MINISTRY OF EDUCATION, SCIENCE, YOUTH
AND SPORTS OF UKRAINE
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BASIC ELECTRICAL ENGINEERING

Part 2

Manual

Kharkiv «KhAI» 2011

УДК 621.3(075.8)
З-62

Навчальний посібник написано згідно з програмою підготовки бакалаврів і магістрів. У другій частині розглянуто питання конструкції, принцип дії, основні характеристики і параметри електричних машин постійного і змінного струму і трансформаторів, а також особливості електричних машин, які експлуатуються на літальних апаратах.

Кожний розділ включає теоретичну частину, задачі, контрольні запитання, перевірні тести, тлумачний словник ключових термінів.

Для студентів механічних та електротехнічних спеціальностей вищих навчальних закладів з поглибленим вивченням технічної англійської мови.

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З-62 Basic Electrical Engineering: manual/ A. Ya. Zimovin, V. N. Postnikov, L. I. Volchanskaya. – Kharkiv: National Aerospace University "Kharkiv Aviation Institute", 2011. – P.2. – 116 p.

The second part of the manual has been prepared to suit bachelor and master syllabus. This part is devoted to studying structure, principle of operation, performance characteristics and main parameters of rotating electrical machines and stationary transformers. The emphasis is paid to electric machines used on an aircraft.

Each section contains theory, glossary, test questions and problems, as well as tests for control check.

The textbook can be useful for everyone in the field of engineering for improving technical English.

UDC 621.3 (075.8)

Illustrations 56. Tables 1. Bibliography: 7 references

INTRODUCTION

Today modern airplanes, large airliners and other flying vehicles are equipped with numerous electrical systems, each requiring a substantial amount of electric energy. That is, besides numerous avionic devices which are principally electrically operated, a lot of operations on board the aircraft is done with the help of electric drives like transformers, relays, motors etc.

Generators were the first means of supplying electric power for aircraft. Currently, generators or generator derivatives called alternators are found in a variety of sizes and output capacities. An electric generator can be defined as a machine that converts mechanical energy into electric energy. On aircraft the mechanical energy is usually provided by the aircraft engines. Light aircraft use 14- or 28-V DC generators. Large aircraft typically employ generators that produce an alternating current of 208 or 117 V at 400 Hz. Compared with 28-V DC system, a higher voltage AC system develops several times as much power for the same weight.

The objective of this manual is to introduce the basic operation of main electromagnetic devices, emphasizing the discussion on rotating machines which are aimed either to fulfill some kind of mechanical work if they are electrically energized or, vice versa, to produce electric energy when driven by a rotary actuator. The operation of three major classes of electric machines (DC, synchronous and induction) will be discussed here, taking into account the material of Chapter 1, as well as using fundamental statements of Chapter 5 in previous part of the manual.

The emphasis will be done on explaining the properties of each type of machines, with their advantages and disadvantages regarding other types; and on classifying these machines in terms of their performance characteristics and preferred field of application.

In the end, the student should be able to:

- describe the principles of operation of transformers, DC and AC motors and generators;
- interpret the nameplate data of an electric apparatus;
- interpret the torque-speed characteristic of a rotating machine;
- specify the requirements of a machine for the given application.

Chapter 1

TRANSFORMERS

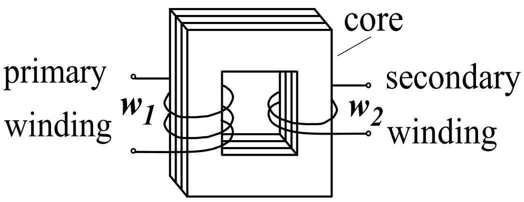
AC circuits are often connected to each other by means of transformers. Due to mutual inductance exposed by two conductors which are closely placed to each other (as in typical transformer arrangement), the arrangement couples two AC circuits magnetically rather than electrically (electrically isolated coupling).

Besides, the transformer displays effective scaling, i.e. increase /decrease of the voltage, current and impedance between one circuit (a source) and the other (a receiver).

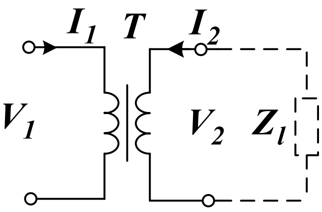
1.1 Transformer Structure

A practical transformer consists of two coils that are coupled to each other by means of a magnetic medium that is usually referred to as a core. ***The ideal transformer is assumed to have the core of infinite permeability and the coils offering negligible resistance to current flow.***

The core provides magnetic circuit with a low reluctance, owing to which the flux is confined to the medium thus linking the flux of both coils perfectly. Fig. 1.1 depicts a sketch of the transformer and its schematic symbol along with the receiving circuit presented by equivalent impedance.



a



b

Fig. 1.1 – Transformer diagrams: structure (a); circuit symbol (b)

The arrangement is realized in a form of **wire coils** wound around a **laminated iron core**. The lamination is intended to introduce tiny, discontinuous air gaps between multiple core layers in order to reduce losses that happen during energy conversion. The core can be made of **ferrite** also, for better working at high frequencies, whereas an **air-core** transformer will collect advantages for very high frequency use.

A transformer coil that is aimed to accept electrical energy from a supply circuit is referred to as **primary winding;** the other coil is commonly named **secondary** because of its purpose to deliver transformed energy to a load circuit.

Presented energy transfer may only occur if the flux in the structure **changes;** due to this condition the transformer action is associated with **alternating (AC)** currents and voltages.

Referring to a particular application, transformers can be classified to single or three-phase **power** transformers, and **voltage** or **current** transformers (among these two types the voltage transformers are most common). Also, one can find **instrumental** (current-measuring) transformers in use, as well as **auto**transformers.

1.2 Transformer Operation

As it was said, the operation of the transformer requires a time-varying current. If an alternating voltage $v_1(t)$ is applied to the primary coil, a corresponding time-varying current $i_1(t)$ will flow in the primary winding. It is followed by producing a magnetizing force $i_1(t) \cdot w_1$. The force causes a time-varying flux $\Phi_1(t)$ in the structure. Due to this flux changing an electromotive force (emf $e(t)$) is induced in both secondary and primary coil according to **Electromagnetic Induction (Faraday's)** Law:

$$e_1 = w_1 \frac{d\Phi}{dt}; \quad e_2 = w_2 \frac{d\Phi}{dt} \quad , \tag{1.1}$$

where w_1 and w_2 are the primary and secondary turns, respectively.

The emf e_1 is a self-induction electromotive force of the primary winding (often termed a counter emf), while associated with the secondary winding emf e_2 presents a mutual induction electromotive force, that is the one induced in the output coil by the primary current $i_1(t)$. This emf can be measured and readily exposed as the open circuit voltage V_2 (effective value) at the output terminals.

It is then seen from (1.1) that

$$\frac{e_2}{e_1} = \frac{w_2}{w_1} = \frac{V_2}{V_1} = k_{21}, \tag{1.2}$$

i.e. the ratio of the output (open-circuit) voltage and input (applied) voltage is determined by the ratio of transformer secondary and primary turns; this specific ratio is named transformer **turn ratio,** or **transformation ratio**.

If $w_2 > w_1$, then $V_2 > V_1$, and we deal with a **step-up** transformer. When coils have a reciprocal turn ratio $(w_2 < w_1)$ the transformer is said to be **step-down** since the output voltage V_2 becomes less than the voltage V_1 applied to the input.

One will recognize the following notations for the electromotive forces induced in transformer coils:

$$E_1 = 4{,}44\, f\, w_1 \Phi_m, \quad E_2 = 4{,}44\, f\, w_2 \Phi_m\ , \tag{1.3}$$

if for conventional sine-wave form $\Phi(t)$ we calculate the derivative in (1.1) and then replace instant-time terms related to emf and flux by their effective and peak amplitude values, correspondingly.

The relationships in (1.3) are known as **transformer emf**.

In the right part of each equation above, f is the frequency of supply sinusoidal voltage; Φ_m implies the magnitude of magnetic flux in the core.

If a load is connected across the secondary coil, an induced load current i_2 produces its own magnetic field which tends to cancel the magnetic field of the primary current i_1. This effort reduces the primary counter-emf e_1 thus allowing more primary current to flow.

The primary current will increase if the load, i.e. the amount of secondary current, increases. Inversely, the less the load current the less the current in primary winding is. Quite small primary current in the no-load transformer is called **exciting** (or magnetizing, or excitation) current – i_{10}. In practice, a particular mean of exciting current is predicted by virtue of Ohm's law, i.e. by the ratio of applied voltage V_1 and primary coil reactance $X_M = 2\pi f L_M$.

For loaded transformer, a quantitative relationship between primary and secondary currents is assumed to depend on turn ratio again. Really, if we resume the fact that the ideal transformer has to conserve the energy entered (since it has no loss, hence dissipates no power), then a relation $I_1 V_1 = I_2 V_2$ will be true and therefore

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = k_{21}. \tag{1.4}$$

On the base of (6.4), a helpful rule that reveals transformer purpose can be formulated: **an ideal transformer multiplies a sinusoidal input voltage by a factor of k_{21} and divides a sinusoidal input current by the same factor.**

Also, a transformer can be viewed as an impedance converter. The problem deals with a question: what actual impedance does the driving source "see" at the input terminals of the transformer? How does the load circuit impedance being reflected to transformer input depend on transformer turn ratio k_{21}?

Let a load that has an impedance Z_l be connected at secondary transformer terminals as shown in Fig. 1.1 b; also let the transformer be ideal one. Then according to the rule stated above we have $V_1 = V_2 / k_{21}$, $I_1 = k_{21} I_2$, and an impedance at the input of the transformer can be written as

$$Z_1 = V_1 / I_1 = Z_l / k_{21}^2 \ , \tag{1.5}$$

since $Z_l = V_2 / I_2$.

That is, an ideal transformer reflects any secondary impedance back to primary circuit by a factor of $1 / k_{21}^2$. This conclusion is equivalent to the statement: an AC circuit that drives a load by means of a transformer accepts this load having k_{21}^2 times transformed impedance. E.g., a step-down transformer increases receiver input impedance, whereas the step-up transformer decreases. This provides the transformer with useful and asked-for ability to match circuits' impedance when they differ much.

1.3 Transformer Losses

For a **practical transformer,** the above mentioned principle of input-output power balance may be applied in approximate form $I_1V_1 \approx I_2V_2$ because of a few losses of power being raised.

In general, transformer losses are not too essential. Therefore, 98% transformer efficiency, for instance, can be easily attained.

Power efficiency is defined by the ratio

$$\eta = \frac{P_2}{P_1},$$

where P_2, P_1 are the output (load) power and the power entered the transformer, correspondingly. Input power, i.e. power drained from a driving source, includes both power P_2 transmitted to the load and power ΔP wasted while transmitting. It is the amount of ΔP, i.e. a **loss**, that results in transformer efficiency.

Some loss is related to imperfect magnetic coupling between coils; this appears in a form of flux leakage. The item is not essential due to state of the art technologies; therefore it is of no practical interest.

Besides leakage loss, transformers are subjected to so-called "**copper**" and "**iron**" losses.

The copper, or ohmic, loss origins from winding wire resistance that always dissipates energy and causes transformer coils heating:

$$P_C = I_1^2 R_1 + I_2^2 R_2 . \tag{1.6}$$

The iron, or magnetic, loss P_M relates to core design and manufacturing and is commonly divided into two groups termed **hysteresis** and **eddy currents losses**.

Hysteresis loss represents the energy lost to heat the core during the process of core magnetizing in different directions when following the applied AC voltage. For a particular core material, the process will require excessive power to demagnetize the core from the portion of induction left in the core after the former half-cycle. The amount of required energy depends on both core material (soft magnetic materials, e.g. stallow or permalloy, will offer the best choice) and the rate of flux change (the latter is defined by the frequency of applied voltage):

$$P_H = \sigma_H f B_m^2 G\,. \tag{1.7}$$

In the above formula, G represents an amount of core iron specified by its weight, B_m is induction peak amplitude, and σ_H is a specific factor of hysteresis.

Eddy current loss is caused by a lot of small circulated electric currents (called "eddy") induced inside conducting core material because of varying magnetic field; these currents, when collecting together, produce extra heat and corresponding power dissipation:

$$P_E = \sigma_E f^2 B_m^2 G\,, \tag{1.8}$$

where σ_E is a specific eddy current factor.

To reduce eddy currents, transformer cores are made of laminations, that is of steel plates separated by a thin layer of insulation. By utilizing laminations the circulation of large induced currents within the core is prevented.

As a result of both hysteresis and eddy current phenomena the core heating in a practical transformer is unavoidable and transformer efficiency drops.

In engineering practice power efficiency can be derived from transformer tests, where the iron ($P_M = P_H + P_E$) and copper (P_C) losses are determined on the basis of appropriate instrumentation (see section 1.5).

1.4 The Law of Magnetic Balance

The law of magnetic balance (magnetic equilibrium) for a transformer circuit states that a magnetic flux within the structure remains constant nevertheless the amount of the load supplied by the output terminals, i.e. it is true that $\Phi_{0m} = const$, when $Z_l = var$ and $V_1 = const$. This can easily be seen from transformer emf equation $E_1 = 4{,}44 f W_1 \Phi_m$ if one assumes that a distinction between the emf E_1 (induced voltage) and the terminal voltage V_1 is inessential (the terms will be exactly the same in the case of ideal transformer). Then, as $E_1 \cong V_1$, nevertheless the secondary current change the magnetic flux Φ_{0m} will not change whilst the supply voltage V_1 remains constant. Therefore, if we introduce Φ_{m1} and Φ_{m2} apart as primary and secondary components of the flux, the following equation

$$\Phi_{0m} = \Phi_{m1} + \Phi_{m2} = const \tag{1.9}$$

can be written in order to present an ability of the transformer to conserve a magnetic flux within the structure when it runs in the range from **open-circuit** to **rated load** and hence Φ_{m1}, Φ_{m2} apart change much.

The equivalent equation for magnetomotive forces balance

$$I_{1o}w_1 = I_1w_1 + I_2w_2 \tag{1.10}$$

can also be written in the form

$$I_1 = I_{1o} - I_2' \tag{1.11}$$

presenting the load (secondary) current reflected to the primary circuit – $I_2' = I_2k_{21}$.

Expression (1.11) emphasizes the fact that the primary current in a loaded transformer consists of two terms: magnetizing current I_{1o} that energizes the core (creates the flux Φ_0 independent of mode), and I_2' term which rises to generate a counter mmf I_1w_1 for opposing the mmf I_2w_2 of the output coil. To manage this duty, the input coil must draw extra current from the source: the more current that the load demands, the more primary current should be.

For practical transformers, the relationship between primary and secondary currents is not quite linear; usually it looks like a graph in Fig.1.2,a.

a b

Fig. 1.2 – Transformer working with a load:
the primary versus secondary current relationship (a);
the load characteristic (b)

In Fig. 1.2,b, performance of a transformer under various practical loads is presented by the sketch of a **load characteristic** which displays a drop in terminal voltage V_R at the rated load compared to the open circuit operation V_{2o}.

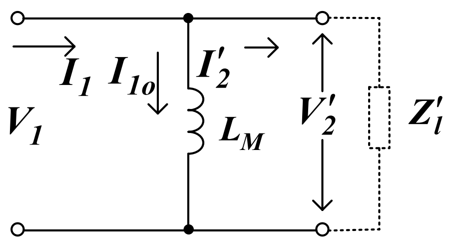
1.5 Transformer Equivalent Circuits

For analyzing circuits that contain transformers it may be convenient to utilize a diagram in which ***magnetic coupling*** has been replaced by an ***equivalent electrical circuit.*** When making such a change, the equivalent circuit must reproduce the same condition for the whole circuit; this means that the load must run at the same power level. Calculation with the equivalent reflected circuit is convenient because all circuit elements can be referred to a single set of variables, i.e. to either primary or secondary voltages or currents.

The equivalent circuit for ***ideal transformer*** may appear in a form shown in Fig. 1.3,a.

In the figure, an inductor $\boldsymbol{L_M}$ represents self-inductance of the transformer magnetizing coil; besides, the input and output transformer terminals are shown as directly connected. For such a connection, actual output voltage must be reduced in order to satisfy condition $V_2' = V_1$. As the actual secondary voltage is determined by $V_2 = k_{21}V_1$, we shall readily see that $V_2' = \dfrac{V_2}{k_{21}}$ in the diagram. Similarly, one can verify that the equivalent (reduced) secondary current will be $I_2' = k_{21}I_2$.

The results obtained for the ideal case do not completely represent the physical nature of transformers. If one wants to include loss mechanisms in a practical transformer model, the equivalent circuit of Fig. 1.3,b must be used.



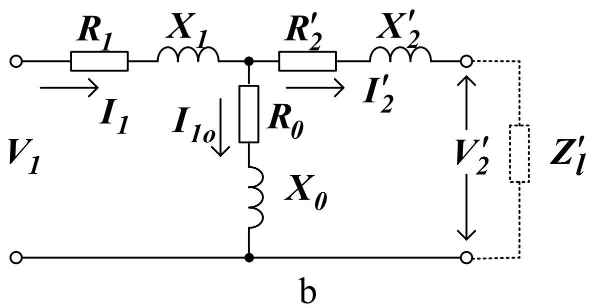


Fig. 1.3 – Transformer equivalent circuits:
the idealized model (a); the practical model (b)

Compared to the simplified circuit, new terms of the practical model introduce the following physical features:

R_1, R_2 – primary and secondary conductor resistances (represent the copper loss);

X_1, X_2 – both primary and secondary reactance originated from transformer flux leakage loss (coils reveal a leakage inductance);

$R_0 = R_H + R_E$ – equivalent resistance representing core losses caused by hysteresis and eddy currents;

reactance X_0 is associated with the storage of flux Φ_0 due to L_M.

For the model, actual parameters of the secondary coil were reflected (transformed by a factor of k_{21}) to the primary circuit in accordance with the principle presented in the beginning of the section: $R_2' = \dfrac{R_2}{k_{21}^2}$, $X_2' = \dfrac{X_2}{k_{21}^2}$.

Voltage regulation. It can be seen from the equivalent diagram that the output transformer voltage will not be kept constant when the load current changes (see Fig. 1.3, b); this is because of voltage drops that occur due to internal loss mechanisms. These drops are directly proportional to current flows; of that, the load (secondary) current flow will dominate. A change of transformer output voltage resulted from variation of the secondary current defines transformer **load,** or **external**, **characteristic**.

Derived from the graph of external curve, a **voltage regulation** of a transformer can be involved to display transformer ability to maintain constant voltage in the face of load variations. Commonly, the voltage regulation implies transformer no-load to rated load running and can be either per-unit or present termed. E.g., percent voltage regulation of the transformer will be defined as follows:

$$\Delta V = \frac{V_{20} - V_{2R}}{V_{20}} 100\%, \tag{1.12}$$

where V_{20}, V_{2R} present output voltages at no load and rated load, respectively. Typical value of voltage regulation for power transformers is about 4…5%.

1.6 Transformer Tests

In order to utilize the transformer properly, e.g. avoid its damage, one may obtain transformer performance characteristics known as transformer ratings. A full list can be found in the data sheet; while the most asked-for items are usually given on the nameplate, which is attached to the housing and indicates the normal operating conditions. The nameplate includes the main three parameters: primary-to-secondary voltage ratio, design operation frequency, and rated output power. The voltage ratio can be used to determine the turn ratio, whereas the rated output power (apparent power, in VA units) represents the continuous power level that can be sustained without overheating.

Besides finding on the nameplate these and other important transformer performance characteristics, e.g. power efficiency and power factor, can be determined experimentally. The investigation is usually referred to as **transformer tests**.

No-load (open-circuit) test

This test is fulfilled with the aim to determine turn ratio and iron loss; also, some parameters of transformer equivalent circuit can be derived.

When performing the no-load test one should supply the transformer with a **rated voltage** V_{1nom} and keep the output terminals **open-circuit**.

To obtain necessary data and then calculate transformer parameters, four conventional instruments are used according to the wiring diagram of Fig. 1.4. These instruments are involved to measure applied voltage V_{1nom}, primary current I_{10}, input power P_{10} and output open-circuit voltage V_{20}.

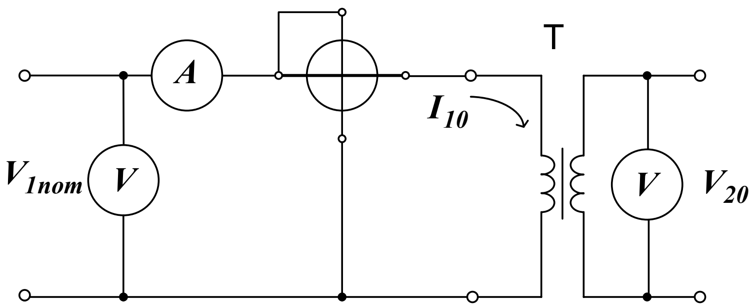


Fig. 1.4 – The circuit diagram for transformer open-circuit test

Here, for example, transformer turn ratio can be simply determined as

$$k_{21} \approx \frac{V_{20}}{V_{1nom}},$$

i.e. with the help of both voltmeters reading. The wattmeter reading can help to determine the iron loss (magnetic loss P_M), as it will be shown further.

Equivalent circuit of the transformer associated with the open circuit test is shown in Fig. 1.5. It allows making useful assumptions related to parameters determination.

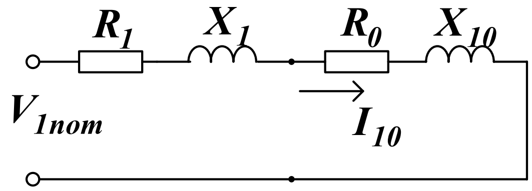


Fig. 1.5 – Transformer open-circuit model

Mind here that R_1 is associated with the active (ohmic) resistance of the primary winding; X_{10},X_1 represent reactance of the winding (both self inductance and leakage inductance); and R_0 implies virtual transformer resistance appeared in the model due to physical core heat (magnetic losses). Commonly, $R_1 \ll R_0$ and $X_{10} \gg X_1$.

The wattmeter in Fig. 1.4 measures full active power P_0 of the circuit. But conventional notation for power dissipation $P_0 = P_2 + P_1$ in the absence of physical load (P_2=0), may be written as $P_0 = P_1 = P_M + P_C$.

Further it can be simplified to $P_0 = P_M = P_H + P_E$, since the copper loss term $P_C = I_{10}^2 R_1$ may be neglected owing to small magnetizing current.

Hence, in the open-circuit test the wattmeter gives a reading very much close to transformer magnetic losses, i.e. $P_0 \cong P_H + P_E$.

It is important to emphasize now that estimated by means of the wattmeter the magnetic losses will be invariant to any transformer loading, if one keeps in mind a condition $V_{1nom} = const$.

Other quantities helpful for practical purpose can be found by using related meter readings and calculation as follows:

– equivalent model resistance $R_{10} = R_1 + R_0 = \dfrac{P_0}{I_{10}^2}$;

– impedance of the open-circuited transformer $Z_{10} = \dfrac{V_{1nom}}{I_{10}}$;

– corresponding reactance $X_{10} = X_1 + X_0 = \sqrt{Z_{10}^2 - R_{10}^2}$;

– transformer power factor $\cos\varphi_0 = \dfrac{P_0}{I_{10}V_1}$.

In the end, we can readily determine primary coil self-inductance $L_1 = X_{10} / 2\pi f$ if we recollect that $X_{10} \gg X_1$; hence $X_0 \approx X_{10}$.

Short-circuit test

In performing this test, the secondary winding is short-circuited while the supply voltage is set at a level much lower than a rated value. A circuit diagram for short-circuit test is shown in Fig. 1.6.

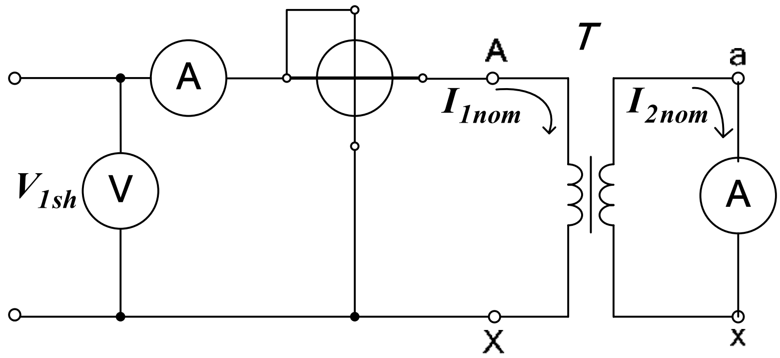


Fig. 1.6 – Transformer short-circuit test diagram

Note: in the diagram, lower case a, x and upper case A, X correspond to low tension and high tension winding, respectively.

In the circuit, for the sake of safety a **low-level voltage** V_{1sh} is rather able to induce a nominal secondary current I_{2nom} in the **short-circuited** winding. Commonly, for safe short-circuited transformer operation the primary voltage should not exceed 5 to10 percent the rated value. The V_{1sh} supply voltage is referred to as short-circuit voltage.

With a very low voltage applied, the magnetizing (core-loss) current becomes small to be taken into account, and the equivalent circuit may be reduced to that of Fig. 1.7.

Again, with the aid of particular measurements major transformer parameters can be determined at once. Namely:

the turn ratio – $k_{21} \approx \dfrac{I_{1nom}}{I_{2nom}}$;

the copper loss – $P_C \cong P_{sh}$ (P_{sh} is the wattmeter reading), since $P_2 = 0$ due to $V_2 = V_{ax} = 0$, and the core loss $P_m \approx 0$ because of small V_{1sh}.

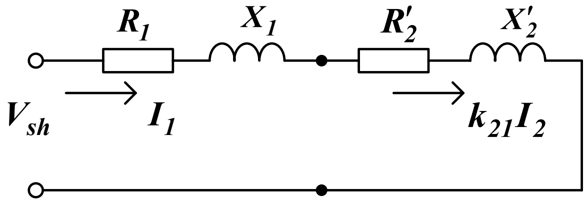


Fig. 1.7 – Equivalent circuit for a short-circuited transformer

In general, one should realize that copper loss is a variable item and depends on load; in the short-circuit test the copper loss is specified for transformer nominal (rated) running.

The rest quantities can be found as follows:

– short-circuit resistance $R_{sh} = \dfrac{P_{sh}}{I_{1nom}^2} = R_1 + R_2'$;

– short-circuit impedance $Z_{sh} = \dfrac{V_{sh}}{I_{1nom}}$;

– short-circuit reactance $X_{sh} = \sqrt{Z_{sh}^2 - R_{sh}^2} = X_1 + X_2'$.

– power factor $\cos\varphi_{sh} = \dfrac{P_{sh}}{I_{1nom} \cdot V_{sh}}$.

In the end one can determine $R_1 \approx R_2' = \dfrac{R_{sh}}{2}$ and $X_1 \approx X_2' = \dfrac{X_{sh}}{2}$ apart.

1.7 Transformer Efficiency

As mentioned, transformer efficiency can be determined with the help of transformer tests. Let us represent the efficiency in a form

$$\eta = \frac{P_2}{P_1} = \frac{P_1 - P_M - P_C}{P_1} = 1 - \frac{P_M + P_C}{P_1} = 1 - \frac{P_M + P_C}{P_2 + P_M + P_C}, \quad (1.13)$$

which utilizes the core P_M and copper P_C loss terms collected during open- and short-circuit tests. The term P_2 denotes (active) receiver power.

Now we investigate whether transformer efficiency depends on the amount of loading.

Active power of the receiver is a distinct function of the consuming current:

$$P_2 = V_2 I_2 \cos\varphi_2 = \frac{I_2}{I_{2nom}} V_2 I_{2nom} \cos\varphi_2 = \beta \cdot S_{nom} \cos\varphi_2 ,$$

where S_{nom} is the rated transformer (apparent) power. Besides, here one can find a specially introduced weighting term $\beta = \frac{I'_2}{I'_{2nom}}$ called the **load factor**.

In (1.13), copper loss component P_C depends upon the load factor as well. In accordance with Fig. 1.7, for a particular output current I_2

$$P_C = I_2'^2 \left(R_1 + R_2' \right) = I_2'^2 \cdot R_{sh} = \frac{I'^2_{2nom}}{I'^2_{2nom}} \cdot I_2'^2 \cdot R_{sh} = \beta^2 P_{sh} ,$$

where P_{sh} is copper loss nominal value collected during the short-circuit test.

In contrary, core loss P_M does not depend on transformer load; it can be estimated with the help of open-circuit test ($P_M = P_0$).

Hence, final substitution of all terms in formula (1.13) yields

$$\eta = 1 - \frac{\beta^2 P_{sh} + P_0}{\beta S_{nom} \cos\varphi_2 + P_0 + \beta^2 P_{sh}} = \frac{\beta S_{nom} \cos\varphi_2}{\beta S_{nom} \cos\varphi_2 + P_0 + \beta^2 P_{sh}} . \quad (1.14)$$

The graph for $\eta = f(\beta)$, as well as loss terms apart are shown in Fig. 1.8.

If the power factor $\cos\varphi_2 = const$, the efficiency depends only on the load factor β. There is a practical need to determine optimal value of the load, i.e. the one for which maximum efficiency η_{max} is attained.

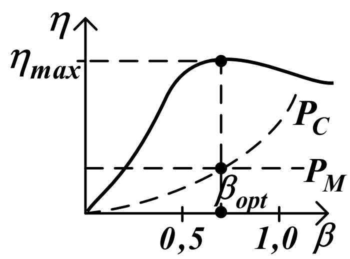


Fig. 1.8 – The relationship $\eta(\beta)$

After solving the equation $P_0 = \beta^2_{opt} P_{sh}$ that is derived from conventional condition $\frac{d\eta}{dt} = 0$ (resulting in $\eta = \eta_{max}$) we can find

$$\beta_{opt} = \sqrt{\frac{P_0}{P_{sh}}} . \quad (1.15)$$

Practical working of various transformers shows that light (small rated power) transformers are recommended for running at rated load because of small value of copper loss (the curve P_C in Fig. 1.9 lies below the curve P_M). In this case one can find that $P_0 \approx P_{sh}$, i.e. $\beta_{opt} = 1$.

In contrary, heavy, i.e. powerful, transformers work effectively in more wide range of loads: $\beta = 0,4...1,5$.

1.8 Particular Transformers

Autotransformer

The autotransformer has only one winding that serves as both primary and secondary (Fig. 1.10). When the primary terminals are connected to AC source the current flows between P_1 and P_2 points. The alternating flux links with all turns on the former inducing a voltage in each. The output is associated with secondary terminals S_1, S_2.

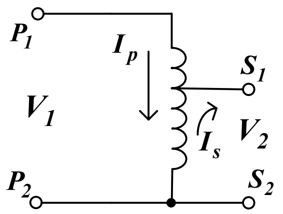


Fig. 1.9 – Autotransformer

The arrangement of Fig. 1.9 represents a step-down auto-transformer.

If it were a step-up type the input and output terminals would change place: P1-P2 – on the right, S1-S2 – on the left.

The effects of different loads on the autotransformer are similar to those for conventional power transformer. However, due to the fact that primary and secondary currents oppose each other in the common portion of the winding, smaller conductors (with small cross-sectional area) may be used in the common portion. This allows a weight saving, especially if the input and output voltages are almost the same.

Autotransformers are used:

– as line boosters to compensate voltage drops in long cable runs;

– for motor starting, i.e., when having a few tapping in sequence, to apply an increasing voltage to the motor;

– for impedance matching;

– to step the ***115 VAC*** aircraft supply down to ***26 VAC*** supply for lighting circuits.

Aviation transformers

Fundamental specification indices defining particular purpose of a transformer are the secondary voltage V_2 and rated power S_{nom} (P_2). At the same time, primary (means supply) voltage V_1 is commonly known because of given (standard) mains.

Aviation transformers obtain particular features, especially as regards the weight. Investigate the problem.

If transformer turns are given, the emf $E_2 = 4.44 f w_2 \Phi_m$ that is induced in the secondary winding will determine a load terminal voltage of the same value.

Magnitude of magnetic flux in the structure, when following the supply voltage $\boldsymbol{V_1}$, depends on both flux density (magnetic induction $\boldsymbol{B_m}$ in the core) and core configuration (cross sectional area $\boldsymbol{S_o}$, mainly) $-\boldsymbol{\Phi_m \cong B_m S_o}$. As a result, in the relationship that defines the output transformer voltage

$$\boldsymbol{V_2 \cong 4.44\, f B_m S_o W_2}$$

we can readily see three essential members (except the number of turns) which determine transformer index. One condition (i.e. $\boldsymbol{B_m}$) relates to specific magnetic properties of the core material (one can find recommended value $\boldsymbol{B_m}$ for any particular case in associated handbook). Other, namely AC frequency $\boldsymbol{f}$, is given due to known mains.

It is the third term (size of the core) that becomes the subject of careful transformer design. Here, the rated power will be a starting point which is designed so that transformer could supply a load with the rated current at nominal voltage. The amount of the load current affects a choice of the wire gauge for secondary, hence primary, coil: the more current is needed, the more conductor cross section should be in order to satisfy permissible density of the coil current.

The wire gauge results in coil size. In practice, the following relationship that aligns geometrical and electromagnetic parameters of a transformer is used:

$$\boldsymbol{S_o S_w \cong \frac{P_0}{2.22\, j B_m f K_{ic}}}, \tag{1.16}$$

where $\boldsymbol{S_w}$ denotes the area of core window in which a former with both coils must be placed; $\boldsymbol{K_{ic} < 1}$ – a factor of constructive and manufacturing imperfection; $\boldsymbol{j}$ – allowable current density.

In order to decrease size, hence transformer mass index for a given power, one may

- increase current density;
- increase flux density (magnetic induction in the core);
- increase operating frequency.

It is because of the latter reason that aviation AC generators adopt relatively high frequency ***400 Hz*** for power supply; the aim is saving the weight and space for various electromagnetic devices including numerous transformers.

The aircraft transformers are also allowed to have noticeably greater values associated with both current and flux density ($\boldsymbol{j = 15...20\ \ A/mm^2}$, $\boldsymbol{B_m = 1.5...1.7\ T}$) compared to the same items for conventional ***50…60 Hz*** industry transformers ($\boldsymbol{j = 5...7\ \ A/mm^2}$, $\boldsymbol{B_m = 0.5...0.6\ \ T}$). However, these stiff parameters adopted make aviation constructors to take special measures against overheating, e.g. forced cooling.

1.9 Terms and Concepts

Transformer is a circuit coupling device that can change electrical energy of a given voltage to energy of a different voltage level; **T**. consists of a few coils wound around an iron core.

Ideal transformer is a model of the transformer with a coupling coefficient that is equal to unity and with primary and secondary reactance very large compared to impedances of connected circuits.

Winding is a term for transformer coil.

Primary winding is that connected to a driving circuit (power supply).

Secondary winding is that driving a receiving circuit (load).

Turn ratio (transformation ratio) is the factor which determines whether the transformer is of a step-up or step-down type (originated from the number of turns of each winding).

Transformer emf (electromotive force) equation is a relationship related to amount of electricity induced in transformer winding ($E = 4{,}44 w\, f\, \Phi_m$).

Transformer losses are items related to certain loss (dissipation) of electric power entered the transformer.

Hysteresis and eddy current losses (iron losses) are those occurred due to transformer core heating.

Lamination is a means of the core design and manufacturing involved to prevent eddy currents within the core; **L**. results in transformer efficiency build-up.

Copper loss is that associated with winding wire (ohmic) resistance.

Flux leakage – as the name implies, the phenomenon results from a fact that not all of the primary flux links with all of the secondary coils. The reduction in flux linkage results in a reduced secondary voltage.

Efficiency of a transformer specifies the amount of energy delivered to receiver comparing to energy entered the transformer from the source. Being an index of perfect design and manufacture, **E.** defines loss of energy during its transferring from the source to the receiver.

Open-circuit is a condition that exists when the current between two terminals is identical with zero.

Short-circuit is a condition that exists when the voltage across two terminals is identical with zero.

Transformer tests are open-circuit and short-circuit experiments aimed to determine major transformer parameters without rated loading the transformer.

Magnetic equilibrium law is a statement applied to transformer ability to serve flux independent of any change in load current; **M.E.L.** assists in realizing physical principle of current flow transfer.

Equivalent circuit of transformer is an entirely electrical model obtained by direct connection of reduced secondary circuit to primary coil; reflection factor is in right (inverse) proportion with the turn ratio, depending on particular parameter.

Voltage regulation of a transformer is a quantitative index defining ability to keep output terminal voltage in full range of specified loads; unit for **V.R.** is percentage (of the rated voltage difference with respect to no-load secondary voltage).

1.10 Review Questions

1. Explain the purpose and operation principle of a transformer.
2. What types of magnetic circuits for single-phase transformers do you know?
3. Write the expression for effective e.m.f. of a transformer.
4. What is the transformation ratio?
5. What losses does a transformer have while working?
6. Sketch the equivalent circuit of an ideal transformer. The same for a practical transformer.
7. What is the purpose of open-circuit and short-circuit tests?
8. Explain how to get the equivalent circuit of a transformer on the base of open-circuit and short-circuit tests?
9. How can eddy current loss be minimized?
10. Name the factors the hysteresis loss depends on.

1.11 Problems

Solved problems

Problem 1. A ***250 kVA***, ***11000-V/415-V, 50 Hz*** single-phase transformer has ***80 turns*** in the secondary winding.

Calculate:

1) the approximate value of the primary and secondary currents;
2) the number of primary turns;
3) the maximum value of the flux.

Solution

1. Approximately for a transformer:

$$S_1 \approx S_2 \text{, or } V_1 I_1 \approx V_2 I_2 ;$$

then full-load primary current

$$I_1 = \frac{S_2}{V_1} = \frac{250 \cdot 1000}{11000} = 22{,}7\ A .$$

2. Full-load secondary current

$$I_2 = \frac{S_2}{V_2} = \frac{250 \cdot 1000}{415} = 602{,}4\ A.$$

3. The number of primary turns can be determined by the transformation ratio from $k_{21} = \frac{V_2}{V_1} = \frac{w_2}{w_1}$;

then $w_1 = \frac{w_2 V_1}{V_2} = \frac{80 \cdot 11000}{415} = 2121$ turns.

4. From the expression $E_2 \approx 4.44 w_2 f \Phi_m \approx V_2$

$$\Phi_m = \frac{V_2}{4.44 w_2 f} = \frac{415}{4.44 \cdot 80 \cdot 50} = 0.0234\ Wb.$$

Problem 2. A single phase transformer with primary to secondary ratio of $220\ V/1100\ V$ has a load impedance of $(6 + j8)$ ohms connected across the terminals of high voltage winding.
Neglecting losses in the transformer, find the current taken by the transformer from supply and power delivered to the load.

Solution

1. The load impedance $Z_2 = \sqrt{R^2 + X^2} = \sqrt{6^2 + 8^2} = 10$ Ohms.
2. Power factor of the load $\cos\varphi_2 = \frac{R}{Z_2} = \frac{6}{10} = 0.6$ (lagging).
3. Current in the secondary winding $I_2 = \frac{V_2}{Z_2} = \frac{1100}{10} = 110\ A$.
4. Therefore, power delivered to the load
   $P_2 = V_2 I_2 \cos\varphi_2 = 1100 \cdot 110 \cdot 0.6 = 72600\ W = 72.6\ kW$.
5. Transformation ratio $k_{21} = \frac{w_2}{w_1} = \frac{V_2}{V_1} = \frac{1100}{220} = 5$.
6. Since there are negligible losses, $I_1 w_1 = I_2 w_2$, and

$$I_1 = \frac{w_2}{w_1} I_2 = 5 \cdot 110 = 550\ A.$$

Problem 3. Obtain the equivalent circuit of a single phase transformer from test data that follow.

Open-circuit test:

$V_{10} = V_{1nom} = 380\ V$.

$V_{20} = 110\ V$; $I_{10} = 0.3\ A$.

$P_0 = 23\ W$.

Short-circuit test:

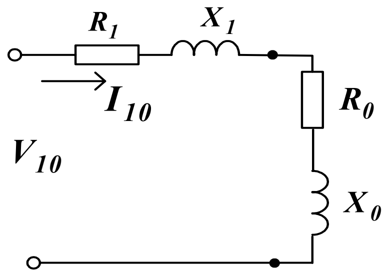
$V_{1sh} = 13\ V$.

$I_{1sh} = I_{1nom} = 5\ A$.

$P_{sh} = 55\ W$.

Solution

I. ***Open-circuit transformer test*** supposes the following equivalent circuit:



1. Impedance of the open-circuit transformer

$$Z_{oc} = \frac{V_{10}}{I_{10}} = \frac{380}{0.3} = 1267 \ \Omega .$$

2. Resistance

$$R_{oc} = R_1 + R_0 = \frac{P_0}{I_{10}^2} = \frac{23}{0.3^2} = 255.5 \ \Omega .$$

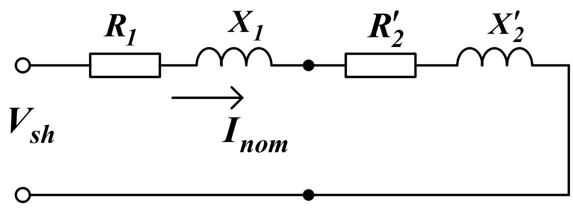
3. Reactance

$$X_{oc} = X_1 + X_0 = \sqrt{Z_{oc}^2 - R_{oc}^2} = \sqrt{1267^2 - 255.5^2} = 1240 \ \Omega .$$

II. ***Short-circuit test***. Here, the equivalent circuit likes that shown in the figure.

1. Impedance of the short-circuit transformer is

$$Z_{sh} = \frac{V_{sh}}{I_{nom}} = \frac{13}{5} = 2.6 \ \Omega .$$



2. Resistance

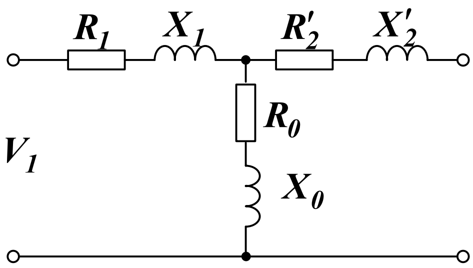
$$R_{sh} = \frac{P_{sh}}{(I_{nom})^2} = \frac{55}{25} = 2.2 \ \Omega .$$

3. Reactance

$$X_{sh} = \sqrt{Z_{sh}^2 - R_{sh}^2} = \sqrt{3^2 - 2.2^2} = 1.4 \ \Omega .$$

III. ***Transformer equivalent circuit for rated performance***

1. On the base of open-circuit and short-circuit calculations have just been made, we determine parameters of the equivalent circuit:



$$R_0 \approx R_{oc} = 255.5\ \Omega;$$
$$X_0 \approx X_{oc} = 1240\ \Omega;$$
$$R_1 \approx R_2' = \frac{R_{sh}}{2} = \frac{2.2}{2} = 1.1\ \Omega;$$
$$X_1 \approx X_2' = \frac{X_{sh}}{2} = \frac{1.4}{2} = 0.7\ \Omega.$$

2. Determine transformation ratio

$$k_{21} = \frac{V_{20}}{V_{10}} = \frac{110}{380} = 0.29.$$

3. Now actual parameters of secondary winding can be found:

$$R_2 = \frac{R_{sh}}{2} k_{21}^2 = 1.1 \times 0.29 = 0.32\ \Omega;\quad X_2 = \frac{X_{sh}}{2} k_{21}^2 = 0.7 \times 0.29 = 0.2\ \Omega.$$

Problem 4. For a single-phase transformer, the given quantities are: $w_1 = 424$, $w_2 = 244$; cross-sectional area of the core $S_{core} = 28.8\ cm^2$; $R_1 = 1.2\ \Omega$; $R_2 = 1.4\ \Omega$; core loss P_M is 1% of P_{1nom}; $V_1 = 220\ V$; $I_{1nom} = 2.95\ A$; $I_{2nom} = 4.85\ A$; open-circuit current I_{10} is 5% of I_{1nom}.

Find B_m, E_2, P_0, P_c, η_{nom}.

Solution

1. From the relationship $V_1 \approx E_1 = 4.44 f w_1 \Phi_m$ the magnetic flux can be calculated:

$$\Phi_m = \frac{V_1}{4.44 f w_1} = \frac{220}{4.44 \cdot 50 \cdot 424} = 0.0023\ Wb.$$

2. Amplitude value of flux density (magnetic induction)

$$B_m = \frac{\Phi_m}{S_{core}} = \frac{0.0023}{26 \cdot 10^{-4}} = 0.88\ T.$$

3. Turn ratio $k_{21} = \dfrac{w_2}{w_1} = \dfrac{244}{424} = 0.58$.

4. Induced secondary emf $E_2 = E_1 k_{21} \simeq V_1 k_{21} = 220 \cdot 0.58 = 127\ V$.

5. Excitation current $I_{10} = 0.05 I_{1nom} = 0.05 \cdot 2.95 = 0.147\,A$.

6. Electrical loss

$$P_C = P_{C1} + P_{C2} = I_1^2 R_1 + I_2^2 R_2 = 2.95^2 \cdot 1.2 + 4.85^2 \cdot 1.4 = 43.3\ W\ .$$

7. Magnetic loss $P_M = P_0 = 0.01 P_1 = 0.01 \cdot 220 \cdot 2.95 = 6.5\ W$.

8. Total losses $\Delta P = P_C + P_M = 43.3 + 6.5 = 49.8\ W$.

9. Input power $P_1 = V_1 I_{1nom} = 220 \cdot 2.95 = 649\ W$.

10. Transformer efficiency for nominal load

$$\eta = \frac{P_1 - \Delta P}{P_1} = \frac{220 \cdot 2.95 - 49.8}{220 \cdot 2.95} = 0.92\ .$$

Supplement problems

Problem 5. A ***25 kVA*** single-phase transformer has ***250 turns*** on the primary and ***40 turns*** on the secondary winding. The primary is connected to ***1500 V***, ***50 Hz*** mains. Calculate:

1. primary and secondary currents on full load;
2. secondary emf;
3. amplitude value of the flux in the core.

Problem 6. Find the number of turns in each winding of a single phase, ***50 Hz*** transformer with a maximum value of flux of about ***0.05 Wb*** and no-load voltage ratio of ***6600/250***.

Problem 7**.** A single-phase transformer has $w_1 = 400$ and $w_2 = 1000$ turns. The net cross-sectional area of the core $S = 60\ cm^2$.

If the primary winding is connected to a ***50 Hz*** supply at $V_1 = 500\ V$, then calculate:

1. peak value of the flux density in the core;
2. voltage induced in the secondary winding.

(Ans.: ***0.94 Wb/m²***; ***1250 V***)

Problem 8. A single-phase transformer has $w_2 = 3000$, $w_1 = 590$, $S_{nom} = 5\ kVA$, $V_{nom} = 130\ V$.

Calculate nominal value of primary and secondary currents and V_{2nom}.

Problem 9. Obtain the equivalent circuit of a single phase transformer from the following test data:

1. open-circuit test: $V_{10} = V_{1nom} = 200\ V$, $I_{10} = 0.7\ A$, $P_0 = 70\ W$, $V_{20} = 380\ V$;
2. short-circuit test: $V_{sh} = 15\ V$, $I_{1nom} = 10\ A$, $P_{sh} = 85\ W$.

Find parameters of transformer equivalent circuit.

Problem 10. A single-phase transformer supplies in the secondary $P_{2nom} = 500\ W$; $P_0 = 10\ W$, $P_{sh} = 40\ W$ for open-circuit and short-circuit tests.

Determine efficiency of the transformer for nominal output power.

Problem 11. A single-phase transformer has $V_{1nom} = 3300\ V$. The investigation of open-circuit and short-circuit tests gave:

a) $V_{20} = 220\ V$, $I_{10} = 0.18\ A$, $P_0 = 70\ W$;

b) $V_{1sh} = 188\ V$, $I_{1nom} = 3.127\ A$, $I_{2nom} = 45.45\ A$, $P_{sh} = 250\ W$.

Calculate P_{2nom} for $\cos\varphi_2 = 1$ and find R_1, R_2, X_1, X_2 for $P_{l1} = P_{l2}$.

Determine transformer efficiency for nominal load; determine two leakage inductances, L_1 and L_2, if mains frequency $f = 50 Hz$.

Tutorial test

1. ***A transformer transforms …***

a) frequency
b) voltage
c) current
d) voltage and current

2. ***Which of the following is not a basic element of a transformer?***

a) core
b) primary winding
c) secondary winding
d) all of the above

3. ***In an ideal transformer***

a) windings have no resistance
b) core has no losses
c) core has infinite permeability
d) all of the above

4. ***The main purpose of using core in a transformer is to***

a) decrease iron losses
b) prevent eddy current loss
c) eliminate magnetic hysteresis
d) decrease reluctance of the common magnetic circuit

5. ***Transformer cores are laminated in order to***

a) simplify its construction
b) minimize eddy current loss
c) reduce cost
d) reduce hysteresis loss

6. ***Transformer having 1000 primary turns is connected to a 250 VAC supply. For a secondary voltage 400 V, the number of secondary turns should be***

a) 1600
b) 250
c) 400
d) 1250

7. ***A step-up transformer increases***

a) voltage
b) current
c) power
d) frequency

8. ***The primary and secondary windings of an ordinary 2-winding transformer always have***

a) different number of turns
b) the same size of copper wire
c) a common magnetic circuit
d) separate magnetic circuits

9. ***In a transformer, the leakage flux of each winding is proportional to the current in that winding because***

a) Ohm's law can be applied to the magnetic circuit
b) the two windings are electrically isolated
c) leakage paths do not saturate
d) mutual flux is confined to the core

10. ***In performing the short-circuit test of a transformer,***

a) high voltage side is usually short circuited
b) low voltage side is usually short circuited
c) any side is short circuited with preference
d) none of the above

11. ***No-load test of a transformer is carried out to determine***

a) copper loss
b) magnetizing current
c) magnetizing current and no-load loss
d) efficiency of the transformer

12. ***During short-circuit test, the iron loss of a transformer is negligible because***

a) the entire input is just efficient to meet copper loss only

b) flux produced is a small fraction of the normal flux

c) iron core becomes fully saturated

d) supply frequency is held constant

13. ***At relatively light loads, transformer efficiency is low because***

a) secondary output is low

b) transformer losses are high

c) fixed loss is high in proportion to the output

d) copper loss is small

14. ***The iron loss of a transformer at 400 Hz is 10 W. Assuming that eddy current and hysteresis losses vary as the square of flux density, the iron loss of the transformer at rated voltage but at 50 Hz would be … watt***

a) 80

b) 640

c) 1.25

d) 100

15. ***In operating a 400 Hz transformer at 50 Hz***

a) only voltage is reduced in the same proportion as the frequency

b) only kVA rating is reduced in the same proportion as the frequency

c) both voltage and kVA rating are reduced in the same proportion as the frequency

d) none of the above.

16. ***The primary and secondary induced emf E1 and E2 in a two-winding transformer are always***

a) equal in magnitude

b) counter-phase with each other

c) in phase with each other

d) determined by load

17. ***In a two-winding transformer, the emf per turn in secondary winding is always … … the induced emf per turn in primary***

a) equal to k times

b) equal to 1/k times

c) equal to

d) greater than

Chapter 2

DC MACHINES

An electrical generator is a machine which converts mechanical energy into electrical energy while, inversely, a motor converts electrical energy of power supply to mechanical energy of rotation.

The energy conversion is based on the principle of dynamically induced (due to motion) electromotive force. As is known, whenever a conductor cuts magnetic flux an induced emf is produced in it in accordance with Faraday's Law (this is referred to as generator principle). This emf causes a current to flow if the conductor circuit is closed.

Likewise the operation of the motor is based on the principle according to which a current carrying conductor being placed in a magnetic field experiences mechanical force.

Hence, two basic and essential parts of any electric machine are, first, a source of magnetic field, and second, a conductor that may move so as to cut the flux. Basically, there is no difference in construction between a generator and a motor; that is the reason why, for example, a term **DC machine** may include the both.

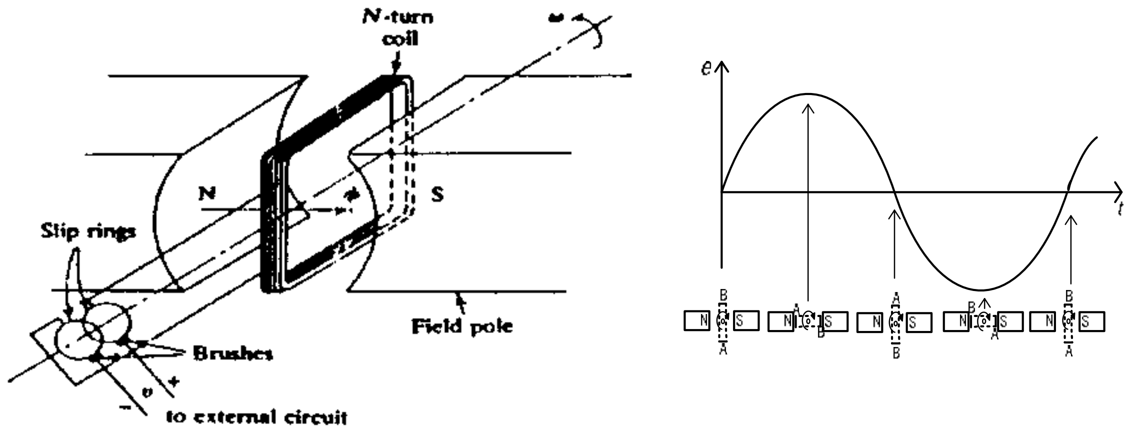
2.1 A Single-loop Generator

In the structure shown in Fig. 2.1, a single-loop coil which is formed of N turns of copper wire rotates with angular speed ω in a magnetic field that is provided by a permanent magnet (alternatively, an electromagnet can be used for that). Each end of the coil is equipped by a **slip ring**; both rings do not have electric contact with the shaft holding the coil. Two contacting brushes (made of carbon or copper) press against the slip rings. Their function is to collect the current induced in the coil and convey it to external circuit (a load).

The rotating coil of the structure is mostly called **armature**, and the magnet system is termed field magnet. Referring to this, a distinct difference can be made between particular windings (coils) of the machine; the distinction follows from the purpose of the current they carry.

If the coil current serves the sole purpose of providing the field and is independent on the load, it is called a magnetizing, or exciting current, and corresponding winding is termed a **field winding**. Field currents are of relatively low power, since their only purpose is to magnetize the core that is made of high permeability, therefore sensible to magnetization material.

On the other hand, if a coil is intended to carry the load current only it is called the armature. In DC machines, separate windings are used to carry the field and armature currents.



a b

Fig. 2.1 – The single-loop generator: structure (a); emf induced (b)

As the winding rotates (i.e. moves) an emf is induced across each side of the wire loop (top and bottom conductors in Fig. 2.1,a). According to ***right hand rule*** each half a revolution induced current flows in opposite direction through the particular side of the loop thus forming resultant closed-loop current of appropriate direction (actual current path depends on the nearest magnet pole - North or South, and a direction of conductor rotation). It is evident that during the following half-turn induced emf of the same conductor changes polarity and loop current flows in opposite direction. Following the induced emf associated brush voltage is depicted in Fig. 2.1,b. As is seen, both emf and corresponding derivatives (terminal voltage and load current) obtain sinusoidal (alternating) waveform.

In general, the induced emf of an l-length conductor that moves at velocity v in uniform magnetic field of B density is defined by

$$e = Blv \sin\vartheta \ , \tag{2.1}$$

where ϑ is the angle between the velocity vector and magnetic flux lines. It is the change of ϑ angle during the loop rotation that forms sinusoidal curve in Fig 2.1,b.

Together with that, collected emf of a number of conductors is determined by the change of total flux linkage, i.e. it depends upon the number of coil turns besides depending on time varying angle $\vartheta = \omega t$. Therefore, for an N-turn coil (it has a couple of conductors in each loop) that rotates with angular velocity ω in a magnetic field one can readily obtain

$$e_g = 2BNlr\omega \sin\omega t \tag{2.2}$$

instead of (2.1) by replacing the linear velocity v by ωr term (r is the radius of the coil). This relationship defines the output of an electric machine which acts as a **generator**.

In turn, a **motor** action is based on Ampere's law. It is commonly formulated as the ***Bli-rule***

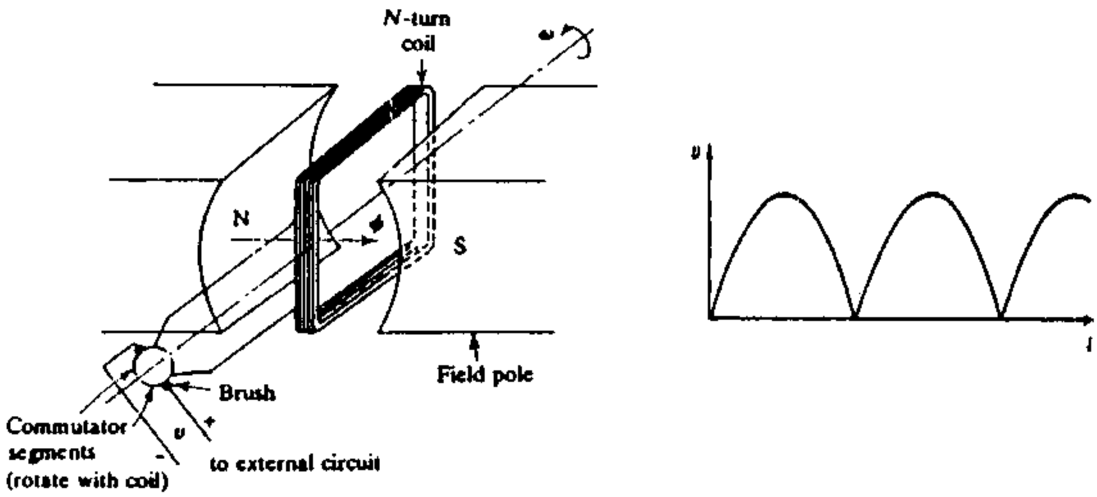
$$\boldsymbol{F = Bli}, \tag{2.3}$$

aimed to determine a force acting on ***i***-carrying ***l***-length conductor which is placed in magnetic field ***B***; the direction of the force may be obtained from the ***left-hand*** rule.

A commutator (split-ring/brush mechanism)

If one deals with a direct current machine (e.g. a DC generator) where the current must flow through the load in the same direction nevertheless alternating polarity of the emf has been induced in the coil, the connection to the external circuit should be switched twice per a revolution to keep a single direction of the current flow.

This can be achieved by splitting the slip rings, i.e. by using a commutator. Here, the slip-ring/brush mechanism of Fig. 2.1,a is modified like that shown in Fig. 2.2,a: instead of two slip rings we can find a single ring splinted into two insulated halves; each is connected to different end of the coil. The halves are known as commutator segments.



a b

Fig. 2.2 – Action of a DC generator

So, the two functions of the commutator are as follows:

1) collecting the current from rotating winding loop and transmitting that current to the stationary external circuit;

2) switching the external circuit periodically so that a single direction of the current flow in it be possible. To avoid communication problems, such a switching is assumed to occur at moments when moving conductors go through a position where they are in parallel with the field lines ($\vartheta = 0$) and therefore have no e.m.f. induced in them (conductor moves in vicinity of its neutral position).

The simplest configuration of a DC generator shown in Fig. 2.2,a is referred to as a single-winding (single loop) generator with the two-segment commutator; the generator produces output voltage depicted in Fig. 2.2,b. This output at the brushes has a nonzero time-average value (DC output).

Although the current flows in one direction through external circuit, it is still of little practical use because the voltage and current fall to zero twice every cycle obtaining much ripples (see Fig. 2.3,a). When using a few loops and multi-segment commutator, lower ripples in the output can be achieved.

For instance, for two loops and four segments the output waveform will look like the sketch in Fig. 2.3,b. Here, the emf no longer falls to zero and only has a small ripple in it. For eight coils and sixteen segments the emf will become practically constant. We should remember, however, that emf induced in generator coils must be alternating.

a b

Fig. 2.3 – The waveform of DC generators:
with 2-segment commutator (a);
for multi-loop armature and multi-segment commutator (b)

It can be verified, by applying the left-hand rule, that in the case of machine action as a motor (when connected to the commutator coil is supplied by external DC source itself) the resulting torque will be unidirectional also.

2.2 Construction of a Practical DC Machine

The physical DC generator does essentially consist of the following parts (Fig. 2.4):

– a **stator** which is a stationary part of electric machine (when the stator has a field system it is named the **inductor**), and

– a **rotor** which is a rotating part of the machine; also it is named the **armature** owing to associated winding in which emf is induced.

Stator parts are as follows: 1) laminated magnetic frame, or yoke; 2) pole cores and pole shoes; 3) pole windings called the field coil. When magnetized by the field current both poles create necessary machine flux.

The yoke serves two purposes: the first is mechanical support of the poles, and the second is carrying the flux produced by the poles.

Some machines carry a few sets of field windings on the same pole core. To facilitate their assembly, the cores of the poles are built of sheet-steel laminations. As the field winding carries direct current, it is not electrically necessary to have the cores laminated. However, it is necessary for the pole faces to be laminated, because of their proximity to the armature windings.

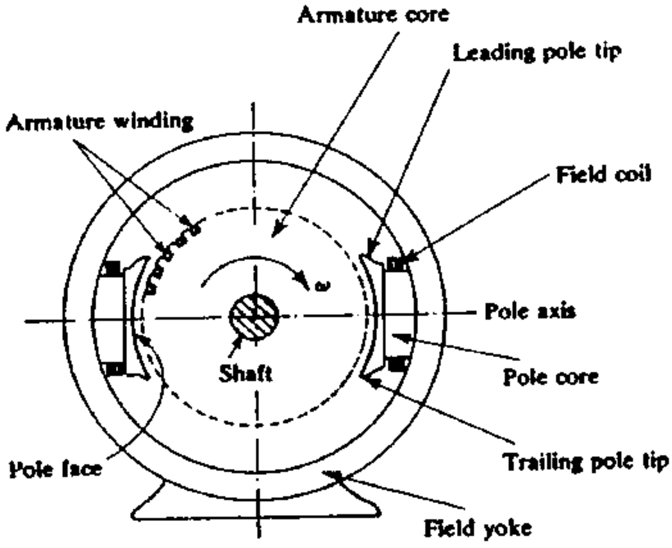
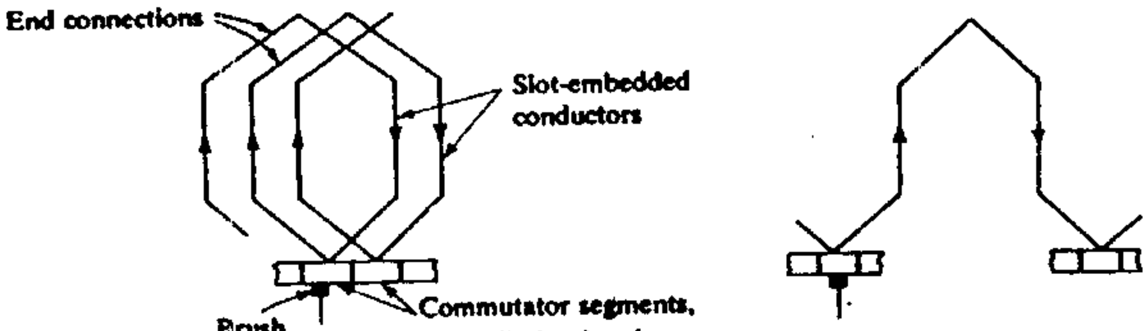


Fig. 2.4 – The parts of a DC machine

The armature consists of: 1) cylindrical laminated armature core; 2) armature winding, which conductors are embedded in armature slots; 3) commutator which is made of hard-drawn copper segments separated one from the other by mica isolator; 4) carbon brushes; 5) mechanical shaft and bearings (the latter can be either ball or roller-type).



a b

Fig. 2.5 – The armature winding configuration:
lap winding (a); wave winding (b)

As it is shown schematically in Fig 2.5, the armature windings are connected to the commutator segments, over which the carbon brushes slide and serve as leads for electrical connection.

The armature winding is the load-carrying winding. It can be either lap or wave-type. In any case, various coils that form the armature winding may be connected in a series-parallel combination. It can be shown then that in the lap-type winding the number of paths in parallel coincides with the number of machine poles ***p***, whereas in case of the wave winding the number of parallel paths equals ***2*** always.

Armature reaction

Assume the flux across machine air gap is uniform.

In the structure, there is a line where rotating conductor does not cut the flux lines (slides along); at this line no emf is induced. To have perfect communication with external wiring the "null" line is the best place where aligning brushes are to be installed since there is no much interference in this case. The line is termed a ***Magnetic Neutral Axis***.

When the generator is connected to a load the current flowing in the armature winding (the load current) creates a magnetic field. This field is superimposed upon the main field that was produced by the current in the field winding. Since lines of force cannot intersect the armature, field causes a distortion of the main field; amount of distortion will vary with the load.

The distorting effect on the main flux by the armature flux is called **armature reaction**. The reaction will develop less affect if the brush installation line is removed in the direction of armature rotation. The angle at which the brushes must be moved from the geometrical neutral axis for better communication and for increasing emf and torque is called the "angle of load".

Another way to minimize the effect of "armature reaction" is to place small auxiliary poles, called inter-poles, between the main field poles. The inter-poles have a few turns of large-gauge (thick) wire connected in series opposite with the armature winding. An extra field of inter poles cancels the armature reaction and improves communications for all loads.

2.3 Machine Equations

All rotating machines being an electromechanical device use the magnetic field for coupling between electrical and mechanical machine components. Remind two aspects of this coupling which play a significant role in operation of an electric machine:

1. Magnetic ***attraction and repulsion forces*** generate mechanical torque due to interaction between energized stator and rotor.

2. The magnetic field ***induces*** a voltage in the machine windings by virtue of Faraday's law.

As regards the latter aspect, once again, the e.m.f. induced in a conductor of l length is proportional to the rate v of conductor movement in the field (see (2.1)):

$$e_{cond} = Blv. \tag{2.4}$$

If a moving conductor system, e.g. a rotor system, integrates N conductors divided into $2a$ parallel branches which, in turn, are connected in series due to implemented commutator, emf of the entire armature system (effective value) will be

$$E_a = E_{cond}\, N/2a\,. \tag{2.5}$$

When substituting (2.4) to (2.5), the latter relationship can be presented in a specific form which involves machine field flux Φ instead of B, and armature rotational speed n (unit for n is rpm – revolution per minute):

$$E_a = \frac{\pi N}{60a} n\Phi = C'_e n\Phi\,. \tag{2.6}$$

The term C_e called armature constant is a coefficient representing machine specific constructive features (size, number of coils, number of pairs of poles, etc). Equation (2.6) defines the effective value of emf induced. It is called the **emf equation,** or **generator equation**. Another form of generator equation utilizes conventional variable ω for angular speed in radians per second:

$$E_a = C_e \omega \Phi\,. \tag{2.6a}$$

If generator magnetic circuit is assumed to be linear, that is if there is no yoke saturation, then the field established in the machine will be in direct proportion to excitation current I_f:

$$\Phi = k_f I_f\,. \tag{2.7}$$

Then (2.6) yields

$$E_a = C_e^* n I_f\,. \tag{2.8}$$

For a nonlinear magnetic circuit (a practical generator), the graph E versus I_f is a nonlinear curve for a given speed, as shown in Fig. 2.6.

A diagram in Fig. 2.6 for a particular armature speed is referred to as no-load saturation characteristic of the generator.

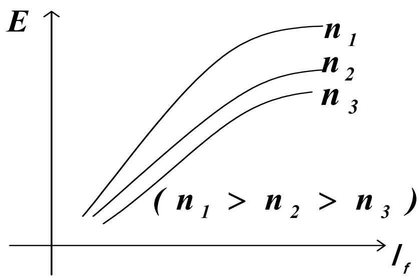


Fig. 2.6 – Magnetic characteristics of a DC generator

Now let us turn to another peculiar characteristic of the electromechanical coupling; we mean the torque generation and performance of the electric machine as a motor. In all electromagnetic devices, the force on a wire with the current I_{cond} in it (e.g., in armature conductor of the machine shown in Fig. 2.7 below) is given by Ampere's law equation:

$$F_{cond} = BlI_{cond}\,. \tag{2.9}$$

If groups of conductors are embedded in a cylindrical structure, for instance in a rotor of ***D/2*** radius, and if the structure is free to rotate, then due to generated torque which is defined as $\boldsymbol{T_a = NF_{con} D/2}$ this rotor will rotate at an angular velocity $\boldsymbol{\omega = \pi n/30}$.

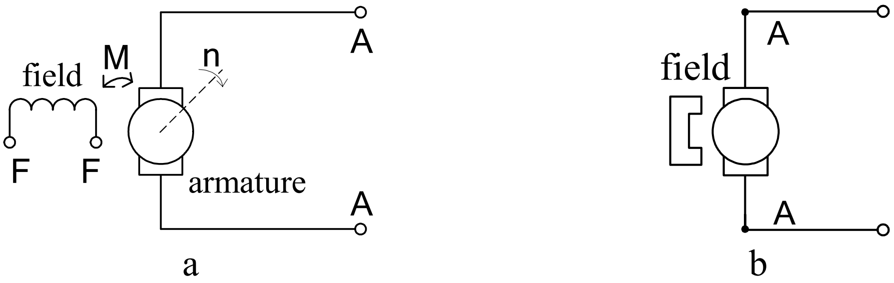


Fig. 2.7 – Separately excited (a) and permanent magnet (b) DC machines

On the other hand, mechanical power developed by the armature will be $\boldsymbol{T_a \omega}$ while the electric power entered the armature has been $\boldsymbol{EI_a}$ ($\boldsymbol{E}$ is the armature supply). Ignoring any loss in the armature during energy conversion we may write equation $\boldsymbol{T_a \omega = I_a E_a}$, where the counter emf of the armature $\boldsymbol{E_a}$ is used. Now we can readily determine the torque that has been generated,

$$\boldsymbol{T_a = C_m I_a \Phi}, \tag{2.10}$$

if we use (2.6).

This relationship collected a form similar to the former notations (2.6), (2.7) related to generator electromotive force; the relationship is known as **torque equation** of an electric machine, or **motor equation**. Here again we can revise the importance of both electromagnetic terms $\boldsymbol{I_a}$, $\boldsymbol{\Phi}$ which associate with machine energizing and its particular construction features (electromechanical constant $\boldsymbol{C_m}$), for developing required mechanical torque.

An electrical circuit model for the armature of the practical DC machine that is useful for engineering calculation can be depicted as shown in Fig. 2.8.

As mentioned above, the notation ***E*** in the diagram implies either induced or back (counter) emf in the armature depending on type of the machine, generator or motor; ***Ra*** represents armature winding resistance.

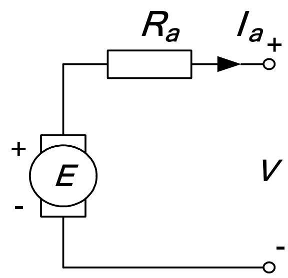


Fig. 2.8 – Armature electric model

With the help of the model, one more relationship related to machine that runs as a motor can be derived.

Under steady state running we have $\boldsymbol{V} - \boldsymbol{E_b} = \boldsymbol{I_a R_a}$, where $\boldsymbol{V}$ is the voltage applied to armature terminals. If expression (2.8) is used for determining the counter emf $\boldsymbol{E}$, then, for a practical motor, we may write a relationship

$$\boldsymbol{n} = \frac{\boldsymbol{V - I_a R_a}}{\boldsymbol{C_e^* I_f}} \tag{2.11}$$

in addition to ideal motor equation (2.10). Again, in (2.11) excitation motor field is assumed to be linear. The relationship (2.11) is known as motor **speed equation**.

Note: when a DC machine is running as a **generator** (the armature is driven by a prime mover, and the induced armature current drives an electrical load), the armature mechanical **reaction torque** appears too. This torque opposes the driving mover effort and therefore is to be overcome by appropriate mover excitation: the more the generator armature current (load current), the more the braking torque is, hence more applied power of the prime mover should be.

2.4 DC Generator Types and Specific Characteristics

Generator configurations

Generators, as well as motors, are usually classified by means of machine field generation. As regards this, generators can be divided into separately-excited, self-excited and permanent magnet generators.

Separately-excited generators have their field supplied from an external source.

By varying the field current (see Fig. 2.9,a), one controls the emf induced (Fig. 2.9,b) hence terminal voltage.

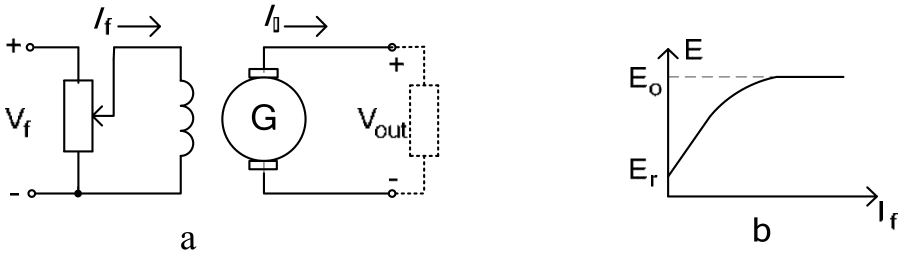


Fig. 2.9 – Separately-excited generator with field control: the diagram (a), the regulating characteristic (b)

Self-excited generators. The generator supplies its own field current from the generator output. The residual magnetism of the yoke and pole pieces is usually sufficient to generate a small voltage at the output terminals; at starting, necessary field builds up rapidly. Again, field current variation may control the output voltage.

There are three types of self-excited generators named in accordance with the manner in which their field winding is connected with the armature (Fig. 2.10).

Shunt-wound generators. The field windings are connected in parallel with the armature conductors and have the full voltage of the generator applied across them. The field winding consists of many turns of fine-gauge, i.e. thin wire. Such generators are in much common use.

Series-wound generators. In this case, the field windings are joined in series with the armature conductors. As they carry full-load current, they consist of relatively few turns of thick wire or strips. Such generators are rarely used except for special purposes; as the booster, for instance.

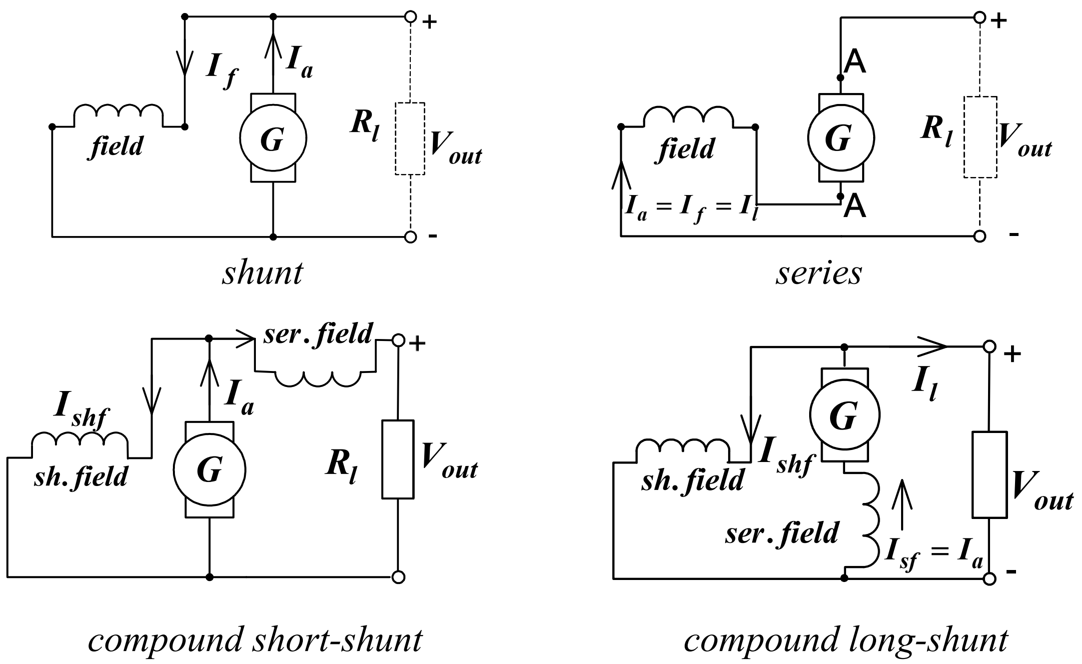


Fig. 2.10 –Self-excited generator configurations

Compound-wound generators. These generators obtain a combination of a few series and shunt windings and can be either short-shunt or long-shunt.

Permanent magnet generators have peculiar features referred to as disadvantages: limited rated power and quite instable output voltage as this voltage depends much on the armature speed.

Particular symbol for the permanent magnet generator was depicted earlier in Fig. 2.7,b.

Aircraft generators are preferably of the self-excited shunt type.

General characteristics

Following are the three most asked-for characteristics of a DC generator.

No-load saturation characteristic. It makes appear a relation between the emf $\boldsymbol{E}$ generated in the armature and the value of the field current $\boldsymbol{I_f}$ **(**generator excitation**)** at a given and fixed armature speed $\boldsymbol{n}$, i.e.

$$\boldsymbol{E = F\left(I_f\right), n = const}. \tag{2.12}$$

The no-load characteristic is also known as generator m**agnetic** characteristic, or **open-circuit** characteristic (OCC). As is seen from the above Fig. 2.6, particular OCC is just a magnetization curve of a material the stator is made of. The curve is practically the same for all generators whether separately- or self-excited.

External characteristic. It is also referred to as **performance** characteristic or sometimes as **voltage regulation curve**:

$$\boldsymbol{V_l = F\left(I_l\right)},\ \boldsymbol{n = const}. \tag{2.13}$$

This characteristic displays how generator terminal voltage is affected by machine loading; it is of great practical importance due to explaining how the generator suits for particular purpose.

Voltage regulation of a generator (when it runs at a constant speed) means a change in generator terminal voltage resulted from a change of the load current. If the voltage has a small change between no-load and full load running, a generator is said to have good regulation.

For example, let the open-circuit and rated-load voltages be 240 V and 220 V, correspondingly; then generator regulation will be

$$\Delta \boldsymbol{U} = \frac{\boldsymbol{240 - 220}}{\boldsymbol{220}} 100\% = 9.1\%.$$

This generator has quite a good voltage regulation.

Field control characteristic:

$$\boldsymbol{I_f = F\left(I_a\right)},\ \boldsymbol{V_l = const, n = const}. \tag{2.14}$$

When the load changes, this characteristic is involved to show how one should vary the excitation current for keeping terminal voltage at the rated level.

2.5 Properties of Particular DC Generators

In this section, most widely used DC generators such as separately-excited generator and self-excited shunt generator will be investigated by analyzing peculiarities of their characteristics.

2.5.1 Separately-excited Generator

First consider distinctive features of separately-excited generators. Typical characteristics of those are presented in Fig. 2.9 and the following Fig. 2.11.

No-load saturation characteristic

The no-load saturation characteristic is the open circuit characteristic. The open circuit voltage of any electrical circuit equals the e.m.f. of equivalent source in the circuit. If one deals with a DC generator (see Fig. 2.9,a) the emf $\boldsymbol{E}$ induced in the armature may be derived from generator equation $\boldsymbol{E = c_e^* n i_f}$. Because of condition $\boldsymbol{n=const}$, resulting $\boldsymbol{E}$ will depend on magnetic flux, whence the field current only. It is clear that a saturation of the curve may occur due to magnetic saturation of the material. Analyzing the graph in Fig. 2.9,b, one can also see a peculiar response in the origin – non-zero emf $\boldsymbol{E_r}$ when no excitation has been established ($\boldsymbol{I_f = 0}$). This is possible due to residual magnetism of the yoke (poles) and running rotor. The residual terminal voltage is commonly 2 to 4 percent of rated value $\boldsymbol{E_0}$.

External characteristic

For separately-excited generator, the exciting current is independent on the armature (load) current, and the induced voltage $\boldsymbol{E_0}$ must be indifferent to any change in the load current (curve 1 in Fig. 2.11,a).

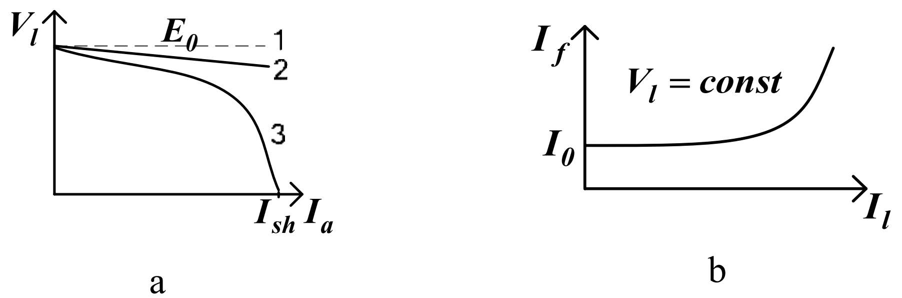


Fig. 2.11 – The separately excited generator:
external characteristics (a); the field curve (b)

Actually, one will recognize a small reduction in output voltage if armature current increases (curve 2 in the figure). This occurs due to armature reaction.

Practical response, however, looks like curve 3 in the figure. This characteristic is defined by equation $\boldsymbol{V_{out} = V_l = E_0 - I_a R_a}$ and discovers much more reduction in the voltage because of excess voltage drop across armature resistance (copper loss is originated from this resistance). Besides, in the curve one can find a particular value of the armature current – a short circuit current which can be attained if output terminals are short-circuited; such experiment, however, should not be applied to machine continuous running because of overloading problems for parts.

Field control characteristic

For separately-excited generator, an increased load requires more strong field in order to compensate excessive voltage drop caused by the increase in armature current, as shown in Fig. 2.11,b.

By changing the field current, good generator regulation can be achieved.

2.5.2 DC Shunt Generator

Open-circuit and field control characteristics of this type of self-excited generator will be similar to those for separately-excited generators although **external characteristic (**Fig. 2.12) will differ.

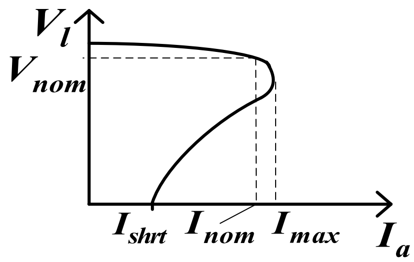


Fig. 2.12 – The external characteristic of a shunt-wound generator

As is known, for any electricity generating device including a DC generator the terminal voltage will fall if the generator is loaded; this is because of internal loss and accompanying processes. In the case of a shunt generator the drop can be explained by following interrelated phenomena:

– with load current increased, the voltage drop across armature winding and brush contact resistance will increase;

– besides, the increase in armature current builds up armature reaction which is followed by a reduction of the field then. Reduction in flux decreases induced emf and hence the terminal voltage;

– the decrease in terminal voltage causes further decrease in field current.

As a result rather sharp decrease of emf and terminal voltage occurs when the load current grows up.

Finally, if the generator terminals are short-circuited (load resistance drops to zero) the terminal voltage is forced to be zero. However, a small current is established in the external circuit since a small voltage is induced in the armature due to residual magnetism.

2.5.3 Particular Application of Various Generators

Separately-excited generators

Since these generators require a separate source of field current they are more expensive than self-excited generators. They are used then and there where self-excited generators are not suited.

A typical example is the ***tachogenerator*** which is designed for measuring angular velocity, therefore is widely used in servomechanisms for speed regulation.

Besides, external excitation provides for better generator stability at low voltage.

Shunt generators with field regulators are used for ordinary lighting and power supply purposes. They are also used for charging batteries since their terminal voltages are almost constant or can be kept constant. As mentioned above, DC shunt generators have found wide application in aircraft engineering.

Series generators are not used for power supplying because of their poor build-up characteristics. However, their particular rising makes them suitable boosters in certain types of distribution system, e.g. in railway service.

Compound generators have series- and shunt-field windings at the same time and conventionally fall into two categories:

- *differential compound generators* (DCG), in which the two field windings (series and shunt) are wound so as to oppose each other;
- *cumulative compound generators* (CCG) in which the series and shunt fields assist each other.

DCG are generally used in applications where a high starting but low running voltages are required. Devices like arc welders or arc lighting may be supplied by this kind of generator.

CCG is the most widely used DC generator since its external characteristic is readily adjusted to obtain voltage regulation. This provides for easy compensation of the voltage drop occurred in the load line due to non-zero conductor resistance. Among various applications of such generators are the motor drives which require DC supply at a constant voltage, other heavy power services.

2.6 DC Motors

Operation

The function and operating principle of DC motors is the reverse of generators, i.e. if an external supply is connected to the terminals it will produce motion of the armature thereby converting electrical energy into mechanical energy.

In details, operation of a DC motor can be explained with the help of Fig 2.2 again.

Assume a coil (i.e. a single loop) rotates between magnet poles. Conductor ends are connected to commutator segments which in turn are connected to a DC power source with the aid of brushes. The commutator in a DC motor is necessary to maintain current flowing in the same direction through the armature while it turns and each half a revolution comes under the action of the opposite magnet pole. Owing to the commutator, at the moment when a half-turn has completed, the direction of current in the same loop side is changed in order to serve loop ***attraction*** with a magnet pole of opposite polarity. Because of that the direction of coil motion (rotation) will remain the same.

Basic equations

When the motor armature rotates the conductors also rotate and hence cut the flux. In accordance with the law of electromagnetic induction, emf is induced in them; its direction being found by the right hand rule (Lenz's law) is in opposition to the applied voltage. Because of opposing direction, it is named **counter emf**, or **back emf**. Back emf value is proportional to established magnetic field and armature speed, i.e.

$$E_b = C_e \omega \Phi , \tag{2.15}$$

like that has been written in generator equation (2.6). Counter emf is always less than applied source voltage.

It is the difference between the applied voltage V_s and back emf that drives the current through the resistance of the armature. This difference is referred to as ***effective*** voltage $I_a R_a = V_s - E_b$, which allows to realize a relationship

$$V_s = E_b + I_a R_a , \tag{2.16}$$

that is known as **voltage equation** of a motor.

For generality, let notation (2.10) for the torque developed by the machine armature be revised here

$$T = C_m I_a \Phi \tag{2.17}$$

with the aim to integrate the set of relationships (2.16), (2.17) related to a steady state operation of a DC motor.

Performance Characteristics

The characteristic curves of the motor (Fig. 2.13) are the graphs which determine relationships between the following motor quantities:

1) the torque and the armature current; this is called an **electrical characteristic** of a motor;

2) the speed and the armature current, i.e. n ***vs.*** I_a characteristic;

3) the armature speed and armature torque; this is known as **mechanical characteristic**. It can be derived from the previous two.

Mechanical characteristic $n = F(T_a)$ is the most asked-for motor characteristic since it implies motor workability at different applied torques (including different loads). The purpose of mechanical characteristic of the motor is similar to that of external characteristic of the generator.

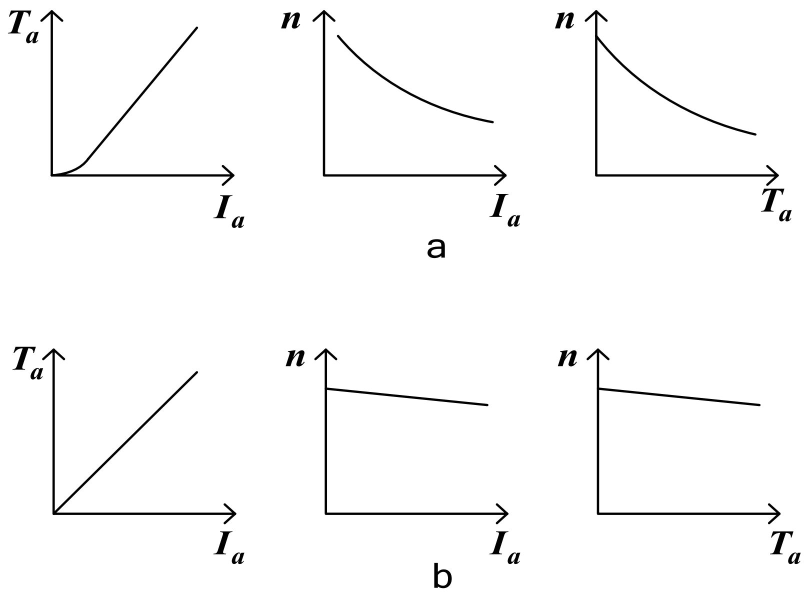


Fig. 2.13 – The characteristic curves for DC motors:
series motor (a); shunt motor (b)

By using motor voltage equation (2.16) and formulas (2.15), (2.17) related to armature back emf and torque we can derive the motor steady state angular speed as a function of the armature torque:

$$\omega = \frac{V_s}{C_e \Phi} - \frac{I_a R_a}{C_e \Phi} = \frac{V_s}{C_e \Phi} - T_a \frac{R_a}{C_e C_m \Phi^2} . \tag{2.18}$$

In terms of rpm the latter relationship can be written as

$$n = n_0 - kT_a , \tag{2.18a}$$

where n_0 represents the no-load motor speed for a given supply; in some cases motor no-load characteristics can be useful.

When writing equations (2.18), (2.18a) for mechanical characteristics we must suppose the excitation of the motor to be fixed, i.e. $I_f = const$.

Load characteristics of electric machines, both motors and generators, are usually of greatest interest in determining potential applications of these machines. Typical loading characteristics for different DC generators are shown in Fig. 2.14,a. Since the compound-connected generator contains both a shunt and a series field winding which make the opportunity to attain a flexible generator regulation depending on particular loads, it is the most general configuration for practical use.

Similarly, a DC motor will adjust to variations in load (load torque) by changing its speed to preserve the balance of mechanical and electrical power. This can be seen from mechanical curves in Fig. 2.13.

For example, as the load torque for the shunt motor is reduced, the armature current will also decrease, causing the speed to increase in accordance with equation (2.18). The change in speed will be on the same order of magnitude as the voltage drop across the armature resistance (resulted from the armature current decrease) and typically takes values around 10 percent.

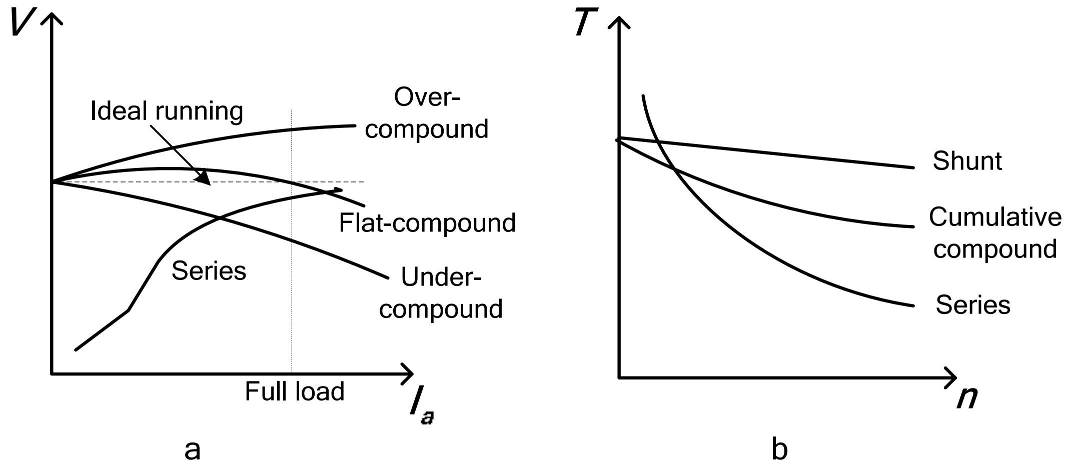


Fig. 2.14 – Generator (a) and motor (b) load characteristics

Fig. 2.14,b illustrates torque-speed characteristics of a DC motor if it has definite particular excitation. The curves show that the speed regulation of the compound connected motor is poorer than that of the shunt motor, i.e. a relatively good speed regulation is an attractive feature of the shunt motors.

2.7 Motor Speed Control. Starting Problem

The great advantage of DC motors is effective and simple speed control. From (2.18) it can be seen that in order to control the motor speed one may vary one of the following:

1. *Armature resistance*. This method of speed control requires connecting a variable resistor (controlling resistor R_{ac}) in series with the armature coil as it is shown in Fig. 2.15,a below (in the figure, R_{sh}, R_f introduce winding resistance and additional resistance of the shunt field). This allows obtaining needed motor speed (Fig. 2.15,b) for a given shaft torque.

However, there are two disadvantages of the method that utilizes armature resistance control.

First, a large amount of power is wasted in controlling resistor since it carries full armature current; as a result the output power and efficiency of the motor are reduced. Besides, an expensive arrangement is necessary to apply in order to dissipate the heat from controlling resistor.

Secondly, the speed will essentially depend on the load. Increased load torque requires more armature current hence more voltage drop across the controlling resistor. This double dependence makes impossible to keep the speed unchanged on a rapidly changing load.

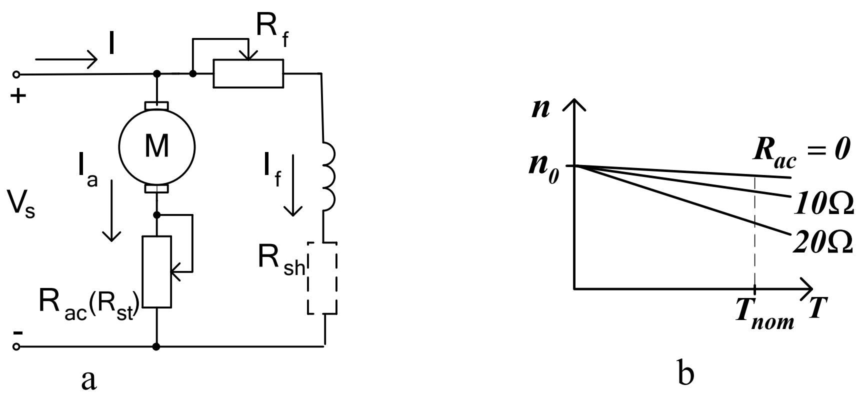


Fig. 2.15 – Speed control of the shunt motor:
general circuit arrangement (a); mechanical characteristics with armature resistance control (b)

The method is therefore employed only if speeds lower than normal are required for a short period of time, that is occasionally. As in the case of printing machines, cranes, and hoists where the motor is continually starting and stopping.

2. *Machine flux*. A variable resistance $\boldsymbol{R_f}$ called shunt field rheostat is placed in series with the shunt field winding (see Fig. 2.15,a). This rheostat reduces the shunt field current $\boldsymbol{I_f}$ and hence the flux $\boldsymbol{\Phi}$.

As the speed is inversely proportional to the flux, the motor runs at a speed higher than normal with the rheostat involved. In any case, only speeds higher than normal can be obtained now (see Fig. 2.16,a) since total resistance of the field circuit cannot be reduced below the value of shunt field resistance $\boldsymbol{R_{sh}}$.

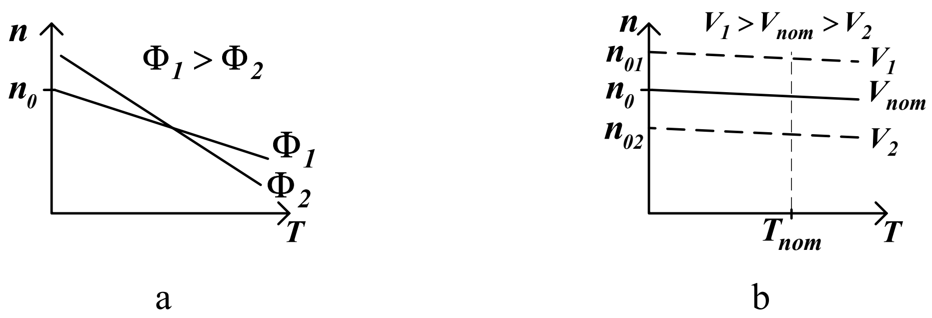


Fig. 2.16 – Mechanical characteristics of a shunt motor:
under the flux control (a) and supply voltage control (b)

However, one can see differing characteristic slopes when the field varies. Besides, if the flux decreases too much, commutation becomes poorer. Therefore, there is a limit to the maximum speed obtainable with this method.

The method offers the following advantages:

a) speed control is independent on the load applied to machine;

b) it is easy, convenient and economical service since very little power is wasted in the shunt field rheostat as heat due to comparatively small value of the field current.

3. *Supply voltage.* The method is called ***Multiple Voltage Control*** sometimes. When utilizing the method, the shunt field of the motor is connected permanently to a fixed excitation, but the armature has an opportunity to be energized by different supply voltages due to application of suitable switchgear. When the armature speed is proportional to the discrete voltage applied (Fig. 2.16,b), it can be set approximately with this method; if needed, the exact speed is attained with the help of shunt field regulator. The method is not used much, however.

Motor starting problem

For better understanding the problem, let us revise Fig. 2.10,a where we replace generator ***symbol G*** by motor notation ***M*** and connect a ***supply*** voltage V_s across the armature terminals (instead of electrical load). By this way, readily presented by Fig. 2.17, we illustrate reciprocity of a DC machine which can run either as a generator or a motor.

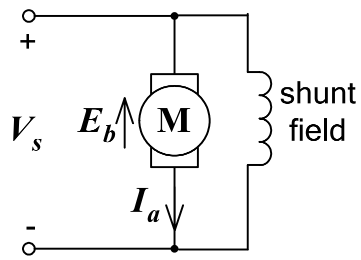


Fig. 2.17 – The simplest DC motor supply

A serious problem arising before the shunt-excited motor at the very start is as follows. When a DC motor starts running the back emf has not been yet induced, i.e. $E_b = 0$. It follows then from (2.16) that starting magnitude of the armature current $I_{st} = \frac{V_s}{R_a}$ is large enough and may exceed the nominal value 10 to 20 times. This is dangerous to machine workability.

There are three appropriate ways for providing the motor with effective and safe start.

1. Simple "**Direct on line**" start (see Fig. 2.17). Because of the reason mentioned, this method may be applied for motors having not more than several

hundred watts rated power (low-power or light motors); for these, a criterion $I_{st} = (4...6)I_a$ can be taken into practical use.

2. Usage of **starting rheostat**; this resistive element is placed in series with the armature winding in order to limit the armature current (see Fig. 2.15,a):

$$I_{st} = \frac{V_s}{R_a + R_{st}}.$$

Commonly, rheostat resistance R_{st} is selected so that the current $I_{st} = (1.4...2)I_a$ would be available.

3. **Gradual build-up** of the feeding voltage; for energizing the motor, output control from the power supply must be available here.

Motor reversing

In a number of applications which involve motors it is required that the direction of motor rotation be reversed in order to perform a particular function, e.g. opening and closing a valve by an actuator. This is achieved by reversing the direction of the current flow or magnetic field polarity in either the armature or the field winding.

2.8 Comparative Overview of Various DC Machines

Shunt Motor peculiar features are as follows:

1. Speed of a shunt motor is sufficiently constant (advantage).
2. For the same current input, its starting torque is not as high as that of series motor (disadvantage).

The shunt motor is therefore used:

– when a load speed has to be maintained as constant as possible;

– when it is required to drive the load at various speeds; any unique speed is required to be constant for a relatively long period of time.

Examples of shunt motors application:

– constant speed line shaft drivers
– lathes
– centrifugal pumps
– machine tools
– blowers and fans
– reciprocating pumps.

Series Motors. These have

1. Relatively large starting torques (advantage).
2. Good accelerating torques (advantage).
3. Low speed at high loads and dangerously high speed at low loads (disadvantage).

Hence, these motors are used:

– when a large starting torque is required, i.e. for driving hoists, cranes, trams, conveyors, etc;

– when the motor can be directly coupled to a load, whose torque increases with speed (e.g. fan).

Inversely, the series motor should not be used if a need of releasing the motor from large load is periodically risen (an idle running is quite probable).

2.9 Losses and Efficiency of DC Machines

Typical application of an electric machine as the energy-conversion device must take into consideration various energy losses which result in performance efficiency associated with these devices.

The efficiency of a DC machine is defined by a known general relation

$$\eta = \frac{P_2}{P_1} = \frac{P_2}{P_2 + \Delta P},$$

where P_2 is the output power; P_1 is the input power; ΔP is the losses. For convenience, various loss mechanisms are summarized in Fig. 2.18, where $P_{\dots}$ is the specific loss and T_s denotes a shaft torque either applied to or developed by the electric machine depending on if it acts as the generator or the motor.

Efficiency may be determined either experimentally from load tests (i.e. by physical input/output power measurement when the machine runs at the rated load) or theoretically by evaluating probable machine losses.

Various losses are divided into three fundamental groups: electrical (or copper) losses, magnetic (core) losses and mechanical losses.

Electrical losses include:

– ohmic (or I^2R) loss in machine coils, including armature and field windings;

– loss due to contact resistance of the brush (the slip rings/commutator assembly).

For both generators and motors, the electrical losses comprise from 4 to 11 percent of total energy wastage.

Mechanical losses**.** These occur because of a friction in the bearings and brushes; also they happen due to windage (the latter means the air drag force which opposes the motion of the rotor).

Magnetic losses consist of hysteresis and eddy current losses which have the same nature as transformer losses in Chapter 1. We may consider these losses if associated magnetic circuits of the armature and pole faces are energized by the field coil, hence we can estimate them while no-load running. It is that why they are also referred to as open-circuit core losses. These losses are often summed with friction and windage mechanical losses to give rise to the no-load rotational loss. Commonly, open-circuit losses are 3 to 15 percent of the machine output power.

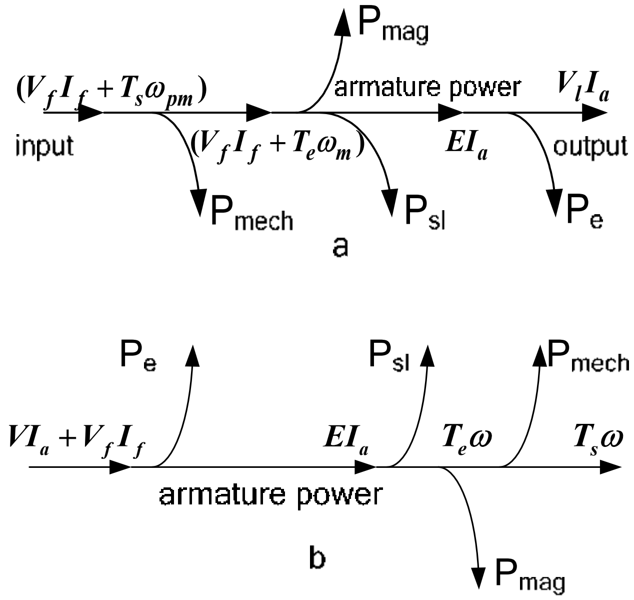


Fig. 2.18 – Power flows in DC machines:
DC generator (a); DC motor (b)

Stray-load losses. There are depended-on-load losses not covered above. They are difficult to determine exactly and are often assumed to be equal to 1% of output power.

2.10 Terms and Concepts

Armature is a part of electrical machine in which emf is induced; the winding that carries the load current of DC machine.

Armature reaction is the interaction of the armature field with the main field of a generator or motor, resulting in distortion of the field.

Brush is a device designed to provide an electric contact between a stationary conductor and a rotating element.

Commutator is a rotating contacting unit in the armature of DC generator or motor; in effect, it changes the alternating current flowing in armature windings to a DC current in external circuit.

Compound winding is a combination of series and parallel, or shunt windings to provide the magnetic field for generator or motor.

Counter (back) electromotive force is a voltage developed in the armature of a motor that opposes the applied emf. The same principle is applied to any inductor through which an alternating current is flowing.

Electric machine is an electromechanical device in which the magnetic field provides a form of coupling between electrical and mechanical systems; the systems involve a stationary section (stator) and moving part (rotor) used to perform energy conversion.

Electric motor is a rotating machine for converting electric energy into mechanical energy.

Excitation is an electric current applied to the field winding of a generator or motor to produce the magnetic field.

Field winding is a coil that carries the current (comparatively small and independent of the load) which serves the sole purpose of providing machine flux.

Generator is a rotating machine designed to produce a certain type and quantity of voltage and current. **G**. is the device that converts mechanical energy from a prime mover to electrical form by exploiting laws of electromagnetic induction.

Inter-pole is a small magnetic pole inserted between the main field poles of a generator or motor in series with the load circuit to compensate for the effect of armature reaction.

Motor is an electric machine that converts electrical energy to mechanical (rotational type) form by generation of the torque when magnetic attraction forces act.

Rotor is the rotating part of an electric machine.

Self-excited machine obtains its field excitation from the armature voltage (can be shunt-connected, sires-connected, and compound-connected).

Separately-excited DC machine is that in which the magnetizing (field) current is provided by an external source.

Slip rings are conducting rings used with brushes to conduct electric current to or from a rotating unit.

Stator is the stationary part of an electric machine.

2.11 Review Questions

1. What laws is the performance of an electric machine based on?
2. Describe the structure of a DC machine.
3. What is the commutator/brush system used for?
4. What do the e.m.f. and torque equations explain?
5. Write and explain the speed equation.
6. What ways of machine excitation do you know?
7. How are various machine losses classified?
8. How can the speed of a DC motor be controlled?
9. What is the armature reaction in a DC machine?
10. How can the effects of armature reaction be minimized?
11. How can DC motor rotation be reversed?
12. How can you explain the term “back emf”?

2.12 Problems

Solved problems

Problem 1. A $\boldsymbol{440\ V}$ shunt motor (see Fig. 2.10,a) has armature resistance of $\boldsymbol{0.8}\ \Omega$ and field resistance of $\boldsymbol{200}\ \Omega$.

Determine the back emf $\boldsymbol{E_b}$ when giving an output of $\boldsymbol{7.46\ kW}$ at $\boldsymbol{85}$ percent efficiency.

Known quantities: $V = 440\ V$, $R_a = 0.8\ \Omega$, $R_f = 200\ \Omega$, $P_2 = 7.46\ kW$, $\eta = 0.85$.

Solution

1. Motor input power $P_1 = \dfrac{P_2}{\eta} = \dfrac{7.46 \cdot 10^3}{0.85} \approx 8.78\ kW$.
2. $I_{in} = \dfrac{P_1}{V} = \dfrac{8.78 \cdot 10^3}{440} = 19.95\ A$.
3. $I_f = \dfrac{V}{R_f} = \dfrac{440}{200} = 2.2\ A$.
4. Armature current $I_a = I_{in} - I_f = 19.95 - 2.2 = 17.75\ A$.
5. Then using voltage equation of a motor

   $$E_b = V - I_a R_a = 440 - 0.8 \cdot 17.75 = 425.8\ V.$$

Problem 2. A $\boldsymbol{25\ kW}$, $\boldsymbol{250\ V}$ DC shunt generator has armature and field resistances of $\boldsymbol{0.06}\ \Omega$ and $\boldsymbol{100}\ \Omega$ respectively.

Determine the total armature power developed when working

a) as a generator delivering $\boldsymbol{25\ kW}$ output (see Fig. 2.10,a);

b) as a motor taking $\boldsymbol{25\ kW}$ input (see Fig. 2.15,a).

Solution

I. <u>The generator</u>

1. Output current $I_l = \dfrac{P_2}{V_l} = \dfrac{25 \cdot 10^3}{250} = 100\ A$.
2. Field current $I_f = \dfrac{V_l}{R_f} = \dfrac{250}{100} = 2.5\ A$.
3. Armature current $I_a = I_l + I_f = 100 + 2.5 = 102.5\ A$.
4. Generated emf in armature

   $$E_a = V + I_a R_a = 250 + 102.5 \cdot 0.06 = 256.2\ V.$$

5. Power developed in armature

   $$P_1 = E_a I_a = 256.2 \cdot 102.5 = 26.25\ kW.$$

II. The motor

1. Motor input current $I_{in} = \frac{P_1}{V} = \frac{25 \cdot 1000}{250} = 100\ A$.
2. Field current $I_f = \frac{V}{R_f} = \frac{250}{100} = 2.5\ A$.
3. Armature current $I_a = I_{in} - I_f = 100 - 2.5 = 97.5\ A$.
4. The back emf $E_b = V - I_a R_a = 250 - 97.5 \cdot 0.06 = 244.15\ V$.
5. Power developed in armature

$$P_2 = E_b I_a = 244.15 \cdot 97.5 = 23.8\ kW.$$

Problem 3. A shunt generator delivers $450\ A$ at $230\ V$; the resistance of the shunt field and armature are $50\ \Omega$ and $0.03\ \Omega$ respectively.

Calculate the generated emf.

Solution

Generator circuit is shown in Fig. 2. 10,a.

1. Current through shunt field winding is $I_{sh} = \frac{V}{R_f} = \frac{230}{50} = 4.6 A$.
2. Load current $I_l = 450\ A$, then armature current

$$I_a = I_l + I_{sh} = 450 + 4.6 = 454.6\ A.$$

3. Armature voltage drop $V_a = I_a R_a = 454.6 \cdot 0.03 = 13.6\ V$.
4. Now emf generated in the armature

$$E_g = V + I_a R_a = 230 + 13.6 = 243.6\ V.$$

Problem 4. A separately excited DC generator (see Fig. 2.7), when running at $n_1 = 1200\ rpm$ supplies $I_a = 200\ A$ at $V_l = 125\ V$ to a circuit of constant resistance.

What will the current be when the speed drops to $n_2 = 1000\ rpm$ and the field current is reduced to 80%?

Armature resistance $0.04\ \Omega$ and total drop at brushes $V_{br} = 2\ V$. Ignore saturation and armature reaction.

Solution

1. We find the generated emf when the load current is $200\ A$:

$$E_1 = V + V_{br} + I_a R_a = 120 + 2 + 200 \cdot 0.04 = 135\ V.$$

2. As $\Phi_1 n_1 \rightarrow E_1$ and $\Phi_2 n_2 \rightarrow E_2$, then $\frac{E_2}{E_1} = \frac{\Phi_2 n_2}{\Phi_1 n_1}$ or $\frac{E_2}{135} = 0.8 \frac{1000}{1220}$,

since $\Phi_2 = 0.8\Phi_1$. Hence, $E_2 = 90\ V$.

3. If we assume that small voltage drop across the brushes remains the same for different load current, then

$$E_2 = V_{l2} + V_{br} + I_{a2}R_a = I_{a2}\frac{V}{I_a} + V_{br} + I_{a2}R_a .$$

4. This yields $I_{a2} = \dfrac{90-2}{0.6+0.04} = 137.5\ A$.

Problem 5. A ***10 hp*** (***hp*** means ***horse power*** unit), ***230 V*** shunt motor (see Fig. 2.15) takes a full-load line current of ***40 A***. The armature and field resistances are ***0.25 Ω*** and ***230 Ω***, respectively. The total brush-contact drop is ***2 V*** and the core and friction losses are ***380 W***.

Calculate the efficiency of the motor. Assume that stray-load loss is 1% of the output.

Solution

6. Input power $P_1 = I_l V = 40 \cdot 230 = 9200\ W$.
7. Field-resistance loss

$$P_f = I_f^2 R_f = \left(\frac{V}{R_f}\right)^2 R_f = \left(\frac{230}{230}\right)^2 \cdot 230 = 230\ W .$$

3. Armature-resistance loss

$$P_{a\,loss} = \left(I_l - I_f\right)^2 R_a = (40-1)^2 \cdot 0.25 = 380\ W .$$

4. Core and friction losses $P_m + P_{fr} = 380\ W$.
5. Brush-contact loss $P_{br} = I_a V_{br} = 39 \cdot 2 = 78\ W$.
6. Taking that ***1hp=746 W*** calculate stray-load loss

$$P_{str} = 0.01 \cdot 10 \cdot 746 \simeq 75W .$$

7. Total losses

$$P_t = P_f + P_{a\,loss} + P_m + P_{fr} + P_{br} + P_{str} =$$
$$= 230 + 380 + 380 + 78 + 75 = 1143\ W.$$

8. Output power $P_2 = P_1 - P_t = 9200 - 1143 = 8057W$.
9. Efficiency $\eta = \dfrac{8057}{9200} = 87.6\%$.

Supplement problems

Problem 6. A DC motor connected to ***460 V*** supply has an armature resistance of ***0.25 Ω***.

Calculate

11. The value of back emf when the armature current is $\boldsymbol{120\ A}$.
12. The value of armature current when the back emf is $\boldsymbol{447.4\ V}$.

(Ans. $\boldsymbol{E_b = 440\ V;\ I_a = 84\ A}$)

Problem 7. A DC shunt generator supplies a load of $\boldsymbol{15\ kW}$ at $\boldsymbol{200\ V}$ through feeders having resistance $\boldsymbol{0.08\ \Omega}$. Resistance of shunt field winding is $\boldsymbol{80\ \Omega}$ while that of the armature is $\boldsymbol{0.04\ \Omega}$.

Find the terminal voltage and the emf generated.

(Ans. $\boldsymbol{E_g = 209.1\ V;\ V = 206\ V}$)

Problem 8. Calculate the electromagnetic torque developed by the armature ($\boldsymbol{I_{a\,max} = 100\ A}$) of a DC machine running at $\boldsymbol{1800\ rpm}$. The 4-pole lap-wound armature has $\boldsymbol{728}$ active conductors. The flux per pole is $\boldsymbol{30\ Wb}$.

(Ans. $\boldsymbol{347.6\ Nm}$)

Problem 9. At what speed (rpm) must the armature of a DC machine run to develop $\boldsymbol{572\ kW}$ at a torque of $\boldsymbol{4605\ Nm}$?

(Ans. $\boldsymbol{1187\ rpm}$)

Problem 10. A self-excited shunt generator supplies a load of $\boldsymbol{12.5\ kW}$ at $\boldsymbol{125\ V}$. The field resistance is $\boldsymbol{25\ \Omega}$ and the armature resistance is $\boldsymbol{0.1\ \Omega}$. The total voltage drop because of brush contact and armature reaction at this load is $\boldsymbol{3.5\ V}$.

Calculate the induced armature voltage.

(Ans. $\boldsymbol{E_g = 139\ V}$)

Problem 11. A $\boldsymbol{230\ V}$ shunt motor, having an armature resistance of $\boldsymbol{0.05\ \Omega}$ and field resistance of $\boldsymbol{75\ \Omega}$, draws a line current of $\boldsymbol{7\ A}$ while running at $\boldsymbol{1120\ rpm}$.

For a load at which the line current is $\boldsymbol{46\ A}$, determine: a) the motor speed, b) motor efficiency, c) total core and mechanical losses.

(Ans. $\boldsymbol{1110.5\ rpm}$ (a), $\boldsymbol{83.9\%}$ (b), $\boldsymbol{903.9\ W}$)

Problem 12. A shunt machine, connected to $\boldsymbol{250\ V}$ mains, has an armature resistance (including brushes) of $\boldsymbol{0.12\ \Omega}$; the resistance of the fields circuit is $\boldsymbol{100\ \Omega}$.

Find the ratio of the speed as a generator to the speed as a motor, if the line current in each case is $\boldsymbol{80\ A}$.

(Ans. $\boldsymbol{1.08}$)

Tutorial test

1. ***The external characteristic of a shunt generator can be obtained directly from its………………characteristic***

a) internal b) open circuit

c) load-saturation d) performance

2. ***Load saturation characteristic of a DC generator gives relation between***

a) V and I_a b) E and I_a

c) E_0 and I_f d) V and I_f

3. ***For the voltage build-up of a self-excited DC generator, which of the following is not an essential condition?***

a) there must be some residual flux

b) field winding mmf must aid the residual flux

c) total field circuit resistance must be less than the critical value

d) armature speed must be very high

4. ***The voltage build-up process of a DC generator is***

a) difficult b) delayed

c) cumulative d) infinite

5. ***Which of the following DC generators cannot build up on open circuit running?***

a) shunt b) series

c) short-shunt d) long-shunt

6. ***If a self-excited DC generator fails after being installed, to build it up for the first run, the first thing to do is to***

a) increase the field resistance b) check armature insulation

c) reverse field connections d) increase the speed of prime mover

7. ***If residual magnetism of a shunt generator is destroyed accidentally, it may be restored by connecting its shunt field***

a) to earth b) to an AC source

c) in reverse d) to a DC source

8. ***The three factors which cause decrease in the terminal voltage of a shunt generator are***

a) armature reactance	b) armature resistance
c) armature leakage	d) reduction in field current

9. ***If field resistance of a DC shunt generator is increased beyond its critical value, the generator***

a) output voltage will exceed its rating	b) will not build up
c) may burn out if loaded to its name-plate rating	d) power out may exceed its name-plate rating

10. ***An ideal DC generator is one that has ……………… voltage regulation***

a) low	b) zero
c) positive	d) negative

11. ***The …………… generator has the poorest voltage regulation***

a) series	b) shunt
c) compound	d) high

12. ***The voltage regulation of an over-compounded DC generator is always***

a) positive	b) negative
c) zero	d) high

13. ***Most commercial compound generators are normally supplied by the manufacturer as over compound machines because***

a) they are ideally suited for transmission of DC energy to remote loads

b) degree of compounding can be adjusted by using a diverter

c) they are more cost-effective than shunt generators

d) they have zero percent regulation

14. ***In a DC motor, unidirectional torque is produced with the help of***

a) brushes	b) commutator
c) end plates	d) brushes and commutator

15. ***A DC motor can be looked upon as DC generator with the power flow***

a) reduced b) reversed

c) increased d) modified

16. ***The induced emf in the armature conductors of a DC motor is***

a) sinusoidal b) trapezoidal

c) rectangular d) alternating

17. ***In a DC motor, the mechanical output power actually comes from***

a) field system b) air gap flux

c) back emf d) electric input power

18. ***Which of the following quantity maintains the same direction whether a DC machine runs as a generator or as a motor?***

a) induced emf b) armature current

c) back emf d) armature current and back emf

19. ***Under constant load conditions, the speed of DC motor is affected by***

a) field flux b) armature current

c) back emf d) armature current and back e.m.f.

20. ***If load on DC shunt motor is increased, its speed is decreased due primarily to***

a) increase in the flux b) decrease in back emf

c) increase in armature current d) increase in back emf

21. ***As the load increases the speed of a DC shunt motor***

a) increases proportionately b) remains constant

c) increases slightly d) reduces slightly

22. ***A shunt DC motor works on AC mains … … …***

a) unsatisfactorily b) satisfactorily

c) not at all d) none of the above

Chapter 3

INDUCTION MOTORS

The field of application of AC motors has widened considerably during the recent years. As a result, motor manufactures have tried to perfect various types of AC motors for all classes of industrial drives and for both single and three phase AC supply.

Numerous AC motors may be classified into various groups.

As regards their principle of operation, there are **asynchronous motors** and **synchronous motors.**

Asynchronous motors are subdivided into two divisions – **induction motors** and **commutator motors**. In turn, the induction motors can be **squirrel cage** (single or double) and **slip-ring**.

Regarding the type of AC current one can find **single-phase** and **three-phase** motors.

As a general rule, conversion of electric power into mechanical power takes place in the rotating part of an electric motor. In ***DC motor***, the electric power is conducted directly to the armature (the rotating part) through brushes and commutator. It is that why a DC motor can be called a ***conduction motor***.

In contrary, in AC motors, the rotor does not receive electric power by conduction but does it by induction in exactly the same way as the secondary of a two-winding transformer receives its power from the primary. That is that why such motors are known as ***induction motor***s.

An induction motor can be treated as rotating transformer, in which primary winding is stationary but the secondary is free to rotate.

Induction motors are extensively used for various kinds of industrial drives.

They have the following main advantages and also some disadvantages.

Advantages:

1) very simple, unbreakable construction (especially squirrel-cage type motors);

2) low cost and very high reliability;

3) high efficiency; they have no brushes, hence frictional losses are reduced;

4) small weight and space;

5) their starting arrangement is simple, especially for a squirrel-cage motor;

6) there is no need to use special starting devices or excitation from an auxiliary source;

7) they will handle a wide range of loads.

Disadvantages:

1) motor speed cannot be varied without sacrificing some of its efficiency;

2) just like a DC shunt motor, the speed decreases with increase in load;

3) starting torque is somewhat inferior to that of a DC shunt motor.

3.1 AC Motor Constructions

An induction motor consists of two main parts: a stator and a rotor.

The stator is made up of stampings, which are slotted to receive the windings (Fig. 3.1).

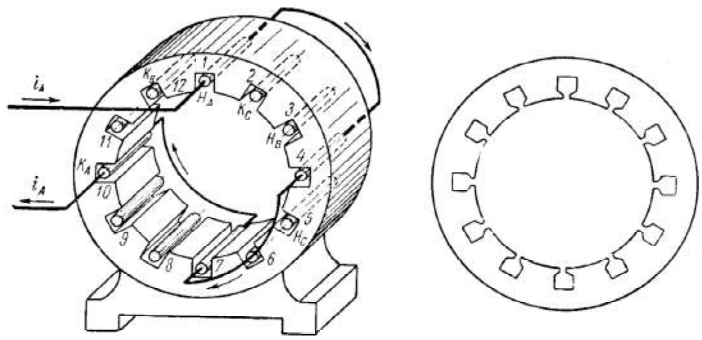


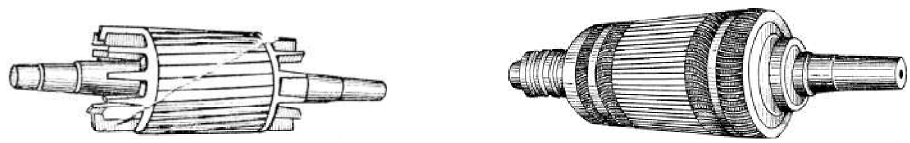
Fig. 3.1 – The stator of induction motor

The stator carries a 3-phase winding and is fed from a 3-phase supply. It produces a rotating magnetic flux, which is of constant magnitude. The revolving magnetic flux induces an emf in the rotor by mutual induction.

As to the rotor, the following two types are used:

a) **squirrel-cage** rotor (Fig. 3.2,a); motors employing this type of a rotor are known as squirrel-cage induction motors;

b) **phase-wound or wound** rotor (Fig. 3.2,b); these are known as phase-wound motors or simply 'wound' motors variously; sometimes wound motors are called 'slip-ring' motors.



a b

Fig. 3.2 – The squirrel-cage (a) and wound (b) rotors

Squirrel-cage rotor

Almost 90 % of all induction motors are squirrel-cage type, because this type of rotor has the simplest and most rugged construction. The rotor consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors which are not wires but heavy bars of copper, aluminum or alloys.

It should be noted that the rotor bars are **permanently short-circuited** on themselves**;** hence it is not possible to add any external resistance in series with the rotor circuit, e.g. for starting purposes.

The rotor slots are usually not quite parallel to the shaft but are purposely given a slight skew (see Fig. 3.2,a)

This is useful in two ways:

1) it helps to make the motor run quietly, by reducing the magnetic hum;

2) it helps in reducing the locking tendency of the rotor.

Phase-wound rotor

This type of rotor is provided with 3-phase distributed winding consisting of three coils. The rotor is wound for as many poles as the number of stator poles and is always wound 3-phased even when the stator is wound two-phased.

The three phases are starred internally (Fig. 3.3). The other three winding terminals are brought out and connected to three insulated slip-rings mounted on the shaft with brushes resting on them.

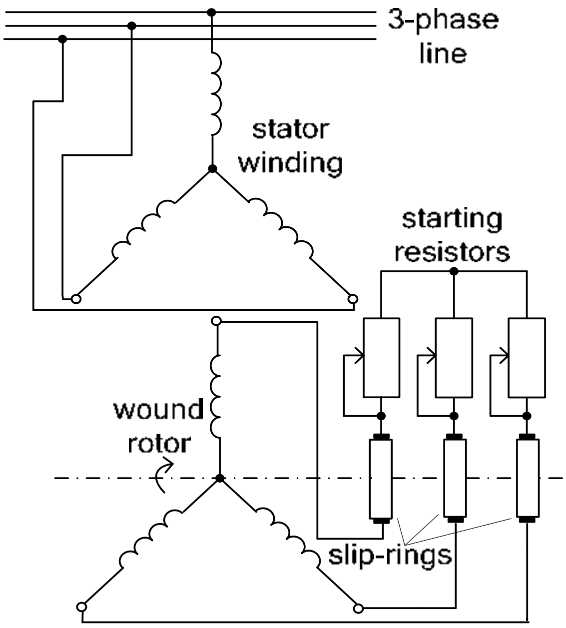


Fig. 3.3 – Induction motor with the phase-wound rotor

These three brushes are externally connected to a 3-phase star-connected rheostat. This makes possible the introduction of additional resistance in the rotor circuit during the starting period for increasing the starting torque of the motor and for changing its speed/torque vs. current characteristics.

When running under normal conditions, the slip-rings are automatically short-circuited by means of a metal collar, which is pushed along the shaft and connects all the rings together. Next, the brushes are automatically lifted from the slip-rings to reduce the frictional losses and wear and tear. Hence, it is seen that under normal running condition, the wound rotor is short-circuited on itself just like the squirrel-cage rotor.

3.2 Producing a Rotating Field

Let us show that when stationary coils wound for two or three phases are supplied by two- or three-phase supply respectively, a uniformly-rotating (revolving) magnetic flux of constant value is produced.

When three-phase winding in a form of three starred coils which are displaced in space by 120^0 (A–X, B–Y, C–Z in Fig. 3.4,a,b,c) is fed by three-phase AC currents displaced (Fig. 3.4,d) in time, it produces a resultant magnetic flux, which rotates in space as if actual magnetic poles were being rotated mechanically. In our case, the induction motor has two poles (or one pair of poles, p=1).

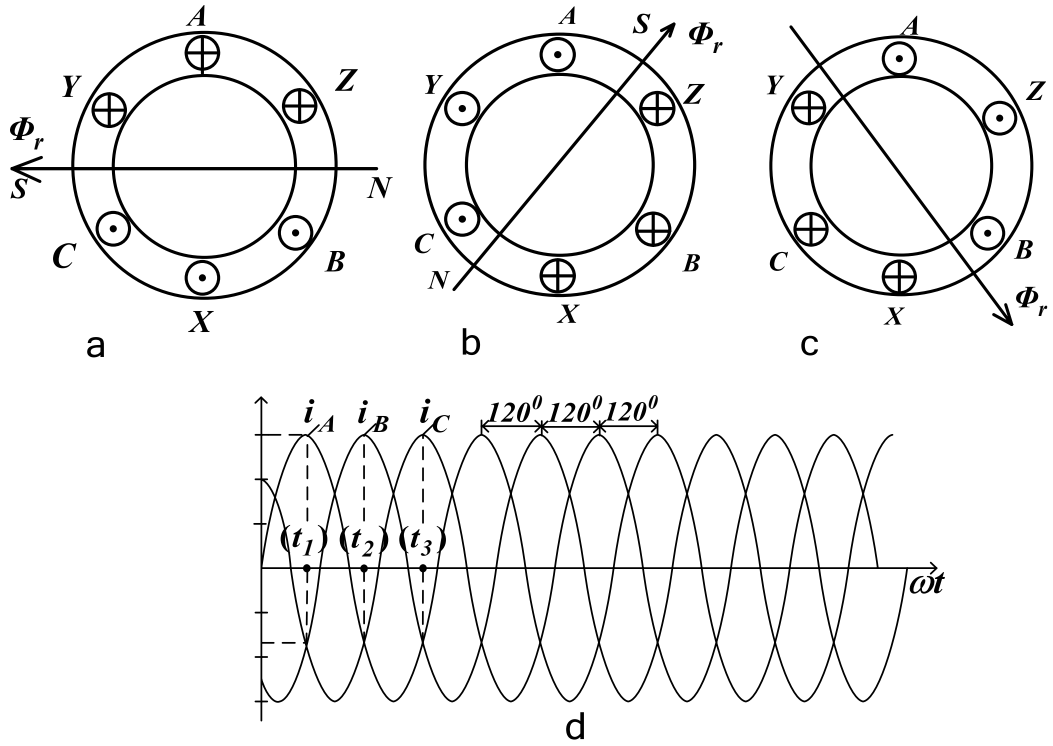


Fig. 3.4 – Rotating magnetic flux (a,b,c) and 3-phase excitation (d)

In sketches (a), (b) and (c) of Fig. 3.4, the direction of resultant magnetic flux is shown for each of t_1, t_2, t_3 time instances pointed in diagram (d).

Because of three phases, the resultant flux Φ_r at any instant can be found by a vector sum of individual fluxes, each having peak value Φ_m. Calculation for the instant will result in the same constant value

$$\Phi_r = \frac{3}{2}\Phi_m . \qquad (3.1)$$

Owing to varying in time field current the flux Φ_r rotates clockwise around the stator; closing for each $1/3$ excitation cycle $T = 1 / f_1$ the space angle of 120^0. Hence, the axis of Φ_r rotates around the stator with the speed n_1 called synchronous: $\boldsymbol{n_1 = f_1}$ (rps) or $\boldsymbol{n_1 = 60 f_1}$ (rpm). Herein, $\boldsymbol{f_1}$ is the frequency of AC mains.

In common case of multi-pole machine we should have

$$\boldsymbol{n_1 = \frac{60 f_1}{p}}. \tag{3.2}$$

For instance: if $\boldsymbol{f_1 = 50 Hz}$, and $\boldsymbol{p = 2}$, then speed of the stator field rotation will be $\boldsymbol{n_1 = \frac{60 \cdot 50}{2} = 1500\ rpm}$.

Hereafter, the '1' index will be referred to as that designating a stator parameter whereas that of the rotor obtains the '2' mark.

Hence we conclude that:

a) the resultant flux is of a constant magnitude which does not change in time;

b) the flux produces a field rotating clockwise. If needed, the rotation of a three-phase induction motor can be ***reversed***; for this, two of three mains leads must change their place.

3.3 Peculiar Operation of an Induction Motor

There are three main questions to be answered to as regards the motor operation.

1. **Why does the rotor rotate?** The reason why the rotor of an induction motor is set into rotation is explained as follows.

When the rotor is placed in a rotating, i.e. moving magnetic flux, the bars (or conductors) are cut by the running flux. The rotor conductors are yet stationary. Due to the relative speed between the rotating flux and the stationary conductors, an emf is induced in the latter, according to Faraday's law of electromagnetic induction. The frequency of induced emf is the same as the supply frequency. Its magnitude is proportional to the relative velocity between the flux and the conductors and its direction is given by right-hand rule. Since the rotor bars (conductors) form a closed circuit a rotor current is produced. According to Ampere's law a force acts on the conductors that carry the current.

2. **What direction does this electromechanical force have?**

This direction being given by Lenz's law is such as to oppose the very cause producing it. It is a relative velocity between rotating flux of the stator and the stationary rotor conductors that produces the emf, hence causes the conductor current. In order to reduce (when opposing) the relative speed, the rotor starts running in the **same direction** as that of the flux and tries to catch up with the rotating flux.

3. Does the moving rotor catch up the rotating flux?

In practice, the rotor never succeeds in 'catching up' with the stator field. If it really did so, then there would be no relative speed between the two, hence no rotor emf would be induced, no rotor current and so no torque to maintain the rotation. That is why the rotor runs at a speed which is always less than the speed of the stator field (dynamic equilibrium). The difference in speed depends upon the load on the motor.

Slip

The difference between the synchronous speed $\boldsymbol{n_1}$ (speed of rotating flux) and the actual speed $\boldsymbol{n_2}$ of the rotor is known as **slip**. Though it may be expressed in so many revolutions per second, yet it is usual practice to express it as a percentage of synchronous speed:

$$s\% = \frac{n_1 - n_2}{n_1} 100 .$$

Actually, the term «slip» is descriptive means to make appear a way in which the rotor «slips back» from synchronism.

Sometimes the term $\left(n_1 - n_2\right)$ is called **slip speed**. It is obvious that rotor (or motor) speed in terms of rpm is

$$n_2 = n_1\left(1 - s\right) = \frac{60 f_1}{p}\left(1 - s\right). \qquad (3.3)$$

It must be kept in mind also that rotating flux is revolving synchronously relative to the stator (the latter being stationary in space) but at slip-speed relative to the rotor.

Rotor current frequency

When the rotor is stationary, the frequency of rotor current is the same as the field supply frequency. But when the rotor begins revolving, then the frequency becomes dependent on the relative, or slip speed $n_1 - n_2$. Although the frequency of stator current is $f_1 = \frac{pn_1}{60}$, for rotor current we find somewhat different frequency meaning:

$$f_2 = \frac{p\left(n_1 - n_2\right)}{60} = \frac{p\left(n_1 - n_2\right)}{60}\frac{n_1}{n_1} = f_1 s . \qquad (3.4)$$

Interrelation of stator and rotor fields

As relation (3.4) shows, rotor bar currents obtain a frequency affected by rotor slip and therefore known as **slip frequency**. The changing currents produce rotating magnetic field of the rotor; its rotational speed is

$$n_2' = \frac{60 f_2}{p} = \frac{60 f_1}{p} s = n_1 s\,. \quad (3.5)$$

However, as the rotor runs in space with speed n_2, resulting space velocity of the rotor field will be

$$n_2' + n_2 = s n_1 + n_1 (1 - s) = n_1\,.$$

This means that no matter what the value of the slip is, rotor currents and stator currents each produce running in space magnetic field of constant magnitude (sinusoidal time-varying waveform due to AC supply). Following mains frequency, the fields run with the angular velocity n_1 rpm. In other words, both the rotor and stator fields rotate synchronously, i.e. they are stationary with respect to each other. These two synchronously rotating magnetic fields, in fact, superimpose on each other and give rise to the actually existing rotating field which follows in form a magnetizing current of the stator winding.

Stator and rotor electromotive forces

Effective value of ***stator emf*** in one phase is

$$E_1 = 4.44 f_1 w_1 \Phi_m k_1\,. \quad (3.6)$$

The same for the ***resting rotor*** will be

$$E_2 = 4.44 f_1 w_2 \Phi_m k_2\,, \quad (3.7)$$

whereas for ***revolving rotor*** (which currents collect slip frequency $s f_1$) the induced emf is

$$E_{2S} = 4.44 f_1 w_2 \Phi_m k_2 s = E_2 s\,. \quad (3.8)$$

In the above relationships, terms k_1 and k_2 are winding coefficients ($0.92 \ldots 0.98$).

Rotor current

Let E_2 be rotor-at-rest phase emf, R_2 – the rotor per-phase resistance; X_2, $Z_2 = \sqrt{R_2^2 + X_2^2}$ – standstill rotor reactance and impedance for one phase, respectively.

Then for running rotor, the magnitude of the rotor alternating current (its frequency is $f_1 s$) can be derived from

$$I_2 = \frac{E_{2S}}{Z_{2S}} = \frac{E_2 s}{\sqrt{R_2^2 + X_{2S}^2}} = \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + X_2^2}}\,. \quad (3.9)$$

Relationship (3.9) is followed by the running rotor equivalent circuit shown in Fig. 3.5 (per phase).

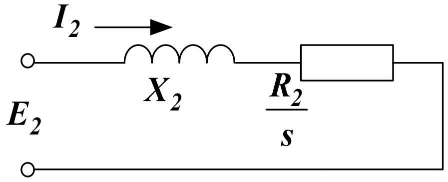


Fig. 3.5 – The rotor equivalent circuit

Field (stator winding) current

To incorporate the stator current and investigate this circuit, the induction ***motor*** may be viewed as a ***transformer with air gap***; this particular transformer must have a varying and dependent on motor speed resistance in the secondary (see (3.9) and Fig. 3.6).

Considering the rotor to be coupled with the stator like the secondary of the transformer is coupled to its primary, the circuit model for induction motor may be depicted as shown in Fig. 3.6.

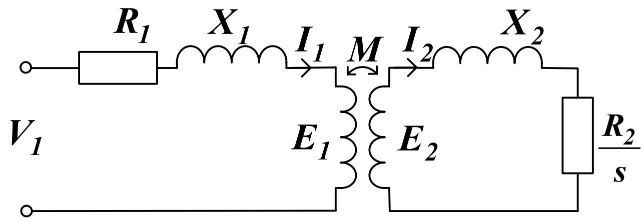


Fig. 3.6 – The circuit model for induction motor

Like a conventional transformer, a motor has mutual flux linking both the stator and the rotor. In the model, this flux is presented by *magnetizing reactance* (with associated coupling factor ***M***); besides, a *leakage reactance* is to represent various flux losses due to leakage reason. It is X_1, X_2 terms in Fig. 3.7 that represent total leakage loss of the motor. As to magnetizing reactance X_0 (due to ***M***), its value tends to be low as compared to a true transformer because of the air gap which results in relatively small amount of ***M***.

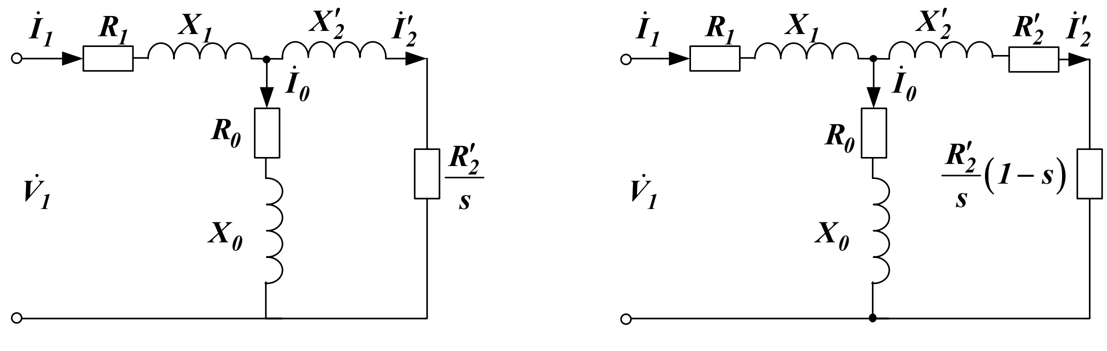


Fig. 3.7 – The reduced equivalent circuits of an induction motor

To develop this circuit model further, we reflect rotor quantities (as in the case of a transformer) to the stator part thus arriving to a reduced equivalent circuit (per phase) shown in Fig. 3.7,a. When doing so, let us realize that all parameters in the model are standstill values and therefore can be readily estimated.

Parameters reflected to the field (stator) circuit have accepted motor transformation factor k_{21} that can be derived from winding coefficients k_1, k_2 have been appeared earlier in relations (3.6), (3.7).

For reason that will immediately become clear, $\frac{R_2'}{s}$ can be split as

$$\frac{R_2'}{s} = R_2' + \frac{R_2'}{s}\left(1 - s\right) \tag{3.10}$$

to obtain the circuit shown in Fig. 3.7,b. Here, R_2' is simply the per-phase standstill rotor resistance referred to the stator, and $R_2'\left(1 - s\right)/s$ is a per-phase dynamic resistance that depends on rotor speed and relates to the actual load on the motor. Hereby, induction motor currents, both rotor and stator, are readily put in dependence on motor shaft loading.

It appears from the above Fig. 3.6 and the following Fig. 3.7 that for input motor circuit a relationship similar to the primary equation of a transformer can be applied:

$$\dot{V}_1 = -\dot{E}_1 + \dot{I}_1\left(R_1 + jX_1\right), \tag{3.11}$$

where E_1 is the back emf of the field winding and R_1, X_1 are winding ***Cu***-resistance and leakage reactance per phase.

In practice, $\dot{V}_1 \approx -\dot{E}_1$.

Consequently, if supply voltage $V_1 = const$, then $E_1 = const$.

It follows therefore that $\Phi_m = const$ too, and a statement

$$\dot{I}_0 w_1 k_1 = \dot{I}_1 w_1 k_1 + \dot{I}_2 w_2 k_2$$

is true according to magnetic equilibrium law.

Now, the stator field current (hence, input power) may be realized as consisting of two terms:

$$\dot{I}_1 = \dot{I}_0 - I_2'. \tag{3.12}$$

The I_0 term represents an own stator (at the rotor disengaged) excitation power, whereas the term

$$I_2' = \frac{w_1 k_1}{w_2 k_2} I_2 = k_{21}\dot{I}_2 \tag{3.13}$$

makes appear a current that is reflected to the field circuit from the rotor; it, first, has a considerable magnitude due to specific rotor short-circuit construction and, secondly, depends of the load on the motor greatly.

3.4 Rotor Torques

As it is known from DC motors study, motor shaft torque T_s is proportional to armature current and amount of the machine flux per pole, i.e. the torque results from the product ΦI_a.

Similarly, in the case of an induction motor, the torque is proportional to product of the flux per stator pole and the rotor current. However, yet there is a factor that has to be taken into account, namely the power factor of the rotor. Therefore,

$$T_s = k\Phi I_2 \cos\varphi_2 .$$

Here, I_2 is the rotor current at standstill; φ_2 is the phase angle between rotor emf and rotor current; k is a constant.

Let us mind that emf induced in the rotor due to M (see Fig. 3.7) is defined by magnetic flux Φ. For this reason, if the emf at standstill is E_2, we may write

$$T_s = k_1 E_2 I_2 \cos\varphi_2 , \tag{3.14}$$

where $k_1 = 3 / 2\pi n_1$ is a new constant that is given here without deriving, for simplicity. Besides it is obvious that induced emf in the rotor will take a sine waveform if alternating flux in the stator is sinusoidal due to appropriate excitation.

From expression (3.14), it becomes clear that as phase shift in rotor circuit increases (power factor decreases) the torque developed by the rotor decreases and vice versa.

Starting torque

The torque developing by the motor at the very starting instant is called **starting torque**. In some cases, it is greater than the normal running torque, whereas in some other cases it is somewhat less.

Let E_2, R_2, X_2 and Z_2 be rotor electric parameters at standstill. These yields

$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}; \ \cos\varphi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} .$$

The standstill (or starting) torque is

$$T_{st} = k_1 E_2 \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

and final expression for finding induction motor starting torque will look like

$$T_{st} = \frac{3}{2\pi n_1} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}. \tag{3.15}$$

If supply voltage V_1 is known to be constant then the flux Φ hence emf E_2 both will be constant; these will result in the following simple relationship as regards the motor starting torque:

$$T_{st} = k_2 \frac{R_2}{R_2^2 + X_2^2} = k_2 \frac{R_2}{Z_2^2}, \tag{3.15a}$$

where k_2 is one more new constant.

Starting torque in a squirrel-cage motor. The resistance of a squirrel-cage motor is fixed and small as compared to its reactance which is very large, especially at the start, because at standstill, frequency of the rotor currents equals the power supply frequency, since s =1.

Hence, the starting current I_2 of the rotor, though very large in magnitude, lags by a very large angle behind E_2 with the result that the starting torque per ampere is very poor. It is roughly 1.5 times the full-load torque, although the starting current is 5…7 times the full-load current. Hence, such motors are not useful where the motor has to start against heavy loads.

Starting torque of a slip-ring motor. The starting torque of such a motor can be increased by improving its power factor due to added external resistance in the rotor circuit. Practical implementation is the star-connected rheostat (see Fig. 3.4), with the resistance being progressively cut out as the motor gathers speed. Addition of external resistance, however, increases the rotor impedance and so reduces the rotor current. At first, the effect of improved power factor predominates the current-decreasing effect of impedance. Hence, starting torque is increased. But after a certain point, the effect of increased impedance predominates over the effect of improved power factor and so the torque begins decreasing.

A condition for maximum starting torque. It can be proved that starting torque collects maximum when rotor resistance equals the rotor reactance.

Actually, if in accordance with (3.15a)

$$T_{st} = \frac{k_2 R_2}{R_2^2 + X_2^2},$$

then a known condition

$$\frac{dT_{st}}{dR_2} = k_2 \left[\frac{1}{R_2^2 + X_2^2} - \frac{R_2\left(2R_2\right)}{\left(R_2^2 + X_2^2\right)^2} \right] = 0$$

yields

$$R_2^2 + X_2^2 = 2R_2^2, \text{ or } R_2 = X_2.$$

This will result in $T_{st\max} = \dfrac{k_2}{2R_2}$.

Effect of supply voltage regulation on starting torque. On the basis of (3.15) one can readily find that starting torque is proportional to supply voltage squared:

$$T_{st} = \frac{k_3 V_1^2 R_2}{R_2^2 + X_2^2} = \frac{k_3 V_1^2 R_2}{Z_2^2}, \tag{3.16}$$

since $E_2 \cong k_{21}^{'} V_1$.

Also, we can see k_3 as yet another constant in (3.16).

It is evident that the torque is subject to any changes in supply voltage. A change of 5 percent in supply voltage, for example, will produce a change of approximately 10% in rotor torque. This fact is of importance in commonly used wye-delta and autotransformer arrangements for motor starting.

Torque under running condition can be determined if one takes into account slip frequency for rotor currents and corresponding change in circuit impedances:

$$T_s = \frac{k\Phi s E_2 R_2}{Z_{2S}^2}. \tag{3.17}$$

Another form utilizes the rotor emf E_2 alone:

$$T_s = \frac{k_1 s E_2^2 R_2}{R_2^2 + \left(sX_2\right)^2}. \tag{3.18}$$

Both forms show clear torque dependence on the rotor slip. Because of varying slip AC current frequency changes thus affecting both emf induced (numerator of (3.18)) and rotor impedance (denominator). It is obvious that motor slip, in turn, depends on load torque: the more the load, the more the slip, the more the motor torque must be developed to manage the increased load. It is also clear that this dependence on the slip is not linear. There exists a slip for which a maximum torque is attained. The condition can be derived from (3.18) by a conventional procedure.

In the end, we get $R_2 = sX_2$, i.e. the slip corresponding to the maximum torque and called **breakdown** or **pull-out** slip will be

$$s_b = \frac{R_2}{X_2}.$$

Rotor torque and breakdown slip. A relative rotor torque (with respect to $T_{\max}$ at any slip) can be expressed in terms of breakdown slip by an empiric equation:

$$T_s\,(s) = \frac{T_{\max}}{\frac{s_b}{s} + \frac{s}{s_b}}. \tag{3.19}$$

A family of torque vs. slip curves for different magnitude of rotor resistance (as compared to rotor reactance) is shown in Fig. 3.8,a.

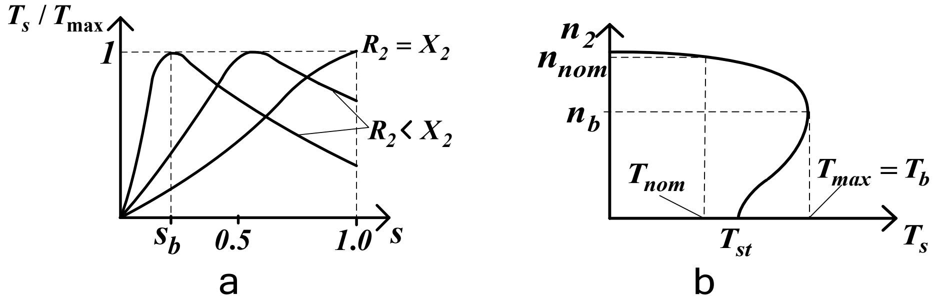


Fig. 3.8 – The torque/slip (a) and speed/torque (b) curves

When adopted, condition $R_2 = sX_2$ gives a relationship for the biggest motor torque:

$$T_{\max} = \frac{3}{2\pi n_1}\frac{E_2^2}{2X_2}. \tag{3.20}$$

The torque developed by a conventional 3-phase motor depends on its speed but the relation between the two cannot be represented by a simple equation. It is easer to show the relationship in the form of a curve (Fig. 3.8,b).

The starting torque (at $n_2 = 0$) is estimated as $T_{st} = 1.5T_{nom}$ and the maximum torque (also called breakdown torque) as $T_{\max} = 2.5T_{nom}$.

A full-load motor runs at a speed of n_{nom}. When mechanical load increases, motor speed decreases till the motor torque again becomes equal to the load torque. As long as two torques are in balance, the motor will run at constant (but lower) speed. However, if the load torque exceeds $2.5T_{nom}$, the motor will suddenly stop (breaks down).

On the basis of investigation above, we resume:

1. The maximum torque is independent of rotor resistance as such.
2. By varying rotor resistance, we are sure to get maximum torque at any desired slip (or motor speed).
3. Maximum torque varies inversely as the standstill reactance.
4. Maximum torque varies directly as the square of applied voltage.
5. For obtaining maximum torque at starting (s=1), rotor resistance must be equal to rotor reactance.

3.5 Losses and Efficiency

The losses taking place in an induction motor are the same as in DC machines.

The iron losses of the rotor are negligible compared to those of the stator (P_{m1}) because frequency of rotor currents is always small.

An induction motor develops gross torque T_g due to gross power P_{mech} being remained in the rotor assembly after wasting copper loss P_{c2}. The shaft torque might be

$$T_g = \frac{P_{mech}}{\omega_1} = \frac{P_{mech}}{2\pi n_1}, \qquad (3.21)$$

if no mechanical loss occurred.

Output rotor power P_2 is less than P_{mech} because of rotor friction and windage losses. Therefore at the output the motor develops a less actual torque as compared to T_g:

$$T_s = \frac{P_2}{\omega_2} = \frac{P_2}{2\pi n_2}. \qquad (3.22)$$

In (3.21) and (3.22), n_1 and n_2 have not commonly used *rpm* but *rps* unit.

However, if they are in rpm, previous relationships associated with obtainable motor torques will accept forms:

$$T_g = \frac{P_{mech}}{2\pi\, n_1/60} = \frac{60}{2\pi}\frac{P_{mech}}{n_1} = 9.55\frac{P_{mech}}{n_1}\ (Nm) \qquad (3.21a)$$

and

$$T_s = \frac{P_2}{2\pi\, n_1/60} = \frac{60}{2\pi}\frac{P_2}{n_2} = 9.55\frac{P_2}{n_2}\ (Nm). \qquad (3.22a)$$

Power distribution diagram for an induction motor is given in Fig. 3.9 (relates to one motor phase).

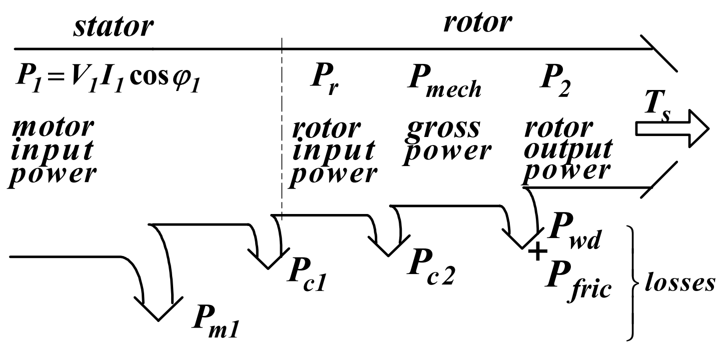


Fig. 3.9 – Power distribution diagram of an induction motor

Induction motor efficiency $\eta = \frac{P_2}{P_1}$ is much dependent on the output power. Conventionally, low power motors obtain efficiency from 40 to 60 percent. Motors having power in the range of 1…100 kW are more effective ones: η varies from 70 % to 90 %. The most powerful motors can have the efficiency from 92% to 96 %.

3.6 Practical Starting of Different Motors

If normal voltage is applied to the stationary motor, then a very large initial current is taken by the stator winding, at least, for a short while. It should be remembered that exactly similar conditions exist in the case of DC motor, if it is put directly across the supply line, e.g. at the moment of starting it, there is no back emf to oppose the initial current rush. Induction motors when switched directly, take five to seven times their full-load current meanwhile developing only 1.5 to 2.5 times their full-load torque.

This initial excessive current is objectionable because it will produce large line-voltage drop that, in turn, will affect the operation of other electrical equipment connected to the same lines.

Ways for starting induction motors may vary depending on type of the rotor and particular purpose of the motor or rated power.

The squirrel-cage motor

Squirrel-cage motors employ a few methods and appropriate auxiliaries enumerated below.

1. Direct-switching or line starting.
2. Primary resistors (rheostat) or reactor.
3. Autotransformer.
4. Star-delta switches.

In 2, 3, 4 methods, terminal voltage of the squirrel-cage motor is being reduced during starting interval.

Primary resistors. Their purpose is to drop some voltage and hence reduce the voltage applied across the motor terminals. In this way the initial current drawn by the motor is reduced. However, it should be noted that whereas current varies directly as voltage, the torque varies as square of applied voltage. If the applied voltage is reduced by 50%, starting current is reduced by 50%, but torque is reduced to 25% of the full-voltage value.

This method is useful for smooth starting of small machines only.

Autotransformers. Such starters, known variously as auto-starters or compensators, consist of an autotransformer equipped with necessary switches.

The method can be used both for wye- and delta-connected motors.

Wye–delta starter. This method is used for motors which are built to run normally with the stator winding that is delta-connected. The arrangement involves a two-way switch which connects the motor in ***star*** for ***starting*** and then in ***delta*** for normal ***running***.

The method is cheap and effective in cases when the starting torque is required not to be more than 1.5 times the full-load torque. Hence, it is used for machine tools, pumps, motor-generators etc.

The slip-ring motor

As a rule, the only method is employed for these motors, namely usage of rotor rheostats.

These motors are practically always started with full line voltage, applied across the stator terminals. The value of starting current is adjusted by introducing a variable resistance in the rotor circuit. The controlling resistance is in a form of star (or wye)-connected rheostat (see previous Fig. 3.3).

It has been already shown that when increasing the rotor resistance, we not only reduce the rotor (hence stator) current at starting but also increase the starting torque due to improved power factor. Such motors can be started under load.

The additional resistance is utilized for starting purpose only. Later on, when motor begins running under normal condition the rings are short-circuited and brushes are lifted above them.

Double squirrel-cage Motor

The main disadvantage of a squirrel-cage motor is its poor starting torque, because of its low rotor resistance. The starting torque could be increased by having a cage of high resistance, but then the motor will have poor efficiency under normal running conditions.

The difficulty with a cage motor is that its cage is permanently short-circuited, so no external resistance can be introduced temporarily in its rotor circuit during starting interval.

A double-squirrel cage rotor was designed to avoid the difficulty just mentioned. The assembly contains two independent cages on the same rotor, one inside the other (Fig. 3.10).

The outer cage consists of high-resistance metal bars (1), whereas the inner cage is made of low-resistance copper bars (2).

Therefore, outer cage has high resistance and low ratio of X_2/R_2 ,

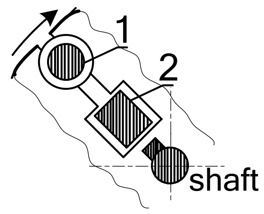


Fig. 3.10 – The double cage

Meanwhile, the inner cage has low resistance but, being placed deep in the rotor, has a large ratio of X_2/R_2 .

Hence the outer cage develops maximum torque at starting, while the inner cage does so at about 15% slip.

At starting and at large slips, frequency of induced emf in the rotor is high. So the reactance of the inner cage ($2\pi f_2 L$) and therefore its impedance are both high. Hence, very little current flows in it. Most of the starting current is confined to outer cage.

As the speed increases, the frequency of the rotor emf decreases, so that the reactance and hence the impedance of inner cage decreases and becomes very small under normal running conditions. Most of the current then flows through it and hence it develops the greater part of the motor torque.

In fact, when speed is normal, frequency of rotor emf is so small that the reactance of both cages is practically negligible. The current is carried by two cages in parallel, giving a low combined resistance. Hence, it has been made possible to construct a single machine which has a good starting torque with reasonable starting current and which maintains high efficiency and good speed regulation under normal operating conditions.

The torque-speed characteristic of a double cage motor may be approximately taken to be the sum of two motors, one having a high-resistance rotor and the other a low-resistance one.

Such motors are particularly useful when frequent starting under heavy loads is required.

3.7 Speed Control

A 3-phase induction motor is practically a constant-speed machine like a DC shunt motor. The speed regulation of induction motor (having low equivalent rotor impedance) is usually less than 5 % at full-load.

Different methods, by which speed control of induction motors is achieved, may be grouped under two main items.

1. Control as viewed from the stator side.

This is performed by:

a) changing the applied voltage;
b) changing the applied frequency;
c) changing the number of stator poles.

2. Control as viewed from the rotor side.

In this case we may arrive to:

a) using the rheostat control;
b) operating two motors in concatenation (cascade motor performance),
c) introducing an extra emf in the rotor circuit.

Consider the particular features of each.

Group 1

Changing the applied voltage. The method, though the cheapest and the easiest, is rarely used because:

– a large change in voltage is required for a relatively small change in speed;

– this large change in voltage will result in a large change in the flux density thereby seriously disturbing the magnetic conditions of the motor.

Changing the applied frequency. This method is also used very rarely. Synchronous speed of an induction motor is given by $n_1 = \frac{60 f_1}{p}$ and rotor speed is $n_2 = \frac{60 f_1}{p}(1-s)$.

Obviously, the shaft speed n_2 can be changed by changing the supply frequency f_1. But the range over which the motor speed may be varied is limited. This method is used for applications where a unique mains is possible to supply the motor (small rated power, basically).

Changing the number of stator poles. From the above equation it is clear that the synchronous (and hence the running) speed of an induction motor could also vary if we change the number of stator poles. This method is easily applicable to squirrel-cage motors because the squirrel-cage rotor adapts itself to any reasonable number of stator poles. This change of poles number is achieved by having two or more entirely independent stator winding in the same slots.

Each winding gives a different number of poles and hence different synchronous speed.

This method has been used for elevator motors, traction motors and also for small motors that drive machine tools.

Group 2

Rotor rheostat control. In this method, which is applicable to slip-ring motors only, the motor speed is reduced by introducing an external resistance in the rotor circuit (see Fig. 3.3 again). It is obvious that for a given torque, the slip can be increased, i.e. the speed can be decreased by increasing the rotor resistance $\boldsymbol{R_2}$.

To apply the method, a rotor starter may be used.

One serious disadvantage of this method is that with increase in rotor resistance, Cu-loss ($\boldsymbol{I_2^2 R_2}$) also increases; this decreases the operating efficiency of the motor. In fact, the loss is directly proportional to the reduction in the speed.

The second disadvantage is that the speed is affected by amount of the rotor and load resistance simultaneously, i.e. it depends essentially on two varying parameters at once.

Control with the help of the rotor rheostat is used where and when a speed change is needed for a short time only.

3.8 Terms and Concepts

Induction motor is a motor, in which the rotor does not receive electric power by conduction, but by induction like the transformer secondary does.

Squirrel-cage rotor is a cylindrical iron core with parallel slots for carrying the rotor conductors, which are not wires, but consist of heavy bars of low-resistance metals or alloys.

Slip-ring (wound) rotor is that wound for as many poles as the number of stator poles.

Rotating field is that having constant magnetic flux and running around the stator at synchronous as regards the AC supply speed.

Slip is a relative difference between synchronous and actual speed of the rotor. Absolute value of such a difference is called the **slip speed**.

Starting (or **breakaway) torque** is the torque developed by the motor at the instant of starting when the rotor is yet "locked". **S.T.** Commonly, **S.T.** is 1.2 to 1.5 times the nominal torque developed while rated running.

Mechanical (speed-torque) characteristic, $\boldsymbol{n_2 = f(T_s)}$**,** is aimed to introduce the most important property of a motor to collect the required speed depending upon the developing/loading torque on the motor shaft.

Breakdown (else, **Stalling, Maximum**, P**ull-out) torque** $\boldsymbol{T_b}$ is a specific torque associated with maximum loading; due to appropriate slip, maximum torque developed is able to provide the motor with a steady rotation at minimum speed.

3.9 Review Questions

1. What are the basic principles of an induction motor?
2. How can the direction of AC motor rotation be reversed?
3. What is the difference between cage-type and wound-rotor motors?

4. Why do we call induction motor asynchronous?

5. How does the slip depend on the load?

6. What factors affect the torque of an induction motor?

7. Depict a typical speed-torque characteristic of an induction motor. What peculiar points can we see in the graph?

8. What are the ways of starting a three-phase induction motor?

9. What motor may a control resistor aimed to vary motor speed be applied for?

10. What are the fundamentals of 3-phase induction motor operation?

3.10 Problems

Solved problems

Problem 1. A slip-ring induction motor runs at ***290 rpm*** at full load, when connected to ***50 Hz*** supply.

Determine the number of poles and slip.

Solution

Since n_2 is ***290 rpm***, speed n_1 has to be near it, say ***300 rpm***.

1. If n_2 is assumed as ***300 rpm***, then using formula $n_1 = \frac{60 f_1}{p}$ the number of pole pairs can be found:

$$p = \frac{60 f_1}{n_1} = \frac{60 \cdot 50}{300} = 10 .$$

2. Slip $s = \frac{n_1 - n_2}{n_1} = \frac{300 - 290}{300} = 0.033$, or ***3,33*** %.

Problem 2. The power input to the rotor of ***440 V***, ***50 Hz***, ***6***-poles, ***3***-phase induction motor is ***80 kW***. The electromotive force is observed to make ***100*** complete alternations per minute.

Calculate the slip, the rotor speed, rotor copper loss per phase.

Solution

1. ***100*** alternations per minute equal $100/60$ cycle/sec, that is $f_2 = 1.67$ ***Hz***. As $f_2 = sf_1$, then the slip is

$$s = \frac{f_2}{f_1} = \frac{1.67}{50} = 0.033 \text{ , or } 3.3\% .$$

2. Rotor speed

$$n_2 = n_1(1 - s) = \frac{60 f_1}{p}(1 - s) = \frac{60 \cdot 50}{3}(1 - 0.033) = 966.7 \ rpm .$$

3. Rotor copper loss per phase is equal to $\frac{1}{3}sP_1$ (P_1 is full rotor input power), that is

$$P_c = \frac{1}{3} \cdot 80 \cdot 10^3 \cdot 0.033 = 888.8\ W\ .$$

Problem 3. A *3*-phase induction motor with a ***wye***-connected rotor has an induced emf of ***80 V*** between slip-rings at standstill in open-circuit mode. The rotor has a resistance and reactance per phase of ***1Ω*** and ***4Ω*** respectively.

Calculate phase current and power factor when:

1) slip-rings are short-circuited;
2) slip-rings are connected to a wye-connected rheostat with $R_r = 3\Omega$ per phase.

Solution

1. Determine: a) standstill phase emf of the rotor $E_{ph} = \frac{E_{load}}{\sqrt{3}} = \frac{80}{1.73} = 46.2\ V$;

b) rotor impedance $Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{1^2 + 4^2} = 4.12\ \Omega$;

c) rotor phase current $I_2 = \frac{V_{ph}}{Z_2} = \frac{46.2}{4.12} = 11.2\ \Omega$;

d) power factor $\cos\varphi_2 = \frac{R_2}{Z_2} = \frac{1}{4.12} = 0.243$.

As power factor is low, the starting torque is small.

2. If wye-connected rheostat is connected to slip-rings then (per phase):

a) rotor circuit resistance $R_{2r} = R_2 + R_r = 1 + 3 = 4\ \Omega$;

b) rotor impedance $Z_{2r} = \sqrt{R_{2r}^2 + X_2^2} = \sqrt{4^2 + 4^2} = 5.66\ \Omega$;

c) rotor phase current $I_{2r} = \frac{E_{ph}}{Z_{2r}} = \frac{46.2}{5.66} = 8.16\ A$;

d) power factor $\cos\varphi_{2r} = \frac{R_{2r}}{Z_{2r}} = \frac{4}{5.66} = 0.707$.

It can be seen that improvement (***2.6*** times) in the power factor is greater than the decrease (***1.5*** times) in current because of increased impedance. Hence, the starting torque will increase with the wye-connected rheostat due to improvement in the power factor.

Problem 4. The power supplied to a three-phase induction motor is ***40 kW*** and the corresponding stator losses are ***1.5 kW***.

Calculate:

1) total mechanical power developed and the rotor Cu-loss if slip s is 0.4 per unit;

2) output power of the motor if friction and windage losses are $0.8\ kW$;

3) efficiency of the motor.

Neglect rotor core loss.

Solution

1. Input power to the rotor $P_{in\ r} = P_1 - P_{st} = 40 - 1.5 = 38.5\ kW$. From $\frac{P_{r\ c}}{P_{in\ r}} = s$ determine rotor losses:

$$P_{rc} = sP_{in\ r} = 0.04 \cdot 38.5 = 1.54\ kW .$$

Hence, mechanical power developed by the rotor

$$P_g = P_{in\ r} - P_{r\ c} = 38.5 - 1.54 = 36.96\ kW .$$

2. Output power of the motor

$$P_2 = P_g - P_{mech} = 36.96 - 0.8 = 36.16\ kW .$$

3. Efficiency of the motor $\eta = \frac{P_2}{P_1} = \frac{36.16}{40} = 0.904 = 90.4\%$.

Problem 5. The per-phase parameters of equivalent circuit (Fig. 3.7a) for a $400\ V$, $60\ Hz$, 3-phase, wye-connected, 4-pole induction motor are: $R_1 = 2R'_2 = 0.2\ \Omega$; $X_1 = 0.5\ \Omega$; $X'_2 = 0.2\ \Omega$; $X_0 = 20\ \Omega$; $R_0 = 0$.

If mechanical and iron losses at $1755\ rpm$ are $800\ W$, compute:

1) input current, 2) input power, 3) output power, 4) output torque, 5) efficiency (all for $1755\ rpm$).

Solution

1. Synchronous speed $n_1 = \frac{60 \cdot 60}{2} = 1800\, rpm$.

2. Slip $s = \frac{n_1 - n_2}{n_1} = \frac{1800 - 1755}{1800} = 0.025$.

3. From the circuit given, the equivalent impedance per-phase is

$$\underline{Z}_g = \left(R_1 + jX_1\right) + \frac{\left(\frac{R'_2}{s} + jX'_2\right) \cdot jX_0}{\frac{R'_2}{s} + j\left(X_0 + X'_2\right)} = \left(0.2 + j0.5\right) + \frac{\left(\frac{0.1}{0.025} + j0.2\right)\left(j20\right)}{\frac{0.1}{0.025} + j\left(20 + 0.2\right)} =$$

$$= \left(0.2 + j0.5\right) + \left(3.77 + j0.944\right) = 3.97 + j1.44 = 4.22e^{j20^\circ}\ \Omega\ ,$$

i.e. impedance absolute value is $\boldsymbol{4.22}\ \Omega$ and impedance phase angle $\varphi_1 = 20^o$.

4. The phase voltage $V_{ph} = \frac{V_1}{\sqrt{3}} = \frac{400}{1.73} = 231\ V$.

5. Input current $I_1 = \frac{231}{4.22} = 54.65\ A$.

6. Total input power

$$P_1 = \sqrt{3} V_1 I_1 \cos\varphi_1 = 1.73 \cdot 400 \cdot 54.65 \cdot \cos 20^\circ = 35.58\ kW.$$

7. Total power P_g transmitted via the air gap (entered the rotor circuit) is equal to the power dissipated by $\mathrm{Re}\,\underline{Z}'_2 = 3.77\ \Omega$ resistance (real part of reflected impedance of the rotor):

$$P_g = 3 I_2^2 R_2' = 3 \cdot (54.65)^2 \cdot 3.77 = 33.79\ kW.$$

Differently, the same rotor input power can be found as

$$P_g = P_1 - P_{1loss} = 35.580 - 3 \cdot (54.65)^2 \cdot 0.2 = 33.79\ kW.$$

8. Mechanical power developed in the rotor is

$$P_d = (1 - s) P_g = 0.975 \cdot 33.79 = 32.94\ kW,$$

whereas the total output power $P_2 = P_d - P_{mech} = 32.94 - 0.8 = 32.14\ kW$.

9. Output torque

$$T_s = \frac{P_2}{\omega_2} = \frac{32140}{2\pi n_2/60} = \frac{32140}{2 \cdot 3.14 \cdot 1755/60} = 174.9\ Nm.$$

10. Efficiency $\eta = \frac{P_2}{P_1} = \frac{32.14}{35.58} = 90.3\%$.

Supplement problems

Problem 6. The nameplate on ***380 V***, ***50 hp***, ***50 Hz***, four-pole induction motor indicates that its speed at rated load is ***1450 rpm***. Assume the motor operating at rated load.

Determine: 1) the slip of the motor; 2) the frequency of the rotor current; 3) the angular velocity of the stator-produced air-gap flux wave with respect to the stator; 4) the same with respect to the rotor; 5) the angular velocity of the rotor-produced air-gap flux wave with respect to the stator; 6) the same with respect to the rotor.

Problem 7. A ***150 kW***, ***3000 V***, ***50 Hz***, ***6***-pole wye-connected induction motor has a wye-connected slip-ring rotor with a transformation ratio of ***3.6*** ($w_1 : w_2$). The rotor resistance is ***0.1*** Ω per phase and its per phase leakage inductance is ***3.61 mH***. The stator impedance may be neglected.

Find the starting current and starting torque on rated voltage with short-circuited slip rings.

(Ans. ***117.4 A***, ***513 Nm***)

Problem 8. A three-phase induction motor is wound for ***4*** poles and is supplied from a ***50 Hz*** system.

Calculate: 1) the synchronous speed; 2) the speed of the rotor when the slip is ***4*** percent; 3) the rotor frequency when the speed of the rotor is ***600 rpm***.

(Ans. ***1500 rpm***, ***1440 rpm***, ***30 Hz***)

Problem 9. The power input to a ***3***-phase induction motor is ***60 kW***. The stator total losses are ***1 kW***.

Find the total mechanical power developed and the rotor copper loss per phase, if the motor is running with a slip of ***3%***.

(Ans. ***57.2 kW***; ***590 W***)

Problem 10. A ***500 V***, ***3***-phase induction motor has a stator impedance of ***(0.062 +j0.21) Ω***. The equivalent rotor impedance at standstill is the same. The magnetizing current is ***36 A***, the core loss is ***1500 W***, and the mechanical loss is ***750 W***.

Evaluate the output power, efficiency and power factor at a slip of ***2%***.

(Ans. ***100 hp***; ***0.926***; ***0.89***)

Problem 11. A ***3***-phase, ***60 Hz***, 4-pole induction motor has a rotor leakage reactance of ***0.8 Ω*** per phase and a rotor resistance of ***0.1 Ω*** per phase.

What additional resistance is to be inserted in the rotor circuit so that the motor could have the maximum starting torque? Use the rotor circuit at Fig. 3.3 for your calculation.

(Ans. ***0.7 Ω***)

Problem 12. A ***20 hp***, ***3***-phase, ***400 V***, ***60 Hz***, ***4***-pole induction motor delivers full-load at ***5%*** slip. The mechanical rotating loss is ***400 W***.

Calculate: 1) the electromagnetic torque; 2) the shaft torque; and 3) the rotor copper loss.

(Ans. ***85.5 Nm*** ; ***83.3 Nm***; ***806.3 W***)

Problem 13. A ***three***-phase induction motor has ***six*** poles.

1. Calculate the speed of the magnetic field in rev/min, if the line frequency is ***60 Hz***.

2. Repeat the calculation if the frequency is changed to ***50 Hz***.

Problem 14. A ***four***-pole induction motor operating at a frequency of ***60 Hz*** has a full-load slip of ***4*** percent.

Find the frequency of the voltage induced in the rotor at the instant of starting and at full load.

Tutorial test

1. ***An induction motor is so called because its operation depends on the phenomenon of***

a) self-induction b) mutual induction

c) eddy currents d) hysteresis

2. ***The no-load speed of an induction motor depends on***

a) the supply frequency b) the number of its poles

c) the maximum flux/phase only (a) and (b)

3. ***The frequency of rotor current in a 6-pole, 50 Hz, 3-phases induction motor running at 950 rpm is … Hz***

a) 2.5 b) 1.5

c) 5.0 d) 0.05

4. ***The torque of an induction motor is proportional to***

a) $1/V$ b) $\sqrt{V}$

c) V^2 d) V^3

5. ***In an induction motor, the ratio of rotor copper loss and rotor input is given by***

a) $1/s$ b) s

c) $(1-s)$ d) $s/(1-s)$

6. ***The speed regulation of a squirrel-cage induction motor is very good primarily because of its relatively low … under normal operation conditions***

a) rotor impedance b) losses

c) power factor d) rotor frequency

7. ***Regarding skewing of motor bars in a squirrel-cage induction motor (SCIM) which statement is false?***

a) it prevents cogging b) it increases starting torque

c) it produces more uniform torque d) it reduces motor “hum”

8. ***The principle of operation of a 3-phase induction motor is most similar to that of a***

a) synchronous motor
b) repulsion start induction motor
c) transformer with a shorted secondary
d) capacitor-start, induction-run motor

9. ***In a 3-phase induction motor, the relative speed of stator flux with respect to … is zero***

a) stator winding
b) rotor
c) rotor flux
d) space

10. ***A 3-phase, 4-pole, 50 Hz induction motor runs at a speed of 1440 rpm. The rotating field produced by the rotor rotates at a speed of … rpm with respect to the rotor***

a) 1500
b) 1440
c) 60
d) 0

11. ***In a 3-phase induction motor, the rotor field rotates at synchronous speed with respect to***

a) stator
b) rotor
c) stator flux
d) none of the above

12. ***In a case of a 3-phase induction motor having $n_1 = 1500$ rpm and running with $s = 0.04$***

a) revolving speed of the stator flux is space is … rpm
b) rotor speed is … rpm
c) speed of rotor flux relative to the rotor is … rpm
d) speed of rotor flux with respect to the stator is … rpm

13. ***A 6-pole, 50 Hz, 3-phase induction motor is running at 950 rpm and has rotor copper loss of 5 kW. Its rotor input is … kW***

a) 100
b) 10
c) 95
d) 5.3

14. ***If the stator voltage and frequency of an induction motor are reduced proportionally, its***

a) locked rotor current is reduced
b) torque developed is increased
c) magnetizing current is decreased
d) both (a) and (b)

15. ***A double squirrel-cage motor (DSCM) scores over SCIM (squirrel-cage induction motor) in the matter of***

a) stating torque
b) high efficiency under running conditions
c) speed regulation under normal operating conditions
d) all of the above

16. ***Fill in the blanks, selecting correct answers from the following items:*** 'increases', 'decreases'

a) speed …
b) slip …
c) rotor induced emf …
d) rotor current …

17. ***The fractional slip of an induction motor is the ratio***

a) rotor Cu-loss/rotor input
b) stator Cu-loss/stator input
c) rotor Cu-loss/rotor output
d) rotor Cu-loss/stator Cu-loss

18. ***The power factor of a squirrel-cage induction motor is***

a) low at light loads only
b) low at heavy loads only
c) low at light and heavy loads both
d) low at rated load only

19. ***The efficiency of a three-phase induction motor is approximately proportional to***

a) (1-s)
b) s
c) n_2
d) n_1

Chapter 4

SYNCRONOUS MACHINES

Synchronous machines are called so because they operate at constant speed and constant frequency under steady state. Like most rotating machines a synchronous machine can function either as a motor or as a generator. AC power systems produce more power per weight of the equipment than DC systems; however, all AC generators require constant-speed drive to maintain a constant AC frequency.

4.1 Construction of a Synchronous Machine

AC synchronous (synch) generators, or alternators (as they are usually called), operate on the same fundamental principles of electromagnetic induction as DC machines. The simplest form of an AC generator is a coil being rotated in a magnetic field by a prime mover and producing the electric power. In practice, most machines operate in opposite way, i.e. the rotating part (rotor) serves as source of the field (typically based on an electromagnet fed from a DC supply) while the coils of the stator have emf induced in them to provide the output.

Examples for both particular arrangements of a four-pole machine are shown in Fig. 4.1.

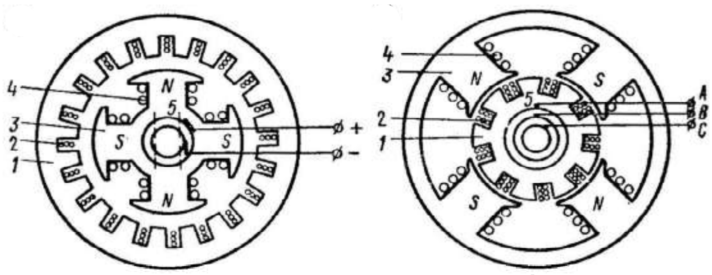


Fig. 4.1 – Synchronous machines:
with rotating field (a); with rotating armature (b)

A standard construction of the configuration presented by Fig. 4.1,a consists of the armature winding (2) mounted on a stationary element (stator 1) and field windings (4) on a rotor (3), while that of Fig. 4.1,b obtains field windings (4) on the stator (3). Here, the rotor (1) carries the armature (or power) winding (2). Both rotors are fed from (in Fig. 4.1,a) or supply to (Fig. 4.1,b) external network with the help of slip-ring/brush mechanism (5). In contrary to DC generators, a synchronous generator does not have a commutator.

A construction along with some details explaining principles of a three-phase synchronous generator typically employed as industrial alternator are shown in Fig. 4.2. The stator (Fig. 4.2,a) consists of cast-iron frame, which supports the armature core 1, having slots on its inner periphery for housing the armature conductors 3.

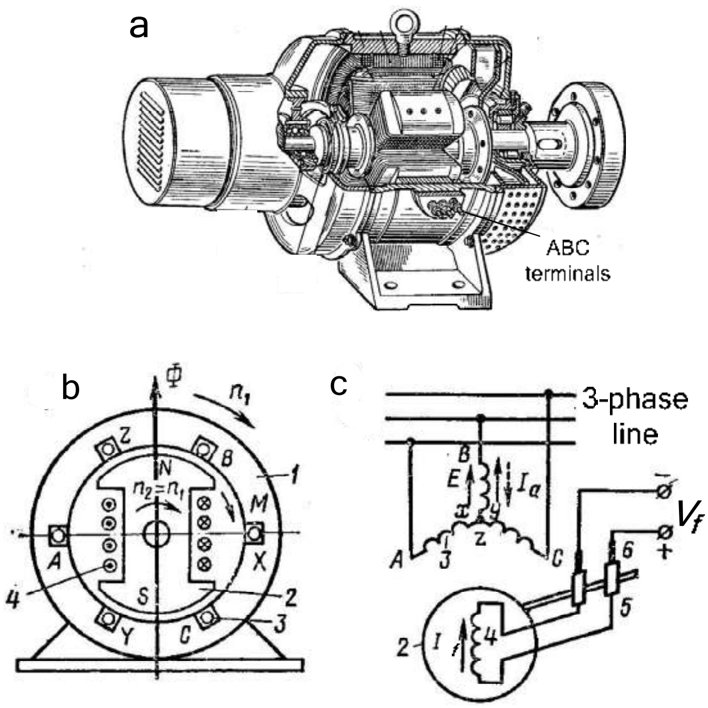


Fig. 4.2 – The 3-phase alternator:
general view (a); structure (b); wiring diagram (c)

The rotor with field winding 4 is like a fly wheel having alternate N and S poles fixed to its outer rim. The magnetic poles are excited from the direct current that is supplied by a DC source (Fig. 4.2,b).

In most cases, necessary exciting current is obtained from a small DC shunt generator which is belted or mounted on the shaft of the alternator itself. Because the field magnets are rotating, this current is supplied through two slip-rings. As the exciting voltage is relatively small, the slip-rings and brush gear (parts 5 and 6) are of light construction. Brushless excitation systems have recently been developed in which a 3-phase AC exciter and a group of rectifiers supply DC excitation to the alternator. Hence, brushes, slip-rings and commutator are eliminated.

When the rotor rotates, the stator conductors, although being stationary, are cut by moving magnetic flux, hence they have emf induced in them. Because the magnetic poles are alternately N and S, they induce such emf and hence current in armature conductors that first flows in one direction and then in the other.

Hereby, an alternating emf whose frequency depends on the number of N and S poles moving past a conductor in one second and whose direction is given by right-hand rule is produced in the stator conductors. This gives rise to an AC current produced by the alternator and further consumed by the load.

Resume again, this type of a machine is known as a ***rotating field*** AC generator. It has a few advantages over the ***rotating armature*** type presented by Fig. 4.1,b:

1. Because the output windings are stationary they are no subject to high centrifugal forces and can therefore be larger.
2. By having the output windings on the outside of the machine there is more unexpended (less expended) way for good insulation and higher voltages can be used.
3. With output windings on the outside of the machine they are more easily cooled and may therefore carry larger currents.
4. Usage of a rotating field requires using only two slip-rings and two brushes; also the field current required is relatively small.

These advantages show that a larger output can be obtained from a smaller machine.

Types of alternator rotors

There are two types of rotor used in AC generators: a salient pole rotor and a cylindrical or round rotor.

Salient pole rotor (Fig. 4.3,a). The poles in this rotor are built up of lamination. Each pole (four pieces in the figure) carries a field coil, which is fed with DC. Because of high windage losses, this type of machines is used at low speed and generally has a large number of poles.

Fig. 4.3 – The rotors of a synchronous machine: salient pole rotor (a); cylindrical rotor (b)

Cylindrical Rotor (Fig. 4.3,b). This type of a rotor is often met in aircraft AC generators and suits for turbo-alternators, which run at very high speeds. The rotor consists of a smooth solid forge-steel cylinder, having a number of slots milled out at intervals along the outer periphery (and parallel to the shaft) for accommodating field coils. It obtains less windage losses and is less susceptible to centrifugal forces than the salient pole rotor. The cylindrical rotor also provides for improved magnetic flux distribution.

Both the salient pole and cylindrical rotors are also used in AC motors. Among the two, the cylindrical rotor is preferred again.

4.2 Three-phase Generator Basic Quantities

Induced electromotive forces

A three phase synch generator (see Fig. 4.2) has three sets of output windings, each physically displaced from the other two by 120° (this can be seen clearly in Fig. 4.2,b). Let the rotor be also similar to that depicted in Fig. 4.3,a.

The three-phase alternator can be viewed really as three single-phase generators on one stator, all using a common field. Due to construction of the machine, the emf generated in each winding is displaced by phase of $\pm 120^o$ with respect to each other:

$$e_A = E_m \sin 2\pi ft;$$
$$e_B = E_m \sin\left(2\pi ft - 120^\circ\right); \qquad (4.1)$$
$$e_C = E_m \sin\left(2\pi ft + 120^\circ\right).$$

Waveforms in Fig. 4.4 illustrate such a displacement in time domain where one sine wave either leads or lags the others by one third the duration of the full cycle.

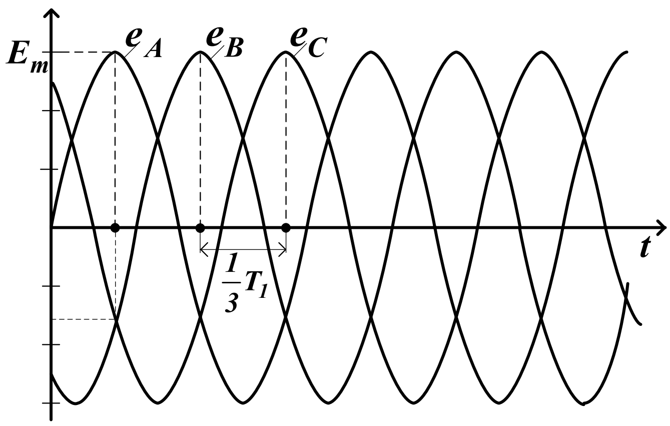


Fig. 4.4 – The emf of three-phase generator

The phase windings are commonly star- (alternative name ‘wye’) or delta-connected.

Note: when the voltage of one phase is at its peak value, the voltages in other two are at half peak value and have inverse direction. The sum of the voltages is therefore zero at any one instant and this statement is true for all instantaneous values of three-phase voltage.

Three phase systems are most generally used in generation, transmission and distribution of electric power.

The advantages, if compared with one or two phase systems, are as follows:

1. Higher power-to-weight ratio.
2. Three phase machines can operate in parallel more readily.
3. Loads can be arranged to share power from the generator.

Rotor speed and frequency of generated AC power

In a synchronous generator, there exists a definite relationship between the speed of rotor rotation $\boldsymbol{n_2}$, the frequency of generated emf $\boldsymbol{f_1}$ and the number of rotor poles (pair poles $\boldsymbol{p}$):

$$f_1 = \frac{pn_2}{60}\ Hz\ . \tag{4.2}$$

Let us discuss formula (4.2).

Since one cycle of emf is induced in a conductor when one pair of magnet poles passed over it, the number of emf cycles produced for one revolution of the rotor is equal to the number of pair poles and the number of revolutions per second. But for electric machines, rotational speed $\boldsymbol{n_2}$ is conventionally termed in rpm; that is why we can see the specific number in denominator of (4.2).

A direction of the induced emf is given by right hand rule.

In formula (4.2), variable $\boldsymbol{n_2}$ presenting the rotor speed is known as synchronous speed, because it is the speed at which the alternator must run in order to generate emf of the required frequency.

Actually, for frequency and number of poles given, the speed is fixed. For example, to produce a frequency of 50 Hz, the alternator will have to operate at different speeds, depending on a particular construction of its rotor (Table 4.1).

Table 4.1

Number of pair poles	1	2	3	6	12	24
Rotor speed (rpm)	3000	1500	1000	500	250	125

4.3 Alternator Power Angle. Voltage Regulation

Vector diagrams

If a load is applied to an alternator and this load varies, terminal voltage $\boldsymbol{V}$ is also found to vary as in DC generators. But there is some difference associated with emf $\boldsymbol{E}$ induced.

The variation in both the terminal voltage and the emf occurs due to the following reasons:

1. Voltage drop because of armature resistance $\boldsymbol{R_a}$.
2. Voltage drop due to armature leakage reactance X_L.
3. Voltage drop because of armature reaction.

Armature Resistance reason. The armature (per phase) resistance $\boldsymbol{R_a}$ causes a voltage drop of $\boldsymbol{IR_a}$ which is in phase with the armature current $\boldsymbol{I_a}$.

However, this drop is practically negligible since $\boldsymbol{R_a}$ is very small.

Armature Leakage Reactance. When current flows through the armature conductors, there are fluxes which do not cross the air-gap, but take different paths. Such fluxes are known as **leakage fluxes**. The leakage flux is practically independent of saturation, but is dependent on $\boldsymbol{I_a}$ and collects a phase angle as regarding the terminal voltage $\boldsymbol{V}$. This leakage flux sets up an emf of self-inductance which is known as **reactance emf** and which is ahead of $\boldsymbol{I_a}$ by 90°. Hence, armature winding is assumed to possess leakage reactance $\boldsymbol{X_L}$ which results in voltage drop $\boldsymbol{I_a X_L}$. As a result, a part of full generated emf (armature self-inductance emf) is wasted to overcome the reactance emf. We can write therefore:

$$\dot{\boldsymbol{E}} = \dot{\boldsymbol{V}} + \dot{\boldsymbol{I}}_a \boldsymbol{R_a} + j\dot{\boldsymbol{I}}_a \boldsymbol{X_s} = \dot{\boldsymbol{V}} + \dot{\boldsymbol{I}}_a \underline{\boldsymbol{Z}}_s . \tag{4.3}$$

Equation (4.3) is illustrated by the vector diagram in Fig. 4.5.

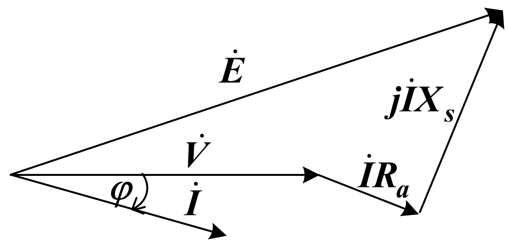


Fig. 4.5 – Vector diagram for a synchronous generator

In the above equation (4.3), the impedance $\underline{\boldsymbol{Z}}_s$ is known as the synchronous impedance; it integrates armature resistance $\boldsymbol{R_a}$ and armature total so-called synchronous reactance $\boldsymbol{X_s} = \boldsymbol{X_a} + \boldsymbol{X_L}$, thus incorporating both specified armature and load reactance.

For a round-rotor generator, the synchronous reactance can readily be measured, since it does not depend on the rotor position. In salient-pole generators, however, the synchronous reactance depends on the rotor position.

Armature Reaction. As in DC generators, armature reaction is a phenomenon raised when the armature flux influences the main field flux. Unlike the DC, in the case of alternator a power factor of the load $\boldsymbol{\cos\varphi_L}$ (or $\boldsymbol{\cos\varphi_2}$ as it is conventionally labeled in electric machine application) has a considerable effect on the armature reaction.

Consider three possible cases (Fig. 4.6):

(a) load power factor (PF) is unity ($\varphi_2 = \boldsymbol{0^o}$),

(b) PF is zero lagging ($\varphi_2 = \boldsymbol{-90^o}$), and

(c) PF is zero leading ($\varphi_2 = \boldsymbol{+90^o}$).

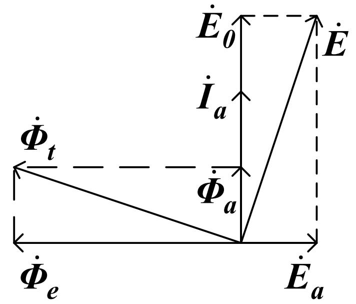


Fig. 4.6 – Armature reaction with $\boldsymbol{\cos\varphi_2 = 1}$

For ***unity power factor*** the armature flux Φ_a is cross-magnetizing (the right angle between armature Φ_a and field Φ_e fluxes).

The result is that the flux at the leading tips of poles is reduced while it increases at the trailing tips. However, these two effects nearly offset each other thus leaving the average field strength constant. It can be seen that resulted emf E is shifted as compared to no-load emf E_0 besides being changed in magnitude. In other words, armature reaction for unity PF is distortional.

Zero PF lagging (Fig. 4.7, on the left hand). Here, the armature flux is in direct opposition to the main flux. Hence, the main flux is decreased.

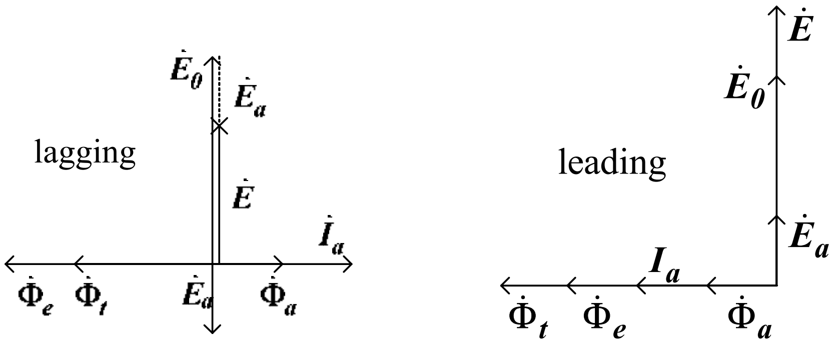


Fig. 4.7 – Armature reaction at zero margin power factors

Zero PF leading. In this case (Fig. 4.7, on the right) the armature flux is in phase with the main flux. This results in the added main flux.

Alternators regulation

As it has just been shown with a change in the load, a change of alternator output (terminal) voltage happens. This change depends not only on the amount of the load but also on the load power factor.

Similar to a DC generator, the voltage regulation for an alternator is defined as the relative rise in voltage when full-load is removed (alternator excitation and running speed are assumed to remain the same):

$$\Delta V\% = \frac{E_0 - V_t}{V_t} 100 . \tag{4.4}$$

In (4.4), E_0 and V_t are the open-circuit or no-load and the full-load (rated) terminal voltages.

Unlike what happens in a DC generator, the voltage regulation of a synchronous generator may become zero or even negative, depending upon the PF ($\cos\varphi_2$) and the load as such.

When neglecting the armature resistance and following an instance of Fig. 4.5 above, we illustrate the latter by two diagrams shown in Fig. 4.8.

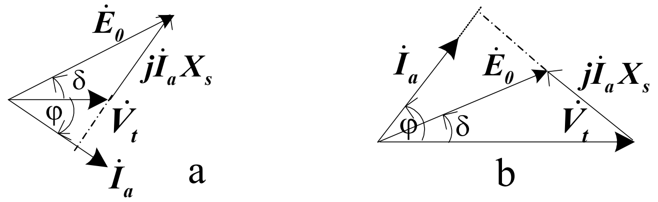


Fig. 4.8 – Vector diagrams for different loads:
lagging power factor (a), leading power factor (b)

If PF is lagging (Fig. 4.8,a), the terminal voltage is always less than E_0 and the alternator obtains clear positive regulation, i.e. terminal voltage increases when the load is removed.

In the case of leading PF, the terminal voltage may score open-circuit emf E_0 as shown in Fig. 4.8,b. It will fall down on removing the full-load, hence the regulation is negative in this case.

Power angle

Besides the angle φ of complex load impedance (resulting in an AC circuit ***power factor***), one more particular angle named 'δ-angle' can be found in the diagrams of Fig. 4.8. This term specifies alternator's ***power angle***. Physically, it is a phase shift appeared between the induced emf (open-circuit terminal voltage) and the voltage at same terminals when a load with its particular impedance, hence power factor, was thrown across them. Power development and further transfer to the load depend much on this term. That is the reason why the angle was named so.

It can be readily justified, when studying similar triangles depicted in either diagram of Fig. 4.8. Actually, one can find that

$$I_a X_s \cos\varphi = E_0 \sin\delta . \tag{4.5}$$

At the same time, the power developed by the generator and supplied to the load is

$$P_2 = V_t I_a \cos\varphi . \tag{4.6}$$

Substitution (4.5) in (4.6) yields

$$P_2 = \frac{E_0 V_t}{X_s} \sin\delta . \tag{4.7}$$

This proves that power developed by the alternator is proportional to the shift that is intended to arise between two voltages to align synch generator (as well as synch motor, as it will be shown later) and the load.

4.4 Synchronous Generators Excitation

When generator field winding (inductor) is placed in the stator and the rotor carries the armature winding the construction allows using generator case as a core. However, the brush/slip-ring mechanism can not collect a large amount of load current reliably and without loss. That is why high power generators (with more than 15 kVA rated power) are inversely arranged, i.e. have an excitation applied to the rotor and output armature windings mounted in the stator.

AC generators may have the following two types of excitation:

– separate excitation fed from a DC power source;

– the excitation taken from a special energizing unit (exciter) which is placed on the same shaft with the basic generator.

Schematic diagram for the first type generator is shown in Fig. 4.9. Here, the excitation winding 2 is connected to DC mains (DC power distribution system) by means of rheostat 3 which is used to control power.

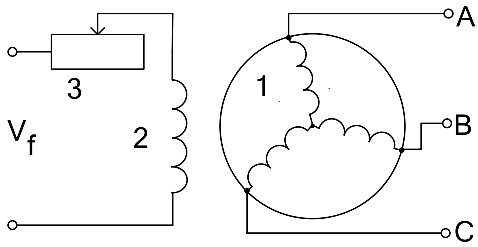


Fig. 4.9 – Synch generator with independent DC excitation

Disadvantages of the system above are as follows:

– need for a stable DC power supply;

– large energy waist when making output control;

– large-weight and expensive regulator due to considerable value of the field current required.

Therefore such excitation system is applied for generators of relatively small rated power (7.5…30 kVA).

More powerful alternators employ special exciters to produce magnetic field in the system. A schematic diagram for this type of excitation is shown in Fig. 4.10.

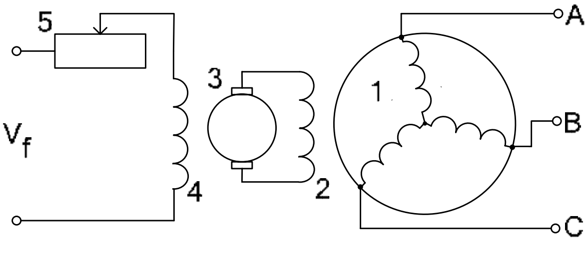


Fig. 4.10 – Schematic diagram of an alternator with the exciter:
1 – generator armature; 2 – excitation winding; 3 – exciter;
4 – exciter field winding; 5 – rheostat

The advantage of this excitation system is that it may be self-contained; if needed it requires no other supply except for powering the prime mover (not difficult problem). For this purpose, the exciter can simply be designed as a shunt-wound generator.

Excessive weight and size is a disadvantage of the system.

For aircraft application, both three-phase and single-phase alternators have been engineered. Three-phase generators of GSS type are rated at powers of 5, 7, 30, 40, 90, 120 kVA for linear voltage of 120, 208 and 360 V. Also, they are met to be adapted either for variable or constant frequency running.

Single-phase generators are basically configured from the three-phase ones.

To do this, the single-phase powering is carried out by connecting a load across any two armature terminals of three-phase machine as shown in Fig. 4.11. The third phase is in no use then.

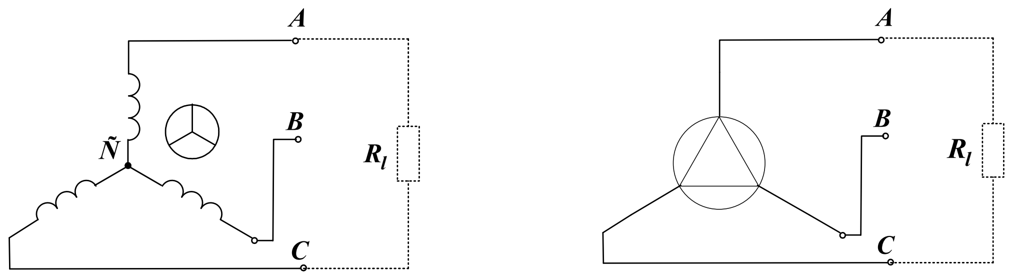


Fig. 4.11 – Single-phase supply based on a three-phase generator

In electric machines, reliability and service life are determined by three main factors: electric insulation, bearings, brush-contact reliability. To eliminate unreliable brush/slip-ring mechanism, one may use a non-contact generator.

Let us consider the design principle for achieving the non-contact generator when utilizing a ***rotating rectifier*** (Fig. 4.12).

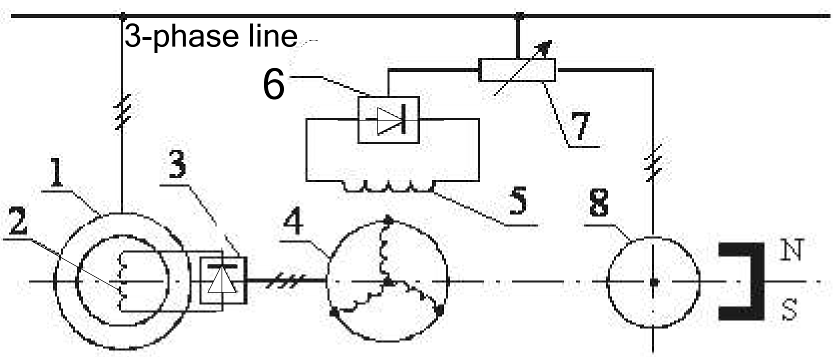


Fig. 4.12 – Diagram of synchronous generator with rotating rectifier:
1 –three-phase synch generator; 2 – excitation winding;
3, 6 – rectifying units; 4 – three-phase synchronous exciter;
5 – exciter field winding; 7 – voltage regulator; 8 – three-phase electromagnetic pilot exciter

In this case, the generator is an assembly of the following electric machines driven by a single prime mover: three-phase synchronous generator (basic alternator); three-phase synchronous exciter; three-phase electromagnetic pilot exciter. Once again, all these units are mounted on the same shaft.

When the common shaft rotates the magnetic field of the pilot exciter rotor 8 induces three-phase alternating current in windings of its stator. This current flows through the voltage regulator 7 and further through rectifier 6 to energize field winding 5 of the exciter. Due to DC power applied the exciter rotor starts to rotate in a magnetic field of the winding; as a result, a three-phase AC is induced in exciter (4) rotor. This current is applied to the main stage i.e. basic alternator 1. Rectifier 3 converts AC to DC and feeds excitation winding 2 of the main generator. The field winding is on rotating inductor. When the inductor driven by the prime mover rotates within the three-phase armature winding 1 AC power of required frequency is induced and supplies 3-phase distribution line. For example, in this way an aircraft power generating system works when supplying 115 VAC, 400 Hz 3-phase onboard mains.

4.5 Performance Characteristics

Like DC generators, peculiar property of an AC generator is presented by specific performance characteristic for better understanding the principles of energy conversion with the help of a given electric machine and realizing the conditions based on numerous requirements how to solve the problem when performing such a conversion.

Typical curves associated with synch generator operation under various distinctive conditions are shown in Fig. 4.13.

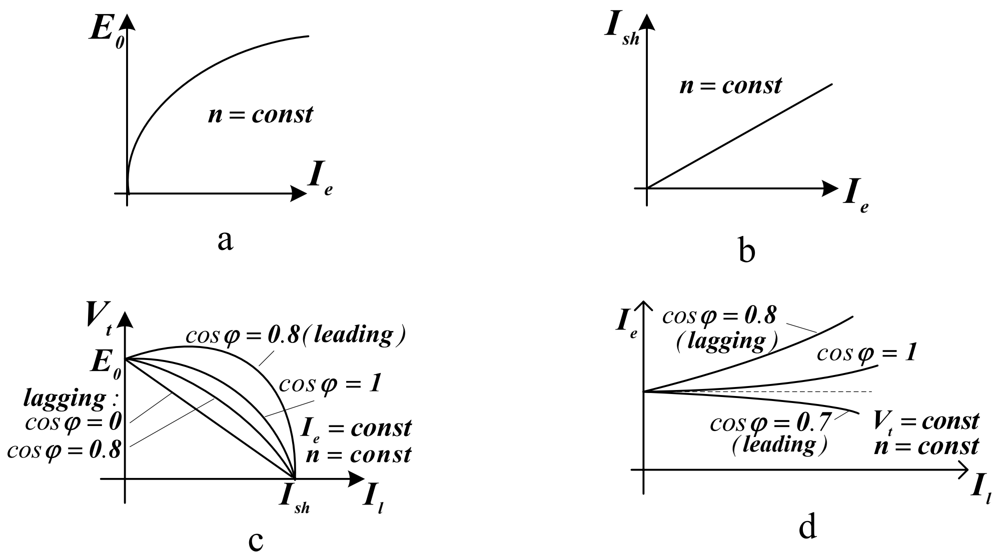


Fig. 4.13 – Basic characteristics of a synchronous generator

Here we can see:

a) no-load (open circuit) characteristic;
b) short-circuit characteristic;
c) external characteristic;
d) control characteristic.

By way of illustration, we can say that a control characteristic of the generator, if we have it at our disposal, explains which change of the field current you need to make in your generator in order to keep the required terminal voltage at given running of the rotor when a change in the load including power factor change occurs.

Among all the above mentioned characteristics, no-load and short-circuit ones are mostly aimed for particular generator selection at the very first stage of electric system design. The two latter characteristics allow to decide whether the selected machine is able to perform the task according to given requirements in various working conditions, e.g. in case when there is a need to supply power to the single mains from a few alternators simultaneously.

Parallel running of synchronous generators

In order to increase the load power, synchronous generators can be connected for parallel work. Certain requirements should be met for this running to avoid abnormal operation:

1) generated currents must have the same frequency, ($f_r = f_c$);

2) no phase shift between the generators should take place (this means the synchronization of the induced emf on phase, $\varphi_r = \varphi_c$);

3) terminal (rated) voltage of the alternators being integrated should be equal, ($E_r = V_c$); the components must be selected and tested properly;

4) the same ABC phase sequence rule for all alternators must be observed.

If needed, these requirements for generators integration we can satisfy by separate adjustment of the field in each system and careful speed control in any place of connection. If we meet these requirements we will provide synchronization of additional alternators with the main one. Besides, special auxiliary devices (synchronizers) were designed for this purpose.

4.6 Synchronous Motor. Fundamentals

The synchronous motor (synch motor) is electrically identical with the alternator. Really, any taken synchronous machine may be used at least theoretically as an alternator when driven mechanically or as a motor when driven electrically, just as in the case of DC machines.

The distinctions of a synch motor are as follows:

1. It runs either at synchronous speed or not at all. The only way to change its speed is to vary the supply frequency since relationship $n_2 = \dfrac{60 f_1}{p}$ is valid both for alternator and motor.

2. It is not inherently self-starting; it has to be run up to synchronous speed by some means before it can be synchronized with the supply.

3. It is capable of being operated under a wide range of power factors, both lagging and leading. Hence, it can be used for power correction purposes, in addition to supplying torque to drive loads.

Basic operation

When a 3-phase winding is connected to a 3-phase supply, then rotating at synchronous speed magnetic flux of constant magnitude is produced. Consider a two-pole machine (Fig. 4.14,a) in which two stator poles (depicted as N_s and S_s, respectively) are shown rotating at synchronous speed in clockwise direction.

With the rotor position, as shown in the picture, suppose the stator poles being at points A and B at that instant. The two similar poles ***N*** (of the rotor) and N_s (of the stator) as well as inverse poles S and S_s will repel each other and the rotor tends to rotate in the counterclockwise direction.

But half a period later, stator poles, having rotated around, interchanged their position, that is, N_s became at point B and S_s at point A. Under these conditions N_s attracts S and S_s attracts N. Hence, rotor tends to rotate clockwise (which is just the reverse of the first direction).

Therefore, we find that due to continuous and rapid rotation of stator poles the rotor is subjected to torque which tends to move it first in one direction and then in the opposite direction. Owing to its large inertia, the rotor cannot instantaneously respond to such quickly-reversing torque, that is why it remains stationary.

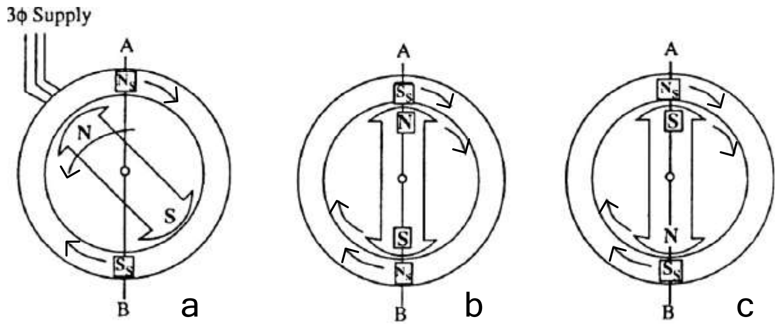


Fig. 4.14 – Unidirectional torque of a two-pole synch motor

Now consider the condition shown in Fig. 4.14,b; the stator and rotor poles are attracting each other. Suppose that the rotor is not stationary but is rotating clockwise with such a speed that it turns through one pole-pitch by the time the stator poles have interchanged their positions, as shown in Fig. 4.14,c. Here, again the stator and rotor poles attract each other.

It means that if the rotor poles also shift their positions along with the stator poles, then they will continuously experience a unidirectional torque, that is, clockwise torque, as shown in Fig. 4.14,b.

Unique synch motor start and running

The rotor which is as yet unexcited is speeded up to near synchronous speed by some arrangement; then it is excited by the DC source. To the moment this (near) synchronously rotating rotor is excited it is magnetically locked into correct position with the stator, that is, rotor poles are engaged with the stator poles and both run synchronously in the same direction. It is because of this interlocking of stator and rotor poles that the motor has either to run synchronously or not to run at all. The synchronous speed (***rpm***) is given by the known relation $n_2 = \frac{60f}{p}$, where f is the power supply frequency; p is the number of machine pole pairs (e.g. $p=1$ for our instance).

However, it is important to realize that the alignment between the stator and rotor poles is not an absolutely rigid one. As the load on the motor is increased, the rotor progressively tends to fall back in phase (not in speed, as in the case of DC motor) by some angle but it still continues to run synchronously. In other words, a value of the above mentioned angle, which have already been introduced in alternator section and called load or **coupling angle**, depends upon the amount of the load met by the motor. Pay attention, the working of synchronous motor is in many ways similar to the transmission of mechanical power by the shaft that is made of a rubber cord.

Electrically, the load angle defines a phase shift (Fig. 4.15) between armature terminal, i.e. supply voltage and induced emf (due to relative motion of stator and rotor fields) in armature winding. As we know, in the field of motor application this emf is called back emf.

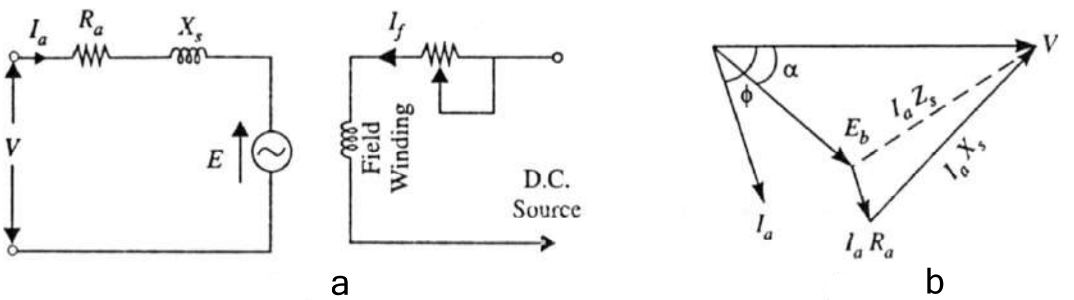


Fig. 4.15 – Equivalent circuit of synchronous motor (a) and associated vector diagram (b)

Hereinafter, let α character designates the load angle while describing the motor operation.

Revise again that the load (coupling) angle α affects the torque developed by synch motor greatly.

4.7 Power Flow and Torques within a Synchronous Motor

The synch motor circuit model

Fig. 4.15 depicts an electrical model for one armature phase of a synchronous motor and related vector diagram.

It is seen from Fig. 4.15,b that applied voltage V is the vector sum of the inverted back emf E_b and the voltage drop $I_a Z_a$ occurred due to armature impedance, i.e. $\dot{V} = -\dot{E}_b + \dot{I}_a \underline{Z}_s$.

Because of $R_a \ll X_s$, for practical motors we can use the simplified notation

$$\dot{V} \approx -\dot{E}_b + \dot{I}_a X_s. \tag{4.8}$$

Power developed by a synch motor

The graph in Fig. 4.16 illustrates energy conversion within a synchronous motor. The sketch includes typical power losses which, if needed, could be evaluated in the manner adopted for other types of electric machines.

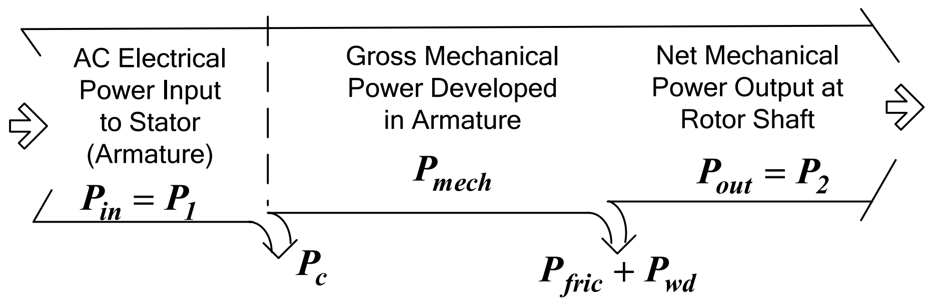


Fig. 4.16 – The synchronous motor power flow and losses

Except for very small machines, the armature resistance of a synch motor is negligible as compared to its synchronous reactance. Hence, it is seen from vector diagram in Fig. 4.15,b that $AB = E_b \sin\alpha = I_a X_a \cos\varphi$,

or

$$V I_a \cos\varphi = E_b I_a \sin\alpha = E_b \frac{V}{X_a} \sin\alpha,$$

The left part of the latter equation represents motor input power P_{in} per phase which, if followed by the right part, can alternatively be termed via motor supply voltage and armature back emf shifted with respect to the former by the load angle. For the entire three-phase armature this relation will look like

$$P_{a\ in} = 3\frac{E_b V}{X_a} \sin\alpha \,.$$

If we neglect copper loss of the stator, then $P_{a\ in}$ may also represent the gross mechanical power P_{mech} developed by the motor:

$$P_{mech} \cong \frac{3E_b V}{X_a} \sin\alpha \,. \tag{4.9}$$

Synch motor torques

The gross torque developed by the motor is

$$T_g = 9{,}55\, P_{mech} / n_1 \,. \tag{4.10}$$

Other various torques associated with a synch motor are as follows:

1. **Starting torque**. It is the torque (or turning effort) developed by the motor when full voltage is applied to its stator (armature) winding. It is sometimes called **breakaway** torque.

2. **Running torque**. As its name indicates it is the torque developed by the motor under running conditions. It is determined by the horse-power and speed specification of the driven machine (of the load driven by the motor). The peak horsepower determines the maximum torque that would be required by the driven machine. The motor must have a **breakdown** or a maximum running torque greater than this value in order to avoid stalling.

3. P**ull-in torque**. A synch motor is started as induction motor till it runs 2 to 5 percent below the synchronous speed. Afterwards, excitation is switched on and the rotor pulls into step with the synchronously-rotating stator field. The amount of torque at which the motor will pull into step is called the pull-in torque.

4. **Pull-out torque**. The maximum torque which the motor can develop without pulling out of step, or synchronism, is called the pull-out torque.

As we know, normally, when load on the motor is increased its rotor progressively tends to fall in phase by the load angle behind the synchronously-revolving stator magnetic field though it keeps running synchronously. The motor develops maximum torque (see (4.9), (4.10), Fig. 4.17) when its rotor is retarded by the angle of 90˚.

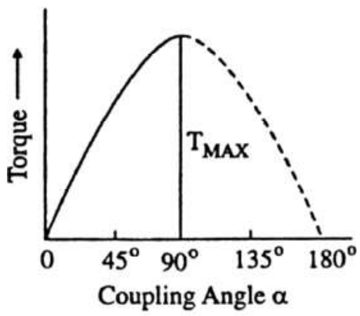


Fig. 4.17 – Torque/load angle relationship

Usually, for synchronous motors and small loads coupling angle α is close to 0^o, and the motor torque is just sufficient to overcome its own windage and friction losses; as the load increases the rotor field falls further out of phase with the stator field (although the two are still rotating at the same speed), until α reaches a maximum at 90^o.

In other words, the rotor has shifted backward by a distance equal to half the distance between adjacent poles. Any further increase in load will cause the motor to pull out of step for synchronism and stop.

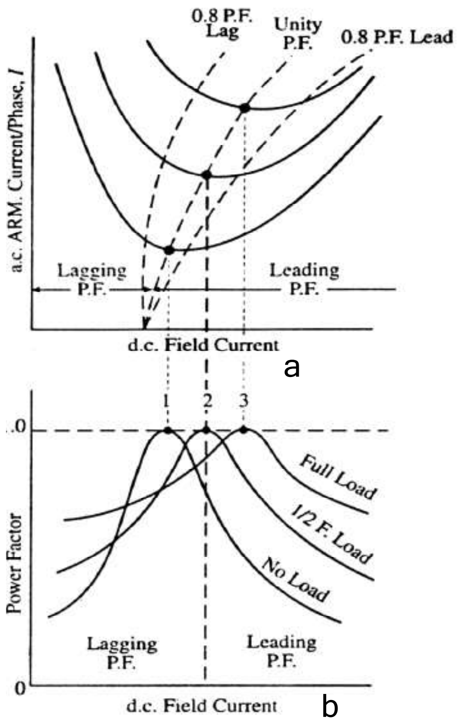


Fig. 4.18 – V-curves

When considering running of the motor in which the mechanical load is constant, two significant moments are to be pointed out as regarding the amount of excitation applied (DC field current supplied).

1. The magnitude of armature current varies with excitation. The current collects a large value both for low and high excitation level (though it is lagging for low excitation and leading for higher excitation). In between, it has minimum value corresponding to a certain excitation. The variations of I_a with excitation are shown in Fig. 4.18,a; they are known as 'V curves' because of their shape.
2. For the same field, armature current varies over a wide range, so it causes the power factor also to vary accordingly. When over-excited, motor runs with leading PF; inversely, it runs with lagging PF, when under-excited. In between the PF is unity. The variations of PF with excitation are shown in Fig. 4.17,b. The curve for a certain PF looks like inverted **V** curve. It would be noted that minimum armature current corresponds to unity power factor.

Synchronous capacitor

When a synch motor is over-excited its generated, or back, emf E_b is much greater than supply voltage V. When such a motor operates without load, the load angle α is small. Meanwhile, the power factor is leading. It means that the synch motor acts as a capacitor and can be used to correct the PF of the lagging load.

Such a motor designed for carrying no load is called a **synchronous capacitor**. It is used where the load is so large that construction of a static capacitor is impractical.

In plants utilizing ***induction*** (not synchronous) motors as main drivers for different loads, the PF does not exceed 85 percent. Taking into account a poor power factor of such motors and also that not all the motors operate at full-load, the overall practical PF is as low as 60 percent. As shown in Fig. 4.19, installation of the synchronous capacitor in parallel with the induction motors improves the PF of the line and hence of supply generators and transformers.

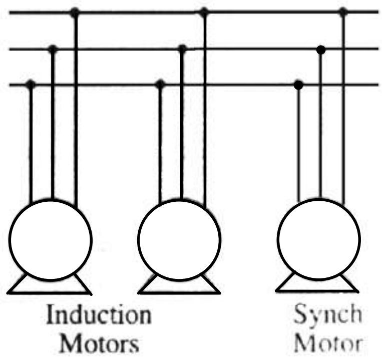


Fig. 4.19 – The synch motor in a role of the capacitor

It is obvious that in order to raise the PF to unity, a current taken by the capacitor must be equal to the total reactive current drawn by the induction motors.

4.8 Distinctive Synchronous Motor Handling

Almost all synch motors are equipped with dampers or squirrel cage windings consisting of copper bars embedded in the pole-shoes and short-circuited at both ends. Such a motor starts readily, acting as an induction motor during the starting period.

The process is as follows. The line voltage is applied to the armature (stator) terminals but the field (excitation) circuit of the rotor remains unexcited. Motor starts as an induction motor and while it reaches nearly 95 % of its synchronous speed the DC field is applied. At that moment the stator and rotor poles get engaged, or interlocked, with each other and hence pull the motor into synchronism.

However, two points should be noted:

1. At the beginning, when the line voltage is applied, the rotor is stationary. The rotating field of the stator winding has induced a very large emf in the rotor during the starting period, though the value of this emf is decreasing as the rotor gathers speed.

Normally, the field windings are designed for ***220 V*** (or ***380 V*** for large machines) but during starting period there are many thousands of volts induced in them. Hence, the rotor windings have to be highly insulated for withstanding such voltages.

2. When full-line voltage is switch on to the armature at rest, a very large current, usually 5 to 7 times the full-load armature current is drawn by the motor.

In some cases, the both items may not be objectionable but where it is not the applied voltage at starting is reduced by using autotransformers. However, the voltage should not be reduced to a very low value of rated voltage. Usually a value of 50 % to 80 % of the full-line voltage is satisfactory.

Practical implementation for motor speeding-up

The following sequence of actions is adopted while starting a modern synch motor which damper windings are provided for.

1. First, main field winding is short-circuited.
2. Then a reduced line voltage (with the help of autotransformer) may be applied across stator terminals. The motor starts up.
3. When it reaches a steady speed (as judged by its sound), a weak DC excitation is applied by removing the short circuit from the main field winding. If excitation is sufficient, then the machine will be pulled into synchronism.
4. Full supply voltage is applied across stator terminals by cutting out the autotransformer.
5. The motor may be operated at any desired power factor by changing the DC excitation.

Comparison between the Synchronous and Induction Motors

As explained previously, AC machines are constrained to fix-speed or near-fixed speed operation when supplied by a constant-frequency source. Only a few simple methods exist to provide limited speed an AC induction machines. More complex methods, involving the use of advanced power electronics circuits can be used if the intended application requires wide-bandwidth control of motor speed and torque.

When comparing the induction and synch motors, we may resume:

- whereas a synch motor (SM) runs only at synchronous speed, an induction motor (IM) never runs with synchronous speed;
- SM can be operated under a wide range of power factors both lagging and leading. But IM always runs with lagging PF;
- SM is not inherently self-starting whereas an IM is;
- the torque of a SM is much less affected by changes in applied voltage than that of an IM;
- a DC excitation is required by SM but not by IM;
- SM is usually more costly and complicated than IM of the same rating.

4.9 Terms and Concepts

Alternator is an electric generator designed to produce alternating current. **A.** performance is based on principles of mutual inductance between two synchronously rotating magnetic fluxes.

Synchronous machine is a rotating AC apparatus whose rotor is synchronized with the rotating field of the stator. The ***speed*** of rotation (in ***synch motor SM***) or ***frequency*** of the current produced (***synch generator SG***) is always ***in time*** with the frequency of the applied alternating current (***SM***) or the speed of the prime mover (***SG***).

Round (**cylindrical**) **rotor** is a type of the rotating part of a synch machine built up on the basis of solid forge-steel cylinder with accommodated winding coils. **R.R.** is intended to obtain smaller windage loss along with as low as possible susceptibility to centrifugal forces. **R.R.** results in flux hence efficiency improvement at very high rotational speeds.

Salient pole rotor is a type of the rotor that is convenient to provide a multi pole construction of synchronous machine. **S.P.R.** allow machine running at lower speeds.

Zero voltage regulation is a property of a synchronous generator to remain terminal voltage nevertheless a certain full-load is (applied across the terminals or) removed from the terminals. **Z.V.R.** is achieved by load power factor improvement.

Power angle is a particular term associated with synchronous machine and load interaction causing a shift between synchronously rotating flux and running rotor.

Contact synchronous generator is that which obtains a sliding contact to supply electric current to the rotor.

Non-contact generator is that having electromagnetic, hence non-contact means of generator excitation. **N.C.G**. is constructed as a machine with rotating by mechanical device (rotor) field.

Synchronization of SM is a set of actions directed towards the order of powering synch machine components; procedure involves rotor start, speed-up and pulling into synchronism proved by steady final running.

Synchronous capacitor is a synch motor designed to carry no load and having the leading power factor. **S.C.** being connected in parallel with a lagging load improves the line power factor.

Synchronous reactance is a resulting reactance of the synch machine (being under load) which compiles both machine armature reactance and load reactance. **S.R.** affects machine power angle hence generator voltage regulation or loading capacity of the motor strongly.

V-curve of the synchronous motor is a graph presenting dependence of the armature current needed for steady rotating a given mechanical load on the amount of exciting current; the motor requires different excitation if armature power factor varies from lagging via zero-mean to leading.

4.10 Review Questions

1. What is the difference between a synchronous motor and an induction motor?
2. How does the power factor and armature current of a synch motor that has to work at constant mechanical load depend on its excitation?
3. What are peculiar features of a synchronous generator design?
4. Is a synchronous motor self-starting? If not, what are the methods used for starting it?
5. Explain the essential difference between cylindrical (smooth) and salient-pole rotors used in large alternators. What type of a rotor can be installed into a 2-pole machine? The same for a 12-pole machine?
6. How is the synchronous speed of a synch motor defined?
7. What type of alternator excitation is the most reliable?
8. Enumerate and define basic alternator characteristics.
9. What are the requirements for alternators expected to run in parallel?
10. What interrelations do external and control characteristics of a synch generator reveal?
11. Draw the complete equivalent circuit of a synchronous generator and its vector (phasor) diagram.

4.11 Problems

Solved problems

Problem 1. A 4-pole induction motor, running with ***5 %*** slip, is supplied by a ***60 Hz*** synchronous generator.

1. Calculate the speed of the motor.
2. What is the generator speed if it has six poles?

Solution

1. The speed of the motor

$$n_2 = n_1(1-s) = \frac{60 \cdot f_1}{P}(1-s) = \frac{60 \cdot 60}{2}(1-0.05) =$$

$$= 1800 \cdot 0.95 = 1710 \ rpm.$$

2. The generator speed for 6 poles:

$$n_{2SG} = \frac{60 f_1}{3} = \frac{60 \cdot 60}{3} = 1200 \ rpm \ .$$

Problem 2. A *3*-phase ***star***-connected alternator supplies a load of ***10 MW*** at PF ***0.85*** lagging and at ***11 kV*** terminal voltage. Its resistance is ***0.1*** Ω per phase and synchronous reactance ***0.66*** Ω per phase.

Calculate the value of emf generated.

Solution

1. Full-load output current

$$I_a = \frac{P_2}{\sqrt{3}V_l \cos\varphi_2} = \frac{10 \cdot 10^6}{1.73 \times 11.000 \times 0.85} = 618\ A.$$

2. Voltage drops across R_a and synchronous reactance:

$$V_R = I_a R_a = 618 \times 0.1 = 61.8\ V.$$

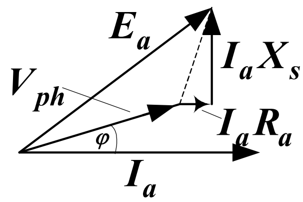
$$V_X = I_a X_s = 618 \times 0.66 = 408\ V.$$

3. Terminal phase voltage

$$V_{ph} = \frac{V_l}{\sqrt{3}} = 6350\ V.$$

4. Since $cos\varphi = 0.85;\ \varphi = 31.8^\circ$, then $sin\varphi = 0.527$.

5. Phase emf can be determined from vector diagram on the right where phase voltage and armature current are given:



$$E_{ph} = \sqrt{\left(V_{ph} cos\varphi + I_a R_a\right)^2 + \left(V_{ph} sin\varphi + I_a X_s\right)^2} =$$

$$= \sqrt{\left(6350 \cdot 0.85 + 61.8\right)^2 + \left(6350 \cdot 0.527 + 408\right)^2} = 6625\ V.$$

6. Then ***line*** emf

$$E_l = \sqrt{3} E_{ph} = 1.73 \cdot 6625 = 11486\ V.$$

Problem 3. A given ***3 MVA***, ***50 Hz***, ***11 kV***, 3-phase, ***Y***-connected alternator when supplying ***100 A*** at zero PF leading has a line-to-line voltage of ***12370 V***; when the load is removed, the terminal voltage falls down to ***11000 V***.

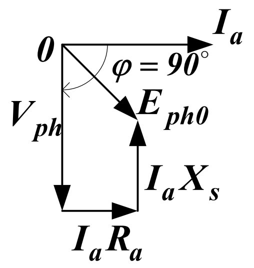
Predict the regulation of the alternator when supplying full load at PF ***0.8*** lag. Assume an effective resistance of ***0.4*** Ω per phase.

Solution

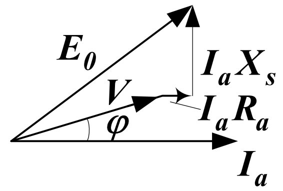
1) As seen from the diagram (a) below,

$$E_{ph}^2 = \left(V_{ph} cos\varphi + I_a R_a\right)^2 + \left(V_{ph} sin\varphi - I_a X_s\right)^2$$

at zero PF leading ($\varphi = 90^\circ$).



a



b

2) Open circuit emf per phase $E_{ph0} = \dfrac{E_l}{\sqrt{3}} = \dfrac{11000}{1.73} = 7142\ V$.

3) Because of $\cos\varphi = 0,\ \sin\varphi = 1$ we can find X_s by using

$$6350^2 = (0 + 100 \times 0.4)^2 + (7142 - 100X_s)^2 :$$

$$100X_s = 790, \text{ or } X_s = 7.9\ \Omega.$$

4) Full current $I_{al} = \dfrac{P_2}{\sqrt{3}E_l} = \dfrac{3 \cdot 10^6}{1.73 \cdot 11000} = 157\ A$.

5) $I_{al}R_a = 157 \times 0.4 = 63\ V$.

6) $I_{al}X_s = 157 \times 7.9 = 1240\ V$.

7) Then according to diagram (b)

$$E_0 = \sqrt{\left(E_{ph0} \cdot \cos\varphi_2 + I_aR_a\right)^2 + \left(E_{ph0} \cdot \sin\varphi_2 + I_aX_s\right)^2} =$$

$$= \sqrt{(6350 \cdot 0.8 + 63)^2 + (6350 \cdot 0.6 + 1240)^2} = 7210\ V.$$

8) The regulation of the alternator

$$\Delta V\% = \frac{7210 - 6350}{6350} \cdot 100 = 13.5\%.$$

Problem 4. A ***75 kW***, 3-phase, Y-connected, ***50 Hz***, p=2, 440 V condition with 0.8 PF leading. The motor efficiency excluding field and stator losses, is ***95%*** and $X_a = X_s = 2.5\ \Omega$.

Calculate: 1) mechanical power developed; 2) armature current; 3) back emf; 4) power angle and 5) maximum (pull-out) torque of the motor.

Solution

1) $n_1 = n_2 = n_s = \dfrac{60 \cdot f_1}{p} = \dfrac{60 \cdot 50}{2} = 1500\,rpm = 25\,rps$.

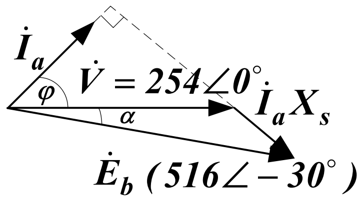
$$P_{mech} \simeq P_{in} = P_1 = \frac{P_2}{\eta} = \frac{75 \cdot 10^3}{0.95} = 78.950\ W\ .$$

2) Since input power $P_1 = \sqrt{3} V_l I_a \cos\varphi$, then

$$I_a = \frac{P_1}{\sqrt{3} V_l \cos\varphi} = \frac{78.950}{1.71 \cdot 440 \cdot 0.8} = 129\ A\ .$$

3) Applied phase voltage $V_{ph} = V_l / \sqrt{3} = 440/1.73 = 254\ V$.

Let it be $\dot{V} = V\angle 0^\circ$. Then the following vector diagram can be depicted:



In accordance with the diagram $\dot{V} = E_b + j\dot{I}_a X_s$, or

$$\dot{E}_b = \dot{V} - j\dot{I}_a X_s =$$
$$= 254\angle 0^\circ - 129\angle 36.9^\circ \times 2.5\angle 90^\circ =$$
$$= 250\angle 0^\circ - 322\angle 126.9^\circ =$$
$$= 254 - 322\left(\cos 126.9^\circ + j\sin 126.9^\circ\right) =$$
$$= 254 - 322\left(-0.6 + j0.8\right) = 516\angle -30^\circ$$, i.e. $\alpha = -30^\circ$.

4) Pull-out torque occurs when $\alpha = 90^\circ$ and

$$P_m = 3\frac{E_b V}{X_s}\sin\alpha = 3\frac{256 \cdot 516}{2.5}\sin 90 = 157.275\ W\ .$$

This yields for the torque:

$$T_g = 9.55\frac{P_m}{n_1} = 9.55\frac{157.275}{1500} = 1000\ Nm\ .$$

Problem 5. A 3-phase, ***440 V***, ***50 Hz***, ***wye***-connected synchronous motor taken ***7.46 kW*** from the armature winding is ***0.5*** ohm. The motor operates at a PF of ***0.75 lag***. Iron and mechanical losses amount to ***500*** watts. The excitation loss is ***650*** watts (assume a separate DC source for excitation).

Calculate: 1) armature current, 2) power supplied to the motor, 3) efficiency of the motor.

Solution

1. A 3-phase synchronous motor receives power from two sources:
   a) 3-phase AC source feeding power to the armature;

b) DC source that feeds electrical power only to the field winding (excitation).

2. Thus, power received from the DC source is utilized only to meet the copper loss of the field winding.

3. 3-phase AC source feeds electrical power to the armature for the following needs (including losses):

a) net mechanical output from the shaft;

b) copper loss in armature winding;

c) friction and armature core (iron) losses.

4. In call of the given problem:

a) total power entered the armature for driving the load

$$P_a = \sqrt{3} I_a V_l \cdot \cos\varphi_1 ,$$

whence

$$I_a = \frac{P_1}{1.73 V_l \cdot \cos\varphi_1} = \frac{7460}{1.73 \cdot 440 \cdot 0.75} = 13.052\ A ;$$

b) total copper losses in armature winding is

$$P_c = 3 I_a^2 R_a = 3 \cdot \left(13.052\right)^2 \cdot 0.5 = 255\ W .$$

5. Power supplied to the motor

$$P_1 = P_a + P_f = 7460 + 650 = 8110\ W .$$

6. Output power from the shaft

$$P_2 = P_1 - P_c - (P_{fric} + P_{iron}) = 7460 - 255 - 500 = 6705\ W .$$

6. Motor efficiency

$$\eta = \frac{P_2}{P_1} = \frac{6705}{8110} = 82.7\% .$$

Supplement problems

Problem 6. What is the maximum speed at which (1) a ***60 Hz -***, (2) a ***50 Hz -*** synchronous machine can operate?

(Ans. (1) ***3600 rpm***; (2) ***3000 rpm***)

Problem 7. A ***500-kVA***, **6**-pole, ***500-V***, ***3***-phase, wye-connected synchronous generator has a synchronous impedance of $(0.1 + j1.5)\Omega$ per phase. If the generator is driven at ***1000 rpm***, what is the frequency of generated voltage?

Determine the excitation voltage and the power angle on full-load and ***0.8 lagging*** power factor.

Problem 8. A ***star***-connected alternator supplies a delta connected load. The impedance of the load branch is $(8 + j6)$ ***ohms/phase***. The line voltage is ***230 V***.

Determine: 1) current in the load branch; 2) power consumed by the load; 3) power factor of the load; 4) load reactive power.

Problem 9. A ***500-V***, ***50-kVA***, ***1***-phase alternator has an effective resistance of $0.2\ \Omega$. A field current of ***10 A*** produced an armature current of ***200 A*** on short-circuit and an emf of ***450 V*** on open-circuit.

Calculate the full load regulation at PH ***0.8 lag***.

(Ans. ***34.4%***)

Problem 10. A ***3***-phase, ***400-V***, synchronous motor takes ***52.5 A*** at power factor 0.8 leading.

Calculate the power supplied and the induced emf. The motor impedance per phase is $(0.25 + j3.2)$ ***ohm***.

(Ans. ***29.1 kW***; ***670 V***)

Problem 11. A synchronous motor takes ***25 kW*** from ***400-V*** supply mains. The synchronous reactance of the motor is ***4*** ohms.

Calculate the power factor at which the motor would operate when the field excitation is so adjusted that generated emf is ***500 V***.

(Ans. ***0.666***)

Problem 12. The excitation of ***415 V***, ***3***-phase, delta connected synchronous motor is such that the induced emf is ***520 V***. The impedance per phase is $(0.5 + j4.0)$ ***ohm***.

If the friction and iron losses are constant at ***1000 W***, calculate the power output, line current, power factor and efficiency for maximum power output.

(Ans. ***134.69 kW***; ***268.5 A***; ***0.89*** (lead); ***78.45%***)

Tutorial test

1. ***The frequency of voltage generated by an alternator having 4-poles and rotating at 1800 rpm is … hertz.***

a) 60 b) 7200

c) 120 d) 450

2. ***A 50-Hz alternator will run at the greatest possible speed if it is wound for … poles.***

a) 8 b) 6

c) 4 c) 2

3. ***Three phase alternators are invariably Y-connected because***

a) magnetic losses are minimized b) less turns of wire are required

c) smaller conductors can be used c) smaller conductors can be used

4. ***When speed of an alternator is changed from 3600 rpm to 1800 rpm, the generated EMF/phase will become***

a) one-half b) twice

c) four times d) one-fourth

5. ***The magnitude of the three voltage drops in an alternator due to armature resistance, leakage reactance and armature reaction are solely determined by***

a) load current I_a b) PF of the load

c) whether it is a lagging or leading PF load d) field construction of the alternator

6. ***Armature reaction in an alternator primarily affects***

a) rotor speed b) terminal voltage per phase

c) frequency of armature current d) generated voltage per phase

7. ***Under no-load condition power drawn by the prime mover of an alternator goes to***

a) produce induced emf in armature winding b) meet Cu losses both in armature and rotor windings

c) produce power in the armature d) meet no-load losses

8. ***As load PF of an alternator becomes more leading, the value of generated voltage required to give rated terminal voltage***

a) increases b) remains unchanged

c) decreases d) varies with rotor speed

9. ***At leading PF, the armature flux in an alternator ….. the rotor flux***

a) opposes b) aids

c) distorts d) does not effect

10. ***The power factor of an alternator is determined by its***

a) speed b) load

c) excitation d) prime mover

11. ***For proper parallel operation AC multiphase alternators must have the same***

a) speed b) voltage rating

c***) kVA*** rating d) excitation

12. ***Of the following conditions, the one which does not have to be met by alternators working in parallel is***

a) terminal voltage of each machine must be the same

b) the machines must have the same phase rotation

c) the machines must operate at the same frequency

d) the machines must have equal rotating

13. ***For a machine of infinite bus, active power can be varied by***

a) changing field excitation b) changing of prime mover speed

c) both a) and b) above d) none of the above

14. ***In a synchronous motor, the magnitude of stator back EMF*** E_b ***depends on***

a) speed of the motor b) load of the motor

c) both the speed and rotor flux d) DC excitation only

15. ***An electric motor in which both the rotor and stator fields rotates with same speed is called a/an … motor***

a) DC b) charge

c) synchronous d) universal

16. ***The direction of rotation of a synchronous motor can be reversed by reversing***

a) current to the field winding b) supply phase sequence

c) polarity of rotor poles d) none of the above

17. ***The angle between the synchronously-rotating stator flux and rotor poles of a synchronous motor is called … angle***

a) synchronizing b) torque

c) power factor d) slip

18. ***The maximum value of torque angle in a synchronous motor is … degrees electrical***

a) 45 b) 90

c) between 45 and 90 d) below 60

19. ***A synchronous motor running with normal excitation adjusts to load increases essentially by increase in its***

a) power factor b) torque angle

c) back emf d) armature current

20. ***If excitation of a synchronous motor running with constant load is increased, its torque angle must necessarily***

a) decrease b) increase

c) remain constant d) become twice the no-load value

21. ***A synchronous machine is called a doubly-excited machine because***

a) it can be overexcited b) it has two sets of rotor poles

c) both its rotor and stator are excited d) it needs twice the normal exciting current

22. ***Synchronous capacitor is***

a) an ordinary static capacitor bank

b) an over-excited synchronous motor driving mechanical load

c) an over-excited synchronous motor running without mechanical load

d) none of the above (a, b, c).

23. ***Armature reaction in an alternator primarily affects***

a) rotor speed b) terminal voltage per phase

c) frequency of armature current d) generating voltage per phase

24. ***The power factor of an alternator is determined by its***

a) speed b) load

c) excitation d) prime mover.

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Волчанська Любов Іванівна

ОСНОВИ ЕЛЕКТРОТЕХНІКИ
Частина 2
(Англійською мовою)

Редактор Л.І. Волчанська
Технічний редактор Л.О. Кузьменко
Комп’ютерне верстання: Т.В. Григор’єва

Зв. план, 2011
Підписано до друку 31.08.2011
Формат 60x84 1/16. Папір. офс. № 2. Офс. друк
Ум. друк арк. 6,5. Обл.-вид. арк. 7,37 Наклад 100 пр.
Замовлення 253. Ціна вільна

Національний аерокосмічний університет ім. М.Є. Жуковського
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