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ON ESTIMATES OF COEFFICIENTS OF GENERALIZED ATOMIC WAVELETS EXPANSIONS AND THEIR APPLICATION TO DATA PROCESSING

Discrete atomic compression (DAC) of digital images is considered. It is a lossy compression algorithm. The aim of this paper is to obtain a mechanism for control of quality loss. Among a large number of different metrics, which are used to assess loss of quality, the maximum absolute deviation or the MAD-metric is chosen, since it is the most sensitive to any even the most minor changes of processed data. In DAC, the main loss of quality is got in the process of quantizing atomic wavelet coefficients that is the ***subject matter*** *of this paper. The* ***goal*** *is to investigate the effect of the quantization procedure on atomic wavelet coefficients. We solve the following* ***task****: to obtain estimates of these coefficients. In the current research, we use the* ***methods*** *of atomic function theory and digital image processing. Using the properties of the generalized atomic wavelets, we get estimates of generalized atomic wavelet expansion coefficients. These inequalities provide dependence of quality loss measured by the MAD-metric on the parameters of quantization in the form of upper bounds. They are confirmed by the DAC-processing of the test images. Also, loss of quality measured by root mean square (RMS) and peak signal to noise ratio (PSNR) is computed. Analyzing the results of experiments, which are carried out using the computer program "Discrete Atomic Compression: Research Kit", we obtain the following* ***results****: 1) the deviation of the expected value of MAD from its real value in some cases is large; 2) accuracy of the estimates depends on parameters of quantization, as well as depth of atomic wavelet expansion and type of the digital image (full color or grayscale); 3) discrepancies can be reduced by applying a correction coefficient; 4) the ratio of the expected value of MAD to its real value behaves relatively constant and the ratio of the expected value of MAD to RMS and PSNR do not.* ***Conclusions:*** *discrete atomic compression of digital images in combination with the proposed method of quality loss control provide obtaining results of the desired quality and its further development, research and application are promising.*

Keywords: *lossy image compression; discrete atomic compression; generalized atomic wavelets; maximum absolute deviation; quality loss control.*

Introduction

Images have become an essential part of our life [1]. They are acquired by customer digital cameras [2] airborne and spaceborne remote sensing sensors [3], medical imaging devices [4]. Average size of images and periodicity of their acquiring increase rapidly leading to a great amount of data that have to be stored and transferred via communication lines. This explains why there is an obvious necessity in efficient image compression. Lossless compression is often not able to satisfy the main requirements since compression ratio is too small. Then, one has to apply lossy compression that introduces distortions.

Lossy compression can be based on different principles where the existing standards and other modern compression techniques usually employ orthogonal transforms like discrete cosine transform or wavelets. Type and properties of wavelets have considerable impact on performance of the corresponding compression method [1]. Because of this, selection of a wavelet basis and thorough analysis of its properties is an important task in lossy image compression.

In [5 – 7], infinitely differentiable wavelets with a compact support, which are finite linear combinations of the atomic functions

$$\mathrm{up}_s(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{itx} \prod_{k=1}^{\infty} \frac{\sin^2\left(\frac{st}{(2s)^k}\right)}{\frac{t}{(2s)^k} s^2 \sin\left(\frac{t}{(2s)^k}\right)} dt, \ s = 2, 3, \ldots,$$

were introduced. These wavelets are called atomic wavelets. Their generalizations, called generalized atomic wavelets, were constructed in [8].

Atomic wavelets and generalized atomic wavelets combine a number of useful properties that makes them a promising tool for data analysis and processing (a detailed discussion is given in [9]). Digital image compression is one of the applications of these functions [10 – 12]. Discrete atomic compression (DAC) of digital images, which was presented in these papers, is based on the following classic compression scheme: discrete transform → quantization → encoding. In Fig. 1, discrete atomic compression of full color digital images is shown.

Application of the quantization procedure provides the possibility of compression, as well as quality loss,

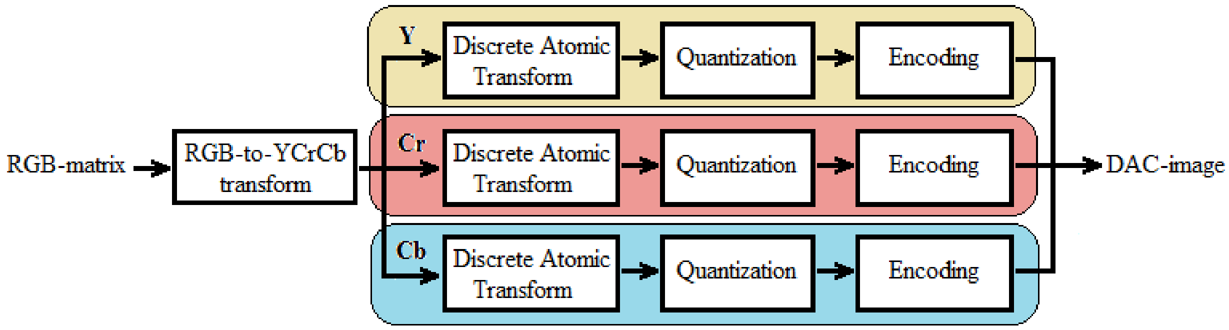

Fig. 1. Discrete atomic compression of full color digital image

which can be measured using various metrics. The degree of acceptability of these data changes depends directly on the area of application of the compression algorithm. Obviously, the following question is of interest: what parameters of the compression algorithm should be used to provide loss of quality that is acceptable in the sense of some given metric?

It is clear that search for a mechanism, which can be applied to management of quality loss, is not trivial. Among the papers focused on this problem, we note [13 – 15]. Most of them are concentrated on providing a desired mean square error or peak signal-to-noise ratio. However, the use of other criteria is possible. In particular, preservation of spectral signatures is important in near-lossless compression of multichannel remote sensing data [16] and minimal distortions of diagnostically valuable information is acceptable in lossy compression of medical images [17].

The aim of this paper is to obtain a technique for managing of quality loss that appears when using the DAC algorithm of digital images.

1. Formulation of the problem

Consider the so-called uniform metric or maximum absolute deviation (MAD)

$$\mathrm{MAD} = \max_{i=1,2,\ldots,n} |x_i - y_i|,$$

where $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$ are the source data and the reconstructed data after their processing respectively.

The main feature of this metric is its extremely high sensitivity to any local changes. This means that if at least one element x_k is different from the corresponding value of y_k, then this immediately affects the MAD-metric. Although a high value of MAD does not mean high visual quality loss (see Fig. 2). Nevertheless, if this value is small, then difference between each pair x_j and y_j is also small, i.e. loss of quality is small.

The main task of this paper is to obtain estimates of quality loss measured by the metric MAD.

Discrete atomic transform, which is a core of the algorithm DAC, is a transform of the source data $\{d_1, d_2, \ldots, d_n\}$ to the set of atomic wavelets coefficients. In DAC, these coefficients are quantized and then encoded. It is at this stage that the main loss of quality appears. Therefore, to solve the current problem, atomic wavelets coefficients should be investigated and their estimates should be obtained.

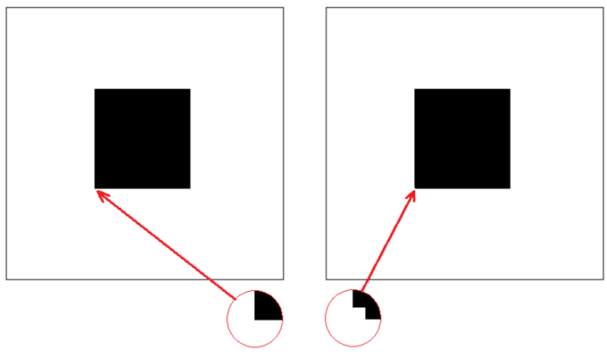

Fig. 2. Two 24-bit images: the white pixel instead of the black one in the lower left corner of the black square is the only difference between these images. Visually these images might seem identical, but $\mathrm{MAD} = 255$

2. Solution of the problem

2.1. Generalized atomic wavelets

Consider the set of functions $\{v_k(x)\}_{k=0}^{\infty}$ such that

$$v_k(x) = \frac{1}{2\pi}\int_{\mathbb{R}} e^{itx} V_k(t)\,dt,$$

where

$$V_k(t) = \frac{\sin\left(\frac{2^k t}{N}\right)}{\frac{2^k t}{N}} \cdot \prod_{j=0}^{k-1} \cos\left(\frac{2^j t}{N}\right) \cdot F\left(\frac{t}{N}\right),$$

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$N \neq 0$ and $F(t)$ is the Fourier transform of the function $f(x) \in C(\mathbb{R})$ that combines the following properties:

1) $\operatorname{supp} f(x) = [-1,1]$;

2) $f(x)$ is an even function;

3) $f(x) > 0$ if $|x| < 1$;

4) $\int_{\mathbb{R}} f(x)dx = 1$.

Here, $\operatorname{supp} f(x)$ is a support of the function $f(x)$, i.e. $\operatorname{supp} f(x) = \overline{\{x: f(x) \neq 0\}}$.

It is clear that

$$\operatorname{supp} v_k(x) = \left[-\frac{2^{k+1}}{N}, \frac{2^{k+1}}{N}\right]. \quad (1)$$

In the paper [4], generalized atomic wavelets $\{w_k(x)\}$ based on these functions were constructed:

$$w_k(x) = \sum_{j=1}^{5} c_{k,j} v_{k-1}\left(x - \frac{2^k j}{N}\right), \quad (2)$$

where $c_{k,1} = c_{k,5} = -b_{k-1}$, $c_2 = c_4 = a_{k-1} + 2b_{k-1}$, $c_3 = -2(a_{k-1} + b_{k-1})$ and

$$a_{k-1} = \int_{\mathbb{R}} v_{k-1}^2(x)dx,$$

$$b_{k-1} = \int_{\mathbb{R}} v_{k-1}(x) v_{k-1}\left(x - \frac{2^k}{N}\right)dx.$$

The main properties of $w_k(x)$ are

1) compactness of the support:

$$\operatorname{supp} w_k(x) = \left[0; \frac{6 \cdot 2^k}{N}\right];$$

2) smoothness, the order of which depends on the choice of the function $f(x)$.

In DAC, atomic wavelets constructed using the functions $up_{2^m}(x)$ are applied. We note that these functions are infinitely differentiable and non-analytic [18]. These features are important in processing of digital images with smooth color changes. Moreover, non-analyticity means that atomic wavelets are less smooth than trigonometric polynomials $\{\cos(nx), \sin(nx)\}$. Hence, the following hypothesis is plausible: atomic wavelets provide better compression of images with contrast changes of color than trigonometric polynomials. Although this statement requires careful analysis and verification.

The system of functions

$$\left\{w_k\left(x - \frac{2^{k+1} j}{N}\right), v_n\left(x - \frac{2^{n+1} j}{N}\right)\right\}_{k=1,\ldots,n;\ j \in \mathbb{Z}}$$

constitutes a basis of the space

$$L = \left\{f(x): f(x) = \sum_j c_j v_0\left(x - \frac{2j}{N}\right)\right\}.$$

In terms of the A.N. Kolmogorov width, this space has good approximation properties [19]. Largely due to this very property, DAC provides good compression with insignificant loss of quality [10 – 12].

Further, we consider some data presented by the function

$$d(x) = \sum_{k=1}^{n} \sum_j \omega_{k,j} w_k\left(x - \frac{2^{k+1} j}{N}\right) + \sum_j \upsilon_j v_n\left(x - \frac{2^{n+1} j}{N}\right). \quad (3)$$

In Subsection 2.3, we obtain estimates of the coefficients $\{\omega_{k,j}; \upsilon_j\}$ that can be used to study the effect of quantization of these coefficients on quality loss in the sense of the MAD-metric. For this purpose we need some properties of the functions $v_k(x),\ k = 0,1,2,\ldots$.

2.2. Properties of $v_k(x)$

For any $k = 0,1,2,\ldots$ the following holds:

1) $v_k(x) = 0$ if $|x| \geq \frac{2^{k+1}}{N}$;

2) $v_k(x) > 0$ if $|x| < \frac{2^{k+1}}{N}$ and $v_k(x)$ is an even function;

3) $\int_{\mathbb{R}} v_k(x)dx = 1$;

4) $\sum_{j \in \mathbb{Z}} v_k\left(x - \frac{2^{k+1} j}{N}\right) \equiv \frac{N}{2^{k+1}}$ and $v_k(0) = \frac{N}{2^{k+1}}$;

5) $a_k + 2b_k = \frac{N}{2^{k+1}}$.

The properties 1) and 2) are satisfied by the construction of $v_k(x)$.

Further, since $V_k(0) = 1$, we get the property 3).

Also, $v'_k(-x) = -v'_k(x)$. Combining this with 1) and 2), we obtain $\sum_{j \in \mathbb{Z}} v'_k\left(x - \frac{2^{k+1} j}{N}\right) = 0$ for any $x \in \mathbb{R}$.

Therefore, $\sum_{j \in \mathbb{Z}} v_k\left(x - \frac{2^{k+1} j}{N}\right) \equiv c$, where c is some constant. If we combine this equality with the property 1), we get $v_k(x) + v_k\left(x - \frac{2^{k+1}}{N}\right) \equiv c$ on the closed interval $\left[0, \frac{2^{k+1}}{N}\right]$. By integrating the left part of this equality, we obtain

$$\int_0^{\frac{2^{k+1}}{N}} \left(v_k(x) + v_k\left(x - \frac{2^{k+1}}{N}\right)\right)dx = \int_{-\frac{2^{k+1}}{N}}^{\frac{2^{k+1}}{N}} v_k(x)dx = 1.$$

If we equate this to integral of the right part, we see that $c\cdot\frac{2^{k+1}}{N}=1$. Hence, $c=\frac{N}{2^{k+1}}$. This completes the proof of the property 4).

In particular, we have shown that

$$v_k\left(x-\frac{2^{k+1}j}{N}\right)+v_k\left(x-\frac{2^{k+1}(j+1)}{N}\right)\equiv\frac{N}{2^{k+1}}$$

for any $x\in\left[\frac{2^{k+1}j}{N},\frac{2^{k+1}(j+1)}{N}\right]$ and $j\in\mathbb{Z}$. Besides, if $j=0$ and $x=0$, then $v_k(0)=\frac{N}{2^{k+1}}$.

Finally, the last property can be proved as follows:

$$a_k+2b_k=2\int_0^{\frac{2^{k+1}}{N}}\left(v_k^2(x)+v_k(x)\cdot v_k\left(x-\frac{2^{k+1}}{N}\right)\right)dx=$$

$$=2\int_0^{\frac{2^{k+1}}{N}}v_k(x)\underbrace{\left(v_k(x)+v_k\left(x-\frac{2^{k+1}}{N}\right)\right)}_{\frac{N}{2^{k+1}}}dx=$$

$$=\frac{N}{2^{k+1}}\cdot 2\int_0^{\frac{2^{k+1}}{N}}v_k(x)dx=\frac{N}{2^{k+1}}\int_{\mathbb{R}}v_k(x)dx=\frac{N}{2^{k+1}}.$$

Of course, not all properties of $v_k(x)$ are considered here. The probabilistic properties of these functions are of particular interest, but this is a topic for another research.

2.3. Estimates of wavelet coefficients

Expansion (3) can be expressed as follows:

$$d(x)=\sum_{k=1}^{n}\ell_k(x)+m(x),$$

where

$$\ell_k(x)=\sum_j\omega_{k,j}w_k\left(x-\frac{2^{k+1}j}{N}\right)$$

and

$$m(x)=\sum_j\upsilon_j v_n\left(x-\frac{2^{n+1}j}{N}\right).$$

Here, $m(x)$ is the main value or trend of the data $d(x)$. We note that the graph of this function looks like a small copy of the graph of $d(x)$. In other words, the set of coefficients $\{\upsilon_j\}$ describes a small copy of the source data.

We call n **the depth** of generalized atomic wavelets decomposition. The function $\ell_k(x)$ is said to be its **k-th level**.

Let $E_k=\{\sigma_{k,j}:\ j\in\mathbb{Z}\}$ be bounded set of real numbers for any $k=1,2,\ldots,n+1$.

Consider $\varepsilon_k=\sup_{j\in\mathbb{Z}}\left|\sigma_{k,j}\right|$.

Let

$$m(x)=\sum_j\upsilon_j v_n\left(x-\frac{2^{n+1}j}{N}\right),$$

where $\upsilon_j=\upsilon_j+\sigma_{n+1,j}$.

It follows from the properties of the function $v_n(x)$ that for any $p\in\mathbb{Z}$ and $x\in\left[\frac{2^{n+1}p}{N},\frac{2^{n+1}(p+1)}{N}\right)$ the following holds:

$$\left|m(x)-m(x)\right|=\left|\sum_j\sigma_{n+1,j}v_n\left(x-\frac{2^{n+1}j}{N}\right)\right|=$$

$$=\left|\sigma_{n+1,p}v_n\left(x-\frac{2^{n+1}p}{N}\right)+\sigma_{n+1,p+1}v_n\left(x-\frac{2^{n+1}(p+1)}{N}\right)\right|\le$$

$$\le\varepsilon_{n+1}\left(v_n\left(x-\frac{2^{n+1}p}{N}\right)+v_n\left(x-\frac{2^{n+1}(p+1)}{N}\right)\right)=\frac{\varepsilon_{n+1}N}{2^{n+1}}.$$

Using the notation $\delta_{n+1}=\frac{\varepsilon_{n+1}N}{2^{n+1}}$, we get

$$\max_{x\in\mathbb{R}}\left|m(x)-m(x)\right|\le\delta_{n+1}. \qquad (5)$$

Hence, if ε_{n+1} is the greatest lower bound of the absolute calculation error of $\{\upsilon_j\}$,then the maximum absolute deviation of the function m(x) from $m(x)$ does not exceed δ_{n+1}.

Consider any $k=1,2,\ldots,n$.

Let

$$\tilde{\ell}_k(x)=\sum_j\omega_{k,j}w_k\left(x-\frac{2^{k+1}j}{N}\right),$$

where $\omega_{k,j}=\omega_{k,j}+\sigma_{k,j}$.

Then for each $p\in\mathbb{Z}$ and $x\in\left[\frac{2^k p}{N},\frac{2^k(p+1)}{N}\right)$ we get

$$\left|\tilde{\ell}_k(x)-\ell_k(x)\right|\le\varepsilon_k\tfrac{1}{2}\left(a_{k-1}+2b_{k-1}\right)\times$$

$$\times\left(v_{k-1}\left(x-\frac{2^k p}{N}\right)+v_{k-1}\left(x-\frac{2^k(p+1)}{N}\right)\right)=\varepsilon_k\frac{N^2}{2^{2k+1}}=\delta_k.$$

This implies that

$$\max_{x\in\mathbb{R}}\left|\tilde{\ell}_k(x)-\ell_k(x)\right|\le\delta_k. \qquad (6)$$

Combining this with (5), we obtain that the maximum absolute deviation of the function d(x) from

$$d(x)=\sum_{k=1}^{n}\tilde{\ell}_k(x)+m(x)$$

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can be estimated as follows:

$$\max_{x\in\mathbb{R}}\left|d(x)-d(x)\right| \le \sum_{k=1}^{n+1}\delta_k \;. \qquad (7)$$

This means that the maximum absolute deviation of the source data from the data, which were obtained with some errors, does not exceed the sum of errors of all levels.

Note that the estimate (7) significantly depends on the depth of decomposition and not just on the errors on each of level.

In practice, the data function d(x) is usually considered on the bounded subset of $\mathbb{R}$. This yields that only finite set of wavelet coefficients is used. Therefore, $\varepsilon_k = \max_j\left|\sigma_{k,j}\right|$.

2.4. Quantization of wavelet coefficients

The relations presented in the previous subsection can be used as the basis of the mechanism for managing quality losses that occur during the quantization of wavelet coefficients.

Let $\{\delta_1, \delta_2, ..., \delta_{n+1}\}$ be a set of levels errors.

Consider the following wavelet coefficients quantization approach:

$$\xi_{k,j} = \mathrm{Round}\left(\omega_{k,j}\cdot\frac{N^2}{\delta_k 2^{2k+1}}\right) \qquad (8)$$

for any $k = 1, 2, ..., n$, $j \in \mathbb{Z}$ and

$$\eta_j = \mathrm{Round}\left(\upsilon_j\cdot\frac{N}{\delta_{n+1}2^{n+1}}\right) \qquad (9)$$

for each $j \in \mathbb{Z}$.

These values can be used to reconstruct wavelet coefficients as follows:

$$\omega_{k,j} = \xi_{k,j}\cdot\frac{\delta_k 2^{2k+1}}{N^2}, \qquad (10)$$

$$\upsilon_j = \eta_j\cdot\frac{\delta_{n+1}2^{n+1}}{N}. \qquad (11)$$

Then the estimate (7) is satisfied, i.e.

$$\mathrm{MAD} \le \delta_1 + \delta_2 + ... + \delta_{n+1}. \qquad (12)$$

We note that this relation is an upper bound and the value of MAD can be significantly less than the sum of levels errors.

A similar approach can be used in the multidimensional case. The main difference is described below.

One-dimensional data d(x) is presented by one-dimensional array of wavelets coefficients (see Fig. 3).

$$d(x) \longleftrightarrow \begin{array}{ccccc} \ell_1(x) & \ell_2(x) & & \ell_n(x) & m(x) \\ \boxed{\omega_{1,j}} & \boxed{\omega_{2,j}} & \cdots & \boxed{\omega_{n,j}} & \boxed{\upsilon_j} \\ \uparrow & \uparrow & & \uparrow & \uparrow \\ \delta_1 & \delta_2 & & \delta_n & \delta_{n+1} \end{array}$$

Fig. 3. One-dimensional data presentation

Description of two-dimensional data d(x,y) is provided by the matrix of blocks B_{ij} corresponding to wavelet levels with respect to x and y (see Fig. 4).

$$d(x,y) \longleftrightarrow \begin{pmatrix} & \vdots & \\ \cdots & \boxed{B_{ij}} & \cdots \\ & \vdots & \end{pmatrix}, \text{ where } \overset{\ell_{ij}(x,y)}{\boxed{B_{ij}}}$$

Fig. 4. Two-dimensional data presentation

Each element of the block B_{ij} is quantized using formulas that are similar to (8) and (9). In this case, we get

$$\mathrm{MAD} \le \sum_{i=1}^{n+1}\sum_{j=1}^{p+1}\delta_{ij},$$

where δ_{ij} is an error of block B_{ij}, n and p are depths of wavelet decomposition with respect to each variable. We call the right part of this inequality an upper bound of maximum absolute deviation and denote it by UBMAD. In this notation, we obtain

$$\mathrm{MAD} \le \mathrm{UBMAD}, \qquad (13)$$

where

$$\mathrm{UBMAD} = \sum_{i=1}^{n+1}\sum_{j=1}^{p+1}\delta_{ij}. \qquad (14)$$

We see that using the quantization method described above, we guarantee that quality loss measured by MAD does not exceed the predetermined value.

In discrete atomic compression of full color digital images, the DAT-procedure is applied to processing of the matrices Y, Cr and Cb. Obviously, wavelet coefficients, which are obtained at this stage, describe Y, Cr and Cb, but not the RGB-matrix of the source image. Nevertheless, if we apply the proposed approach to quantization, then we get the following estimates:

$$\mathrm{MAD}^{[Y]} \le \sum_{i=1}^{n+1}\sum_{j=1}^{p+1}\delta_{ij}^{[Y]},$$

$$\mathrm{MAD}^{[Cr]} \le \sum_{i=1}^{n+1}\sum_{j=1}^{p+1}\delta_{ij}^{[Cb]},$$

$$\mathrm{MAD}^{[Cb]} \le \sum_{i=1}^{n+1}\sum_{j=1}^{p+1}\delta_{ij}^{[Cb]},$$

where $MAD^{[Y]}, MAD^{[Cr]}, MAD^{[Cb]}$ are maximum absolute deviations of Y, Cr, Cb from the reconstructed after quantization matrices $\tilde{Y}, \tilde{C}r, \tilde{C}b$ respectively. Here, $\delta_{ij}^{[Y]}, \delta_{ij}^{[Cr]}, \delta_{ij}^{[Cb]}$ are the corresponding block errors. It can be easily shown that maximum absolute deviation of RGB-matrix A of the source image from RGB-matrix $\tilde{A}$ of the reconstructed image satisfies the following inequality:

$$MAD \le UBMAD, \tag{15}$$

where

$$UBMAD = \max\left\{\sum_{i,j}\delta_{ij}^{[Y]}, \sum_{i,j}\delta_{ij}^{[Cr]}, \sum_{i,j}\delta_{ij}^{[Cb]}\right\}. \tag{16}$$

We note that

$$MAD = \max_{i,j}\left\{\left|a_{ij}^{[R]} - \tilde{a}_{ij}^{[R]}\right|, \left|a_{ij}^{[G]} - \tilde{a}_{ij}^{[G]}\right|, \left|a_{ij}^{[B]} - \tilde{a}_{ij}^{[B]}\right|\right\},$$

where $a_{ij} = \left(a_{ij}^{[R]}, a_{ij}^{[G]}, a_{ij}^{[B]}\right)$ and $\tilde{a}_{ij} = \left(\tilde{a}_{ij}^{[R]}, \tilde{a}_{ij}^{[G]}, \tilde{a}_{ij}^{[B]}\right)$ are elements of RGB-matrices A and $\tilde{A}$ respectively.

2.5. Experiments

In this subsection, we illustrate the approach proposed above. Atomic wavelets constructed using the function $up_{32}(x)$ are applied in discrete atomic compression of grayscale test images "Baboon", "Boats", "Frisco", "Lenna" and their full color versions (see Fig. 5, 6). These images were downloaded from the USC-SIPI images database [20].

The computer program "Discrete Atomic Compression: Research Kit" is applied [21].

In the current experiments, we consider two cases $n = p = 1$ and $n = p = 5$, where n, p are depths of wavelet expansions. The value of UBMAD, which is defined by (14) and (16) in the cases of grayscale test images processing and full color test images processing respectively, is varied. In a similar way, other values of n and p can be used. Actually, each of these parameters can be any natural number.

Test images processing results are presented in Tables 1 – 4. In these Tables, values of root mean square (RMS) and peak signal to noise ratio (PSNR) are given. Also, dependence of MAD-metric on UBMAD is visualized in Figures 7 – 10 (note that values of UBMAD are given on the x-axis).

2.6. Discussion of the results

Analyzing the results, we see the following:

1. Upper estimates (13) and (15) are confirmed by DAC-processing of test images.

2. Equality between MAD and UBMAD is achieved only in few cases (see Table 2: UBMAD = 4, test images "Boats" and "Lenna"; UBMAD = 7, test image "Lenna"). In all other cases, $MAD < UBMAD$.

a b c d

Fig. 5. Grayscale test images: a – "Baboon", b – "Boats", c – "Frisco", d – "Lenna"

a b c d

Fig. 6. Full color test images: a – "Baboon", b – "Boats", c – "Frisco", d – "Lenna"

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Table 1

DAC-processing of grayscale test images: n = p = 1

UBMAD	**Baboon**			**Boats**			**Frisco**			**Lenna**		
	MAD	RMS	PSNR	MAD	RMS	PSNR	MAD	RMS	PSNR	MAD	RMS	PSNR
4	2	0,703	51,2	2	0,697	51,27	2	0,686	51,4	2	0,699	51,244
5	2	0,739	50,75	2	0,734	50,82	2	0,689	51,36	2	0,734	50,817
7	3	0,891	49,13	3	0,885	49,2	3	0,784	50,25	3	0,884	49,204
8	3	0,964	48,45	3	0,961	48,47	3	0,864	49,4	3	0,96	48,488
9	4	1,061	47,62	4	1,058	47,64	4	0,876	49,28	4	1,047	47,73
11	5	1,236	46,29	5	1,235	46,3	4	0,967	48,43	5	1,226	46,362
12	5	1,33	45,65	5	1,325	45,69	5	1,096	47,34	5	1,318	45,731
15	6	1,591	44,1	6	1,585	44,13	5	1,179	46,7	6	1,56	44,267
19	7	1,982	42,19	7	1,98	42,2	8	1,393	45,25	8	1,932	42,413
23	10	2,365	40,65	9	2,356	40,69	8	1,616	43,96	9	2,227	41,177
27	11	2,762	39,31	10	2,76	39,31	10	1,836	42,85	10	2,513	40,127
31	13	3,14	38,19	12	3,136	38,2	12	2,088	41,74	12	2,781	39,246
35	14	3,531	37,17	14	3,534	37,17	13	2,289	40,94	13	3,024	38,518
39	15	3,903	36,3	16	3,908	36,29	13	2,596	39,85	15	3,273	37,831
43	17	4,276	35,51	17	4,32	35,42	16	2,751	39,34	18	3,502	37,245
47	18	4,64	34,8	19	4,697	34,69	16	3,016	38,54	18	3,726	36,706
51	20	4,999	34,15	21	5,084	34,01	17	3,149	38,17	18	3,956	36,186
59	24	5,703	33,01	23	5,843	32,8	23	4,097	35,88	24	4,401	35,259

Table 2

DAC-processing of full color test images: n = p = 1

UBMAD	**Baboon**			**Boats**			**Frisco**			**Lenna**		
	MAD	RMS	PSNR	MAD	RMS	PSNR	MAD	RMS	PSNR	MAD	RMS	PSNR
4	3	0,733	50,83	4	0,716	51,03	3	0,731	50,85	4	0,734	50,81
5	4	0,912	48,93	4	0,836	49,68	4	0,909	48,96	4	0,913	48,92
7	6	1,229	46,34	6	1,123	47,13	6	1,226	46,36	7	1,229	46,34
8	7	1,381	45,32	7	1,282	45,97	7	1,378	45,35	7	1,381	45,32
9	8	1,548	44,33	7	1,345	45,55	8	1,54	44,38	8	1,539	44,38
11	9	1,878	42,66	9	1,596	44,07	9	1,858	42,75	10	1,861	42,74
12	10	2,047	41,91	10	1,785	43,1	10	2,026	42	10	2,03	41,98
15	13	2,541	40,03	13	2,021	42,02	12	2,438	40,39	12	2,47	40,28
19	16	3,204	38,02	14	2,414	40,47	16	2,96	38,7	16	3,016	38,54
23	19	3,863	36,39	15	2,793	39,21	19	3,424	37,44	18	3,498	37,26
27	26	4,517	35,03	18	3,163	38,13	21	3,877	36,36	21	3,911	36,28
31	28	5,146	33,9	21	3,568	37,08	25	4,289	35,48	24	4,307	35,45
35	30	5,764	32,92	23	3,977	36,14	28	4,61	34,86	29	4,685	34,72
39	33	6,362	32,06	26	4,379	35,3	32	5,076	34,02	33	5,042	34,08
43	38	6,946	31,3	27	4,76	34,58	35	5,355	33,56	31	5,382	33,51
47	42	7,505	30,62	33	5,164	33,87	35	5,979	32,6	34	5,736	32,96
51	44	8,066	30	29	5,6	33,17	39	6,056	32,49	38	6,156	32,34
59	48	9,099	28,95	34	6,365	32,06	45	7,119	31,08	44	6,908	31,34

Table 3

DAC-processing of grayscale test images: $n = p = 5$

UBMAD	Baboon			Boats			Frisco			Lenna		
	MAD	RMS	PSNR	MAD	RMS	PSNR	MAD	RMS	PSNR	MAD	RMS	PSNR
36	4	0,908	48,97	5	0,915	48,9	5	0,881	49,23	4	0,914	48,91
39	4	1,019	47,96	5	1,025	47,91	4	0,951	48,57	5	1,024	47,92
46	5	1,136	47,03	6	1,14	46,99	6	1,053	47,68	6	1,139	47
57	6	1,317	45,74	6	1,32	45,72	7	1,17	46,76	6	1,304	45,82
66	7	1,422	45,07	7	1,426	45,05	7	1,281	45,98	7	1,409	45,15
71	7	1,487	44,68	8	1,492	44,65	7	1,352	45,51	8	1,475	44,76
77	8	1,752	43,26	10	1,757	43,24	8	1,437	44,98	8	1,729	43,37
91	12	2,017	42,04	11	2,021	42,02	9	1,62	43,94	10	1,974	42,22
113	12	2,408	40,5	12	2,405	40,51	12	1,846	42,81	13	2,232	41,16
125	14	3,033	38,49	14	3,027	38,51	13	1,97	42,24	14	2,664	39,62
161	21	4,298	35,47	20	4,389	35,28	17	2,286	40,95	17	3,204	38,02
189	24	4,513	35,04	21	4,597	34,88	20	2,585	39,88	19	3,461	37,35

Table 4

DAC-processing of full color test images: $n = p = 5$

UBMAD	Baboon			Boats			Frisco			Lenna		
	MAD	RMS	PSNR	MAD	RMS	PSNR	MAD	RMS	PSNR	MAD	RMS	PSNR
36	8	1,216	46,43	8	1,171	46,76	7	1,213	46,46	8	1,221	46,39
39	11	1,446	44,93	8	1,345	45,56	9	1,439	44,97	8	1,448	44,92
46	10	1,671	43,67	11	1,542	44,37	10	1,661	43,72	10	1,67	43,68
57	12	2,008	42,07	11	1,778	43,13	12	1,925	42,44	12	1,956	42,3
66	14	2,207	41,26	14	1,95	42,33	13	2,086	41,74	13	2,14	41,52
71	15	2,326	40,8	13	2,071	41,81	15	2,197	41,3	15	2,261	41,05
77	16	2,8	39,19	15	2,278	40,98	16	2,645	39,68	16	2,704	39,49
91	20	3,259	37,87	19	2,586	39,88	21	3,016	38,54	19	3,093	38,32
113	25	3,91	36,29	22	2,923	38,81	23	3,35	37,63	21	3,425	37,44
125	28	4,947	34,24	25	3,115	38,26	26	3,94	36,22	25	4,005	36,08
161	37	6,88	31,38	28	3,521	37,2	44	4,643	34,79	34	4,608	34,86
189	43	7,253	30,92	30	4,004	36,08	45	4,975	34,19	34	5,018	34,12

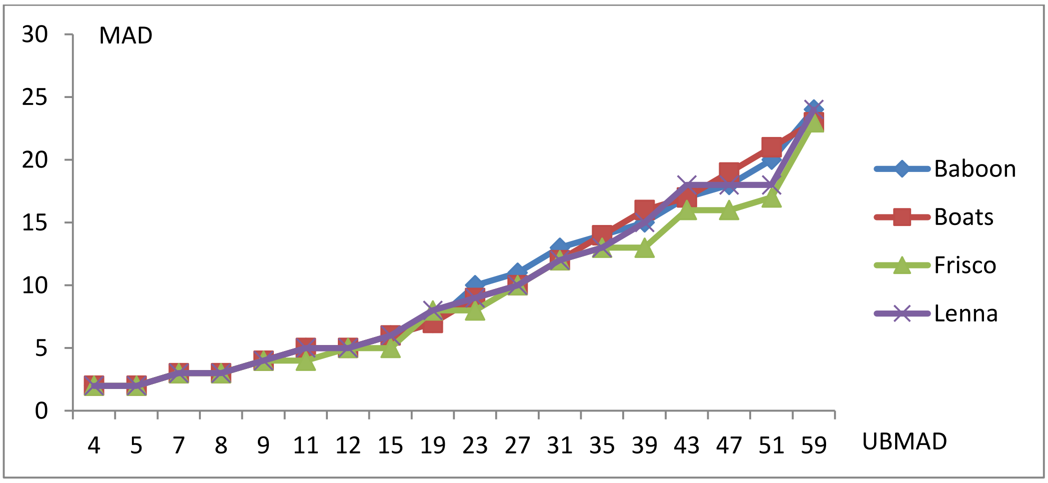

Fig. 7. Dependence of MAD-metric on UBMAD: grayscale test images, $n = p = 1$

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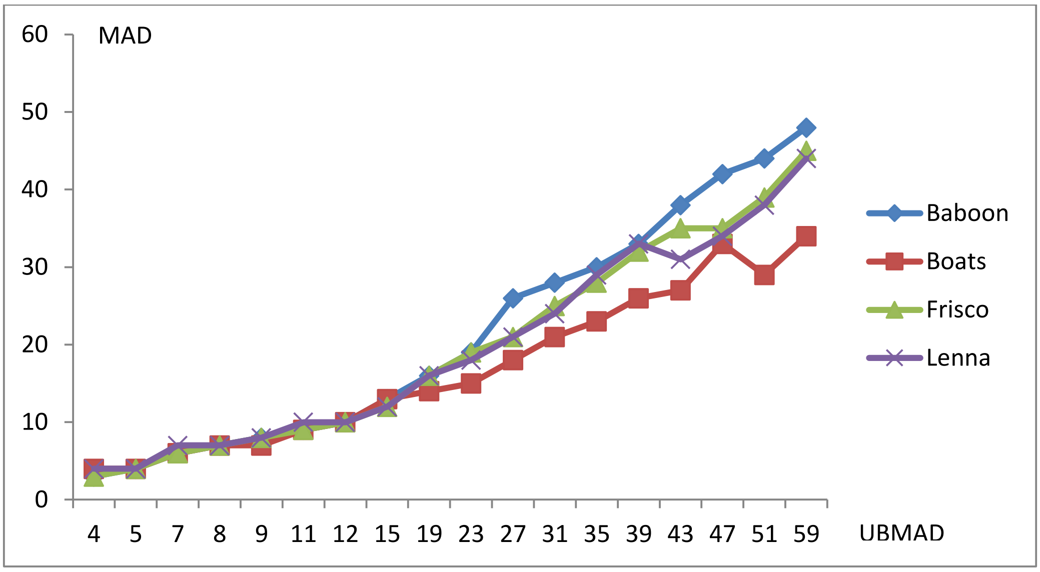

Fig. 8. Dependence of MAD-metric on UBMAD: full color test images, n = p = 1

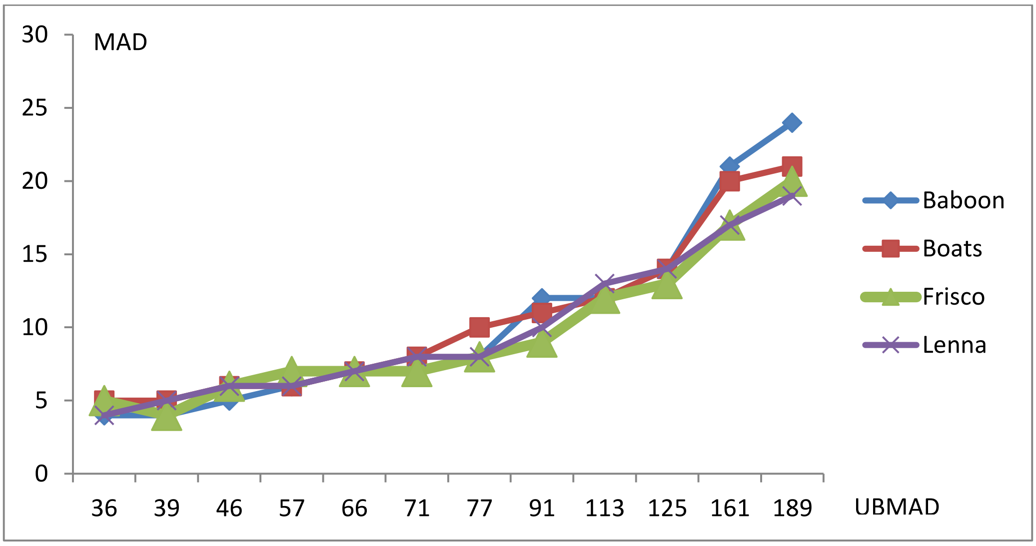

Fig. 9. Dependence of MAD-metric on UBMAD: grayscale test images, n = p = 5

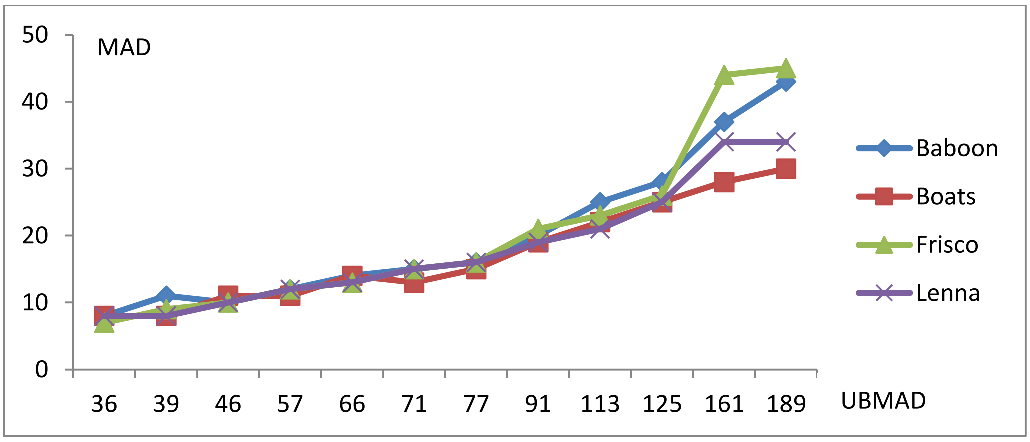

Fig. 10. Dependence of MAD-metric on UBMAD: full color test images, n = p = 5

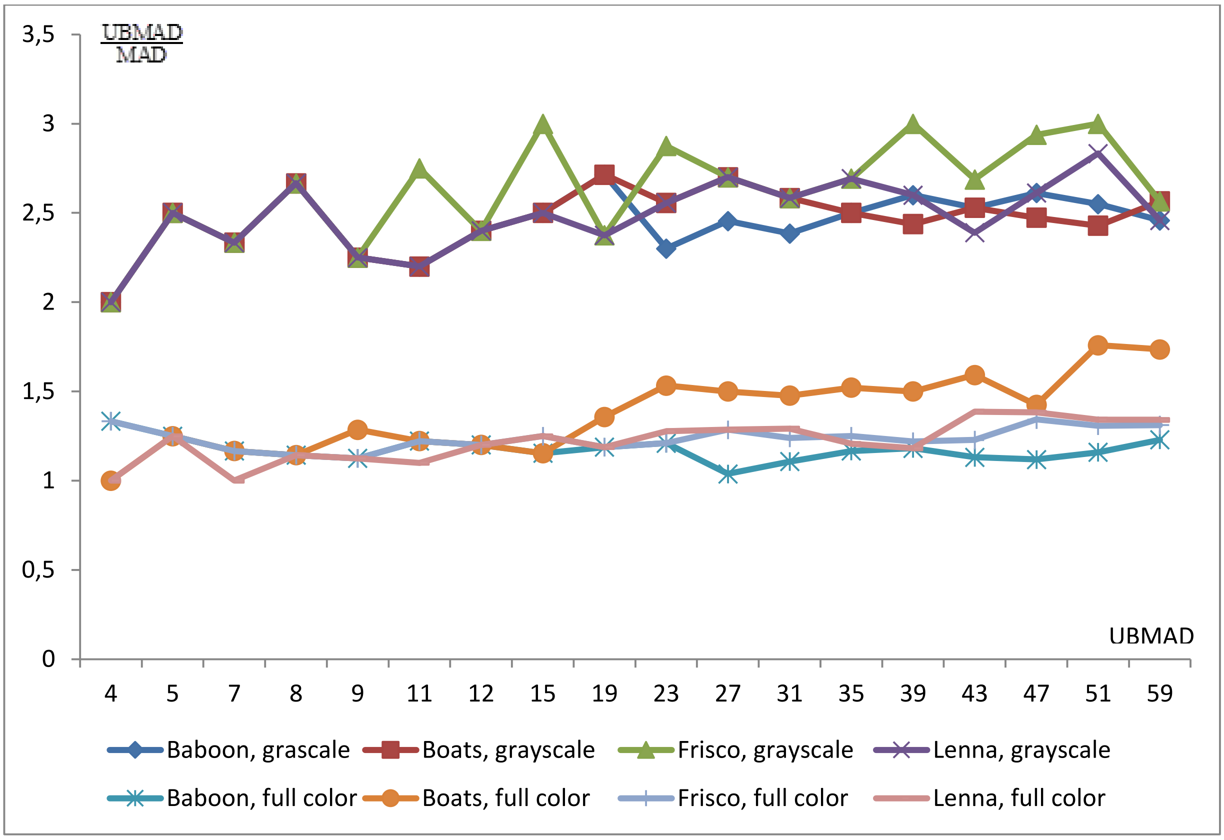

Fig. 11. Graphs of the ratio $\frac{\text{UBMAD}}{\text{MAD}}$ for the case $n = p = 1$

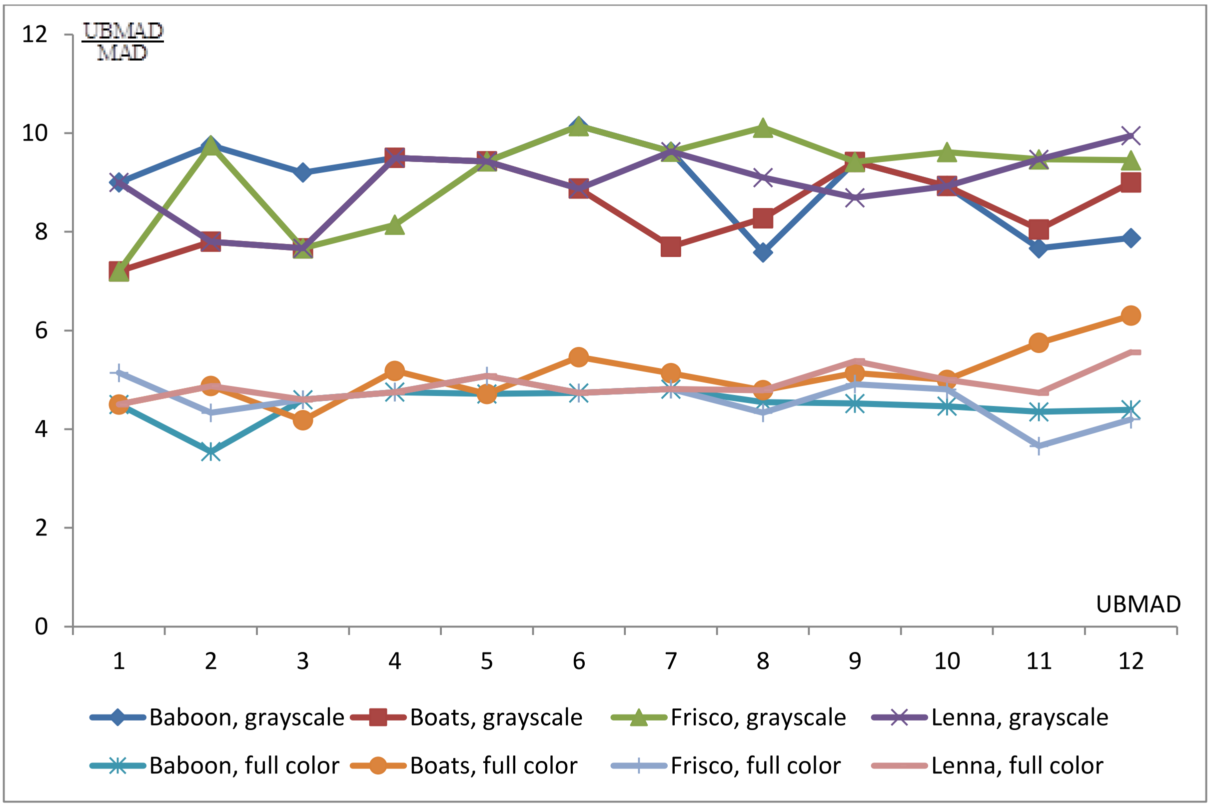

Fig. 12. Graphs of the ratio $\frac{\text{UBMAD}}{\text{MAD}}$ for the case $n = p = 5$

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This is primarily due to how this estimate was obtained. It is obvious that sometimes the difference between the left and right parts of the inequality

$$|a+b| \le |a| + |b|$$

is quite significant. In the proof of (13) an (15), replacing of $|a+b|$ with $|a|+|b|$ is used.

This is the reason for the inaccuracy of the estimates. Note also that such a fundamental property of the applied functions as the compactness plays an important role. It is clear that

$$\left|\sum_j \sigma_{n+1,j} v_n\left(x - \frac{2^{n+1} j}{N}\right)\right| \le \sum_j \left|\sigma_{n+1,j} v_n\left(x - \frac{2^{n+1} j}{N}\right)\right|.$$

Nevertheless, compactness of the function $v_n(x)$ provides more accurate estimation

$$\left|\sum_j \sigma_{n+1,j} v_n\left(x - \frac{2^{n+1} j}{N}\right)\right| \le$$

$$\le \left|\sigma_{n+1,p} v_n\left(x - \frac{2^{n+1} p}{N}\right)\right| + \left|\sigma_{n+1,p+1} v_n\left(x - \frac{2^{n+1}(p+1)}{N}\right)\right|,$$

where p is such integer that $x \in \left[\frac{2^{n+1} p}{N}, \frac{2^{n+1}(p+1)}{N}\right)$.

3. The maximum difference between MAD and UBMAD is observed for in the case of DAC-processing of full color test images and $n = p = 5$. DAC-processing with $n = p = 1$ of full color test images provides the minimum difference between MAD and UBMAD. This means that depths n and p of the applied wavelet expansion affect the accuracy of the obtained estimates.

4. The following inequality is satisfied:

$$MAD \le c \cdot UBMAD,$$

where c is a correction value, which actually depends on a number of factors. From Tables 1, 3 and 4, it follows that

$$MAD \le \tfrac{1}{2} UBMAD$$

in the case of processing grayscale test images and $n = p = 1$;

$$MAD \le \tfrac{5}{36} UBMAD$$

in the case of processing grayscale test images and $n = p = 5$;

$$MAD \le \tfrac{11}{39} UBMAD$$

in the case of processing full color test images and $n = p = 5$. Although the search for correction value requires a detailed investigation.

Hence, the following approach can be applied.

Denote by $MAD^{[desired]}$ the desired loss of quality measured by the MAD-metric.

Let

$$UBMAD = \frac{MAD^{[desired]}}{c},$$

where c is a correction value defined in Table 5.

Table 5

Values of correction value in the case $n = p$

n, p	Type of image	
	grayscale	full color
1	$\frac{1}{2}$	1
5	$\frac{5}{36}$	$\frac{11}{39}$

This implies that the inequality

$$MAD \le MAD^{[desired]}$$

is satisfied. In other words, quality, which is not worse than required, is guaranteed.

5. In all cases, $\frac{UBMAD}{MAD} \approx const$, i.e. this ratio behaves relatively constant (see Fig. 11, 12). It is not hard to show that $\frac{UBMAD}{RMS}$ and $\frac{UBMAD}{PSNR}$ do not have this property. This means that dependence of quality loss measured by RMS and PSNR on UBMAD is more complex.

Conclusions

Quantization mechanism, which guarantees that quality loss measured by MAD-metric do not exceed the given value UBMAD, is the main result of this paper. It is based on the obtained upper estimates of coefficients of the generalized atomic wavelet expansions. These results have been confirmed by a number of experiments. When processing full color test images using the algorithm DAC with wavelet expansions depths, which are equal to 1, the value of MAD is closest to the corresponding value of UBMAD. In other cases, accuracy of the estimates can be improved by applying of the correction coefficients. Thus, DAC provides the ability to obtain the desired loss of quality.

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ПРО ОЦІНКИ КОЕФІЦІЄНТІВ РОЗВИНЕНЬ ЗА УЗАГАЛЬНЕНИМИ АТОМАРНИМИ ВЕЙВЛЕТАМИ ТА ЇХ ЗАСТОСУВАННЯ В ОБРОБЦІ ДАНИХ

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У роботі розглянуто дискретне атомарне стиснення (ДАС) цифрових зображень. Цей алгоритм є алгоритмом стиснення з втратами якості. Основна мета даної роботи – отримати механізм управління втратами якості. Серед великої кількості метрик, що використовуються для оцінки втрат якості, обрано MAD-метрику (maximum absolute deviation). Важливою особливістю цієї метрики є дуже висока чутливість до будь-яких змін даних, що оброблюються. В алгоритмі ДАС основні втрати якості відбуваються під час квантування вейвлет-коефіцієнтів атомарних розвинень, що є **предметом** даного дослідження. **Метою** є дослідження впливу параметрів квантування на атомарні коефіцієнти. **Завдання:** отримати оцінки цих коефіцієнтів. У роботі використовуються **методи** теорії атомарних функцій та цифрової обробки зображень. З використанням властивостей узагальнених атомарних вейвлетів отримано оцінки коефіцієнтів розвинень за узагальненими атомарними вейвлетами. Ці нерівності подають залежність втрат якості від параметрів квантування у вигляді верхніх оцінок. Їх також підтверджено ДАС-обробкою тестових зображень. Окрім того, обчислено значення RMS (root mean square) та PSNR (peak signal to noise ratio). Аналізуючи результати експериментів, які було проведено за допомогою комп'ютерної програми "Discrete Atomic Compression: Research Kit", було отримано такі **результати**: 1) відхилення очікуваного значення метрики MAD від реального значення у деяких випадках є досить значним; 2) точність оцінок суттєво залежить від параметрів квантування, а також глибини розвинення за атомарними вейвлетами та типу зображення (повнокольорові або у градаціях сірого); 3) розбіжність можна зменшити за допомогою використання поправочного коефіцієнту; 4) відношення очікуваного значення MAD до його реального значення поводить себе відносно постійно, а відношення прогнозованого значення MAD до RMS та PSNR – ні. **Висновки**: дискретне атомарне стиснення цифрових зображень у поєднанні з запропонованим методом керування втратами якості надає можливість отримати результати потрібної якості, що робить його подальші дослідження та застосування перспективними.

Ключові слова: стиснення з втратами якості; дискретне атомарне стиснення; узагальнені атомарні вейвлети; максимальне абсолютне відхилення; управління втратами якості.

ОБ ОЦЕНКАХ КОЭФФИЦИЕНТОВ РАЗЛОЖЕНИЙ ПО ОБОБЩЕННЫМ АТОМАРНЫМ ВЕЙВЛЕТАМ И ИХ ПРИМЕНЕНИЕ В ОБРАБОТКЕ ДАННЫХ

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В роботе рассмотрено дискретное атомарное сжатие (ДАС) цифровых изображений. Этот алгоритм является алгоритмом сжатия с потерями качества. Основная цель данной работы – получить механизм управления потерями качества. Среди большого числа метрик, которые используются для оценки потерь качества, выбрана MAD-метрика (maximum absolute deviation). Важной особенностью этой метрики является ее высокая чувствительность к любым изменениям обрабатываемых данных. В алгоритме ДАС основные потери качества получаются в процессе квантования вейвлет-коэффициентов атомарных разложений, которые являются **предметом** данного исследования. **Целью** является исследование влияния параметров квантования на атомарные коэффициенты. **Задача** исследования: получить оценки этих коэффициентов. В работе используются **методы** теории атомарных функций и цифровой обработки изображений. С использованием свойств обобщенных атомарных вейвлетов получены оценки коэффициентов соответствующих разложений. Эти неравенства представляют зависимость в форме верхних оценок потерь качества от параметров квантования. Путем обработки тестовых изображений получено их экспериментальное подтверждение. Кроме того, найдены значения RMS (root mean square) и PSNR (peak signal to noise ratio). Анализируя результаты экспериментов, которые проводились при помощи компьютерной программы "Discrete Atomic Compression: Research Kit", были получены такие **результаты**: 1) отклонение ожидаемого значения MAD-метрики от реального в некоторых случаях является значительным; 2) точность оценок существенно зависит не только от параметров квантования, но и от глубины преобразования и типа обрабатываемого изображение (полноцветное или в градациях серого); 3) имеющиеся расхождения можно уменьшить с помощью поправочного вспомогательного коэффициента; 4) отношение ожидаемого значения MAD к его реальному значению вдет себя относительно постоянно. а отношение ожидаемого значения MAD к RMS и PSNR – нет. **Выводы**: дискретное атомарное сжатие цифровых изображений в сочетании с предложенным методом управления потерями качества позволяет получить результаты требуемого качества, что делает его дальнейшее развитие и применения перспективными.

Ключевые слова: сжатие с потерями качества; дискретное атомарное сжатие; обобщенные атомарные вейвлеты; максимальное абсолютное отклонение; управление потерями качества.

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