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CHAOTIC BEHAVIOR OF TAKAGI-SUGENO RECURRENT MODELS WITH LARGE-SCALE TIME DELAY

In this paper we consider the dynamic systems, which are represented by recurrent Takagi-Sugeno (TS) fuzzy rule bases. We found the conditions for the recurrent rule base with large-scale time delay that cause the chaotic behavior in the sense of Li-Yorke. We will find the rules for consequent of 0th order Takagi-Sugeno model that produce chaotic orbits. We also propose the methods of finding chaotic behavior for arbitrary number of rules in these models. For this purpose we generalize the scalar case of Kloeden theorem to vector one.

recurrent fuzzy model, chaos, rule base

Introduction

For description of many complex dynamic processes we often use 0th order recurrent Takagi-Sugeno (TS) fuzzy rule bases. These rules have the following form:

$$\begin{aligned}
&R_1 : if \quad x_k \quad is \quad L_{1j_k} \quad and \quad x_{k+1} \quad is \quad L_{1j_{k+1}} and\ldots \\
&\ldots and \quad x_{k+n} \quad is \quad L_{1j_{k+n}} \quad then \quad x_{k+n+1} = A_1, \\
&R_2 : if \quad x_k \quad is \quad L_{2j_k} \quad and \quad x_{k+1} \quad is \quad L_{2j_{k+1}} and\ldots \\
&\ldots and \quad x_{k+n} \quad is \quad L_{2j_{k+n}} \quad then \quad x_{k+n+1} = A_2, \\
&\vdots \\
&R_M : if \quad x_k \quad is \quad L_{Mj_k} and \quad x_{k+1} \quad is \quad L_{Mj_{k+1}} and\ldots \\
&\ldots and \; x_{k+n} \quad is \quad L_{Mj_{k+n}} \quad then \quad x_{k+n+1} = A_M,
\end{aligned} \qquad (1)$$

where L_{ij_q} are linguistic variables (terms) and A_i are numerical constants.

Such rule bases can demonstrate the chaotic behavior in sense of Li and Yorke [1]. So it is important to find the properties of the consequents that could help to recognize chaos. In paper [2] we have considered the one time delay case of system (1) for $n = 0$ and found the conditions for coefficients A_i that deliver chaotic behavior of recurrent rule base. We proved the statement that minimum number of rules that are necessary for chaos in TS models is three for triangular membership functions. Moreover, we obtained the necessary and sufficient conditions for coefficients A_i in such rule base. The main result was following. For rule base

$$\begin{aligned}
&R_1 : if \quad x_k \quad is \quad L_1 \quad then \quad x_{k+1} = A_1, \\
&R_2 : if \quad x_k \quad is \quad L_2 \quad then \quad x_{k+1} = A_2, \\
&R_3 : if \quad x_k \quad is \quad L_3 \quad then \quad x_{k+1} = A_3,
\end{aligned} \qquad (2)$$

let the core positions $a_1,\ a_2, a_3 \in I = [0,1]$ of linguistic variables $L_1,\ L_2,\ L_3$ fulfill the inequality $0 \le a_1 < a_2 < a_3 \le 1$. Then the following theorem is hold.

Theorem 1. A rule base (2) with mapping $f : I \to I$ is chaotic on $x \in [a_1, a_3] \subset I$ in the sense of Li and Yorke if the following conditions are satisfied:

a) $A_1 \in [a_1, a_2]$;

b) $A_2 = a_3$;

c) $A_3 \in [a_1, a_2)$.

One important question of investigations of TS model is about incomplete rule base. It means that for one-time delay case we can have zero value as in nominator and denominator of fraction of transition function in (1)

$$f(x) = \frac{\sum_{i=1}^{M} \mu_i(x) \cdot A_i}{\sum_{i=1}^{M} \mu_i(x)}$$

(it take place because we haven't any rule to work with some input). Thus it is necessary to settle this contradiction. The simplest way that is widely used in practice (for example, in MatLab Fuzzy Logic Toolbox as default) is assume that fraction is 0.5. In this case the problem of incomplete rule base became the very simple problem of general case of TS model and is an exclude case in this paper.

Much more interesting problem is large-scale time delay TS models that are really important in task of time series analysis and simulation modeling. We will consider at first the case of two time delay model, then three time delay and at last general case. We consider here the mapping

$$f : x, x, \ldots, x \to x$$

on the set $x \in I = [0,1]$.

1. Chaos in two time delay TS model

In this case model (1) can be rewritten so

$$\begin{aligned}
&R_1 : \; if \;\; x_k \;\; is \;\; L_{1j_k} \;\; and \;\; x_{k+1} \;\; is \;\; L_{1j_{k+1}} \\
&then \;\; x_{k+2} = A_1, \\
&R_2 : \; if \;\; x_k \;\; is \;\; L_{2j_k} \;\; and \;\; x_{k+1} \;\; is \;\; L_{2j_{k+1}} \\
&then \;\; x_{k+2} = A_2, \\
&\ldots \\
&R_M : \; if \;\; x_k \;\; is \;\; L_{Mj_k} \;\; and \;\; x_{k+1} \;\; is \;\; L_{Mj_{k+1}} \\
&then \;\; x_{k+2} = A_M.
\end{aligned} \tag{3}$$

It is easy to see that following proposition is holds. Three rules in two time delay TS rule base are necessary and sufficient for producing chaos. Let us consider the following rule base:

$$\begin{aligned}
&R_1 : \; if \;\; x_k \;\; is \;\; L_1 \;\; then \;\; x_{k+2} = A_1, \\
&R_2 : \; if \;\; x_k \;\; is \;\; L_2 \;\; then \;\; x_{k+2} = A_2, \\
&R_3 : \; if \;\; x_k \;\; is \;\; L_3 \;\; then \;\; x_{k+2} = A_3.
\end{aligned} \tag{4}$$

If the linguistic variables L_i and consequents A_i in (4) are satisfied the conditions of Theorem 1 for one time delay case, we have chaotic orbit $X = \{x_0, x_2, \ldots, x_{2n}, \ldots\}$. In general case of two time delay TS model the rule base is as follow:

$$\begin{aligned}
&R_1 : if \; x_k \; is \, L_1 \; and \; x_{k+1} \; is \, L_1 \; then \, x_{k+2} = A_{11}, \\
&R_2 : if \; x_k \; is \, L_1 \; and \; x_{k+1} \; is \, L_2 \; then \, x_{k+2} = A_{12}, \\
&R_3 : if \; x_k \; is \, L_1 \; and \; x_{k+1} \; is \, L_3 \; then \, x_{k+2} = A_{13}, \\
&R_4 : if \; x_k \; is \, L_2 \; and \; x_{k+1} \; is \, L_1 \; then \, x_{k+2} = A_{21}, \\
&R_5 : if \; x_k \; is \, L_2 \; and \; x_{k+1} \; is \, L_2 \; then \, x_{k+2} = A_{22}, \\
&R_6 : if \; x_k \; is \, L_2 \; and \; x_{k+1} \; is \, L_3 \; then \, x_{k+2} = A_{23}, \\
&R_7 : if \; x_k \; is \, L_3 \; and \; x_{k+1} \; is \, L_1 \; then \, x_{k+2} = A_{31}, \\
&R_8 : if \; x_k \; is \, L_3 \; and \; x_{k+1} \; is \, L_2 \; then \, x_{k+2} = A_{32}, \\
&R_9 : if \; x_k \; is \, L_3 \; and \; x_{k+1} \; is \, L_3 \; then \, x_{k+2} = A_{33}.
\end{aligned} \tag{5}$$

Let a_1, a_2, a_3 are the core positions of appropriate linguistic variables L_1, L_2, L_3. Then rule base (5) can be considered as lattice with coordinates a_1, a_2, a_3 on each axe (see example on Fig. 1). The nodes of the lattice contains the appropriate values $A_{11}, A_{12}, \ldots, A_{33}$.

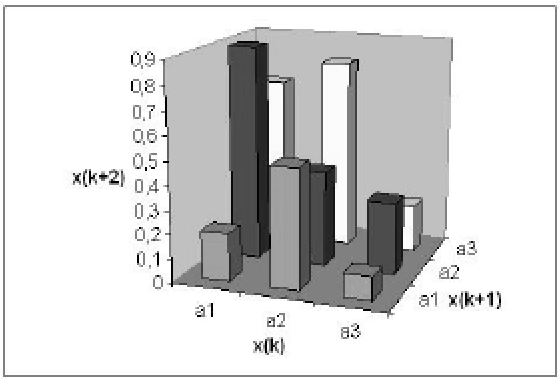

Fig. 1. Example of rule base (5)

The following theorem gives sufficient conditions for the existence of chaos for Banach space mappings and is suited for TS models as well.

Theorem 2 (Kloeden) [3]. Let $f : I \to I$ be a continuous mapping of a Banach space I into itself and suppose that there exists non-empty compact subsets A and B of I, and integers $n_1, n_2 \geq 1$ such that:

A is homeomorphic to a convex subset of I,

$$A \subseteq f(A),$$

f is expanding on A, that is there exists a constant $\lambda > 1$ such that

$$\lambda \left\| x - y \right\| \le \left\| f(x) - f(y) \right\|$$

for all $x, y \in A$;

$B \subset A$; $f^{n_1}(B) \cap A = \varnothing$; $A \subseteq f^{n_1+n_2}(B)$;

$f^{n_1+n_2}$ is injective on B (one-to-one).

Then the mapping f is chaotic in the sense of Li and Yorke.

We use this theorem for general case of two-time delay TS model (5). Let us try to generalize the Theorem 1 on the vector case $f : x, x \to x$. Then we have the following theorem.

Theorem 3. A rule base (5) with mapping

$$f : \left[a_1, a_3\right] \times \left[a_1, a_3\right] \to \left[a_1, a_3\right]$$

is chaotic on $x \in \left[a_1, a_3\right] \subset I$ in the sense of Li and Yorke if the following conditions are satisfied:

a) $A_{22} = a_3$;

b) $\min(A_{11}, A_{12}, A_{13}) \in \left[a_1, a_2\right]$;

c) $\min(A_{31}, A_{32}, A_{33}) \in \left[a_1, a_2\right)$.

Proof. According to the Theorem 2 it is necessary to find the appropriate sets A and B for the transition function $f : \left[a_1, a_3\right] \times \left[a_1, a_3\right] \to \left[a_1, a_3\right]$. Let us extend the conditions of Theorem 2 on vector case. Assume that $A = \left[\left[\xi, \psi\right] \times \left[\theta, \psi\right]\right]$ and $B = \left[\left[\xi, \psi\right] \times \left[\theta, a_2\right]\right] \subset A$ (see Fig. 2) with ξ, ψ, θ to be determined. Let $\theta < a_2$, $\psi > \xi > a_2$ as well.

We can rewrite general mapping $f : x_k, x_{k+1} \to x_{k+2}$ in the state space $X = \left(x_1, x_2\right)$, where $x_1 = x_k$, $x_2 = x_{k+1}$. Then we can write

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{k+1} = \begin{pmatrix} x_2 \\ f\left(x_1, x_2\right) \end{pmatrix}_k . \qquad (6)$$

It is clear, that according to (6) we obtain $X_{k+1} = F\left(X_k\right)$ where $F = \begin{pmatrix} x_2 \\ f\left(x_1, x_2\right) \end{pmatrix}$. Then we can consider the Theorem 2 in vector case.

Let $A = \begin{pmatrix} \left[\xi, \psi\right] \\ \left[\theta, \psi\right] \end{pmatrix}$. Then

$$F\left(A\right) = \begin{pmatrix} \left[\theta, \psi\right] \\ \left[f\left(\psi, \circ\right), f\left(\xi, a_2\right)\right] \end{pmatrix}.$$

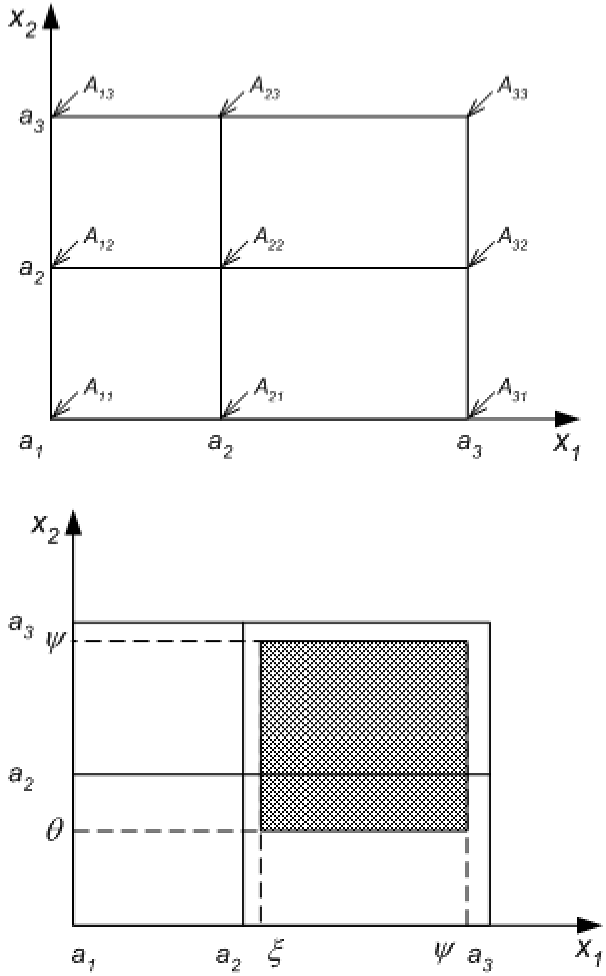

Fig. 2. Rule base representation and illustration of sets A, B

Here $f\left(\xi, a_2\right)$ is a value of function F on the straight line $x_1 = \xi$ and $f\left(\psi, \circ\right)$ means the some value of F on the straight line $x_1 = \psi$. Let us assume that $f\left(\xi, a_2\right) = \psi$. To provide the Condition 2 of the

Theorem 2 in a vector case $A \subseteq F(A)$ we have following inequality

$$f(\psi, \circ) \le \theta < a_2.$$

Because

$$B = \begin{pmatrix} [\xi, \psi] \\ [\theta, a_2] \end{pmatrix},$$

we can find

$$F(B) = \begin{pmatrix} [\theta, a_2] \\ [f(\psi, \circ), f(\xi, a_2) = \psi] \end{pmatrix}.$$

It is clear that $F(B) \cap A = \varnothing$ (Condition 5 of Theorem 2 for $n_1 = 1$). Let us find $F^2(B)$. We have here

$$F^2(B) = \begin{pmatrix} [f(\psi, \circ), f(\xi, a_2) = \psi] \\ [f(\theta, \circ), a_3] \end{pmatrix}.$$

To provide the Condition 6 of Theorem 2 we demand that $f(\theta, \circ) \le \theta \le a_2$. Then conditions

$$f(\theta, \circ) \le \theta \le a_2,\ f(\xi, a_2) = \psi,\ f(\psi, \circ) < a_2$$

define the conditions (1) – (3) of the Theorem 3. The rest conditions of Theorem 2 are evident and aren't proved here.

2. Chaos in three and above time delay TS model

We propose the same approach in investigation of higher order TS model. Let us consider the general case of three time delay TS model. We have here the following rule base:

$$\begin{aligned}
&R_1 : \textit{if } x_k \textit{ is } L_1 \textit{ and } x_{k+1} \textit{ is } L_1 \\
&\textit{and } x_{k+2} \textit{ is } L_1 \textit{ then } x_{k+3} = A_{11}, \\
&R_2 : \textit{if } x_k \textit{ is } L_1 \textit{ and } x_{k+1} \textit{ is } L_1 \\
&\textit{and } x_{k+2} \textit{ is } L_2 \textit{ then } x_{k+3} = A_{12}, \qquad (7) \\
&\vdots \\
&R_{27} : \textit{if } x_k \textit{ is } L_3 \textit{ and } x_{k+1} \textit{ is } L_3 \\
&\textit{and } x_{k+2} \textit{ is } L_3 \textit{ then } x_{k+3} = A_{33}.
\end{aligned}$$

We consider now the mapping

$$f : x_k, x_{k+1}, x_{k+2} \to x_{k+3}.$$

First of all it is necessary to present this model in the state space form $X_{k+1} = F(X_k)$, where $X = (x_1, x_2, x_3)$. We have

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{k+1} = \begin{pmatrix} x_2 \\ x_3 \\ f(x_1, x_2, x_3) \end{pmatrix}_k. \qquad (8)$$

According to proposed approach we have theorem.

Theorem 4. A rule base (7) with mapping

$$f : [a_1, a_3] \times [a_1, a_3] \times [a_1, a_3] \to [a_1, a_3]$$

is chaotic on $x \in [a_1, a_3] \subset I$ in the sense of Li and Yorke if the following conditions are satisfied:

a) $A_{222} = a_3$;

b) $\min(A_{111}, A_{112}, \ldots, A_{133}) \in [a_1, a_2]$;

c) $\min(A_{311}, A_{312}, \ldots, A_{333}) \in [a_1, a_2)$.

Proof. According to the Theorem 2 it is necessary to find the appropriate sets A and B for the transition function $f : [a_1, a_3] \times [a_1, a_3] \times [a_1, a_3] \to [a_1, a_3]$. Assume that $A = [[\xi, \psi] \times [\theta, \psi] \times [\theta, \psi]]$ and $B = [[\xi, \psi] \times [\theta, a_2] \times [\xi, \psi]] \subset A$ (see Fig.3) with ξ, ψ, θ to be determined. Let $\theta < a_2, \psi > \xi > a_2$ as well.

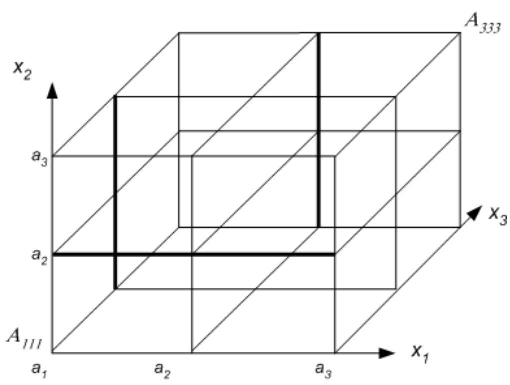

Fig. 3. Rule base representation for three time delay model

According to (8) we can write

$$F = \begin{pmatrix} x_2 \\ x_3 \\ f(x_1, x_2, x_3) \end{pmatrix}.$$

Let $A = \begin{pmatrix} [\xi, \psi] \\ [\theta, \psi] \\ [\theta, \psi] \end{pmatrix}$. Then $F(A) = \begin{pmatrix} [\theta, \psi] \\ [\theta, \psi] \\ [f(\psi, \circ, \circ), \psi] \end{pmatrix}$.

To provide the Condition 2 of the Theorem 2 in a vector case $A \subseteq F(A)$ we have following inequality

$$f(\psi, \circ, \circ) \le \theta < a_2.$$

Because

$$B = \begin{pmatrix} [\xi, \psi] \\ [\theta, a_2] \\ [\xi, \psi] \end{pmatrix},$$

we can find

$$F(B) = \begin{pmatrix} [\theta, a_2] \\ [\xi, \psi] \\ [f(\psi, \circ, \circ), \psi] \end{pmatrix}.$$

It is clear that $F(B) \cap A = \varnothing$ (Condition 5 of Theorem 2 for $n_1 = 1$). Let us find $F^2(B)$. We have here

$$F^2(B) = \begin{pmatrix} [\xi, \psi] \\ [f(\psi, \circ, \circ), \psi] \\ [f(\theta, \circ, \circ), a_3] \end{pmatrix}.$$

To provide the Condition 6 of Theorem 2 we demand that $f(\theta, \circ, \circ) \le \theta \le a_2$. Then conditions:

$$f(\theta, \circ, \circ) \le \theta \le a_2;$$

$$f(\xi, a_2, a_2) = \psi;$$

$$f(\psi, \circ, \circ) \le \theta < a_2$$

define the conditions (1) – (3) of the Theorem 4. The rest conditions of Theorem 2 are evident and aren't proved here.

Thus we can generalize our investigations in the following Theorem.

Theorem 5. *A rule base (1) with mapping*

$$f : [a_1, a_3] \times [a_1, a_3] \times \ldots \times [a_1, a_3] \to [a_1, a_3]$$

is chaotic on $x \in [a_1, a_3] \subset I$ *in the sense of Li and Yorke if the following conditions are satisfied:*

a) $A_{22\ldots2} = a_3$;

b) $\min(A_{11\ldots1}, A_{11\ldots2}, \ldots, A_{13\ldots3}) \in [a_1, a_2]$;

c) $\min(A_{311\ldots1}, A_{31\ldots2}, \ldots, A_{33\ldots3}) \in [a_1, a_2)$.

Conclusion

We consider in this paper only TS models of 0th order. We have plans to the expansion of proposed approach to the TS model of 1st order. Proposed Theorems can help to identify chaotic behavior only by values of consequent and may be useful in practice of series analysis. If we have incomplete rule base it means that we have constant values in appropriate nodes of the lattice and use the general approach.

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