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ATOMIC FUNCTIONS AND THEIR GENERALIZATIONS IN DATA PROCESSING: FUNCTION THEORY APPROACH

The atomic function is solutions with a compact support of the linear functional differential equations with constant coefficients and linear transforms of the argument. The atomic function theory was created in the 70's of the 20th century due to the necessity to solve different applied problems, in particular, boundary value problems. One of the reasons for the appearance of atomic functions and some other classes of functions was the inability to apply such classic approximation tools as algebraic and trigonometric polynomials. V.A. Rvachev up-function is the most famous and widely applies atomic function. The development of technologies changes the existing problems and fundamentally new problems appear. Nowadays big data processing is one of the most important problems. It should be mentioned that suitable mathematical tools must be applied to obtain the desired result. This paper is devoted to the fundamentals of applications of some atomic functions and their generalizations in data processing and data reduction. In this paper, we consider the main properties of these functions from the function theory point of view and give their interpretation with respect to information processing. Smoothness, compact support, and good approximation properties are the main advantages of atomic functions. Moreover, the spaces of atomic functions and the spaces of generalized Fup-functions, which are the natural generalization of V.A. Rvachev Fup-functions, are asymptotically extremal for the approximation of periodic differentiable functions. This means that in the terms of A.N. Kolmogorov width these functions are just as effective as classic trigonometric polynomials {1, cos(nx), sin(nx)}. Hence, the replacement of discrete transforms based on trigonometric functions on similar transforms based on atomic functions and generalized Fup-functions is quite promising. For this purpose, we introduce discrete atomic transform and generalized discrete atomic transform. We also discuss the dependence of data processing results on order of smoothness and size of support of the applied functions. The theoretical justification of the application of some atomic functions and generalized Fup-functions to data processing and, in particular, data reduction is the main result of this paper.

Keywords: *data processing, data compression, atomic functions, up-function, Fup-function, generalized Fup-function, discrete atomic transform, generalized discrete atomic transform.*

Introduction

Theory of atomic functions appeared in the 70’s of the 20th century because of necessity to solve different applied problems, especially boundary value problems [1]. The first atomic function is V. A. Rvachev up-function

$$\mathrm{up}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{itx}\prod_{k=1}^{\infty}\frac{\sin\left(t\cdot 2^{-k}\right)}{t\cdot 2^{-k}}dt .$$

This function is the most famous and widely used atomic function. Later many other different atomic functions were constructed and investigated. One of the reasons for the appearance and further development of atomic functions and some other classes of new functions was the inability to use in numerical methods such classic constructive tools as trigonometric and algebraic polynomials [1].

Now fundamentally new technical capabilities have been created and science community has a significant number of totally new problems. Processing of big data is one of them.

It is well-known that complexity and efficiency of algorithms of big data sets processing depend on properties of used mathematical tools. Atomic functions combine a huge number of convenient properties. For this reason they have many different applications (note that a list of current application is presented in [4]; also comprehensive survey, which was actual at the time of writing, can be found in [5]). It should be mentioned that at the current time atomic functions are mostly used as a tool for analysis of functions of real or complex variables. But now data is often discrete (for example, digital image is a matrix). Therefore, it is the development of algorithms for discrete data processing that is one of the most important problems. Hence, the following question is quite natural: **is it rational to use atomic functions in processing of discrete data?** In this paper we get an answer on this question. For this purpose the following approach is used: we consider the main properties of some atomic functions and their generalizations from the function theory point of view and obtain the interpretation with respect to information processing.

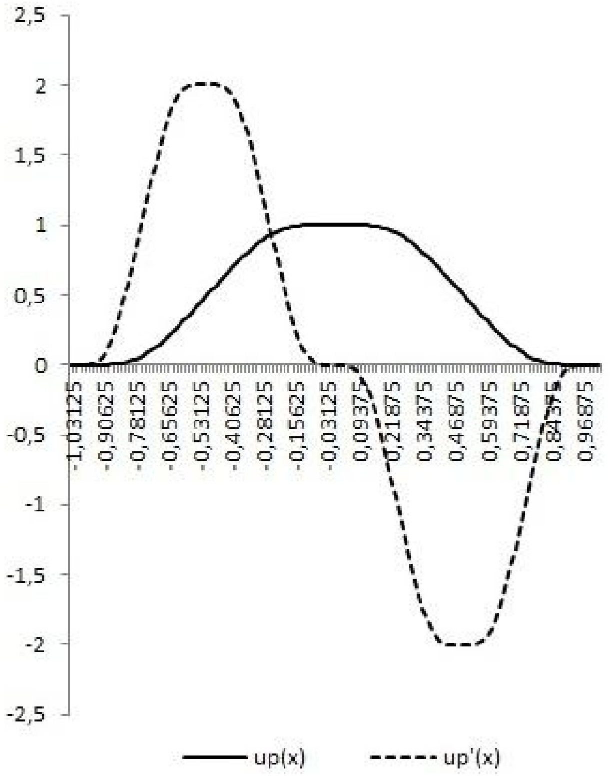

Fig. 1. The graph of up-function and its derivative

Formulation of the problem

Consider the space of up-function shifts

$$UP_n = \left\{ \varphi(x): \ \varphi(x) = \sum_k c_k \, up\left(x - \frac{k}{2^n} \right) \right\}, \ n = 0,1,2,\dots.$$

Approximation properties of these spaces were investigated in [1 - 3]. It was also shown that in the space UP_n there exists a basis that consists of translates of the function

$$Fup_n(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{itx} \left(\frac{\sin\left(t2^{-n-1}\right)}{t2^{-n-1}} \right)^n F\left(\frac{t}{2^n} \right) dt,$$

where $F(t)$ is the Fourier transform of $up(x)$ with a local support.

These results were generalized in [12] for the case of spaces

$$UP_{s,n} = \left\{ \varphi: \ \varphi(x) = \sum_k c_k up_{s,n} \left(x - \frac{k}{(2s)^n} \right) \right\},$$

where $s = 2,3,4,\dots$, $n = 0,1,2,\dots$ and

$$up_s(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{itx} \prod_{k=1}^{\infty} \frac{\sin^2\left(st(2s)^{-k}\right)}{s^2 (2s)^{-k} \sin\left(t(2s)^{-k}\right)} dt.$$

In particular, it was proved that shifts of the function

$$Fup_{s,n}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{itx} \left(\frac{\sin \frac{t}{2(2s)^n}}{\frac{t}{2(2s)^n}} \right)^n F_s\left(\frac{t}{(2s)^n} \right) dt,$$

where $F_s(t)$ is the Fourier transform of $up_s(x)$, constitute a basis of the space $UP_{s,n}$.

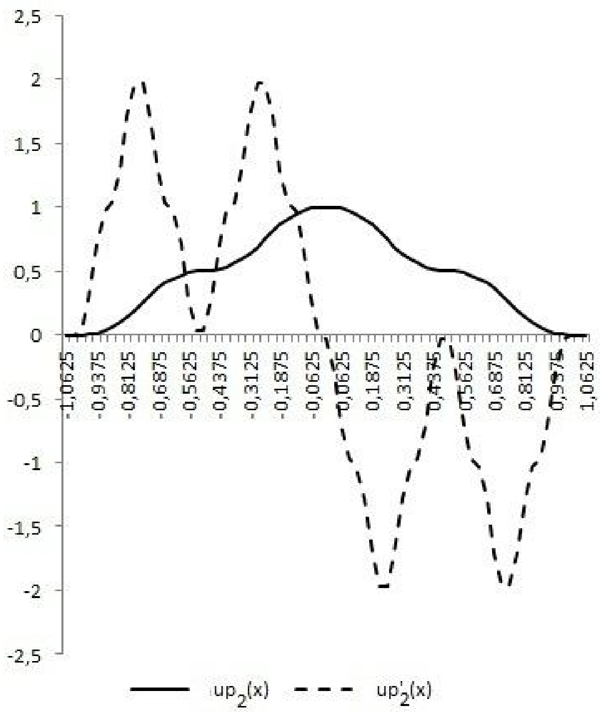

Fig. 2. The graph of $up_2(x)$ and its derivative

Consider the function

$$f_{N,m}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{itx} \left(\frac{\sin(t/N)}{t/N} \right)^{m+1} F(t/N) dt,$$

where F(t) is the Fourier transform of the positive even function $f(x) \in L_2(\mathbb{R})$ with a support $\operatorname{supp} f(x) = [-1,1]$, $\int_{-\infty}^{\infty} f(x)dx = 1$, $N \neq 0$ and $m \in \mathbb{N}$. The function $f_{N,m}(x)$ is called generalized Fup-function and $f(x)$ is its mother function [6, 7].

Generalized Fup-function is a natural generalization of V. A. Rvachev Fup-functions and $Fup_{s,n}(x)$.

It follows from [1 – 3, 6 – 8] that UP_n, $UP_{s,n}$ and the spaces of generalized Fup-functions shifts combine the following advantages:

1) these spaces are asymptotically extremal for approximation of periodic differentiable functions;

2) the spaces UP_n are extremal for approximation of periodic differentiable functions;

3) these spaces have a locally supported bases;

4) elements of these spaces have high order of smoothness.

This implies that atomic functions $up(x)$, $up_s(x)$ and generalized Fup-functions are convenient for applications, in particular, for construction of different numerical methods. In other words, from function theory point of view these functions are excellent mathematical tools.

The aim of this paper is to prove the efficiency of $up(x)$, $up_s(x)$ and generalized Fup-functions in discrete data processing.

Discrete data transform

The spaces of atomic functions and generalized Fup-functions can be easily used for representation of real or complex variable functions. Let us introduce the following scheme of discrete data representation.

We consider the case of one-dimensional of real-valued data.

Let $D = \{d_0, d_1, \dots, d_M\}$ be some data. The value of d_i can be the result of measuring a certain indicator. For example, d_i is the temperature or the exchange rate at the i-th moment in time.

Let us construct the function $d(x)$ such that the data D is a set of values of this function. For this purpose it is sufficient to construct the scale of the independent variable x.

By $[a,b]$ denote some segment. Let $x_i = a + i \cdot h$, where $h = (b-a)/M$ and $i = 0,1,\dots,M$.

Further, denote by L the space of atomic functions UP_n or $UP_{s,n}$.

Finally, let the system of functions $\{\varphi_j(x)\}$ be a basis of the space L.

Using classic decomposition procedure, we can find the function

$$d(x) = \sum_k \omega_k \varphi_k(x) \quad (1)$$

such that

$$d(x_i) = d_i \quad (2)$$

for any $i = 0,1,\dots,M$.

Note that the set of atomic coefficients $\Omega = \{\omega_k\}$ uniquely identifies the data set $D = \{d_j\}$.

In this way we construct the procedure for the transformation of some discrete data D into atomic coefficients Ω. We call this procedure the **discrete atomic transform** (DAT) and the set of coefficients the **DAT-coefficients**. If L is a space of generalized Fup-functions, we call this procedure the **generalized discrete atomic transform**.

It is obvious that the basis $\{\varphi_j(x)\}$ of the linear space L can be chosen in different ways. The simplest approach is to choose atomic wavelets or generalized atomic wavelets, which were constructed in [4, 9 – 11], as a basis. In this case we get wavelet expansion of the discrete data D.

In the same way we can construct DAT-procedure and generalized DAT-procedure for representation of complex-valued and multi-dimensional discrete data D.

Dependence of data processing complexity on the support of the basic functions

In the previous section we introduced the special numerical scheme that can be used for discrete data processing. It is evident that the following questions are of particular interest:

1) what is the complexity of DAT and generalized DAT procedures?

2) what is the complexity of algorithm for obtaining initial data?

It is obvious that complexity of these algorithms principally depends on the properties of the basis $\{\varphi_j(x)\}$.

Existence of the locally supported basis in the spaces of atomic functions and generalized Fup-functions is a convenient feature of these spaces. For instance,

$$\operatorname{supp} Fup_n(x) = \left[-\frac{n+2}{2^{n+1}}, \frac{n+2}{2^{n+1}} \right] \quad (3)$$

and the system of functions

$$\left\{ Fup_n \left(x - \frac{2k+n}{2^{n+1}} \right) \right\}_k \quad (4)$$

is a basis of the space UP_n [1, 3];

$$\operatorname{supp} Fup_{s,n}(x) = \left[-\frac{n+2}{2(2s)^n}, \frac{n+2}{2(2s)^n} \right] \quad (5)$$

and the system

$$\left\{ Fup_{s,n} \left(x - \frac{2k+n}{2(2s)^n} \right) \right\}_k \quad (6)$$

constitutes a basis in the space $UP_{s,n}$ [12];

finally,

$$\operatorname{supp} f_{N,m}(x) = \left[-\frac{m+2}{N}, \frac{m+2}{N} \right] \quad (7)$$

and spaces of these generalized Fup-functions have a basis

$$\left\{ f_{N,m}\left(x - \frac{2k+m}{N} \right) \right\}_k \qquad (8)$$

(actually, the spaces of generalized Fup-functions are constructed as a linear span of the system (8) [8]).

To obtain DAT-coefficient Ω we put $x = x_i$ for $i = 0,1,...,N$ in expansion (1). Using (2), we get the system of linear algebraic equations. If (4), (6) or (8) is a basis of the space L, then it follows from (3), (5) and (7) that matrix of this system is band. Hence, complexity of solution is $\underline{O}(M)$. In other words, the algorithm of transform of D into the coefficients Ω has linear complexity in the size of initial data. This means that **DAT and generalized DAT procedures have linear complexity in the size of data**. By the same argument, **complexity of the inverse procedure is also linear**. Indeed, to obtain the value d_i we should put $x = x_i$ in (1). From (3), (5) and (7) it follows that the most part of the terms on the right side of equality (1) is equal to zero. So, we need to calculate the sum of several terms.

If we choose atomic wavelets or generalized atomic wavelets as a basis of the linear space L, we obtain just the same situation. It was shown in [4, 9 - 11] that these wavelets are locally supported functions. Therefore discrete atomic transform and generalized atomic transform of the data D into the coefficients Ω have linear complexity. Also, the inverse procedures are the algorithms with a linear complexity.

Processing of multi-dimensional data using atomic functions and their generalizations has the same convenient feature.

Influence of approximation properties on the data representation

Allocating useful information from initial data is one of the data processing problems. It is often required that the extracted data adequately represent the initial information [12 – 15]. Obviously, the efficiency of such extraction algorithms depends on many factors, in particular, on the approximation properties of the applied mathematical tools.

For example, algorithm JPEG is de-facto a standard for compression of digital images. In this algorithm, compression is achieved due to the fact that most of the quantized coefficients, which are obtained using discrete cosine transform (DCT), are equal to zero. This means that a few DCT-coefficients describe all information. This important property is based on the well-known approximation properties of trigonometric polynomials $\{1, \cos(nx), \sin(nx)\}$.

It was proved in [1 – 3, 6, 8] that the best approximation of wide classes of periodic differentiable functions by spaces of linear combinations of atomic functions $up(x)$, $up_s(x)$ and generalized Fup-functions $f_{N,m}(x)$ almost coincides with the corresponding A.N. Kolmogorov width (we note also that in some cases the value of best approximation is equal to the width). This means that in the terms of A.N. Kolmogorov width these functions are just as effective as classic trigonometric polynomials. Hence, it is enough to operate with an insignificant number of DAT or generalized DAT coefficients in the process of data analysis. So, **DAT-coefficients and generalized DAT-coefficients are convenient tools for processing of discrete data**.

Dependence of data representation on the order of smoothness

The wide use of classic trigonometric polynomials is due to many factors. The following one is the most important: there are many physical processes and phenomena, which are well described using trigonometric functions.

But in some cases application of $\cos(x)$ and $\sin(x)$ leads to undesirable effects. These functions are analytic. For this reason processing of data with smooth changes using trigonometric functions is so useful. For example, application of discrete cosine transform in JPEG algorithm allows achieving a high compression ratio. At the same time, it can be easily checked that if we recompress JPEG-image using JPEG algorithm, we get degradation in quality or increase the file size. What is the reason for this effect? The answer is quantization of DCT-coefficients. Indeed, this procedure is used to obtain integer numbers instead of real-valued DCT-coefficients. But in the process of decompression we get data with less smooth changes. So, the decompressed image is less analytical than the original one. This yields that JPEG recompression becomes less effective.

Atomic functions $up(x)$ and $up_s(x)$ are **not** analytic. Moreover, these functions are non-quasianalytic (for more details see, for example, [2]). By construction, order of smoothness of the generalized Fup-function depends on the properties of the mother function. For example, if the mother function $f(x)$ is non-quasianalytic, then $f_{N,m}(x)$ is non-quasianalytic too. Therefore, application of atomic functions and generalized Fup-functions to processing of data with low order of smoothness is quite promising.

Besides, it was shown in [1, 6, 8] that there exists almost trigonometric basis in the spaces of atomic functions and generalized Fup-functions. This means that these spaces are just as good approximation tools as trigonometric polynomials. Combining this with the previous conclusion, we see that **application of DAT-procedure and generalized DAT-procedure is much more effective than discrete cosine transform**.

Application perspectives

In this section we consider some actual problems that can be solved or partially solved using DAT and generalized DAT.

As it was shown above, DAT and generalized DAT procedures can be effectively used instead of DCT. Hence, it is quite natural to use them, for example, in lossy data compression (notice that a detailed discussion of application of atomic functions $\mathrm{up}_s(x)$ to lossy image compression is presented in [16]). For this purpose the following scheme can be used:

1) preliminary data processing (for example, RGB-to-YCrCb transformation, block splitting procedure etc.);

2) discrete atomic transform (or generalized discrete atomic transform);

3) quantization of DAT-coefficients (or generalized DAT-coefficients);

4) lossless compression of quantized coefficients.

Another application is an analysis of time series. For this purpose generalized atomic wavelet expansion, which was introduced in [4], can be used.

This is a partial list of practical applications. It is clear that investigation of all possible applications of discrete atomic transform and generalized discrete atomic transform is a topic for a special research.

Conclusions

In this paper we have introduced discrete atomic transform and generalized discrete atomic transform that can be used in discrete data processing. It have been shown that

1) DAT-procedure and generalized DAT-procedure have linear complexity in the size of initial data;

2) DAT-coefficients and generalized DAT-coefficients are convenient for discrete data processing and analysis;

3) procedures, which are inverse to DAT and generalized DAT, have linear complexity;

4) application of DAT-procedure and generalized DAT-procedures is much more promising than DCT-procedure.

We also have presented some possible applications of DAT-procedure and generalized DAT-procedures.

In general, the main result of the current paper is a fundamental justification of application of atomic functions $\mathrm{up}(x)$, $\mathrm{up}_s(x)$ and generalized Fup-functions $f_{N,m}(x)$ to the processing of discrete data and lossy data compression.

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АТОМАРНІ ФУНКЦІЇ ТА ЇХ УЗАГАЛЬНЕННЯ В ОБРОБЦІ ДАНИХ: ТЕОРЕТИКО-ФУНКЦІОНАЛЬНИЙ ПІДХІД

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Атомарна функція – це розв'язок з компактним носієм лінійного функціонально-диференціального рівняння з постійними коефіцієнтами та лінійними перетвореннями аргументу. Теорія атомарних функцій виникла у 70-х роках 20-го сторіччя через необхідність розв'язувати різноманітні прикладні проблеми, зокрема крайові задачі. Однією з причин виникнення атомарних функцій та багатьох інших класів функцій є неможливість використання таких класичних засобів наближення, як алгебраїчні та тригонометричні поліноми. Однією з найбільш відомих атомарних функцій є up-функція В. О. Рвачова. З розвитком технічних засобів змінювалися існуючі проблеми та виникали принципово нові. Так, зараз однією з найбільш важливих проблем є обробка великих об'ємів даних. При цьому ефективність алгоритмів здебільшого залежить від властивостей математичного апарату, що використовується. Ця стаття присвячена базовим принципам застосування деяких атомарних функцій та їх узагальнень в обробці даних та стисненні інформації з втратами якості. У роботі ми розглянемо основні теоретико–функціональні властивості цих функцій та надамо їх інтерпретацію стосовно обробки інформації. Основними перевагами атомарних функцій є гладкість, компактність носія та гарні апроксимаційні властивості. Окрім того, простори атомарних функцій та узагальнених Fup-функцій, які є цілком природним узагальненням Fup-функцій В. О. Рвачова, асимптотично екстремальні для наближення періодичних диференційованих функцій. У термінах поперечника за А. Н. Колмогоровим це означає, що ці функції так само ефективні, як і класичні тригонометричні поліноми {1, cos(nx), sin(nx)}. Тому заміна дискретних перетворень, що базуються на тригонометричних функціях, на відповідні перетворення, які будуються за допомогою атомарних функцій та узагальнених Fup-функцій, є перспективною. З цією метою ми вводимо у розгляд дискретне атомарне перетворення та узагальнене дискретне атомарне перетворення. Ми також розглянемо залежність результатів обробки даних від порядку гладкості та розміру носія. Основним результатом роботи є теоретичне обґрунтування ефективності атомарних функцій та узагальнених Fup-функцій в обробці даних та стисненні інформації.

Ключові слова: обробка даних, стиснення інформації, атомарні функції, up-функція, Fup-функція, узагальнена Fup-функція, дискретне атомарне перетворення, узагальнене дискретне атомарне перетворення.

АТОМАРНЫЕ ФУНКЦИИ И ИХ ОБОБЩЕНИЯ В ОБРАБОТКЕ ДАННЫХ: ТЕОРЕТИКО-ФУНКЦИОНАЛЬНЫЙ ПОДХОД

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Атомарной функцией называется решение с компактным носителем линейного функционально-дифференциального уравнения с постоянными коэффициентами и линейными преобразованиями аргумента. Теория атомарных функций возникла в 70-х годах 20-го столетия в связи с необходимостью решать различные прикладные задачи, в частности, краевые задачи. Одной из причин возникновения атомарных функций и многих других классов функций является невозможность применения таких классических средств приближения, как алгебраические и тригонометрические полиномы. Одной из наиболее известных атомарных функций является up-функция В. А. Рвачева. С развитием технических средств меняются существующие проблемы и появляются принципиально новые. Так, в настоящее время одной из наиболее важных проблем является обработка больших данных. При этом эффективность алгоритмов существенным образом зависит от применяемого математического аппарата. Данная статья посвящена базовым принципам применения атомарных функций и их обобщений в обработке и сжатии информации. В работе мы рассмотрим основные теоретико-функциональные свойства этих функций и приведём их интерпретацию с позиции обработки информации. Основными преимуществами атомарных функций являются гладкость, компактность носителя и хорошие аппроксимационные свойства. Кроме того, пространства атомарных функций и обобщённых Fup-функций, которые являются естественным обобщением Fup-функций В. А. Рвачева, асимптотически экстремальны для приближения периодических дифференцируемых функций. В терминах поперечника по А. Н. Колмогорову это значит, что такие функции являются такими же эффективными, как и классические тригонометрические полиномы {1, cos(nx), sin(nx)}. Поэтому замена дискретных преобразований, которые базируются на тригонометрических функциях, на соответствующие преобразования, которые строятся с помощью атомарных функций и обобщённых Fup-функций, является перспективной. С этой целью мы вводим дискретное атомарное преобразование и обобщённое дискретное атомарное преобразование. Также мы рассмотрим зависимость результатов обработки данных от порядка гладкости и размера носителя. Основным результатом работы является теоретическое обоснование эффективности атомарных функций и обобщённых Fup-функций в обработке данных и сжатии информации.

Ключевые слова: обработка данных, сжатие информации, атомарные функции, up-функция, Fup-функция, обобщённая Fup-функция, дискретное атомарное преобразование, обобщённое дискретное атомарное преобразование.

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