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LOW SNR TRESHOLD IN RAPID ESTIMATORS OF COMPLEX SINUSOIDALS

A task of estimation of complex sinusoid frequency is considered. A particular but practically important case of low signal-to-noise ratio (SNR) is studied. The low SNR threshold, commonly overlooked in development of the rapid estimator of complex sinusoidals, is addressed. Signals of different length are considered and SNR is varied in wide limits. It is demonstrated that a simple interpolation with factor 2 lowers the SNR threshold by 1.5dB for the most complicated practical situations. Further interpolation does not bring any improvement. This allows proposing a compromise practical algorithm that provides accuracy close to the limit and is still very simple and fast.

Keywords: DFT, frequency estimators, SNR threshold.

Introduction

Estimation of sinusoidal signal parameters represents one of the most important traditional signal processing problems. First theoretically grounded results in this field can be followed from seminal paper of Rife and Boorstyn [1]. In addition to statistical accuracy analysis and achievable bounds in the sinusoidals' parameter estimation, Rife and Boorstyn have proposed estimation of the sinusoidal parameters in the rough stage using the DFT peak position in an initial stage. In the second stage, some refinement is applied to reduce the mean squared error (MSE) toward the Cramer-Rao Lower Bound (CRLB). Over course of time, this strategy becomes dominant in the field with numerous extensions. This refinement (interpolation) is performed by using neighbor samples of the DFT peak $\hat{k} \pm \Delta k$, where $\hat{k}$ is frequency bin corresponding to the DFT peak while Δk could be small integers $\Delta k = 1$ or $\Delta k = 2$, or some fractions like for example $\Delta k = 1/2$. The goal is to keep refinement/interpolation as simple as possible producing the MSE as close as possible to the CRLB.

One of the first attempts to design efficient frequency estimators in this direction was technique proposed by Quinn [2, 3]. It has been generalized for estimation of all parameters of complex sinusoidals in [4]. This technique includes nonlinear optimization and switching rule; so, numerous simplifications are proposed in the last two decades. Candan has proposed frequency interpolator using three DFT samples [5, 6] with evaluation of trigonometric functions. Similar approach has been proposed in [7] with combination of three frequency bins without need for iterative

procedures. An alternative approaches have been proposed by Zakharov and co-authors in [8] and by Provencher in [9]. Currently the most popular and the most efficient estimator for rapid frequency estimation is Aboutanios-Mulgrew approach proposed in [10, 11]. Fine stage of this algorithm consists of two very simple iterations without need for employing switching rules or evaluation of trigonometric or other nonlinear functions. It has been generalized for decaying sinusoidals in [12].

An important advantage of this algorithm is ability to work in non-Gaussian impulsive environments [13, 14] with significantly smaller number of iterations than alternatives [15]. As other advantages of this technique, it should be mentioned simple realization for the real-valued signals without bias commonly appearing due to effect of the spectral component at negative frequencies [16]. In addition, it has been shown as an excellent tool for estimation of sinusoidal signal parameters in the wireless sensor network setup [17]. In [18, 19] an overview of recent advances in this field has been given. Different approaches for rapid estimation of the frequency are summarized in [20].

Problem statement

In this paper, we are going to discuss how to have as low as possible value of the SNR threshold in these estimators. The SNR threshold is value of the input SNR where the algorithm breaks down giving unreliable results. Simultaneously, we try to keep the estimator as simple as possible. These important issues are rarely discussed in the open literature with only several notable exceptions (threshold

behavior of the frequency estimators has been briefly studied in [21, 22]).

The paper is organized as follows. In the next section, we present the Aboutanios and Mulgrew algorithm as a representative of the considered estimators class. In Section III, requirement for evaluation of the DFT over denser frequency grid in order to reduce the SNR threshold is been described. Simulations are given in Section IV followed by conclusions.

1. Aboutanios and Mulgrew estimator

A particularly popular rapid frequency estimator has been proposed recently by Aboutanios and Mulgrew [10]. Note that this important estimator is used here just for illustration purpose while the same conclusions are confirmed in simulations for other similar estimators [11], [15].

The estimator can be summarized with the following simple steps:

Input: a complex sinusoidal corrupted by the Gaussian noise $x(n) = A \exp(j\omega_0 n + j\varphi_0) + \nu(n)$, $n \in [0, N)$. Number of samples N for a given observation interval is selected that the Nyquist criterion is satisfied. The input SNR for a considered task can be defined as $SNR_{in} = A^2 / \sigma^2$ where A is amplitude of sinusoidal while σ^2 is variance of an additive Gaussian noise.

Rough stage: Evaluation of the DFT and its maximization:

$$\hat{k} = \arg \max_k |X(k)|, \qquad (1)$$

$$X(k) = \sum_{n=0}^{N-1} x(n) \exp(-j2\pi \frac{nk}{N}). \qquad (2)$$

It is supposed that the DFT (2) is evaluated by the fast Fourier transform (FFT) algorithm of complexity $O(N \log_2 N)$.

Fine stage: Fine stage is performed in two iterations

$\delta = 0$
for iter=1:2

$$\delta = \delta + \frac{1}{2} \frac{\left| X(\hat{k} + \frac{1}{2} + \delta) \right| - \left| X(\hat{k} - \frac{1}{2} + \delta) \right|}{\left| X(\hat{k} + \frac{1}{2} + \delta) \right| + \left| X(\hat{k} - \frac{1}{2} + \delta) \right|} \qquad (3)$$

end

Output: Final frequency estimate is $\hat{\omega} = 2\pi \hat{k} / N$, where $\hat{k} = \hat{k} + \delta$.

More iterations could be helpful only in the case of high SNR (above $SNR > 40dB$) while for low SNR considered here (which are of more interest for different

practical situations) two iterations are enough to reach the CRLB [10].

Obviously, the refinement/interpolation requires 4 additional evaluations of the DFT using direct application of sum. The proposed technique for wide range of the input $SNR_{in} = A^2 / \sigma^2$ gives the MSE, $SNR_{out} = E\left\{ [\hat{\omega} - \omega_0]^2 \right\}$, that is very close to the CRLB (only about 0.06dB above the CRLB).

2. SNR threshold

A particularly interesting issue quite often overlooked is how to lower the SNR threshold and price for such an activity. The DFT peak magnitude for frequency corresponding to the sinusoidal should be as high as possible in order to obtain low SNR threshold. Namely, if the sinusoid has frequency on the frequency grid $\omega_0 = 2k\pi / N, k \in [-N / 2, N / 2)$, expected value of the DFT maximum is proportional to NA (hereafter expected values for DFT peak magnitude are given for noiseless signal). The DFT variance is proportional to $N\sigma^2$. Simplified analysis is given with mathematical rigor reduced to a necessary level.

Then, the ratio between power of the sinusoid and noise variance is proportional to $\approx NA^2 / \sigma^2$. However, in the case when frequency is not on the grid $\omega_0 \neq 2\pi k / N$, $k \in Z$, the magnitude of the peak DFT is smaller than NA. The most critical case is for frequencies in the middle between two frequency bins $\omega_0 = 2(k + \frac{1}{2})\pi / N$ with magnitude of the DFT peak $\approx NA \sin(\pi / 2) / (\pi / 2) = 2NA / \pi \approx 0.637NA$. This means that the resulting SNR is approximately $\approx 0.405NA^2 / \sigma^2$, i.e., less than half than for the frequency on the grid. Since rough estimation of the frequency is performed based on the DFT peak position, this significantly reduces ability to recognize peak for high noise (low SNR) environments. This effect is only partially compensated with the fact that there are now two frequency bins with similar magnitudes corresponding to the considered sinusoidal. The ratio between peak DFT magnitude and resulting noise standard deviation is crucial for achieving low SNR threshold and high estimator accuracy in the rough stage.

Therefore, the Aboutanios-Mulgrew algorithm can be modified by evaluating the DFT in the initial stage over ρ times denser grid:

Rough stage: Evaluation of the DFT and its maximization:

$$\hat{k} = \frac{1}{\rho} \arg \max_k |Y(k)|, \qquad (4)$$

$$Y(k) = \sum_{n=0}^{N-1} x(n) \exp(j2\pi \frac{n(k/\rho)}{N}), k \in [0, \rho N), \quad (5)$$

followed by the same two iterations for refinement/interpolation of the frequency estimate. Complexity of the rough stage is increased by ρ times with respect to the previous algorithm.

However, for example, consider $\rho = 2$. After determination of the position of the maxima $Y(\hat{k})$ two neighbor bins $Y(\hat{k} \pm 1)$ are on the position corresponding to $X(\hat{k} \pm 1/2)$ for N frequency bins (2). Therefore, we have already calculated two points for the first iteration (3) and displacement for it can be calculated as

$$\delta = \frac{1}{2} \frac{\left| Y(\hat{k}+1) \right| - \left| Y(\hat{k}+1) \right|}{\left| Y(\hat{k}+1) \right| + \left| Y(\hat{k}+1) \right|}. \quad (6)$$

So, the overall complexity is increased for (big O notation is dropped due to simplicity) $N \log_2 N - 2N = N \log_2(N/4)$. In this way, the smallest DFT peak magnitude is achieved for displacement of $\pm 1/4$ from the grid where amplitude of DFT is $\approx NA \sin(\pi/4)/(\pi/4) \approx 0.9NA$ what is $\sqrt{2}$ times larger than for the most critical case for the DFT evaluated over N frequency bins.

The SNR between DFT on the peak and output noise in the most critical case is now $\approx 0.81NA^2/\sigma^2$ (twice than in the case of the most critical situation when the DFT is evaluated over N frequency bins).

We can proceed further with $\rho = 4$ increasing complexity for additional $2N \log_2 N$ operations in the rough stage but this is not bringing further decrease of complexity in the fine stage. In that case, minimal DFT peak magnitude is $\approx 0.9745NA$ with resulting SNR $\approx 0.95NA^2/\sigma^2$. As it is shown in simulations, this additional calculation burden cannot be justified with significant improvement since the largest benefit comes from the DFT interpolation with the factor $\rho = 2$. Note that the this analysis is very rough but, as it will be seen in the next section, it is good indication what can be achieved without going in hard and precise characterization of the noise effects in the interpolated DFT that is not white anymore. Again, even this rough analysis is quite useful and pointing to correct results that are confirmed by extensive simulations in the next section.

3. Simulation tests

Experiments with sinusoids of various number of samples $N = 2^m, m \in [5, 14]$ are conducted. In each trial, frequency is generated randomly. Number of trials was 10000 for each SNR_{in} and signal length. Signal lengths are represented with different colors (see the plots in Fig. 1). The considered SNR range was $SNR_{in} \in [-30, 5]$dB with step 1dB while around breakdown point smaller step of 0.1dB was used.

The obtained dependences are given in Figure 1. Dashed lines represent the case when the DFT in the rough stage is evaluated over N frequency bins, solid lines are for the DFT evaluated in the initial stage over 2N frequency bins, while dotted lines correspond to the DFT evaluated in the rough stage over 4N frequency bins. It can be seen that the SNR threshold (position when the MSE increases abruptly) is approximately 1.5dB lower in the case of the DFT evaluated over 2N samples in the rough stage than in the case when evaluation is performed for N samples. Also, it can be seen that further increasing number of frequency bins does not bring almost any benefit in term of the SNR threshold while it increases burden (dotted lines are hardly visible since results obtained with 4N are almost the same as for 2N frequency bins).

Similar results are obtained for the dichotomous estimators [15] and the hybrid estimator from [11]. In both of these estimators, the situation is the same. Evaluation of the DFT with 2N points in the rough stage reduces the SNR threshold as in the case of the considered estimator.

It can be noted that the SNR threshold decreases approximately linearly as logarithm of signal length increases. It can be approximated for the DFT evaluated over N frequency bins as [1, 21, 22]

$$SNR_{thresh} = 10 \log_{10} \frac{A_{thresh}^2}{\sigma_{thresh}^2} [dB] \approx$$

$$\approx [-2.8 \log_2 N + 14] dB$$

Since there are techniques (for example [23]) that can be used for the SNR_{in} estimation, it can be arranged that for SNR_{in} above the threshold SNR_{thresh} the DFT in the rough stage is evaluated over N frequency bins while around SNR_{thresh} it can be arranged evaluation of the DFT with 2 N frequency bins in rough stage (for example in the range of $SNR_{thresh} \pm 2dB$).

Conclusion

Simple conclusion follows that it is worth evaluating the DFT over 2N frequency bins in the rough stage of the rapid frequency estimators. Further interpolation does not bring any visible improvement so it should be avoided.
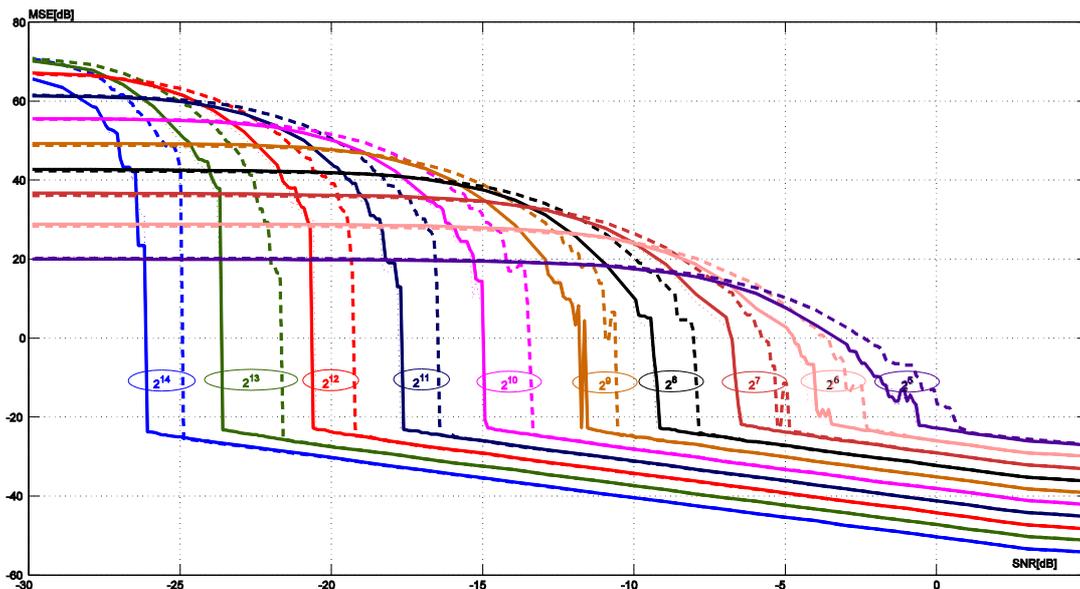
Fig. 1. MSE in frequency estimation for the Aboutanios-Mulgrew algorithm. Dashed lines - DFT in initial stage evaluated over N frequency bins (N is signal length); Solid lines - DFT in initial stage interpolated to 2N frequency bins; Dotted lines - DFT in initial stage interpolated to 4N frequency bins. Signal lengths are represented with various colors and indicated on the figure

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НИЖНИЙ ПОРОГ ОСШ ПРИ БЫСТРОМ ОЦЕНИВАНИИ КОМПЛЕКСНЫХ СИНУСОИД

И. Джурович, В. В. Лукин

Рассмотрена задача оценки частоты комплексной синусоиды. Изучен частный, но важный на практике случай низкого отношения сигнал-шум (ОСШ). Особое внимание уделено нижнему пределу ОСШ, обычно не рассматриваемому при разработке методов быстрого оценивания параметров комплексных синусоид. Рассмотрены сигналы различной длительности, ОСШ варьируется в широких пределах. Показано, что простая интерполяция в два раза уменьшает нижний предел на 1,5 дБ для наиболее сложных практических ситуаций. Дальнейшая интерполяция не приводит к положительным результатам. Это позволяет предложить компромиссный практический алгоритм, который обеспечивает точность, близкую к предельной, но при этом остается очень простым и быстрым.

Ключевые слова: ДПФ, оценивание частоты, порог ОСШ.

НИЖНІЙ ПОРІГ ВСШ ПРИ ШВИДКОМУ ОЦІНЮВАННІ КОМПЛЕКСНИХ СИНУСОЇД

І. Джурович, В. В. Лукін

Розглянуто задачу оцінки частоти комплексної синусоїди. Вивчено частковий, але важливий для практики випадок низького відношення сигнал-шум (ВСШ). Особливу увагу приділено нижній межі ВСШ, яку зазвичай не розглядають під час розробки методів швидкого оцінювання параметрів комплексних синусоїд. Розглянуто сигнали різної довжини, ВСШ варіюється в широких межах. Показано, що проста інтерполяція в два рази зменшує нижню межу на 1,5 дБ для найбільш складних практичних ситуацій. Подальша інтерполяція не призводить до позитивних наслідків. Це дозволяє запропонувати компромісний практичний алгоритм, який забезпечує точність, близьку до межі, але при цьому залишається дуже простим і швидким.

Ключові слова: ДПФ, оцінювання частоти, поріг ВСШ.

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