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PHYSICS FOR ENGINEERS

WAVE OPTICS

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Подано теоретичну інформацію з оптики – науки, що вивчає світло, яке є частиною спектра електромагнітного випромінювання.

Описано хвильову модель електромагнітного випромінювання, один з двох можливих підходів до аналізу оптичних явищ, що є важливим для розуміння загальних процесів в оптиці: поляризації, інтерференції й дифракції.

Висвітлено важливі практичні застосування фізичної оптики: інтерферометрію, дифракційні ґрати в прецизійних вимірюваннях, рентгенівську кристалографію й спектроскопію, голографію.

Наведено велику кількість практичних завдань та вправ, пов’язаних з розглянутими теоретичними питаннями. Важливі приклади містять розв’язки, які допомагають удосконалити вміння і навички читача.

Для студентів вищих технічних навчальних закладів, а також студентів, які навчаються за спеціальністю «Прикладна лінгвістика», при вивченні англійської мови (технічний переклад). Може бути корисним для студентів, що готуються до стажування в технічних університетах Європи й США, а також іноземних громадян, які навчаються в Україні.

Іл. 80. Табл. 26. Бібліогр.: 15 назв

The textbook contains theoretical information on optics, the science studying light, which is known to be a part of electromagnetic radiation spectrum.

Wave model of electromagnetic radiation as one of two possible approaches in optical phenomena analysis is important for understanding general optical processes such as polarization, interference, and diffraction.

Important practical applications of physical optics – interferometry, diffraction gatings, precise measuring, x-ray crystallography, spectroscopy, and holography are considered.

The book includes a wide range of questions, problems, exercises, connected with theoretical issues. Important examples with solutions help to improve the reader’s performance and skills.

The textbook is intended for the students of higher technical universities, as well as for those students who are trained in “Applied Linguistics” field, when learning English (technical translation). It may also be useful for those students who are preparing for their practical training at universities of Europe and the USA, as well as for the foreign citizens who are trained in Ukraine.

Illustrations 80. Tables 26. Bibliographical references: 15 names

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АБОНЕМЕНТ НАУКОВОЇ ЛІТЕРАТУРИ

CONTENTS

Preface

Physics is the most fundamental of all natural sciences. Following engineering disciplines are based on fundamental physical principles. That's why it is important, that students would understand and be able to apply the physical concepts and theories discussed in the textbook. It should be better for student to read carefully the textbook before attending lectures on the covered material to accomplish the understanding the basic concepts and principles perfectly.

The authors had attempted to present an interesting, helpful textbook. Our general goal is to provide a broad, rigorous introduction to fundamentals of optics. Because this textbook is intended for students majoring in science and engineering, the material covers fundamental topics in wave optics and provides an introduction to modern physics.

We emphasize on the physical principles and the development of problem-solving ability, rather than on historical background or specialized applications. The use of easily understandable language is at the core of our effort without sacrificing of the subject. In addition, it is important for the student to see relevant application of the material to everyday life.

This book is divided in four parts. Chapter 1 deals with interference, chapter 2 covers diffraction, chapter 3 – polarization and chapter 4 – interaction of light with substance.

The text contains a variety of aids to the students that should make their study of physical science more effective. These aids are included to help students clearly understand the concepts and principles that serve as the foundation of physical science. They are the following.

Introductory overview. Each chapter begins with an introductory overview. It helps students to organize their thoughts for the coming chapter materials.

Bold words. As students read each chapter they will notice that some words appear in bolds. They signify an important terms that students will need to understand and remember to fully comprehend the material in which these terms appear. The important terms are defined in text the first time they are used.

Italics words. Italics words are meant to emphasize their importance in understanding explanation of ideas and concepts discussed.

Solved examples. Most important types of problems are included as solved examples in order to clarify the concepts of the chapter. Each topic discussed contains one or more solved examples of varying difficulty to promote student's understanding of the concepts. In many cases, the examples serve as models for solving the end-of–chapter problems. Some of them serve to understand the relevance of physical science to conform many issues we face in our day-to-day life.

End-of–chapter exercises. At the end of each chapter students will find questions and exercises that can be performed at home or in the classroom to demonstrate important concepts and reinforce their understanding of them. There are two groups of exercises. One of them has answers in the text. The exercises of other group are similar to that of the first group and easier for solving, they don't contain answers in the text. More than 200 questions and exercises are included. Special attention has been paid for providing 80 illustrative figures.

Key terms. Each chapter contains a list of key terms that reviews the important concepts discussed.

Summary. Each chapter contains a summary that reviews the important concepts discussed.

SI units have been used. At the end of the book a lot of referenced data, which can be useful for solving problem, are placed.

Indexes. At the end of the textbook there is a list of all important terms that are page-referenced, where students can find the terms defined in text.

We welcome suggestions and comments from our users, especially the students and teachers. We wish our readers a great success in studying of physics science.

The authors

Introduction

The nature and properties of light has been a subject of great interest and speculation since ancient times. The Greeks believed that light consisted of tiny particles (corpuscles) that were emitted by a light source and that these particles stimulated the perception of vision upon striking the observer's eye. Newton used this corpuscular theory to explain the reflection and refraction of light. In 1678, one of Newton's contemporaries, the Dutch scientist Christian Huygens, was able to explain many other properties of light by proposing that light is a wave. In 1801, Thomas Young showed that light beams can interfere with one another, giving strong support to the wave theory.

In 1865, Maxwell developed a brilliant theory that electromagnetic waves travel with the speed of light. He showed that light is an electromagnetic wave – and thus that optics, the study of visible light, is a branch of electromagnetism. Spurred on by Maxwell's work, Heinrich Hertz discovered what we now call radio waves and verified that they move at the same speed as visible light. By that time, the wave theory of light seemed to be firmly established.

However, at the beginning of the 20th century, Max Planck returned to the corpuscular theory of light to explain the radiation emitted by hot objects. Einstein then used the corpuscular theory to explain how electrons are emitted by a metal exposed to light. Today, scientists view light as having a dual nature - that is, light exhibits characteristics of a wave in some situations and characteristics of a particle in the others.

We now know a wide spectrum (or range) of electromagnetic waves, referred to by one imaginative writer as "Maxwell's rainbow." We are bathed in electromagnetic waves throughout this spectrum. The Sun, whose radiations define the environment in which we as a species have evolved and adapted, is the dominant source. We are also crisscrossed by radio and television signals. Microwaves from radar systems and from telephone relay systems may reach us. There are electromagnetic waves from lightbulbs, from the heated engine blocks of automobiles, from X-ray machines, from lightning flashes, and from buried radioactive materials. Beyond this, radiation reaches us from stars and other objects in our galaxy and from other galaxies. Electromagnetic waves also travel in the other direction. Television signals, transmitted from Earth since about 1950, have now taken news about us to whatever technically sophisticated inhabitants there may be on whatever planets may encircle the nearest 400 or so stars.

The visible region of the spectrum is of course of particular interest to us. Figure I.1 shows the relative sensitivity of the human eye to light of various wavelengths. The center of the visible region is about 555 nm, which produces the sensation that we call yellow-green.

The limits of this visible spectrum are not well defined because the eye sensitivity curve approaches the zero-sensitivity line asymptotically at both long and short wavelengths. If we take the limits arbitrarily, as the wavelengths at which eye sensitivity has dropped to 1% of its maximum value, these limits are about 430 and 690 nm; however, the eye can detect electromagnetic waves somewhat beyond these limits if they are intense enough.

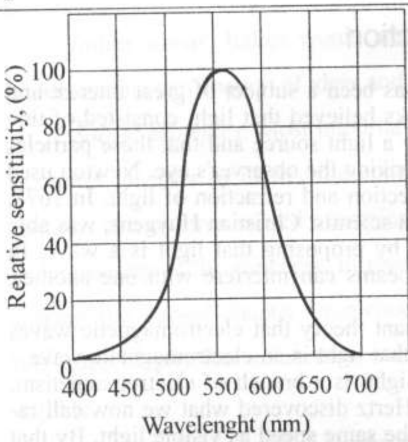

Figure I.1 The relative sensitivity of the average human eye to electromagnetic waves at different wavelength

In secondary school we began to study optics from the special case where the wave picture can be simplified further by representing light in terms of **rays**. The ray representation forms the basis of **geometrical optics** and is the model used to analyze mirrors, lenses, and common optical instruments. Geometrical optics is more limited in its scope than the general wave picture, but it is much simpler. We used it to study the optical behavior of several practical devices, including cameras, projectors, optical systems, the human eye, and various kinds of microscopes and telescopes.

The field of geometric optics involves the study of the propagation of light, with the assumption that light travels in a fixed direction in a straight line as it passes through uniform medium and changes its direction when it meets the surface of medium or if the optical properties of the medium are not uniform in either space or time. In the wave approximation, we assume that a wave moving through a medium travels in a straight line in the direction of its rays.

If the wave meets a barrier in which there is a circular opening whose diameter d is much larger than the wavelength ($d \gg \lambda$), the wave emerging from the opening continues to move in a straight line (apart from some small edge effects); hence, the ray approximation is valid. If the diameter of the opening is of the order of the wavelength ($d \approx \lambda$), the waves spread out from the opening in all directions. Finally, if the opening is much smaller than the wavelength ($d \ll \lambda$), the opening can be approximated as a point source of waves.

In this volume of "Physics for Engineers" we proceed to several optical phenomena that require the more general wave description of light for their understanding. Interference and diffraction phenomena show departures of light from the straight-line propagation that would occur if the ray picture were exactly correct. The corresponding branch of optics is called **physical optics**, to distinguish it from the more restricted geometrical optics. Interference effects enable us to measure wavelengths of light, design nonreflective coatings for lenses, measure atomic spacing in crystal lattices, and understand the fundamental limitations on the resolution of optical instruments. Finally, we study the principles of holography, one of the most exciting and useful developments in modern optics and a striking application of the wave nature of light.

We know that light waves, as any electromagnetic waves, are transverse. In this volume we study polarization of light waves, wide–range application of this phenomenon for investigation of different objects from tiny viruses to huge galaxies.

Next we examine the processes which take place in a medium when light propagates through it. We consider dispersion and understand why rainbow and diamond are so beautiful. We study the process of light absorption and understand why the sky is red in the morning and evening and blue at noon.

Chapter 1

Interference of Light Waves

The term *interference* refers to any situation in which *two or more coherent waves overlap in space*. To understand interference, we must go beyond the restrictions of geometrical optics and employ the full power of wave optics. In fact, as you will see, the existence of interference phenomena is perhaps our most convincing evidence that light is wave – because interference cannot be explained other than with waves.

Sunlight, as the rainbow shows us, is a composite of all the colors of the visible spectrum. The colors reveal themselves in the rainbow because the incident wavelengths are bent through different angles as they pass through raindrops that produce the bow. However, soap bubbles and oil slicks can also show striking colors, produced not by refraction but by constructive and **destructive interference** of light. The interfering waves combine either to enhance or to suppress certain colors in the spectrum of the incident sunlight. This selective enhancement or suppression of wavelengths has many applications. When light encounters an ordinary glass surface, for example, about 4% of the incident energy is reflected, thus weakening the transmitted beam by that amount. This unwanted loss of light can be a real problem in optical systems with many components. A thin, transparent "interference film," deposited on the glass surface, can reduce the amount of reflected light (and thus enhance the transmitted light) by destructive interference. The bluish cast of a camera lens reveals the presence of such a coating. Interference coatings can also be used to enhance – rather than reduce – the ability of a surface to reflect light.

1.1 Nature of Light

Before the beginning of the 19th century, light was considered to be a stream of particles that either was emitted by the object being viewed or emanated from the eyes of the viewer. Newton, the chief architect of the particle theory of light, held that particles were emitted from a light source and that these particles stimulated the sense of light upon entering the eye. Using this idea, he was able to explain reflection and refraction.

Most scientists accepted Newton's particle theory. During his lifetime, however, another theory was proposed – one that argued that light might be some sort of wave motion. In 1678, the Dutch physicist and astronomer Christian Huygens showed that a wave theory of light could also explain *reflection* and *refraction*. The wave theory did not receive immediate acceptance. It was argued that, if light were some form of wave, it would bend around obstacles; hence, we should be able to see around corners. It is now known that light does indeed bend around the edges of objects. This phenomenon, known as diffraction, is not easy to observe because light waves have such short wavelengths. Thus, although Francesco Grimaldi (1618 – 1663) provided experimental evidence for diffraction in approximately 1660, most scientists rejected the wave theory and adhered to Newton's particle theory for more than a century.

In 1801, Thomas Young (1773 – 1829) provided the first clear demonstration of the wave nature of light. Young showed that, under appropriate conditions, light rays interfere with each other. Such behavior could not be explained by a particle theory because there was no conceivable way in which two or more particles could come together and cancel one another. Several years later, a French physicist, Augustin Fresnel (1788 – 1827), performed a number of experiments dealing with interference and diffraction. In 1850, Jean Foucault (1819 – 1868) provided further evidence of the inadequacy of the particle theory by showing that the speed of light in liquids is less than its speed in air. According to the particle model, the speed of light would be higher in liquids than in air. Additional developments during the 19th century led to the general acceptance of the wave theory of light, the most important resulting from the work of Maxwell, who in 1873 asserted that light was a form of high–frequency electromagnetic wave. In 1887 Henrich Hertz provided experimental confirmation of Maxwell's theory by producing and detecting electromagnetic waves. Furthermore, Hertz showed that these waves underwent reflection and refraction and exhibited all the other characteristic properties of waves.

Although the wave model and the classical theory of electricity and magnetism were able to explain most known properties of light, they could not explain some subsequent experiments. The most striking of these is the photoelectric effect, also discovered by Hertz: when light strikes a metal surface, electrons are sometimes ejected from the surface. As one example of the difficulties that arose, experiments showed that the kinetic energy of an ejected electron is independent of the light intensity. This finding contradicted the wave theory, which held that a more intense beam of light should add more energy to the electron. An explanation of the photoelectric effect was proposed by Einstein in 1905 in a theory that used the concept of quantization developed by Max Planck (1858 – 1947) in 1900. The quantization model assumes that the energy of a light wave is present in bundles called *photons*; hence, the energy is said to be quantized. According to Einstein's theory, the energy of a photon is proportional to the frequency of the electromagnetic wave:

$$E = h\nu ,$$

where the constant of proportionality $h = 6.63 \times 10^{-34}$ J·s is Planck's constant. It is important to note that this theory retains some features of both the wave theory and the particle theory. The photoelectric effect is the result of energy transfer from a single photon to an electron in the metal, and yet this photon has wave-like characteristics because its energy is determined by the frequency (a wave-like quantity). In view of these developments, light must be regarded as having a dual nature: *Light exhibits the characteristics of a wave in some situations and the characteristics of a particle in other situations.* Therefore, the question "is light a wave or a particle?" is an inappropriate one. Sometimes light acts like a wave, and at other times it acts like a particle.

Question

1.1 Though quantum theory of light can explain a number of phenomena observed with light, it is necessary to retain the wave nature of light to explain the phenomenon of

a) photoelectric effect;
b) diffraction;
c) Compton's effect;
d) black body radiation.

1.2 Sources of Light

The Sun, stars, lightbutbs, and burning materials all give off light. When something produces light it is said to be *luminous*. The Sun is a luminous object that provides almost all of the natural light on the Earth. A small amount of light does reach the earth from the stars but not really enough to see by on a moonless night. The moon and planets shine by reflected light and do not produce their own light, so they are not luminous.

Burning has been used as a source of artificial light for thousands of years. A wood fire and a candle flame are luminous because of their high temperatures. When visible light is given off as a result of high temperatures, the light source is said to be *incandescent*. A flame from any burning source, an ordinary lightbulb, and the sun are all incandescent sources because of high temperatures. All heated bodies emit electromagnetic radiation as a result of thermal motion of their molecules; this radiation, called **thermal radiation**, is a mixture of different wavelengths. One of the brightest sources of light is the carbon arc. Two carbon rods, typically 10 to 20 cm long and 1 cm in diameter, are connected to a 120 or 240 V dc source. They are touched together momentarily and then pulled apart a few millimeters. An arc forms, and the resulting intense electron bombardment of the positive rod causes an extremely hot crater to form at its end. This crater, whose temperature is typically 4000°C, is the source of light. Carbon–arc lights are used in most theater motion–picture projectors and in large searchlights and lighthouses.

Some light sources use an arc discharge through a conducting metal vapor, such as mercury or sodium. The vapor is contained in a sealed bulb with two electrodes, which are connected to a power source. Argon is sometimes added to permit a glow discharge that helps vaporize and ionize the metal. The bluish light of mercury–arc lamps and the bright orange–yellow of sodium–vapor lamps are familiar in highway and other outdoor lighting.

An important variation of the mercury–arc lamp is the *fluorescent* lamp, consisting of a glass tube containing argon and mercury vapor, with tungsten electrodes. When an electric discharge takes place in the mercury–argon mixture, the emitted radiation is mostly in the ultraviolet region. The ultraviolet radiation is absorbed in a thin layer of material, called a *phosphor*, which is the white coating on the interior walls of the glass tube. The phosphor has the property *of fluorescence*, which means that it emits visible light when illuminated by ultraviolet radiation. Various phosphors can be used to obtain various colors of light. Fluorescent lamps have much higher efficiency of conversion of electrical energy to visible light than do incandescent lamps.

Phosphorus, oxidized in air, glows due to energy which liberates at chemical transformations. Such type of a glowing called *hemiluminescence*. Glowing, occurring at any type of self-maintained discharge is called *electroluminescence*. Glowing due to electromagnetic radiation absorbed by a body is called *photoluminescence*.

A special light source that has attained prominence in the last years is the *laser*. It can produce a very narrow beam of enormously intense radiation. High–intensity lasers have been used to cut through steel, fuse high–melting–point materials, and bring about many other effects that are important in physics, chemistry, biology, and engineering. An equally significant characteristic of laser light is that it is much more nearly *monochromatic*, or single–frequency, than any other light source.

Questions

1.2. What is the fundamental source of all electromagnetic radiation?
1.3. What kind of light sources do you now?

1.3 Speed of Light

Light travels at such a high speed ($c \approx 3\times10^8$ m/s) that early attempts to measure its speed were unsuccessful. Galileo attempted to measure the speed of light by positioning two observers in towers separated by approximately 10 km. Each observer carried a shuttered lantern. One observer would open his lantern first, and then the other would open his lantern at the moment he saw the light from the first lantern. Galileo reasoned that, knowing the transit time of the light beams from one lantern to the other, he could obtain the speed of light. His results were inconclusive. Today we realize (as Galileo concluded) that it is impossible to measure the speed of light in this manner because the transit time is so much less than the reaction time of the observers.

A lot of attempts were made to determine the speed of light and now we describe two of them.

Roemer's Method

In 1675, the Danish astronomer Ole Roemer (1644 – 1710) made the first successful estimate of the speed of light. Roemer's technique involved astronomical observations of one of the moons of Jupiter, Io, which has a period of revolution around Jupiter of approximately 42.5 h. The period of revolution of Jupiter around the Sun is about 12 year; thus, as the Earth moves through 90° around the Sun, Jupiter revolves through only (1/12)90° = 7.5° (Figure 1.1).

An observer using the orbital motion of Io as a clock would expect the orbit to have a constant period. However, Roemer, after collecting data for more than a year, observed a systematic variation in Io's period. He found that the periods were longer than average when the Earth was receding from Jupiter and shorter than average when the Earth was approaching Jupiter. If Io had a constant period, Roemer should have seen it become eclipsed by Jupiter at a particular instant and should have been able to predict the time of the next eclipse. However, when he checked the time of the second eclipse as the Earth receded from Jupiter, he found that the eclipse was late. If the interval between his observations was three months, then the delay was approximately 600 s.

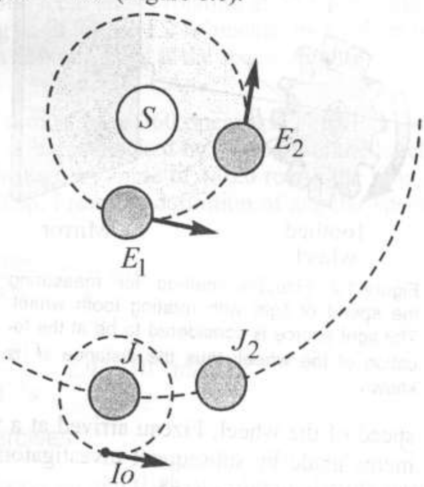

Figure 1.1 Roemer's method for measuring the speed of light. In the time it takes the Earth to travel 90° around the Sun (three months), Jupiter travels about only 7.5°

Roemer attributed this variation in period to the fact that the distance between the Earth and Jupiter changed from one observation to the next. In three months (one quarter of the period of revolution of the Earth around the Sun), the light from Jupiter must travel an additional distance equal to the radius of the Earth's orbit.

Using Roemer's data, Huygens estimated the lower limit for the speed of light to be approximately 2.3×10^8 m/s. This experiment is important historically because it demonstrated that light does have a finite speed and gave an estimate of this speed.

Fizeau's Method

The first successful method for measuring the speed of light by means of purely terrestrial techniques was developed in 1849 by Armand L. Fizeau (1819 – 1896). Figure 1.2 represents a simplified diagram of Fizeau's apparatus. The basic procedure is to measure the total time it takes light to travel from some point to a distant mirror and back. If d is the distance between the light source (considered to be at the location of the wheel) and the mirror and if the transit time for one round trip is t, then the speed of light is $c = 2d/t$.

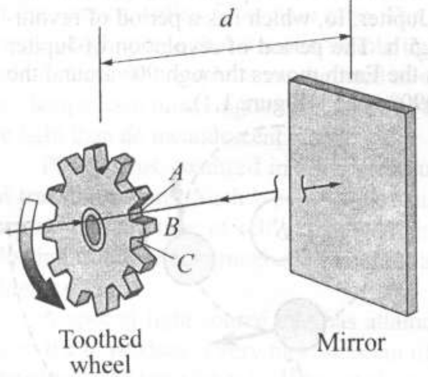

Figure 1.2 Fizeau's method for measuring the speed of light with rotating tooth wheel. The light source is considered to be at the location of the wheel; thus the distance d is known

To measure the transit time, Fizeau used a rotating toothed wheel, which converts a continuous beam of light into a series of light pulses. The rotation of such a wheel controls what an observer at the light source sees. For example, if the pulse traveling toward the mirror and passing the opening at point A in Figure 1.2 should return to the wheel at the instant, when tooth B had rotated into position to cover the return path, the pulse would not reach the observer. At a greater rate of rotation, the opening at point C could move into position to allow the reflected pulse to reach the observer. Knowing the distance d, the number of teeth in the wheel, and the angular speed of the wheel, Fizeau arrived at a value $c = 3.1 \times 10^8$ m/s. Similar measurements made by subsequent investigators yielded more precise values for c, approximately 2.9979×10^8 m/s.

Fizeau's apparatus was modified by Foucault, who replaced the toothed wheel with a rotating mirror. The most precise measurements by the Foucault method were made by Albert A. Michelson (1852 – 1931). His first experiments were performed in 1878; the last, underway at the time of his death, were completed in 1935 by Pease and Pearson.

Today the value for the speed of light is determined as

$$c = 2.99792458 \times 10^8 \text{ m/s}.$$

This number is based on the definition of the meter in terms of the krypton wavelength and the definition of the second in terms of the cesium clock. The definition of the second is precise to within one part in 10 trillion (10^{13}), whereas the definition of the meter is much less precise, about four parts in a billion (10^9).

According to Maxwell's electromagnetic theory, speed of any electromagnetic wave, including light, in empty space is:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}, \tag{1.1}$$

where permittivity constant $\varepsilon_0 = 8.85 \times 10^{-12}$ and permeability constant $\mu_0 = 4\pi \times 10^{-7}$ H/m. In any medium light travels with a less speed, namely:

$$v = \frac{c}{n}, \tag{1.2}$$

where n is *index of refraction* of the medium.

Examples

Example 1.1

Assume that Fizeau's wheel has 360 teeth and is rotating at 27.5 rev/s when a pulse of light passing through opening A in Figure 1.2 is blocked by tooth B on its return. If the distance to the mirror is 7500 m, what is the speed of light?

Solution

The wheel has 360 teeth, and so it must have 360 openings. Therefore, because the light passes through opening A but is blocked by the tooth immediately adjacent to A, the wheel must rotate through an angle of 1/720 rev in the time it takes the light pulse to make its round trip. From the definition of angular speed, that time is

$$t = \frac{\theta}{\omega} = \frac{(1/720)\,\text{rev}}{27.5\,\text{rev/s}} = 5.05 \times 10^{-5}\ \text{s}.$$

Hence, the speed of light is

$$c = \frac{2d}{t} = \frac{2(7500\,\text{m})}{5.05 \times 10^{-5}\,\text{s}} = 2.97 \times 10^{8}\ \text{m/s}.$$

Exercises

1.4. During a thunderstorm we always see the flash of lightning before hearing the accompanying thunder. Discuss this in terms of the various wave speeds. Can this phenomenon be used to determine how far away the storm is?

1.5. Fizeau's measurements of the speed of light were continued by Cornu, using Fizeau's apparatus but with the distance from the toothed wheel to the mirror increased to 22.9 km. One of the toothed wheels used was 40 mm in diameter and has 180 teeth. Find the angular velocity at which it should rotate so that light transmitted though one opening will return through the next.

1.4 Waves, Wave Surfaces, Wave Fronts and Rays

The concepts of **wave surface** and **wave front** provide a convenient language for describing the propagation of any kind of wave. We define a *wave surface as the locus of all points at which the phase of vibration of a physical quan-*

tity associated with the wave is the same. A familiar example is the crest of a water wave; when we drop a pebble in a calm pool, the expanding circles formed by the wave crests are wave surfaces. When sound waves spread out in all directions from a pointlike source, any spherical surface concentric with the source is a wave surface. The surfaces over which the pressure is maximum and those over which it is minimum form sets of expanding spheres as the wave travels outward from the source. The phase of the pressure variation is the same at all points on one of the spherical surfaces.

In diagrams of wave motion we usually draw only a few wave surfaces, often those that correspond to the maxima and minima of the disturbance, such as the *crests* and *troughs* of a water wave. For a sinusoidal wave, wave surface corresponding to maximum displacements in opposite directions are separated from each other by one–half wavelength. Two consecutive wave surfaces corresponding to maximum displacement in the same direction are separated by one wavelength. Wave surface, which separates the region of space, where the wave is present and region, which wave doesn't reach yet, is called a *wave front.* In a given instant of time there is only one wave front, whereas there are a lot of wave surfaces.

For a light wave (or any other electromagnetic wave), the quantity that corresponds to the pressure in a sound wave is the electric or magnetic field. Often it is not necessary to indicate in a diagram either the magnitude or the direction of the field; instead we simply show the shapes of the wave fronts or their intersections with some reference plane. For example, the electromagnetic waves radiated by isolated small source of light may be represented by spherical surfaces concentric with the source or, as in Figure 1.3 (a), by the intersections of these surfaces with the plane of the diagram. The equation of spherical wave is:

$$E = \frac{E_0}{r}\cos(\omega t - \vec{k}\vec{r} + \alpha), \qquad (1.3)$$

where E_0 is the initial amplitude of light wave, $\frac{E_0}{r}$ is wave amplitude at the point under consideration, $\vec{r}$ is the position vector from the center of light wave to the point, $\vec{k}$ is wave vector and α – phase constant.

At a sufficiently great distance from the source, where the radii of the spheres become very large, the spherical surfaces can be considered as planes and we have a plane wave, as in Figure 1.3 (b). The equation of a plane wave has form:

$$E = E_0\cos(\omega t - \vec{k}\vec{r} + \alpha). \qquad (1.4)$$

For some phenomena, it is convenient to represent a light wave by *rays* rather than by wave fronts. From the wave viewpoint, a *ray is an imaginary line drawn in the direction in which the wave is traveling.* Thus in Figure 1.3 (a) the rays are the radii of the spherical wave surfaces, and in Figure 1.3 (b) they are the straight lines perpendicular to the wave surfaces. In fact, in every case where waves travel in a homogeneous isotropic material, the rays are straight lines normal to the wave fronts. At a boundary surface between two materials, such as the surface between a glass plate and the air outside it, the direction of a ray changes, but the portions in the air and in the glass are straight lines.

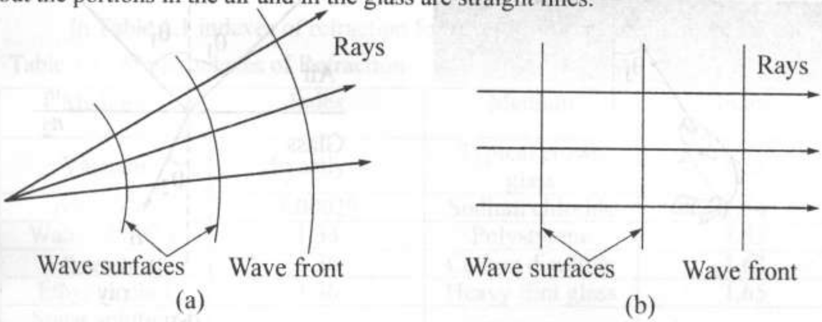

Figure 1.3 Wave fronts, wave surfaces and rays. (a) When the wave surfaces are spherical, the rays radiate out from the center of the spheres; (b) When the wave surfaces are planes, the rays are parallel

Although the ray picture provides an adequate description of many reflection and refraction phenomena found in mirrors and lenses, several other optical phenomena, such as polarization and diffraction, require a more detailed wave theory for their understanding.

Exercises

1.6. What is wave surface?

1.7. What is wave front? Show on a diagram, the direction of a ray relative to a wave front.

1.5 Index of Refraction

The **index of refraction** n of a medium is the ratio

$$n = \frac{speed \quad of \quad light \quad in \quad vacuum}{speed \quad of \quad light \quad in \quad a \quad medium} = \frac{c}{v}. \tag{1.5}$$

From this definition, we see that the index of refraction n is a dimensionless number greater than unity because v is always less than c. Furthermore, n is equal to unity for vacuum.

From the other hand, according to the Snell's law of refraction (incident and refracted rays and the normal to the surface all lie in the same plane and the ratio of the sine of the angle of incidence and sine of the angle of refraction equals to the index of refraction, Figure 1.4), the index of refraction can also be expressed as

$$n = \frac{\sin\theta_1}{\sin\theta_2} \tag{1.6}$$

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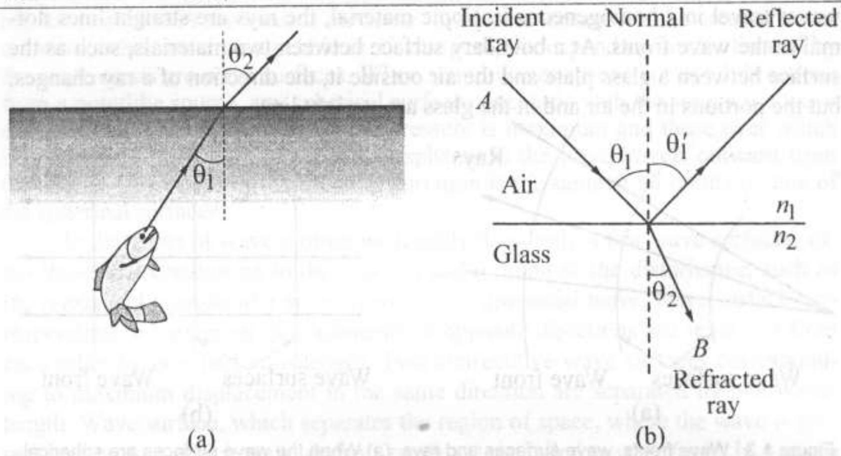


Figure 1.4 (a) A view from the fish's eye: $\theta_2 > \theta_1$; (b) $n_2 > n_1$. All rays and normal lie in the same plane

When light interacts with matter, the electrons in the material absorb energy from the light and undergo vibration motion with the same frequency as the light. This motion causes reradiation of the energy *with the same frequency*. When light passes from one material to another, its frequency f does not change but its wavelength λ does (Figure 1.5). The relationship $v = \lambda f$ must be valid in both media and because $f_1 = f_2 = f$, we see that

$$v_1 = \lambda_1 f \quad \text{and} \quad v_2 = \lambda_2 f.$$

Because $v_1 \neq v_2$, it follows that

$$\lambda_1 \neq \lambda_2.$$

We can obtain a relationship between index of refraction and wavelength:

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}.$$

This gives

$$\lambda_1 n_1 = \lambda_2 n_2.$$

If medium 1 is vacuum, or for all practical purposes, air, then $n_1 = 1$ and it follows,

$$\lambda_0 = \lambda_n n,$$

where λ_0 is wavelength in vacuum and λ_n is wavelength in a medium with index n, that is, the wavelength λ_n of

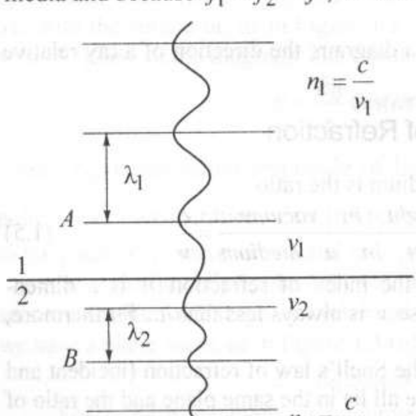


Figure 1.5 As a wave moves from medium 1 to medium 2, its wavelengths changes but its frequency remains constant

light in a material is less than the wavelength λ_0 of the same light in vacuum, by a factor n:

$$\lambda_n = \frac{\lambda_0}{n}. \tag{1.7}$$

In Table 1.1 indexes of refraction for several substances are represented.

Table 1.1 Some Indexes of Refraction

Medium	Index	Medium	Index
Vacuum	Exactly 1	Typical crown glass	1.52
Air (STR)	1.00029	Sodium chloride	1.54
Water (20° C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

Example 1.2

Laser Light in a Compact Disc. A laser in a compact disc player generates light that has a wavelength of 780 nm in air.

a) Find the speed of this light once it enters the plastic of a compact disc ($n = 1.55$).

Solution

We expect to find a value less than 3×10^8 m/s because $n > 1$. We can obtain the speed of light in the plastic by using Eq. (1.2)

$$v = \frac{c}{n} = \frac{3.0\times10^8 \text{ m/s}}{1.55} = 1.94\times10^8 \text{ m/s}.$$

b) What is the wavelength of this light in the plastic?

Solution

We use Eq. (1.7) to calculate the wavelength in plastic, noting that we are given the wavelength in air to be $\lambda_0 = 780$ nm:

$$\lambda_n = \frac{\lambda_0}{n} = \frac{780\,\text{nm}}{1.55} = 503 \text{ nm}.$$

Exercises

1.8. Sunlight or starlight passing though the earth's atmosphere is always bent toward the vertical. Why? Does this mean that a star is not really where it appears to be?

1.9. An object submerged in water appears to be closer to the surface than it actually is. Why?

1.10. Does a light ray traveling from one medium into another always bend toward the normal? Explain.

1.11. As light travels from one medium to another, does the wavelength of the light change? Does the frequency change? Does the speed change? Explain.

1.12. Two light pulses are emitted simultaneously from a source. Both pulses travel to a detector, but one first passes through 6.20 cm of crown glass. Determine the difference in the pulses' times of arrival at the detector. (Ans.: 1.74×10^{-10} s).

1.13. The speed of yellow light (from a sodium lamp) in a certain liquid is measured to be 1.92×10^{8} m/s. What is the index of refraction of this liquid for the light? (Ans.: 1.56).

1.14. Light of wavelength 436 nm in air enters a fishbowl filled with water and then exits through the crown glass wall of the container (Figure 1.4.) What is the wavelength of the light (a) in the water and (b) in the glass? (Ans. $\lambda_w = 328$ nm, $\lambda_g = 287$ nm.)

1.6 Huygens' Principle

The principles of reflection and refraction of light rays were discovered experimentally long before the wave nature of light was firmly established. These principles may, however, be derived from wave considerations and thus shown to be consistent with the wave nature of light. To establish this connection we use a principle called **Huygens' principle**. This principle, stated originally by Christian Huygens in 1678, is a geometrical method for finding, from the known shape of a wave front at some instant, the shape of the wave front at some later time. Huygens assumed that: *All points on a given wave front are taken as point sources for the production of spherical secondary waves, called wavelets, which propagate outward through a medium with speed characteristics of wave in that medium. After some time has elapsed, the new position of wave front is the surface tangent to the wavelets.*

Let us consider a plane wave moving through free space, as shown in Figure 1.6 (a). At $t=0$, the wave front is indicated by the plane labeled AA'. In Huygens' construction, each point on this wave front is considered a point source. For clarity, only three points on AA' are shown. With these points as a sources for the wavelets, we draw circles, each of radius $c\Delta t$, where c is the speed of light in free space and Δt is the time of propagation from one wave front to the next. The surface drawn tangent to these wavelets is the plane BB', which is parallel to AA'. In a similar manner, Figure 1.6 (b) shows Huygens's construction for a spherical wave.

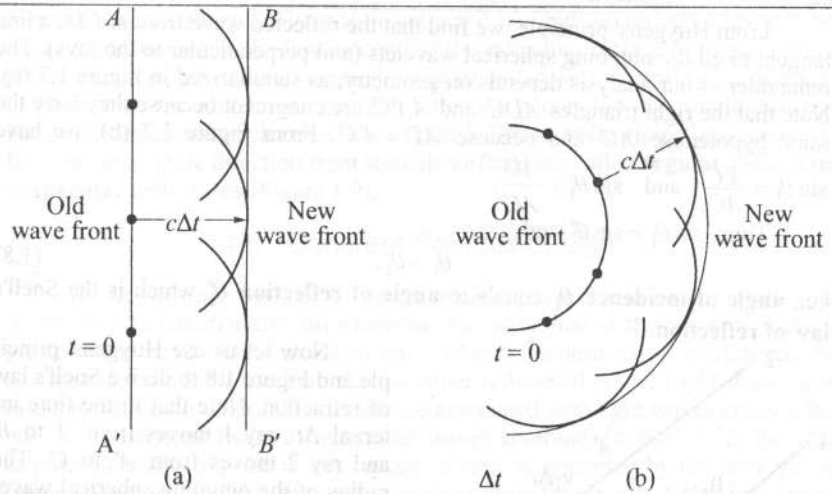


Figure 1.6 Huygens's construction for (a) a plane wave propagating to the right and (b) a spherical wave propagating to the right

Question

1.15. Why does the light appear to travel in a straight line in spite of its wave nature?

1.7 Huygens' Principle Applied to Reflection and Refraction

The Snell's *laws of reflection* and refraction were stated experimentally without proof. We now derive these laws, using Huygens's principle.

For the law of reflection refer to Figure 1.7 (a). The line AA' represents a wave front of the incident light. As ray 3 travels from A' to C, ray 1 reflects from A and produces a spherical wavelet of radius AD. (Recall that the radius of a Huygens wavelet is $c\Delta t$.) Because the two wavelets having radii $A'C$ and AD are in same medium, they have the same speed c; therefore, $A'C = AD$.

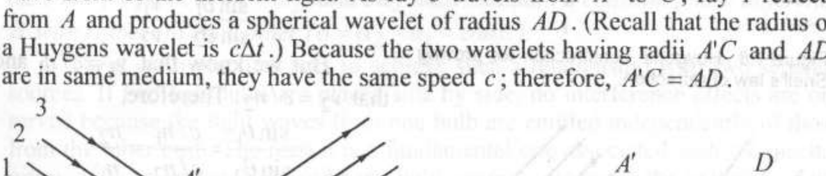


Figure 1.7 (a) Huygens' construction for proving the law of reflection; (b) Triangle ADC is congruent to triangle $AA'C$

From Huygens' principle, we find that the reflected wave front is CD, a line tangent to all the outgoing spherical wavelets (and perpendicular to the rays). The remainder of our analysis depends on geometry, as summarized in Figure 1.7 (b). Note that the right triangles ADC and $AA'C$ are congruent because they have the same hypotenuse AC and because $AD = A'C$. From Figure 1.7 (b), we have

$$\sin\theta_1 = \frac{A'C}{AC} \quad \text{and} \quad \sin\theta_1' = \frac{AD}{AC}.$$

Thus, $\sin\theta_1 = \sin\theta_1'$ and

$$\theta_1 = \theta_1', \tag{1.8}$$

i.e., **angle of incidence** θ_1 equals to **angle of reflection** θ_1' which is the Snell's **law of reflection**.

Now let us use Huygens' principle and Figure 1.8 to derive Snell's law of refraction. Note that in the time interval Δt, ray 1 moves from A to B and ray 2 moves from A' to C. The radius of the outgoing spherical wavelet centered at A is equal to $v_2\Delta t$. The distance $A'C$ is equal to $v_1\Delta t$. Geometric considerations show that angle $A'AC$ equals θ_1 and that angle ACB equals θ_2. From triangles $AA'C$ and ACB, we find that

$$\sin\theta_1 = \frac{v_1\Delta t}{AC} \quad \text{and} \quad \sin\theta_2 = \frac{v_2\Delta t}{AC}.$$

If we divide the first equation by the second, we obtain

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}.$$

But we know that $v_1 = c/n_1$ and that $v_2 = c/n_2$. Therefore,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1},$$

and, finally,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1},$$

which is Snell's **law of refraction.**

Two final comments about reflection and refraction need to be made. First, reflection occurs at a highly polished surface of an opaque

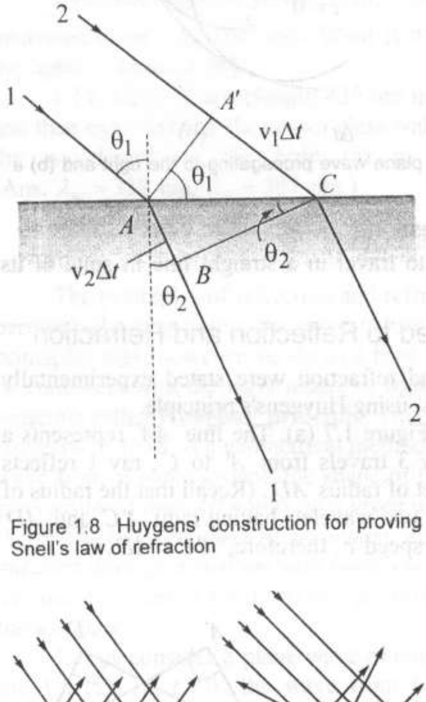


Figure 1.8 Huygens' construction for proving Snell's law of refraction

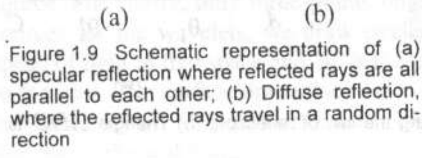


Figure 1.9 Schematic representation of (a) specular reflection where reflected rays are all parallel to each other; (b) Diffuse reflection, where the reflected rays travel in a random direction

material such as a metal. There is no refracted ray, but the reflected ray still behaves according to Snell's law. Second, if the reflecting surface of either a transparent or an opaque material is rough, with irregularities of a scale comparable to or larger than the wavelength of light, reflection occurs not in a single direction but in all directions; such reflection is called **diffuse reflection**. Conversely, reflections in a single direction from smooth surfaces are called **regular reflections** or **specular reflections** (Figure 1.9).

1.8 Coherent Sources of Light

We that adding together of two mechanical waves can be *constructive* or *destructive*. In **constructive interference**, the amplitude of the resulting wave is greater than that of either individual wave, whereas in destructive interference, the resultant amplitude is less than that of either individual wave. Light waves also interfere with each other. All interference associated with light waves arises when the electromagnetic fields of individual waves combine. In such cases the total displacement at any point at any instant of time is governed by the *principle of linear superposition*. This principle, the most important in all of physical optics, states *that when two or more waves overlap, the resultant displacement at any point and at any instant may be found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone*. The term displacement means the magnitude of the electric or magnetic field. When light of extremely high intensity passes through matter, the principle of linear superposition is not precisely obeyed, and the resulting phenomena are classified under the heading **nonlinear optics**.

Interference effects in light waves are not easy to observe because of the short wavelengths involved (from 4×10^{-7} to 7×10^{-7} m). For sustained interference in light waves to be observed, the sources must be **coherent** – *that is, have the same frequency ($\omega_1 = \omega_2$) and they must maintain a constant phase difference δ with respect to each other ($\delta = \alpha_2 - \alpha_1 = const$).*

There is no practical way to achieve such a relationship with two separate sources. If two lightbulbs are placed side by side, no interference effects are observed because the light waves from one bulb are emitted independently of those from the other bulb. The reason is a fundamental one associated with the mechanisms of light emission. In ordinary light sources, atoms of the material of the source are given excess energy by thermal agitation or by impact with accelerated electrons. An atom thus "excited" begins to radiate energy and continues until it has lost all the energy it can, typically in a time of the order of 10^{-8} s. A source ordinarily contains a very large number of atoms, and they radiate in an unsynchronized and random phase relationship. Thus emission from two such sources has a rapidly varying phase relation. The result is an interference pattern that constantly changes in a random manner, and ordinary observation does not show a visible interference pattern at all. Such light sources are said to be **incoherent**.

The common methods of producing coherent sources use the division of wave front. In these methods the wave front is divided into two or more parts with help of mirrors or prisms. The common methods are: Young's double–slit arrangement, Fresnel's biprism method, Lloyd's mirror method, and Fresnel's mirror method. One of these methods we describe now, with the others we will meet later.

Fresnel's Biprism Method

A biprism is essentially a combination of two acute prisms placed base to base. In fact this combination is obtained from plane glass plate by proper grinding and polishing. A narrow adjustable slit S is illuminated by a monochromatic source of light in Figure 1.10. Biprism is adjusted parallel to the edge of the slit. Light passes through biprism, whose upper and lower parts creates virtual images S_1 and S_2 respectively. The images S_1 and S_2 are in the same vertical plane and serve as two coherent sources. The interference fringes are obtained in the region of overlapping AB on the screen. Let n be the index of refraction of the prism, β is the angle of refraction of the prism, a is the length of CO and b is the length of SC. Each part of biprism deflects the paraxial ray at angle $(n-1)\beta$. The distance d between images S_1 and S_2 equals to $d = S_1S_2 = 2b(n-1)\beta$ whereas the angular distance between them is $\alpha = d(a+b)$. Then the width of fringe will be

$$\Delta x = \frac{\lambda}{\alpha} = \frac{\lambda(a+b)}{2b(n-1)\beta}.$$

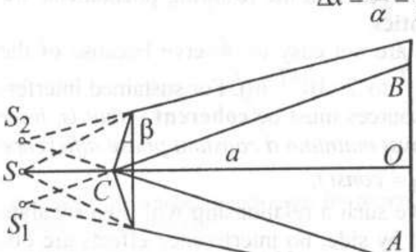


Figure 1.10 Fresnel's biprism apparatus for obtaining two coherent source of light

In all of the methods used, however, if the light from a single source is split so that parts of it emerge from two or more regions of space, forming two or more *secondary sources,* any random phase change in the source affects these secondary sources equally and does not change their relative phase. Two such sources derived from a single source and having a definite phase relation are coherent.

1.9 Interference Pattern due Two Light Sources

Suppose that the two slits represent coherent sources of light waves such that the two waves from the slits have the same angular frequency ω and a constant phase difference δ. The total magnitude of the electric field at point P on the screen in Figure 1.11 is the vector superposition of the two waves. Assuming that the two waves have the amplitudes E_1 and E_2 respectively, we can write the magnitude of the electric field at point P due to each wave separately as

$$E_1\cos(\omega t+\alpha_1) \quad \text{and} \quad E_2\cos(\omega t+\alpha_2).$$

Using the phasor addition of waves, we can obtain the magnitude of the resultant electric field at point P:

$$E^2 = E_1^{\ 2} + E_2^{\ 2} + 2E_1E_2\cos\delta, \tag{1.9}$$

where $\delta = \alpha_2 - \alpha_1$ is *phase difference.*

As intensity of light is proportional to the squared amplitude, the resultant intensity is:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta. \tag{1.10}$$

The latest term in Eq. (1.10) is so-called **interference term.** At the points with $\cos\delta > 0$, the resultant intensity $I > I_1 + I_2$, whereas at the points where $\cos\delta < 0$, the resultant intensity $I < I_1 + I_2$. Therefore the redistribution of energy in space occurs at interference phenomena. Maximum of intensity appears at certain points in space and minimum of intensity appears at the others as a result of this redistribution of energy. For comparison, if we add two incoherent light waves, the resultant intensity is simply the sum of the individual intensities:

$$I = I_1 + I_2.$$

Let's return to Figure 1.11. In general case two waves can propagate through different substances which are characterized by different indices of refraction n_1 and n_2, respectively. Let the distance between two sleets is d, that between two–slits screen and viewing screen is L and that from the axis of symmetry and point P is y. To arrive at point P the first wave travels the path l_1 in a medium with index of refraction n_1, whereas the second wave travels the path l_2 in a medium with index of refraction n_2. If oscillations have the same phases equal to ωt at the both slits, then the oscillation at P due to the first wave will be $E_1\cos\omega(t - l_1/v_1)$, and that due to the second wave will be $E_2\cos\omega(t - l_2/v_2)$.

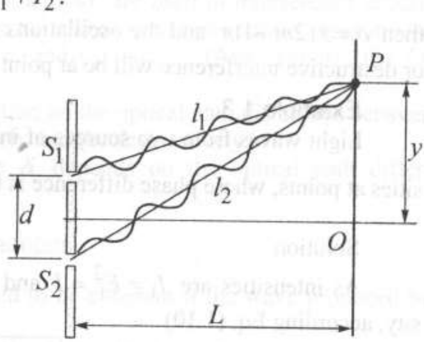


Figure 1.11 Construction for analyzing two-source interference pattern. A bright fringe, or intensity maximum, is observed at O.

Therefore the **phase difference** of these oscillations at P is

$$\delta = \omega\left(\frac{l_2}{v_2} - \frac{l_1}{v_1}\right) = \frac{\omega}{c}(n_2 l_2 - n_1 l_1).$$

After substitution $\frac{\omega}{c}$ as $\frac{2\pi f}{c} = \frac{2\pi}{\lambda_0}$, ($\lambda_0$ – wavelength of light in vacuum) the previous expression we can rewrite as:

$$\delta = \frac{2\pi}{\lambda_0}\Delta, \tag{1.11}$$

where

$$\Delta = n_2 l_2 - n_1 l_1. \tag{1.12}$$

Parameter Δ is so-called **optical path difference** and nl is the **optical path** of wave.

It is clear from Eqs. (1.11) and (1.12), that when optical path difference equals the integer number of wavelength in vacuum

$$\Delta = \pm m\lambda_0, \quad m = 0, 1, 2, \ldots \tag{1.13}$$

then $\delta = \pm 2\pi m$, and hence the oscillations at P have the same phase and constructive interference occurs at this point (i.e. this point is the point of maximum). Integer m is called **order of spectrum** or **order number**.

When optical path difference is:

$$\Delta = \left(m + \frac{1}{2}\right)\lambda_0, \tag{1.14}$$

then $\delta = \pm(2m+1)\pi$ and the oscillations come at P out of phase and minimum, or destructive interference will be at point P.

Example 1.3

Light waves from two sources of intensities I and $4I$ interfere. Find intensities at points, where phase difference is (a) $\frac{\pi}{2}$ and (b) π.

Solution

As intensities are $I_1 = E_1^2 = I$ and $I_2 = E_2^2 = 4I$, then the resultant intensity, according Eq. (1.10)

$$I_R = I + 4I + 2\sqrt{I \times 4I}\cos\delta = 5I + 4I\cos\delta.$$

(a) When $\delta = \frac{\pi}{2}$, resultant intensity is $I_R = 5I + 4I\cos\frac{\pi}{2} = 5I$.

(b) When $\delta = \pi$, resultant intensity is $I_R = 5I + 4I\cos\pi = I$.

Example 1.4

Two waves of amplitudes E_1 and E_2 interfere with each other. Find the ratio of intensities of maxima to that of minima.

Solution

When two light waves of amplitudes E_1 and E_2, differing by phase δ, interfere, the intensity of the resultant light is given by

$$I = E_1^{\ 2} + E_2^{\ 2} + 2E_1E_2\cos\delta .$$

The intensity of light will be maximum, when $\delta = 0$. Hence,

$$I_{\max} = E_1^{\ 2} + E_2^{\ 2} + 2E_1E_2\cos 0 = (E_1 + E_2)^2 .$$

On the other hand, intensity of light will be minimum, when $\delta = \pi$. Therefore,

$$I_{\min} = E_1^{\ 2} + E_2^{\ 2} + 2E_1E_2\cos\pi = (E_1 - E_2)^2 .$$

Hence, $\dfrac{I_{\max}}{I_{\min}} = \dfrac{(E_1 + E_2)^2}{(E_1 - E_2)^2}$.

Exercises

1.16. Can two independent sources of light produce interference?

1.17. At points of constructive interference between waves of equal amplitude, the intensity is four times that of either individual wave. Does this violate the energy conservation? If not, why not?

1.18. Two coherent sources whose intensity ratio is 16:1 produce interference fringes. Calculate the ratio of intensity of maxima and minima in the fringe system. (Ans.: 9:1).

1.19. Two sources of intensities I and $4I$ are used in interference experiment. Obtain intensities at points, where the waves from two sources superimpose with a phase difference of (a) 0, (b) π and (c) $\pi/2$. (Ans.: (a) $9I$, (b) $5I$ and (c) I).

1.20. What is the necessary condition on the optical path difference between two waves that interfere (a) constructively and (b) destructively?

1.21. How the phase difference δ depends on the optical path difference Δ?

Questions

1.22. Two sources of light are said to be coherent if the wave produced by them have the same:

a) wavelength,

b) amplitude,

c) frequency and a constant phase difference,

d) frequency and the same amplitude.

1.10 Young's Double – Slit Experiment

Interference in light waves from two sources was first demonstrated by Thomas Young in 1801. A schematic diagram of the apparatus that Young used is shown in Figure 1.12 (a).

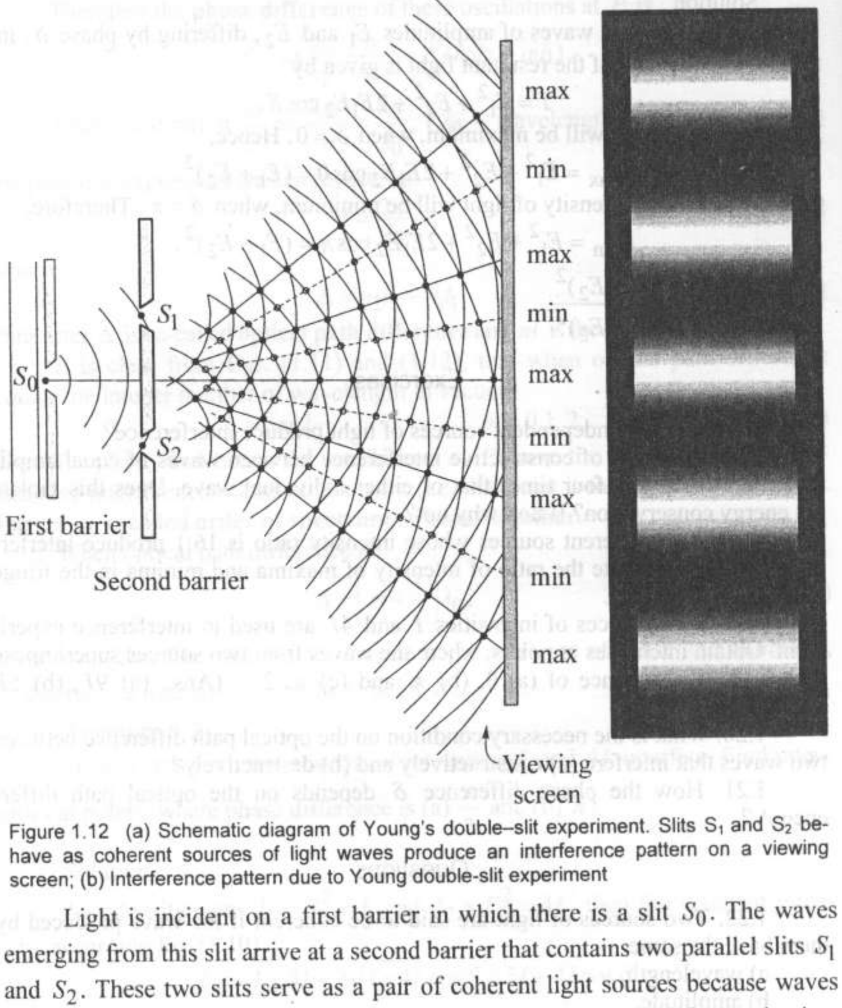


Figure 1.12 (a) Schematic diagram of Young's double–slit experiment. Slits S_1 and S_2 behave as coherent sources of light waves produce an interference pattern on a viewing screen; (b) Interference pattern due to Young double-slit experiment

Light is incident on a first barrier in which there is a slit S_0. The waves emerging from this slit arrive at a second barrier that contains two parallel slits S_1 and S_2. These two slits serve as a pair of coherent light sources because waves emerging from them originate from the same wave front and therefore maintain a constant phase relationship. The light from S_1 and S_2 produces on a viewing screen a visible pattern of bright and dark parallel bands called *fringes*. (See Figure 1.12. (b)) When the light from S_1 and that from S_2 arrive in phase at a point on the viewing screen, constructive interference occurs at that location, a bright fringe appears. When the light from the two slits combines destructively at any location on the screen dark **fringes** results.

Figure 1.13 shows some of the ways in which two waves can combine at the screen. In Figure 1.13 (a) the two waves, which leave the two slits in phase, strike the screen at the central point P. Because both waves travel the same distance in the same medium, they arrive at P in phase. As a result, constructive interference occurs at this location, and a bright fringe is observed. In Figure 1.13 (b), the two waves also start in phase, but in this case the upper wave has to travel one wavelength farther than the lower wave to reach point Q. Because the upper wave falls behind the lower one by exactly one wavelength, they still arrive in phase at Q, and so a second bright fringe appears at this location. At point R in Figure 1.13 (c), however, midway between points P and Q, the upper wave has fallen half a wavelength behind the lower wave. This means that a trough of the lower wave overlaps a crest of the upper wave; this gives rise to destructive interference at point R. For this reason, a dark fringe is observed at this location.

We can describe Young's experiment quantitatively with the help of Figure 1.14.

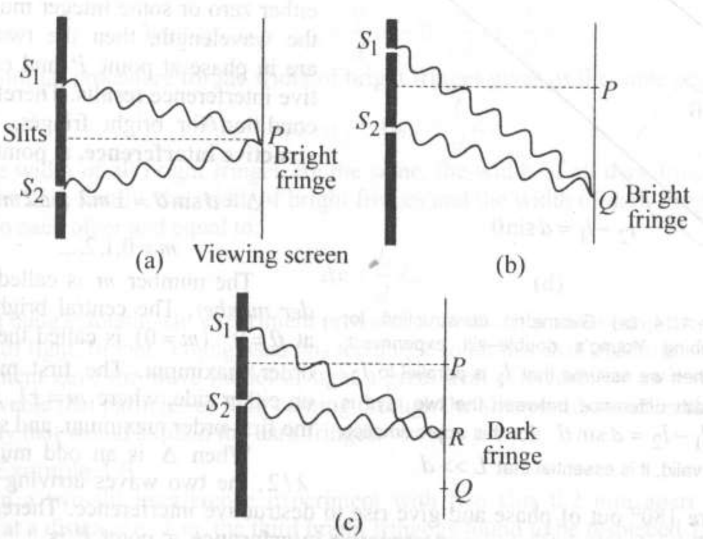
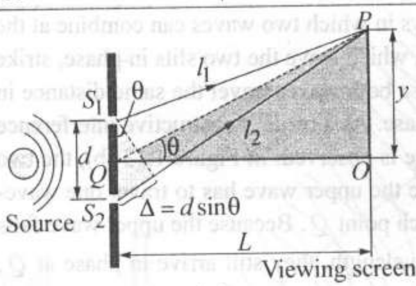
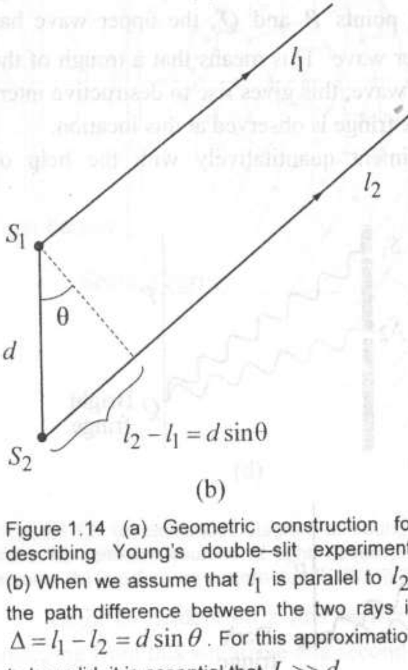


Figure 1.13 (a) Constructive interference occurs at point P when the waves combine; (b) Constructive interference occurs at point Q because the upper wave falls a wavelength behind the lower wave; (c) Destructive interference occurs at R when two waves combine because the upper wave falls half a wavelength behind the lower wave



(a)



(b)

Figure 1.14 (a) Geometric construction for describing Young's double–slit experiment; (b) When we assume that l_1 is parallel to l_2. the path difference between the two rays is $\Delta = l_1 - l_2 = d\sin\theta$. For this approximation to be valid, it is essential that $L >> d$

The viewing screen is located a perpendicular distance L from the double–slit barrier. S_1 and S_2 are separated by a distance d, and the source is monochromatic. To reach any arbitrary point P, a wave from the lower slit travels farther than a wave from the upper slit by a distance $d\sin\theta$. This distance is the *optical path difference* Δ. If we assume that l_1 and l_2 are parallel, which is approximately true because L is much greater than d, then Δ is given by

$$\Delta = l_2 - l_1 = d\sin\theta . \qquad (1.15)$$

The value of Δ determines whether the two waves are in phase when they arrive at point P. If Δ is either zero or some integer multiple of the wavelength, then the two waves are in phase at point P and constructive interference results. Therefore, the condition for bright fringes, or **constructive interference,** at point P is

$$\Delta = d\sin\theta = \pm m\lambda = \pm 2m\frac{\lambda}{2}, \quad m = 0,1,2,... \qquad (1.16)$$

The number m is called the *order number*. The central bright fringe at $\theta = 0$ $(m = 0)$ is called the zeroth-order maximum. The first maximum on either side, where $m = \pm 1$, is called the first-order maximum, and so on.

When Δ is an odd multiple of $\lambda/2$, the two waves arriving at point P are 180° out of phase and give rise to destructive interference. Therefore, the *condition for dark fringes,* or **destructive interference,** at point P is

$$\Delta = d\sin\theta = \pm(2m+1)\frac{\lambda}{2}, \quad m = 0,1,2,... \qquad (1.17)$$

It is useful to obtain expressions for the positions of the bright and dark fringes measured vertically from O to P. In addition to our assumption that $L >> d$ we assume that $d >> \lambda$. These can be valid assumptions because in practice L is often of the order of 1 m, d a fraction of a millimeter, and λ a fraction of a micrometer for visible light. Under these conditions, θ is small; thus, we can use the approximation $\sin\theta = \tan\theta$. Then, from triangle OPQ in Figure 1.14 (a), we see that

$$y = L\tan\theta = L\sin\theta. \tag{1.18}$$

Solving Eq. (1.16) for $\sin\theta$ and substituting the result into Eq. (1.18), we see that the positions of the bright fringes measured from O are given by the expression

$$y_b = \frac{\lambda L}{d}m. \tag{1.19}$$

Using Eqs. (1.17) and (1.18), we find that the dark fringes are located at

$$y_d = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right). \tag{1.20}$$

Now we obtain the width of bright and dark fringes. The m-th bright fringe is formed when $y_{mb} = \frac{\lambda L}{d}m$. Similarly the position of $(m+1)$ bright fringe is: $y_{(m+1)_b} = \frac{\lambda L}{d}(m+1)$. Hence the width of the m-th dark fringe is given by

$$\Delta y_b = y_{(m+1)_b} - y_{mb} = \frac{\lambda L}{d}(m+1) - \frac{\lambda L}{d}m = \frac{L}{d}\lambda.$$

Similar procedure for the width of bright fringes gives us the same result:

$$\Delta y_d = y_{(m+1)_d} - y_{md} = \frac{L}{d}\lambda,$$

i.e., the width of all bright fringes are the same, the width of all dark fringes are the same and, finally, the width of bright fringes and the width of dark fringes are equal to each other and equal to:

$$\Delta y = \frac{L}{d}\lambda. \tag{1.21}$$

Young's double–slit experiment provides a method for measuring the wavelength of light. In fact, Young used this technique to do just that. Additionally, the experiment gave the wave model of light a great deal of credibility as it was inconceivable that particles of light coming through the slits could cancel each other in a way that would explain the dark fringes.

Example 1.5

In a two-slit interference experiment with two slits 0.2 mm apart and a screen at a distance of 1 m, the third bright fringe is found to be displaced 7.5 mm from the central fringe. Find the wavelength of the light used.

Solution

Let λ be the unknown wavelength. Then

$$\lambda = \frac{y_m d}{mL} = \frac{75\times 0.2\ \text{mm}}{3\times 1000\,\text{mm}} = 5\times 10^{-5}\ \text{cm}.$$

Example 1.6

A radio station operating at a frequency of $1500\ \text{kHz} = 1.5\times10^6$ Hz has two identical vertical dipole antennas spaced 400 m apart. Where are the intensity maxima and minima in the resulting radiation pattern?

Solution

The wavelength is $\lambda = c/f = 200$ m. The directions of the intensity maxima are those for which the path difference is zero or an integer number of wavelengths, as given by Eq. (1.16). Inserting the numerical values, we find

$$\sin\theta = \frac{m\lambda}{d} = \frac{m\,200\,\text{m}}{400\,\text{m}} = \frac{m}{2}, \quad \theta = 0, \pm30°, \pm90°.$$

In this example, values of m greater than 2 give values of $\sin\theta$ greater than unity and thus have no meaning; there is *no* direction for which the path difference is three or more wavelengths. Similarly, the directions having zero intensity (complete destructive interference) are given by:

$$\sin\theta = \frac{(2m+1)\frac{\lambda}{2}}{d} \approx \frac{(2m+1)\frac{200\,\text{m}}{2}}{400\,\text{m}}, \quad \theta = \pm14.5°, +48.5°.$$

In this case values of m greater than 1 have no meaning, for the reason just mentioned.

Example 1.7

Measuring the Wavelength of a Light Source.

A viewing screen is separated from a double-slit source by 1.2 m. The distance between the two slits is 0.030 mm. The second-order bright fringe $(m=2)$ is 4.5 cm from the center line.

a) Determine the wavelength of the light.

Solution

We can use Eq. (1.19), with $(m=2)$, $y_2 = 4.5\times10^{-2}$ m, $L=1.2$ m, and $d = 3.0\times10^{-5}$ m:

$$\lambda = \frac{dy_2}{mL} = \frac{(3.0\times10^{-5}\,\text{m})(4.5\times10^{-2}\,\text{m})}{2(1.2\,\text{m})} = 5.6\times10^{-7}\ \text{m}.$$

b) Calculate the distance between adjacent bright fringes.

Solution

From Eq. (1.19) and the results of part (a), we obtain

$$y_{m+1} - y_m = \frac{\lambda L(m+1)}{d} - \frac{\lambda Lm}{d} = \frac{\lambda L}{d} = \frac{(5.6\times10^{-7}\,\text{m})(1.2\text{m})}{3.0\times10^{-5}\,\text{m}} = 2.2\times10^{-2}\ \text{m}.$$

Note that the spacing between all fringes is equal.

Example 1.8

A light source emits visible light of two wavelengths: $\lambda = 430$ nm and $\lambda' = 510$ nm. The source is used in a double-slit interference experiment in which $L = 1.5$ m and $d = 0.025$ mm. Find the separation distance between the third-order bright fringes.

Solution

Using Eq. (1.19), with $m = 3$, we find that the fringe positions corresponding to these two wavelengths are

$$y_3 = \frac{\lambda L}{d} m = 3\frac{\lambda L}{d} = 7.74 \times 10^{-2} \text{ m},$$

$$y_3' = \frac{\lambda' L}{d} m = 3\frac{\lambda' L}{d} = 9.18 \times 10^{-2} \text{ m}.$$

Hence, the separation distance between the two fringes is

$$\Delta y = y_3' - y_3 = 9.18 \times 10^{-2} \text{ m} - 7.74 \times 10^{-2} \text{ m} = 1.4 \times 10^{-2} \text{ m}.$$

Exercises

1.23. A double-slit arrangement produces interference fringes for sodium light ($\lambda = 5890$ Å) that are 0.20° apart. What is the angular fringe separation if the entire arrangement is immersed in water? (Ans.: 0.15°).

1.24. Light with a wavelength of 442 nm passes through a double-slit system that has a slit separation $d = 0.40$ mm. Determine how far away a screen must be placed so that a dark fringe appears directly opposite both slits, with just one bright fringe between them.

1.25. Aleksey asserted that it is impossible to observe interference fringes in a two-source experiment if the distance between sources is less than half the wavelength of the wave. Do you agree? Explain.

1.26. An amateur scientist proposed to record a two-source interference pattern by using only one source, placing it first in position S_1 in Figure 1.14 (a) and turning it on for a certain time, then placing it at S_2 and turning it on for an equal time. Does this work?

1.27. If a two-slit interference experiment were done with white light, what would be seen?

1.28. In Young's double-slit experiment, the monochromatic source of light is replaced by white light source. What would be the central fringe?

1.29. Does the fringe width for dark fringe is different from that for white fringe?

1.30. What happens to the interference pattern when the entire arrangement of double-slit experiment is dipped in water?

1.31. What will be the effect on interference pattern obtained in Young's double-slit experiment if

1) one slit is covered,

2) a source of light of higher frequency is used,

3) distance between two slits be increased,

4) distance between screen and double-slitted barrier is increased.

1.32. Two slits are separated by 0.320 mm. A beam of 500-nm light strikes the slits and produces an interference pattern. Determine the number of maxima observed in the angular range $-30.0° < \varphi < 30.0°$.

1.33. In the double-slit arrangement of Figure 1.14, $d = 0.150$ mm, $L = 140$ cm, $\lambda = 643$ nm, and $y = 1.80$ cm. (a) What is the path difference Δ for the rays from the two slits arriving at point P? (b) Express this path difference in terms of λ. (c) Does point P correspond to a maximum, a minimum, or an intermediate condition? (Ans.: (a) 1.93 μm, (b) 3.00 λ, (c) maximum).

1.34. Two slits are spaced 0.3 mm apart and are placed 50 cm from a screen. What is the distance between the second and third dark lines of the interference pattern when the slits are illuminated with light of 600-nm wavelength? (Ans.: 1.0 mm).

1.35. Young's experiment is performed with sodium light ($\lambda = 589$ nm). Fringes are measured carefully on a screen 100 cm away from the double-slit, and the center of the twentieth fringe is found to be 11.78 mm from the center of the zeroth fringe. What is the separation of the two slits?

1.36. Light from a mercury-arc lamp is passed through a filter that blocks everything except for one spectrum line in the green region of the spectrum. It then falls on two slits separated by 0.6 mm. In the resulting interference pattern on a screen 2.5 m away, adjacent bright fringes are separated by 2.27 mm. What is the wavelength? (Ans.: 545 nm).

1.37. In Young's double-slit experiment, why do we use monochromatic light? If white light is used, how would the pattern change?

1.38. A laser beam ($\lambda = 632.8$ nm) is incident on two slits 0.200 mm apart. How far apart are the bright interference fringes on a screen 5.00 m away from the slits? (Ans.: 1.58 cm).

1.39. A Young's interference experiment is performed with monochromatic light. The separation between the slits is 0.500 mm, and the interference pattern on a screen 3.30 m away shows the first maximum 3.40 mm from the center of the pattern. What is the wavelength? (Ans.: 5.5×10^{-7} m).

1.40. Young's double-slit experiment is performed with 589-nm light and a slit-to-screen distance of 2.00 m. The tenth interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits. (Ans.: 1.54 mm).

1.41. A pair of narrow, parallel slits separated by 0.250 mm is illuminated by green light ($\lambda = 546.1$ nm). The interference pattern is observed on a screen 1.20 m away from the plane of the slits. Calculate the distance (a) from the central maximum to the first bright region on either side of the central maximum and (b) between the first and second dark bands. (Ans.: (a) 2.62 mm, (b) 2.62 mm.)

1.11 Intensity Distribution of the Double-Slit Interference Pattern

Note that the edges of the bright fringes in Figure 1.12 (b) are fuzzy. So far we have discussed the locations of only the centers of the bright and dark fringes on a distant screen. We now direct our attention to the intensity of the light at other points between the positions of maximum of constructive and destructive interference. In other words, we now calculate the distribution of light intensity associated with the double-slit interference pattern.

Again, suppose that the two slits represent coherent sources of sinusoidal waves such that the two waves from the slits have the same angular frequency ω and a constant phase difference δ. The total magnitude of the electric field at point P on the screen in Figure 1.14 (a) is the vector superposition of the two waves. Assuming that the two waves have the same amplitude E_0, we can write the magnitude of the electric field at point P due to each wave separately as

$$E_1 = E_0 \sin \omega t \quad \text{and} \quad E_2 = E_0 \sin(\omega t + \delta).$$

Although the waves are in phase at the slits, their phase difference δ at point P depends on the optical path difference $\Delta = l_1 - l_2 = d \sin\theta$. Because a path difference of λ (constructive interference) corresponds to a phase difference of 2π rad, we obtain the ratio

$$\frac{\Delta}{\lambda} = \frac{\delta}{2\pi},$$

$$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda} d \sin\theta.$$

This equation tells us precisely how the phase difference δ depends on the angle θ.

Using the superposition principle, we can obtain the magnitude of the resultant electric field at point P:

$$E_P = E_1 + E_2 = E_0[\sin\omega t + \sin(\omega t + \delta)].$$

To simplify this expression, we use the trigonometric identity $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$. Taking $A = \omega t + \delta$ and $B = \omega t$, we can write the expression for E_P in the form

$$E_P = 2E_0 \cos\left(\frac{\delta}{2}\right)\sin\left(\omega t + \frac{\delta}{2}\right). \tag{1.22}$$

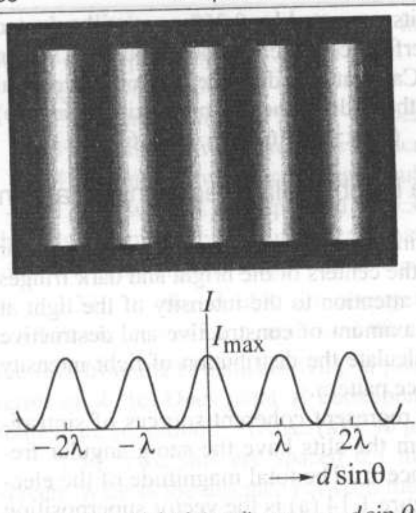


Figure 1.15 Light intensity versus $d\sin\theta$ for a double-slit interference pattern when the screen is far from the slits ($L >> d$)

This result indicates that the electric field at point P has the same frequency ω as the light at the slits, but that the amplitude of the field is multiplied by the factor $2\cos(\delta/2)$. To check the consistency of this result, note that if $\delta = 0, 2\pi, 4\pi, \ldots$, then the electric field at point P is $2E_0$, corresponding to the condition for constructive interference. These values of δ are consistent with Eq. (1.16) for constructive interference. Likewise, if $\delta = \pi, 3\pi, 5\pi, \ldots$, then the magnitude of the electric field at point P is zero; this is consistent with Eq. (1.17) for destructive interference.

Finally, to obtain an expression for the light intensity at point P, recall that the **intensity of a wave** is proportional to the square of the resultant electric field magnitude at that point.

We can therefore express the light intensity at point P as

$$I \propto E_P^2 = E_0^2 \cos^2\left(\frac{\delta}{2}\right)\sin^2\left(\omega t + \frac{\delta}{2}\right). \tag{1.23}$$

Most light-detecting instruments measure time-averaged light intensity, and the time-averaged value of $\sin^2(\omega t + \delta/2)$ over one cycle is $1/2$. Therefore, we can write the average light intensity at point P as

$$I = I_{\max}\cos^2\left(\frac{\delta}{2}\right), \tag{1.24}$$

where $I_{\max}$ is the maximum intensity on the screen and the expression represents the time average. Substituting the value for δ given by Eq. (1.11) into this expression, we find that

$$I = I_{\max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right). \tag{1.25}$$

Alternatively, because $\sin\theta = y/L$ for small values of θ in Figure 1.14, we can write the intensity in the form

$$I = I_{\max}\cos^2\left(\frac{\pi d}{\lambda L}y\right).$$

Constructive interference, which produces light intensity maxima, occurs when the quantity $\pi dy/\lambda L$ is an integral multiple of π, corresponding to $y=(\lambda L/d)m$. This is consistent with Eq. (1.19).

We have seen that the interference phenomena arising from two sources depend on the relative phase of the waves at a given point. Furthermore, the phase difference at a given point depends on the path difference between the two waves. The resultant light intensity at a point is proportional to the square of the resultant electric field at that point. That is, the light intensity is proportional to $(E_1+E_2)^2$. It would be incorrect to calculate the light intensity by adding the intensities of the individual waves. This procedure would give $E_1^2+E_2^2$, which of course is not the same as $(E_1+E_2)^2$. Note, however, that $(E_1+E_2)^2$ has the same average value as $E_1^2+E_2^2$ when the time average is taken over all values of the phase difference between E_1 and E_2. Hence, the law of conservation of energy is not violated.

We can apply phasor method for description of interference pattern. Figure 1.16 represents the phasors diagrams for various values of the phase difference δ and the corresponding values of the path difference Δ. The light intensity at a point is a maximum when E_R is a maximum; this occurs at $\delta=0,2\pi,4\pi,...$ The light intensity at some point is zero when E_R is zero; this occurs at $\delta=\pi,3\pi,5\pi,...$ These results are in complete agreement with analytical procedure described earlier.

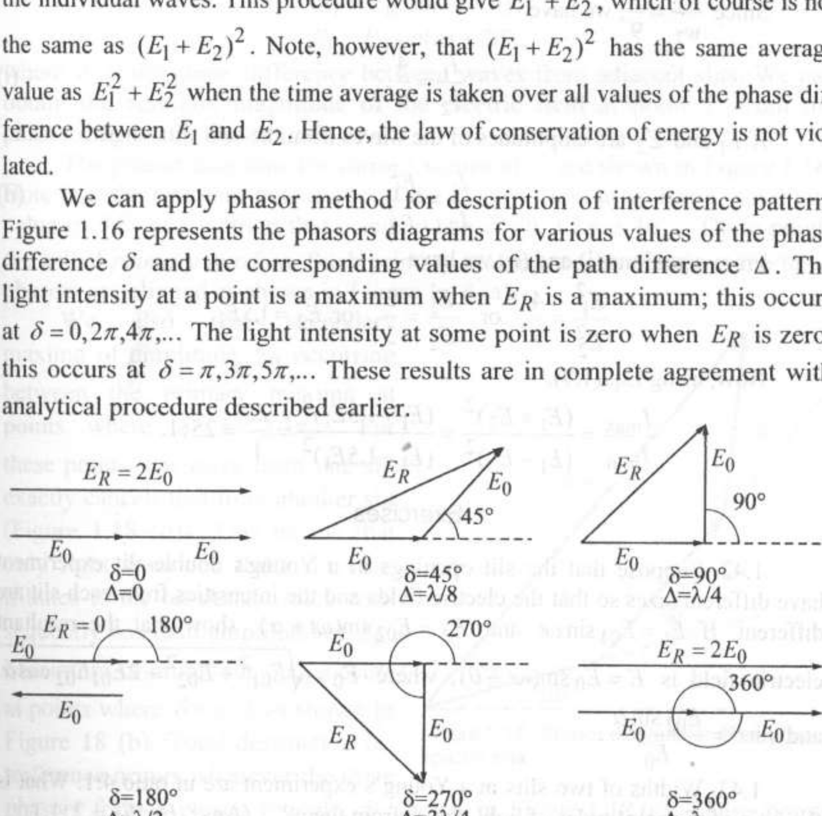


Figure 1.16 Phasor diagram for a double-slit interference pattern. The resultant phasor E_R is a maximum when $\delta=0,\pm 2\pi,\pm 4\pi,...$ and is zero when $\delta=\pm\pi,\pm 3\pi,...$

Example 1.9

If the two slits in Young's experiment have width in the ratio $w_1 : w_2 = 4:9$, then find the ratio of intensity at the maximum to the intensity at the minimum in the interference pattern.

Solution

The intensity of light due to a slit is directly proportional to the width of the slit. Therefore,

$$\frac{I_1}{I_2} = \frac{w_1}{w_2}.$$

Since $\frac{w_1}{w_2} = \frac{4}{9}$, we have

$$\frac{I_1}{I_2} = \frac{4}{9}. \qquad \text{(i)}$$

If E_1 and E_2 are amplitudes of the waves from the two slits, then

$$\frac{I_1}{I_2} = \frac{E_1^2}{E_2^2} \qquad \text{(ii)}$$

From equations (i) and (ii) we have

$$\frac{E_1^2}{E_2^2} = \frac{4}{9}, \text{ or } \frac{E_1}{E_2} = \frac{2}{3}, \text{ or } E_2 = 1{,}5E_1.$$

Now, using Eq. (1.9),

$$\frac{I_{max}}{I_{min}} = \frac{(E_1+E_2)^2}{(E_1-E_2)^2} = \frac{(E_1+1.5E_1)^2}{(E_1-1.5E_1)^2} = \frac{25}{1} = 25:1.$$

Exercises

1.42. Suppose that the slit openings in a Young's double-slit experiment have different sizes so that the electric fields and the intensities from each slit are different. If $E_1 = E_{01}\sin\omega t$ and $E_2 = E_{02}\sin(\omega t + \alpha)$, show that the resultant electric field is $E = E_0\sin(\omega t + \theta)$, where $E_0 = \sqrt{E_{01}{}^2 + E_{02}{}^2 + 2E_{01}E_{02}\cos\alpha}$ and $\sin\theta = \frac{E_{02}\sin\alpha}{E_0}$.

1.43. Widths of two slits in a Young's experiment are in ratio 4:1. What is the ratio of the amplitudes of light waves from them? (Ans.: $E_1 : E_2 = 2:1$.)

1.44. What is the ratio of slits widths, when the amplitudes of light waves from them have a ratio $\sqrt{2}:1$? (Ans.: $w_1 : w_2 = 2:1$.)

1.45. If the two slits in Young's double–slit experiment have width ratio 4:1, deduce the ratio of intensities at minima and maxima in the interference pattern. (Ans.: $I_1 : I_2 = 9:1$).

1.12 *N*-Slit Interference Pattern

Using phasor diagrams let us analyze the interference pattern caused by N equally spaced slits. For simplicity we'll take $N = 3$. We can express the electric field components at a point P on the screen caused by waves from the individual slits as

$$E_1 = E_0 \sin \omega t\,,$$
$$E_1 = E_0 \sin(\omega t + \delta)\,,$$
$$E_1 = E_0 \sin(\omega t + 2\delta)\,,$$

where δ is the phase difference between waves from adjacent slits. We can obtain the resultant magnitude of the electric field at point P from the phasor diagram in Figure 1.17.

The phasor diagrams for various values of δ are shown in Figure 1.18. Note that the resultant magnitude of the electric field at P has a maximum value of $3E_0$ at condition that occurs when $\delta = 0, \pm 2\pi, \pm 4\pi, \ldots$ These points are called *primary maxima*. Such primary maxima occur whenever the three phasors are aligned as shown in Figure 1.18 (a).

We also find secondary maxima of amplitude E_0 occurring between the primary maxima at points where $\delta = \pm\pi, \pm 3\pi, \ldots$ For these points, the wave from one slit exactly cancels that from another slit (Figure 1.18 (d)). This means that only light from the third slit contributes to the resultant, which consequently has total amplitude of E_0. Maximum of amplitude $2E_0$ occurs at points where $\delta = \pi/3$ as shown in Figure 18 (b). Total destructive interference occurs whenever the three

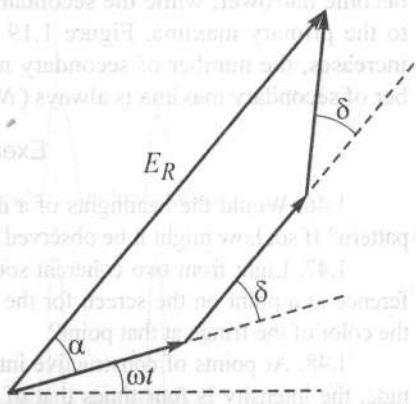


Figure 1.17 Phasor diagram for three equally spaced slits

phasors form a closed triangle, as shown in Figure 1.18 (c). These points where $E_R = 0$ correspond to $\delta = \pm 2\pi/3, \pm 4\pi/3, \ldots$ You should be able to construct other phasor diagrams for values of δ greater than π.

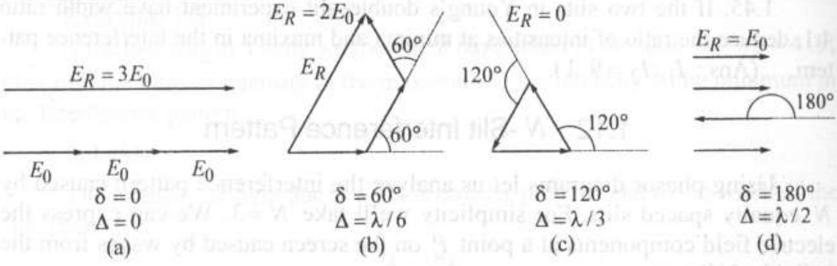


Figure 1.18 Phasor diagrams for three equally spaced slits at various values of δ. Note from (a) that there are primary maxima of amplitude $3E_0$ and from (d) that there are secondary maxima of amplitude E_0

Figure 1.19 shows multiple-slit interference patterns for a number of configurations. For three slits, note that the primary maxima are nine times more intense than the secondary maxima as measured by the height of the curve.

This is because the intensity varies as $E_R{}^2$. For N slits, the intensity of the primary maxima is N^2 times greater than that due to a single slit. As the number of slits increases, the primary maxima increase in intensity and become narrower, while the secondary maxima decrease in intensity relative to the primary maxima. Figure 1.19 also shows that as the number of slits increases, the number of secondary maxima also increases. In fact, the number of secondary maxima is always $(N-2)$, where N is the number of slits.

Exercises

1.46. Would the headlights of a distant car form a two-source interference pattern? If so, how might it be observed? If not, why not?

1.47. Light from two coherent sources is reaching a screen. If the path difference at a point on the screen for the yellow light be $3\lambda/2$, then what will be the color of the fringe at that point?

1.48. At points of constructive interference between waves of equal amplitude, the intensity is four times that of either individual wave. Does this violate energy conservation? If not, why not?

1.49. In using the superposition principle to calculate intensities in interference and diffraction patterns, could one add the intensities of the waves instead of their amplitudes? What is the difference?

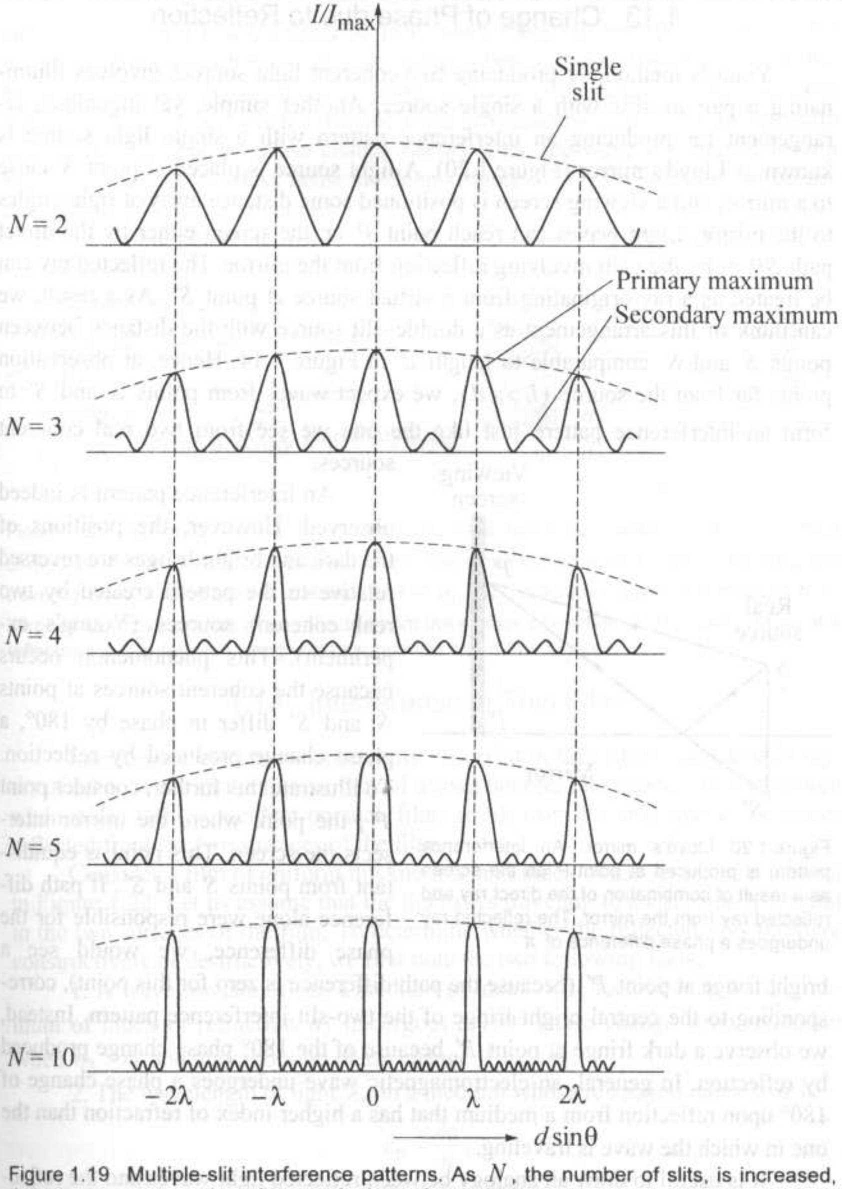


Figure 1.19 Multiple-slit interference patterns. As N, the number of slits, is increased, the primary maxima (the tallest peaks in each graph) become narrower but remain fixed in position, and the number of secondary maxima increases

1.13 Change of Phase due to Reflection

Young's method for producing two coherent light sources involves illuminating a pair of slits with a single source. Another simple, yet ingenious, arrangement for producing an interference pattern with a single light source is known as Lloyd's mirror (Figure 1.20). A light source is placed at point S close to a mirror, and a viewing screen is positioned some distance away at right angles to the mirror. Light waves can reach point P on the screen either by the direct path SP or by the path involving reflection from the mirror. The reflected ray can be treated as a ray originating from a virtual source at point S'. As a result, we can think of this arrangement as a double–slit source with the distance between points S and S' comparable to length d in Figure 1.14. Hence, at observation points far from the source $(L >> d)$, we expect waves from points S and S' to form an interference pattern just like the one we see from two real coherent sources.

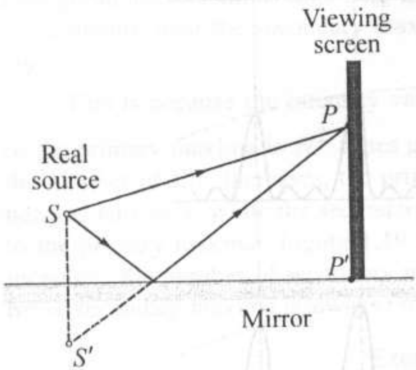


Figure 1.20 Lloyd's mirror. An interference pattern is produced at point P on the screen as a result of combination of the direct ray and reflected ray from the mirror. The reflected ray undergoes a phase difference of π

An interference pattern is indeed observed. However, the positions of the dark and bright fringes are reversed relative to the pattern created by two real coherent sources (Young's experiment). This phenomenon occurs because the coherent sources at points S and S' differ in phase by 180°, a phase change produced by reflection. To illustrate this further, consider point P', the point where the mirror intersects the screen. This point is equidistant from points S and S'. If path difference alone were responsible for the phase difference, we would see a bright fringe at point P' (because the path difference is zero for this point), corresponding to the central bright fringe of the two-slit interference pattern. Instead, we observe a dark fringe at point P' because of the 180° phase change produced by reflection. In general, an electromagnetic wave undergoes a phase change of 180° upon reflection from a medium that has a higher index of refraction than the one in which the wave is traveling.

It is useful to draw an analogy between reflected light waves and the reflections of a transverse wave pulse on a stretched string. The reflected pulse on a string undergoes a phase change of 180° when reflected from the boundary of a denser medium, but no phase change occurs when the pulse is reflected from the boundary of a less dense medium. Similarly, an electromagnetic wave undergoes a 180° phase change when reflected from a boundary leading to an optically denser medium, but no phase change occurs when the wave is reflected from a boundary leading to a less dense medium (Figure 1.21). These rules can be deduced from Maxwell's equations.

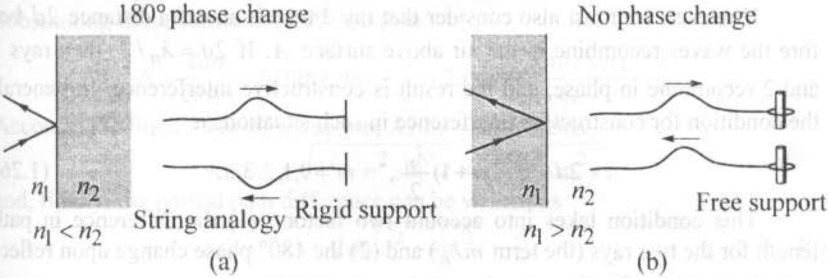


Figure 1.21 (a) For $n_1 < n_2$, a light ray traveling in medium 1 when reflected from the surface of medium 2 undergoes a 180° phase change. The same thing happens with a reflected pulse traveling along a string fixed at one end; (b) For $n_1 > n_2$, a light ray traveling in medium 1 undergoes no phase change when reflected from the surface of medium 2. The same is true of a reflected wave pulse on a string whose supported end is free to move

1.14 Interference in Thin Film

Interference effects are commonly observed in thin films, such as thin layers of oil on water or the thin surface of a soap bubble. The varied colors observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film.

Consider a film of uniform thickness d and index of refraction n, as shown in Figure 1.22. Let us assume that the light rays traveling in air are nearly normal to the two surfaces of the film. To determine whether the reflected rays interfere constructively or destructively, we first note the two following facts:

1. A wave traveling from a medium of index of refraction n_1 toward a medium of index of refraction n_2 undergoes a 180° phase change upon reflection when $n_2 > n_1$ and undergoes no phase change if $n_2 < n_1$.

2. The wavelength of light λ_n in a medium whose refraction index is n is

$$\lambda_n = \frac{\lambda_0}{n},$$

where λ_0 is the wavelength of the light in free space.

Let us apply these rules to the film of Figure 1.22, where $n_{film} > n_{air}$. Reflected ray 1, which is reflected from the upper surface (A), undergoes a phase change of 180° with respect to the incident wave. Reflected ray 2, which is reflected from the lower film surface (B), undergoes no phase change because it is reflected from a medium (air) that has a lower index of refraction. Therefore ray 1 is 180° out of phase with ray 2, which is equivalent to a path difference of $\lambda/2$.

However, we must also consider that ray 2 travels an extra distance $2d$ before the waves recombine in the air above surface A. If $2d = \lambda_n/2$, then rays 1 and 2 recombine in phase, and the result is constructive interference. In general, the condition for constructive interference in such situations is

$$2d = \pm(2m+1)\frac{\lambda_n}{2}, \quad m = 0,1,2,3,... \quad (1.26)$$

This condition takes into account two factors: (1) the difference in path length for the two rays (the term $m\lambda_n$) and (2) the 180° phase change upon reflection (the term $\lambda_n/2$). Because $\lambda_n = \lambda_0/n$, we can write:

$$2nd = \pm(2m+1)\frac{\lambda_0}{2} \quad m = 0,1,2,3,... \quad (1.27)$$

If the extra distance $2d$ traveled by ray 2 corresponds to a multiple of λ_n, then the two waves combine out of phase, and the result is destructive interference. The general equation for destructive interference is

$$2nd = \pm m\lambda \quad m = 1,2,3,.... \quad (1.28)$$

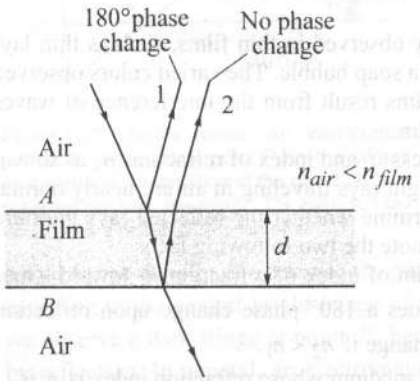


Figure 1.22 Interference in light reflected from a thin film is due to a combination of rays reflected from the upper and lower surface of the film

Consider a general case of interference in thin film, when angle of incidence is nonzero (Figure 1.23). AB is a wave front. Ray 1 travels extra distance BC in air before the waves recombine in point C. Additionally it is reflected from the medium with larger index of refraction and hence undergoes a 180° phase change with respect to the incident wave, i.e. its optical path is $(BC)n_{air} + \lambda/2$. Optical path of the ray 2 is $(AO+OC)n_{film}$. Then optical path difference

$$\Delta = (AO + OC)n_{film} - (BC)n_{air} - \frac{\lambda}{2}. \quad (1.29)$$

It is clear from Figure 1.23 that

$$BC = AC\sin i = 2d\tan r\sin i$$

and

$$(AO + OC) = \frac{2d}{\cos r}.$$

Substituting these values in (1.29) we obtain

$$\Delta = \frac{2dn}{\cos r} - 2d\tan r\sin i - \frac{\lambda}{2} = 2d\frac{n^2 - n\sin r\sin i}{n\cos r} - \frac{\lambda}{2}.$$

According to Snell's law of refraction, $n\sin r = \sin i$, hence

$$n\cos r = \sqrt{n^2 - n^2\sin^2 r} = \sqrt{n^2 - \sin^2 i},$$

and, finally, the optical path difference can be written as

$$\Delta = 2d\sqrt{n^2 - \sin^2 i} - \frac{\lambda}{2}. \quad (1.30)$$

The foregoing conditions for constructive and destructive interference are valid when the medium above the top surface of the film is the same as the medium below the bottom surface. The medium surrounding the film may have a refractive index less than or greater than that of the film. In either case, the rays reflected from the two surfaces are out of phase by 180°. If the film is placed between two different media, one with $n < n_{film}$ and the other with $n > n_{film}$ then the conditions for constructive and destructive interference are reversed. In this case, either there is a phase change of 180° for both ray 1 reflecting from surface A and ray 2 reflecting from surface B or there is no phase change for either ray; hence, the net change in relative phase due to the reflections is zero.

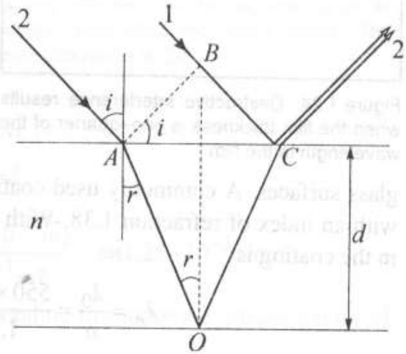


Figure 1.23 Interference in thin film when angle of incidence is not zero

The thin–film interference is used in nonreflective coatings for lenses. A thin layer or film of hard transparent material with an index of refraction smaller than that of the glass is deposited on the surface of the glass, as in Figure 1.24. If the coating has the proper index of refraction, equal quantities of light will be reflected from its outer surface and from the boundary surface between it and the glass. Furthermore, since in both reflections the light is reflected from a medium of greater index than that in which it is traveling, the same phase change occurs in each reflection. It follows that if the film thickness is 1/4 wavelength in the film (normal incidence is assumed), the light reflected from the first surface will be 180° out of phase with that reflected from the second, and complete destructive interference will result.

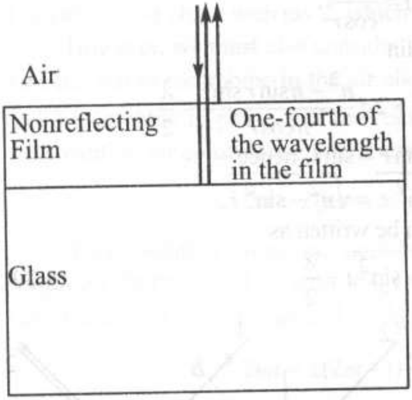


Figure 1.24 Destructive interference results when the film thickness is one–quarter of the wavelength in the film

Of course, the thickness can be 1/4 wavelength for only one particular wavelength. This is usually chosen in the yellow–green portion of the spectrum (about 550 nm), where the eye is most sensitive. Some reflection then takes place at both longer and shorter wavelengths, and the reflected light has a purple hue. The overall reflection from a lens or prism surface can be reduced in this way from 4% to a fraction of 1%. The treatment is extremely effective in eliminating stray reflected light and increasing the contrast in an image formed by highly corrected lenses having a large number of air–glass surfaces. A commonly used coating material is magnesium fluoride, MgF_2, with an index of refraction 1.38. With this coating, the wavelength of green light in the coating is

$$\lambda_n = \frac{\lambda_0}{n} = \frac{550\times10^{-9}}{1.38} = 4\times10^{-5}\ \text{cm}$$

and the thickness of a **"nonreflecting" film** of MgF_2 is 10^{-5} cm.

If a material whose index of refraction is greater than that of glass is deposited on glass to a thickness of wavelength, then the reflectivity is increased. For example, a coating of index 2.5 will allow 38% of the incident energy to be reflected, instead of the usual 4% when there is no coating. By use of multilayer coatings, it is possible to achieve reflectivity for a particular wavelength of almost 100%. These coatings are used for "one–way" windows and reflective sunglasses.

Example 1.10

Suppose the two glass plates in Figure 1.25 are two microscope slides 10 cm long. At one end they are in contact, and at the other end they are separated by a thin piece of tissue paper 0.02 mm thick. What is the spacing of the resulting interference fringes? Is the fringe adjacent to the line of contact bright or dark? Assume monochromatic light with $\lambda = 500$ nm.

Solution

To answer the second question first, the fringe at the line of contact is dark because the wave reflected from the lower surface of the air wedge has undergone a half-cycle phase shift, while that from the upper surface has not. For this reason, the condition for *destructive* interference (a dark fringe) is that the path difference $2d$ be an integer number of wavelengths:

$$2d = m\lambda, \quad (m = 0, 1, 2, 3, \ldots).$$

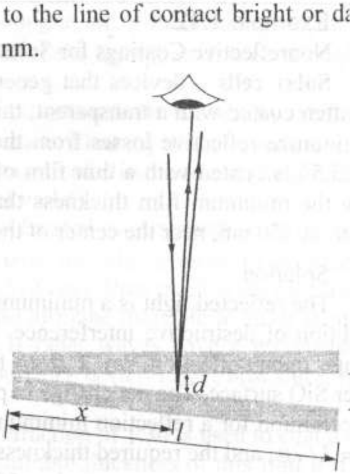


Figure 1.25 Interference between two light waves reflected from the two sides of an air wedge separating two glass plates. The path difference is $2d$

From similar triangles in Figure 1.25, d is proportional to the distance x from the line of contact:

$$\frac{d}{x} = \frac{h}{l}.$$

Combining this with path difference, we find

$$\frac{2xh}{l} = m\lambda,$$

$$\text{or } x = m\frac{l\lambda}{2h} = m\frac{(0.1\,\text{m})\times(500\times10^{-9}\,\text{m})}{2\times(0.02\times10^{-3}\,\text{m})} = m(1.25\times10^{-3})\ \text{m}.$$

Thus successive dark fringes, corresponding to successive integer values of m, are spaced 1.25 mm apart.

Example 1.11

Calculate the minimum thickness of a soap-bubble film ($n = 1.33$) that results in constructive interference in the reflected light if the film is illuminated with light whose wavelength in free space is $\lambda = 600$ nm.

Solution.

The minimum film thickness for constructive interference in the reflected light corresponds to $m = 0$ in Eq. (1.27). This gives $2nd = \lambda/2$, or

$$d = \frac{\lambda}{4n} = \frac{600\,\text{nm}}{4(1.33)} = 113\ \text{nm}.$$

Example 1.12

Nonreflective Coatings for Solar Cells.

Solar cells – devices that generate electricity when exposed to sunlight – are often coated with a transparent, thin film of silicon monoxide (SiO, $n = 1.45$) to minimize reflective losses from the surface. Suppose that a silicon solar cell ($n = 3.5$) is coated with a thin film of silicon monoxide for this purpose. Determine the minimum film thickness that produces the least reflection at a wavelength of 550 nm, near the center of the visible spectrum.

Solution

The reflected light is a minimum when rays 1 and 2 in Figure 1.23 meet the condition of destructive interference. Note that both rays undergo a 180° phase change upon reflection: ray 1 from the upper SiO surface and ray 2 from the lower SiO surface. The net change in phase due to reflection is therefore zero, and the condition for a reflection minimum requires a path difference of $\lambda/2$. Hence $2d = \lambda/2n$, and the required thickness is

$$d = \frac{\lambda}{4n} = \frac{550\,\text{nm}}{4(1.45)} = 94.8 \text{ nm}.$$

A typical uncoated solar cell has reflective losses as high as 30%; a SiO coating can reduce this value to about 10%. This significant decrease in reflective losses increases the cell's efficiency because less reflection means that more sunlight enters the silicon to create charge carriers in the cell. No coating can ever be made perfectly nonreflecting because the required thickness is wavelength-dependent and the incident light covers a wide range of wavelengths.

Glass lenses used in cameras and other optical instruments are usually coated with a transparent thin film to reduce or eliminate unwanted reflection and enhance the transmission of light through the lenses.

Exercises

1.50. A colorless clean layer of oil is spread on water surface. When white light is incident on the layer, the reflected light appears green. What is the reason?

1.51. What is the optical path difference at interference in thin film?

1.52. When a thin oil film spreads out on a puddle of water, the thinnest part of the film looks lightest in the resulting interference pattern. What does this tell you about the relative magnitudes of the refractive indexes of oil and water?

1.53. A glass windowpane with a thin film of water on it reflects less than when it is perfectly dry. Why?

1.54. White light reflected at perpendicular incidence from a soap film has, in the visible spectrum, an interference maximum at 6000 Å and a minimum at 4500 Å, with no minimum in between. If $n = 1.33$ for the film, what is the film thickness, assumed uniform?

1.55. A soap bubble ($n = 1.33$) is floating in air. If the thickness of the bubble wall is 115 nm, what is the wavelength of the light that is most strongly reflected?

1.56. An oil film ($n = 1.45$) floating on water is illuminated by white light at normal incidence. The film is 280 nm thick. Find (a) the dominant observed color in the reflected light and (b) the dominant color in the transmitted light. Explain your reasoning. (Ans.: (a) green, (b) violet).

1.57. A thin film of oil ($n = 1.25$) is located on a smooth, wet pavement. When viewed perpendicular to the pavement, the film appears to be predominantly red (640 nm) and has no blue color (512 nm). How thick is the oil film?

1.58. A possible means for making an airplane invisible to radar is to coat the plane with an antireflective polymer. If radar waves have a wavelength of 3.0 cm and the index of refraction of the polymer is ($n = 1.50$), how thick would you make the coating? (Ans.: 0.50 cm).

1.59. A material having an index of refraction of 1.30 is used to coat a piece of glass ($n = 1.50$). What should be the minimum thickness of this film if it is to minimize reflection of 500-nm light?

1.60. A film of MgF_2 ($n = 1.38$) having a thickness of 1.0×10^{-5} cm is used to coat a camera lens. Are any wavelengths in the visible spectrum intensified in the reflected light? (Ans.: no reflection maxima in the visible spectra).

1.61. A plane wave of monochromatic light is incident normally on a uniform thin film of oil that covers a glass plate. The wavelength of the source can be varied continuously. Fully destructive interference of the reflected light is observed for wavelength of 500 and 700 nm and for no wavelengths in between. If the index of refraction of the oil is 1.50, find the thickness of the oil film.

1.62. An air wedge is formed between two glass plates separated at one edge by a very fine wire, as shown in Figure 1.25. When the wedge is illuminated from above by 600-nm light, 30 dark fringes are observed. Calculate the radius of the wire. (Ans.: 4.35 μm).

1.63. Two rectangular flat glass plates ($n = 1.52$) are in contact along one end and separated along the other end by a sheet of paper 4.0×10^{-3} cm thick (see Figure 1.25). The top plate is illuminated by monochromatic light ($\lambda = 546.1$ nm). Calculate the number of dark parallel bands crossing the top plate (include the dark band at zero thickness along the edge of contact between the two plates).

1.64. Light of wavelength 500 nm is incident perpendicularly from air on a film 1×10^{-4} cm thick and of refractive index 1.375. Part of the light is reflected from the first surface of the film, and part enters the film and is reflected back at the second surface.

a) How many waves are contained along the path of this second part of the light in the film?

b) What is the phase difference between these two parts of the light as they leave the film?

1.65. In Example 1.7 suppose the top plate is glass with $n = 1.4$, the wedge is filled with silicone grease having $n = 1.5$, and the bottom plate is glass with $n = 1.6$. Calculate the spacing between the dark fringes. (Ans.: 0.833 mm).

1.66. A sheet of glass 10 cm long is placed in contact with a second sheet and is held at a small angle with it by a metal strip 0.1 mm thick placed under one end. The glass is illuminated from above with light of 546-nm wavelength. How many interference fringes are observed per centimeter in the reflected light?

1.67. Two rectangular pieces of plane glass are laid one upon the other on a table. A thin strip of paper is placed between them at one edge so that a very thin wedge of air is formed. The plates are illuminated by a beam of sodium light at normal incidence ($\lambda = 589$ nm). Interference fringes are formed, with ten fringes per centimeter length of wedge measured normal to the edges in contact. Find the angle of the wedge. (Ans.: 2.94×10^{-4} rad $= 0.0168°$).

1.68. Is a thin film of quartz suitable as a nonreflecting coating for fabulite? If so, what is the minimum thickness of the film required?

1.69. What is the thinnest film of a 1.40 refractive index coating on glass ($n = 1.5$) for which destructive interference of the violet component (400 nm) of an incident white light beam in air can take place by reflection? (Ans.: 7.14×10^{-8} m).

1.70. Consider a dark fringe in an interference pattern, at which almost no light is arriving. Light from both slits is arriving at this point, but the waves are canceling. Where does the energy go?

1.71. An oil film on water appears brightest at the outer regions, where it is thinnest. From this information, what can you say about the index of refraction of oil relative to that of water?

1.72. In our discussion of thin-film interference, we looked at light reflecting from a thin film. Consider one light ray, the direct ray that transmits through the film without reflecting. Consider a second ray, the reflected ray that transmits through the first surface, reflects from the second, reflects again from the first, and then transmits out into the air, parallel to the direct ray. For normal incidence, how thick must the film be, in terms of the wavelength of light, for the outgoing rays to interfere destructively? Is it the same thickness as for reflected destructive interference?

1.73. If we are to observe interference in a thin film, why must the film not be very thick (on the order of a few wavelengths)?

1.74. Why is the lens on a high-quality camera coated with a thin film?

1.75. A broad source of light ($\lambda = 6800$ Å) illuminates normally two glass plates 12 cm long that touch at one end and are separated by a wire 0.048 mm in a diameter at the other. How many bright fringes appear over 12-cm distance?

1.76. A thin film 4×10^{-5} cm thick is illuminated by white light normal to its surface. Its index of refraction is 1.5. What wavelengths within the visible spectrum will be intensified in the reflected beam? (Ans.: $\lambda = 4800$ Å).

1.15 Newton's Rings

Another method for observing interference in light waves is to place a planoconvex lens on top of a flat glass surface, as shown in Figure 1.26 (a). With this arrangement, the air film between the glass surfaces varies in thickness from zero at the point of contact to some value b at point P. If the radius of curvature R of the lens is much greater than the distance r, and if the system is viewed from above using light of a single wavelength λ, a pattern of light and dark rings is observed, as shown in Figure 1.26 (b). These circular fringes, discovered by Newton, are called **Newton's rings**.

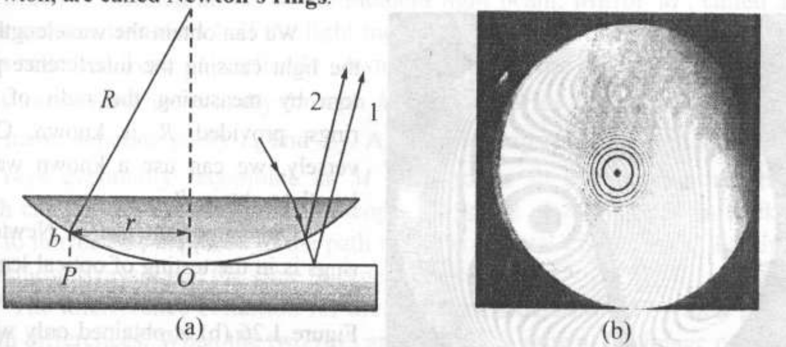


Figure 1.26 (a) The interference effect is due to the combination of ray 1, reflected from the flat plate, with ray 2, reflected from the curved surface of the lens. Ray 1 undergoes a phase change of 180° upon reflection (because it is reflected from a medium of higher refractive index), whereas ray 2 undergoes no phase change (because it is reflected from a medium of lower refractive index); (b) Newton's rings

The contact point at O is dark, as seen in Figure 1.26 (a), because ray 1 undergoes a 180° phase change upon external reflection (from the flat surface); in contrast, ray 2 undergoes no phase change upon internal reflection (from the curved surface). Using the geometry shown in Figure 1.26 (a), we can obtain expressions for the radii of the bright and dark bands in terms of the radius of curvature R and wavelength λ.

$$R^2 = (R-b)^2 + r^2 = R^2 - 2Rb + b^2 + r^2 .$$

As $b^2 << r$, we can neglect b^2 and then

$$R^2 = R^2 - 2Rb + r^2 , \quad \text{or}$$

$$b = \frac{r^2}{2R} .$$

Hence the optical path difference is

$$\Delta = 2bn + \frac{\lambda}{2} = \frac{r^2 n}{R} + \frac{\lambda}{2} . \tag{1.31}$$

In the locations with $\Delta = m\lambda$, bright circulars appear and at the points, in that with $\Delta = (2m+1)\lambda/2$, dark circulars appears.

From Eq. (1.31) is clear that the bright rings have radii given by the expression

$$r = \sqrt{\frac{R\lambda}{2}(2m-1)}, \qquad (1.32)$$

whereas dark rings have radii:

$$r = \sqrt{R\lambda m}. \qquad (1.33)$$

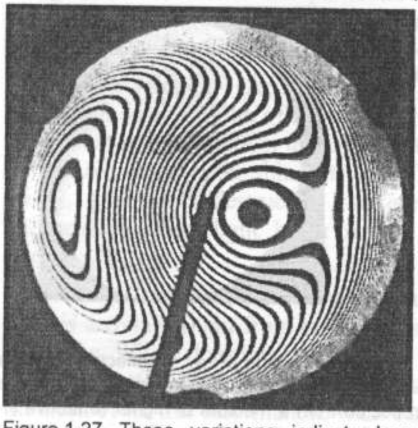


Figure 1.27 These variations indicate how the lens must be repolished to remove the imperfections. Variations in film thickness produce the interesting color pattern

We can obtain the wavelength of the light causing the interference pattern by measuring the radii of the rings, provided R is known. Conversely, we can use a known wavelength to obtain R.

One important use of Newton's rings is in the testing of optical lenses. A circular pattern like that pictured in Figure 1.26 (b) is obtained only when the lens has perfectly symmetric curvature. Variations from such symmetry might produce a pattern like that shown in Figure 1.27.

Exercises

1.77. The diameter of the tenth bright ring in a Newton's rings apparatus changes from 1.40 to 1.27 cm as a liquid is introduced between the lenses and the plate. Find the index of refraction of the liquid. (Ans.: $n = 1.21$).

1.78 A lens with outer radius of curvature R and index of refraction n, rests on a flat glass plate, and the combination is illuminated with white light from above. Is there a dark spot or a light spot at the center of the lens? What does it mean if the observed rings are noncircular?

1.79. In a Newton's rings experiment, a plano-convex lens having a diameter 10 cm is placed on a flat plate. When a 630-nm light is incident normally, 55 bright rings are observed, with the last ring right on the edge of the lens. What is the radius of the curvature of the lens?

1.16 The Michelson's Interferometer

The **interferometer,** invented by A. A. Michelson (1852 – 1931), splits a light beam into two parts and then recombines the parts to form an interference pattern. The device can be used to measure wavelengths or other lengths with great precision.

A schematic diagram of the interferometer is shown in Figure 1.28. A ray of light from a monochromatic source is split into two rays by mirror M, which is inclined at 45° to the incident light beam. Mirror M, called a *beam splitter*, transmits half the light incident on it and reflects the rest. One ray is reflected from M vertically upward toward mirror M_1; and the second ray is transmitted horizontally through M toward mirror M_2. Hence, the two rays travel separate paths L_1 and L_2. After reflecting from M_1 and M_2, the two rays eventually recombine at M to produce an interference pattern, which can be viewed through a telescope. The glass plate P, equal in thickness to mirror M, is placed in the path of the horizontal ray to ensure that the two returning rays travel the same thickness of glass.

The interference condition for the two rays is determined by their path length differences. When the two rays are viewed as shown, the image of M_2 produced by the mirror M is at M_2', which is nearly parallel to M_1. (Because M_1 and M_2 are not exactly perpendicular to each other, the image M_2' is at a slight angle to M_1). Hence, the space between M_2' and M_1 is the equivalent of a wedge-shaped air film. The effective thickness of the air film is varied by moving mirror M_1 parallel to itself with a finely threaded screw adjustment. Under these conditions, the interference pattern is a series of bright and dark parallel fringes. As M_1 is moved, the fringe pattern shifts. For example, if a dark fringe appears in the field of view (corresponding to destructive interference) and M_1 is then moved a distance $\lambda/4$ toward M, the path difference changes by $\lambda/2$ (twice the separation between M_1 and M_2'). What was a dark fringe now becomes a bright fringe. As M_1 is moved an additional distance $\lambda/4$ toward M, the bright fringe becomes a dark fringe. Thus, the fringe pattern shifts by one-half fringe each time M_1 is moved a distance $\lambda/4$. The wavelength of light is then measured by counting the number of fringe shifts for a given displacement of M_1. If the wavelength is accurately known (as with a laser beam) mirror displacements can be measured to within a fraction of the wavelength.

If the fringes are observed through a telescope whose eyepiece is equipped with a cross hair, and m fringes cross the cross hair when the mirror is moved a distance x, then

$$x = m\frac{\lambda}{2} \quad \text{or} \quad \lambda = \frac{2x}{m}.$$

If m is several thousand, the distance x is large enough so that it can be measured with good precision, and hence a precise value of the wavelength λ can be obtained.

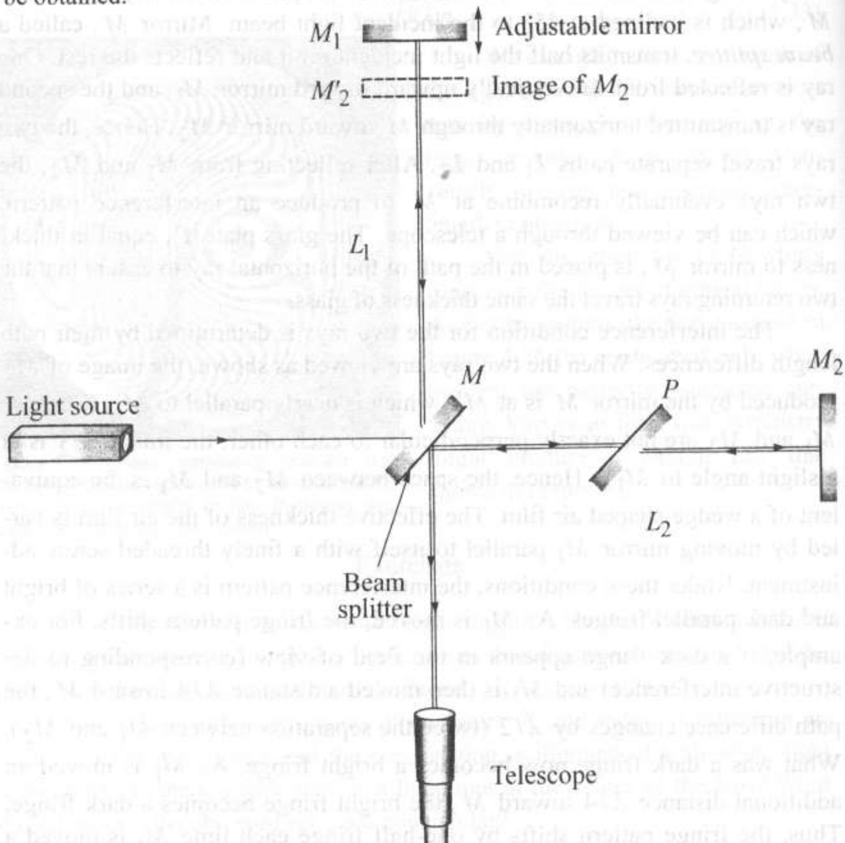


Figure 1.28 Diagram of the Michelson interferometer. A single ray of light is split into two rays by mirror M, which is called a beam splitter. The path difference between the two rays is varied with the adjustable mirror M_1. As M_1 is moved toward M, an interference pattern moves across the field of view

Until recently the meter was defined as a length equal to a specified number of wavelengths of the orange–red light of krypton–86. Before this standard could be established, it was necessary to measure as accurately as possible the number of these wavelengths in the former standard meter, defined as the distance between two scratches on a bar of platinum–iridium. The measurement was made with a modified Michelson interferometer many times and under very carefully controlled conditions. The number of wavelengths in a distance equal to the old standard meter was found to be 1 650 763.73 wavelengths. The meter was then defined as exactly this number of wavelengths. As we mentioned, this definition has recently been superseded by a new length standard based on the unit of time.

Another application of the Michelson interferometer with considerable historical interest is the Michelson–Morley experiment. To understand the purpose of this experiment, we must recall that before the electromagnetic theory of light and Einstein's special theory of relativity became established, physicists believed that the propagation of light waves occurred in a medium called the **ether,** which was believed to permeate all space. In 1887 Michelson and Morley used the Michelson interferometer in an attempt to detect the motion of the Earth through the ether.

Suppose the interferometer in Figure 1.28 was moving from left to right relative to the ether. According to nineteenth-century theory, this would lead to changes in the speed of light. There would be fringe shifts relative to the positions the fringes would have if the instrument were at rest in the ether. Then, when the entire instrument was rotated 90°, the other portions of the paths would be similarly affected, giving a fringe shift in the opposite direction.

Michelson and Morley expected a fringe shift of about four-tenths of a fringe when the instrument was rotated. The shift actually observed was less than a hundredth of a fringe and, within the limits of experimental uncertainty, appeared to be exactly zero. Despite its orbital velocity, the Earth appeared to be at rest relative to the ether. This negative result baffled physicists of the time, and to this day the Michelson-Morley experiment is the most significant "negative-result" experiment ever performed.

Understanding of this result had to wait for Einstein's special theory of relativity, published in 1905. Einstein realized that the velocity of a light wave has the same magnitude c relative to *all* reference frames, whatever their velocity may be relative to each other. The presumed ether then plays no role, and the very concept of ether has been given up. The theory of relativity is a well-established cornerstone of modern physics.

Exercises

1.80. A thin film with $n = 1.40$ for a light of wavelength 5890 A is placed in one arm of a Michelson's interferometer. If a shift of 7.0 fringes occurs, what is the film thickness?

1.81. How far must the mirror M_2 (Figure 1.28) of the Michelson's interferometer be moved so that 3000 fringes of krypton-86 light ($\lambda = 606$ nm) will move across a line in the field of view?

1.82. Light of wavelength 550.5 nm is used to calibrate a Michelson's interferometer, and mirror M_1 is moved 0.180 mm. How many dark fringes are counted?

1.83. Mirror M_1 in Figure 1.28 is displaced a distance ΔL. During this displacement, 250 fringe reversals (formation of successive dark or bright bands) are counted. The light being used has a wavelength of 632.8 nm. Calculate the displacement ΔL. (Ans.: 39.6 μm).

1.84. Monochromatic light is beamed into a Michelson's interferometer. The movable mirror is displaced 0.382 mm; this causes the interferometer pattern to reproduce itself 1 700 times. Determine the wavelength and the color of the light.

1.85. One leg of a Michelson's interferometer contains an evacuated cylinder 3.00 cm long having glass plates on each end. A gas is slowly leaked into the cylinder until a pressure of 1 atm is reached. If 35 bright fringes pass on the screen when light of wavelength 633 nm is used, what is the index of refraction of the gas? (Ans.: 1.000369).

1.86. One leg of a Michelson's interferometer contains an evacuated cylinder of length L having glass plates on each end. A gas is slowly leaked into the cylinder until a pressure of 1 atm is reached. If N bright fringes pass on the screen when light of wavelength λ is used, what is the index of refraction of the gas?

1.87. If mirror M_2 in Michelson's interferometer is moved through 0.233 mm, 792 fringes are counted. What is the wavelength of the light?

1.17 Holography

One of the most interesting applications of interference is holography, which is used to create three-dimensional images found practically everywhere, from credit cards to postage stamps.

Holography is a technique for recording and reproducing an image of an object without the use of lenses. Unlike the two-dimensional images recorded by an ordinary photograph or television system, a holographic image is truly three-dimensional. Such an image can be viewed from different directions to reveal different sides, and from various distances to reveal changing perspective.

The basic procedure for making a hologram is very simple in principle. A possible arrangement is shown in Figure 1.29 (a). We illuminate the object to be holographed with monochromatic light, and we place a photographic film so that it is struck by scattered light from the object and also by direct light from the source. In practice, the source must be a laser, for reasons to be discussed later. Interference between the direct and scattered light leads to the formation and recording of a complex interference pattern on the film. To form the images, we simply project laser light through the developed film, as shown in Figure 1.29 (b). Two images are formed, a virtual image on the side of the film nearer the source, and a real image on the opposite side. A complete analysis of holography is beyond our scope, but we can gain some insight into the process by examining how a single point is holographed and imaged. Consider the interference pattern formed on a photographic film by the superposition of an incident plane wave and a spherical wave, as shown in Figure 1.30 (a). The spherical wave originates at a point source P a distance d_0 from the film; P may in fact be a small object that scatters part of the incident plane wave. In any event, we assume that the two waves are monochromatic and coherent, and that the phase relation is such that constructive interference occurs at point O on the diagram.

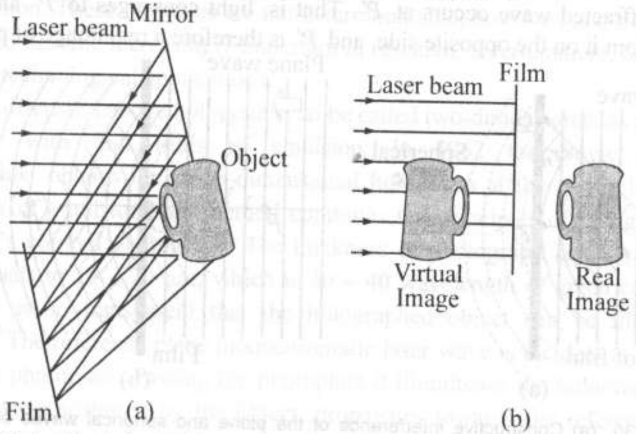


Figure 1.29 (a) The hologram is record on film of the interference pattern formed with light directly from the source and light scattered from the object; (b) Images are formed when light is projected through the hologram

Then constructive interference will also occur at any point Q on the film that is farther from P than O is, by an integer number of wavelengths. That is, if

$d_m - d_0 = m\lambda$, where m is an integer, then constructive interference occurs. The points where this condition is satisfied form circles centered at O, with radii r_n given by

$$d_m - d_0 = \sqrt{d_0^2 + r_m^2} - d_0 = m\lambda.$$

Solving this equation for r_m^2, we find

$$r_m^2 = \lambda(2dm_0 + m^2\lambda).$$

Ordinarily d_0 is very much larger than λ, so we neglect the second term in parentheses, obtaining

$$r_m = \sqrt{2m\lambda d_0}.$$

Since m must be an integer, the interference pattern consists of a series of concentric bright circular fringes, with the radii of the brightest regions. Betwecn these bright fringes are darker fringes.

Now we develop the film and make a transparent positive print, so the bright-fringe areas have the greatest transparency on the film. It is then illuminated with monochromatic plane-wave light of the same wavelength as that used initially. In Figure 1.30 (b), consider a point P' at a distance d_0 along the axis from the film. The centers of successive bright fringes differ in their distances from P' by an integer number of wavelengths, and therefore a strong maximum in the diffracted wave occurs at P'. That is, light converges to P' and then diverges from it on the opposite side, and P' is therefore a real image of point P.

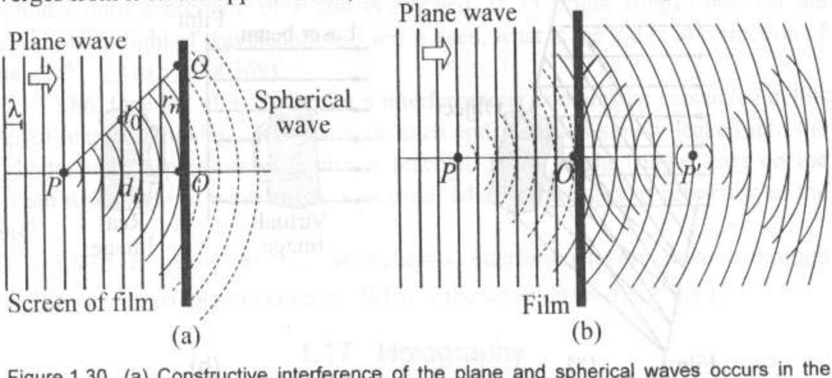


Figure 1.30 (a) Constructive interference of the plane and spherical waves occurs in the plane of the film at every point Q for which the distance d_n from P is greater than the distance d_0 from P to Q by an integer number of wavelengths $m\lambda$. For the point shown, $m = 2$; (b) When the plane wave strikes the developed film, the diffracted wave consists of a wave converging to P' and then diverging again, and a diverging wave that appears to originate at P. These waves form the real and virtual images, respectively

This is not the entire diffracted wave, however; there is also a diverging spherical wave, which would represent a continuation of the wave originally emanating from P if the film had not been present. Thus the total diffracted wave is a superposition of a converging spherical wave forming a real image at P' and a diverging spherical wave shaped as though it had originated at P, forming a virtual image at P.

Because of the principle of linear superposition, what is true for the imaging of a single point is also true for the imaging of any number of points. The film records the superposed interference pattern from the various points, and when light is projected through the film the various image points are reproduced simultaneously. Thus the images of an extended object can be recorded and reproduced just as for a single point object.

In making a hologram, several practical problems must be overcome. First, the light used must be coherent over distances that are large compared to the dimensions of the object and its distance from the film. Ordinary light sources do not satisfy this requirement, for reasons discussed earlier, and laser light is essential. Second, extreme mechanical stability is needed. If any relative motion of source, object, or film occurs during exposure, even by as much as a wavelength, the interference pattern on the film is blurred enough to prevent satisfactory image formation. These obstacles are not insurmountable, however, and holography promises to become increasingly important in research, entertainment, and a wide variety of technological applications.

The discussed above holograms can be called two-dimensional, as they used photoplate with thin films of emulsion. In 1962 the Soviet scientist J.N Denisiuk. obtains the three–dimensional holograms at the photoplates with thick films of emulsion. His method combines the principles of holograph and Flippmant's colored photograph. The thickness of photograph layer is approximately equals to 15 – 20 μm, which is 30 – 40 wavelength of green color. The photolayer is so transparent that the holographed object can be illuminated through it. The reference plane monochromatic laser wave is incident at the glass side of the photoplate. Passing the photoplate it illuminates the holographed object. The wave scattered by the object, propagates towards the reference wave giving interference in the thickness of photoemulsion. Interference pattern represents the standing waves superposed by intricate figure of fine component due to maximum and minimum, as only reference wave among interfering waves is the plane one. The developed and fixed photoplate serves as a Denisiuk hologram.

Reactivation of the object wave is reactivated by diverging beam of white light. Each layer of rejected silver serves as two-dimensional hologram and gives weak virtual and real images of the object. At multi-beam interference the reinforcement of the waves with wavelengths equal to that of the laser, and in the direction where the phase difference between the waves due to the adjacent silver layers equals 2π occurs only. As a result images of same color as that of laser light emerges. The rest images damp each other at interference.

The Denisiuk's method, similarly to the tree-colored photography, allows obtaining the image of objects in the natural colors. For that the hologram due to three lasers is obtained at the same phtopolate. The lasers are different in wavelength in such a way that they reproduce the color of the object perfectly at mixing.

The holography is independent, rapidly developing branch of science, technique and art, which has a brilliant future.

Summary

Monochromatic light is light having a single definite frequency. Coherent waves are the waves of the same frequency and unchanging phase relationship between them. When two coherent waves overlap they produce an interference pattern. Interference is the redistribution of intensity in the region of superposition of two or more coherent waves. The principle of linear superposition states that the total wave disturbance at a point at any instant is the sum of the disturbances from the separate waves. When the sources are in phase, constructive interference at a point occurs when the optical path difference

$$\Delta = n_1 l_1 - n_2 l_2$$

from the two sources is zero or an integer number of wavelengths:

$$\Delta = \pm 2m\frac{\lambda}{2},\ m = 0,1,2,3,...$$

Destructive interference occurs when the optical path difference is a half-integer number of wavelengths

$$\Delta = \pm(2m+1)\frac{\lambda}{2},\ m = 0,1,2,3,...$$

In the Young double-slit experiment when the line from the sources to a point P makes an angle θ with the line perpendicular to the line of the sources, and when the distance between sources is d, the condition for constructive interference is

$$d\sin\theta = \pm m\lambda \quad (m = 0,1,2,3,...),$$

and the condition for destructive interference is

$$d\sin\theta = \pm(m+\frac{1}{2})\lambda \quad m = 0, 1, 2, 3, ...$$

When θ is very small, the position y_m of the m-th bright fringe is given by

$$y_{bright} = \frac{\lambda L}{d} m$$

and the position y_m of the mth dark fringe is

$$y_{darc} = \frac{\lambda L}{d}(m+\frac{1}{2}).$$

When two sources emit waves in phase, the phase difference δ of the waves arriving at point P is related to the optical difference in path length Δ by

$$\delta = \frac{2\pi}{\lambda}(n_1 l_1 - n_2 l_2) = k(n_1 l_1 - n_2 s_2) = k\Delta,$$

where $k = 2\pi / \lambda$ is wave number.

When two sinusoidal waves of amplitude E and phase difference δ are superposed, the resultant amplitude E_p is

$$E_p{}^2 = 4E^2 \cos^2(\delta/2),$$

and the intensity I s given by

$$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta.$$

The optical path difference for the case of thin film is expressed as

$$\Delta = \frac{2dn}{\cos r} - 2d\tan r\sin i = 2d\frac{n^2 - n\sin r\sin i}{n\cos r} - \frac{\lambda}{2}.$$

The Michelson interferometer uses an extended monochromatic source and can be used for high-precision measurements of wavelengths. Its original purpose was to detect motion of the Earth relative to hypothetical ether, the supposed medium for electromagnetic waves. The concept of ether has been abandoned; the speed of light is the same relative to all observers. This is part of the foundation of the special theory of relativity.

A hologram is photographic record of an interference pattern formed by light scattered from an object and light coming directly from the source. It can be used to form three-dimensional images of the object.

Key Terms

index of refraction	показатель преломления	показник заломлення
monochromatic wave	монохроматическая волна	монохроматична хвиля
coherent wave	когерентная волна	когерентна хвиля
interference	интерференция	інтерференція
Huygens's principle	Гюйгенса принцип	Гюйгенса принцип
optical path difference	оптическая разность хода	оптична різниця ходу
constructive interference	усиливающая интерференция	підсилювальна інтерференція
destructive interference	ослабляющая интерференция	послаблювальна інтерференція
Young double-slit experiment	Юнга опыт с двумя щелями	Юнга дослід з двома щілинами
Newton's rings	Ньютона кольца	Ньютона кільця
change phase due to reflection	изменение фазы при отражении	зміна фази при віддзеркаленні
Michelson interferometer	Майкельсона интерферометр	Майкельсона інтерферометр
holography	голография	голографія

Chapter 2

Diffraction

In this chapter we shall discuss diffraction of light waves. Its essence is this: if a wave meets an obstacle that has an opening of dimensions comparable with its wavelength, the part of the wave that passes through the opening will flare (spread) out – will diffract– into the region beyond the barrier. The flaring is consistent with the spreading of wavelets in the Huygens' construction. Diffraction occurs for waves of all types, not just light waves.

Diffraction limits geometrical optics, in which we represent an electromagnetic wave with a ray. If we actually try to form a ray by sending light through a narrow slit, or through a series of narrow slits, diffraction will always defeat our effort because it always causes the light to spread. Indeed, the narrower we make the slits (in the hope of producing a narrower beam), the greater the spreading is. Thus, geometrical optics holds only when slits or other apertures that might be located in the path of light do not have dimensions comparable to or smaller than the wavelength of the light.

2.1 Introduction to Diffraction

When light waves pass through a small aperture, an interference pattern is observed rather than a sharp spot of light. This behavior indicates that light, once it has passed through the aperture, spreads beyond the narrow path defined by the aperture into regions that would be in shadow if light traveled along straight lines (Figure 2.1). Instead, Huygens' principle requires that the waves spread out from the slits as shown in Figure 2.2. In other words, the light deviates from a straight line path and enters the region that would otherwise be shadowed. This divergence of light from its initial line of traveling is called **diffraction**. Other waves, such as sound waves and water waves also have this property of spreading when passing through apertures or by sharp edges. This phenomenon can be described only with wave model.

In general, diffraction occurs when waves pass through small openings, around obstacles, or small sharp edges, as shown in Figure 2.3. When an opaque object is placed between a point source of light and a screen, no sharp boundary exists on the screen between a shadowed region and an illuminated region. The illuminated region above the shadow of the object contains alternating light and dark fringes. Such a display is called a **diffraction pattern**.

The essential features observed in diffraction effects can be predicted with the help of Huygens' principle: Every point of a wave surface can be considered the source of secondary wavelets that spread out in all directions. At every point we must combine the displacements that would be produced by the secondary wavelets, taking into account their amplitudes and relative phases. The mathematical operations are often quite complicated.

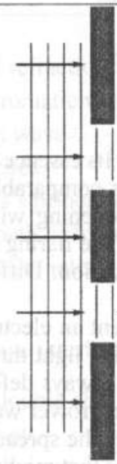


Figure 2.1 According to geometrical optics, the transmitted beam should have the same cross section as the slit

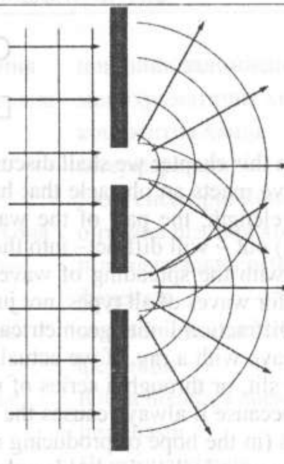


Figure 2.2 The light waves from two slits overlap as they spread out, filling what we expect to be shadowed region with light and producing interference fringes

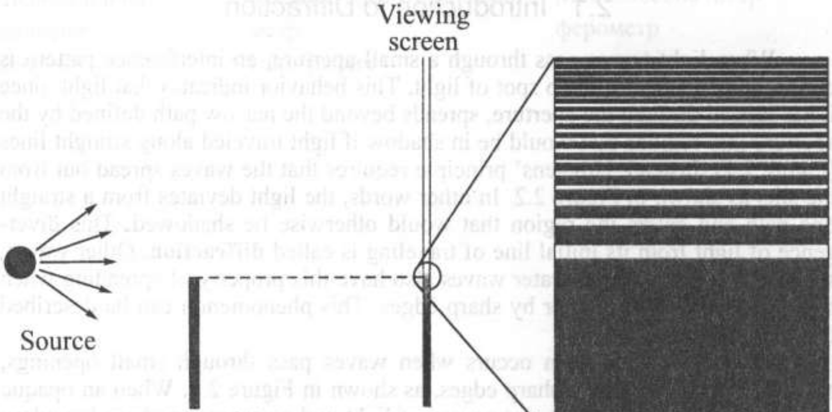


Figure 2.3 Light from a small source passed by the edge of an opaque object. We might expect no light to appear on the screen below the position of the edge of the object. In reality, light bends around the top edge of the object and enter this region. Because of these effects, a diffraction pattern consisting of bright and dark fringes appears in the region above the edge of the object

Diffraction patterns are not commonly observed in everyday life because most ordinary light sources are not point sources of monochromatic light. If the shadow of obstacles cast by an incandescent lamp, for example, the light from every point of the surface of the lamp forms its own diffraction pattern, but these overlap to such an extent that no individual pattern can be observed.

The term "diffraction" is applied to problems involving the resultant effect produced by a limited portion of a wave front. Since in most diffraction problems some light is found within the region of geometrical shadow, diffraction is sometimes defined as "*the bending of light around an obstacle*".

It should be emphasized, however, that the process by which diffraction effects are produced is going on continuously in the propagation of every wave. Only if part of the wave is cut off by some obstacle do we observe diffraction effects. But every optical instrument uses only a limited portion of a wave; for example, a telescope uses only that portion of a wave admitted by the objective lens. Thus diffraction plays a role in practically all optical phenomena.

Diffraction phenomena are divided into two classes. When wave front is sphere (both the point source and the screen are at finite distances from the obstacle forming the diffraction pattern), this situation is described as **Fresnel diffraction** (after Augustin Jean Fresnel, 1788 – 1827), and the resulting pattern on the screen is called a Fresnel diffraction pattern (Figure 2.4). If the wave front is plane (source, obstacle and screen are far enough away so that the lines from the source to the obstacle and from the obstacle to a point in the pattern formed on the screen can be considered to be parallel), the phenomenon is called **Fraungofer diffraction** (after Joseph von Fraungofer, 1787 – 1826) (Figure 2.5). The latter situation is simpler to treat in detail.

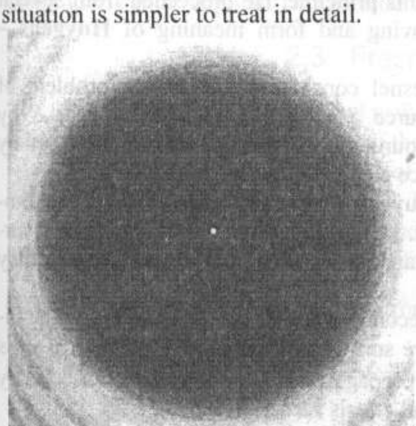


Figure 2.4 Diffraction pattern created by the illumination of a small ball, with the ball positioned midway between the screen and light source

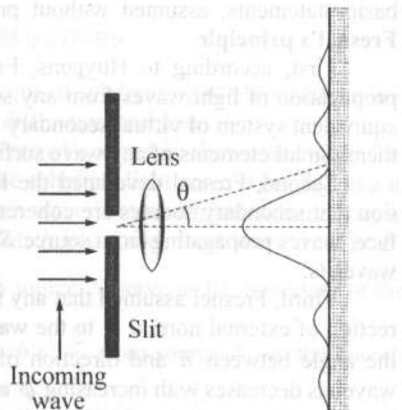


Figure 2.5 Fraungofer's diffraction from a single slit

Exercises

2.1. Why can you hear around corners but not see around them?

2.2. Observe the shadow of your book when it is held a few centimeters above a table while illuminated by a lamp several meters above it. Why is the shadow somewhat fuzzy at the edges?

2.3. Knowing that radio waves travel at the speed of light and that a typical AM radio frequency is 1 000 kHz while an FM radio frequency might be 100 MHz, estimate the wavelengths of typical AM and FM radio signals. Use this information to explain why FM radio stations often fade out when you drive through a short tunnel or underpass but AM radio stations do not.

2.4. State the condition for diffraction of light to occur.

2.2 Huygens – Fresnel's Principle

In the beginning of XIX century Fresnel developed a technique, which could explain diffraction phenomenon and created method for its quantitative calculation. He was lucky to show that the law of rectilinear propagation of light is the approximate solution of general problem of diffraction of light. It was found, that this law, as well as all the geometrical optics is valid only in limit when wavelength of light $\lambda \to 0$ in comparison with dimensions of obstacles.

Huygens's principle says nothing about the amplitude and, hence, intensity of light waves. Fresnel complemented this principle. He proceeded from several basic statements, assumed without proving and form meaning of **Huygens – Fresnel's principle**.

First, according to Huygens, Fresnel considered that in the problem of propagation of light waves from any source S, one can replace the source S by equivalent system of virtual secondary sources and secondary wavelets excited by them. Small elements of any wave surface can be chosen as these sources.

Second, Fresnel developed the Huygens' principle essentially by assumption that secondary sources are coherent. Hence at any point out of the wave surface, waves propagating from source S are the result of interference of secondary wavelets.

Third, Fresnel assumed that any secondary source radiates mainly in the direction of external normal $\vec{n}$ to the wave surface at that point. Therefore, if φ is the angle between $\vec{n}$ and direction of propagation, the amplitude of secondary wavelets decreases with increasing φ and equals zero at $\varphi \geq \pi/2$.

Hence according to Huygens-Fresnel's principle *every element of wave surface serves as the source of secondary wavelets with amplitude proportional to the area dA of this element.* The amplitude of spherical wave decreases with distance r from the source as $1/r$. Hence, the oscillation

$$dE = K\frac{E_0}{r}\cos(\omega t - \vec{k}\vec{r} + \phi)dA, \tag{2.1}$$

due to any surface element dA is coming at point P located beyond the wave surface. In expression (2.1) the $(\omega t+\phi)$ is the phase of oscillation at the wave surface A, $\vec{k}$ is the wave vector, $\vec{r}$ – the vector from element dA to the point P. The amplitude of light oscillation at dA is defined as E_0. The coefficient K depends on the angle φ between the normal $\vec{n}$ to the surface dA and direction from $\vec{n}$ to point P, i.e., vector $\vec{r}$. The coefficient K has maximum value for $\varphi=0$ and minimum for $\varphi=\frac{\pi}{2}$.

Theoretically we could compute the intensity at any point P of the diffraction pattern by dividing the area of the wave front into small elements, finding the resulting wave amplitude and phase at P, and then integrating over all the area of the wave front to find the resultant amplitude and intensity. Analytical expression of Huygens-Fresnel's principle has form:

$$E=\int dE=\int_A K\frac{E_0}{r}\cos(\omega t-\vec{k}\vec{r}+\phi)dA. \qquad (2.2)$$

In practice, the integration cannot be carried out in terms of elementary functions and has to be done by numerical approximation.

Calculation according to Eq. (2.2) is rather difficult, but Fresnel showed that for some symmetric cases the geometrical or algebraic summing rather then integration can be applied. This method is known as Fresnel's zones method.

2.3 Fresnel's zones

Fresnel suggested the original technique of subdividing of wave surface into zones, which simplifies the solving of the diffraction problem.

The technique for construction of Fresnel's zones is shown in Figure 2.6. Let point source of light is placed at S. The spherical light wave is propagating in an isotropic medium. Wave front of such a wave is symmetrical with respect to line SP. The arc is the spherical wave front, b is distance from wave front to P. Points located at distance $b_1=b+\frac{\lambda}{2}$ from source S serve as the boundary of the first (central) zone. Points at distance $b_2=b+2\frac{\lambda}{2}$ from source S are the boundary of the second zone and so on ($b_m=b+m\frac{\lambda}{2}$). Obviously the oscillations at P due to two adjacent zones are out of phase as their optical path difference to point P is $\lambda/2$. Hence at superposition they interfere destructively at P.

As amplitude is proportional to the area of zone, we need areas of zones. The path difference between waves coming to point P from similar point of adjacent zones is $\lambda/2$ (Figure 2.6).

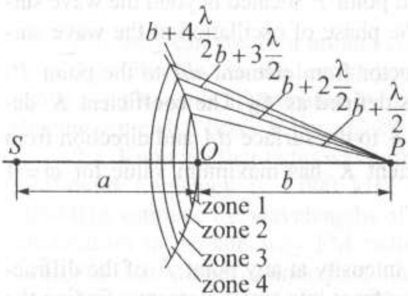


Figure 2.6 Construction of Fresnel's zones

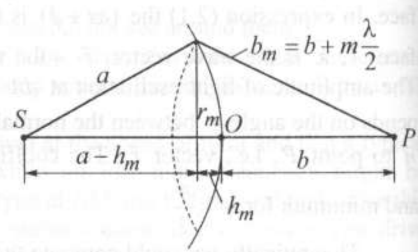


Figure 2.7 Fresnel's zone of radius r_m

Area of m-th zone can be defined as

$$\Delta A = A_m - A_{m-1},$$

where A_m and A_{m-1} are the areas of the spherical segments due to outer boundary of m and $m-1$ zones, respectively. From Figure 2.7 it is clear, that

$$r_m^{\ 2} = a^2 - (a - h_m)^2,$$

$$r_m^{\ 2} = \left(b + m\frac{\lambda}{2}\right)^2 - (b + h_m)^2,$$

where a is the radius of the wave front, r_m is the radius of outer boundary of m-th zone. The previous expressions can be rewritten as follows:

$$r_m^{\ 2} = 2ah_m - h_m^{\ 2}, \quad r_m^{\ 2} = bm\lambda + m^2\left(\frac{\lambda}{2}\right)^2 - 2bh_m - h_m^{\ 2}. \tag{2.3}$$

Then

$$h_m = \frac{bm\lambda + m^2(\lambda/2)^2}{2(a+b)}, \text{ or}$$

$$h_m = \frac{bm\lambda}{2(a+b)}. \tag{2.4}$$

(As λ is small, we neglect the term with λ^2). The area of spherical segments is $A = 2\pi Rh$ (R is the radius of the sphere and h is the height of segment). Hence

$$A_m = 2\pi a h_m = \frac{\pi ab}{a+b} m\lambda,$$

and the area of m-th zone equals to

$$\Delta A_m = A_m - A_{m-1} = \frac{\pi ab}{a+b}\lambda.$$

This expression doesn't depend on m. Hence (when m is not very large), Fresnel's zones are approximately equal in area.

The radii of Fresnel's zones can be determined from Eq. (2.3). As $h_m << a$ we can assume $r_m^2 = 2ah_m$, and then radius of m-th Fresnel's zone will be:

$$r_m = \sqrt{\frac{ab}{a+b} m\lambda} . \quad (2.5)$$

The areas of Fresnel's zones are approximately equal, angle φ between the normal to the surface and direction to the point P increases with m, the distance b_m also increases with m, that is why the amplitude E_m of oscillation due to m-th zone monotonically decreases with m:

$$E_1 > E_2 > E_3 > ... > E_m > ...$$

As the oscillations due to adjacent zones are out of phase then the resultant amplitude at point P can be written as:

$$E_p = E_1 - E_2 + E_3 - E_4 + ... \quad \text{or}$$

$$E = \frac{E_1}{2} + \left(\frac{E_1}{2} - E_2 + \frac{E_3}{2}\right) + \left(\frac{E_3}{2} - E_4 + \frac{E_5}{2}\right) + ... \quad (2.6)$$

Due to monotonous decreasing of E we can write that

$$E_m = \frac{E_{m-1} + E_{m+1}}{2} .$$

Then the expressions in brackets in Eq. (2.6) equal zero and it reduces to

$$E = \frac{E_1}{2} + \frac{E_{m-1}}{2} - E_m \quad (m\text{-even});$$

$$E = \frac{E_1}{2} + \frac{E_m}{2} \quad (m\text{-odd}). \quad (2.7)$$

Hence the amplitude at point P due to the whole wave surface is one half of the amplitude due to the first zone plus one-half of the E_m amplitude (for m–odd). At large m the amplitude E_m is small and the resultant amplitude E at point P due to whole spherical surface equals to one-half of amplitude due to only one central zone.

Examples

Example 2.1

Calculate radius of the first (central) Fresnel's zone for the case when the distance from the light source to the wave surface is $a = 1$ m, the distance from wave surface to the viewing point is $b = 1$ m and wavelength of light is $\lambda = 0.5$ μm.

Solution

The radius of the first ($m=1$) zone is

$$r_1=\sqrt{\frac{ab}{a+b}m\lambda}=\sqrt{\frac{1\times 1}{1+1}1\times 0.5\times 10^{-6}}=0.50\,\text{mm}=5\times 10^{-4}\ \text{m}.$$

Radii of the following zones increase as $\sqrt{m}$.

Exercise

2.5. Calculate the radii of the first five Fresnel's zones $m=(1...5)$ for plane wave, if the distance from wave surface to the viewing point is $b=1$ m and wavelength of light is $\lambda=0.5$ μm. (Note, that for plane wave $a=\infty$ and

$$r_m=\sqrt{\frac{ab}{a+b}m\lambda}=\sqrt{\frac{\infty\times b}{\infty+b}m\lambda}=\sqrt{bm\lambda}).$$

(Ans.: $r_1=0.71\,\text{mm}$, $r_2=1.0\,\text{mm}$, $r_3=1.22\,\text{mm}$, $r_4=1.41\,\text{mm}$, $r_5=1.58\,\text{mm}$.)

2.4 Fresnel's Diffraction from Circular Aperture

Let's place an opaque screen with circular aperture on the way of the spherical light wave. When the radius of aperture r_0 is much less than both a and b, then the length a can be treated as the distance from light source S to the obstacle and the length b as the distance from obstacle to the point P (Figure 2.8 (a)). If the distances a and b satisfy the expression:

$$r_0=\sqrt{\frac{ab}{a+b}m\lambda},$$

where m is an integer, then the aperture remains m first Fresnel's zones opened. Hence, the number of unclosed Fresnel's zones is defined as:

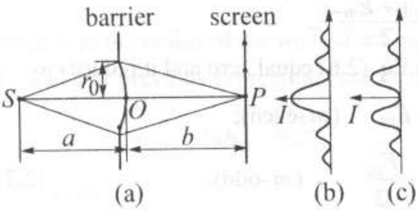


Figure 2.8 (a) The aperture opens m Fresnel's zones; intensity distribution for (b) odd m; (c) even m

$$m=\frac{r_0^{\,2}}{\lambda}\left(\frac{1}{a}+\frac{1}{b}\right). \tag{2.8}$$

According to (2.6) the resultant amplitude at point P will be equal:

$$E=E_1-E_2+E_3-..\pm E_m. \tag{2.9}$$

The E_m is positive when m is odd and negative when m is even. Then

$$E=\frac{E_1}{2}+\frac{E_m}{2} \qquad (m\text{-odd}) \text{ (Figure 2.8 (b))};$$

$$E=\frac{E_1}{2}+\frac{E_{m-1}}{2}-E_m \qquad (m\text{-even}) \text{ (Figure 2.8 (c))}.$$

The amplitudes of adjacent zones are approximately equal, so the expression $\left(\frac{E_{m-1}}{2} - E_m\right)$ can be substituted by $\left(\frac{E_m}{2}\right)$. As a result we obtain:

$$E = \frac{E_1}{2} \pm \frac{E_m}{2}. \qquad (2.10)$$

When m is small, the amplitudes E_1 and E_m are approximately equal to each other.

For odd m the amplitude at P will be approximately E_1 and for even m will be zero. If we take off the obstacle, the amplitude in the point P become equal to $E_1/2$. Hence, the obstacle, which opens several Fresnel's zones, does not only reduce the intensity, but instead, multiplies up amplitude twice and, as a result, intensity – four times.

The diffraction pattern formed by a circular aperture consists of a series of bright and dark rings. Central spot will be bright when m is odd, or dark when m is even (Figure 2.9).

If we move the obstacle toward or from the source of light, we obtain at the screen an alternating dark and bright central spots, as we change the distance b between the viewing screen and obstacle, and as a results the number of Fresnel's zone passing through the aperture.

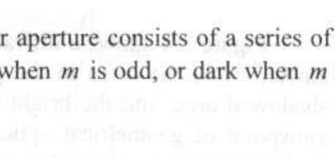


Figure 2.9 The diffraction pattern of circular aperture consists of a central bright disk, surrounded by concentric bright and dark rings

The oscillations from odd and even zones come to the point P out of phase and therefore lead to interference minimum. If plate which would block all odd or all even zones is placed on the way of light, then the intensity of light at point P increases. Such a plate, which is called **amplitude zone plate**, serves as a convex lens. We can achieve even more impressive effect, if we would not block zones, but, instead, change their phase by π. This can be made by use of a transparent plate, thickness of which varies in a certain way in the places which corresponds to odd or even zones. Such a plate is called **phase zone plate**. In comparison with amplitude zone plate, it gives additional growth in amplitude and intensity.

Exercises

2.6. The diffraction pattern is observed at the screen located at distance $l = 4$ m from point source of light with $\lambda = 500$ nm. Midway the screen and light source there is an obstacle with aperture of radius R. Find the radius R, if the disk at the viewing screen, surrounded by concentric bright and dark rings is darkest. (Ans.: $R = r_2 = \sqrt{2ab\lambda/(a+b)} = 1$ mm).

2.7. Beam of parallel rays from monochromatic source of light ($\lambda = 0.6\ \mu m$) is incident normally on the circular aperture of diameter $D = 6$ mm. Viewing screen is placed at distance $b = 3$ m. How many Fresnel's zones can pass through the aperture? Will the center of diffraction pattern be bright or dark?

2.5 Fresnel Diffraction from a Disk

Figure 2.4 shows a diffraction pattern formed by a ball about 2 mm in diameter. Note the rings in the pattern, both outside and inside the geometrical shadowed area, and the bright spot at the very center of the shadow. From the viewpoint of geometrical optics (in which light is viewed as rays traveling in straight lines), we expect the center of the shadow to be dark because that part of the viewing screen is completely shielded by the ball.

The existence of this spot was predicted in 1818 by the French mathematician Simeon Poisson. Ironically, Poisson himself was not a believer in the wave theory of light, and he published this "apparently absurd result" as a final ridicule of the wave theory. But almost immediately, the bright spot was observed experimentally by Arago. It had, in fact, been observed much earlier, in 1723 by Maraldi, but its significance was not recognized then.

Let's place an opaque disk between light source and point P. If the disk shuts m first Fresnel's zones, then the amplitude at point P will be:

$$E = E_{m+1} - E_{m+2} + E_{m+3} - ... =$$
$$= \frac{E_{m+1}}{2} + \left(\frac{E_{m+1}}{2} - E_{m+2} + \frac{E_{m+3}}{2} \right) + ...$$

The expression in bracket is approximately zero, therefore the resultant amplitude is

$$E = \frac{E_{m+1}}{2}. \tag{2.11}$$

When the number m of shut Fresnel's zones is small, the amplitude E_{m+1} is approximately equal to amplitude E_1 and the intensity at point P will be approximately the same as that without any obstacle. The diffraction pattern formed by a disk consists of a central bright spot surrounded by a series of bright and dark rings (Figure 2.4).

Exercise

2.8. The diffraction pattern is observed at the screen located at distance l m from point source of light ($\lambda = 600$ nm). An opaque obstacle with diameter D is placed at distance $a = 0.5l$ from the light source. Calculate the distance l if disk shuts only central Fresnel's zone. (Ans.: $l = 167$ m).

2.6 Fraunhofer Diffraction from a Single Slit

Suppose a monochromatic plane wave (in the ray picture, a beam of parallel rays) is incident on an opaque plate having a narrow slit. According to geometrical optics, the transmitted beam should have the same cross section as the slit, and a screen in the path of the beam would be illuminated uniformly over an area of the same size and shape as the slit, as in Figure 2.1. What is actually observed is the pattern shown in Figure 2.10. The beam spreads out after passing through the slit, and the diffraction pattern consists of a central bright band, which may be much wider than the slit width, bordered by alternating dark and bright bands of decreasing intensity. You can easily observe a diffraction pattern of this sort by looking at a point source such as a distant street light through a narrow slit formed between two fingers in front of your eye. The retina of your eye then corresponds to the screen.

According to Huygens' principle, each element of area of the slit opening can be considered as a source of secondary wavelets. In particular, we may imagine elements of area formed by subdividing the slit into narrow strips parallel to the long edges, perpendicular to the page in Figure 2.11.

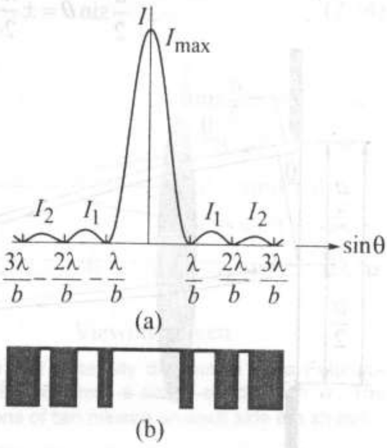


Figure 2.10 (a) Fraunhofer diffraction pattern from a single slit. The pattern consists of a central bright fringe flanked by much weaker maxima alternating with dark fringes. θ is the angle of diffraction; (b) Photograph of a single-slit Fraungofer's diffraction pattern

In Figure 2.11 a screen is placed to the right of the slit. The resultant intensity at a point P is calculated by adding the contributions from the individual wavelets, taking proper account of their various phases and amplitudes. The problem becomes much simpler if the screen is far away, so the rays from the slit to the screen are parallel, or when a lens is placed as in Figure 2.11, in which case the rays to the lens are parallel and the lens forms in its focal plane an image of the pattern that would be formed on an infinitely distant screen without the lens.

You might ask whether the lens introduces additional phase shifts that are different for different parts of the wave front; it can be shown quite generally that the lens does not cause any additional phase shifts. Some aspects of Fraunhofer diffraction from a single slit can be deduced easily. First consider two narrow strips, one just below the top edge of the slit and one at its center, as in Figure 2.11. The difference in path length to point P is $(a/2)\sin\theta$, where the slit width is a. Suppose this path difference happens to be equal to $\lambda/2$; then light from these two strips arrives at point P with a half-cycle phase difference, and cancellation occurs. Similarly, light from two strips just below these two will also arrive a half-cycle out of phase; and, in fact, light from every strip in the top half cancels out that from a corresponding strip in the bottom half, resulting in complete cancellation and giving a dark fringe in the interference pattern. Thus a dark fringe occurs whenever

$$\frac{a}{2}\sin\theta = \pm\frac{\lambda}{2}, \quad \text{or} \quad \sin\theta = \pm\frac{\lambda}{a}.$$

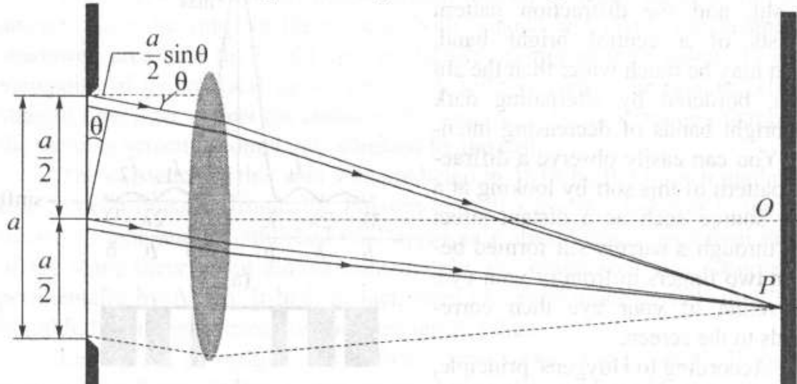


Figure 2.11 Fraunhofer diffraction by a single slit. The wave front is divided into a large number of narrow strips

We may also divide the slit into quarters, sixths, and so on, and use the argument above to show that a dark fringe occurs whenever $\sin\theta = \pm 2\lambda/a$, $\pm 3\lambda/a$ and so on. Hence the condition for a dark fringe is

$$\sin\theta = \pm m\frac{\lambda}{a}. \tag{2.12}$$

Between the dark fringes there are bright fringes. We also note that $\sin\theta = 0$ is a bright band, since then light from the entire slit arrives at P in phase. The central bright fringe is therefore twice as wide as the others, as Figure 2.10 (a) shows. A photograph of an actual diffraction pattern is shown in Figure 2.10 (b).

Note that the intensity of the central maximum is much greater than at any of the others, and that the peak intensities drop off rapidly as we go away from the center of the pattern.

The central peak occurs when $\theta = 0$. It can be shown (from certain transcendent equation) that side maxima occur approximately when

$$\sin\theta = \pm(2m+1)\frac{\lambda}{2a} \quad (m = 1,2,3,...). \tag{2.13}$$

Note that these maximum intensities occur between the minima that we found earlier.

The wavelength of light λ is ordinarily much smaller than the slit width a and the approximation $\sin\theta = \theta$ is very good. With this approximation, the position θ of the first minimum beside the central maximum,

$$\theta_1 = \frac{\lambda}{a}. \tag{2.14}$$

When a is of the order of a centimeter or more, θ is so small that we can consider practically all the light to be concentrated at the geometrical focus.

The general features of the intensity distribution are shown in Figure 2.12. A broad central bright fringe is observed; this fringe is flanked by much weaker bright fringes alternating with dark fringes. The various dark fringes occur at the values of θ that satisfy Eq. (2.12). Note that the central bright maximum is twice as wide as the secondary maxima.

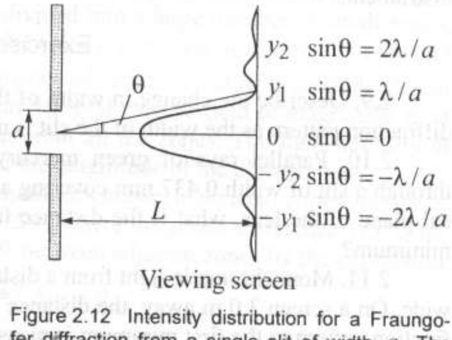


Figure 2.12 Intensity distribution for a Fraungofer diffraction from a single slit of width a. The positions of two minima on each side are shown

Example 2.2

Light of wavelength 580 nm is incident on a slit having a width of 0.300 mm. The viewing screen is 2.00 m from the slit. Find the positions of the first dark fringes and the width of the central bright fringe.

Solution

The two dark fringes that flank the central bright fringe correspond to $m = \pm 1$ in Eq. (2.12). Hence, we find that

$$\sin\theta = \pm\frac{\lambda}{a} = \pm\frac{5.8\times10^{-7}\,\text{m}}{0.30\times10^{-5}\,\text{m}} = \pm1.93\times10^{-3}.$$

From the triangle in Figure 2.12, note that $\tan\theta = y_1 / L$. Because θ is very small, we can use the approximation $\sin\theta \approx \tan\theta$; thus, $\sin\theta \approx y_1 / L$. Therefore, the positions of the first minima measured from the central axis are given by

$$y_1 \approx L\sin\theta = \pm L\frac{\lambda}{a} = \pm 3.87\times 10^{-3} \text{ m.}$$

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe. Hence, the width of the central bright fringe is equal to $2|y_1| = 7.74\times 10^{-3}\text{ m} = 7.74$ mm. Note that this value is much greater than the width of the slit. However, as the slit width is increased, the diffraction pattern narrows, corresponding to smaller values of θ. In fact, for large values of a, the various maxima and minima are so closely spaced that only a large central bright area resembling the geometric image of the slit is observed. This is of great importance in the design of lenses used in telescopes, microscopes, and other optical instruments.

Exercises

2.9. Describe the change in width of the central maximum of the single-slit diffraction pattern as the width of the slit is made narrower.

2.10. Parallel rays of green mercury light of wavelength 546 nm pass through a slit of width 0.437 mm covering a lens of focal length 40 cm. In the focal plane of the lens, what is the distance from the central maximum to the first minimum?

2.11. Monochromatic light from a distant source is incident on a slit 0.8 mm wide. On a screen 3.0 m away, the distance from the central maximum of the diffraction pattern to the first minimum is measured to be 2 mm. Calculate the wavelength of the light. (Ans.: 533 nm).

2.12. Light of wavelength 589 nm from a distant source is incident on a slit 1.0 mm wide, and the resulting diffraction pattern is observed on a screen 2.0 m away. What is the distance between the two dark fringes on either side of the central bright fringe?

2.13. Helium-neon laser light ($\lambda = 632.8$ nm) is sent through a 0.300-mm-wide single slit. What is the width of the central maximum on a screen 1.00 m from the slit? (Ans.: 4.22 mm).

2.14. A beam of green light is diffracted by a slit with a width of 0.550 mm. The diffraction pattern forms on a wall 2.06 m beyond the slit. The distance between the positions of zero intensity on both sides of the central bright fringe is 4.10 mm. Calculate the wavelength of the laser light.

2.15. A screen is placed 50.0 cm from a single slit, which is illuminated with 690-nm light. If the distance between the first and third minima in the diffraction pattern is 3.00 mm, what is the width of the slit? (Ans.: 0.230 mm).

2.16. Coherent microwaves of wavelength 5.00 cm enter a long, narrow window in a building otherwise essentially opaque to the microwaves. If the window is 36.0 cm wide, what is the distance from the central maximum to the first-order minimum along a wall 6.50 m from the window?

2.17. Light with a wavelength of 587.5 nm illuminates a single slit 0.750 mm in width. (a) At what distance from the slit should a screen be located if the first minimum in the diffraction pattern is to be 0.850 mm from the center of the screen? (b) What is the width of the central maximum?

2.18. The second-order bright fringe in a single-slit diffraction pattern is 1.40 mm from the center of the central maximum. The screen is 80.0 cm from a slit of width 0.800 mm. Assuming that the incident light is monochromatic calculate the light's approximate wavelength.

2.7 Intensity of Single-Slit Diffraction Pattern

We can use phasors to determine the light intensity distribution for a single-slit diffraction pattern. Imagine a slit divided into a large number of small zones, each of width Δy as shown in Figure 2.13. Each zone acts as a source of coherent radiation, and each contributes an incremental electric field of magnitude ΔE at some point P on the screen. We obtain the total electric field of magnitude E at point P by summing the contributions from all the zones. The light intensity at point P is proportional to the square of the magnitude of the electric field.

The incremental electric field magnitudes between adjacent zones are out of phase with one another by an amount $\Delta\delta$, where the phase difference $\Delta\delta$ is related to the path difference $\Delta = \Delta y \sin\theta$ between adjacent zones by the expression

$$\Delta\delta = \frac{2\pi}{\lambda}\Delta y \sin\theta .$$

To find the magnitude of the total electric field on the screen at any angle θ, we sum the incremental magnitudes ΔE due to each zone. For small values of θ, we can assume that all the ΔE values are the same. It is convenient to use phasor diagrams for various angles, as shown in Figure 2.14. When $\theta = 0$, all phasors are aligned as shown in Figure 2.14 (a) because all the waves from the various zones are in phase. In this case, the total electric field at the center of the screen is $E_0 = N\Delta E$, where N is the number of zones. The resultant magnitude E_R at some small angle θ is shown in Figure 2.14 (b), where each phasor differs in phase from an adjacent one by an amount $\Delta\delta$. In this case, E_R is the vector sum of the incremental magnitudes and hence is given by the length of the chord. Therefore, $E_R < E_0$. The total phase difference δ between waves from the top and bottom portions of the slit is

$$\delta = N\Delta\delta = \frac{2\pi}{\lambda} N\Delta y \sin\theta = \frac{2\pi}{\lambda} a \sin\theta , \qquad (2.15)$$

where $a = N\Delta y$ is the width of the slit.

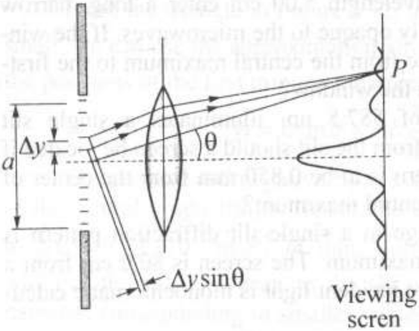


Figure 2.13 Fraungofer diffraction from a single slit. The light intensity at point P is the resultant of all the incremental electric field magnitudes from zones of width Δy

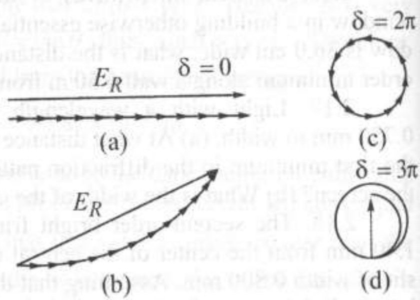


Figure 2.14 Phasor diagrams for obtaining the various maxima and minima of a single-slit diffraction pattern

As θ increases, the chain of phasors eventually forms the closed path shown in Figure 2.14 (c). At this point, the vector sum is zero, and so $E_R = 0$, corresponding to the first minimum on the screen. Noting that $\delta = N\Delta\delta = 2\pi$ in this situation, we see from Eq. (2.15) that

$$2\pi = \frac{2\pi}{\lambda} a \sin\theta \text{ and } \sin\theta = \frac{\lambda}{a}.$$

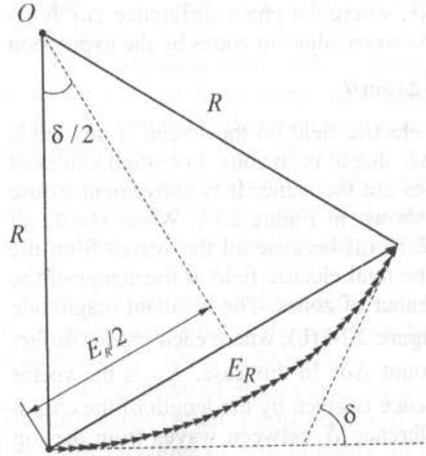


Figure 2.15 Phasor diagram for a large number of coherent sources. All the ends of the phasors lie on the circular arc of radius R. The resultant electric field magnitude E_R equals the length of the chord

That is, the first minimum in the diffraction pattern occurs where $\sin\theta = \lambda / a$; this is in agreement with Eq. (2.12).

At greater values of θ, the spiral chain of phasors tightens. For example, Figure 2.14 (d) represents the situation corresponding to the second maximum, which occurs when $\delta = 360° + 180° = 540°$ (3π rad). The second minimum (two complete circles, not shown) corresponds to $\delta = 720°$ (4π rad), which satisfies the condition $\sin\theta = 2\lambda / a$.

We can obtain the total electric field magnitude E_R and light intensity I at any point P on the screen in Figure 2.13 by considering the limiting case in which Δy becomes infinitesimal (dy) and N approaches ∞. In this limit, the phasor chains in Figure 2.14 (b) become the curve of Figure 2.15. The arc length of the curve is E_0 because it is the sum of the magnitudes of the phasors (which is the total electric field magnitude at the center of the screen). From this figure, we see that at some angle θ, the resultant electric field magnitude E_R on the screen is equal to the chord length. From the triangle containing the angle $\delta/2$, we see that

$$\sin\frac{\delta}{2}=\frac{E_R/2}{R},$$

where R is the radius of curvature. But the arc length E_0 is equal to the product $R\delta$, where δ is measured in radians. Combining this information with the previous expression gives

$$E_R=2R\sin\frac{\delta}{2}=2\left(\frac{E_0}{\delta}\right)\sin\frac{\delta}{2}=E_0\left[\frac{\sin(\delta/2)}{\delta/2}\right].$$

Because the resultant light intensity I at point P on the screen is proportional to the square of the magnitude E_R, we find that

$$I=I_{\max}\left[\frac{\sin(\delta/2)}{\delta/2}\right]^2, \tag{2.16}$$

where $I_{\max}$ is the intensity at $\theta=0$ (the central maximum). Substituting the expression for δ, defined by Eq. (2.15), into Eq. (2.16), we have

$$I=I_{\max}\left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2. \tag{2.17}$$

From this result, we see that minima occur when

$$\frac{\pi a\sin\theta}{\lambda}=m\pi,$$

or

$$\sin\theta=m\frac{\lambda}{a},\qquad m=\pm1,\pm2,\pm3,\ldots,$$

which is in agreement with Eq. (2.12).

Figure 2.10 (a) represents a plot of Eq. (2.17), and Figure 2.10 (b) is a photograph of a single-slit Fraunhofer diffraction pattern. Note that most of the light intensity is concentrated in the central bright fringe.

It can be shown that the intensity at the m-th side maximum is

$$I=\frac{I_0}{\left(m+\frac{1}{2}\right)^2\pi^2}.$$

Putting $m = 1$, $m = 2$, we find the series of intensities $0.0450\, I_0$, $0.0162\, I_0$, $0.0083\, I_0$ and so on. So even the first side peak has less than 5% of the intensity of the central peak, and the intensities drop off rapidly. This is in contrast to the two-slit interference pattern, where the side maxima were approximately as intense as the central maximum. Also, the central maximum in the single-slit pattern is twice as wide as the others, an effect not seen in the two-slit pattern.

Example 2.3

Find the ratio of the intensities of the secondary maxima to the intensity of the central maximum for the single-slit Fraungofer diffraction pattern.

Solution

To a good approximation, the secondary maxima lie midway between the zero points. From Figure 2.10 (a), we see that this corresponds to $\delta / 2$ values of $3\pi / 2$, $5\pi / 2$, $7\pi / 2$,... Substituting these values into Eq. (2.16) gives for the first two ratios

$$\frac{I_1}{I_{max}} = \left[\frac{\sin(3\pi/2)}{(3\pi/2)}\right]^2 = \frac{1}{9\pi^2/4} = 0.045,$$

$$\frac{I_2}{I_{max}} = \left[\frac{\sin(5\pi/2)}{(5\pi/2)}\right]^2 = \frac{1}{25\pi^2/4} = 0.016.$$

That is, the first secondary maxima (the ones adjacent to the central maximum) have an intensity of 4.5% that of the central maximum, and the next secondary maxima have an intensity of 1.6% that of the central maximum.

Exercises

2.19. A diffraction pattern is formed on a screen 120 cm away from a 0.400-mm-wide slit. Monochromatic 546.1 nm light is used. Calculate the fractional intensity I / I_0 at a point on the screen 4.10 mm from the center of the principal maximum.

2.20. Determine the intensity relative to the central maximum, of the secondary maxima corresponding to $m = \pm 3$ for the single-slit Fraunhofer diffraction pattern.

2.21. Coherent light with a wavelength of 501.5 nm is sent through two parallel slits in a large flat wall. Each slit is 0.700 μm wide, and the slits' centers are 2.80 μm apart. The light falls on a semi–cylindrical screen, with its axis at the midline between the slits. (a) Predict the direction of each interference maximum on the screen, as an angle away from the bisector of the line joining the slits; (b) Describe the pattern of light on the screen, specifying the number of bright fringes and the location of each; (c) Find the intensity of light on the screen at the center of each bright fringe, expressed as a fraction of the light intensity I_0 at the center of the pattern.

2.22. The intensity of light in the Fraunhofer diffraction pattern of a single slit is $I = I_{\max}(\sin\beta/\beta)^2$, where $\beta = (\pi a \sin\theta/\lambda)$, Show that the equation for the values of β at which I is a maximum is $\tan\beta = \beta$.

2.8 Intensity of Two-Slit Diffraction Pattern

When more than one slit is present, we must consider not only diffraction due to the individual slits but also the interference of the waves coming from different slits. To determine the effects of both interference and diffraction, we simply combine Eq. (1.25) and Eq. (2.17):

$$I = I_{\max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)\left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2.$$

Although this formula looks complicated, it merely represents the diffraction pattern (the factor in brackets) acting as an "envelope" for a two-slit interference pattern (the cosine-squared factor), as shown in Figure 2.16.

Equation for the conditions for interference maxima is Eq. (1.19): $d\sin\theta = m\lambda$, where d is the distance between the two slits. Eq. (2.12) specifies that the first diffraction minimum occurs when $a\sin\theta = \lambda$, where a is the slit width. Dividing Eq. (1.19) by Eq. (2.12) (with $m = 1$) allows us to determine which interference maximum coincides with the first diffraction minimum:

$$\frac{d\sin\theta}{a\sin\theta} = \frac{m\lambda}{\lambda},$$

$$\frac{d}{a} = m.$$

In Figure 2.16 $\frac{d}{a} = \frac{18\ \mu m}{3.0\ \mu m} = 6$. Thus, the sixth interference maximum (if we count the central maximum as $m = 0$) is aligned with the first diffraction minimum and cannot be seen.

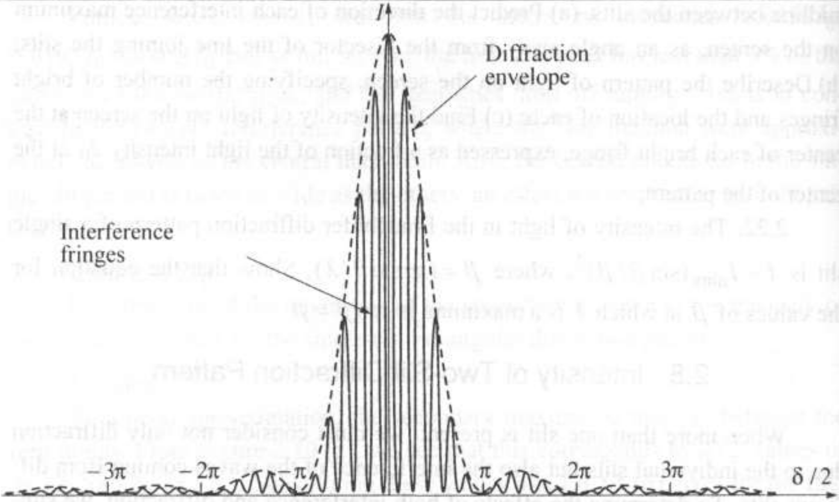


Figure 2.16 The combined effects of diffraction and interference. This is the pattern produced when 600-nm light waves pass through two 3.5-μm slits that are 20 μm apart. Notice how the diffraction pattern acts as an "envelope" and controls the intensity of the regularly spaced interference maxima

2.9 Fraunhofer Diffraction from Diffraction Grating

Suppose that instead of a single slit or two slits side by side, we have a very large number of parallel slits, all with the same width and spaced equal distances apart. Such an arrangement is called a **diffraction grating**; the first one was constructed by Fraungofer with fine wires. Gratings are now made by using a diamond point to scratch a large number of equally spaced grooves on a glass or metal surface, or by photographic reduction of a black-and-white pattern drawn with a pen.

In Figure 2.17, *GG* represents the grating; the slits are perpendicular to the plane of the page. Only five slits are shown in the diagram, but an actual grating may contain several thousands, with spacing d of the order of 0.002 mm. A plane monochromatic wave is incident normally on the grating from the left side. The problem of finding the intensity pattern in the light transmitted by the grating then combines the principles of interference and diffraction. Each slit creates a diffraction pattern, and these then interfere with each other to produce the final pattern. The lens is included so we can view the pattern on a screen at a finite distance from the grating and still meet the conditions for Fraungofer diffraction, that is, parallel rays emerging from the grating.

Let us assume that the slits are very narrow, so the diffracted beam from each slit spreads out over a wide enough angles for it to interfere with all the other diffracted beams. Consider first the light proceeding from elements of infinitesimal width at the lower edges of each opening and traveling in a direction making an angle θ with that of the incident beam, as in Figure 2.17 (a). A lens at the right of the grating forms in its focal plane a diffraction pattern similar to that which would appear on a screen at infinity. Suppose the angle θ in Figure 2.17 (a) is taken so that the distance *ab* equals λ, the wavelength of the incident light. Then $cd = 2\lambda$, $ef = 3\lambda$, and so on. The waves from all these elements, since they are in phase at the plane of the grating, are also in phase along the plane *AA* and therefore reach the point *P* in phase. The same holds true for any set of elements in corresponding positions in the various slits.

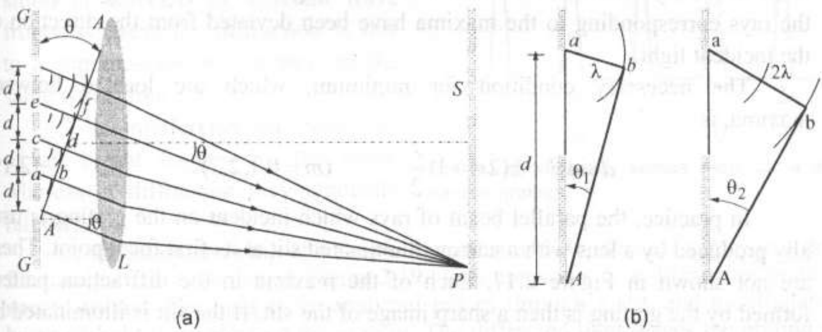


Figure 2.17 (a) The diffraction grating; (b) First-order maximum occurs when $ab = \lambda$, second-order – when $ab = 2\lambda$ and so on

If the angle θ is increased slightly, the waves from the various grating slits no longer arrive at *AA* in phase, and even an extremely small change in angle results in almost complete destructive interference among them, provided there are a large number of slits in the grating. Roughly speaking, for each slit in the grating we can find another slit whose wave arrives a half-cycle out of phase with the first one and cancels it. Hence the maximum at the angle θ is an extremely sharp one, differing from the rather broad maxima that result from interference or diffraction effects with a small number of openings.

As the angle θ is increased still further, a position is eventually reached in which the distance *ab* in Figure 2.17 (b) becomes equal to 2λ. Then *cd* equals 4λ, *ef* equals 6λ, and so on. The waves at *AA* are again all in phase; the path difference between adjacent waves is now 2λ, and another maximum results.

Still others appear when $ab = 3\lambda, 4\lambda,...$ We also find maxima at corresponding angles on the opposite side of the grating normal, as well as along the normal itself, since in the latter position the phase difference between waves reaching AA is zero.

From Figure 2.17 (b) we can find the angles of deviation at which the maxima occur. Consider the right triangle Aba. Let d be the distance between successive grating elements, called the **grating spacing**. The necessary condition for a maximum is that $ab = m\lambda$, where $m = 0, \pm 1, \pm 2, ...$, and so on. It follows that

$$d \sin\theta = \pm m\lambda \qquad (m = 0,1,2,...), \tag{2.18}$$

$$\text{or} \quad \sin\theta = \pm m\frac{\lambda}{d}$$

is the necessary condition for a maximum. The angle θ is also the angle by which the rays corresponding to the maxima have been deviated from the direction of the incident light.

The necessary condition for minimum, which are located between maxima, is

$$d \sin\theta = \pm(2m+1)\frac{\lambda}{2} \qquad (m = 0,1,2,..). \tag{2.19}$$

In practice, the parallel beam of rays which incident on the grating is usually produced by a lens with a narrow illuminated slit at its first focal point. These are not shown in Figure 2.17. Each of the maxima in the diffraction pattern formed by the grating is then a sharp image of the slit. If the slit is illuminated by light consisting of a mixture of two or more wavelengths, the grating forms two or more series of images of the slit in different positions; each wavelength in the original light gives rise to a set of slit images deviated by the appropriate angles. If the slit is illuminated with white light, the diffraction grating forms a continuous group of images side by side. That is, white light is dispersed into continuous spectra. In contrast with the single spectrum produced by a prism, a grating forms several spectra on either side of the normal. Those that correspond to $m = \pm 1$ in Eq. (2.18) are called **first order**; those that correspond to $m = \pm 2$ are called **second order**; and so on. Since for $m = 0$ the deviation is zero, all colors combine to produce a white image of the slit in the direction of the incident beam.

As Eq. (2.18) shows, the sine of the deviation angles of the maxima are proportional to the ratio $\lambda / 2$. Thus for substantial deviation to occur, the grating spacing d should be of the same order of magnitude as the wavelength λ. Gratings for use in or near the visible spectrum are ruled with from about 500 to 1500 lines per millimeter. Period of diffraction grating d is related to number of lines N in diffraction grating as $d = 1/N$.

The diffraction grating is widely used in spectrometry, instead of a prism, as a means of dispersing a light beam into spectra. If the grating spacing is known, then from a measurement of the angle of deviation of any wavelength, the value of this wavelength may be computed. This is not true for a prism; the angles of deviation are not related in any simple way to the wavelengths but depend on the characteristics of the prism material.

The intensity distribution for a diffraction grating obtained with the use of a monochromatic source is shown in Figure 2.18. Note the sharpness of the principal maxima and the broadness of the dark areas.

It wòrth to be emphasized the distinction between interference and diffraction:

1. Interference is due to superposition of wavelets of different wave fronts, whereas the diffraction is due to a superposition of wavelets of the same wave front.

2. In interference the intensities of all bright fringes are the same, whereas at diffraction they gradually fall off.

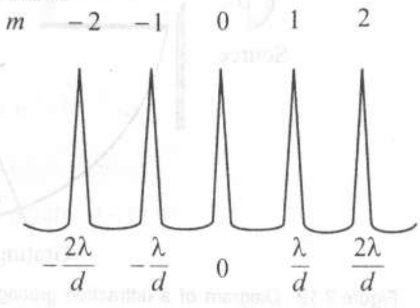


Figure 2.18 Intensity versus $\sin\theta$ for a diffraction grating

A schematic drawing of a simple apparatus used to measure angles in a diffraction pattern is shown in Figure 2.19. This apparatus is a diffraction grating **spectrometer**. The light to be analyzed passes through a slit, and a collimated beam of light is incident on the grating. The diffracted light leaves the grating at angles that satisfy Eq. (2.18), and a telescope is used to view the image of the slit. The wavelength can be determined by measuring the precise angles at which the images of the slit appear for the various orders.

Example 2.4

Light reflected from the surface of a compact disc is multicolored. The colors and their intensities depend on the orientation of the disc relative to the eye and relative to the light source. Explain how this works.

Solution

The surface of a compact disc has a spiral grooved track (with adjacent grooves having a separation on the order of 1 μm). Thus, the surface acts as a reflection grating. The light reflecting from the regions between these closely spaced grooves interferes constructively only in certain directions that depend on the wavelength and on the direction of the incident light.

Any one section of the disc serves as a diffraction grating for white light, sending different colors in different directions. The different colors you see when viewing one section change as the light source, the disc, or you move to change the angles of incidence or diffraction.

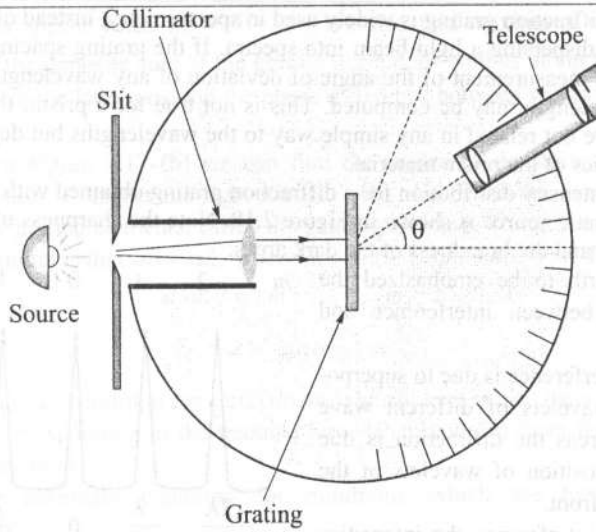


Figure 2.19 Diagram of a diffraction grating spectrometer. The collimated beam incident on the grating is diffracted into the various orders at the angles θ that satisfy the equation $d\sin\theta = mn$, where $m = \pm 0,1,2,...$

Example 2.5

Monochromatic light from a helium-neon laser ($\lambda = 632.8$ nm) is incident normally on a diffraction grating containing 6 000 lines per centimeter. Find the angles at which the first-order, second-order, and third-order maxima are observed.

Solution

First, we must calculate the slit separation, which is equal to the inverse of the number of lines per centimeter:

$$d = \frac{1}{6000} = 1.667 \times 10^{-4} \text{ cm}.$$

For the first-order maximum ($m = 1$), we obtain

$$\sin\theta_1 = \frac{\lambda}{d} = \frac{632.8}{1667} = 0.3796,$$

$$\theta_1 = 22.31°.$$

For the second-order maximum ($m = 2$), we find

$$\sin\theta_2 = \frac{2\lambda}{d} = \frac{2(632.8)}{1667} = 0.7592,$$

$$\theta_2 = 49.39°.$$

For $m=3$, we find that $\sin\theta_3 = 1.139$. Because $\sin\theta$ cannot exceed unity, this does not represent a realistic solution. Hence, only zeroth-, first-, and second-order maxima are observed for this situation.

Example 2.6

The wavelengths of the visible spectrum are approximately 400 to 700 nm. Find the angular breadth of the first-order visible spectrum produced by a plane grating having 6000 lines per centimeter, when white light is incident normally on the grating.

Solution

The grating spacing d is

$$d = \frac{1}{6000\ \text{cm}^{-1}} = 1.67\times10^{-6}\ \text{m}.$$

The angular deviation of the violet is

$$\sin\theta = \frac{400\times10^{-9}\ \text{m}}{1.67\times10^{-6}\text{m}} = 0.240\,.\ \theta = 13.9°.$$

The angular deviation of the red is

$$\sin\theta = \frac{700\times10^{-9}\ \text{m}}{1.67\times10^{-6}\text{m}} = 0.420.\ \theta = 24.8°.$$

Hence the first-order visible spectrum includes an angle of

$$24.8° - 13.9° = 10.9°.$$

Example 2.7

Show that the violet of the third-order spectrum overlaps the red of the second-order spectrum.

Solution

The angular deviation of the third-order violet is

$$\sin\theta = \frac{(3)(400\times10^{-9})}{d},$$

and of the second-order red it is

$$\sin\theta = \frac{(2)(700\times10^{-9})}{d}.$$

Since the first angle is smaller than the second, whatever the grating spacing, the third order will always overlap the second.

Exercises

Note: In the following problems, assume that the light is incident normally on the gratings.

2.23. (a) What is the wavelength of light that is deviated in the first order through an angle of 20° by a transmission grating having 6000 lines/cm?

(b) What is the second-order deviation of this wavelength?

2.24. A plane transmission grating is ruled with 4000 lines/cm. Compute the angular separation in degrees between the α and δ lines of atomic hydrogen in the second-order spectrum. The wavelengths of these lines are, respectively, 656 nm and 410 nm. (Ans.: 12.5°).

2.25. The wavelengths of the visible spectrum are approximately 400 nm to 700 nm. Find the angular breadth of the first-order visible spectrum produced by a plane grating having 6000 lines per centimeter.

2.26. White light is spread out into its spectral components by a diffraction grating. If the grating has 2 000 lines per centimeter, at what angle does red light of wavelength 640 nm appear in first order? (Ans.: 7.35°).

2.27. Light from an argon laser strikes a diffraction grating that has 5 310 lines per centimeter. The central and first-order principal maxima are separated by 0.488 m on a wall 1.72 m from the grating. Determine the wavelength of the laser light.

2.28. The hydrogen spectrum has a red line at 656 nm and a violet line at 434 nm. What is the angular separation between two spectral lines obtained with a diffraction grating that has 4 500 lines per centimeter? (Ans.: 5.91° in first order, 13.2° in the second order, 26.5° in third order).

2.29. A helium-neon laser ($\lambda = 632.8$ nm) is used to calibrate a diffraction grating. If the first-order maximum occurs at 20.5°, what is the spacing between adjacent grooves in the grating?

2.30. Three discrete spectral lines occur at angles of 10.09, 13.71, and 14.77° in the first-order spectrum of a grating spectroscope. (a) If the grating has 3 660 slits per centimeter, what are the wavelengths of the light? (b) At what angles are these lines found in the second-order spectrum? (Ans.: (a) 478.7, 647.6, and 696.6 nm; (b) 20.51, 28.30, 30.66°).

2.31. A diffraction grating has 800 rulings per millimeter. A beam of light containing wavelengths from 500 to 700 nm hits the grating. Do the spectra of different orders overlap? Explain.

2.32. A grating with 250 lines per millimeter is used with an incandescent light source. Assume that the visible spectrum ranges in wavelength from 400 to 700 nm. In how many orders can one see (a) the entire visible spectrum and (b) the short-wavelength region? (Ans.: (a) 5 orders; (b) 10 orders.)

2.33. Show that, whenever white light is passed through a diffraction grating of any spacing size, the violet end of the continuous visible spectrum in third order always overlaps the red light at the other end of the second-order spectrum.

2.34. Light of wavelength 600 nm are incident normally on a plane transmission grating having 500 lines/mm. Find the angles of deviation in the first, second, and third orders. (Ans.: 17.5, 36.9, 64.2°).

2.10 Resolution of Single Slit and Circular Aperture

The ability of optical systems to distinguish between closely spaced objects is limited because of the wave nature of light. To understand this difficulty, let us consider Figure 2.20, which shows two light sources far from a narrow slit of width a. The sources can be considered as two noncoherent point sources S_1 and S_2 – for example, they could be two distant stars. If no diffraction occurred, two distinct bright spots (or images) would be observed on the viewing screen. However, because of diffraction, each source is imaged as a bright central region flanked by weaker bright and dark fringes. What is observed on the screen is the sum of two diffraction patterns: one from S_1, and the other from S_2.

If the two sources are far enough apart to keep their central maxima from overlapping, as shown in Figure 2.20 (a), their images can be distinguished and are said to *be resolved*. If the sources are close together, however, as shown in Figure 2.20 (b), the two central maxima overlap, and the images are not resolved. In determining whether two images are resolved, the following condition is often used: *when the central maximum of one image falls on the first minimum of the other image, the images are said to be just resolved*. This limiting condition of resolution is known as **Raleigh's criterion**.

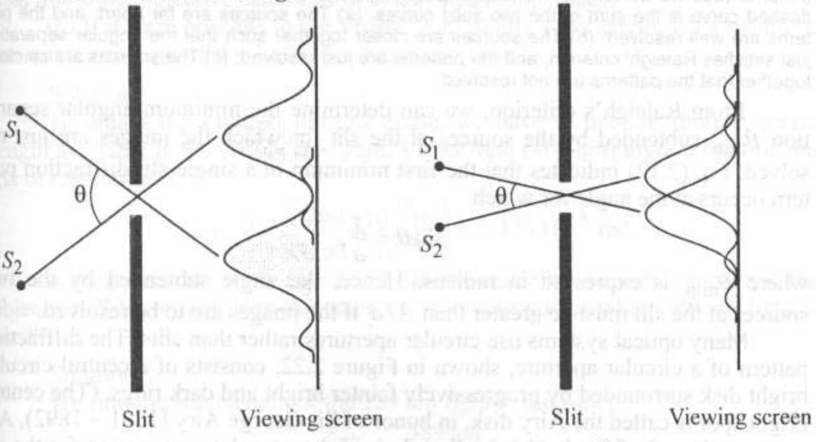


Figure 2.20 Two point sources far from a narrow slit each produce a diffraction pattern. (a) The angle subtended by the sources at the slit is large enough for diffraction pattern to be distinguishable; (b) The angle subtended by the sources is so small that their diffraction pattern is overlapping, and the images are not resolved

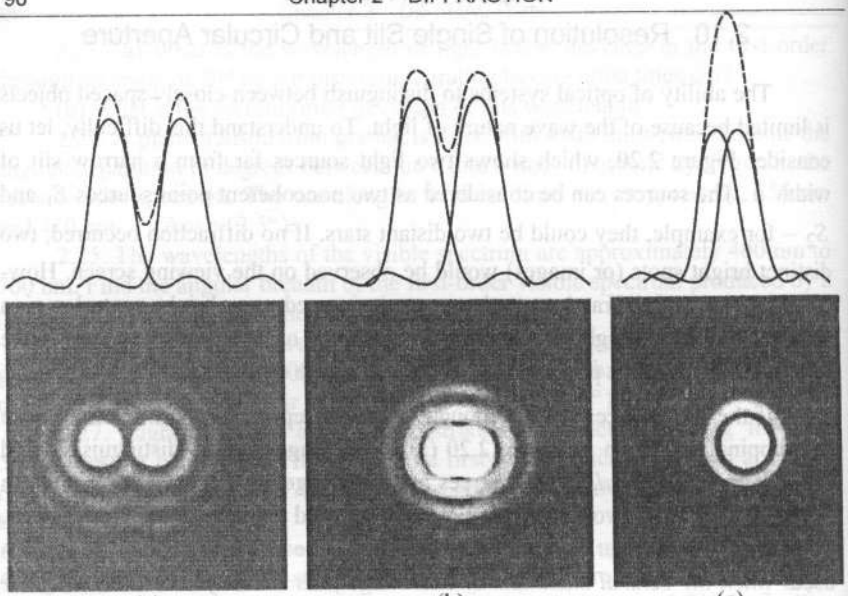


Figure 2.21 Individual diffraction patterns of two point sources (solid curves) and the resultant patterns (dashed curves) for various angular separations of the sources. In each case, the dashed curve is the sum of the two solid curves. (a) The sources are far apart, and the patterns are well resolved; (b) The sources are closer together such that the angular separation just satisfies Raleigh' criterion, and the patterns are just resolved; (c) The sources are so close together that the patterns are not resolved

From Raleigh's criterion, we can determine the minimum angular separation $\theta_{\min}$ subtended by the sources at the slit for which the images are just resolved. Eq. (2.12) indicates that the first minimum in a single-slit diffraction pattern occurs at the angle for which

$$\sin\theta = \frac{\lambda}{a},$$

where $\theta_{\min}$ is expressed in radians. Hence, the angle subtended by the two sources at the slit must be greater than λ / a if the images are to be resolved.

Many optical systems use circular apertures rather than slits. The diffraction pattern of a circular aperture, shown in Figure 2.22, consists of a central circular bright disk surrounded by progressively fainter bright and dark rings. (The central bright spot is called the Airy disk, in honor of Sir George Airy (1801 – 1892), Astronomer Royal of England, who first derived the complete expression for the intensity of the pattern.)

Analysis shows that the **limiting angle of resolution** of the circular aperture is

$$\theta_{\min} = 1.22\frac{\lambda}{D}, \tag{2.21}$$

where D is the diameter of the aperture. Note that this expression is similar to Eq. (2.20) except for the factor 1.22, which arises from a complex mathematical analysis of diffraction from the circular aperture.

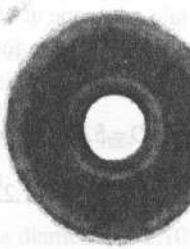


Figure 2.22 The diffraction pattern of a circular aperture consists of a central bright disk surrounded by concentric bright and dark rings

Example 2.8

Resolution of a microscope.

Light of wavelength 589 nm is used to view an object under a microscope. If the aperture of the objective has a diameter of 0.900 cm, (a) what is the limiting angle of resolution?

Solution

(a) Using Eq. (2.21), we find that the limiting angle of resolution is

$$\theta_{\min} = 1.22\left(\frac{589\times10^{-9}\,\text{m}}{0.90\times10^{-2}\,\text{m}}\right) = 7.98\times10^{-5}\ \text{rad}.$$

This means that any two points on the object subtending an angle smaller than this at the objective cannot be distinguished in the image.

(b) If it were possible to use visible light of any wavelength, what would be the maximum limit of resolution for this microscope?

Solution

To obtain the smallest limiting angle, we have to use the shortest wavelength available in the visible spectrum. Violet light (400 nm) gives a limiting angle of resolution of

$$\theta_{\min} = 1.22\left(\frac{400\times10^{-9}\,\text{m}}{0.90\times10^{-2}\,\text{m}}\right) = 5.42\times10^{-5}\ \text{rad}.$$

(c) Suppose that water ($n = 1.33$) fills the space between the object and the objective. What effect does this have on resolving power when 589 nm light is used?

Solution

We find the wavelength of the 589 nm light in the water

$$\lambda_{water} = \frac{\lambda_{air}}{n_{water}} = \frac{589\,\text{nm}}{1.33} = 443\ \text{nm}.$$

The limiting angle of resolution at this wavelength is now smaller than that calculated in part (a):

$$\theta_{\min} = 1.22\left(\frac{443\times10^{-9}\,\text{m}}{0.90\times10^{-2}\,\text{m}}\right) = 6.00\times10^{-5}\ \text{rad}.$$

Example 2.9

Resolution of a telescope.

The Hale telescope at Mount Palomar has a diameter of 5.08 m. What is its limiting angle of resolution for 600 nm light?

Solution

Because $D = 5.08$ m and $\lambda = 6.00\times10^{-7}$ m, Eq. (2.21) gives

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{6.00\times10^{-7}\,\text{m}}{5.08\text{m}}\right) = 1.44\times10^{-7}\,\text{rad} \approx 0.03''.$$

Any two stars that subtend an angle greater than or equal to this value are resolved (if atmospheric conditions are ideal).

The Hale telescope can never reach its diffraction limit because the limiting angle of resolution is always set by atmospheric blurring. This seeing limit is usually about 1 s of arc and is never smaller than about 0.1 s of arc. (This is one of the reasons for the superiority of photographs from the Hubble Space Telescope, which views celestial objects from an orbital position above the atmosphere.)

Example 2.10

Estimate the limiting angle of resolution for the human eye, assuming its resolution is limited only by diffraction.

Solution

Let us choose a wavelength of 500 nm, near the center of the visible spectrum. Although pupil diameter varies from person to person, we estimate a diameter of 2 mm. We use Eq. (2.21), taking $\lambda = 500$ nm and $D = 2$ mm:

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{5.00\times10^{-7}\,\text{m}}{2\times10^{-3}\,\text{m}}\right) = 3\times10^{-4}\,\text{rad} \approx 1'.$$

We can use this result to determine the minimum separation distance d between two point sources that the eye can distinguish if they are a distance L from the observer. Because $\theta_{\min}$ is small, we see that

$$\sin\theta_{\min} \approx \theta_{\min} \approx \frac{d}{L},$$

$$d \approx L\theta_{\min}.$$

For example, if the point sources are 25 cm from the eye (the near point), then

$$d \approx (25\text{cm})(3\times10^{-4}\,\text{rad}) = 8\times10^{-3}\,\text{cm}.$$

This is approximately equal to the thickness of a human hair.

Exercises

2.35. A child is standing at the edge of a straight highway watching her grandparents' car driving away at 20.0 m/s. The air is perfectly clear and steady, and after 10.0 min the car's two taillights appear to merge into one. Assuming the diameter of the child's pupils is 5.00 mm, estimate the width of the car. (Ans.: 1.90 m if the predominant wavelength is 650 nm).

2.36. A binary star system in the constellation Orion has an angular interstellar separation of 1.00×10^{-5} rad. If $\lambda = 500$ nm, what is the smallest diameter a telescope must have to just resolve the two stars.

2.37. A circular radar antenna on a ship has a diameter of 2.10 m and radiates at a frequency of 15.0 GHz. Two small boats are located 9.00 km away from the ship. How close together could the boats be and still be detected as two objects? (Ans.: 105 m).

2.38. If we were to send a ruby laser beam ($\lambda = 694.3$ nm) outward from the barrel of a 2.70-m-diameter telescope, what would be the diameter of the big red spot when the beam hit the Moon 384 000 km away? (neglect atmospheric dispersion.)

2.39. The angular resolution of a radio telescope is to be 0.100° when the incident waves have a wavelength of 3.00 mm. What minimum diameter is required for the telescope's receiving dish? (Ans.: 210 m).

2.40. When Mars is nearest the Earth, the distance separating the two planets is 88.6×10^6 km. Mars is viewed through a telescope whose mirror has a diameter of 30.0 cm. (a) If the wavelength of the light is 590 nm, what is the angular resolution of the telescope? (b) What is the smallest distance that can be resolved between two points on Mars?

2.41. Find the radius of a star image formed on the retina of the eye if the aperture diameter (the pupil) at night is 0.700 cm and the length of the eye is 3.00 cm. Assume that the representative wavelength of starlight in the eye is 500 nm.

2.41. A helium-neon laser emits light that has a wavelength of 632.8 nm. The circular aperture through which the beam emerges has a diameter of 0.500 cm. Estimate the diameter of the beam 10.0 km from the laser. (Ans.: 3.09 m).

2.42. The Impressionist painter Georges Seurat created paintings with an enormous number of dots of pure pigment, each of which was approximately 2.00 mm in diameter. The idea was to locate colors such as red and green next to each other to form a scintillating canvas. Outside what distance would one be unable to discern individual dots on the canvas? (Assume that $\lambda = 500$ nm and that the pupil diameter is 4.00 mm.)

2.43. Suppose that you are standing on a straight highway and watching a car moving away from you at a speed v. The air is perfectly clear and steady, and after a time t the taillights appear to merge into one. Assuming the diameter of your pupil is d, estimate the width of the car.

2.44. The pupil of a cat's eye narrows to a vertical slit of width 0.500 mm in daylight. What is the angular resolution for horizontally separated mice? Assume that the average wavelength of the light is 500 nm. (Ans.: 1.00 rad).

2.11 Resolving Power of the Diffraction Grating

The diffraction grating is most useful for measuring wavelengths accurately. Like the prism, the diffraction grating can be used to disperse a spectrum into its wavelength components. Of the two devices, the grating is the more precise if one wants to distinguish two closely spaced wavelengths.

For two nearly equal wavelengths λ_1 and λ_2 between which a diffraction grating can just barely distinguish, the **resolving power** R of the grating is defined as

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda}, \tag{2.22}$$

where $\lambda = (\lambda_1 + \lambda_2)/2$ and $\Delta\lambda = \lambda_2 - \lambda_1$. Thus, a grating that has a high resolving power R can distinguish small differences in wavelength. If N lines of the grating are illuminated, it can be shown that the resolving power in the m-th-order diffraction is

$$R = Nm. \tag{2.23}$$

Thus, resolving power increases with increasing order number and with increasing number of illuminated slits.

Note that $R = 0$ for $m = 0$; this signifies that all wavelengths are indistinguishable for the zeroth-order maximum. However, consider the second-order diffraction pattern ($m = 2$) of a grating that has 5 000 rulings illuminated by the light source. The resolving power of such a grating in second order is $R = 5000 \times 2 = 10000$. Therefore, for a mean wavelength of, for example, 600 nm, the minimum wavelength separation between two spectral lines that can be just resolved is $\Delta\lambda = \lambda / R = 6.00 \times 10^{-2}$ nm. For the third-order maximum $R = 15000$ and $\Delta\lambda = 4.0 \times 10^{-2}$ nm, and so on.

Example 2.11

When an element is raised to a very high temperature, the atoms emit radiation having discrete wavelengths. The set of wavelengths for a given element is called its atomic spectrum. Two strong components in the atomic spectrum of sodium have wavelengths of 589.00 and 589.59 nm.

a) What must be the resolving power of a grating if these wavelengths are to be distinguished?

Solution

$$R=\frac{\lambda}{\Delta\lambda}=\frac{589.3\text{nm}}{589.59\text{nm}-589.0\text{nm}}=\frac{589.3\text{nm}}{0.59\text{nm}}=999.$$

b) To resolve these lines in the second-order spectrum, how many lines of the grating must be illuminated?

Solution

From Eq. (2.23) and the results to part (a), we find that

$$N=\frac{R}{m}=\frac{999}{2}=500\text{ lines.}$$

Exercises

2.45. A diffraction grating with a width of 4.00 cm has been ruled with 3000 grooves per centimeter. (a) What is the resolving power of this grating in the first three orders? (b) If two monochromatic waves incident on this grating have a mean wavelength of 400 nm, what is their wavelength separation if they are just resolved in the third order? (Ans.: (a) 12000, 24000, 36000; (b) 11.1 pm).

2.46. A source emits 531.62 and 531.81 nm light. (a) What minimum number of lines is required for a grating that resolves the two wavelengths in the first-order spectrum? (b) Determine the slit spacing for a grating 1.32 cm wide that has the required minimum number of lines. (Ans.: (a) 2800 lines; (b) 4.72 μm).

2.47. Two wavelength λ and $\lambda+\Delta\lambda$ (with $\Delta\lambda \ll \lambda$) are incident on a diffraction grating. Show that the angular separation between the spectral lines in the m-th order spectrum is

$$\Delta\theta=\frac{\Delta\lambda}{\sqrt{(d/m)^2-\lambda^2}},$$

where d is the slit spacing and m is the order number.

2.48. The yellow sodium D lines are a doublet with wavelength of 589.0 and 589.6 nm and equal intensities. How many lines/cm are required for a diffraction grating to resolve these two lines in the first-order spectrum? Assume the grating is placed 0.5 m from a screen and the source image is 0.1 mm wide, so that resolution of the lines means their centers are 0.1 mm apart on the screen.

2.49. A source emits 531.62 and 531.81 nm light. What minimum number of lines is required for a grating that resolves the two wavelengths in the first-order spectrum? (Ans.: 2800 lines).

2.12 X-ray Diffraction

X-rays were discovered by Roentgen in 1895, and early experiments suggested that they were electromagnetic waves with wavelengths of the order of 10^{-10} m. It was also strongly suspected at this time that in a crystalline solid the

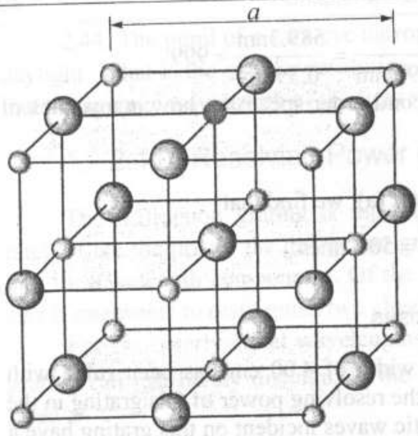


Figure 2.23 Crystalline structure of sodium chloride (NaCl). The length of the cube edge is $a = 0.562737$ nm

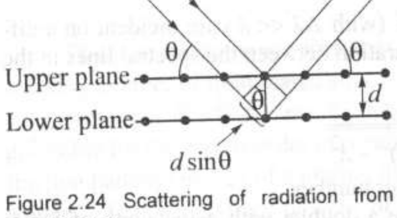


Figure 2.24 Scattering of radiation from a square array

atoms are arranged in a lattice in a regular repeating pattern, with spacing between adjacent atoms also of the order of 10^{-10} m. Putting these two ideas together, in 1913 Russian scientist G.Wolf and English physicist Max von Lauer suggested independently of each other that a crystal might serve as a kind of three-dimensional diffraction grating for x-rays. That is, a beam of x-rays might be scattered by the individual atoms in a crystal, and the scattered waves might interfere just as the individual waves from a diffraction grating interfere.

The first **x-ray diffraction** experiments were performed by Frederic and Knipping, and interference effects were observed. These experiments thus verified in a single stroke the hypothesis that x-rays are waves, or at least have wavelike properties, and that the atoms in a crystal are arranged in a regular pattern. Since that time, the phenomenon of x-ray diffraction by a crystal has proved an invaluable research tool, both as a way to measure x-ray wavelengths and as a method of studying the structure of crystals. Figure 2.23 is a diagram of the structure of a familiar crystal, sodium chloride.

To introduce the basic idea in a simple context, we consider first a two-dimensional scattering situation, as shown in Figure 2.24, where a plane wave is incident on a square array of scattering centers. The situation might be a ripple tank with an array of small posts, or 3 cm microwaves with an array of small conducting spheres, or x-rays with an array of atoms. In the case of electromagnetic waves, the wave induces an oscillating electric dipole moment in each scatterer, and each one emits a scattered wave. The total interference pattern is the superposition of all these scattered waves. To compute its nature we have to consider the total path differences for the various scattered waves, including the distances both from source to scatterer and from scatterer to observer.

As Figure 2.24 shows, the path length from source to observer is the same for all the scatterers in a single row, as shown. Scattered radiation from adjacent rows is also in phase if the path difference is an integer number of wavelengths. Figure shows that the path difference for adjacent rows is $2d\sin\theta$, where θ is the angle between the incident x-ray and the atomic row. Thus the condition for radiation from the entire array to reach the observer in phase is that the path difference for adjacent rows must equal $m\lambda$, where m is an integer. We can express the condition as

$$2d\sin\theta = \pm m\lambda, \quad (m = 1, 2, 3, ...). \tag{2.24}$$

At points where this condition is satisfied, a strong maximum in the interference pattern is observed; otherwise no strong maximum is seen because there is no point where the radiation from all scatterers arrives in phase. We can describe this interference in terms of reflections of the wave from the horizontal rows of scatterers in Figure 2.24. Strong interference occurs at angles such that Eq. (2.24) is satisfied.

We can extend this discussion to a three-dimensional array. Instead of rows, we consider planes of scatterers. Waves from all the scatterers in a given plane interfere constructively.

There is also constructive interference between planes when Eq. (2.24) is satisfied, where d is now the distance between adjacent planes. Because there are many different sets of parallel planes, there are also many values of d and many sets of angles corresponding to constructive interference for the whole crystal lattice. This phenomenon is called Wolf-Bragg reflection, and Eq. (2.24) is called the **Wolff-Bragg condition**, in honor of Russia crystallographer George Wolf and Sir William Bragg and his son Laurence Bragg, the pioneers in x-ray analysis. We must not let the term reflection obscure the fact that we are dealing with an interference effect.

Figure 2.25 is one of the experimental arrangements for observing x-ray diffraction from a crystal. The diffracted beams are very intense in certain directions, corresponding to constructive interference from waves reflected from layers of atoms in the crystal. The diffracted beams can be detected by a photographic film, and they form an array of spots known as a Laue pattern.

Diffraction of x-rays on crystals is basis of two vitally important branches of science and engineering.

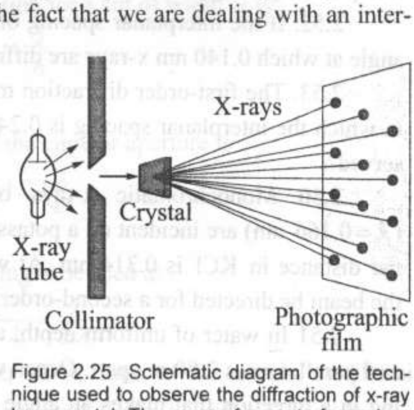


Figure 2.25 Schematic diagram of the technique used to observe the diffraction of x-ray by a crystal. The array of a spots formed on the film is called a Laue pattern

1. **X-ray spectroscopy**. If the crystal lattice spacing is known, we can determine the wavelength of radiation under investigation from the diffraction pattern, just as we determined wavelengths of visible light from measurements of diffraction patterns from slits or gratings.

2. **X-ray structure analysis**. Conversely, if we know the x-ray wavelength, we can use x-ray diffraction to explore the structure and lattice spacing of crystals of unknown structure. One can deduce the crystalline structure by analyzing the positions and intensities of the various spots in the pattern.

Indeed, x-ray diffraction has been by far the most important experimental tool in the investigation of crystal structure of solids. Atomic spacing in crystals can be measured precisely, and the details of the lattice arrangement of complex crystals can be determined. More recently, x-ray diffraction has played an important role in studies of the structures of liquids and of organic molecules.

Exercises

2.50. Potassium iodide (KI) has the same crystalline structure as NaCl, with $d = 0.353$ nm. A monochromatic x-ray beam shows diffraction maximum when the grazing angle is 7.60°. Calculate the x-ray wavelength (assume first order).

2.51. A wavelength of 0.129 nm characterizes K_a x-rays from zinc. When a beam of these x-rays is incident on the surface of a crystal whose structure is similar to that of NaCl, a first-order maximum is observed at 8.15°. Calculate the interplanar spacing.

2.52. If the interplanar spacing of NaCl is 0.281 nm, what is the predicted angle at which 0.140 nm x-rays are diffracted in a first-order maximum?

2.53. The first-order diffraction maximum is observed at 12.6° for a crystal in which the interplanar spacing is 0.240 nm. How many other orders can be observed?

2.50 Monochromatic x-rays of the K_a line from a nickel target ($\lambda = 0.166$ nm) are incident on a potassium chloride crystal surface. The interplanar distance in KCl is 0.314 nm. At what angle (relative to the surface) should the beam be directed for a second-order maximum to be observed?

2.51 In water of uniform depth, a wide pier is supported on pilings in several parallel rows 2.80 m apart. Ocean waves of uniform wavelength roll in, moving in a direction that makes an angle of 80.0° with the rows of posts. Find the three longest wavelengths of waves that will be strongly reflected by the pilings.

Summary

Diffraction of light is deviation from straight line path arising when light passes through an aperture, around a sharp edge or propagates through the inhomogeneous medium. When source and observer are so far away from the obstructing surface that the outgoing rays can be considered parallel, it is called Fraunhofer diffraction, and when source and observer is at a finite distance it is Fresnel diffraction. For a single narrow slit of width a, the condition for destructive interference at a point P at an angle θ from the perpendicular to the surface of the slit is

$$a\sin\theta = \pm m\lambda \qquad (m = 1, 2, 3, ...).$$

A diffraction grating consists of a large number of thin parallel slits, spaced a distance d apart. The condition for maximum intensity in the diffraction pattern s

$$d\sin\theta = \pm m\lambda \qquad (m = 0, 1, 2, 3, ...).$$

A crystal serves as a three-dimensional diffraction grating for waves having wavelengths of the order of magnitude of the lattice spacing; namely, x-rays. For a set of crystal planes spaced a distance d apart, constructive interference occurs when

$$2d\sin\theta = \pm m\lambda \qquad (m = 0, 1, 2, 3, ...).$$

This is so-called the Wolff-Bragg condition.

Diffraction sets the ultimate limit on resolution (image sharpness) of optical instruments. The limiting angle of resolution for a slit of width a is

$$\theta_{\min} = \frac{\lambda}{a},$$

where $\theta_{\min}$ is expressed in radians.

The limiting angle of resolution of the circular aperture is

$$\theta_{\min} = 1.22\frac{\lambda}{D},$$

where D is the diameter of the aperture.

The resolving power R of the grating is defined as

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda},$$

where $\lambda = (\lambda_1 + \lambda_2)/2$ and $\Delta\lambda = \lambda_2 - \lambda_1$. If N lines of the grating are illuminated, it can be shown that the resolving power in the m-th-order diffraction is

$$R = Nm.$$

Key Terms

diffraction	дифракция	дифракція
Fresnel diffraction	Френеля дифракция	Френеля дифракція
Fraungofer diffraction	Фраунгофера дифракция	Фраунгофера дифракція
Fresnel's zones	Френеля зоны	Френеля зони
diffraction grating	дифракционная решетка	дифракційні ґрати
grating spacing	период решетки	період ґрат
x-ray diffraction	дифракция рентгеновских лучей	дифракція рентгенівських променів
Wolff – Bragg condition	Вульфа – Брегга условие	Вульфа – Брегга умова
Raleigh's criterion	Рэлея критерий	Релея критерій
limit of resolution	предел разрешения	межа дозволу
resolving power	разрешающая способность	роздільна здатність
x-ray spectroscopy	рентгеновская спектроскопия	рентгенівська спектроскопія
x-ray structure analysis	рентгеноструктурный анализ	рентгенівський структурний аналіз

Chapter 3

Polarization of Light Waves

Waves are of two types:

1. Longitudinal waves, in which the particles oscillate along the direction of propagation of the waves.

2. Transverse waves, in which the direction of oscillation of particles is perpendicular to the direction of propagation of the waves.

Both types of these waves exhibit the phenomena of reflection, refraction, diffraction and interference but polarization of the waves can only be exhibited by transverse waves. This is the only phenomenon, where the two types of waves essentially differ from one another.

According to Maxwell, light is electromagnetic in nature. An electromagnetic wave consists of varying electric and magnetic fields, such that the two fields are mutually perpendicular to each other and to the direction of propagation of waves. The optical phenomena, i.e. phenomena concerning light, are described by the electric field vector in electromagnetic waves. Therefore, transverse nature of light may be attributed to the vibrations of electric field vector in a direction perpendicular to the direction of propagation of light.

3.1 Polarized and Unpolarized Light

Light, like all electromagnetic radiation, is predicted by electromagnetic theory to be a *transverse* wave, the directions of the vibrating electric and magnetic vectors being at right angles to the direction of propagation. The transverse nature of light cannot be predicted from the interference or diffraction experiments so far described because *longitudinal* waves such as sound also show these effects. An experimental basis for believing that light waves are transverse was predicted by Tomas Young in 1817. Two of his contemporaries, Dominique-Francois Arago (1786 – 1853) and Augustin Jean Fresnel (1788 – 1827), were able, by allowing a light beam to fall on a crystal of calcite, to produce two separate beams. Astonishingly, these beams although coherent, produced no interference fringes but only a uniform illumination. Young deduced from this that light must be a transverse wave and that the planes of vibration in the two beams must be at right angles to each other, because wave disturbances that act at right angles to each other cannot show interference effect.

An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source. Each atom produces a wave having some particular orientation of the electric field vector $\bar{E}$, corresponding to the direction of atomic vibration. The **plane of vibration** of each individual wave is defined to be the

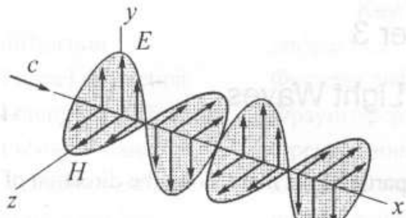


Figure 3.1 Schematic diagram of an electromagnetic wave propagating in the x-direction. The electric field vibrates in the xy-plane, and the magnetic field vibrates in the xz plane

plane in which the electric field is vibrating. In Figure 3.1, this direction happens to lie along the xy plane. However, an individual electro-magnetic wave could have its $\vec{E}$ vector in the yz plane, making any possible angle with the y axis. The plane formed by $\vec{H}$ and the direction of propagation is called the **plane of polarization** of the wave. Figure 3.1 represents the resultant of all individual waves; the plane of polarization is the xz plane. Plane of polarization is perpendicular to the plane of vibration.

The phenomenon, due to which the vibrations of light are restricted in a particular plane, is called the **polarization of light**.

Light from ordinary sources is not polarized for a slightly subtle reason. The "antennas" that radiate light waves are the molecules of which the light sources are composed. The electrically charged particles in the molecules acquire energy in some way and radiate this energy as electromagnetic waves of short wavelength. The waves from any one molecule may be linearly polarized, but since any actual light source contains a tremendous number of molecules, oriented at random, the light emitted is a random mixture of waves linearly polarized in all possible transverse direction. Thus the light propagated in a given direction consists of independent wave–trains whose planes of vibration are randomly oriented about the direction of propagation. Such light, though still transverse, is *unpolarized*. Such an unpolarized light beam is represented in Figure 3.2 (a). The direction of wave propagation in this figure is perpendicular to the page. The arrows show a few possible directions of the electric field vectors for the individual waves making up the resultant beam. At any given point and at some instant of time, all these individual electric field vectors add to give one resultant electric field vector.

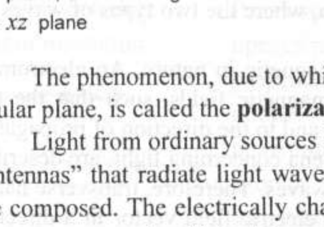


Figure 3.2 (a) An unpolarized light beam viewed along the direction of propagation (perpendicular to the page). The transverse electric field can vibrate in any direction in the plane of the page with equal probability; (b) A linearly polarized light beam with the electric field vibrating in the vertical direction

A wave is said to be **linearly polarized** if the resultant electric field $\vec{E}$ vibrates in the same direction at a particular point, as shown in Figure 3.2 (b). Sometimes, such a wave is called plane-polarized, or simply polarized.

Consider two mutually perpendicular electric field oscillations, which vibrate along x and y axis, correspondingly, and have phase difference δ:

$$E_x = E_1 \cos \omega t \quad \text{and} \quad E_y = E_2 \cos(\omega t + \delta). \tag{3.1}$$

The resultant vector $\vec{E}$ is the vector sum of $\vec{E}_x$ and $\vec{E}_y$ (Figure 3.3).

The angle θ between the vectors $\vec{E}$ and $\vec{E}_x$ is defined by the expression:

$$\tan\theta = \frac{E_y}{E_x} = \frac{E_2 \cos(\omega t + \delta)}{E_1 \cos \omega t}.$$

When the phase difference δ changes randomly then the angle θ, i.e. the direction of light vector changes intermittently and randomly also. Therefore the **natural (unpolarized** *or polarized randomly*) **light** can be represented as a superposition of two incoherent electromagnetic waves of the same intensity which are polarized in two mutually perpendicular planes. Such representation simplifies essentially the study of passing of unpolarized light through polarizing devises.

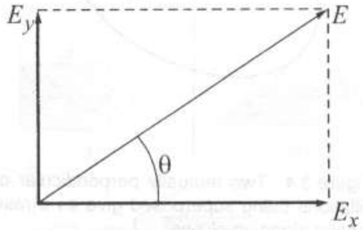


Figure 3.3 The resulting electric field intensity $\vec{E}$ is the vector sum of $\vec{E}_x$ and $\vec{E}_y$

Let the light waves E_x and E_y are coherent and the phase difference δ between them equals to 0 or π. Then according to Eq. (3.1)

$$\tan\theta = \pm\frac{E_2}{E_1} = const. \tag{3.2}$$

Hence the resultant vector vibrates in a fixed direction and the wave occurs to be plane-polarized.

In the particular case when $E_1 = E_2$ and $\delta = \pm\pi/2$:

$$\tan\varphi = \pm\tan\omega t,$$

$[\cos(\omega t \pm \pi/2) = \mp\sin\omega t]$. It follows that the plane of polarization rotates about the direction of ray with angular velocity equal to the frequency of oscillations ω. In this situation light occurs to be circular polarized.

To study the character of resulting oscillation in the case of arbitrary but constant δ, take into account that E_x and E_y in Eq. (3.1) can be treated as coordinates of the tip of the resultant vector $\vec{E}$ (Figure 3.4).

From the theory of oscillations it is known that two mutually perpendicular harmonic oscillations of the same frequency being superposed give as a result a motion along an ellipse in general case.

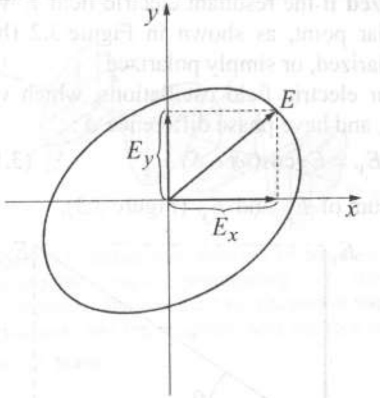


Figure 3.4 Two mutually perpendicular oscillations being superposed give as a result motion along an ellipse

In particular cases motion along a straight line or along a circle can be obtained. Similarly, the point with coordinates, defined by Eqs. (3.1), that is, the tip of light vector $\vec{E}$, moves along ellipse. Therefore, two coherent plane-polarized light waves with mutually perpendicular planes of vibration being superposed give elliptically polarized light wave.

When phase difference δ equals to zero or π ellipse degenerates into straight line and plane-polarized light occurs as a result. When $\delta = \pm\pi/2$ and amplitudes E_1 and E_2 are equal, ellipse transforms into circumference and we obtain circular polarized light.

It is possible to obtain linearly polarized beam from an unpolarized beam by removing all waves from the beam except those whose electric field vectors oscillate in a particular plane. There are four processes for producing polarized light from unpolarized light:

1. Polarization by selective absorption.
2. Polarization by reflection.
3. Polarization by double refraction.
4. Polarization by scattering.

Exercises

3.1. Sound waves cannot be polarized. Why?

3.2. Will ultrasonic waves show any polarization? Give reason for your answer.

3.3. What information does one obtain from polarization about the nature of light?

3.4. Does the value of wavelength of light have any role in polarization?

3.5. What is plane-polarized light?

3.6. Which plane is defined as the plane of polarization in a plane-polarized electromagnetic wave?

3.2 Polarizers

In this book we study electromagnetic waves, especially light, but to introduce basic concepts we first consider the mechanical example of transverse waves on a string. For a string whose equilibrium position is along the x-axis, the displacements may be along the y-direction, as in Figure 3.5 (a). In this case the string always lies in the xy-plane. But the displacements might instead be along the z-axis, as in Figure 3.5 (b), so that the string lies in the xz-plane.

A wave having only y-displacements in the discussion above is said to be linearly polarized in the y-direction, and the one with only z-displacements is linearly polarized in the z-direction. It is easy in principle to construct a mechanical filter that permits only waves with a certain polarization direction to pass. An example is shown in Figure 3.5 (c); the string can slide vertically in the slot without friction, but no horizontal motion is possible. This filter passes waves polarized in the y-direction but blocks those polarized in the z-direction. This same language can be applied to light and other electromagnetic waves. As we have learned, an electromagnetic wave consists of fluctuating electric and magnetic fields, perpendicular to each other and to the direction of propagation. By convention, the direction of polarization is taken to be that of the electric-field vector, not the magnetic field, because most mechanisms for detecting electromagnetic waves employ principally the electric-field forces on electrons in materials. That is, the most common manifestations of electromagnetic radiation are due chiefly to the electric-field force, not the magnetic-field force.

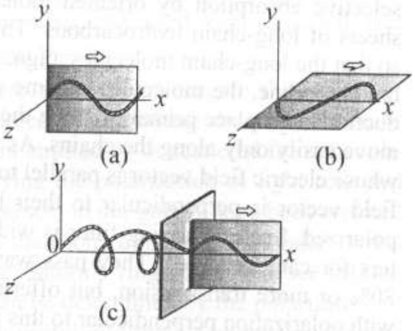


Figure 3.5 (a) Transverse wave on a string polarized in the y-direction; (b) Wave polarized in the z-direction; (c) Barrier with a frictionless vertical slot passes components polarized in the y-direction but blocks those polarized in the z-direction, acting as a polarizing filter

Polarizing filters, or **polarizes**, can be made for electromagnetic waves; the details of construction depend on the wavelength. For microwaves having a wavelength of a few centimeters, a grid of closely spaced, parallel conducting wires insulated from each other will pass waves whose $\vec{E}$ fields are perpendicular to the wires but not those with $\vec{E}$ fields parallel to the wires.

A Nicol prism or tourmaline crystal cut light parallel to its crystallographic axis behaves in a similar manner for light waves as the slit does in case of the wave motion produced in the string.

The most common technique for producing polarized light is to use a material that transmits waves whose electric fields vibrate in a plane parallel to a certain direction (called the *optical axis* or **polarizing axis**) and that absorbs waves whose electric fields vibrate in all other directions. In 1938, E. H. Land (1909 – 1991) discovered a material, which he called **polaroid** that polarizes light through selective absorption by oriented molecules. This material is fabricated in thin sheets of long-chain hydrocarbons. The sheets are stretched during manufacture so that the long-chain molecules align. After a sheet is dipped into a solution containing iodine, the molecules become good electrical conductors. However, conduction takes place primarily along the hydrocarbon chains because electrons can move easily only along the chains. As a result, the molecules readily absorb light whose electric field vector is parallel to their length and allow light whose electric field vector is perpendicular to their length. The emerging light will be plane–polarized. Such polarizing filter is widely used for sunglasses and polarizing filters for camera lenses. They pass waves polarized parallel polarizing axis with 80% or more transmission, but offering only 1% or less transmission to waves with polarization perpendicular to this axis.

Artificial colloid films serve for obtaining a polarized light. Herapathite, which is compound of iodine and quinine, is common material for preparation of polaroid. The compound is introduced in a celluloid or gelatin film. In the film ultramicroscopic herapathite crystals by any way (usually mechanically) orient theirs axis along certain direction. The obtained mass, similarly to tourmaline act as single crystal and absorbs light oscillations, which have electrical vector, perpendicular to the optical axis.

An ideal polarizer (or simply polarizer) has the property that it passes 100% of the incident light polarized in the direction of the filter's polarizing axis but blocks completely all light polarized perpendicular to this axis. Such a device is an unattainable idealization, but the concept is useful in clarifying the basic ideas.

In an ideal polarizer, all light with $\vec{E}$ parallel to the transmission axis is transmitted, and all light with $\vec{E}$ perpendicular to the transmission axis is absorbed.

The polarizer which blocks not all the light polarized perpendicular to transmission axis is called **imperfect polarizer**. The light emerging from the imperfect polarizer is not plane-polarized completely. There exist oscillations of all direction, but direction parallel to the axis of polarizer is predominated. Such a light is called **partially polarized light**. It can be represented as a mixture of polarized and unpolarized light. Partially polarized light, just like unpolarized one, can be represented as a superposition of two incoherent waves with mutually perpendicular planes of vibration. But in unpolarized light these waves are of equal intensities, whereas in the partially polarized light the wave intensities are different.

If partially polarized light passes through polarizer, then at rotation of the polarizer about the direction of light beam, the intensity of emerging light varies from I_{max} up to I_{min}, though transition from one of the above values to the other will happen at turning at angle $\theta = \pi/2$. That is during one complete revolution maximum value will be reached twice as well as minimum value does. The expression

$$P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \tag{3.3}$$

defines *a* **degree of polarization**. For the plane-polarized light $I_{min} = 0$ and $P = 1$. For unpolarized light $I_{max} = I_{min}$ and $P = 0$.

Historically polarization studies were made to investigate the nature of light. Today we reverse the procedure and deduce something about the nature of an object from the polarization state of light emitted by or scattered from that object. It has been possible to deduce, studying the polarization of light reflected from them, that the grains of cosmic dust present in our galaxy have been oriented in the weak galactic magnetic field so that their long dimension is parallel to this field. Polarization studies have shown that Saturn's rings consist of ice crystal. The size and the shape of virus particles can be determined by the polarization of ultraviolet light scattered from them. Much useful information about the structure of atoms and nuclei is gained from the polarization studies of their emitted radiations in all parts of the electromagnetic spectrum. Thus we have a useful research technique for structures ranging in size from a galaxy ($\sim 10^{+20}$ m) to a nucleus ($\sim 10^{-14}$m). Polarized light also has many practical applications in industry and in engineering science.

Polaroid sheets whose polarizing direction is vertical are used in sunglasses for reducing the intensity of direct sunlight and reduce glare of reflected light. Most reflected light are partially plane-polarized. Hence a major component of reflected light is absorbed and the transmitted light is of reduced intensity.

In three-dimensional moves, two pictures of the same scene are projected slightly displaced from each other on a curved screen. The scene on the screen looks blurred to the naked eye. When viewed with a pair of glasses made of polaroids, one with vertical polarizing direction and other with horizontal polarizing direction, each eye sees a separate scene. The brain interprets the two pictures as a single picture with depth.

In natural light the random orientation of the planes of vibration produces symmetry about the propagation direction, which, on casual study, conceals the true transverse nature of the waves. To study this transverse nature, a way must be found to distinguish the different planes of vibration. Device which serves for transformation of unpolarized or partially polarized light into plane-polarized light is called polarizer.

Exercises

3.7. Why do sunglasses made of polarizing material have a marked advantage over those that simply depend on absorption effect?

3.8. Derive a way to identify the polarizing direction of a sheet of polaroid.

3.9. Can you detect by naked eye, whether a given light is polarized or not?

3.10. What is a polaroid?

3.11. Write several uses of polaroids.

3.3 Polarization by Selective Absorption. Malus's Law

Let oscillation of amplitude E_0 vibrates in a plane that makes angle θ with plane of polarizer. The oscillation can be resolved into two mutually perpendicular oscillations with amplitudes $E_1 = E_0 \cos\theta$ and $E_2 = E_0 \sin\theta$, respectively (Figure 3.6). The first oscillation passes through the device, whereas the second one is blocked.

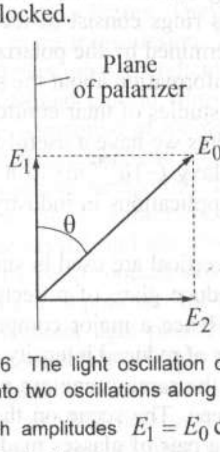


Figure 3.6 The light oscillation can be resolved into two oscillations along x and y axes with amplitudes $E_1 = E_0 \cos\theta$ and $E_2 = E_0 \sin\theta$, respectively

Unpolarized light has equiprobable value of θ. Hence, the part of light, transmitted through the polarizer is proportional to the average value of $\cos^2\theta$, i.e. $1/2$. Therefore when polarizer rotates about the direction of unpolarized light beam, then the intensity of transmitted light remains constant.

Figure 3.7 represents an unpolarized light beam E^* incident on a polarizing sheet, called the *polarizer*. The component of E^* parallel to the polarizing axis passes through the polarizer, whereas the component, perpendicular to the polarizing axis, is blocked. Intensity varies as the square of amplitude, so we can conclude that intensity of this plane-polarized light will be one-half of the unpolarized light intensity, i.e. $I_0 = \frac{1}{2} I^*$.

After polarizer the unpolarized light emerges as the plane-polarized one E_0. Because the transmission axis is oriented vertically in the figure, the light transmitted through this sheet is polarized vertically.

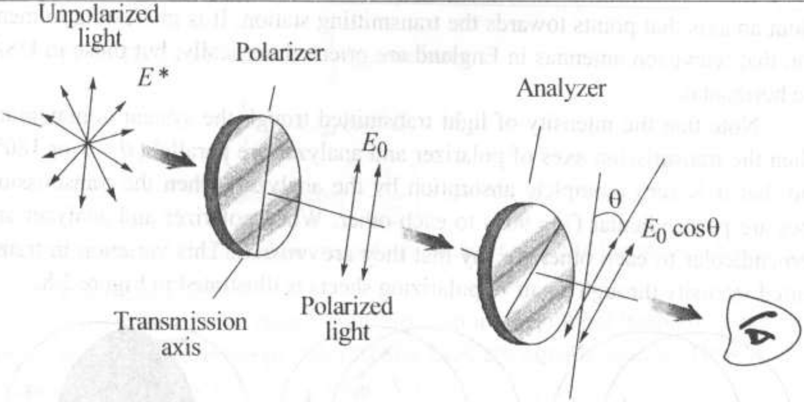


Figure 3.7 Two polarizing sheets whose transmission axes make an angle θ with each other. Only a fraction of the polarized light incident on the analyzer is transmitted through it

A second polarizing sheet, (usually called, when so used, an **analyzer**), intercepts the beam. In Figure 3.7, the analyzer transmission axis is set at an angle θ to the polarizer axis. The component of E_0 parallel to the analyzer axis, which is allowed through the analyzer, is $E = E_0 \cos\theta$, and intensity of light transmitted through the analyzer will be

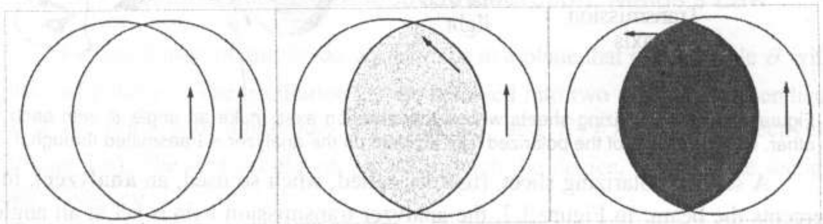
$$I = I_0 \cos^2\theta = \frac{1}{2} I^* \cos^2\theta. \tag{3.4}$$

Eq. (3.4), called the **law of Malus**, was discovered by Etienne Louiz Malus (1775 – 1812) experimentally in 1809. Eq. (3.4) describes precisely the lack of symmetry about the propagation direction that must be exhibited by plane-polarized transverse waves.

Electromagnetic waves in the radio and microwave range exhibit polarization character readily. Such a wave, generated by the surging of charge up and down in the dipole that forms the transmitting antenna has (at large distances from the dipole and at right angles to it) an electric field vector parallel to the dipole axis. When the plane-polarized wave falls on a second dipole connected to a microwave detector, the alternating electric component of the wave will cause electrons to surge back and forth in the receiving antenna, producing a reading on the detector. If we turn the receiving antenna through 90° about the direction of propagation, the detector reading drops to zero. In this orientation the electric field vector is not able to cause charge to move along the dipole axis because it points at right angles to this axis. We can reproduce this experiment by turning the receiving antenna of a TV set (assumed an electric dipole type) through 90°

about an axis that points towards the transmitting station. It is interesting to mention that television antennas in England are oriented vertically, but those in USA are horizontal.

Note that the intensity of light transmitted trough the system is maximum when the transmission axes of polarizer and analyzer are parallel ($\theta = 0$ or $180°$) and that it is zero (complete absorption by the analyzer) when the transmission axes are perpendicular ($\theta = 90°$) to each other. When polarizer and analyzer are perpendicular to each other we say that they are *crossed*. This variation in transmitted intensity through a pair of polarizing sheets is illustrated in Figure 3.8.



(a) (b) (c)

Figure 3.8 The intensity of light transmitted through two polarizes depends on the relative orientation of their transmitted axes. (a) The transmitted light has maximum intensity when the transmission axis is aligned with each other; (b) The transmitted light has lesser intensity when the transmission axis is at an angle of $45°$ with each other; (c) The transmitted light has minimum intensity when the transmission axes are at right angle with each other

Examples

Example 3.1

In Figure 3.7 the incident unpolarized light has intensity I^*. Find the intensity transmitted by the first polarizer and by the second if the angle θ is $30°$.

Solution

As explained above, the intensity after the first filter is $I^*/2$. According to Eq. (3.4), the second filter reduces the intensity by a factor of $\cos^2 30° = 3/4$. Thus the intensity transmitted by the second polarizer is $\left(\frac{I^*}{2}\right)\left(\frac{3}{4}\right) = \frac{3}{8}I^*$.

Example 3.2

Two polarizing sheets have their polarizing directions parallel so that the intensity I_m of the transmitted light is a maximum. Through what angle must either sheet be turned if the intensity is to drop by one-half?

Solution

From Eq. (3.4), since $I=\frac{1}{2}I_m$, we have

$$\frac{1}{2}I_m = I_m \cos^2\theta, \quad \text{or}$$

$$\theta = \arccos\left(\pm\frac{1}{\sqrt{2}}\right) = \pm 45°;\ 135°.$$

The same effect is obtained no matter which sheet is rotated or in which direction.

Example 3.3

Two polaroids are placed at 90° to each it other. What happens, when $N-1$ more polaroids are between them? Their axes are equally spaced. How does the transmitted intensity behave for large N?

Solution.

When $N-1$ polaroids are inserted between two polaroids, the total number of polaroids becomes $N+1$. The axes of all the polaroids are equally spaced. If θ is angle between the axes of two adjacent polaroids, then

$$\theta_1+\theta_2+\theta_3+\ldots.+\theta_N = 90°, \quad \text{or} \quad N\theta = 90°, \quad \text{or} \quad \theta=\frac{\pi}{2N}.$$

According to Malus law, the intensity of light on passing through a pair of polaroid is proportional to $\cos^2\theta$. Before the light passes out of the last polaroid, this change in intensity will be repeated N times. If I_0 is intensity of the incident light and I the intensity of light after passing through all the polaroids then

$$I = I_0(\cos^2\theta)^N = I_0(\cos\theta)^{2N},$$

or

$$I = I_0\left(\cos\frac{\pi}{2N}\right)^{2N}.$$

When N is very large, angle $\theta=\frac{\pi}{2N}$ approaches zero and hence $\left(\cos\frac{\pi}{2N}\right)$ approaches 1. Therefore, when N is very large, I will approach I_0.

Exercises

3.12. How can one distinguish between an unpolarized light beam and a linearly polarized beam using a polaroid?

3.13. What do you mean by term crossed polaroids?

3.14. Unpolarized light falls on two polarizing sheets so oriented that no light is transmitted. If a third polarizing sheet is placed between them, can light be transmitted?

3.15. Unpolarized light passes through two polarized sheets. The axis of the first is vertical, and that of the second is 30° to the vertical. What fraction of the initial light is transmitted? (Ans.: 3/8).

3.16. Unpolarized light of intensity I^* is incident on a polarizing filter, and the emerging light strikes a second polarizing filter with its axis 45° to that of the first. Determine (a) the intensity of the emerging beam; (b) its state of polarization. (Ans.: (a) $I_0/4$, (b) Linearly polarized, parallel to axis of second polarizer).

3.17. A polarizer and an analyzer are oriented so that the maximum amount of light is transmitted. To what fraction of its maximum value is the intensity of the transmitted light reduced when the analyzer is rotated through:

a) 30°?

b) 45°?

c) 60°?

3.18. Three polarizing filters are stacked, with the polarizing axes of the second and third at 45° and 90°, respectively, with that of the first. (a) If unpolarized light of intensity I^* is incident on the stack, find the intensity and state of polarization of light emerging from each filter. (b) If the second filter is removed, how does the situation change? (Ans.: (a) $I^*/2$. $I^*/4$, $I^*/8$. In each case the light is linearly polarized, along the axis of the polarizer; (b) $I^*/2$ for the first, 0 for the next).

3.19. A beam of unpolarized light of intensity 10 mW/m^2 is sent through a polarizing sheet. (a) Find the maximum value of the electric field of the transmitted beam. (b) What radiation pressure is exerted on the polarizing sheet? (Ans.: (a) 1.9 V/m; (b) 1.7×10^{-11} Pa).

3.20. Plane-polarized light is incident on a single polarizing disk, with the direction of E_0 parallel to the direction of the transmission axis. Through what angle should the disk be rotated so that the intensity in the transmitted beam is reduced by a factor of (a) 3.00, (b) 5.00, (c) 10 .00? (Ans.: 54.7, 63.4, 71.6°).

3.21. An unpolarized beam of light is incident on a group of four polarizing sheets which are lined up so that the characteristic direction of each is rotated by 30°clockwise with respect to the preceding sheet. What fraction of the incident intensity is transmitted?

3.22. A horizontal beam of vertically polarized light of intensity 43 W/m^2 is sent through two polarizing sheets. The polarizing direction of the first is at 70° to the vertical, and that of the second is horizontal. What is the intensity of light transmitted by the pair of sheets? (Ans.: 4.4 W/m^2).

3.4 Polarization by Reflection. Brewster's Law

When an unpolarized light beam is reflected from a surface, the reflected light may be completely polarized, partially polarized, or unpolarized, depending on the angle of incidence. Anyone who has watched the sun's reflection in water, while wearing a pair of sunglasses made of polarizing sheet, has probably noticed the effect. It is necessary only to tilt the head from side to side, thus rotating the polarizing sheets, to observe that the intensity of the reflected sunlight passes through a minimum. If the angle of incidence is 0°, the reflected beam is unpolarized. For other angles of incidence, the reflected light is polarized to some extent, and for one particular angle of incidence, the reflected light is completely polarized. Let us now investigate reflection at that special angle.

Suppose that an unpolarized light beam is incident on a surface, as shown in Figure 3.9 (a).

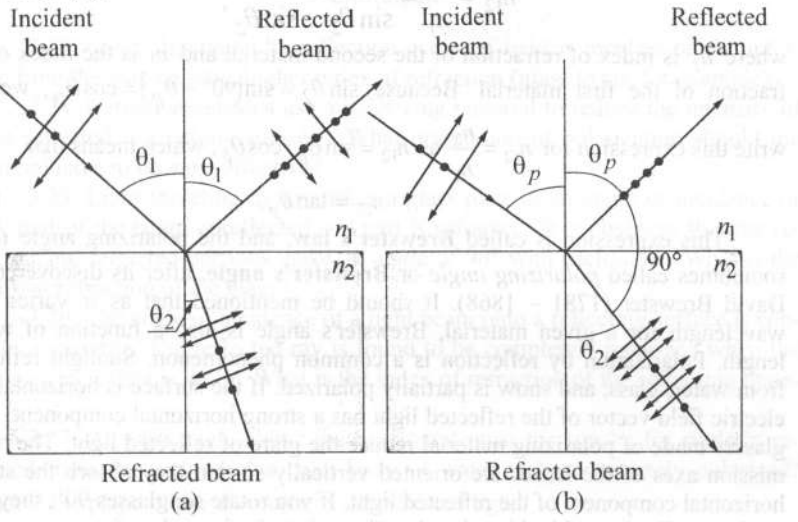


Figure 3.9 (a) When unpolarized light is incident on a reflecting surface, the reflected and refracted beams are partially polarized; (b) The reflected beam is completely polarized when the angle of incidence equals the polarizing angle u_p, which satisfies the Eq. $n_{12} = \tan\theta_p$

Each individual electric field vector can be resolved into two components: one parallel to the surface (and perpendicular to the page in Figure 3.9, represented by the dots), and the other (represented by the arrows) perpendicular both to the first component and to the direction of propagation. Thus, the polarization of the entire beam can be described by two electric field components in these directions. It is found experimentally that the parallel component reflects more strongly than the perpendicular component, and these results in a partially polarized reflected beam. Furthermore, the refracted beam is also partially polarized.

Now suppose that the angle of incidence θ_1 is varied until the angle between the reflected and refracted beams is 90°, as shown in Figure 3.9 (b). At this particular angle of incidence, the reflected beam is completely polarized (with its electric field vector parallel to the surface), and the refracted beam is still only partially polarized. The angle of incidence at which this polarization occurs is called the **polarizing angle** θ_p.

We can obtain an expression relating the polarizing angle to the index of refraction of the reflecting material by using Figure 3.9 (b). From this figure, we see that $\theta_p + \frac{\pi}{2} + \theta_2 = \pi$; thus, $\theta_2 = \frac{\pi}{2} - \theta_p$. Using Snell's law of refraction, we have

$$n_{12} = \frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin\theta_p}{\sin\theta_2},$$

where n_2 is index of refraction of the second material and n_1 is the index of refraction of the first material. Because $\sin\theta_2 = \sin(90° - \theta_p) = \cos\theta_p$, we can write this expression for $n_{12} = \frac{n_2}{n_1}$ as $n_{12} = \sin\theta_p / \cos\theta_p$, which means that

$$n_{12} = \tan\theta_p. \tag{3.5}$$

This expression is called **Brewster's law**, and the polarizing angle θ_p is sometimes called *polarizing angle* or **Brewster's angle**, after its discoverer, Sir David Brewster (1781 – 1868). It should be mentioned that as n varies with wavelength for a given material, Brewster's angle is also a function of wavelength. Polarization by reflection is a common phenomenon. Sunlight reflected from water, glass, and snow is partially polarized. If the surface is horizontal, the electric field vector of the reflected light has a strong horizontal component. Sunglasses made of polarizing material reduce the glare of reflected light. The transmission axes of the lenses are oriented vertically so that they absorb the strong horizontal component of the reflected light. If you rotate sunglasses 90°, they will not be as effective at blocking the glare from shiny horizontal surfaces.

Example 3.4

A ray of light is incident on the surface of a glass plate of refractive index 1.536 at the polarizing angle. Calculate the angle of refraction.

Solution

Here, $n = 1.536$. If θ_p is polarizing angle, then

$$\tan\theta_p = 1.536 \quad \text{or} \quad \theta_p = 56°56'.$$

If θ_2 is the angle of refraction, then

$$\theta_2 = 90° - 56°56' = 33°4'.$$

Example 3.5

We wish to use a plate of glass ($n = 1.5$) as a polarizer. What is the polarizing angle? What is the angle of refraction?

Solution

Polarizing angle θ_p according Brewster law is:

$$\theta_p = \arctan 1.5 = 56.3^\circ .$$

The angle of refraction θ_r follows from Snell's law:

$$\sin\theta_p = n\sin\theta_r$$

or

$$\sin\theta_r = \frac{\sin 56.3^\circ}{1.5} = 0.555, \quad \theta_r = 33.7^\circ .$$

Exercises

3.23. Can polarization by reflection occurs if light is incident on the interface from the side with the higher index of refraction (glass to air, for example).

3.24. Certain sunglasses use a polarizing material to reduce the intensity of light reflected from shiny surfaces. What orientation of polarization should the material have to be most effective?

3.25. Light traveling in air strikes a glass plate at an angle of incidence of 60°; part of the beam is reflected and part is refracted. It is observed that the reflected and refracted portions make an angle of 90° with each other. What is the index of refraction of the glass?

3.26. The angle of incidence of a light beam onto a reflecting surface is continuous variable. The reflected ray is found to be completely polarized when the angle of incidence is 48.0°. What is the index of refraction of the refracting material?

3.27. At what angle above the horizontal must the sun be for sunlight reflected from the surface of a calm body of water to be completely polarized? What is the plane of the $\vec{E}$ vector in the reflected light?

3.28. A parallel beam of unpolarized light is incident at an angle of 58° (with respect to the normal) on a plane glass surface. The reflected beam is completely linearly polarized. (a) What is the refractive index of the glass? (Ans.: 1.6). (b) What is the angle of refraction of the transmitted beam? (Ans.: 32.0°).

3.5 Polarization by Double Refraction

Although direct sunlight is unpolarized, light from much of the sky is at least partially polarized by scattering. Bees use the polarization of sky light in navigation to and from their hives. Similarly, the Vikings used it to navigate across the North Sea when the daytime Sun was below the horizon because of the high latitude of the North Sea). These early seafarers had discovered certain crystals (now called cordierite) that changed color when rotated in polarized light. By looking at the sky through such a crystal while rotating it about their line of sight, they could locate the hidden Sun and thus determine which way was sough.

In previous chapters we assumed that the speed of light, and hence, the dielectric permittivity and, hence, index of refraction, is independent on the direction of propagation in the medium and of the state of polarization of light. Liquids, amorphous solids such as glass, and crystalline solids having cubic symmetry normally show this behavior and said to be *optically isotropic*.

In certain anisotropic crystalline materials, however, such as calcite and quartz, the speed of light is not the same in all directions. Such materials are characterized by two indexes of refraction. Hence, they are often referred to as **double-refracting** or **birefringent materials**.

Upon entering a calcite crystal, unpolarized light splits into two plane-polarized beams that travel with different velocities, corresponding to two angles of refraction, as shown in Figure 3.10. The two beams are polarized in two mutually perpendicular directions, as indicated by the dots and arrows. One beam, called the **ordinary (O) beam,** is characterized by an index of refraction n_o that is the same in all directions.

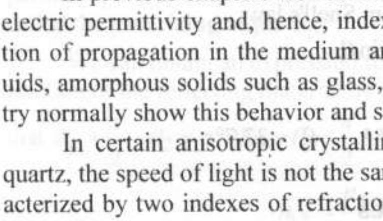


Figure 3.10 Unpolarized light incident on a calcite crystal splits into an ordinary (O) ray and an extraordinary (E) ray. These two rays are polarized in mutually perpendicular directions

This means that if one could place a point source of light inside the crystal, as shown in Figure 3.11, the ordinary waves would spread out from the source as spheres. Ordinary beam does obey the laws of geometric optic.

The second plane-polarized beam, called the **extraordinary (E) beam**, travels with different speeds in different directions and hence is characterized by an index of refraction n_e that varies with the direction of propagation. The point source in Figure 3.11 sends out an extraordinary wave having wave fronts that are elliptical in cross-section. Extraordinary beam does not obey the laws of geometric optic.

Note from Figure 3.11 that there is one direction, called the **optical axis**, along which the ordinary and extraordinary beams have the same speed, corresponding to the direction for which $n_o = n_e$. It should be mentioned that optical axis is not a straight line but a certain *direction* in a crystal and any straight line parallel to this direction is called an optical axis. Any plane constructed through the optical axis named a **principal cross section** or **principal plane**. The oscillation plane of ordinary ray is perpendicular to the principal plane, whereas the oscillation plane of extraordinary one is parallel to it. After emerging the crystal the both rays differ only in the direction of polarization, hence the concepts of ordinary and extraordinary rays are important only inside the crystal.

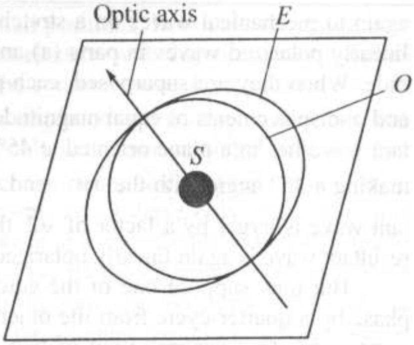


Figure 3.11 A point source S inside a double-refracting crystal produces a spherical wave front corresponding to the ordinary ray and an elliptical wave front corresponding to the extraordinary ray. The two waves propagate with the same velocity along the optical axis

The difference in speed for the two beams is a maximum in the direction perpendicular to the optical axis. For example, in calcite, $n_o = 1.658$ at a wavelength of 589.3 nm, and n_e varies from 1.658 along the optic axis to 1.486 perpendicular to the optic axis. If we place a piece of calcite on a sheet of paper and then look through the crystal at any writing on the paper, we see two images. These two images correspond to one formed by the ordinary beam and second formed by the extraordinary beam.

Exercises

3.29. Is the optical axis of a doubly refracting crystal simply a line or a direction in space?

3.30. Does the E-wave in doubly refracting crystals always travel at a speed given c/n_e?

3.31. What is birefringent material?

3.32. What happens when unpolarized light enters birefringent material?

3.33. In what direction the difference in speed for ordinary and extraordinary beams is a maximum?

3.34 Explain how the cross-polarized light with different speeds can exists in a birefringent material?

3.6 Circular and Elliptical Polarization

Up to this point we have discussed polarization phenomena in terms of linearly polarized light. Light (and all other electromagnetic radiation) may also have circular or elliptical polarization. To understand these new concepts, we return again to mechanical waves on a stretched string. In Figure 3.5, suppose the two linearly polarized waves in parts (a) and (b) are in phase and have equal amplitude. When they are superposed, each point in the string has simultaneously y- and z-displacements of equal magnitude, and a little thought shows that the resultant wave lies in a plane oriented at 45° to the $y-$ and $z-$ axes (i.e., in a plane making a 45° angle with the $xy-$ and $xz-$ planes). The amplitude of the resultant wave is larger by a factor of $\sqrt{2}$ than that of both component wave and the resultant wave is again linearly polarized.

But now suppose one of the equal-amplitude component waves differs in phase by a quarter-cycle from the other. Then the resultant motion of each point corresponds to a superposition of two simple harmonic motions at right angles with a quarter-cycle phase difference. The motion is then no longer confined to a single plane, and it can be shown that each point on the rope moves in a circle in a plane parallel to the $yz-$plane. Successive points on the rope have successive phase differences, and the overall motion of the string then has the appearance of a rotating helix. This particular superposition of two linearly polarized waves is called **circular polarization**. By convention, the wave is said to be *right circularly polarized* when the motion of a particle of the string, to an observer looking backward along the direction of propagation, is clockwise. The wave is *left circularly polarized* when it appears counterclockwise to that observer. Left circular polarization would be the result if the phase difference between $y-$ and $z-$ components were opposite to that in our example.

If the phase difference between the two component waves is something other than a quarter-cycle, or if the two component waves have different amplitudes, then each point on the string traces out not a circle but an ellipse. The resulting wave is said to be **elliptically polarized**.

For electromagnetic waves of radio frequencies, circular or elliptical polarization can be produced by using two antennas at right angles, fed from the same transmitter but with a phase-shifting network that introduces the appropriate phase difference. For light, the phase shift can be introduced by use of a birefringent material.

We now seek to understand, in terms of the atomic structure of optically anisotropic crystals, how cross-polarized light wave with different speeds can exist. Light is propagated through a crystal by the action of the vibrating $\vec{E}$ vectors of the wave on the electrons in the crystal. These electrons, which experience electrostatic restoring forces if they are moved from their equilibrium positions, are set into forced periodic oscillation and pass along the transverse wave disturbance that constitutes the light wave. The strength of the restoring forces may be measured by a force constant k, as for simple harmonic oscillation. In optically isotropic materials the force constant k is the same for all directions of displacement of the electrons from their equilibrium positions. In doubly refracting crystals, however, k varies with direction. For electron with displacements in a plane at right angles to the optical axis k has the constant value k_o, no matter how the displacement is oriented in this plane. For displacements parallel to the optic axis, k has the larger value k_e. Also, for crystals with three principal indexes of refraction, there will be three principal force constants. Such crystals have two optics axes and are called **biaxial**. The crystals which have only one single optic axis are called **uniaxial**. The speed of a wave in a crystal is determined by the direction in which the $\vec{E}$ vectors vibrate and not by the direction of propagation. It is the transverse $\vec{E}$-vector vibrations call the restoring forces into play and thus determine the wave speed. Note too, that the stronger the restoring force, that is, the larger k, the faster the wave. For waves traveling along a stretched cord, for example, the restoring force for the transverse displacements is determined by the tension in the cord.

Let's discuss a linearly polarized light passing through a plate of anisotropic substance of thickness d. Before entering the plate the electric field intensities E_o and E_e of ordinary and extraordinary beams vibrates in the same phase, giving linearly polarized light $\vec{E} = \vec{E}_o + \vec{E}_e$ as a result. Inside the plate ordinary and extraordinary waves propagate with different velocities and after the plate electric field vectors E_o and E_e have optical path difference

$$\Delta = d(n_o - n_e)$$

and corresponding phase difference:

$$\Delta\varphi = \frac{2\pi\Delta}{\lambda_0} = \frac{2\pi d}{\lambda_0}(n_o - n_e),$$

where λ_0 is the wavelength of light in vacuum. Hence after passing the plate the light becomes elliptically polarized in general case. Depending on thickness of a plate several particular cases are possible:

1. **A quarter-wave plate**. Thickness of such a plate satisfies the condition:

$$d(n_o - n_e) = \pm\left(m + \frac{1}{4}\right)\lambda_0,$$

where $m = 0,1,2,...$ After such a plate vectors E_o and E_e have phase difference of $\pi/2$. If additionally $\theta = \pi/4$ light will be circularly polarized.

2. **A half-wave plate.** $d(n_o - n_e) = \pm\left(m + \frac{1}{2}\right)\lambda_0$. After such a plate the phase difference is π. Emerging light remains linearly polarized but directions of polarizations of incident and emerging light are symmetric about the principal plane of polarization.

3. **A wave plate**. For this case $d(n_o - n_e) = \pm m\lambda_0$. Emerging light remains linearly polarized and directions of polarization remain the same.

Some doubly refracting crystals have the interesting property, called **dichroism**, in which one of the polarization components is strongly absorbed within the crystal, the other being transmitted with little loss. Dichroism, illustrated in Figure 3.12 is the basic operating principle of the commercial polaroid sheet. The many small crystallites, imbedded in a plastic sheet with their optical axes parallel, have a polarizing action equivalent to that of a single large crystal slab.

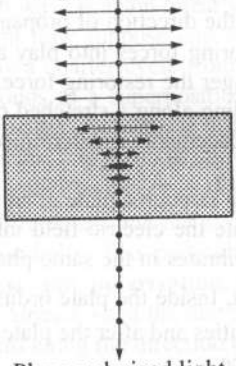


Figure 3.12 Showing the absorption of one polarization component inside a dichroic crystal of the type used in Polaroid sheets θ

Example 3.6

A quartz quarter-wave plate is said to be used with sodium light $\lambda = 5890\,\text{Å}$. What must its thickness be?

Solution

Two waves travel through the slab at speed corresponding to the principal indexes of refraction $n_e = 1.553$ and $n_o = 1.544$. If the crystal thickness is x, the number of wavelength of the first wave contained in the crystal is:

$$N_e = \frac{x}{\lambda_e} = \frac{xn_e}{\lambda},$$

where λ_e is the wavelength of the E-wave in the crystal and λ is the wavelength in air.

For the second wave the number of wavelengths is

$$N_o = \frac{x}{\lambda_o} = \frac{xn_o}{\lambda},$$

where λ_o is the wavelength of the O – wave in the crystal. The difference $N_e - N_o$ must be one-fourth, or

$$\frac{1}{4} = \frac{x}{\lambda}(n_e - n_o).$$

This equation yields

$$x = \frac{\lambda}{4(n_e - n_o)} = \frac{5890\,A}{4(1.544 - 1.533)} = 0.016 \text{ mm}.$$

This plate is rather thin: most quarter-wave plates are made from mica, splitting the sheet to the correct thickness by trial and error.

Example 3.7

A beam of circularly polarized light falls on a polarizing sheet. Describe the emerging beam.

Solution

The circularly polarized light, as it enters the sheet, can be represented by

$$E_x = E_m \sin \omega t \quad \text{and} \quad E_y = E_m \cos \omega t ,$$

where x and y represent arbitrary perpendicular axes. These equations correctly represent the fact that a circularly polarized wave is equivalent to two plane-polarized waves with equal amplitude and a 90° phase difference.

The resultant amplitude in the incident circularly polarized wave is

$$E_{cp} = \sqrt{E_x{}^2 + E_y{}^2} = \sqrt{E_m{}^2(\sin^2 \omega t + \cos^2 \omega t)} = E_m,$$

an expected result if the circularly polarized wave is represented as a rotating vector. The resulting intensity in the incident circularly polarized wave is proportional to $E_m{}^2$, or

$$I_{cp} \propto E_m{}^2 . \tag{3.6}$$

Let the polarizing direction of the sheet make an arbitrary angle θ with x axis. The instantaneous value of the plane-polarized wave transmitted by the sheet is

$$E = E_y \sin\theta + E_x \cos\theta = E_m \cos\omega t \sin\theta + E_m \sin\omega t \cos\theta = E_m \sin(\omega t + \theta).$$

The intensity of the wave transmitted by the sheet is proportional to E^2, or

$$I \propto E_m{}^2 \sin^2(\omega t + \theta).$$

The eye and other measuring instruments respond only to the average intensity I, which is found by replacing $\sin^2(\omega t + \theta)$ by its average value over one or more cycles (=1/2), or

$$I \propto \frac{1}{2} E_m{}^2 .$$

Comparison with (3.6) shows inserting the polarizing sheet reduces the intensity by one-half. The orientation of the sheet makes no difference, since θ does not appear in this expression; this is to be expected if circularly polarized light is represented by a rotating vector, all azimuths about the propagation direction being equivalent. Inserting a polarizing sheet in an unpolarized beam has just the same effect, so that a simple polarizing sheet cannot be used to distinguish between unpolarized and circularly polarized light.

Example 3.8

A beam of light is thought to be circularly polarized. How may this be verified?

Solution

Insert a quarter-wave plate. If the beam is circularly polarized, the two components will have a phase difference of 90° between them. The quarter-wave plate will introduce a further phase difference of ±90° so that the emerging light will have a phase difference of either zero or 180°. In either case the light will now be plane-polarized and can be made to suffer complete extinction by rotating a polarizer in its path.

Exercises

3.35. Explain the existence of elliptically and circularly polarized light.

3.36. Distinguish between right circularly polarized light and left circularly polarized light.

3.37. What is quarter-wave plate?

3.38. When does rotation of plane of polarization take place?

3.39. Derive a way to identify the direction of the optic axis in a quarter-wave plate.

3.40. If plane-polarized light falls on a quarter-wave plate with its plane of vibration making an angle of 0°, or 90° with the axis of plate, describe the transmitted light. If this angle is arbitrary chosen, the transmitted light is called elliptically polarized; describe such light.

3.41. What would be the action of a half-wave plate (that is, a plate twice as thick as a quarter-wave plate) on (a) plane-polarized light (assume the phase of vibration to be at 45° to the axis of the plate); (b) circular polarized light and (c) unpolarized light?

3.42. You are given an object which may be (a) a disk of grey glass, (b) a polarizing sheet, (c) a quarter-wave plate, or (d) a half-wave plate. How could you identify it?

3.43. Would you expect a quarter-wave plate made from calcite to be thicker than one made from a quartz?

3.44. What is the state of polarization of the light transmitted by a quarter-wave plate when the electric vector of the incident linearly polarized light makes an angle of 30° with the optic axis?

3.45. A beam of right circularly polarized light is reflected at normal incidence from a reflecting surface. Is the reflected beam right or left circularly polarized? Explain.

3.46. Can a plane-polarized light be represented as a sum of two circularly polarized light beam of opposite rotation? What effect has changing the phase of one the circular components on the resultant beam?

3.47. A beam of light is said to be unpolarized, linearly polarized, or circularly polarized. How could you choose among them experimentally?

3.48. What is dichroism?

3.7 Polarization by Scattering

When light is incident on any material, the electrons in the material can absorb and reradiate part of the light. Such absorption and reradiation of light by electrons in the gas molecules that make up air is what causes sunlight reaching an observer on the Earth to be partially polarized. You can observe this effect – called **scattering** – by looking directly up at the sky through a pair of sunglasses whose lenses are made of polarizing material. Less light passes through at certain orientations of the lenses than at others.

Figure 3.13 illustrates how sunlight becomes polarized when it is scattered. An unpolarized beam of sunlight traveling in the horizontal direction (parallel to the ground) strikes a molecule of one of the gases that makes up air, setting the electrons of the molecule into vibration. These vibrating charges act like the vibrating charges in an antenna. The horizontal component of the electric field vector in the incident wave results in a horizontal component of the vibration of the charges whereas the vertical component of the vector results in a vertical component of vibration. If the observer in Figure 3.13 is looking straight up (perpendicular to the original direction of propagation of the light), the vertical oscillations of the charges send no radiation toward the observer. Thus, the observer sees light that is completely polarized in the horizontal direction, as indicated by the arrows. If the observer looks in other directions, the light is partially polarized in the horizontal direction.

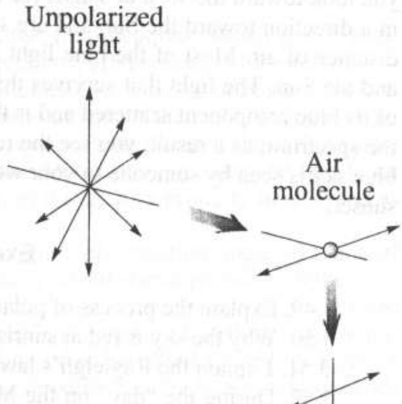


Figure 3.13 The scattering of unpolarized sunlight by air molecules. The scattered light traveling perpendicular to the incident light is plane-polarized because the vertical vibrations of the charges in the air molecule send no light in this direction

Some phenomena involving the scattering of light in the atmosphere can be understood as follows. When light of various wavelengths λ is incident on gas molecules of diameter d, where $d << \lambda$, the intensity of the scattered light varies according to Raleigh law as $1/\lambda^4$.

The condition $d \ll \lambda$ is satisfied for scattering from oxygen (O_2) and nitrogen (N_2) molecules in the atmosphere, whose diameters are about 0.2 nm. Hence, short wavelengths (blue light) are scattered more efficiently than long wavelengths (red light). Therefore, when sunlight is scattered by gas molecules in the air, the short-wavelength radiation (blue) is scattered more intensely than the long-wavelength radiation (red).

When you look up into the sky in a direction that is not toward the Sun, you see the scattered light, which is predominantly blue; hence, you see a blue sky. If you look toward the west at sunset (or toward the east at sunrise), you are looking in a direction toward the Sun and are seeing light that has passed through a large distance of air. Most of the blue light has been scattered by the air between you and the Sun. The light that survives this trip through the air to you has had much of its blue component scattered and is thus heavily weighted toward the red end of the spectrum; as a result, you see the red and orange colors of sunset. However, a blue sky is seen by someone to your west for whom it is still a quarter hour before sunset.

Exercises

3.49. Explain the process of polarization at scattering.

3.50. Why the sky is red at sunrise and sunset and blue at daytime?

3.51. Explain the Rayleigh's law.

3.52. During the "day" on the Moon (that is, when the Sun is visible), you see a black sky and the stars are clearly visible. During the day on the Earth, you see a blue sky and no stars. Explain this difference.

3.53. You can make the path of a light beam visible by placing dust in the air (perhaps by shaking a blackboard eraser in the path of the light beam). Explain why you can see the beam under these circumstances.

3.54. Is light from the sky polarized? Why is it that clouds seen through Polaroid glasses stand out in bold contrast to the sky?

3.55. When observing a clear sky through a polarizing sheet, one finds that the intensity varies by a factor of two on rotating the sheet. This does not happen when one views a cloud through the sheet. Can you devise an explanation?

3.8 Optical Activity

Rotation of Plane of Polarization. In 1811 D. Arago discovered a beam of linearly polarized light was sent through crystal of quartz along its optical axis, the direction of polarization of the emerging linearly polarized light is *found to be different from the original direction.* If the plane-parallel plate of quartz, cut perpendicularly to its optical axis is placed between crossed polarizer and analyzer, the field of view of analyzer clarifies. For complete darkening of the field of view the analyzer should be turned about the beam at certain angle θ, which is equal to the rotation angle in the quartz plate. This phenomenon is called *the rotation of the direction of polarization* and substance that exhibit this effect are said to be **optically active.**

Further investigations show that the phenomenon is inherent to several other substances. Not only other birefringent crystals (cinnabar), but some optically isotropic crystals, solutions (for example, camphor in benzol, water solution of sugar, glucose) and pure liquids (turpentine, nicotine) are optically active. It was found that all the substances which are optically active in liquid state (including solutions), display the same property in crystalline state. At the same time some substances, which are optically active in the crystalline state, are not optically active in liquid state (melted quartz).

For the most optically active crystals two modifications, which ensure rotation of plane of polarization in opposite directions, are revealed. One of them implements clockwise rotation, the other – anticlockwise rotation of plane of polarization. Those that rotate the direction of polarization to the right, looking along the advancing beam, are called *dextrorotatory, or right-handed*; those that rotate it to the left are *levorotatory, or left-handed*. Optical activity may be due to an asymmetry of the molecules of a substance, or it may be a property of a crystal as a whole.

In optically active crystals and pure liquids the rotation angle of plane of polarization θ is proportional to the thickness l of substance passed by light:

$$\theta = \alpha l, \tag{3.8}$$

where constant α is so-called *specific rotational constant*. Coefficient α is numerically equal to the rotation angle of plane of polarization caused by layer of optically active substance of unit thickness. Specific rotational constant depends on nature of substance, temperature, and wavelength of light (rotational dispersion). Far from regions of absorption of light by substance, dependence of α on λ satisfies law of Bio: $\alpha \sim 1/\lambda^2$. For dextrorotatory and levorotatory modifications of the same substance the values of α differ only in sign.

It was shown by J. Bio that in solutions the angle of rotation depends on the property of active substance $[\alpha]$, concentration of solution c and the length l of the light path through it:

$$\theta = [\alpha]cl, \tag{3.7}$$

where $[\alpha]$ is *specific rotational constant of solution*. It depends on nature of the optically active substance and its solvent, temperature, and wavelength of light. The rotation of plane of polarization by a sugar solution is used commercially as a method of determining the proportion of sugar in a given sample.

Crystalline quartz is also optically active; some natural crystals are right-handed and others left-handed. Here the optical activity is a result of the crystalline structure, since the activity disappears when the quartz is melted and allowed to resolidify into a glassy, noncrystalline state called fused quartz. The angle through which the light is rotated by a crystal depends on the property of a crystal and length of the path:

$$\theta = \alpha l, \tag{3.8}$$

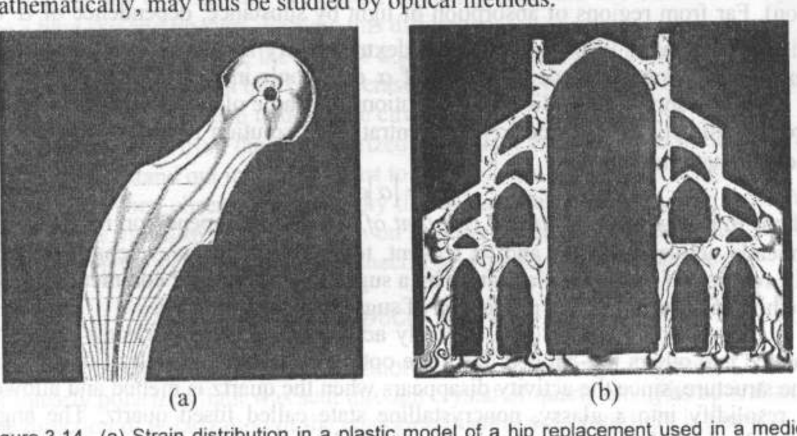
here α is so-called *rotational constant*.

Photoelastivity. Molecular asymmetry determines whether a material is optically active. For example, some proteins are optically active because of their spiral shape. Other materials, such as glass and plastic, become optically active when being stressed. This is the basis of **photoelastivity**. Suppose that an unstressed piece of plastic is placed between a polarizer and an analyzer so that light passes from polarizer through plastic to analyzer. When the plastic is unstressed and the analyzer axis is perpendicular to the polarizer axis, none of the polarized light passes through the analyzer. In other words, the unstressed plastic has no effect on the light passing through it. If the plastic is stressed, however, it becomes optically active due to difference of indexes of refraction of ordinary and extraordinary rays and the regions of greatest stress rotate the polarized light through the largest angles. Hence, a series of bright and dark bands is observed in the transmitted light, with the bright bands corresponding to regions of greatest stress. There was experimentally established that difference of indexes of refraction is proportional to the stress σ at a given point:

$$n_o - n_e = k\sigma,$$

where k is the coefficient of proportionality which depends on the property of the substance.

Scientists and engineers often use this technique, called **optical stress analysis**, in designing structures ranging from bridges to small tools. They build a plastic model and analyze it under different load conditions to determine regions of potential weakness and failure under stress. Some examples of a plastic model under stress are shown in Figure 3.14. Very complicated stress distributions, such as those around a hole or gear tooth, that are practically impossible to analyze mathematically, may thus be studied by optical methods.



(a) (b)

Figure 3.14 (a) Strain distribution in a plastic model of a hip replacement used in a medical research laboratory. The pattern is produced when the plastic model is viewed between a polarizer and analyzer oriented perpendicular to each other; (b) A plastic model of an arch structure under load conditions observed between perpendiculars polarizes. Such patterns are useful in the optimum design of architectural components

Kerr Effect. Liquids are not usually doubly refracting, but some of them obtain the property when an electric field is established within them. This phenomenon is known as *Kerr effect*, after John Kerr (1824 – 1907). The liquid crystal displays found in most calculators have their optical activity changed by the application of electric potential across different parts of the display.

Kerr cell with transparent walls contains the liquid between a pair of parallel plates. The cell is inserted between crossed polaroids. Light is transmitted when an electric field is set up between the plates and is cut off when the field is removed. Experiments proved that liquid polarized and get properties of uniaxial birefringent crystal when is inserted between plates of parallel–plate capacitor. Optical axis of liquid coincides with direction of electric field strength $\vec{E}$ of capacitor.

The resulting difference of indexes of refraction for ordinary and extraordinary beams of monochromatic light in direction perpendicular to the vector $\vec{E}$ is proportional to the squared electric field intensity:

$$n_o - n_e = B\lambda E^2,$$

where λ is wavelength of light, B – so–called *Kerr constant*. Kerr constant depends on the nature of substance, wavelength λ and temperature and usually decreases with increasing of the last.

Kerr cell placed between crossed polarizer and analyzer, acts on the light in the same manner as plane–parallel plate. It produces phase difference between ordinary and exraordinary beams and the corresponding phase difference can be expressed as

$$\delta = \frac{2\pi d}{\lambda}(n_o - n_e) = -2\pi d B E^2 = -2\pi d B \frac{V^2}{a^2}, \qquad (3.9)$$

Where d is the length of cell, which is equal to the length of plates of capacitor, $V = Ea$ – potential difference between the plates of capacitor, and a – distance between the plates of capacitor. At $V = 0$ the cell is completely isotropic and does not change the character of polarization of light, incident at it. Therefore light does not pass through the cell. As far as potential difference increases, the phase difference increases according to Eq. (3.9). As a result the intensity of light emerging from analyzer, increases, and reaches maximum at V corresponding to $\delta = -\pi$:

$$V = \frac{a^2}{2dB}.$$

Kerr effect is inertialess practically; the duration of processes of transition from isotropic state into anisotropic one and reverse does not exceed $10^{-9} - 10^{-10}$ s. Therefore giving alternative voltage V on the plates of capacitor, we have opportunity to modulate the intensity of light through the analyzer according to oscillation of V. The existence of the Kerr effect makes it possible to construct an electrically controlled "light valve", high–speed photography and so on.

Liquid crystals. Displays for digital watches and calculators use liquid crystals. Try turning a piece of polaroid directly over a liquid crystal display (LCD). The contrast between the displayed information and the background is changed. Liquid crystals are not rigid and can flow, but molecules are in a regular order just like atoms in a solid crystal. Each LCD is a matrix of segments, each segment activated by a tiny electric current which make it darker than its background. So by activating a suitable combination of segments, the LCD can be made to display any number required.

The reason why a segment becomes darker when activated is that it does not reflect light when it is activated. The segment is constructed from a cell of liquid crystal between crossed polaroids. The underside of the top piece of the cell is marked with lots of fine parallel lines along which the molecules of the liquid crystal line up. The bottom of the cell is also marked with fine parallel lines, but positioned at right angles to the top set, so the line–up of molecules changes by 90° from the top to the bottom. As a result, light which passes through the cell has its plane of polarization turned through 90°. Therefore even though the cell is between crossed polaroids, light can pass through it. With no voltage across the cell, light from the room passes through it, reflects off a mirror underneath, and passes back out again: so the cell allows the light to be reflected. When a voltage is applied across the cell, the molecules line up along the electric field instead of the twisted arrangement explained above. The light cannot pass through since the new line does not rotate the plane of polarization. Hence the cell is dark when activated.

Cotton – Mutton effect. In 1907 E. Cotton and X. Mutton discovered that isotropic substances display optical anisotropy when placed in strong magnetic field. The difference between indexes of refraction of ordinary and extraordinary beams advancing perpendicular to the direction of optical axis, i.e., vector $\vec{H}$ of uniform magnetic field has the form:

$$n_o - n_e = -C\lambda H^2 .$$

Faraday's Effect. Some optically inactive substance obtains the property to rotate plane of polarization when inserted in magnetic field. This phenomenon was discovered by Faraday and is called *Faraday effect.* It can be observed only when light advances along direction of magnetic field. Therefore for observation of Faraday's effect several apertures are drilled through pole lead of electromagnet to let the light beam. The substance under investigation is placed between the poles of electromagnet.

The rotation angle of polarization plane φ is proportional to the path l, passed by light in substance, and magnetic field strength H:

$$\varphi = VlH\,, \tag{3.10}$$

where the coefficient V is so-called *Verde constant* or *specific magnetic rotation*. Verde constant depends on the nature of substance and wavelength of light.

The direction of rotation is defined by direction of magnetic field, it doesn't depend on the direction of light ray. Therefore when light reflected from a mirror the angle of rotation is doubled. Magnetic rotation is caused by precession of electron orbits in magnetic field.

Optical active substances being exposed to magnetic field gain additional ability to rotate the plane of polarization, which is added to their own one.

Exercises

3.56. What substances are called optically active and why they have such a property?

3.57. Write and explain angle of rotation of plane of polarization for solutions and solids.

3.58. Explain the photoelastivity phenomena.

3.59. What is optical stress analysis?

3.60. Explain the Kerr effect.

3.61. Explain the Faraday effect.

Summary

Electromagnetic waves, like all transverse waves, exhibit polarization. The direction of polarization of a linearly polarized wave is defined as the direction of the $\vec{E}$ field. A polarizing filter passes radiation that is linearly polarized in the direction of its polarizing axis, and blocks radiation polarized perpendicularly to that axis. When linearly polarized light is incident on a polarizing filter with its axis at an angle θ to the direction of polarization, the transmitted light intensity is

$$I = I_{\max}\cos^2\theta\,,$$

where $I_{\max}$ is the intensity at $\theta = 0$. This relation is called Malus' law. When unpolarized light strikes an interface between two materials, the reflected light is completely polarized perpendicular to the plane of incidence if the angle of incidence θ_p is given by

$$\tan\theta_p = \frac{n_2}{n_1}.$$

This relation is called Brewster's law.

Materials having different indexes of refraction for two perpendicular directions of polarization are said to be birefringent. Those that show preferential absorption for one polarization direction are dichroic; they are used in polarizing filters for light. Some materials become birefringent under mechanical stress; these form the basis of photoelastic stress analysis.

Light can be scattered by air molecules. The scattered light is preferentially polarized.

When two linearly polarized waves with a phase difference are superposed, the result is circularly or elliptically polarized light. In this case the $\vec{E}$ vector is not confined to a plane containing the direction of propagation but describes a circle or ellipse in the planes perpendicular to the direction of propagation.

Key Terms

lineary polarized light	поляризованный свет	поляризоване світло
polarizer	поляризатор	поляризатор
polarizing axis	ось поляризатора	вісь поляризатора
Malus' law	Малюса закон	Малюса закон
Brewster's law	Брюстера закон	Брюстера закон
polarizing angle	угол поляризации (Брюстера угол)	кут поляризації (Брюстера кут)
birefringent	двойное лучепреломление	подвійне променезаломлення
dichroism	дихроизм	дихроїзм
circular polarization	круговая поляризация	кругова поляризація
Kerr effect	Керра эффект	Керра ефект
Faraday effect	Фарадея эффект	Фарадея ефект

Chapter 4 Propagation of Light through Substance

4.1 Dispersion of Light

When materials are exposed to electromagnetic radiation, it is important to be able to predict optical properties of different materials, and understand the mechanisms responsible for their optical behaviors. In this chapter we discuss phenomena of dispersion of light, absorption of light and Vavilov–Cherenkov's radiation.

4.1.1 Introduction to Dispersion

It had been known for centuries that small fragments of colorless glass and precious stones glittered in bright colors when white light passes through them, but it was not until the middle of the seventeenth century that Sir Isaac Newton investigated the problem systematically. Newton's work on this subject arose out of the need for finding a way of removing coloration from the images seen through a telescope.

Most light beams are a superposition of waves with different wavelengths extending throughout the visible spectrum. The speed of light in vacuum is the same for all wavelengths but the speed of light in a material is different for different ones.

It means that index of refraction n encountered by light in any medium except vacuum depends on the wavelength of the light. The dependence of n on wavelength implies that when a light beam consists of rays of different wavelengths, the rays will be refracted at different angles by a surface; that is, the light will be spread out by the refraction. This spreading of light is called **chromatic dispersion,** in which "chromatic" refers to the colors associated with the individual wavelengths and **"dispersion"** refers to the spreading of the light according to its wavelengths or colors.

The dispersion can be characterized by functions:

$$n = f(\lambda) \quad \text{or} \quad n = f(\omega).$$

This dependence is not linear, i.e., $\frac{dn}{d\lambda} \neq const$. In the region of visible light (which is usually far from absorption region), for all transparent substances $\frac{dn}{d\lambda} < 0$ or $\frac{dn}{d\omega} > 0$ (Figure 4.1). Such dispersion is called **normal dispersion**.

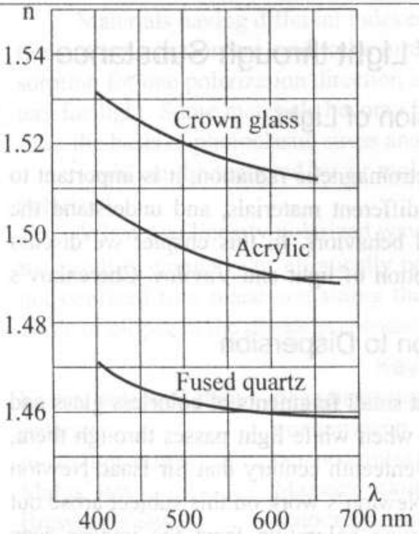


Figure 4.1 Variation of index of refraction n with vacuum wavelength for three materials

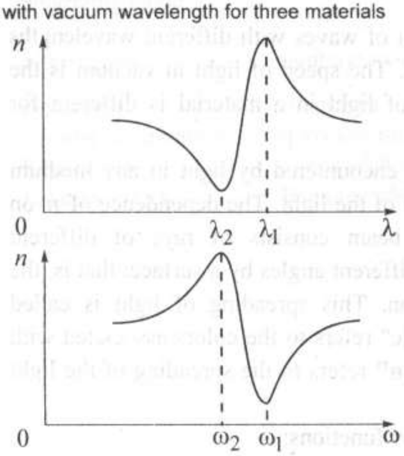


Figure 4.2 Typical dependence of n on λ and ω

If substance partially absorbs incident light, then in absorption region $\frac{dn}{d\lambda} > 0$ or $\frac{dn}{d\omega} < 0$ and dispersion is known as **abnormal dispersion**. Figure 4.2 shows typical dependence of n on λ and ω. Region from λ_1 and λ_2 or from ω_1 to ω_2, respectively, corresponds to abnormal dispersion. For example, absorption regions of glass lie in ultraviolet and infrared regions and there will be abnormal dispersion.

In general, index of refraction of a given medium is *greater* for a shorter wavelength (corresponding to, say, violet light) than for a longer wavelength (say, red light). Thus light of longer wavelength usually has greater speed in a material than does light of shorter wavelength. It means that blue light bends more than red light does when passing into a refracting medium.

As an example, Figure 4.1 shows how the index of refraction of crown glass, acrylic and fused quartz depends on the wavelength of light. To understand the effect that dispersion exerts on light, let's consider what happens when light strikes a prism (Figure 4.3). A ray of single-wavelength light incident on the prism from the left emerges refracted from its original direction of travel by an angle δ, called the **angle of deviation**.

Now suppose that a beam of white light (a combination of all visible wavelengths) is incident on a prism, as illustrated in Figure 4.4. The rays that emerge spread out in a series of colors known as the visible spectrum. These colors, in order of decreasing wavelength, are red, orange, yellow, green, blue, indigo and violet. Clearly, the angle of deviation δ depends on wavelength.

Violet light deviates the most, red the least, and the remaining colors in the visible spectrum fall between these extremes. Newton showed that each color has a particular angle of deviation and that the colors can be recombined to form the original white light.

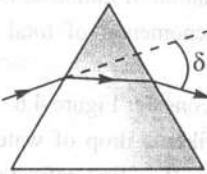


Figure 4.3 A prism refracts a single-wavelength light ray through an angle δ

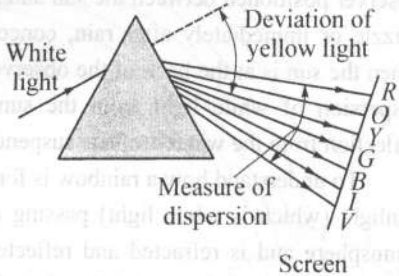


Figure 4.4 Dispersion by a prism. The band of colors on the screen is called a spectrum

Since the index of refraction of optical glass varies more rapidly at the violet end than at the red end of the spectrum, the spectrum formed by a prism is always spread out more at the violet end than it is at the red. Also, while a prism deviates red light the least and violet the most, the reverse is true for a diffraction grating.

A prism is often used in an instrument known as a spectrometer, the essential elements of which are shown in Figure 4.5. The instrument is commonly used to study the wavelengths emitted by a light source. Light from the source is sent through a narrow, adjustable slit to produce a parallel, or collimated, beam. The light then passes through the prism and is dispersed into a spectrum. The dispersed light is observed through a telescope. The experimenter sees an image of the slit through the eyepiece of the telescope. The telescope can be moved or the prism rotated so that the various images formed by different wavelengths at different angles of deviation can be viewed.

All hot, low-pressure gases emit their own characteristic spectra. Thus, one use of a prism spectrometer is to identify gases. For example, sodium emits two wavelengths, 589.0 and 589.6 nm, in the visible spectrum, which appear as two closely spaced yellow lines. Thus, a gas emitting these colors can be identified as having sodium as one of its constituents. Likewise, mercury vapor has its own characteristic spectrum, dominated "fingerprints" of that gas.

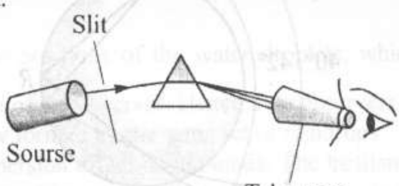


Figure 4.5 Diagram of a prism spectrometer. The various colors in the visible spectrum are viewed through telescope

The rainbow. The dispersion of light into a spectrum is demonstrated most charming in nature by the formation of a rainbow, which is often seen by an observer positioned between the sun and a rain shower. Sometimes, during a light drizzle or immediately after rain, concentric colored arcs are seen in the sky, when the sun is at the back of the observer. The formation of rainbows is due the dispersion of white light from the sun due to phenomenon of total internal reflection from the water droplets suspended in the air.

To understand how a rainbow is formed, let us consider Figure 4.6. A ray of sunlight (which is white light) passing overhead strikes a drop of water in the atmosphere and is refracted and reflected as follows: It is first refracted at the front surface of the drop, with the violet light deviating the most and the red light the least. At the back surface of the drop, the light is reflected and returns to the front surface, where it again undergoes refraction as it moves from water into air. The rays leave the drop such that the angle between the incident white light and the most intense returning violet ray is 40° and the angle between the white light and the most intense returning red ray is 42°. This small angular difference between the returning rays causes us to see a colored bow (Figure 4.7).

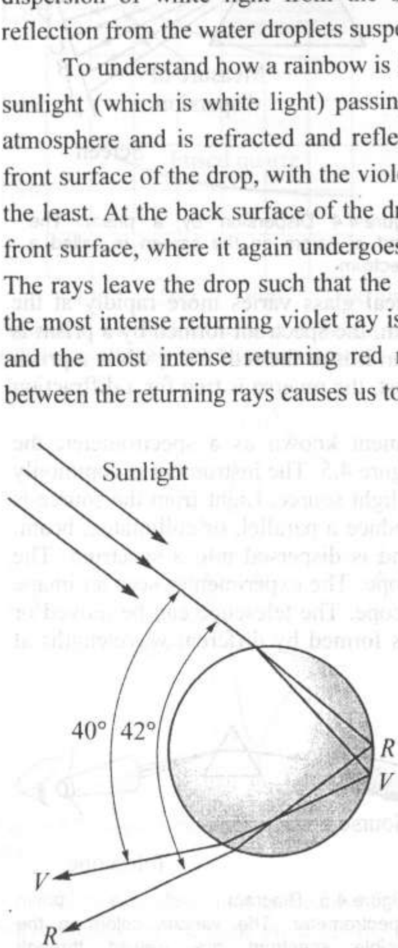


Figure 4.6 Refraction of sunlight by a spherical raindrop

Sometimes two rainbows, called the *primary rainbow* and the *secondary rainbow* are seen. However, the common centre of both rainbows lies on the line joining the sun and the observer.

Primary rainbow. The primary rainbow has violet color on the inner edge and the red color on the outer edge of the rainbow, as shown in Figure 4.7. It is formed, when the light from the sun after undergoing dispersion from the water droplets reaches the observer after suffering one internal reflection. It can be mathematically proved that an observer receives intense red light in a direction making angle of 42° and intense violet light in a direction making an angle of 40° with the line joining the sun and the observer. This line is called the *axis of rainbow*.

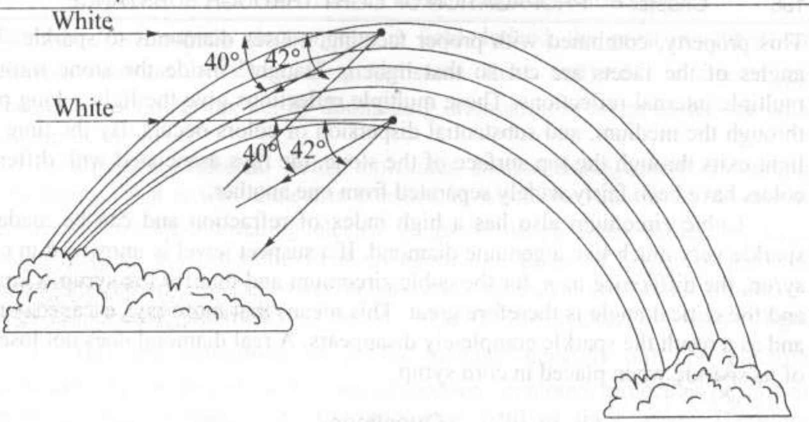


Figure 4.7 The formation of a rainbow

Secondary rainbow. The secondary rainbow has red color on the inner edge and the violet color on the outer edge of the rainbow, i.e. the sequence of colors in the secondary rainbow is just reverse of that in the primary rainbow. It is formed, when light from the sun undergoing dispersion from the suspended water droplets, reaches the observer after suffering two internal reflections. The calculations show that the violet light is received by the observer in a direction making an angle of 55° and red light in a direction making an angle of 52° with the axis of the rainbow.

It is found that the region of the sky between the two rainbows is comparatively darker and the regions below the primary rainbow and above the secondary rainbow are comparatively brighter than the rest of the sky.

Two observers cannot see the same rainbow. For a primary rainbow to be seen by an observer, the rays emerging out from water droplets must subtend a mean angle of $41° = \dfrac{40° + 42°}{2}$ at the observer's eye and for a secondary rainbow, a mean angle of $53.5° = \dfrac{52° + 55°}{2}$. The positions of the water droplets, which send such rays, depend upon the position of the observer. Hence, two observers at different positions do not see the rainbow formed by the same set of raindrops.

Another attractive example of dispersion effect is diamonds. The brilliance of diamond is due partly to its large dispersion and partly to its unusually large index of refraction. As index of refraction of the diamond $n_1 = 2.4$ is considerably greater than the index of refraction n_2 for air, the critical angle for total internal reflection is small: $\theta = 24°$. Any ray inside the diamond that approaches the surface at an angle greater than this is completely reflected back into the stone.

This property, combined with proper faceting, causes diamonds to sparkle. The angles of the facets are cut so that light is "caught" inside the stone through multiple internal reflections. These multiple reflections give the light a long path through the medium, and substantial dispersion of colors occurs. By the time the light exits through the top surface of the stone, the rays associated with different colors have been fairly widely separated from one another.

Cubic zirconium also has a high index of refraction and can be made to sparkle very much like a genuine diamond. If a suspect jewel is immersed in corn syrup, the difference in n for the cubic zirconium and that for the syrup is small, and the critical angle is therefore great. This means that more rays escape sooner, and as a result the sparkle completely disappears. A real diamond does not lose all of its sparkle when placed in corn syrup.

Questions

4.1. What is dispersion?

4.2. What is normal and abnormal dispersion?

4.3. Show typical dependence of n on λ or ω.

4.4. What wavelength has greater speed a material: longer or shorter?

4.5. What light: red or violet bends more when passing into a refracting material?

4.6. At what end the spectrum formed by a prism is always spread out more: at the violet or at the red?

4.7. What is the difference between dispersion and diffraction spectrums?

4.8. Explain the formation of a rainbow.

4.9. When two colors X and Y are sent through a glass prism, X is bent more than Y. Which color travels more slowly in the prism?

4.10. Why does the arc of a rainbow appear with red on top and violet on the bottom?

4.1.2 Classical Theory of Dispersion

The dispersion phenomenon is the result of interaction between electromagnetic waves with charged particles of substance. Classical theory of dispersion was developed only after creation of electron theory of substance by G. Lorenz. It follows from Maxwell's theory that index of refraction of substance n is related to the dielectric constant of the substance as:

$$n = \sqrt{\varepsilon}\,. \qquad (4.1)$$

At first sight it seems that Eq. (4.1) is in contradiction with experimental dates. For example, for water $\varepsilon = 81$. But its index of refraction for visible light is 1.33, not 9. This "contradiction" is result of neglecting of dispersion, that is, it is incorrect to use Eq. (4.1). The quantity ε and hence n has to depend on frequency f of varying electromagnetic field. Indeed, the great magnitude of dielectric constant of water in electrostatic field $\varepsilon(0) = 81$ is caused by orientational polarization, i.e. by dominating orientation of water molecules with great dipole moment.

In alternating electric fields molecules cannot change their orientation immediately. That is why dielectric constant ε would be equal to $\varepsilon(0)$ only at low frequencies of alternating electric fields at which water molecules have enough time to reorient along field. In alternating fields of higher frequencies orientational polarization of water or any other dielectric with polar molecules must disappeared practically. That is why in the region of visible light ($f \sim 10^{15}$ Hz) the magnitude of ε for medium depends only on electron polarization, i.e. on forced oscillations of electrons in atoms, molecules or ions of substance under influence of electromagnetic field of light wave. Therefore $\varepsilon(f) < \varepsilon(0)$ and $n = \sqrt{\varepsilon(f)} < 9$.

When light of frequency f is incident on the substance, it forces electrons in the substance to oscillate with the same frequency. As a result electric dipole moments of atoms vary periodically with frequency f. Therefore atoms radiate secondary waves of the same frequency. Average distances between particles of substance are much less then extent of one wave–train. Hence secondary waves radiated by a lot of adjacent atoms of homogeneous medium are coherent as with themselves as with a primary wave. Being superimposed, they interfere and the result of interference depends on relationship of amplitudes and phases.

In the case of optically inhomogeneous medium the superposition of primary and secondary waves leads to scattering of light. And at last, when light falls on the interface of two different substances the interference causes not only transmitted wave, but reflected wave as well.

Let's discuss classical theory of dispersion of light in homogenous dielectric in details. From electromagnetic theory it follows that dispersion can be formally treated as a consequence of dependence of dielectric constant ε on frequency f of light waves.

Lorenz proved that hypothesis about existence of electrons bounded quasi-elastically in atoms, is enough for qualitative understanding of many optical phenomena. Being displaced from equilibrium positions these electrons begin to oscillate, gradually losing energy for radiation of electromagnetic waves. As a result the oscillations will be dumped. The dumping can be taken into account by introducing so–called "friction force of radiation", which is proportional to the velocity.

When electromagnetic wave passes through the substance, each electron is exerted by Lorenz's force

$$\vec{F} = -e\vec{E} - e\left[\vec{v}\vec{B}\right] = -e\vec{E} - e\mu_0\left[\vec{v}\vec{H}\right], \qquad (4.2)$$

($-e$ – is the charge of electron). In electromagnetic wave the ratio of magnetic field strength H to the electric field strength E is

$$\frac{H}{E}=\sqrt{\frac{\varepsilon_0}{\mu_0}}.$$

Hence the ratio of magnetic force to electric force experienced by electron is defined as

$$\frac{\mu_0 vH}{E}=\mu_0 v\sqrt{\frac{\varepsilon_0}{\mu_0}}=v\sqrt{\varepsilon_0\mu_0}=\frac{v}{c}.$$

Even if the amplitude of electron's oscillation would reach Å (10^{-10}m), (the order of atom magnitude), the amplitude of electron's speed $A\omega$ would be approximately

$$10^{-10}\times 3\times 10^{15}\,\text{m/s}=3\times 10^{5}\,\text{m/s}.$$

($\omega=2\pi f$ is approximately equals to 3×10^{-15} s^{-1}). Hence, $\frac{v}{c}\ll 10^{-3}$ and we can neglect the second term in Eq. (4.2).

Thus, when electromagnetic wave passes through substance, each electron experiences the force

$$F=-eE_0\cos(\omega t+\alpha),$$

where E_0 is the amplitude of electric field in electromagnetic wave, α – quantity defined by the coordinates of a given electron.

The visible light acts more strongly on the outer, so-called *valence* or *optical* electrons because the natural frequencies of inner electrons differ very much from the frequencies of optical waves and hence the oscillations of inner electrons by a visible light are not exited.

Next, for the simplicity we assume that there is only one optical electron in atom. We assume also that atoms don't interact with each other (which are valid for gases). Additionally, in the very beginning of derivation we neglect damping due to radiation. Later we'll take account of the dumping by introducing certain corrections into the obtained formulas.

In such a case the equation of motion of electron has form:

$$\frac{d^2r(t)}{dt^2}+\omega_0^2 r(t)=-(e/m)E_0\cos(\omega t+\alpha),$$

where ω_0 is the *natural frequency* of electron. It follows from the theory of differential equations that the solution of this equation is:

$$r(t)=\frac{-(e/m)E_0}{\omega_0{}^2-\omega^2}\cos(\omega t+\alpha)=\frac{-(e/m)}{\omega_0{}^2-\omega^2}E(t).$$

As the mass of electron is much less than nuclear mass we can neglect the displacement of nuclear from its equilibrium position due to electric field of light wave. Then the molecular dipole moment can be expressed as

$$p(t) = -er(t) = \frac{e^2/m}{{\omega_0}^2 - \omega^2} E(t).$$

Let's denote the number of molecules per unit volume as N. Then the product $Np(t)$ gives the polarization of substance $P(t)$.

$$P(t) = \frac{N(e^2/m)}{{\omega_0}^2 - \omega^2} E(t).$$

As in an isotropic dielectrics the polarization P is related to the electric field strength E by simple relationship

$$P = \chi\varepsilon_0 E,$$

where χ is susceptibility, then the dielectric constant ε is

$$\varepsilon = 1 + \chi = 1 + \frac{P(t)}{\varepsilon_0 E(t)} = 1 + \frac{N}{\varepsilon_0}\frac{p(t)}{E(t)}.$$

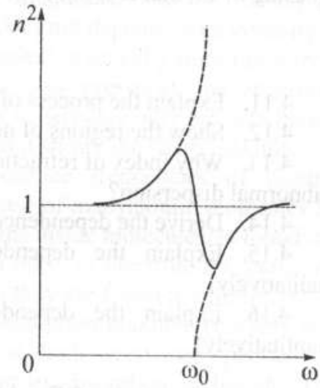


Figure 4.8 The dependence of n^2 on ω in the region of absorption

And, finally, recalling that $\varepsilon = n^2$, we obtain:

$$n^2 = 1 + \frac{N}{\varepsilon_0}\frac{e^2/m}{{\omega_0}^2 - \omega^2}. \quad (4.3)$$

When frequencies ω differ greatly from the natural frequency ω_0, the second term in the expression (4.3) is small in comparison with unity, and that is why $n^2 \approx 1$. In the vicinity of the natural frequency ω_0 the function (4.3) approaches to $+\infty$ when ω comes from the left and approaches to $-\infty$ when it comes from the right (Figure 4.8).

Such behavior of theoretical curve is caused by neglecting of dumping of radiation. When we take into account this dumping, we obtain function as that at Figure 4.9.

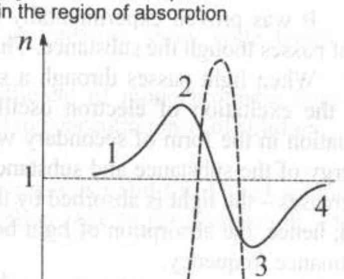


Figure 4.9 n as a function of ω in the vicinity of the region of absorption. Dispersion curve. (1-2) and (3-4) corresponds to the normal dispersion, (2-3) corresponds to the abnormal dispersion. Dash line shows dependence of absorption coefficient in the absorption region

For all transparent substances the function $n(\omega)$ has a form, shown in Figure 4.9. When frequency increases, the index of refraction increases too, i.e. $dn/d\omega > 0$. The sections (1 – 2) and (3 – 4) at Figure 4.9 correspond to the normal dispersion. If substance absorbs some part of light, then in the region of absorption the dispersion shows anomaly – index of refraction decreases with increasing of ω, that is $dn/d\omega < 0$ (2 – 3).

Exercises

4.11. Explain the process of interaction of light with substance.

4.12. Show the regions of normal and abnormal dispersion.

4.13. Why index of refraction decreases with increasing of ω in the region of abnormal dispersion?

4.14. Derive the dependence of index of refraction on frequency.

4.15. Explain the dependence of index of refraction on frequency (qualitatively)

4.16. Explain the dependence of index of refraction on frequency (quantitatively)

4.2 Absorption of Light

It was proved experimentally that intensity of light wave decreases when light passes though the substance. This phenomenon is called **absorption of light**.

When light passes through a substance, some part of wave energy spends for the excitation of electron oscillations. Partially this energy returns to the radiation in the form of secondary waves. Partially it transforms into the internal energy of the substance and substance is heated. As a result the intensity of light decreases – the light is absorbed by the substance. Forced oscillations of electrons and, hence, the absorption of light becomes especially intensive at the vicinity of resonance frequency.

There was established experimentally that intensity of light decreases exponentially when light is passing through the substance:

$$I = I_0 \exp(-\alpha l). \quad (4.4)$$

Expression (4.4) is called **Burger's law,** after French scientist Pier Burger (1698 – 1758). Here I_0 is the initial intensity of light that enters the absorbing layer (at the boundary or any point inside the substance), l – thickness of absorbing layer, α-absorption coefficient, depending on properties of absorbing substance and wavelength or frequency of light.

Taking the derivative of (4.4) we obtain

$$dI = -\alpha I_0 \exp(-\alpha l)dl = -\alpha I dl. \quad (4.5)$$

From Eq. (4.5) it follows that the loss of intensity at the path dl is proportional to the length of this path and intensity I itself. The absorption coefficient α serves as a coefficient of proportionality.

According to (4.4) the intensity I become e times smaller I_0 at distance $l = 1/\alpha$. It means that the absorption coefficient α is the *physical quantity inversely proportional to the thickness of absorption layer corresponding to the e times decreasing of intensity*. It should be mentioned that the absorption coefficient depends on wavelength of light and does not depend on its intensity.

For substances, in which atoms (or molecules) practically does not interact with each others, (gases, vapors of metals at low pressure), the absorption coefficient for the most wavelengths approaches to zero and only for very narrow spectral bands (about several angstroms in width) it shows the sharp maxima.

These maxima correspond to the resonance frequencies of electron oscillations inside atoms (Figure 4.10). In the case of manyatomic molecules the frequencies, corresponding to oscillations of atoms inside molecules, is found out. As masses of molecules are much greater then mass of electron, the molecular frequencies are much less then the atomic ones – they are found in infrared region of spectrum. When pressure of gas increases, absorption maxima, (initially very narrow), become broader and broader and at high pressures spectrum of absorption of gases approaches to the spectrum of absorption of liquids. This proves that broadening of the absorption band is the result of interaction between atoms.

Solid bodies, liquids and gases at high pressure show wide bands of absorption as shown in Figure 4.11.

Experiments show that absorption coefficient of monochromatic light in solution of absorbing substance is proportional to concentration c of solution:

$$\alpha = \alpha_1 c.$$

This expression is **law of Bar**. The Bar's law is valid for diluted solution. The absorption coefficient α_1 depends on the frequency ω (or wavelength λ) of the light.

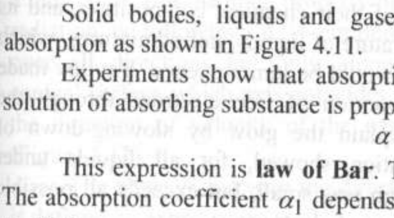


Figure 4.10 The dependence of absorption coefficient on frequency for gases and vapors of metals at low pressure

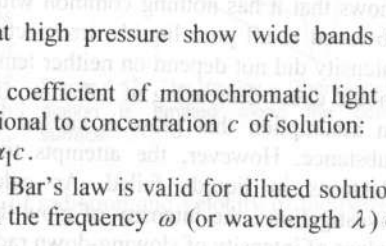


Figure 4.11 Gases at high pressure, liquids and solid states give a broad absorption band

And finally, it should be mentioned that metals are practically opaque for light (their absorption coefficient α has the magnitude of order 10^6 m^{-1}; whereas the absorption coefficient for glass is $\alpha \approx 1$). This is due to existence of free electrons in metals. Free electrons go into motion under electric field of electromagnetic wave. As a result rapidly varying currents with joule heating appear in metal and energy of light wave decreases rapidly, being transformed into the internal energy of metal.

Questions

4.17. Why solid states and liquids have broader band of absorption then gases?

4.18. Write and explain Burger's law.

4.19. Determine the physical meaning of absorption coefficient.

4.20. Write and explain Bar's law.

4.3 Vavilov – Cherenkov's Effect

In 1934 two soviet scientists Pavel Cherenkov and Igor Vavilov discovered that certain substances being exposed by γ and β-rays emit light. This radiation differs from usual type of luminescence. Analysis of properties of the radiation shows that it has nothing common with luminescence. Thus for example, it was observed in all pure liquids, irrespective of their chemical composition, and its intensity did not depend on neither temperature of liquids, nor admixtures, which should cause decreasing of radiation, if it would be luminescence. Vavilov made an assumption that glow is caused by motion of free electrons through the substance. However, the attempts to explain the glow by slowing-down of electrons in liquids failed. As calculation showed, for all liquids under investigations, the intensity of glow though was small, but exceeds all possible values of intensity of slowing-down radiation of electrons.

The effect has following explanation. According to electromagnetic theory, the moving charge radiates the energy when it moves with acceleration and doesn't radiate the energy when it moves with constant speed. But proof of this statement is based on the assumption that no particle cannot move with the speed greater than the speed of light. This assumption seemed to be so obvious consequence of relativistic theory that nobody even try to analyze it. Actually, the theory of relativity only allow to state that speed v of any charged particle always less then speed of light c in vacuum: i.e. $v < c$. That is why, the charge moving uniformly along straight line in vacuum, really, doesn't radiate electromagnetic waves. In transparent medium phase speed of visible light is always less than c. It is equal to c/n, where $n > 1$ is refractive index of substance. Therefore, in substance charge can move with "superlight" speed $c > v > c/n$. Soviet scientists

Tamm and Franck showed that charge moving in substance with "superlight" speed must radiate electromagnetic waves.

It should be mentioned that during Vavilov – Cherenkov's radiation the energy and speed of radiating free particle decrease, that is, particle decelerates. But it is important to emphasize that unlike slowing-down radiation, which is the result of acceleration of particle, the decrease of speed in Vavilov – Cherenkov effect is rather not the cause but result of radiation. In other words, if decrease of energy at Vavilov – Cherenkov effect would compensated by some way and the particle all the time would move with "superlight" speed, the Vavilov – Cherenkov radiation all the same would take place, whereas any slowing-down radiation would not be.

Charged particle induces short-timed polarization of substance along line of its motion. As a result molecules of substance become short-timed coherent sources of light which produce interference at overlapping. When $v < c/n$ the secondary wavelets interfere destructively and eliminate each other. When $v > c/n$ the secondary wavelets interfere constructively and reinforce each other.

The Vavilov – Cherenkov's effect was experimentally observed in liquids and solids. The short waves are inherent mainly for this radiation that is why it has a blue color. But the main property of this radiation is following: it is emitted not over all directions but only along the conical surface which axis coincides with the direction of velocity of the particle (Figure 4.12).

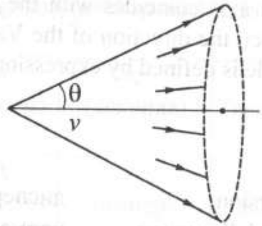


Figure 4.12 The Vavilov – Cherenkov radiation is emitted along the conical surface

The angle θ between the direction of radiation and velocity of the particle is defined by expression:

$$\cos\theta = \frac{c/n}{v} = \frac{c}{nv}. \tag{4.6}$$

The Vavilov – Cherenkov's effect is widely used in experimental techniques for investigations of radioactivity and elementary particles.

Summary

The index of refraction of a material n depends on the wavelength of light λ. It usually decreases with increasing wavelength. This phenomenon is called dispersion. In general, index of refraction of a given medium is greater for a shorter wavelength (corresponding to, say, violet light) than for a longer wavelength (say, red light). It means that blue light bends more than red light does. On the basis of classical electromagnetic theory it can be shown that

$$n^2 = 1 + \frac{N}{\varepsilon 0} \frac{e^2/m}{\omega_0{}^2 - \omega^2}.$$

The intensity of light decreases exponentially when light is passing through the substance according Burger's law:

$$I = I_0 \exp(-kl).$$

Experiments show that absorption coefficient of monochromatic light in solution of absorbing substance is proportional to concentration c of solution:

$$\alpha = \alpha_1 c.$$

This expression is law of Bar.

Certain substances being exposed by γ and β-rays emit light. This radiation differs from usual type of luminescence. One of main property of this radiation is: it is emitted not over all directions but only along the conical surface which axis coincides with the direction of velocity of the particle The angle θ between the direction of the Vavilov – Cherenkov's radiation and velocity of the particle is defined by expression:

$$\cos\theta = \frac{c/n}{v} = \frac{c}{nv}.$$

Key Terms

dispersion	дисперсия	дисперсія
normal dispersion	нормальная дисперсия	нормальна дисперсія
abnormal dispersion	аномальная дисперсия	аномальна дисперсія
Burger's law	Бугера закон	Бугера закон
Vavilov – Cherenkov's radiation	Вавилова – Черенкова излучение	Вавілова – Черенкова випромінювання
Bar's law	Бэра зкон	Бера закон

Appendixies

Appendix A Symbols, Dimensions, and Units

Table A.1 Base SI Units
Таблица А.1 Основные единицы СИ
Таблиця А.1 Основні одиниці СІ

No.	Sym-bol	Name of Value English Russian Ukrainian	Dimen-sions	SI units		
				English	Russian	Ukrainian
1	*l*	length длина довжина	L	m (meter)	м (метр)	м (метр)
2	*m*	mass масса маса	M	kg (kilogram)	кг (килограмм)	кг (кілограм)
3	*t*	time время час	T	s (second)	с (секунда)	с (секунда)
4	*I*	electric current сила электрического тока сила електричного струму	I	A (ampere)	А (ампер)	А (ампер)
5	*T*	thermodynamic temperature термодинамическая температура термодинамічна температура	θ	K (kelvin)	К (кельвин)	К (кельвін)
6	*N*	amount of substance количество вещества кількість речовини	N	mol	моль	моль
7	*I*	luminous intensity сила света сила світла	J	cd (candela)	кд (кандела)	кд (кандела)

Table A.2 SI Supplementary Units
Таблица А.2 Дополнительные единицы СИ
Таблиця А.2 Додаткові одиниці СІ

No.	Symbol	Name of Value English Russian Ukrainian	Dimensions	SI units English	Russian	Ukrainian
1	α, β, γ	plane angle плоский угол плоский кут	1	rad (radian)	рад (радиан)	рад (радіан)
2	Ω	solid angle телесный угол тілесний кут	1	sr (steradian)	ср (стерадиан)	ср (стерадіан)

Table A.3 SI Special Named Derivative Units
Таблица А.3 Производные единицы СИ, имеющие специальное название
Таблиця А.3 Похідні одиниці СІ, що мають спеціальні назви

No.	Symbol	Name of Value English Russian Ukrainian	Dimensions	SI units English	Russian	Ukrainian	SI base units relationships
1	2	3	4	5	6	7	8
1	f	frequency частота частота	T^{-1}	Hz (hertz)	Гц (герц)	Гц (герц)	1 Гц = 1 c^{-1}
2	F	force, weight сила, вес сила, вага	LMT^{-2}	N (newton)	Н (ньютон)	Н (ньютон)	1 Н = = 1 кг·м/c^2
3	p	pressure, mechanical stress давление, механическое напряжение тиск, механічне напруження	$L^{-1}MT^{-2}$	Pa (pascal)	Па (паскаль)	Па (паскаль)	1 Па = = 1 Н/$м^2$
4	E, A, W, Q	work, energy, quantity of heat энергия, работа, количество тепла енергія, робота, кількість тепла	L^2MT^{-2}	J (joule)	Дж (джоуль)	Дж (джоуль)	1 Дж = = 1 Н·м

Table A.3 (continued)
Таблица А.3 (продолжение)
Таблиця А.3 (продовження)

1	2	3	4	5	6	7	8
5	*P*	power, energy flux мощность, поток энергии потужність, потік енергії	L^2MT^{-3}	W (watt)	Вт (ватт)	Вт (ватт)	1 Вт = = 1 Дж/с
6	*Q*	charge, quantity of electricity электрический заряд, количество электричества електричний заряд, кількість електрики	TI	C (coulomb)	Кл (кулон)	Кл (кулон)	1 Кл = = 1 А·с
7	*φ, U, E*	potential difference, electromotive force (emf), voltage электрический потенциал, разность электрических потенциалов, электрическое напряжение, электродвижущая сила (ЭДС) електричний потенціал, різниця електричних потенціалів, електрична напруга, електрорушійна сила (ЕРС)	$L^2MT^{-3}I^{-1}$	V (volt)	В (вольт)	В (вольт)	1 В = = 1 Вт/А
8	*C*	electric(al) capacitance электрическая емкость електрична ємність	$L^{-2}M^{-1}T^4I^2$	F (farad)	Ф (фарада)	Ф (фарада)	1 Ф = = 1 Кл/В

Table A.3 (continued)
Таблица А.3 (продолжение)
Таблиця А.3 (продовження)

1	2	3	4	5	6	7	8
9	R	electric(al) resistance электрическое сопротивление електричний опір	$L^2MT^{-3}I^{-2}$	Ω (ohm)	Ом (ом)	Ом (ом)	1 Ом = = 1 В/А
10	G	electric(al) conductivity электрическая проводимость електрична провідність	$L^{-2}M^{-1}T^3I^2$	S (siemens)	См (сименс)	См (сіменс)	1 См = = 1 Ом$^{-1}$
11	Φ	magnetic flux магнитный поток магнітний потік	$L^2MT^{-2}I^{-1}$	Wb (weber)	Вб (вебер)	Вб (вебер)	1 Вб = = 1 В·с
12	B	induction (magnetic flux density) магнитная индукция (плотность магнитного потока) магнітна індукція (густина магнітного потоку)	$MT^{-2}I^{-1}$	T (tesla)	Тл (тесла)	Тл (тесла)	1 Тл = = 1 Вб/м^2
13	L, L_m	inductance индуктивность, взаимная индуктивность індуктивність, взаємоіндуктивність	$L^2MT^{-2}I^{-2}$	H (henry)	Гн (генри)	Гн (генрі)	1 Гн = = 1 Вб/А
14	Φ_v	luminous (light) flux световой поток світловий потік	J	lm (lumen)	лм (люмен)	лм (люмен)	1 лм = = 1 кд·ср
15	E_v	illuminance освещенность освітленість	$L^{-2}J$	lx (lux)	лк (люкс)	лк (люкс)	1 лк = = 1 лм/м^2

Table A.4 SI Derived Units of Space and Time
Таблица А.4 Производные единицы величин, описывающих пространство и время
Таблиця А.4 Похідні одиниці величин, що описують простір і час

No.	Symbol	Name of Value English Russian Ukrainian	Dimensions	SI units: English	Russian	Ukrainian
1	ω	angular velocity угловая (круговая) частота, угловая скорость кутова (кругова) частота, кутова швидкість	T^{-1}	S^{-1} (rad/s)	C^{-1} (рад/с)	C^{-1} (рад/с)
2	α	angular acceleration угловое ускорение кутове прискорення	T^{-2}	rad/s^2	рад/$с^2$	рад/$с^2$
3	v	velocity скорость швидкість	LT^{-1}	m/s	м/с	м/с
4	a	acceleration ускорение прискорення	LT^{-2}	m/s^2	м/$с^2$	м/$с^2$
5	g	acceleration due to gravity ускорение свободного падения, гравитационное ускорение прискорення вільного падіння, гравітаційне прискорення	LT^{-2}	m/s^2	м/$с^2$	м/$с^2$
6	α, β, γ	plane angle плоский угол плоский кут	1	rad (radian)	рад (радиан)	рад (радіан)
7	Ω	solid angle телесный угол тілесний кут	1	sr (steradian)	ср (стерадиан)	ср (стерадіан)
8	A, S	area площадь площа	L^2	m^2	$м^2$	$м^2$
9	V	volume объем об'єм	L^3	m^3	$м^3$	$м^3$

Table A.5 SI Derived Units of Periodic Processes and Phenomena
Таблица А.5 Производные единицы величин, описывающих периодические процессы и явления
Таблиця А.5 Похідні одиниці величин, що описують періодичні процеси та явища

No.	Symbol	Name of Value English Russian Ukrainian	Dimensions	SI units English	 Russian	 Ukrainian
1	2	3	4	5	6	7
1	f	frequency частота частота	T^{-1}	Hz (hertz)	Гц (герц)	Гц (герц)
2	n	rotational frequency частота вращения частота обертання	T^{-1}	r/s	об/с	об/с
3	t	period период період	T	s (second)	с (секунда)	с (секунда)
4	τ	relaxation time время релаксации час релаксації	T	s	с	с
5	ω	angular velocity угловая (круговая) частота, угловая скорость кутова (кругова) частота, кутова швидкість	T^{-1}	S^{-1} (rad/s)	$с^{-1}$ (рад/с)	$с^{-1}$ (рад/с)
6	λ	wavelength длина волны довжина хвилі	L	m, X	м	м
7	δ	damping coefficient коэффициент затухания коефіцієнт згасання	T^{-1}	s^{-1}	$с^{-1}$	$с^{-1}$
8	Λ	damping decrement (logarithm) логарифмический декремент затухания логарифмічний декремент згасання	–	–	–	–

Table A.6 SI Derived Units of Electric and Magnetic Values
Таблица А.6 Производные единицы электрических и магнитных величин
Таблиця А.6 Похідні одиниці електричних і магнітних величин

No.	Symbol	Name of Value English Russian Ukrainian	Dimensions	SI units		
				English	Russian	Ukrainian
1	2	3	4	5	6	7
1	Q	charge, quantity of electricity электрический заряд, количество электричества електричний заряд, кількість електрики	TI	C (coulomb)	Кл (кулон)	Кл (кулон)
2	σ	electric charge density (surface) поверхностная плотность электрического заряда поверхнева густина електричного заряду	$L^{-2}TI$	C/m^2	Кл/м^2	Кл/м^2
3	D	electric induction, electric-flux density электрическое смещение (вектор электрической индукции) електричне зміщення (вектор електричної індукції)	$L^{-2}TI$	C/m^2	Кл/м^2	Кл/м^2
4	ρ	electric charge density (volume) (объемная) плотность электрического заряда (об'ємна) густина електричного заряду	$L^{-3}TI$	C/m^3	Кл/м^3	Кл/м^3
5	E	electric field strength напряженность электрического поля напруженість електричного поля	$LMT^{-3}I^{-1}$	V/m	В/м	В/м

Table A.6 (continued)
Таблица A.6 (продолжение)
Таблиця A.6 (продовження)

1	2	3	4	5	6	7
6	V, U	voltage напряжение, разность потенциалов напруженість, різниця потенціалів	$L^2MT^{-3}I^{-1}$	V (volt)	В (вольт)	В (вольт)
7	φ	electric potential электрический потенциал електричний потенціал	$L^2MT^{-3}I^{-1}$	V (volt)	В (вольт)	В (вольт)
8	E	electromotive force электродвижущая сила, ЭДС електрорушійна сила, ЕРС	$L^2MT^{-3}I^{-1}$	V (volt)	В (вольт)	В (вольт)
9	C	electric capacitance электрическая емкость електрична ємність	$L^{-2}M^{-1}T^4I^2$	F (farad)	Ф (фарада)	Ф (фарада)
10	ε_a	dielectric permittivity абсолютная диэлектрическая проницаемость абсолютна діелектрична проникність	$L^{-3}M^{-1}T^4I^2$	F/m	Ф/м	Ф/м
11	ε_r	relative dielectric permittivity относительная диэлектрическая проницаемость відносна діелектрична проникність	–	–	–	–
16	H	magnetic field strength напряженность магнитного поля напруженість магнітного поля	$L^{-1}I$	A/m (ampere/m)	А/м (ампер/метр)	А/м (ампер/метр)

Table A.6 (finished)
Таблица А.6 (окончание)
Таблиця А.6 (закінчення)

1	2	3	4	5	6	7
17	B B_r, B_s	magnetic induction (magnetic flux density), remanent induction, saturation induction магнитная индукция, (плотность магнитного потока), магнітна індукція, (густина магнітного потоку) остаточная магнитная индукция залишкова магнітна індукція магнитная индукция насыщения магнітна індукція насичення	$MT^{-2}I^{-1}$	T = Wb/m^2 (tesla = = weber/m^2)	Тл (тесла)	Тл (тесла)
18	Φ	magnetic flux магнитный поток магнітний потік	$L^2MT^{-2}I^{-1}$	Wb (weber)	Вб (вебер)	Вб (вебер)
19	μ	magnetic permeability магнитная проницаемость магнітна проникність	$LMT^{-2}I^{-2}$	H/m (henry/m)	Гн/м (генри/метр)	Гн/м (генрі/метр)
20	μ_r	relative magnetic permeability относительная магнитная проницаемость відносна магнітна проникність	–	–	–	–
25	R	resistance электрическое сопротивление електричний опір	$L^2MT^{-3}I^{-2}$	Ω (ohm)	Ом	Ом
32	φ	phase angle разность фаз різниця фаз	–	rad	рад	рад

Table A.7 SI Derived Units of Optical Values and Electromagnetic Radiation
Таблица А.7 Производные единицы световых величин оптического излучения
Таблиця А.7 Похідні одиниці світлових величин оптичного випромінювання

No.	Sym-bol	Name of Value English Russian Ukrainian	Dimen-sions	SI units		
				English	Russian	Ukrainian
1	2	3	4	5	6	7
1	f	frequency частота частота	T^{-1}	Hz (hertz)	Гц (герц)	Гц (герц)
2	ω	angular frequency угловая частота кутова частота	T^{-1}	s^{-1} (rad/s)	$с^{-1}$ (рад/с)	$с^{-1}$ (рад/с)
3	λ	wavelength длина волны довжина хвилі	L	m	м	м
4	C	illumination, candela-second освечивание освітлення	TJ	cd·s	кд·с	кд·с
5	Φ_v	luminous flux световой поток світловий потік	J	lm (lumen)	лм (люмен)	лм (люмен)
6	L_v	luminance яркость яскравість	$L^{-2}J$	cd/m^2	кд/м^2	кд/м^2
7	M_v	luminous exitance светимость світність	$L^{-2}J$	lm/m^2	лм/м^2	лм/м^2
8	E_v	illuminance освещенность освітленість	$L^{-2}J$	lx (lux)	лк (люкс)	лк (люкс)
9	D	optical density оптическая плот-ность оптична густина	–	–	–	–

Table A.7 (finished)
Таблица А.7 (окончание)
Таблиця А.7 (закінчення)

1	2	3	4	5	6	7
10	λ	linear absorption coefficient натуральный показатель поглощения, линейный коэффициент поглощения натуральний показник поглинання, лінійний коефіцієнт поглинання	L^{-1}	m^{-1}	$м^{-1}$	$м^{-1}$
11	n	index of refraction коэффициент преломления коефіцієнт заломлення	–	–	–	–

Important SI Units Names

Table A.8 SI Derived Units of Space and Time
Таблица А.8 Производные единицы СИ, используемые для описания пространства и времени
Таблиця А.8 Похідні одиниці СІ, які використовуються для опису простору і часу

Name of unit		Designation		Dimensions
English	Ukrainian	International	Ukrainian	
square meter	кадратний метр	m^2	$м^2$	L^2
cubic meter	кубічний метр	m^3	$м^3$	L^3
meter per second	метр на секунду	m/s	м/с	LT^{-1}
meter per second squared	метр на секунду в квадраті	m/s^2	$м/с^2$	LT^{-2}
second to the minus 2nd power	секунда в мінус другому степені	s^{-2}	$с^{-2}$	T^{-2}
radian per second	радіан на секунду	rad/s	рад/с	T^{-1}
radian per second squared	радіан на секунду в квадраті	rad/s^2	$рад/с^2$	T^{-2}
hertz	герц	Hz	Гц	T^{-1}
second to the minus 1st power	секунда в мінус першому степені	s^{-1}	$с^{-1}$	T^{-1}
meter to the minus 1st power	метр у мінус першому степені	m^{-1}	$м^{-1}$	L^{-1}

Table A.9 SI Derived Units of Optical Values
Таблица А.9 Производные единицы СИ световых величин оптического излучения
Таблиця А.9 Похідні одиниці СІ світлових величин оптичного випромінювання

Name of unit		Designation		Dimensions
English	Ukrainian	International	Ukrainian	
1	2	3	4	5
meter per second	метр на секунду	m/s	м/с	LT^{-1}
joule	джоуль	J	Дж	L^2MT^{-2}
joule per cubic meter	джоуль на кубічний метр	J/m^3	$Дж/м^3$	$L^{-1}MT^{-2}$
watt	ват	W	Вт	L^2MT^{-3}
watt per square meter	ват на квадратний метр	W/m^2	$Вт/м^2$	MT^{-3}
joule per square meter	джоуль на квадратний метр	J/m^2	$Дж/м^2$	MT^{-2}
watt per steradian	ват на стерадіан	W/sr	Вт/ср	L^2MT^{-3}
watt per steradian-square meter	ват на стерадіан-квадратний метр	$W/(sr{\cdot}m^2)$	$Вт/(ср{\cdot}м^2)$	MT^{-3}
watt per square meter-kelvin to the 4th power	ват на квадратний метр-кельвін у четвертому степені	$W/(m^2{\cdot}K^4)$	$Вт/(м^2{\cdot}K^4)$	$MT^{-3}{\cdot}\theta^{-4}$
watt-square meter	ват-квадратний метр	$W{\cdot}m^2$	$Вт{\cdot}м^2$	L^4MT^{-3}
meter-kelvin	метр-кельвін	m·K	м·К	$L\theta$
joule-second	джоуль-секунда	J·s	Дж·с	L^2MT^{-1}
loule per kelvin	джоуль на кельвін	J/K	Дж/К	$L^2MT^{-2}\theta^{-1}$
lumen	люмен	Lm	лм	J
lumen-second	люмен-секунда	Lm·s	лм·с	TJ
lux	люкс	Lx	лк	$L^{-2}J$
lumen per square meter	люмен на квадратний метр	Lm/m^2	$лм/м^2$	$L^{-2}J$
candela per square meter	кандела на квадратний метр	cd/m^2	$кд/м^2$	$L^{-2}J$
lux-second	люкс-секунда	Lx·s	лк·с	$L^{-2}TJ$
lumen per watt	люмен на ват	Lm/W	лм/Вт	$L^{-2}M^{-1}T^3J$
lumen per meter-radian	люмен на метр-радіан	Lm/(m·rad)	лм/(м·рад)	$L^{-1}J$
meter to the minus 1st power	метр у мінус першому степені	m^{-1}	$м^{-1}$	L^{-1}
square meter per kilogram	квадратний метр на кілограм	m^2/kg	$м^2/кг$	L^2M^{-1}
square meter per mole	квадратний метр на моль	m^2/mol	$м^2/моль$	L^2N^{-1}
candela per lux	кандела на люкс	cd/Lx	кд/лк	L^2

Appendix B Conversion Factors

In conversion factors tables the SI units are fully capitalized

Table B.1 Length

cm	METER (m)	km	in	ft	mi
1 centimeter = 1	10^{-2}	10^{-5}	0.3937	3.281×10^{-2}	6.214×10^{-6}
1 METER = 100	1	10^{-3}	39.37	3.281	6.214×10^{-4}
1 kilometer = 10^5	1000	1	3.937×10^4	3281	0.6214
1 inch = 2.540	2.540×10^{-2}	2.540×10^{-5}	1	8.333×10^{-2}	1.578×10^{-5}
1 foot = 30.48	0.3048	3.048×10^{-4}	12	1	1.894×10^{-4}
1 mile = = 1.609×10^5	1609	1.609	6.336×10^4	5280	1
1 angstrom = = 10^{-10} m	1 fermi = 10^{-15} m		1 fathom = 6 ft		1 rod = = 16.5 ft
1 nautical mile = 1852 m =	1 light-year = = 9.460×10^{12} km		1 Bohr radius = = 5.292×10^{-11} m		1 mil = = 10^{-3} in.
= 1.151 miles = 6076 ft	1 parsec = 3.084×10^{13} km		1 yard = 3 ft		1 nm = = 10^{-9} m

Table B.2 Area

METER^2 (m^2)	cm^2	ft^2	in^2
1 SQUARE METER = 1	10^4	10.76	1550
1 square centimeter = 10^{-4}	1	1.076×10^{-3}	0.1550
1 square foot = 9.290×10^{-2}	929.0	1	144
1 square inch = 6.452×10^{-4}	6.452	6.944×10^{-3}	1
1square mile = 2.788×10^{-7} ft^2 = 640 acres		1 acre = 43 560 ft^2	
1 barn = 10^{-28} m^2		1 hectare = 10^4 m^2 = 2.471 acres	

Table B.3 Volume

METER^3 (m^3)	cm^3	L	ft^3	in^3
1 CUBIC METER = 1	10^6	1000	35.31	6.102×10^4
1 cubic centimeter = 10^{-6}	1	1.000×10^{-3}	3.531×10^{-5}	6.102×10^{-2}
1 liter = 1.000×10^{-3}	1000	1	3.531×10^{-2}	61.02
1 cubic foot = 2.832×10^{-2}	2.832×10^{-4}	28.32	1	1728
1 cubic inch = 1.639×10^{-5}	16.39	1.639×10^{-2}	5.787×10^{-4}	1

Table B.4 Mass

Quantities in the dark-out areas are not mass units but are often used as such. When we write, for example, 1 kg "=" 2.205 lb, this means that a kilogram is a *mass* that *weighs* 2.205 pounds at a location where *g* has the standard value of 9.80665 m/s^2.

g	KILOGRAM (kg)	slug	u	oz	lb
1 gram = 1	0.001	6.852×10^{-5}	6.022×10^{23}	3.527×10^{-2}	2.205×10^{-3}
1 KILOGRAM = 1000	1	6.852×10^{-2}	6.022×10^{26}	35.27	2.205
1 slug = $= 1.459\times10^{4}$	14.59	1	8.786×10^{27}	514.8	32.17
1 atomic mass unit = $= 1.661\times10^{-24}$	1.661×10^{-27}	1.138×10^{-28}	1	5.857×10^{-26}	3.662×10^{-27}
1 wince = = 28.35	2.835×10^{-2}	1.943×10^{-3}	1.718×10^{25}	1	6.250×10^{-2}
1 pound = = 453.6	0.4536	3.108×10^{-2}	2.732×10^{26}	16	1
1 ton = $= 9.072\times10^{5}$	907.2	62.16	5.463×10^{29}	3.2×10^{4}	2000

1 metric ton = 1000 kg

Table B.5 Time

y	d	h	min	SECOND (s)
1 year = 1	365.25	8.766×10^{3}	5.259×10^{5}	3.156×10^{7}
1 day = 2.738×10^{-3}	1	24	1440	8.640×10^{4}
1 hour = 1.141×10^{-4}	4.167×10^{-2}	1	60	3600
1 minute = 1.901×10^{-6}	6.944×10^{-4}	1.667×10^{-2}	1	60
1 SECOND = 3.169×10^{-8}	1.157×10^{-5}	2.778×10^{-4}	1.667×10^{-2}	1

Table B.6 Speed

ft/s	km/h	METER/SECOND (m/s)	mi/h	cm/s
1 foot per second = 1	1.097	0.3048	0.6818	30.48
1 kilometer per hour = 0.9113	1	0.2778	0.6214	27.78
1 METER per SECOND = 3.281	3.6	1	2.237	100
1 mile per hour = 1.467	1.609	0.4470	1	44.70
1 centimeter per second = $= 3.281\times10^{-2}$	3.6×10^{-2}	0.01	2.237×10^{-2}	1

Table B.7 Force

Force units in the dark-out areas are now little used. To clarify: 1 gram-force (= 1 gf) is the force of gravity that would act on an object whose mass is 1 gram at a location where g has the standard value of 9.80665 m/s^2.

dyne	NEWTON (N)	lb	pdl	gf	kgf
1 dyne = 1	10^{-5}	2.248×10^{-6}	7.233×10^{-5}	1.020×10^{-3}	1.020×10^{-6}
1 NEWTON = $= 10^5$	1	0.2248	7.233	102.0	0.1020
1 pound = $= 4.448\times10^5$	4.448	1	32.17	453.6	0.4536
1 poundal= $= 1.383\times10^4$	0.1383	3.108×10^{-2}	1	14.10	1.410×10^2
1 gram-force = $= 980.7$	9.807×10^{-3}	2.205×10^{-3}	7.093×10^{-2}	1	0.001
1 kilogram-force = $= 9.807\times10^5$	9.807	2.205	70.93	1000	1

1 ton = 2000 lb

Table B.8 Pressure

atm	dyne/cm^2	inch of water	cmHg	PASCAL (Pa)	lb/in^2	lb/ft^2
1 atmosphere = 1	1.013×10^6	406.8	76	1.013×10^5	14.70	2116
1 dyne per centimeter^2 $= 9.869\times10^{-7}$	1	4.015×10^{-4}	7.501×10^{-5}	0.1	1.405×10^{-5}	2.089×10^{-3}
1 inch of water[1] at 4°C $= 2.158\times10^{-3}$	2491	1	0.1868	249.1	3.613×10^{-2}	5.202
1 centimeter of mercury[1] at 0°C $= 1.316\times10^{-2}$	1.333×10^4	5.353	1	1333	0.1934	27.85
1 PASCAL $= 9.869\times10^{-6}$	10	4.015×10^{-3}	7.501×10^{-4}	1	1.450×10^{-4}	2.089×10^{-2}
1 pound per inch^2 $= 6.805\times10^{-2}$	6.895×10^4	27.68	5.171	6.895×10^3	1	144
1 pound per foot^2 $= 4.725\times10^{-4}$	478.8	0.1922	3.591×10^{-2}	47.88	6.944×10^{-3}	1

[1] Where the acceleration of gravity has the standard value of 9.80665 m/s^2.

1 bar = 10^6 dyne/cm^2 = 0.1 MPa.

1 millibar = 10^3 dyne/cm^2 = 10^2 Pa.

1 torr = 1 mm Hg.

Table B.9 Power

	Btu/h	ftlb/s	hp	cal/s	kW	WATT (W)
1 British thermal unit per hour =	1	0.2161	3.929×10^{-4}	6.998×10^{-2}	2.930×10^{-4}	0.2930
1 foot-pound per second =	4.628	1	1.818×10^{-3}	0.3239	1.356×10^{-3}	1.356
1 horsepower =	2545	550	1	178.1	0.7457	745.7
1 calorie per second =	14.29	3.088	5.615×10^{-3}	1	4.186×10^{-3}	4.186
1 kilowatt =	3413	737.6	1.341	238.9	1	1000
1 WATT =	3.413	0.7376	1.341×10^{-3}	0.2389	0.001	1

Table B.10 Plane Angle

°(degree)	'(minute)	"(second)	RADIAN (rad)	rev
1 degree = 1	60	3600	1.745×10^{-2}	2.778×10^{-3}
1 minute = 1.667×10^{-2}	1	60	2.909×10^{-4}	4.630×10^{-5}
1 second = 2.778×10^{-4}	1.667×10^{-2}	1	4.848×10^{-6}	7.716×10^{-7}
1 RADIAN = 57.30	3438	2.063×10^{5}	1	0.1592
1 revolution = 360	2.16×10^{4}	1.296×10^{6}	6.283	1

Table B.11 Solid Angle

1 sphere = 4π steradians = 12.57 steradians

Table B.12 Magnetic Field

	gauss	TESLA (T)	milligauss
1 gauss =	1	10^{-4}	1000
1 TESLA =	10^{4}	1	10^{7}
1 milligauss =	0.001	10^{-7}	1

1 tesla = 1 weber/meter^2

Table B.13 Magnetic Flux

	maxwell	WEBER (Wb)
1 maxwell =	1	10^{-8}
1 WEBER =	10^{8}	1

Appendix C Some Fundamental Constants

Table C.1 Fundamental Physical Constants
Таблица C.1 Фундаментальные физические постоянные
Таблиця C.1 Фундаментальні фізичні сталі

Name of unit English Russian Ukraine	Symbol	Value
1	2	3
Universal constants Универсальные постоянные Універсальні сталі		
Speed of light in vacuum Скорость света в вакууме Швидкість світла у вакуумі	c	299 792 458 $m \cdot s^{-1}$
Magnetic constant Магнитная постоянная Магнітна стала	μ_0	$4\pi 10^{-7}$ $H \cdot m^{-1}$ = = 12.566 370 614$\cdot 10^{-7}$ $H \cdot m^{-1}$
Electric constant Электрическая постоянная Електрична стала	ε_0	8.854 187 817$\cdot 10^{-12}$ $F \cdot m^{-1}$
Gravitational constant Гравитационная постоянная Гравітаційна стала	G	6.672 59$\times 10^{-11}$$\cdot m^3 \cdot kg^{-1} \cdot s^{-2}$
Planck's constant Постоянная Планка Стала Планка	h	6.626 0755$\cdot 10^{-34}$ J·s
Dirac's constant Постоянная Дирака Стала Дірака	$\hbar$	1.054 572 66$\cdot 10^{-34}$ J·s
Atomic constants Атомные постоянные Атомні сталі		
Rydberg constant Постоянная Ридберга Стала Рідберга	R_∞	10 973 731.534 m^{-1}
Bohr radius Боровский радиус Боровський радіус	a_0	0.529 177 249$\cdot 10^{-10}$ m

Table C.1 (finished)

Таблица С.1 (окончание)

Таблиця С.1 (закінчення)

Electromagnetic constants Электромагнитные постоянные Електромагнитні сталі		
Charge of electron (elementary) Элементарный заряд (заряд электрона) Елементарний заряд (заряд електрона)	e	$1.602\,177\,33 \cdot 10^{-19}$ C
Bohr magneton Магнетон Бора Магнетон Бора	μ_B	$9.274\,0154 \cdot 10^{-24}$ $J \cdot T^{-1}$
Nuclear magneton Ядерный магнетон Ядерний магнетон	μN	$5.050\,7866 \cdot 10^{-27}$ $J \cdot T^{-1}$
Electron Электрон Електрон		
Mass of electron Масса покоя электрона Маса спокою електрона	m_e	$9.109\,3897 \cdot 10^{-31}$ kg
Electron charge-mass ratio Отношение заряда электрона к его массе Відношення заряду електрона до його маси	$-\dfrac{e}{m_e}$	$-1.75881962 \cdot 10^{11}$ $C \cdot kg^{-1}$
Classic electron radius Классический радиус электрона Класичний радіус електрона	r_e	$2.817\,940\,92 \cdot 10^{-15}$ m
Magnetic moment of electron Магнитный момент электрона Магнітний момент електрона	μ_e	$928.477\,01 \cdot 10^{-26}$ $J \cdot T^{-1}$

Appendix D Useful Mathematical Formulae

A. Geometry

Circumference of circle of radius r: $C = 2\pi r$

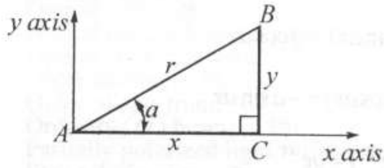
Area of circle of radius r: $A = \pi r^2$

Volume of sphere of radius r: $V = 4\pi r^3/3$

Surface area of sphere of radius r: $A = 4\pi r^2$

Volume of cylinder of radius r and height h: $V = \pi r^2 h$

B. Trigonometry



Pythagorean theorem:

In the right triangle *ABC*, $x^2 + y^2 = r^2$

Definitions of the trigonometric functions of angle θ:

$\sin\theta = y/r$, $\cos\theta = x/r$, $\tan\theta = y/x$, $\mathrm{ctg}\theta = x/y$, $\sec\theta = r/x$, $\csc\theta = r/y$

Trigonometric Identities:

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\sin\theta/\cos\theta = \tan\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta =$

$= 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\sin\frac{1}{2}\theta = \sqrt{\frac{1-\cos\theta}{2}}$

$\cos\frac{1}{2}\theta = \sqrt{\frac{1+\cos\theta}{2}}$

$\sin(\theta \pm \pi/2) = \pm\cos a$

$\cos(\theta \pm \pi/2) = \mp\sin a$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$

$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$

$\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha + \beta)\sin\frac{1}{2}(\alpha - \beta)$

C. Algebra

$a^{-x} = \frac{1}{a^x}$ $\quad a^{(x+y)} = a^x a^y$ $\quad a^{(x-y)} = \frac{a^x}{a^y}$

Logarithms: If $\log a = x$, then $a = 10^x$. $\quad \log a + \log b = \log(ab)$

If $\ln a = x$, then $a = e^x$. $\quad \ln a + \ln b = \ln(ab)$

$\log a - \log b = \log(a/b)$ $\quad \log(a^n) = n\log a$

$\ln a - \ln b = \ln(a/b)$ $\quad \ln(a^n) = n\ln a$

Quadratic formula: If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Binomial theorem:

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + \ldots$$

D. Calculus

Derivatives:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin ax) = a\cos ax$$

$$\frac{d}{dx}(\cos ax) = -a\sin ax$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\sec^2 x$$

$$\frac{d}{dx}(\sec x) = \tan x \sec x$$

$$\frac{d}{dx}(\csc x) = -\cot x \csc x \qquad \frac{d}{dx}(\ln x) = \frac{1}{x}$$

Integrals:

$$\int dx = x$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \frac{dx}{x} = \ln|x|$$

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2+a^2}}$$

$$\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} \ (m \neq -1)$$

$$\int \tan x\,dx = \ln|\sec x|$$

$$\int xe^{-ax}dx = -\frac{1}{a^2}(ax+1)e^{-ax}$$

$$\int e^x\,dx = e^x$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax$$

$$\int e^{-ax}dx = -\frac{1}{a}e^{-ax}$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x+\sqrt{x^2+a^2})$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a^2}\frac{x}{\sqrt{x^2+a^2}}$$

$$\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x$$

$$\int x^2 e^{-ax}dx = -\frac{1}{a^3}(a^2x^2+2ax+2)e^{-ax}$$

Power series (convergent for range of x shown):

a) *trigonometric expansions* (*x in radians*):

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \quad (\text{all } x)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \quad (\text{all } x)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \ldots \ (|x| < \pi/2)$$

b) *exponential expansion*:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \quad (\text{all } x)$$

c) *logarithmic expansion*:

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \ (|x| < 1)$$

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English-Russian-Ukrainian Dictionary of Important Terms

A

abnormal dispersion (syn. anomalous dispersion)	аномальная дисперсия	аномальна дисперсія
abscissa, X-coordinate, axis of abscissas	абсцисса, ось абсцисс	абсциса, вісь абсцис
absolute permeability	абсолютная магнитная проницаемость, магнитная постоянная	абсолютна магнітна проникність, магнітна стала
absolute temperature	абсолютная температура	абсолютна температура
absolute value, absolute magnitude (of phisical quantity)	абсолютное значение (физической величины)	абсолютне значення (фізичної величини)
absorbtion (of light)	поглощение (света)	поглинання (світла)
absorption coefficient (syn. coefficient of absorption, absorptivity, absorption factor)	коэффициент поглощения (среды)	коефіцієнт поглинання (середовища)
absorption curve	кривая поглощения	крива поглинання
absorption edge	край полосы поглощения	край смуги поглинання
absorption factor (see absorption coefficient)		
absorptivity (see absorption coefficient)		
acceleration	ускорение	прискорення
acutance	резкость (контуров изображения) четкость (изображения)	різкість (контурів зображення) чіткість (зображення)
aeolotropy (see anisotropy)		
Airy disk	Айри диск	Айрі диск
alternating current	переменный ток	змінний струм
amplitude	амплитуда	амплітуда
amplitude splitting	разделение амплитуды (один из методов получения когерентных источников света)	розділення амплітуди (один з методів одержання когерентних джерел світла)
analyzer, analyser	анализатор	аналізатор
angle of deviation (of a light beam)	угол отклонения (пучка света)	кут відхилення (пучка світла)
angle of divergence (of a light beam)	угол расхождения (пучка света)	кут розходження (пучка світла)
angle of incidence	угол падения (пучка света)	кут падіння (пучка світла)
angle of reflection	угол отражения (пучка света)	кут відбиття (пучка світла)
angle of refraction	угол преломления (пучка света)	кут заломлення (пучка світла)
angle of view	угол обзора	кут огляду
angular velocity	угловая скорость	кутова швидкість
anisotropy (syn. aeolotropy)	анизотропия	анізотропія
anomalous dispersion (see abnormal dispersion)		
antinode (of oscillations)	пучность (колебаний)	пучність (коливань)
aperture	апертура, отверстие, диафрагма	апертура, отвір, діафрагма
arc discharge	дуговой разряд	дуговий розряд
atmosphere	атмосфера	атмосфера
attractive force	сила притяжения	сила тяжіння
axial symmetry	осевая симметрия	осьова симетрія
axis of ordinates	ось ординат	вісь ординат
axis of symmetry (syn. symmetry axis)	ось симметрии	вісь симетрії

B

band	полоса (в интерференционной или дифракционной картинах)	смуга (у інтерференційній або дифракційній картинах)
beam	луч	промінь
beam divergence	расходимость пучка (света)	розбіжність пучка (світла)
beam splitter	делитель пучка [устройство]	подільник пучка [пристрій]
bending (of light)	отклонение (изгиб) светового пучка	відхилення (вигин) світлового пучка
biprism	бипризма	біпризма
birefringent (syn. double refraction)	двойное лучепреломление	подвійне променезаломлення
blue color	голубой цвет (электромагнитного спектра излучения)	блакитний колір (електромагнітного спектра випромінювання)

Boltzmann's constant	Больцмана постоянная	Больцмана стала
boundary conditions	граничные условия, краевые условия	граничні умови, крайові умови
boundary effect	краевой еффект, граничный эффект	крайовий ефект, граничний ефект
Bragg angle	Брэгга угол	Брегга кут
Bragg reflection	Брэгга отражение	Брегга відбиття
Bragg's equation	уравнение Брэгга – Вульфа	Брегга – Вульфа рівняння
Bragg's law	Брэгга закон	Брегга закон
Brewster's law	Брюстера закон	Брюстера закон
Brewster's angle	Брюстера угол	Брюстера кут
brightness	яркость	яскравість
Buger's law	Бугера закон	Бугера закон

C

Canada balsam	канадский бальзам (вещество для склеивания стопы Столетова)	канадський бальзам (речовина для склеювання стопи Столєтова)
capacitance	емкость (электрическая)	ємність (електрична)
capacitor	электрический конденсатор	електричний конденсатор
cartesian coordinates	декартовы координаты	декартові координати
cathodoluminescence	катодолюминесценция	катодолюмінесценція
cathodophosphorescence	катодофосфоресценция	катодофосфоресценція
Celsius temperature scale (syn. centigrade temperature scale)	Цельсия температурная шкала	Цельсія температурна шкала
center, centre	центр	центр
centigrade temperature scale (see Celsius temperature scale)		
charge	заряд (электр.)	заряд (електр.)
circuit	цепь (электр.)	ланцюг (електр.)
circular birefringence	круговое двулучепреломление	кругове двопроменезаломлення
circular polarization	круговая поляризация	кругова поляризація
coating	покрытие (поверхности)	покриття (поверхні)
coefficient of absorption (see absorbtion coefficient)		
coefficient of damping (syn. damping coefficient, damping factor)	коэффициент затухания	коефіцієнт згасання
coefficient of reflection, reflection coefficient	коэффициент отражения	коефіцієнт відбиття
coefficient of refraction (see index of refraction)		
coherence	когерентность	когерентність
coherence area	область когерентности	область когерентності
coherence distance	длина когерентности	довжина когерентності
coherence time	время когерентности	час когерентності
coherent light	когерентный свет	когерентне світло
coherent radiation	когерентное излучение	когерентне випромінювання
collimator	коллиматор	коліматор
color	цвет	колір
color filter	цветной светофильтр	кольоровий світлофільтр
compensator	компенсатор	компенсатор
condition of continuity	условие непрерывности	умова безперервності
conduction current	ток проводимости	струм провідності
constructive interference	усиливающая интерференция	підсилювальна інтерференція
continuity	непрерывность	безперервність
continuous spectrum	сплошной спектр	безперервний спектр
converging lens	собирающая линза	збиральна лінза
conversion	преобразование	перетворення
corner	угол, край (объекта)	кут, край (об'єкта)
corona	корона	корона
corpuscle	частица, корпускула	частинка, корпускула
corpuscular theory of light	корпускулярная теория света (Ньютона теория)	корпускулярна теорія світла (Ньютона теорія)
Cotton – Mouton birefringence	Коттона – Мутона (двойное) лучепреломление	Коттона – Мутона (подвійне) променезаломлення
Cotton – Mouton constant	Коттона – Мутона постоянная (магнитооптический коэффициент)	Коттона – Мутона стала (магнітооптичний коефіцієнт)
Cotton – Mouton effect	Коттона – Мутона эффект (в магнитооптике)	Коттона – Мутона ефект (у магнітооптиці)
Coulomb force	Кулона сила	Кулона сила
Coulomb's law	Кулона закон	Кулона закон
crest (of a wave)	гребень (волны)	гребінь (хвилі)
critical angle (in polarization)	критический угол, предельный угол (в поляризации)	критичний кут, граничний кут (у поляризації)

cross section	поперечное сечение	поперечний переріз
crystal (crystalline) structure	кристаллическая структура [вещества], кристаллическое строение	кристалічна структура [речовини] кристалічна будова
crystal lattice (see also lattice)	кристаллическая решетка	кристалічні ґрати
crystallogram	рентгенограмма кристалла	рентгенограма кристала
crystallographic axis	кристаллографическая ось	кристалографічна вісь
current (electric)	ток (электрический)	струм (електричний)
curvature	кривизна	кривизна
cyclic frequency	циклическая частота, круговая частота	циклічна частота, кругова частота

D

damage	повреждение	ушкодження, пошкодження
damped harmonic motion	затухающее гармоническое колебание	згасаюче гармонійне коливання
damping	затухание	згасання
damping coefficient (see coefficient of damping)		
damping factor (see coefficient of damping)		
dc (see direct current)		
defect	дефект [структуры, кристаллической решетки и др.]	дефект [структури, кристалічних ґрат та ін.]
deformation	деформация, деформирование (общее понятие)	деформація, деформування (загальне поняття)
degree	градус, порядок, степень	градус, порядок, ступінь
degree of polarization	степень поляризации	ступінь поляризації
density	плотность	густина (рідини), щільність (твердого тіла)
density of charge carriers	плотность носителей заряда	густина носіїв заряду
destructive interference	ослабляющая интерференция	послаблювальна інтерференція
dextrorotatory,or right-handed, substance	вещество со свойством вращения плоскости поляризации вправо	речовина з властивістю обертання площини поляризації вправо
diagram, chart, plot	диаграмма, схема, график	діаграма, схема, графік
diamagnetism	диамагнетизм	діамагнетизм
dichroism	дихроизм	дихроїзм
dielectric constant (syn. electric constant) (e_0)	диэлектрическая постоянная (син. диэлектрическая проникновенность вакуума) (e_0)	діелектрична стала (син. діелектрична проникність вакууму) (e_0)
dielectric displacement (*D*)	электрическое смещение (индукция)	електричне зміщення (індукція)
dielectric permittivity, absolute, F/m	диэлектрическая проницаемость среды, абсолютная, Ф/м	діелектична проникність середовища, абсолютна, Ф/м
dielectric permittivity, relative	диэлектрическая проницаемость, относительная	діелектична проникність, відносна
dielectric susceptibility	диэлектрическая восприимчивость среды	діелектрична сприйнятливість середовища
dielectric, dielectric material	диэлектрик	діелектрик
dielectrical permittivity of a vacuum	диэлектрическая постоянная	діелектрична стала
diffraction	дифракция	дифракція
diffraction analysis	дифракционный анализ	дифракційній аналіз
diffraction grating	дифракционная решетка	дифракційні ґрати
diffraction grating spacing	период дифракционной решетки	період дифракційних ґрат
diffraction pattern	дифракционная картина	дифракційна картина
diffraction scattering	дифракционное рассеяние	дифракційне розсіяння
diffuse illumination	рассеянное освещение	розсіяне освітлення
dipole	диполь	диполь
dipole moment	дипольный момент	дипольний момент
dipole polarization	дипольная поляризация	дипольна поляризація
direct current (syn dc)	постоянный ток	постійний струм
discharge (electric)	разряд (электрический)	розряд (електричний)
discontinuity	нарушение непрерывности разрывность	порушення безперервності розривність
disorder	разупорядочение	розупорядкування
dispersion	дисперсия	дисперсія
dispersion spectrum	дисперсионный спектр	дисперсійний спектр
dissipation	рассеяние	розсіяння
distance	расстояние	відстань
disturbance (of oscillations)	возмущение (колебаний)	збурення коливань
double refraction (see birefringent)		
double-concave lens	двояковогнутая линза	двовгнута лінза
double-convex lens	двояковыпуклая линза	двоопукла лінза

E

eclipse	затмение	затемнення
edge	кромка, край	кромка, край
elastic force	сила упругости	сила пружності
electric charge	электрический заряд	електричний заряд
electric constant (dielectric permittivity for a vacuum), F/m	электрическая постоянная (диэлектрическая проницаемость вакуума), Ф/м	електрична стала (ділектична проникність вакууму), Ф/м
electric current	электрический ток	електричний струм
electric current density	плотность электрического тока	густина електричного струму
electric displacement (syn. dielectric displacement, dielectric flux density, , electric flux density, electric induction)	электрическое смещение	електричний зсув
electric field	электрическое поле	електричне поле
electric insulator, insulator	изолятор, диэлектрик	ізолятор, ділектрик
electric potential	электрический потенциал	електричний потенціал
electric spark	электрическая искра	електрична іскра
electric susceptibility	диэлектрическая восприимчивость [среды]	ділектрична сприйнятливість [середовища]
electrical resistance	электрическое сопротивление	електричний опір
electric-field intensity (electric-field strength)	напряженность электрического поля	напруженість електричного поля
electric-field vector	вектор напряженности электрического поля	вектор напруженості електричного поля
Electroluminescence	Электролюминесценция	електролюмінесценція
electroluminescent phosphor	электролюминесцентный люминофор	електролюмінесцентний люмінофор
electromagnetic	электромагнитный	електромагнітний
electromagnetic field	электромагнитное поле	електромагнітне поле
electromagnetic induction	электромагнитная индукция	електромагнітна індукція
electromagnetic radiation	электромагнитное излучение	електромагнітне випромінювання
electromagnetic theory of light	электромагнитная теория света	електромагнітна теорія світла
electromagnetic wave	электромагнитная волна	електромагнітна хвиля
electromagnetism	электромагнетизм	електромагнетизм
electron	электрон	електрон
electron orbit	орбита электрона	електронна орбіта
electronic polarization	электронная поляризация	електронна поляризація
+electronoluminescence	электролюминесценция	електролюмінесценція
electron-volt (eV)	электронвольт (eV)	електронвольт (eV)
electrophotoluminescence	электрофотолюминесценция	електрофотолюмінесценція
electrostatic attraction	электростатическое притяжение	електростатичне тяжіння
electrostatic field	электростатическое поле	електростатичне поле
electrostatic repulsion	электростатическое отталкивание	електростатичне відштовхування
electrostatics	электростатика	електростатика
elementary charge	элементарный заряд	елементарний заряд
elliptical polarization	эллиптическая поляризация	еліптична поляризація
emf	электродвижущая сила	електрорушійна сила
emission	излучение	випромінювання
emission spectrum	спектр излучения	спектр випромінювання
empirical	эмпирический, опытный	емпіричний, дослідний
energy	энергия	енергія
energy density	плотность энергии	густина енергії
energy dissipation	рассеяние энергии	розсіяння енергії
energy flux	поток энергии	потік енергії
envelope (of wave front)	огибающая (волнового фронта)	обвідна (хвильового фронту)
environment	окружающая среда	навколишнє середовище
equilibrium	равновесие	рівновага
equilibrium process	равновесный процесс	рівноважний процес
equilibrium stale	равновесное состояние	рівноважний стан
error	погрешность, ошибка	похибка, помилка
ether	эфир	ефір
excited state	возбужденное состояние	збуджений стан
external force	внешняя сила	зовнішня сила
extraordinary index	показатель преломления [необыкновенной волны]	показник заломлення [незвичайної хвилі]
extraordinary ray	необыкновенный луч	незвичайний промінь
extraordinary wave	необыкновенная волна	незвичайна хвиля
eyeglasses, eyepiece	очки, окуляр	окуляри, окуляр

F

Fabry – Pérot interferometer	Фабри – Перо интерферометр	Фабрі – Перо інтерферометр
factor (see index)		
Farad (F)	фарада (F)	фарада (F)
Faraday (diamagnetic) effect	Фарадея эффект (вращение плоскости поляризации света в магнитном поле)	Фарадея ефект (обертання площини поляризації світла у магнітному полі)
Faraday birefringence effect	Фарадея эффект двойного лучепреломления	Фарадея ефект подвійного променезаломлення
field of view	поле зрения	поле зору
filament	нить накала	волосок розжарення
film	пленка	плівка
filter	фильтр	фільтр
first-order spectrum	спектр первого порядка	спектр першого порядку
Fizeau interferometer	Физо интерферометр	Фізо інтерферометр
flow	поток	потік
fluid	жидкость	рідина
fluorescence	флуоресценция, флюоресценция	флуоресценція, флюоресценція
focal length	фокусное расстояние	фокусна відстань
focal plane	фокальная плоскость	фокальна площина
forced oscillations	вынужденные колебания	вимушені коливання
Fraungofer diffraction	Фраунгофера дифракция	Фраунгофера дифракція
free electron	свободный электрон	вільний електрон
free oscillation	свободные колебания	вільні коливання
frequency	частота	частота
frequency band	диапазон (полоса) частот	діапазон (смуга) частот
frequency of oscillations	частота колебаний	частота коливань
Fresnel biprism	Френеля бипризма	Френеля біпризма
Fresnel diffraction	Френеля дифракция	Френеля дифракція
Fresnel mirrors	Френеля зеркала	Френеля дзеркала
Fresnel zones	Френеля зоны	Френеля зони
fringe	полоса (интерференционная)	смуга (інтерференційна)
function	функция	функція
fundamental frequency	основная частота	основна частота

G

gamma-ray	гамма-луч	гамма-промінь
glow	свечение	світіння, свічення

H

half-wave plate	полуволновая пластинка	півхвильова пластинка
harmonic oscillations	гармонические колебания	гармонійні коливання
harmonic wave	гармоническая волна	гармонійна хвиля
HF	ВЧ (высокие частоты)	ВЧ (високі частоти)
hologram	голограмма	голограма
holography	голография	голографія
hue	тон, оттенок	тон, відтінок
Huygens' principle	Гюйгенса принцип	Гюйгенса принцип
Huygens – Fresnel principle	Гюйгенса – Френеля принцип	Гюйгенса – Френеля принцип

I

image	изображение	зображення
imperfect polarizer	несовершенный поляризатор	недосконалий поляризатор
incandescent lamp	лампа накаливания	лампа розжарювання
incidence plane	плоскость падения	площина падіння
incoherent light	некогерентный свет	некогерентне світло
incoherent scattering	некогерентное рассеяние	некогерентне розсіювання
index (syn. factor)	показатель, коэффициент	показник, коефіцієнт
index of absorption (see absorption index)		
index of refraction (see refraction coefficient)	показатель преломления [света], коэффициент преломления [света]	показник заломлення [світла], коефіцієнт заломлення [світла]
induced anisotropy	наведенная анизотропия	наведена анізотропія
inductance	индуктивность	індуктивність
inductance coil	катушка индуктивности	котушка індуктивності
inert gas	инертный газ	інертний газ

infrared irradiation, IR irradiation	инфракрасное изучение, ИК излучение	інфрачервоне випромінювання, ІЧ випромінювання
insulator	диэлектрик, изолятор	діелектрик, ізолятор
intensity	интенсивность	інтенсивність
interaction	взаимодействие	взаємодія
interatomic spacing	межатомное расстояние	міжатомна відстань
interference (of light)	интерференция (света)	інтерференція (світла)
interference fringes	интерференционные полосы	інтерференційні смуги
interference pattern	интерференционная картина	інтерференційна картина
interferometer	интерферометр	інтерферометр
ion	ион	іон
ionic bond	ионная связь	іонний зв'язок
ionic crystal	ионный кристалл	іонний кристал
ionic lattice	ионная решетка	іонні ґрати
ionic polarization	ионная поляризация	іонічна поляризація
isotropy	изотропия	ізотропія

K

Kelvin absolute temperature scale	Кельвина абсолютная температурная шкала	Кельвіна абсолютна температурна шкала
Kerr constant	Керра постоянная	Керра стала
Kerr effect	Керра эффект	Керра ефект
Kerr magnetooptic effect	Керра магнитооптический эффект	Керра магнітооптичний ефект
Kerr's cell	Керра ячейка	Керра комірка
kinetic energy	кинетическая энергия	кінетична енергія

L

Lambert's law	Ламберта закон	Ламберта закон
laser	лазер	лазер
lattice parameter	параметр кристаллической решетки	параметр кристалічних ґрат
law of reflection	закон отражения (света)	закон відбиття (світла)
law of refraction	закон преломления (света)	закон заломлення (світла)
layer	слой	шар
left-hand (levorotatory) polarization	левая поляризация	ліва поляризація
length	длина	довжина
lens	линза	лінза
light beam	луч света	промінь світла
light flux	световой поток	світловий потік
light intensity	интенсивность света	інтенсивність світла
light ray	луч света, световой луч	промінь світла, світловий промінь
light scattering	рассеяние света	розсіяння світла
lightning	молния	блискавка
limit	предел	межа, границя
limit of resolution	предел разрешения	межа розділення, роздільна здатність
linear polarization	линейная поляризация	лінійна поляризація
linearly polarized light	линейно поляризованный (плоскополяризованный) свет	лінійно поляризоване (пласкополяризоване) світло
liquid	жидкость	рідина
longitudinal wave	продольная волна	поздовжня хвиля
luminescence	люминесценция	люмінесценція
luminophor	люминофор	люмінофор

M

magnetic constant (*syn.* magnetic permeability of free space)	магнитная постоянная (магнитная проницаемость вакуума)	магнітна стала (магнітна проникність вакууму)
magnetic field	магнитное поле	магнітне поле
magnetic field intensity	напряженность магнитного поля	напруженість магнітного поля
magnetic flux	магнитный поток	магнітний потік
magnetic flux density	плотность магнитного потока, магнитная индукция	густина магнітного потоку, магнітна індукція
magnetic flux density (see magnetic induction)		

magnetic induction (syn. magnetic flux density (B)), T=Wb/m^2	магнитная индукция, плотность магнитного потока (B), Тл=Вб/м^2	магнітна індукція, густина магнітного потоку (B), Тл=Вб/м^2
magnetic permeability	магнитная проницаемость	магнітна проникність
magnetic permeability of a vacuum, H/m	магнитная проницаемость вакуума, Гн/м	магнітна проникність вакууму, Гн/м
magnetic permeability, relative	магнитная проницаемость, относительная	магнітна проникність, відносна
magnetism	магнетизм	магнетизм
magnification	увеличение (особенно с помощью линз микроскопа)	збільшення (особливо за допомогою лінз мікроскопа)
Malus's law (syn. Malus cosine-squared law)	Малюса закон	Малюса закон
Malus's cosine-squared law (see Malus law)		
mass	масса	маса
Maxwell equations	Максвелла уравнения	Максвелла рівняння
medium	среда	середовище
Michelson interferometer	Майкельсона интерферометр	Майкельсона інтерферометр
Michelson – Morley experiment	Майкельсона – Морли эксперимент	Майкельсона – Морлі експеримент
micrometer (unit of length)	микрон (единица длины)	мікрон (одиниця довжини)
microscopy	микроскопия, микроскопические исследования	мікроскопія, мікроскопічні дослідження
mirror	зеркало	дзеркало
mirror image	зеркальное изображение	дзеркальне відображення
mismatch	несоответствие, рассогласование	невідповідність, розузгодження
mode of oscillation	мода колебаний	мода коливання
model	модель	модель
molecule	молекула	молекула
monochromatic light	монохроматический свет	монохроматичне світло
motion, movement	движение	рух
multiple-beam interference	многолучевая интерференция	багатопроменева інтерференція

N

natural frequency	собственная частота	власна частота
Newton's rings	Ньютона кольца	Ньютона кільця
Nicol prism	Николя призма	Ніколя призма
node	узел (стоячей волны)	вузол (стоячої хвилі)
nonlinear oscillations	нелинейные колебания	нелінійні коливання
normal dispersion	нормальная дисперсия	нормальна дисперсія

O

O ray (see ordinary ray)		
opacity	непрозрачность, непроницаемость	непрозорість, непроникність
opaque barrier	непрозрачное препятствие	непрозора перешкода
opening	отверстие	отвір
optic axis	оптическая ось	оптична вісь
optical activity	оптическая активность	оптична активність
optical anisotropy	оптическая анизотропия	оптична анізотропія
optical density (of substance)	оптическая плотность (вещества)	оптична густина (речовини)
optical electron	оптический электрон	оптичний електрон
optical filter	светофильтр	світлофільтр
optical length, optical distance (syn. optical path)	оптическая длина пути	оптична довжина шляху
optical path (see optical length)		
optical path difference	оптическая разность хода	оптична різниця ходу
optically anisotropic crystal	оптически анизотропный кристалл	оптично анізотропний кристал
orange	оранжевый цвет	помаранчевий колір
orbit	орбита	орбіта
order number	порядковый номер (полосы в интерференционной или дифракционной картине)	порядковий номер (смуги в інтерференційній або дифракційній картині)
ordinary ray (syn. O ray)	обыкновенный луч	звичайний промінь
orientation polarization	ориентационная поляризация	орієнтаційна поляризація
origin of coordinates	начало координат	початок координат
oscillation	колебание	коливання

P

perfect crystal	совершенный кристалл	досконалий кристал
period of oscillations	период колебаний	період коливань
periodic lattice	периодическая решетка	періодичні ґрати
periodic wave	периодическая волна	періодична хвиля
phase difference	разность фаз	різниця фаз
phase equilibrium	фазовое равновесие	фазова рівновага
phase of oscillations	фаза колебаний	фаза коливань
phase plate	фазовая пластинка	фазова пластинка
phasor, rotating vector	фазор, вращающийся вектор	фазор, вектор, що обертається
phosphorescence	фосфоресценция	фосфоресценція
phosphorus	фосфор	фосфор
photoelastic effect	эффект фотоупругости, поляризационно-оптический эффект	ефект фотопружності, поляризаційно-оптичний ефект
photoluminescence	фотолюминесценция	фотолюмінесценція
photomagnetoelectric effect	фотоэлектромагнитный эффект	фотомагнітоелектричний ефект
photon	фотон	фотон
piezooptical effect	пьезооптический эффект	п'єзооптичний ефект
plane mirror	плоское зеркало	пласке дзеркало
plane of polarization	плоскость поляризации	площина поляризації
plane of vibration	плоскость колебаний	площина коливань
plane polarization	плоская поляризация	плоска поляризація
plane wave	плоская волна	плоска хвиля
planoconcave lens	плосковыпуклая линза	плоскоопукла лінза
polarization (electronic)	поляризация (электронная)	поляризація (електронна)
polarization (ionic)	поляризация (ионная)	поляризація (іонна)
polarization (orientation)	поляризация (ориентационная)	поляризація (орієнтаційна)
polarization by reflection	поляризация при отражении	поляризація при відбиванні
polarized light	поляризованный свет	поляризоване світло
polarized scattering	поляризационное рассеяние	поляризаційне розсіювання
polarizer	поляризатор	поляризатор
polarizing angle	угол полной поляризации	кут повної поляризації
polarizing axis	ось поляризации	вісь поляризації
polarizing filter	поляризационный светофильтр	поляризаційний світлофільтр
polaroid	поляроид	поляроїд
polycrystal	поликристалл	полікристал
potential energy	потенциальная энергия	потенціальна енергія
power	мощность	потужність
Poynting vector	Пойнтинга вектор, вектор потока электромагнитной энергии, вектор потока энергии	Пойнтінга вектор, вектор потоку електромагнітної енергії, вектор потоку енергії
preferential absorption	избирательное поглощение	вибіркове поглинання
pressure	давление	тиск
primary maxima	главный максимум	головний максимум
primary optic axis	главная оптическая ось	головна оптична вісь
principal axis (of a crystal)	главная ось (кристалла)	головна вісь (кристала)
principal ray	главный луч	головний промінь
principle of superposition (syn. superposition principle)	принцип суперпозиции	принцип суперпозиції
prism	призма	призма
propagation constant	коэффициент распространения, постоянная распространения	коефіцієнт поширення, стала поширення
proton	протон	протон

Q

quarter-wave plate	четвертьволновая пластина	чвертьхвильова пластина
quartz	кварц	кварц

R

radiation	излучение	випромінювання
rainbow	радуга	райдуга, веселка
Raleigh's criterion	Рэлея критерий	Релея критерій
ratio	отношение	відношення
ray	луч	промінь
Rayleigh's law	закон Рэлея	закон Релея
real image	действительное изображение	дійсне зображення
reciprocal wavelength	волновое число	хвильове число

reflected ray	отраженный луч	відбитий промінь
reflected wave	отраженная волна	відбита хвиля
reflection coefficient	коэффициент отражения	коефіцієнт відбиття
reflective coating	отражающее покрытие	відбивне покриття
refracted ray	преломленный луч	заломлений промінь
refracted wave	преломленная волна	заломлена хвиля
refraction coefficient (syn. index of refraction, refractive index)	показатель преломления [света], коэффициент преломления [света]	показник заломлення [світла], коефіцієнт заломлення [світла]
refractive index (see refraction coefficient)		
relation, relationship	соотношение	співвідношення
relative permeability	относительная магнитная проницаемость	відносна магнітна проникність
relative permittivity	относительная диэлектрическая проницаемость	відносна діелектрична проникність
repulsion	отталкивание	відштовхування
repulsive force	сила отталкивания	сила відштовхування
requirement	требование	вимога
resolving power	разрешающая способность	роздільна здатність
resonance	резонанс	резонанс
resonance frequency	резонансная частота	резонансна частота
right angle	прямой угол	прямий кут
right-hand polarization	правая поляризация	права поляризація
roentgen rays	рентгеновские лучи	рентгенівські промені
röntgen	рентген	рентген
rotate	вращаться	обертатися
rotation	вращение	обертання

S

scale	шкала (масштаб)	шкала (масштаб)
scattering	рассеивание	розсіяння
scintillation	мерцание, сцинтилляция	мерехтіння, сцинтиляція
screen	экран	екран
screening	экранирование	екранування
screw	винт	гвинт
searchlight	прожектор	прожектор
second (unit of time)	секунда (1/60 часть минуты)	секунда (1/60 частина хвилини)
secondary	вторичный	вторинний
secondary rainbow	вторичная радуга	вторинна райдуга
secondary wave	вторичная волна	вторинна хвиля
selective absorption	избирательное поглощение	вибіркове (вибірне) поглинання
selective reflection	избирательное отражение	вибіркове відбиття
shadow	тень	тінь
shape, form	форма (геометрическая), конфигурация	форма (геометрична), конфігурація
short-range order	ближний порядок	близький порядок
shutter	затвор	затвор
SI	СИ (система измерений)	СІ (система вимірювань)
simple harmonic motion	гармоническое колебание	гармонійне коливання
sine	синус	синус
sine wave	гармоническая волна, синусоидальная волна	гармонійна хвиля, синусоїдальна хвиля
single crystal	монокристал	монокристал
size, dimension	размер	розмір
slit	диафрагма, щель	діафрагма, щілина
soap-bubble	мыльный пузырь	мильна бульба
solar wind	солнечный ветер	сонячний вітер
solid	твердое тело	тверде тіло
solid state	твердое состояние (вещества)	твердий стан (речовини)
source	источник	джерело
spark	искра	іскра
spark discharge	искровой разряд	іскровий розряд
spark gap	искровой промежуток	іскровий проміжок
spark voltage	напряжение искрового разряда	напруження іскрового розряду
spatial coherence	пространственная когерентность	просторова когерентність

specific	удельный	питомий
spectrometer	спектрометр	спектрометр
spectrum	спектр	спектр
spectrum line	спектральная линия	спектральна лінія
speed (magnitude of velocity)	скорость	швидкість
speed of light	скорость света	швидкість світла
spherical wave	сферическая волна	сферична хвиля
split	расщеплять	розщеплювати
star	звезда	зірка
stationary wave	стоячая волна	стояча хвиля
strenght of a magnetic field	напряженность магнитного поля	напруженість магнітного поля
Strength	прочность	міцність
strength of the electric field	напряженность электрического поля	напруженість електричного поля
stress	напряжение (механическое)	напруження (механічне)
stress	напряжение	напруга
stress analysis	анализ напряжений	розрахунок напруженого стану
stress concentration	концентрация напряжений	концентрація напруг
sum	сумма	сума
superposition	суперпозиция	суперпозиція
superposition principle (see principle of superposition)		
suppress	подавлять	заглушувати
surface	поверхность	поверхня
susceptibility	восприимчивость	сприйнятливість
symmetry axis (see axis of symmetry)		

T

tangent	касательная	дотична
tesla, T	тесла (единица измерения магнитной индукции), Тл	тесла (одиниця вимірювання магнітної індукції), Тл
thermal radiation	тепловое излучение	теплове випромінювання
thick lens	толстая линза	товста лінза
thickness	толщина	товщина
thin lens	тонкая линза	тонка лінза
time	время	час
tissue	ткань	тканина
tooth	зуб	зуб
top	вершина	вершина
transmitted wave	проходящая волна	прохідна хвиля
transparency	прозрачность (способность пропускать свет)	прозорість (здатність пропускати світло)
transparency, transparence	прозрачность, оптическая прозрачность	прозорість, оптична прозорість
transparent	прозрачный	прозорий
transparent medium	прозрачная среда	прозоре середовище
transverse	поперечный	поперечний
transverse electromagnetic wave	поперечная электромагнитная волна	поперечна електромагнітна хвиля
transverse piezoelectric effect	поперечный пьезоэлектрический эффект	поперечний п'єзоелектричний ефект
transverse wave	поперечная волна	поперечна хвиля
traveling wave	бегущая волна	хвиля, що біжить
trough	впадина, углубление	западина, заглиблення
two-beam interference	двухлучевая интерференция	двопроменева інтерференція

U

ultraviolet	ультрафиолетовый	ультрафіолетовий
ultraviolet radiation	ультрофиолетовое излучение	ультрафіолетове випромінювання
undamped	незатухающий	незгасаючий
undergo	испытывать	випробувати
uniaxial crystal	одноосный кристалл	одновісний кристал
unit	единица измерения	одиниця вимірювання
unit cell	элементарная ячейка	елементарна комірка
UV	УФ, ультрафиолетовое излучение	УФ, ультрафіолетове випромінювання

V

vector	вектор	вектор
velocity	скорость	швидкість
Verde constant	постоянная Верде	стала Верде
virtual	мнимый	уявний
virtual image	мнимое изображение	уявне зображення
virus	вирус	вірус
visible spectrum	видимая часть спектра	видима частина спектра
Volt (V)	вольт (единица измерения электрического напряжения)	вольт (одиниця вимірювання електричної напруги)
voltage	электрическое напряжение напряжение (электрическое)	електрична напруга, напруга (електрична)

W

water	вода	вода
watt, W	ватт (единица измерения мощности), Вт	ват (одиниця вимірювання потужності), Вт
wave	волна	хвиля
wave amplitude	амплитуда волны	амплітуда хвилі
wave crest	гребень волны	гребінь хвилі
wave equation	волновое уравнение	хвильове рівняння
wave front	фронт волны	фронт хвилі
wave intensity	интенсивность волны	інтенсивність хвилі
wave interference	интерференция волн	інтерференція хвиль
wave motion	волновое движение	хвильовий рух
wave number	волновое число	хвильове число
wave optics	волновая оптика	хвильова оптика
wave surfaces	волновая поверхность	хвильова поверхня
wave theory of light	волновая теория света	хвильова теорія світла
wave train	цуг волн, последовательность волн	цуг хвиль, послідовність хвиль
wave vector	волновой вектор	хвильовий вектор
waveform	форма волны	формула хвилі
wavefront splitting	разделение волнового фронта	розділення хвильового фронту
wavelength	длина волны	довжина хвилі
wave-particle duality	корпускулярно-волновой дуализм	корпускулярно-хвильовий дуалізм
weber, Wb	Вебер (единица измерения магнитного потока), Вб	Вебер (одиниця вимірювання магнітного потоку), Вб
wedge	клин	клин
Wolff – Bragg condition (syn.Woulf – Bragg law)	условие Вульфа – Брэгга	умова Вульфа – Брегга
work	работа	робота

X

X-ray	рентгеновский луч	рентгенівський промінь
X-ray analysis	рентгеноструктурный анализ	рентгеноструктурний аналіз
X-ray diffraction	дифракция рентгеновских пучей	дифракція рентгенівських променів
X-ray radiography	рентгенография	рентгенографія

Y

yellow	желтый	жовтий
Young's two-slit interference	интерференция от двух щелей	інтерференція від двох щілин

Z

z axis	ось Z	вісь Z
zone	зона	зона
zone plate	зонная пластинка	зонна пластинка

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