\mathbf{\Gamma}}{\partial n}\right)\right\} dl;
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
&\mathbf{\Omega} = -\oint_{(L)}\left\{\frac{Re}{\mu}(\mathbf{n},\nabla\varphi)\left(\rho\frac{v^2}{2} + p\right) - \right. \\
&\left. - \frac{Re}{\nu}V_n(\mathbf{\Omega},\mathbf{\Gamma}) + \left(\frac{\partial\mathbf{\Omega}}{\partial n},\mathbf{\Gamma}\right) + \left(\mathbf{\Omega},\frac{\partial\mathbf{\Gamma}}{\partial n}\right)\right\} dl;
\end{aligned} \tag{2.11}$$

$$\begin{aligned}
&\mathbf{\Psi} = \oint_{(L)}\left\{\left(\left(\rho V_n\mathbf{V} + \mathbf{n}p - \frac{\mu}{Re}[\mathbf{n},\mathbf{\Omega}] - \right.\right.\right. \\
&\left.\left.\left. -2\frac{\mu}{Re}\frac{\partial\mathbf{V}}{\partial n}\right),\mathbf{\Gamma}\right) - \left(\mathbf{\Psi},\frac{\partial\mathbf{\Gamma}}{\partial n}\right)\right\} dl.
\end{aligned} \tag{2.12}$$

Here it is necessary to use the well-known formula of vector analysis

$$\begin{aligned}
&\frac{\partial\mathbf{V}}{\partial n} = (\mathbf{n},\nabla)\mathbf{V} = \frac{1}{2}\left(\nabla(\mathbf{n},\mathbf{V}) - [\nabla, [\mathbf{n},\mathbf{V}]] - \right. \\
&\left. - [\mathbf{n}, [\nabla,\mathbf{V}]] - [\mathbf{V}, [\nabla,\mathbf{n}]] + \mathbf{n}(\nabla,\mathbf{V}) - \mathbf{V}(\nabla,\mathbf{n})\right) = \\
&= \frac{1}{2}\left(\nabla V_n + [\nabla,\mathbf{k}V_s] - [\mathbf{V}, [\nabla,\mathbf{n}]] + \mathbf{V}(\nabla,\mathbf{n})\right).
\end{aligned}$$

Then, when $\mathbf{V} = \mathbf{s}V_s + \mathbf{n}V_n$

$$\begin{aligned}
&\frac{\partial\mathbf{V}}{\partial n} = (\mathbf{n},\nabla)\mathbf{V} = \frac{1}{2}\left\{\frac{\mathbf{n}}{H_n}\frac{\partial V_n}{\partial n} + \frac{\mathbf{n}}{H_s}\frac{\partial V_s}{\partial s} + \right. \\
&+ \mathbf{s}\left(\frac{1}{H_s}\frac{\partial V_n}{\partial s} - \frac{1}{H_n}\frac{\partial V_s}{\partial n}\right) + \frac{\mathbf{s}}{H_sH_n}\left(\frac{\partial(V_sH_s)}{\partial n} - \right. \\
&\left.\left. - \frac{\partial(V_nH_n)}{\partial s}\right) - [\mathbf{V}, [\nabla,\mathbf{n}]] + \mathbf{V}(\nabla,\mathbf{n})\right\}.
\end{aligned} \tag{2.13}$$

Conclusions

The development of the theory of vertical-axial wind turbines in Ukraine is in its infancy for many reasons: the lack of systematic theoretical and experimental studies in wind tunnels and in full-scale conditions of various WOVD schemes, the lack of an

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appropriate experimental base in technical universities, a small number of publications in foreign and domestic literature.

Existing mathematical models for calculation of the aerodynamic and operational characteristics of vertical-axial wind engines, as a rule, are based on linear approximations of the conservation laws of fluid and gas mechanics, which does not allow to fully take into account the drag forces caused by the viscous interaction of the flow with the streamlined elements of the engine, and also do not take into account the change in the speed of the oncoming flow along the rotor or take into account changes in several planes (by analogy with the horizontal-axis wind turbines). Improving accuracy of the velocity field calculation in front of the rotor of the vertical-axial wind engine can be useful in the design of rotors VAWE, the choice of optimal geometric parameters and modes of operation of the VAWE.

The presented work is just devoted to the construction of such a mathematical model based on a complete, nonlinear system of conservation of fluid and gas mechanics, the integral form of which allows, firstly, to set an arbitrarily oriented oncoming flow, and secondly, to vary both the number of bearing elementary profiles, and their shape, orientation and overall dimensions of the VAWE.

The main advantage of the work is, based on the correct generalization of the apparatus of vector-tensor analysis, an integral representation of solutions to the boundary value problem of flow around a system of airfoils. Moreover, the corresponding system of boundary integral equations is linear and allows for convenient algorithmization for the numerical implementation of the computational process, making it possible to determine not only the total aerodynamic characteristics of the structural elements and the engine as a whole, but also distributed ones, such as pressure and friction.

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МАТЕМАТИЧНА МОДЕЛЬ ВЕРТИКАЛЬНО-ОСЬОВОГО ВІТРОДВИГУНА У ПОТОЦІ В'ЯЗКОГО ГАЗУ

О. П. Бобрякова, А. О. Гундарь, С. І. Іванов, Ю. О. Крашаниця, І. О. Рудик, О. В. Сербин, Д. І. Федченко

Розвиток вертикально-осьових вітродвигунів в Україні знаходиться в зародковому стані з багатьох причин: відсутність систематичних теоретичних та експериментальних досліджень аеродинамічних характеристик різних схем вітротурбін, відсутність у технічних університетах проектних організаціях відповідної експериментальної бази, недостатня кількість доступних публікацій у зарубіжній літературі через високу конкуренцію фірмами-монополістами. В даний час для вирішення актуальних завдань аерогідродинаміки широко застосовуються різні чисельні методи, що застосовуються для наближеного вирішення крайових задач у вигляді диференціальних форм математичних моделей. х загальними недоліками є частковість і трудомісткість рішень, високі вимоги до обчислювальних ресурсів і, як наслідок, складність вирішення завдань оптимізації та економічної доцільності. Цих проблем можна уникнути, використовуючи точні або наближені аналітичні залежності, які дозволяють вирішувати деякі актуальні завдання дослідження взаємодії в'язкого газу з несучими елементами літальних апаратів, так і інженерних споруд. Існуючі методики розрахунків аеродинамічних характеристик, засновані на ідеології математичної моделі руху ідеального середовища без в'язкої взаємодії, не відповідають реальним процесам та запитам практики. У статті представлена ідеологія визначення аеродинамічних характеристик взаємодіючої системи тілесних профілів у конфігурації вертикально-осьового вітродвигуна в потоці в'язкого газу. На базі узагальненого векторно-тензорного аналізу побудовано контурні інтегральні представлення рішень основної задачі механіки рідини та газу, пов'язаної з визначенням кінематичних та динамічних характеристик взаємодії. Окрім цього, доведено існування векторного потенціалу тензора напруг і швидкостей деформацій, який зводить, у найпростіших випадках, процес визначення характеристик до інтегрування. Граничні значення цих інтегральних представлень – система граничних інтегральних рівнянь, що допускає елементарну алгоритмізацію і призводить до системи лінійних алгебраїчних рівнянь, що мають єдиний розв'язок.

Ключові слова: в'язкий газ; закони збереження; граничні інтегральні рівняння; система тілесних профілів; вітродвигун; аеродинамічні характеристики.

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ВЕРТИКАЛЬНО-ОСЕВОГО ВЕТРОДВИГАТЕЛЯ В ПОТОКЕ ВЯЗКОГО ГАЗА

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Развитие вертикально-осевых ветродвигателей в Украине находится в зачаточном состоянии по многим причинам: отсутствие систематических теоретических и экспериментальных исследований аэродинамических характеристик различных схем ветротурбин, отсутствие в технических университетах проектных организациях соответствующей экспериментальной базы, недостаточное количество доступных публикаций в зарубежной литературе из-за высокой конкуренции между фирмами-монополистами. В

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настоящее время для решения актуальных задач аэрогидродинамики широко применяются различные численные методы, применяемые для приближенного решения краевых задач в виде дифференциальных форм математических моделей. Их общими недостатками являются частность и трудоемкость решений, высокие требования к вычислительным ресурсам и, как следствие, сложность решения задач оптимизации и экономической целесообразности. Этих проблем можно избежать, используя точные или приближенные аналитические зависимости, которые позволяют решать некоторые актуальные задачи исследования взаимодействия вязкого газа с несущими элементами как летательных аппаратов, так и инженерных сооружений. Существующие методики расчетов аэродинамических характеристик, основанные на идеологии математической модели движения идеальной среды без вязкого взаимодействия, не соответствуют реальным процессам и запросам практики. В статье представлена идеология определения аэродинамических характеристик взаимодействующей системы телесных профилей в конфигурации вертикально-осевого ветродвигателя в потоке вязкого газа. На базе обобщенного векторно-тензорного анализа построены контурные интегральные представления решений основной задачи механики жидкости и газа, связанной с определением кинематических и динамических характеристик взаимодействия. Кроме этого, доказано существование векторного потенциала тензора напряжений и скоростей деформаций, сводящий, в простейших случаях, процесс определения характеристик к интегрированию. Предельные значения этих интегральных представлений – система граничных интегральных уравнений, допускает элементарную алгоритмизацию и приводит к системе линейных алгебраических уравнений, имеющих единственное решение.

Ключевые слова: вязкий газ; законы сохранения; граничные интегральные уравнения; система телесных профилей; ветродвигатель; аэродинамические характеристики.

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