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RADIAL DISTRIBUTION OF ELECTRONS ROTATION MOMENT IN HALL EFFECT AND PLASMA-ION THRUSTERS

The subject matter *of the article is the radial distribution of electrons movement parameters inside electric propulsion thrusters with closed electrons drift. The radial magnetic field in Hall effect thrusters is the limits the axial flow of electrons because of interaction with azimuth electron current. In turn, this azimuth current exists as a result of rivalry between the attempt of the magnetic field to transform electrons current completely closed one and the loss of electrons rotation moment in collisions. Similar processes take place in the ionization chamber of plasma-ion thrusters with the radial magnetic field. The attempts to estimate electrons parameters through only collisions with ions and atoms inside volume have given the value of axial electrons current much lower than really being. This phenomenon is called anomalous electrons conductivity, which was tried to be explained as a consequence of various effects including "near-the-wall-conductivity", which was explained as a result of non-mirror reflection of electrons from the Langmuir layer near the walls of the thruster channel. The disadvantage of this name is the fact that the reflection of the electron occurs before reaching the surface from the potential barrier at the plasma boundary with any environment: the wall, but also with the environment vacuum. The potential distribution in the Langmuir layer is non-stationary and inhomogeneous due to the presence of so-called plasma oscillations. The definition of "conductivity" is just as unfortunate in this name, because the collisions are always not a factor of conductivity, but on the contrary – of resistance. The* ***goal*** *is to solve the task of electrons rotation moment distribution in the thruster channel. The* ***methods*** *used are the formulation of the kinetic equation for electrons distribution function over the velocities, radius, and projections of the coordinates of the instantaneous center of cyclotron rotation; solution of this equation and finding with its use the distribution of the gas-dynamic parameters of electrons along the cross-section of the channel.* ***Conclusions****. A mathematical model of electrons rotation moment dynamics is proposed, which allows using plasma-dynamics equations to analyze its distribution along the cross-section of thruster channel and to estimate the effect of "near-the-wall-conductivity" using appropriate boundary conditions.*

Keywords: *Hall effect thruster; electrons rotation moment; velocity distribution function; kinetic equation; Langmuir layer.*

Introduction

Hall effect thruster (HET) and plasma-ion thruster (PIT) with radial magnetic field relate to electrostatic thrusters where acceleration of ions is made by electrostatic component of electromagnetic field produced by electrode system and with direction of electric field tension E_x almost in parallel to necessary direction of thrust. It is enough for thrust produce but without any other factor it would mean acceleration of electrons in opposite direction with extreme electron current and the most their part in power consumption.

To prevent this the magnetic field is used. A radial magnetic field B_r in the HET and PIT created by a special magnetic system limits the axial current of electrons with a force almost equal to the electric force tending to accelerate the electrons towards the anode.

A moderate axial current of electrons exists due to two factors that prevent the magnetic field from making their motion completely closed: collisions of electrons with atoms and ions in the volume and loss of electron rotational moment as a result of their non-mirror reflection from a potential barrier at the plasma boundary near the channel. The last factor, which was unsuccessfully called "near-the-wall-conductivity", is main one in rarefied plasma of HET and PIT. Thus the question is important about including the description of this effect into mathematical model of the processes in the thruster.

Formulation of the problem

The authors of papers [1, 2] try to describe "near-the-wall-conductivity" with the use of total electron collision frequency without a clear comment about this totality. Such comment is made in paper [3] as the momentum transfer frequency including electron-atom and electron-ion collisions are included, as well as effect of anomalous transport but without presenting a specific mathematical form for this frequency.

The authors of paper [4] propose to estimate this non-mirror reflection with the use of experimental results, which produce the following serious doubts: do the methods exist to measure the electrons flow density in any part of the channel; even yes, is it possible to adequately list the results of measurements in one type-size of device with some own profile of magnetic field for another type-size in any section?

In turn, the authors of work [5] consider the particle-in-cell simulations as a way to solve this problem.

The attempts to use the empiricism or particle-in-cell simulations here can be explained by the fact that the authors are sure that this effect can not be described by plasma-dynamics.

Really it is possible but with understanding of the followings: the equations of plasma-dynamics relate to the point in the volume and this effect must be firstly represented as boundary conditions [6]; the equations of plasma-dynamics themselves must include the parameters, for which these boundary conditions are written.

Gas dynamics equation set is fundamentally opened, and the number of equations is brought into correspondence with the number of unknowns only approximately – as a result of certain assumptions. Due to the rarefaction of the substance in the electric propulsion thrusters the local thermodynamic equilibrium method is insufficient to describe the processes there, and there is a need to use more extended form of equations as it was done in the article [7]. This equations system is enough to describe the most of features of processes in HET. But specific of radial magnetic field action on the component of electrons pressure tensor (trial to revolve $P_e^{(r\varphi)}$ into $P_e^{(xr)}$ and then $P_e^{(xr)}$ into $-P_e^{(r\varphi)}$) does not permit to close equations system with any appropriate supposition.

The goal of this work is to solve the kinetic task about electrons rotation distribution in thruster channel.

Solutions

The necessity to use kinetics approach appears and the channel curvature, volume collisions, electron flux onto the radial boundary and, as a consequence, the radial projection of the electric field tension are neglected in the following entries for simplicity to save the most important aspects in approximate solution of kinetic equation. Also the electric field tension and magnetic induction in the axial size of the order of the cyclotron radius of the electron are considered as constant ones.

The stationary form of the kinetic equation for electrons velocity distribution function $f(x,r,\vec{v})$ is the following one:

$$v_x\frac{\partial f(x,r,\vec{v})}{\partial x}+v_r\frac{\partial f(x,r,\vec{v})}{\partial r}-\frac{eE_x}{m_e}\frac{\partial f(x,r,\vec{v})}{\partial v_x}-$$
$$-\frac{eB_r}{m_e}\left(v_x\frac{\partial f(x,r,\vec{v})}{\partial v_\varphi}-v_\varphi\frac{\partial f(x,r,\vec{v})}{\partial v_x}\right)=0\,,\quad(1)$$

or:

$$v_x\frac{\partial f(x,r,\vec{\upsilon})}{\partial x}+v_r\frac{\partial f(x,r,\vec{\upsilon})}{\partial r}+$$
$$+\omega_c\left(\upsilon_\varphi\frac{\partial f(x,r,\vec{\upsilon})}{\partial v_x}-v_x\frac{\partial f(x,r,\vec{\upsilon})}{\partial \upsilon_\varphi}\right)=0\,,\quad(2)$$

where the cyclotron frequency ω_c, drift velocity V_H and cyclotron rotation velocity $\vec{\upsilon}$ of electrons:

$$\omega_c=\frac{eB_r}{m_e}\,,\ V_H=\frac{E_x}{B_r}\,,\ \upsilon_x=v_x\,,\ \upsilon_\varphi=v_\varphi-V_H\,.\quad(3)$$

The motion equation of a single electron in projections has the form:

$$\frac{dv_r}{dt}=0\,,\ \frac{dv_x}{dt}-\omega_c\upsilon_\varphi=0\,,\ \frac{d\upsilon_\varphi}{dt}+\omega_c v_x=0\,,\quad(4)$$

$$\frac{dr}{dt}=v_r\,,\ \frac{dx}{dt}=v_x\,,\ \frac{da}{dt}=r\frac{d\varphi}{dt}=V_H+\upsilon_\varphi\,,\quad(5)$$

where a – the length of the arc in the azimuth direction.

As a result, the motion of a single electron is described by the following expressions:

$$v_x=v_{xo}\cos\psi+\upsilon_{\varphi o}\sin\psi\,,\quad(6)$$

$$\upsilon_\varphi=-v_{xo}\sin\psi+\upsilon_{\varphi o}\cos\psi\,,\quad(7)$$

$$x=x_c+\frac{v_{xo}\sin\psi-\upsilon_{\varphi o}\cos\psi}{\omega_c}\,,\quad(8)$$

$$a=a_c+\frac{V_H\psi+v_{xo}\cos\psi+\upsilon_{\varphi o}\sin\psi}{\omega_c}\,,\quad(9)$$

where ψ, x_c and a_c – phase and projections of the coordinates of the instantaneous center of cyclotron rotation on a plane transverse to magnetic induction:

$$\psi=\omega_c t=\frac{\omega_c(r-r_o)}{v_r}\,,\quad(10)$$

$$x_c=x_o+\frac{\upsilon_{\varphi o}}{\omega_c}=x+\frac{\upsilon_\varphi}{\omega_c}\,,$$

$$a_c=a_o-\frac{v_{xo}}{\omega_c}=a-\frac{V_H\psi+v_x}{\omega_c}\,.\quad(11)$$

Time is counted from the moment the electron is positioned on the middle axis of the channel.

We can introduce the electrons distribution function over the velocities, radius, and projections of the coordinates of the instantaneous center of cyclotron rotation $F(x_c,r,a_c,v_x,v_r,\upsilon_\varphi)$ – such that the number of electrons in the elementary six-dimensional range of each of its arguments is equal to:

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$$dN = F(x_c, r, a_c, v_x, v_r, v_\varphi)\times$$
$$\times dx_c\, dr\, da_c\, dv_x\, dv_z\, dv_\varphi. \quad (12)$$

Axial symmetry in this problem means the absence of dependence of quantities on the arc coordinate a :

$$F(x_c, r, a_c, v_x, v_r, v_\varphi) = F(x_c, r, v_x, v_r, v_\varphi). \quad (13)$$

On the centerline of the channel, with $r = r_o$:

$$F(x_c, r_o, v_x, v_r, v_\varphi) = F_0(x_c, v_x, v_r, v_\varphi). \quad (14)$$

The motion of electrons without collisions means the conservation of their number on one Hall trajectory with a shift along the radius:

$$F(x_c, r, v_x, v_r, v_\varphi) = F_0(x_c, v_{xo}, v_r, v_{\varphi o}) =$$
$$= F_0(x_c, v_x \cos\psi - v_\varphi \sin\psi,$$
$$v_r, v_x \sin\psi + v_\varphi \cos\psi). \quad (15)$$

At the same point in space with a coordinate x can be electrons, which coordinate x_c the instantaneous center of cyclotron rotation is offset from the coordinate x at different distances depending on the radius and phase of cyclotron rotation. In this case, the relationship between the distribution functions over the actual coordinates $f(x, r, v_x, v_r, v_\varphi)$ and coordinates of the instantaneous center of cyclotron rotation $F(x_c, r, v_x, v_r, v_\varphi)$ is defined by the expression:

$$f(x, r, v'_x, v'_r, v_\varphi)\, dx =$$
$$= \int_{x_c\left(x - \frac{1}{2}dx, v_\varphi\right)}^{x_c\left(x + \frac{1}{2}dx, v_\varphi\right)} F(x'_c, r, v_x, v_r, v_\varphi)\, dx'_c, \quad (16)$$

where $x_c(x, v_\varphi) = x + \dfrac{v_\varphi}{\omega_c}$ – the dependence presented in (11).

In this way:

$$f(x, r, v_x, v_r, v_\varphi) = F\left(x + \frac{v_\varphi}{\omega}, r, v_x, v_r, v_\varphi\right) \quad (17)$$

or, in accordance with (7), (8) and (15):

$$f(x, r, v_x, v_r, v_\varphi) =$$
$$= F_0\left(x + \frac{v_\varphi}{\omega}, v_x \cos\psi - v_\varphi \sin\psi,\right.$$
$$\left. v_r, v_x \sin\psi + v_\varphi \cos\psi\right). \quad (18)$$

In the polar coordinate system, the cyclotron rotation velocity can be represented as follows:

$$v_x = v_c \cos\phi, \quad v_\varphi = v_c \sin\phi, \quad (19)$$

where ϕ – the phase of cyclotron rotation.

The kinetic equation (2) in this case takes the form:

$$v_c \cos\phi \frac{\partial f(x, r, v_r, v_c, \phi)}{\partial x} +$$
$$+ v_r \frac{\partial f(x, r, v_r, v_c, \phi)}{\partial r} - \omega_c \frac{\partial f(x, r, v_r, v_c, \phi)}{\partial \phi} = 0 \quad (20)$$

and the expression (18):

$$f(x, r, v_r, v_c, \phi) = F_0\left(x + \frac{v_c \sin\phi}{\omega_c}, v_r, v_c, \phi + \psi\right). \quad (21)$$

On the centerline of the channel, with $r = r_o$:

$$f_0(x, v_r, v_c, \phi) = f(x, r_o, v_r, v_c, \phi) =$$
$$= F_0\left(x + \frac{v_c \sin\phi}{\omega_c}, v_r, v_c, \phi\right), \quad (22)$$

whence follows:

$$F_0(x, v_r, v_c, \phi) = f_0\left(x - \frac{v_c \sin\phi}{\omega_c}, v_r, v_c, \phi\right), \quad (23)$$

$$F_0(x, v_r, v_c, \phi + \psi) =$$
$$= f_0\left(x - \frac{v_c \sin(\phi + \psi)}{\omega_c}, v_r, v_c, \phi + \psi\right) \quad (24)$$

and

$$f(x, r, v_r, v_c, \phi) = f_0\left(x + \frac{v_c}{\omega_c}\left(\sin\phi - \sin(\phi + \psi)\right),\right.$$
$$\left. v_r, v_c, \phi + \psi\right). \quad (25)$$

From the expressions (10) and (25) it follows:

$$\frac{\partial f}{\partial x} = \frac{\partial f_0}{\partial x}, \quad (26)$$

$$v_r \frac{\partial f}{\partial r} = \omega_c \frac{\partial f}{\partial \psi} = \omega_c \frac{\partial f_0}{\partial \phi} - v_c \cos(\phi + \psi) \frac{\partial f_0}{\partial x}, \quad (27)$$

$$\omega_c \frac{\partial f}{\partial \phi} = \omega_c \frac{\partial f_0}{\partial \phi} + v_c \left(\cos\phi - \cos(\phi + \psi)\right) \frac{\partial f_0}{\partial x}, \quad (28)$$

which really corresponds to equality (20).

Thus, taking into account (25), the problem of changing the velocity distribution function of electrons in the radial direction has been solved.

Value $\dfrac{v_c}{\omega_c}$ in (25) has the order of the cyclotron radius, at the size of which all parameters vary little, which allows us to write:

$$f(x, r, v_r, v_c, \phi) \approx f_0(x, v_r, v_c, \phi + \psi) +$$
$$+ \frac{\partial f_0(x, v_r, v_c, \phi + \psi)}{\partial x} \frac{v_c}{\omega_c}\left(\sin\phi - \sin(\phi + \psi)\right). \quad (29)$$

In this approximation:

$$\frac{\partial f}{\partial x} = \frac{\partial f_0}{\partial x} + \frac{v_c}{\omega_c}\left(\sin\phi - \sin(\phi + \psi)\right) \frac{\partial^2 f_0}{\partial x^2}, \quad (30)$$

$$v_r \frac{\partial f}{\partial r} = \omega_c \frac{\partial f}{\partial \psi} = \omega_c \frac{\partial f_0}{\partial \phi} - v_c \cos(\phi+\psi)\frac{\partial f_0}{\partial x} + $$
$$+ v_c \left(\sin\phi - \sin(\phi+\psi) \right) \frac{\partial^2 f_0}{\partial x \partial \phi}, \qquad (31)$$

$$\omega_c \frac{\partial f}{\partial \phi} = \omega_c \frac{\partial f_0}{\partial \phi} + v_c \left(\cos\phi - \cos(\phi+\psi) \right) \frac{\partial f_0}{\partial x} +$$
$$+ v_c \left(\sin\phi - \sin(\phi+\psi) \right) \frac{\partial^2 f_0}{\partial x \partial \phi} \qquad (32)$$

and

$$v_c \cos\phi \frac{\partial f(x,r,v_r,v_c,\phi)}{\partial x} + v_r \frac{\partial f(x,r,v_r,v_c,\phi)}{\partial r} -$$
$$- \omega_c \frac{\partial f(x,r,v_r,v_c,\phi)}{\partial \phi} = \frac{v_c^2}{\omega_c} \cos\phi \left(\sin\phi - \sin(\phi+\psi) \right) \times$$
$$\times \frac{\partial^2 f_0(x, v_r, v_c, \phi+\psi)}{\partial x^2}. \qquad (33)$$

The ratio of the right-hand side of (33) to the first term of the left-hand side coincides in order with the small ratio of the cyclotron radius to the length of the thruster channel. Thus, neglecting the named small ratio, expression (29) is an approximate solution of the kinetic equation (20).

The symmetry of the problem relative to the midline of the channel means the requirement:

$$f(x,r,v_r,v_c,\phi) = f(x,r',v_r',v_c,\phi)$$
$$\text{at } r' - r_o = r_o - r \text{ and } v_r' = -v_r, \qquad (34)$$

from where, subject to this requirement in sign ψ should also:

$$f_0(x,v_r,v_c,\phi) = f_0(x,-v_r,v_c,\phi) = f_0(x,|v_r|,v_c,\phi). \quad (35)$$

Moreover, in the case of mirror reflection, there would be equality:

$$f^{(m)}(x,r,v_r,v_c,\phi) = f^{(m)}(x,r,-v_r,v_c,\phi), \quad (36)$$

which, in accordance with (29), would mean the absence of a function $f_0(x,v_r,v_c,\phi)$ dependence from phase ϕ and coordinates x:

$$f_0^{(m)}(x,v_r,v_c,\phi) = f_0^{(m)}(|v_r|,v_c). \qquad (37)$$

The deviation from the mirror reflection trajectory is small, and the consequence of this deviation is the appearance of a weak dependence $f_0(x,|v_r|,v_c,\phi)$ from phase ϕ and coordinate x. Therefore in the decomposition $f_0(x,|v_r|,v_c,\phi)$ in the Fourier series, the dependence on x in terms describing the dependence on ϕ and vice versa, dependence on ϕ in the zero term describing the dependence on x:

$$f_0(x,v_r,v_c,\phi) = f_0^{(c)}(x,|v_r|,v_c) +$$
$$+ \sum_{k=1}^{\infty} f_k^{(c)}(|v_r|,v_c) \cos(k\varphi) +$$
$$+ \sum_{k=1}^{\infty} f_k^{(s)}(|v_r|,v_c) \sin(k\varphi) \qquad (38)$$

and correspondingly:

$$f(x,r,v_r,v_c,\phi) \approx f_0^{(c)}(x,|v_r|,v_c) +$$
$$+ \frac{\partial f_0^{(c)}(x,|v_r|,v_c)}{\partial x} \frac{v_c}{\omega_c} \left(\sin\phi - \sin(\phi+\psi) \right) + .$$
$$+ \sum_{k=1}^{\infty} f_k^{(c)}(|v_r|,v_c) \cos(k(\phi+\psi)) +$$
$$+ \sum_{k=1}^{\infty} f_k^{(s)}(|v_r|,v_c) \sin(k(\phi+\psi)). \qquad (39)$$

Periodic by phase ϕ terms (39), when substituted into the expression for the electron concentration, give a zero result:

$$n_e(x,r) = \int f(x,r,v_r,v_c,\phi) v_c d v_c d v_r d\phi =$$
$$= \int f_0^{(c)}(x,|v_r|,v_c) v_c d v_c d v_r d\phi = n_e(x). \quad (40)$$

and the electron concentration is a function of only x.

The expression for the azimuth projection of the electron flux density in the distribution (39) has the form:

$$n_e(x,r)\left(V_{e\varphi}(x,r) - V_H\right) =$$
$$= \frac{1}{2\omega_c} \frac{\partial}{\partial x} \int f_0^{(c)}(x,|v_r|,v_c) v_c^3 d v_c d v_r d\phi -$$
$$- \frac{1}{2\omega_c} \frac{\partial}{\partial x} \int f_0^{(c)}(x,|v_r|,v_c) v_c^3 d v_c \cos\psi \, d v_r d\phi +$$
$$+ \frac{1}{2} \int f_1^{(s)}(|v_r|,v_c) v_c^2 d v_c \cos\psi \, d v_r d\phi. \quad (41)$$

In the second and third terms (41), we can distinguish the cosine of the phase ψ average over the radial velocity projection:

$$\langle \cos\psi \rangle_0^{(c)} = \frac{\int_{-\infty}^{\infty} f_0^{(c)}(x,|v_r|,v_c) \cos\psi \, d v_r}{\int_{-\infty}^{\infty} f_0^{(c)}(x,|v_r|,v_c) d v_r},$$

$$\langle \cos\psi \rangle_1^{(s)} = \frac{\int_{-\infty}^{\infty} f_1^{(s)}(|v_r|,v_c) \cos\psi \, d v_r}{\int_{-\infty}^{\infty} f_1^{(s)}(|v_r|,v_c) d v_r}. \quad (42)$$

Under the assumption of the Maxwell form of functions $f_0^{(c)}(x,|v_r|,v_c)$ and $f_1^{(s)}(x,|v_r|,v_c)$ it means:

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$$\langle \cos\,\psi\rangle = \sqrt{\frac{m_e}{2\pi k T_e}} \times$$

$$\times \int_{-\infty}^{\infty} \exp\left(-\frac{m_e v_r^2}{2kT_e}\right)\cos\left(\frac{\omega\,(r-r_o)}{v_r}\right) d\,v_r \qquad (43)$$

or

$$\langle \cos\,\psi\rangle = \frac{2}{\sqrt{\pi}}\int_0^{\infty} e^{-z^2}\cos\left(\frac{\chi(r)}{z}\right) d\,z\,, \qquad (44)$$

where

$$\chi(r) = \omega\,(r-r_o)\sqrt{\frac{m_e}{2kT_e}}\,. \qquad (45)$$

The results of numerical integration (44) are shown in Figure 1.

You may notice that $\langle\cos\psi\rangle$ getting smaller 0.01 already at $\chi(r)\geq 10$, which is noticeably smaller than the Hall parameter found over the channel width of the HET $\chi(R)$. Notable value of $\langle\cos\psi\rangle$ takes place only near the midline of the channel at $r\to r_0$.

When averaging also over the channel cross section, you can write:

$$\langle\langle \cos\,\psi\rangle\rangle = \frac{1}{\chi(R)}\int_0^{\chi(R)}\langle \cos\,\psi\rangle\, d\,\chi\,. \qquad (46)$$

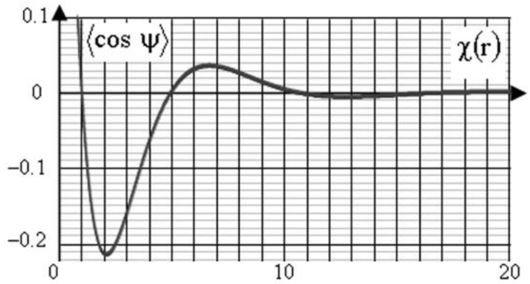

Fig. 1. Cosine phase ψ average by v_r

The results of numerical integration (46) are shown in Figure 2. Cosine of the phase ψ average by v_r and the channel cross section becomes to be less than 0.005 already at $\chi(R)\geq 10$.

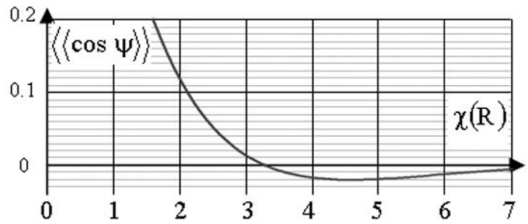

Fig. 2. Cosine phase ψ average by v_r and r

Thus, the velocity distribution function of electrons in this problem can be represented as follows:

$$f(x,r,v_r,v_c,\phi)\approx f_0^{(c)}(x,|v_r|,v_c)+$$

$$+\frac{\partial f_0^{(c)}(x,|v_r|,v_c)}{\partial x}\frac{v_c}{\omega_c}\sin\phi$$

$$+\delta f(x,|v_r|,v_c,\phi+\psi), \qquad (47)$$

where:

$$\delta f(x,|v_r|,v_c,\phi+\psi)=\sum_{k=1}^{\infty} f_k^{(c)}(|v_r|,v_c)\cos\,(k\,(\phi+\psi))+$$

$$+\sum_{k=1}^{\infty} f_k^{(s)}(|v_r|,v_c)\,\sin(k\,(\phi+\psi))-$$

$$-\frac{\partial f_0^{(c)}(x,|v_r|,v_c)}{\partial x}\frac{v_c}{\omega_c}\sin(\phi+\psi)\,. \qquad (48)$$

In this case, the contribution of the function $\delta f(x,|v_r|,v_c,\phi+\psi)$ into the integral characteristics and the characteristics at the boundaries of the plasma with the parietal layer can be neglected.

In this approximation, the expression (41) takes the form:

$$n_e(x)V_{e\varphi}(x,r)=n_e(x)V_H\;+$$

$$+\frac{1}{2\omega_c}\frac{\partial}{\partial x}\int f_0^{(c)}(x,|v_r|,v_c)v_c^3 d\,v_c d\,v_r d\,\phi=$$

$$=n_e(x)V_{e\varphi}(x), \qquad (49)$$

in which the azimuth projection of the electrons mass flow velocity also turns out to be a function of only the axial coordinate x.

The axial-axial and axial-radial components of the electron momentum flux density tensor $\mathbf{\Pi}_e$ in this approximation are equal to:

$$\Pi_e^{(xx)}=\frac{m_e}{2}\int f_0^{(c)}(x,|v_r|,v_c)v_c^3 d\,v_c d\,v_r d\,\phi\,, \quad (50)$$

$$\Pi_e^{(xr)}=0\,. \qquad (51)$$

Moreover, taking into account (3) and (49):

$$\frac{\partial\,\Pi_e^{(xx)}}{\partial\,x}+\frac{\partial\,\Pi_e^{(xr)}}{\partial\,r}+e\,n_e(E_x-V_{e\varphi}B_r)=0\,, \quad (52)$$

which fully corresponds to the axial projection of the motion equation of electrons.

With a more accurate description, taking into account the dependence of the electron concentration on the radial coordinate, the azimuth projection in the expression for the rotation moment of the electrons:

$$M_e(x,r)=n_e(x,r)r\,V_{e\varphi}(x)\,. \qquad (53)$$

can be considered as a function of only the axial coordinate.

Conclusions

The kinetic task is approximately solved with the use of electrons distribution function over the velocities, radius, and projections of the coordinates of the instantaneous center of cyclotron rotation. The results obtained have shown the approximate equivalence of electrons azimuth mass flow velocity on the Langmuir bound and one averaged by channel cross section. The obtained conclusion can be used in a quasi-one-dimensional mathematical model of processes with the Hall effect and a plasma-ion thruster with appropriate boundary conditions.

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РАДІАЛЬНИЙ РОЗПОДІЛ ОБЕРТАЛЬНОГО МОМЕНТУ ЕЛЕКТРОНІВ В ХОЛЛІВСЬКОМУ І ПЛАЗМОВО-ІОННОМУ ДВИГУНІ

Го Цзуншуай

Предметом статті є радіальний розподіл параметрів руху електронів усередині електроракетних двигунів із замкнутим дрейфом електронів. Радіальне магнітне поле в холлівських двигунах обмежує

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осьовий потік електронів через взаємодію з азимутальним електричним струмом. У свою чергу, цей азимутальний струм існує в результаті суперництва між спробою магнітного поля перетворити струм електронів в повністю замкнутий і втратою моменту обертання електронів в зіткненнях. Аналогічні процеси відбуваються в іонізаційній камері плазмово-іонних двигунів з радіальним магнітним полем. Спроби оцінити параметри електронів тільки на основі зіткнень з іонами і атомами в об’ємі дали значення осьового струму електронів набагато нижче, ніж в дійсності. Це явище отримало назву аномальної провідності електронів, яке намагалися пояснити як наслідок різних ефектів, включаючи «пристінкову провідність», яка пояснюється недзеркальним відбиттям електронів від ленгмюрівського шару біля стінок каналу двигуна. Недоліком цієї назви є те, що відбиття електрона відбувається до досягнення поверхні від потенціального бар’єру на межі плазми з будь-яким середовищем: стінкою, але також з вакуумом навколишнього середовища. Розподіл потенціалу в ленгмюрівському шарі є нестаціонарним і неоднорідним через наявність так званих плазмових коливань. Визначення «провідність» в цій назві є настільки ж невдалим, тому що зіткнення завжди є чинником не провідності, а, навпаки, опору. **Метою** є вирішення завдання про розподіл моменту обертання електронів в каналі двигуна. **Використовувані методи**: формулювання кінетичного рівняння для функції розподілу електронів за швидкостями, радіусом і проекцією координат миттєвого центру циклотронного обертання; рішення цього рівняння і знаходження з його допомогою розподілу газодинамічних параметрів електронів по поперечному перетину каналу. **Висновки**. Запропоновано математичну модель динаміки моменту обертання електронів, яка дозволяє за допомогою рівнянь динаміки плазми аналізувати його розподіл по поперечному перетину каналу двигуна і оцінювати ефект «пристінкової провідності» з використанням відповідних граничних умов.

Ключові слова: холлівський двигун; обертальний момент електронів; функція розподілу за швидкостями; кінетичне рівняння; ленгмюрівський прошарок.

РАДИАЛЬНОЕ РАСПРЕДЕЛЕНИЕ ВРАЩАТЕЛЬНОГО МОМЕНТА ЭЛЕКТРОНОВ В ХОЛЛОВСКОМ И ПЛАЗМЕННО-ИОННОМ ДВИГАТЕЛЕ

Го Цзуншуай

Предметом статьи является радиальное распределение параметров движения электронов внутри электроракетных двигателей с замкнутым дрейфом электронов. Радиальное магнитное поле в холловских двигателях ограничивает осевой поток электронов из-за взаимодействия с азимутальным электронным током. В свою очередь, этот азимутальный ток существует в результате соперничества между попыткой магнитного поля преобразовать ток электронов в полностью замкнутый и потерей момента вращения электронов в столкновениях. Аналогичные процессы происходят в ионизационной камере плазменно-ионных двигателей с радиальным магнитным полем. Попытки оценить параметры электронов только на основе столкновений с ионами и атомами в объеме дали значение осевого тока электронов намного ниже, чем в действительности. Это явление получило название аномальной проводимости электронов, которое пытались объяснить как следствие различных эффектов, включая «пристеночную проводимость», которая объясняется незеркальным отражением электронов от ленгмюровского слоя у стенок канала двигателя. Недостатком этого названия является то, что отражение электрона происходит до достижения поверхности от потенциального барьера на границе плазмы с любой средой: стенкой, но также с вакуумом окружающей среды. Распределение потенциала в ленгмюровском слое нестационарно и неоднородно из-за наличия так называемых плазменных колебаний. Определение «проводимость» в этом названии столь же неудачно, потому что столкновения всегда являются фактором не проводимости, а, наоборот, сопротивления. **Целью является** решение задачи о распределении момента вращения электронов в канале двигателя. **Используемые методы**: формулировка кинетического уравнения для функции распределения электронов по скоростям, радиусу и проекциям координат мгновенного центра циклотронного вращения; решение этого уравнения и нахождение с его помощью распределения газодинамических параметров электронов по поперечному сечению канала. **Выводы**. Предложена математическая модель динамики момента вращения электронов, которая позволяет с помощью уравнений динамики плазмы анализировать его распределение по поперечному сечению канала двигателя и оценивать эффект «пристеночной проводимости» с использованием соответствующих граничных условий.

Ключевые слова: холловский двигатель; момент вращения электронов; функция распределения по скоростям; кинетическое уравнение; ленгмюровский слой.

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