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MOTIONS MODELS OF A TWO-WHEELED EXPERIMENTAL SAMPLE

The subject of study is the physical processes of translational and angular motion of a two-wheeled experimental sample. The goal is to develop physical, mathematical, and graphic models of the translational and angular motions of a two-wheeled experimental sample as an object of automatic control. The objectives: to form physical models of a two-wheeled experimental sample; to develop a nonlinear mathematical description of the processes of translational and angular sample`s motions using the Lagrange approach; to obtain a linearized mathematical sample`s description as an object of automatic control in the state space and frequency domain; to generate graphic models in the form of structural diagrams in the time and frequency domains; to analyze the functional properties of an object of automatic control: stability, controllability, observability, structural and signal diagnosability concerning violations of the functional properties of electric drives and sensors of the angular position of the body and wheels. The methods of the study: the Lagrange method, Taylor series, state-space method, Laplace transformations, Lyapunov, Kalman criteria, and diagnosability criterion. The results: physical models of a two-wheeled experimental sample have been obtained in the form of a kinematic diagram of the mechanical part and the electric circuit of an electric drive; mathematical descriptions of translational and angular motions have been developed in nonlinear and linearized forms; structural diagrams have been developed; functional characteristics of a two-wheeled experimental model as an object of automatic control have been analyzed to solve problems of control algorithms synthesis. Conclusions. The scientific novelty lies in obtaining new models that describe the translational and angular motion of a two-wheeled experimental model as an object of automatic control. The obtained models differ from the known ones by considering the dynamic properties of sensors and electric drives, as well as the relationship of movements.

Keywords: *two-wheeled experimental sample; motion models; object of automatic control; mathematical models; state space; controllability; observability; diagnosability.*

Introduction

Motivation. The property of unstable angular motions relative to the center of mass is a feature of a number of aircraft classes. So, to ensure the maneuverability of fighters, the center of pressure of aerodynamic forces is placed in front of the center of mass. The same is true for the design of passenger and transport wide-body aircraft if it is need to ensure higher technical and economic performance indicators. At the initial stage of movement, the launch vehicles are affected by the overturning moment of aerodynamic forces. In space where damping environment is absent flying vehicles are affected by a number of perturbing influences leading to unstable angular motions relative to the center of mass [1].

The simplest dynamically similar prototypes for the study and investigation of unstable angular motions are various devices of the "reverse pendulum" type [2]. Using pendulum devices allows to transfer the study the properties of unstable aircraft angular motions to simpler objects of unstable motions. There are necessary to use simple objects of unstable motions with the aim to study structurally unstable control objects and methods of their stabilization in the courses "Automatic control theory" and "Digital control systems". Such objects are interesting not only because of their unstable and non-linear nature, but also due to their application out of the academic sphere.

Pendulum devices, which are an autonomous two-wheeled vehicle, are used as passenger carriage in urban environments [3], helping people with disabilities [4], luggage transportation [5], working in dangerous conditions for human life and health [6], and in other cases.

Two-wheeled vehicles have a number of advantages over other vehicles: small size, simple and compact design, high maneuverability, low cost, significant environmental friendliness [7].

State of the Art. The need of research on the modeling motion of the two-wheeled vehicle is reasoned by the certain circumstances. In the known models of two-wheeled vehicles, only part of motions are taken into account, which are caused by the angular position instability [8]. To solve the problems of trajectory algorithms synthesis there are needed the models that reflect the relationship of translational and angular motions both in the form of nonlinear and linearized equations. Moreover, for the development of high-precision control system for two-wheeled vehicle, it is necessary to take into account not only its dynamics, but also the dynamic properties of electric drives, angular position sensors and displacement sensors, i.e. to consider it as an object of automatic control.

Objectives and structure. The purpose of the study presented in the article is to build physical, mathematical and graphic models of the translational and angular motions of a two-wheeled experimental sample on a flat surface as an object of automatic control.

The first section of the article presents a general view of a two-wheeled experimental sample and describes the physical models of its mechanical and electromechanical parts. The second section is devoted to the formation of a nonlinear mathematical description using the Lagrange formalism. The linear mathematical description of a two-wheeled experimental sample is given in the third section. It describes mathematical model of the sample as an object of automatic control in the time and frequency domains. In the fourth section of the article, graphic models are given in the form of structural diagrams, which reflect the transformation processes in a two-wheeled experimental sample as an object of automatic control. The fifth section is devoted to the analysis of the functional properties of the object of automatic control. In conclusion, the results of research on the formation of a two-wheeled experimental sample models as an object of automatic control are summarized.

1. Physical models of a two-wheeled experimental sample

The variety of pendulum devices are used at the Aircraft Control Systems Department to research unstable angular motions relative to the center of mass. Fig. 1 shows a two-wheeled experimental sample (TES). The physical model of the rectilinear sample motion with a small angular deviation of the body relative to the vertical axis is shown in fig. 2.

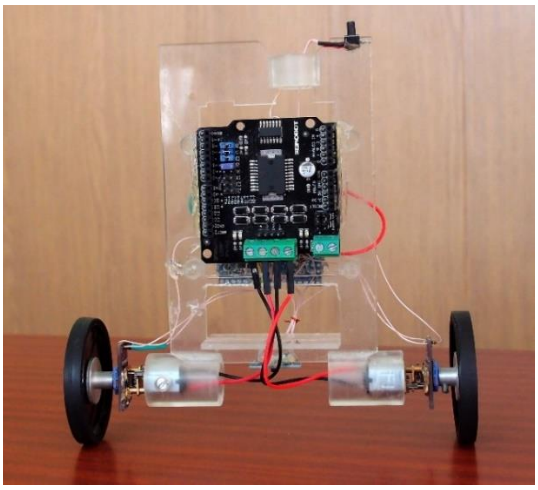

Fig. 1. Two-wheeled experimental sample

The sample is a rectangular parallelepiped, placed on two coaxial wheels, driven by internal DC electric engines with a total torque T_S. The sample is affected by the force of gravity P applied at the center of mass (c.m.) and the force of friction f. The sample makes motions along the Ox axis with speed V.

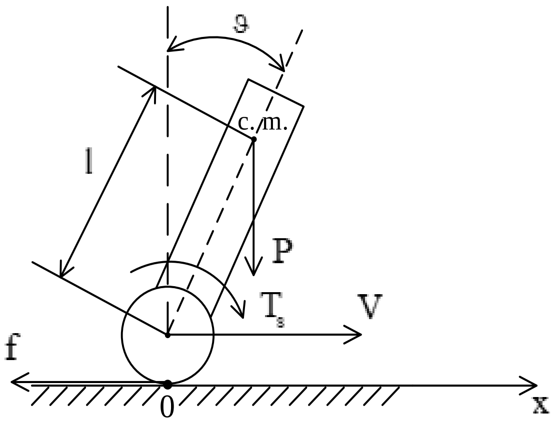

Fig. 2. Physical model of the TES mechanical part

The physical model of the electric wheel drive is shown in fig. 3.

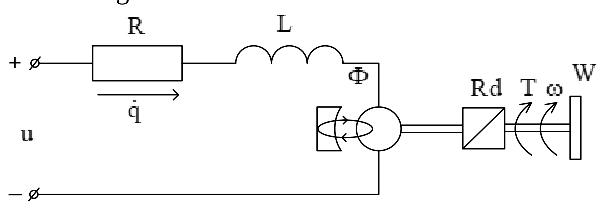

Fig. 3. Physical model of the electric drive

The model uses the following notation: u – control voltage; $\dot{q}$ – armature current; R – armature winding resistance; L – armature winding inductance; Φ – permanent magnet excitation flux; Rd – reducer; ω –angular speed of wheel (W).

2. Nonlinear mathematical description

To formalize the processes of TES motion, there have been used the Lagrange approach [9] in the following form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\lambda}_i}\right) - \frac{\partial T}{\partial \lambda_i} = Q_i^u + Q_i^f \; ; \; i = \overline{1, n} \, , \qquad (1)$$

where T – the kinetic energy of modeling object;

λ_i – the generalized coordinate;

Q_i^u – the generalized force of control impacts;

Q_i^f – the generalized force of disturbances.

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The TES motion has four degrees of freedom and is characterized by such generalized coordinates: displacement – x; angular position of the body – ϑ; angle of wheel rotation – φ; charge quantity – q.

The kinetic energy of the TES is described by the following expression:

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_b\vartheta^2 + \frac{1}{2}J\dot{\varphi}^2 + \frac{1}{2}L\dot{q}^2, \quad (2)$$

here m – TES weight; J_b – TES moment of inertia relative to the axis of rotation; J – total kinetic moment of rotating parts of electric drives.

The generalized forces for each generalized coordinate are described as follows:

$$\begin{gathered} Q_1^u = \frac{T_s}{r} + ml\ddot{\vartheta};\ Q_1^f = ml\ddot{\vartheta} - f; \\ Q_2^u = -ml\ddot{x};\ Q_2^f = Pl\sin\vartheta; \\ Q_3^u = T_s;\ Q_3^f = -fr; \\ Q_4^u = u;\ Q_4^f = -R\dot{q} - e, \end{gathered} \quad (3)$$

where r – wheel radius; e – back-EMF induced in the armature winding of an electric drive when it rotates with an angular velocity $\omega = \dot{\varphi}$.

Performing the appropriate actions in accordance with equation (1), the following system of equations has been obtained:

$$\begin{cases} m\ddot{x} = \frac{T_s}{r} - ml\ddot{\vartheta} - f; \\ J_b\ddot{\vartheta} = Pl\sin\vartheta - ml\ddot{x}; \\ J\ddot{\varphi} = T_s - fr; \\ L\ddot{q} = u - R\dot{q} - e. \end{cases} \quad (4)$$

The moments and forces on the right-hand sides of the equations depend on the generalized coordinates. These dependencies can be described analytically. So, the torque of the electric drive is characterized by the following expression:

$$T_s = \kappa_{rd}c_m\Phi\dot{q} = \kappa_m\dot{q}, \quad (5)$$

where κ_{rd} – the reducer`s gear ratio;

c_m – constructive constant of the electric motor;

κ_m – transmission ratio of electric drive.

Back-EMF is defined as follows:

$$e = c_e\Phi\dot{\varphi} = \kappa_e\dot{\varphi}, \quad (6)$$

where c_e – constructive constant of the electric motor; κ_e – transmission ratio.

Equations (4) – (6) together describe the nonlinear processes of the TES motion. The nonlinear relationship between the variables characterizing the motion of the sample significantly complicates the understanding of the motion processes in a wide range of their variation. To simplify the understanding of the TES motions, it is possible to significantly reduce the range of variables` variation and go to the equations of the first approximation using the method of analytical linearization.

3. Linear mathematical description

The method of analytical linearization is based upon the use of nonlinear function expansion in a Taylor series under the certain initial conditions and discarding the terms of the series with small variables deviations starting from the second terms. As the initial conditions for the variables, the state of unstable equilibrium has been chosen, i.e. $x_0 = 0$; $\vartheta_0 = 0$; $\varphi_0 = 0$ and $q_0 = 0$. As a result of the linearization procedure, the following system of equations has been obtained:

$$\begin{cases} m\Delta\ddot{x} = \frac{\kappa_m}{r}\Delta\dot{q} - ml\Delta\ddot{\vartheta} - \Delta f; \\ J_b\Delta\ddot{\vartheta} = Pl\cos\varphi_0\Delta\vartheta - ml\Delta\ddot{x}; \\ J\Delta\ddot{\varphi} = \kappa_m\Delta\dot{q}; -r\Delta f; \\ L\Delta\ddot{q} = \Delta u - R\Delta\dot{q} - \kappa_e\Delta\dot{\varphi}. \end{cases} \quad (7)$$

Solving the equation for the higher derivatives, obtain in a specific form:

$$\begin{cases} \Delta\ddot{x} = \frac{\kappa_m}{mr}\Delta\dot{q} - l\Delta\ddot{\vartheta} - \frac{1}{m}\Delta f; \\ \Delta\ddot{\vartheta} = \frac{Pl}{J_b}\Delta\vartheta - \frac{ml}{J_b}\Delta\ddot{x}; \\ \Delta\ddot{\varphi} = \frac{\kappa_m}{J}\Delta\dot{q} - \frac{r}{J}\Delta f; \\ \Delta\ddot{q} = \frac{1}{L}\Delta u - \frac{R}{L}\Delta\dot{q} - \frac{\kappa_e}{L}\Delta\dot{\varphi} \end{cases} \quad (8)$$

or in general form

$$\begin{cases} \Delta\ddot{x} = a_{11}\Delta\dot{q} - a_{12}\Delta\ddot{\vartheta} - a_{13}\Delta f; \\ \Delta\ddot{\vartheta} = a_{21}\Delta\vartheta - a_{22}\Delta\ddot{x}; \\ \Delta\ddot{\varphi} = a_{31}\Delta\dot{q} - a_{32}\Delta f; \\ \Delta\ddot{q} = -a_{41}\Delta\dot{q} - a_{42}\Delta\dot{\varphi} + a_{43}\Delta u. \end{cases} \quad (9)$$

Using substitutions, the system of equations (9) can be transformed to the following:

$$\begin{cases} \Delta\ddot{\vartheta} = \dfrac{a_{21}}{1-a_{22}a_{12}}\Delta\vartheta + \dfrac{a_{22}a_{11}}{a_{31}(1-a_{22}a_{12})}\Delta\ddot{\varphi} - \\ -\dfrac{a_{22}(a_{11}a_{32}-a_{31}a_{13})}{a_{31}(1-a_{22}a_{12})}\Delta f; \\ \Delta\dddot{\varphi} = -a_{41}\Delta\ddot{\varphi} + a_{31}a_{42}\Delta\dot{\varphi} + a_{31}a_{43}\Delta u - \\ -a_{32}\Delta\dot{f} - a_{41}a_{32}\Delta f. \end{cases} \tag{10}$$

In accordance with the state space method [1] the resulting system of equations (10) has been represented in the following state variables: $x_1(t) = \Delta\vartheta$; $x_2(t) = \Delta\dot{\vartheta}$; $x_3(t) = \Delta\varphi$; $x_4(t) = \Delta\dot{\varphi}$; $x_5(t) = \Delta\ddot{\varphi}$. The external influences are relabeled as follows: $\Delta u = u(t)$; $f = f(t)$. Assuming constancy of Δf, it is considered that $\Delta\dot{f} = 0$. As a result of the corresponding transformations, a system of equations in vector-matrix form has been obtained:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_5(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ a'_{21} & 0 & 0 & 0 & a'_{25} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a'_{54} & a'_{55} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} + \tag{11}$$

$$+\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ b_5 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ b'_2 \\ 0 \\ 0 \\ b'_5 \end{bmatrix} f(t); \begin{bmatrix} x_1(t_0) \\ x_2(t_0) \\ x_3(t_0) \\ x_4(t_0) \\ x_5(t_0) \end{bmatrix} = \begin{bmatrix} x_{10} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

where $a'_{21} = \dfrac{a_{21}}{1-a_{22}a_{12}}$; $a'_{25} = \dfrac{-a_{22}a_{11}}{a_{31}(1-a_{22}a_{12})}$;

$b'_2 = \dfrac{-a_{22}(a_{11}a_{32}-a_{31}a_{13})}{a_{31}(1-a_{22}a_{12})}$; $a'_{54} = a_{31}a_{42}$; $a'_{55} = -a_{41}$;

$b_5 = a_{31}a_{43}$; $b_5{}' = -a_{41}a_{32}{}'$.

For automatic control of the TES motion, sensors of its angular position ϑ and wheel angle φ sensors are required. Then the output signals of the sensors $u_{s1}(t)$ and $u_{s2}(t)$ are related through the corresponding transmission coefficients κ_{s1} and κ_{s2} with the state vector as follows:

$$\begin{bmatrix} u_{s1}(t) \\ u_{s2}(t) \end{bmatrix} = \begin{bmatrix} \kappa_{s1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \kappa_{s2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix}. \tag{12}$$

In a more compact form, the system of equations (11) can be written as:

$$\begin{aligned} &\dot{x}(t) = Ax(t) + bu(t) + b_f f(t); \\ &x(t_0) = x_0; u_s(t) = Cx(t). \end{aligned} \tag{13}$$

To obtain a mathematical description of the TES motions as an object of automatic control in the frequency domain, the Laplace transform for equations (13) with zero initial conditions was used and, as a result, the following operator equations have been obtained:

$$\begin{aligned} sX(s) = AX(s) + bU(s) + b_f F(s); \\ U_s(s) = CX(s), \end{aligned} \tag{14}$$

where s – Laplace transform variable;

$X(s)$ – state vector image $x(t)$;

$U(s)$ – control impact image;

$F(s)$ – disturbance image;

$U_s(s)$ –image of sensor signals vector;

The operator equation for the image $U_s(s)$ has the form:

$$U_s(s) = C[sJ-A]^{-1} bU(s) + C[sJ-A]^{-1} b_f F(s), \tag{15}$$

where J – (5×5) identity matrix.

This operator equation allows to obtain all transfer functions reflecting the connections between the output and input signals images of the TES as an object of automatic control.

So, according to the control impacts:

$$W_1(s) = \frac{U_{s1}(s)}{U(s)} = \frac{\kappa_{s1}a'_{25}b_5 s}{\left(s^2 - a'_{21}\right)\left(s^2 + a'_{55}s + a'_{54}\right)}; \tag{16}$$

$$W_2(s) = \frac{U_{s2}(s)}{U(s)} = \frac{\kappa_{s2}b_5}{s\left(s^2 + a'_{55}s + a'_{54}\right)}. \tag{17}$$

According to the disturbance:

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$$W_1^f(s) = \frac{U_{s1}(s)}{F(s)} = \frac{\kappa_{s1} b_2'}{(s^2 - a_{21}')}; \qquad (18)$$

$$W_2^f(s) = \frac{U_{s2}(s)}{F(s)} = \frac{-\kappa_{s2} b_5'}{s(s^2 + a_{55}' s + a_{54}')}. \qquad (19)$$

The obtained mathematical descriptions of the TES, as an object of automatic control, in the time domain – equations (11) and (12), and in the frequency domain – transfer functions (16) – (19) can be used in algorithms synthesis for the automatic control device of rectilinear motion with a balancing body position.

4. Structural diagrams of the object of automatic control

The graphical representation of mathematical descriptions allows to visualize the structures of transformation processes for a visual assessment of their properties.

The equations (11) and (12), describing the processes of motion in the time domain, can be graphically represented using the structural diagram as shown in fig. 4.

The structural diagram of the object of automatic control in the frequency domain using transfer functions (16) – (19) is shown in fig. 5.

The given structural diagrams reflect both the composition of the converting elements of mathematical descriptions, and the relationship between the elements and signals characteristics in the mathematical object of automatic control.

5. Analysis of the object of automatic control functional properties

The obtained mathematical descriptions of the transforming properties of TES as an object of automatic control make it possible to evaluate the functional properties using analytical analysis tools. So, to assess the state of equilibrium, it is necessary to form a characteristic equation using the following formula:

$$\det[sJ - A] = 0, \qquad (20)$$

where det – determinant symbol.

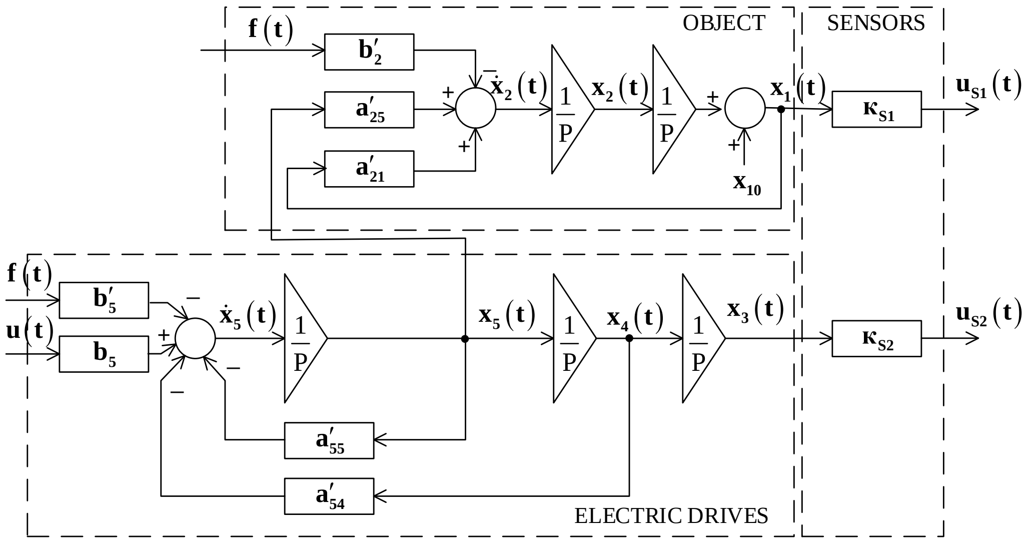

Fig. 4. Structural diagram of the object of automatic control in the time domain

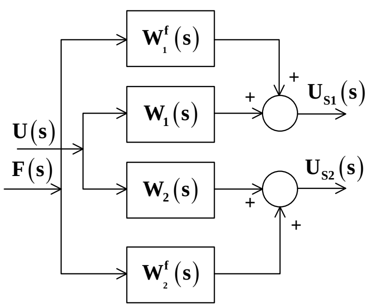

Fig. 5. Structural diagram of the object of automatic control in the frequency domain

To obtain the characteristic equation the determinant of the matrix $[sJ - A]$ should be calculated as following:

$$s\left(s^2 - a'_{21}\right)\left(s^2 + a'_{55}s + a'_{54}\right) = 0. \qquad (21)$$

The roots of the characteristic equation are the following values:

$$s_1 = 0;\; s_{2,3} = \mp\sqrt{a'_{21}};$$
$$s_{4,5} = -\frac{a'_{55}}{2} \pm \sqrt{\frac{\left(a'_{51}\right)^2}{4} - a'_{54}}. \qquad (22)$$

The coefficient a'_{21} in terms of physical characteristics:

$$a'_{21} = \frac{a_{21}}{1 - a_{22}a_{12}} = \frac{Pl}{\left(J_b + ml^2\right)}. \qquad (23)$$

If the coefficient $a'_{21} > 0$, then the roots $s_{2,3}$ are real and have different signs.

Accordingly, the coefficients a'_{55} and a'_{54}:

$$a'_{55} = \frac{a_{41}}{a_{31}} = \frac{RJ}{\kappa_m L};\; a'_{54} = a_{42} = \frac{\kappa_e}{L}. \qquad (24)$$

In steady-state functioning of the electric drives $\kappa_m = \kappa_e$. If, for example, for low-power electric drives $\kappa_m = 50 \cdot 10^{-3}\,\mathrm{Nm/A}$, then $a'_{55} > a'_{54}$, consequently, roots $s_{4,5}$ are real and negative.

Such values of the roots of the characteristic equation indicate an unstable TES state of equilibrium since $s_1 = 0$ and $s_2 = +\sqrt{a'_{21}}$.

The ability of control of all variables of the vector $x(t)$, when we can state and observe all its changes, is estimated using Kalman criterion [1].

Thus, the ability to control the TES estimated using controllability criterion:

$$\mathrm{rang}R = \mathrm{rang}\left[b, Ab, A^2b, A^3b, A^4b\right] = 5, \qquad (25)$$

where rang – controllability matrix rank symbol R; 5 – state vector dimension. Making operations with matrices A and b, according to the criterion, a controllability matrix of size $\dim R = 5\times5$ can be obtained. Among the columns of this matrix, the first two: b and Ab differ from others and among themselves in structure. The column b contains 4 zero components, and the column Ab contains 2 zero components. The columns A^2b, A^3b and A^4b do not contain zero components, but they are linearly independent in terms of components, therefore, the object of automatic control has the property of complete controllability.

In addition to controllability, the object of automatic control must have the property of observability, which means the ability to observe the change of the state vector as the results of available measurements $x(t)$. Observability can be estimate by means of the criterion:

$$\mathrm{rang}Q = \left[C^T\;(CA)^T\;\left(CA^2\right)^T\;\left(CA^3\right)^T\;\left(CA^4\right)^T\right] = 5. \qquad (26)$$

Performing the appropriate transformations with matrices A and C, the observability matrix Q, $\dim Q = 5\times10$ can be obtained. The observability matrix has 5 columns, which are structurally and parametrically linearly independent, that indicates the full observability of the object of automatic control.

Controllability and observability are criteria that reflect the structural properties of an object of automatic control. These properties can be qualitatively assessed using the structural diagram (fig. 5), having analyzed the links of the control action $u(t)$ with the components of the state vector $x(t)$ and its relation with the output vector $u_д(t)$. In the development of linear motion control system with the balancing position of the TES body, the following control principles can be used:

1) disturbance control principle;
2) deviation control principle;
3) diagnostic control principle.

To use the first two principles, an object of automatic control must have obligatory controllability and observability properties. Due to the third principle using the object of automatic control must have the property of diagnosability [10]. Diagnosability of an object of automatic control is understood as a property of its structure and signals, that allow to identify destabilizing influences.

Any changes in the functional characteristics of electric drives and sensors may have destabilizing effects on the considered object of automatic control during functioning. Then the set of destabilizing influences D includes the following events:

$$D = \{d_1, d_2, d_3, d_4\}, \qquad (27)$$

where d_1 – change of inertial properties of electric drives;

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d_2 – reduction of torque of electric drives;

d_3 – reduction of the transmission coefficient of the first sensor;

d_4 – reduction of the transmission coefficient of the second sensor.

By means of parameterization, the following set has been formed:

$$P = \{J, \kappa_m, \kappa_{s1}, \kappa_{s2}\}. \quad (28)$$

To assess the structural properties of the object, the criterion of structural diagnosability [11] has been used with the separate applying to each equation in the state space. The first two parameters are the part of the matrices A and b of the equation (11) and are included into the coefficients $a'_{25} = \frac{a_{22}a_{11}}{a_{31}(1-a_{22}a_{12})} = \frac{Jl}{r\left(J_b + ml^2\right)}$;

$$a'_{55} = -\frac{RJ}{\kappa_m L};\ b_5 = a_{31}a_{43} = \frac{\kappa_m}{JL}.$$

Sensitivity matrices for these parameters looks like following:

$$L_I = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{l}{r\left(J_b + ml^2\right)} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{R}{\kappa_m L} & -\frac{\kappa_m}{J^2 L} \end{bmatrix};$$

$$L_{\kappa_м} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{RJ}{\kappa_m{}^2 L} & \frac{1}{JL} \end{bmatrix}. \quad (29)$$

The parameters κ_{s1} and κ_{s2} are the part of the matrix C of equation (12), therefore

$$L_{\kappa_{s1}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$L_{\kappa_{s2}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \quad (30)$$

In accordance with the criterion of complete structural diagnosability [11], the sensitivity matrices should be linearly independent in all pairwise combinations. Obviously, the reduced matrices (29), (30) are linearly independent; therefore, the object of automatic control is fully diagnosed regarded to the destabilizing effects of the set D.

To assess the signal properties of the object of automatic control, the signal diagnosability criterion [11] has been used with the applying to each sensitivity matrix. The following functions have been obtained:

$$\psi_L = \begin{bmatrix} 0 \\ \frac{l}{r\left(J_b + ml^2\right)} x_s(t) \\ 0 \\ 0 \\ -\frac{R}{\kappa_m L} x_4(t) - \frac{\kappa_m}{J^2 L} x_5(t) \end{bmatrix};$$

$$\psi_{\kappa_м} = \begin{bmatrix} 0 \\ \frac{l}{r\left(J_b + l^2\right)} x_s(t) \\ 0 \\ 0 \\ \frac{RJ}{\kappa_m{}^2 L} x_4(t) + \frac{1}{L} x_5(t) \end{bmatrix}; \quad (31)$$

$$\psi_{\kappa_{s1}} = \begin{bmatrix} x_1(t) \\ 0 \end{bmatrix};\ \psi_{\kappa_{s2}} = \begin{bmatrix} 0 \\ x_3(t) \end{bmatrix}.$$

According to the criterion of complete signal diagnosability [11], the sensitivity functions (31) should be linearly independent in pairwise combinations. Paired combinations of functions ψ_L and ψ_{κ_m}, as well as functions $\psi_{\kappa_{s1}}$ and $\psi_{\kappa_{s2}}$ are linearly independent in transient modes. Thus, the object of automatic control are fully signally diagnosed in transient modes.

Conclusion

The physical models of mechanical and electromechanical processes have been developed as the result of investigations of the translational and angular motions of the TES as a dynamically similar prototype of the decomposed simplest motions of aircraft. These models allow to formalize the processes of TES motion as the mathematical models using the Lagrange approach. The model was developed in the form of a system of four nonlinear differential equations, that relate the linear x and angular υ motions with the control u and disturbing f influences.

Mathematical models of linear approximation have been obtained to study the processes of motion "in small" using the method of analytical linearization. Linear approximation models are represented by a linear differential equations system of second order with constant coefficients. TES as an object of automatic control, including the mechanical part, electric drives and linear and angular sensors, is represented by mathematical models in a five-dimensional state space and using transfer functions. The structural diagrams of linear mathematical models in the time and frequency domains have been developed.

The analysis of the functional properties of the TES as an object of automatic control allow to establish the instability of angular movements, its controllability, observability and the possibility of diagnosing changes in the functional properties of electric drives and sensors in transient modes.

The obtained models and the results of the analysis of the TES functional properties as an object of automatic control might be useful in the educational process during the carrying on of the course and diploma projects. As well the results might be of great use at the stages of conceptual design of automatic control systems for dynamically similar experimental models of modern and advanced autonomous maneuverable aircraft and also two-wheeled vehicles for various purposes.

References (GOST 7.1:2006)

1. Dorf, R. Modern control systems, 13th Edition [Text] / R. Dorf, R. Bishop. – Austin. : University of Texas at Austin Publ., 2017. – 881 p.

2. Advanced sliding mode control techniques for Inverted Pendulum: Modelling and simulation [Text] / S. Irfan, A. Mehmood, M. Razzaq, J. Iqbal // Engineering science and technology, an international journal. – 2018. – V. 21, no. 4. – P. 753-759. DOI: 10.1016/j.jestch.2018.06.010.

3. Modeling, control of a two-wheeled self-balancing robot [Text] / O. Jamil, M. Jamil, Y. Ayaz, K. Ahmad // 2014 International Conference on Robotics and Emerging Allied Technologies in Engineering (iCREATE). – Islamabad, Pakistan, 2014. – P. 191-199. DOI: 10.1109/iCREATE.2014.6828364.

4. Takahashi, Y. Experiments on step climbing and simulations on inverse pendulum control using robotic wheelchair with inverse pendulum control [Text] / Y. Takahashi, S. Ogawa, S. Machida // Transactions of the Institute of Measurement and Control. – 2008. – V. 30(1). – P. 47–61. DOI: 10.1177/0142331206075538.

5. Takei, T. Baggage Transportation and Navigation by a Wheeled Inverted Pendulum Mobile Robot [Text] / T. Takei, R. Imamura, S. Yuta // IEEE Transactions on Industrial Electronics. – 2009. – V. 56, no. 10. – P. 3985-3994. DOI: 10.1109/TIE.2009.2027252.

6. Galicki, M. Robust Task Space Trajectory Tracking Control of Robotic Manipulators [Text] / M. Galicki // International Journal of Applied Mechanics and Engineering. – 2016. – V. 21, No. 3. – P. 547-568. DOI: 10.1515/ijame-2016-0033.

7. Control System Design, Analysis & Implementation of Two Wheeled Self Balancing Robot (TWSBR) [Text] / M. A. Imtiaz, M. Naveed, N. Bibi, S. Aziz, S. Z. H. Naqvi // 2018 IEEE 9th Annual Information Technology, Electronics and Mobile Communication Conference (IEMCON). – Vancouver, BC, Canada, 2018. – P. 431-437. DOI: 10.1109/IEMCON. 2018.8614858.

8. Jimenez, F. Modeling and Control of a Two Wheeled Self-Balancing Robot: a didactic platform for control engineering education [virtual edition] / F. Jimenez, A. Jimenez, I. Ruge // 18th LACCEI International Multi-Conference for Engineering, Education, and Technology: "Engineering, Integration, and Alliances for a Sustainable Development" "Hemispheric Cooperation for Competitiveness and Prosperity on a Knowledge-Based Economy", July 27-31, 2020. – 2020. – Available at: http://laccei.org/LACCEI2020-VirtualEdition/full_papers/FP556.pdf. – 14.12.2020. DOI: 10.18687/LACCEI2020.1.1.556.

9. Morin, P. Control of Wheeled Mobile Robots [Text] / P. Morin, C. Samson // Springer Handbook of Robotics. – Springer, Berlin, Heidelberg. – P. 799-826. DOI: 10.1007/978-3-540-30301-5_35.

10. Кулик, А. С. Сигнально-параметрическое диагностирование систем управления [Текст] : монография / А. С. Кулик ; Гос. аэрокосм. ун-т им. Н. Е. Жуковского «ХАИ». — Харьков : Бизнес Информ, 2000. — 260 с.

11. Kulik, A. S. Rational Control of Operability of Autonomous Aircraft. Part I [Text] / A. S. Kulik // Journal of Automation and Information Sciences. – 2017. V. 49, no. 5, part I. – P. 1-15. DOI: 10.1615/JAutomatInfScien.v49.i5.10.

References (BSI)

1. Dorf, R., Bishop, R. *Modern control systems, 13th Edition.* Austin, University of Texas at Austin Publ., 2017. 881 p.

2. Irfan, S., Mehmood, A., Razzaq, M., Iqbal, J. Advanced sliding mode control techniques for Inverted Pendulum: Modelling and simulation. *Engineering science and technology, an international journal*, 2018, vol. 21, no. 4, pp. 753-759. DOI: 10.1016/j.jestch.2018.06.010.

3. Jamil, O., Jamil, M., Ayaz, Y., Ahmad, K. Modeling, control of a two-wheeled self-balancing robot. *2014 International Conference on Robotics and Emerging Allied Technologies in Engineering (iCREATE)*. Islamabad, Pakistan, 2014, pp. 191-199. DOI: 10.1109/iCREATE.2014.6828364.

4. Takahashi, Y., Ogawa, S., Machida, S. Experiments on step climbing and simulations on inverse pendulum control using robotic wheelchair with inverse pendulum control. *Transactions of the Institute of Measurement and Control*, 2008, vol. 30(1), pp. 47–61. DOI: 10.1177/0142331206075538.

5. Takei, T., Imamura, R., Yuta, S. Baggage Transportation and Navigation by a Wheeled Inverted Pendulum Mobile Robot. *IEEE Transactions on Industrial Electronics,* 2009, vol. 56, no. 10, pp. 3985-3994. DOI: 10.1109/TIE.2009.2027252.

6. Galicki, M. Robust Task Space Trajectory Tracking Control of Robotic Manipulators. *International Journal of Applied Mechanics and Engineering,* 2016, vol. 21, no. 3, pp. 547-568. DOI: 10.1515/ijame-2016-0033.

7. Imtiaz, M. A., Naveed, M., Bibi, N., Aziz, S., Naqvi, S. Z. H. Control System Design, Analysis & Implementation of Two Wheeled Self Balancing Robot (TWSBR). *2018 IEEE 9th Annual Information Technology, Electronics and Mobile Communication Conference (IEMCON)*. Vancouver, BC, Canada, 2018, pp. 431-437. DOI: 10.1109/IEMCON.2018.8614858.

8. Jimenez, F., Jimenez, A., Ruge, I. Modeling and Control of a Two Wheeled Self-Balancing Robot: a didactic platform for control engineering education. *18th LACCEI International Multi-Conference for Engineering, Education, and Technology: "Engineering, Integration, and Alliances for a Sustainable Development" "Hemispheric Cooperation for Competitiveness and Prosperity on a Knowledge-Based Economy",* July 27-31, 2020. DOI: 10.18687/LACCEI2020.1.1.556.

9. Morin, P., Samson, C. Motion Control of Wheeled Mobile Robots, in Siciliano, B., Khatib, O. (eds) *Springer Handbook of Robotics.* Springer, Berlin, Heidelberg, pp. 799-826. DOI: 10.1007/978-3-540-30301-5_35.

10. Kulik, A. S. *Signal'no-parametricheskoe diagnostirovanie sistem upravleniya: monografiya* [Signal-parametric diagnostics of control systems: monograph]. Kharkiv, Biznes Inform Publ., 2000. 260 p.

11. Kulik, A. S. Rational Control of Operability of Autonomous Aircraft. Part I. *Journal of Automation and Information Sciences,* 2017, vol. 49, no. 5, part I, pp. 1-15. DOI: 10.1615/JAutomatInfScien.v49.i5.10.

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МОДЕЛІ РУХІВ ДВОКОЛІСНОГО ЕКСПЕРИМЕНТАЛЬНОГО ЗРАЗКА

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Предметом вивчення в статті є фізичні процеси поступального і кутового рухів двоколісного експериментального зразка. **Мета** полягає в розробці фізичних, математичних і графічних моделей поступального і кутового рухів двоколісного експериментального зразка як об'єкта автоматичного управління. **Завдання:** сформувати фізичні моделі двоколісного експериментального зразка. Розробити за допомогою лагранжевого підходу нелінійний математичний опис процесів поступального і кутового рухів зразка. Отримати лінеаризований математичний опис зразка як об'єкта автоматичного управління в просторі станів і частотній області. Сформувати графічні моделі в формі структурних схем в часовій і частотній областях. Проаналізувати функціональні властивості об'єкта автоматичного управління: стійкість, керованість, спостережність, структурну та сигнальну діагностованість щодо порушень функціональних властивостей електроприводів і датчиків кутового положення корпусу і коліс. **Методи**, що використовувалися: метод Лагранжа, ряд Тейлора, метод простору станів, перетворення Лапласа, критерії Ляпунова, Калмана, діагностованості. Отримані наступні **результати**: сформовані фізичні моделі двоколісного експериментального зразка в формі кінематичної схеми механічної частини і електричної схеми електроприводу; отримані математичні описи поступального і кутового рухів в нелінійній і лінеаризованій формах як об'єкта автоматичного управління; розроблено структурні схеми та проаналізовані функціональні властивості двоколісного експериментального зразка як об'єкта автоматичного управління, що необхідні для вирішення завдань синтезу алгоритмів керування **Висновки.** Наукова новизна полягає в отриманні нових моделей, що описують поступальний і кутовий рух двоколісного експериментального зразка як об'єкта автоматичного управління. Отримані моделі відрізняються від відомих урахуванням динамічних властивостей датчиків і електроприводів, а також урахуванням взаємозв'язків рухів.

Ключові слова: двоколісний експериментальний зразок; моделі рухів; об'єкт автоматичного управління; математичні моделі; простір станів; керованість; спостережність; діагностованість.

МОДЕЛИ ДВИЖЕНИЙ ДВУХКОЛЕСНОГО ЭКСПЕРИМЕНТАЛЬНОГО ОБРАЗЦА

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Предметом изучения в статье являются физические процессы поступательного и углового движений двухколесного экспериментального образца. **Цель** заключается в разработке физических, математических и графических моделей поступательного и углового движений двухколесного экспериментального образца как объекта автоматического управления. **Задачи:** сформировать физические модели двухколесного экспериментального образца. Разработать с помощью лагранжевого подхода нелинейное математическое описание процессов поступательного и углового движений образца. Получить линеаризованное математическое описание образца как объекта автоматического управления в пространстве состояний и частотной области. Сформировать графические модели в форме структурных схем во временной и частотной областях. Проанализировать функциональные свойства объекта автоматического управления: устойчивость, управляемость, наблюдаемость, структурную и сигнальную диагностируемость относительно нарушений функциональных свойств электроприводов и датчиков углового положения корпуса и колес. Используемые **методы**: метод Лагранжа, ряд Тейлора, метод пространства состояний, преобразования Лапласа, критерии Ляпунова, Калмана, диагностируемости. Получены следующие **результаты**: сформированы физические модели двухколесного экспериментального образца в форме кинематической схемы механической части и электрической схемы электропривода; получены математические описания поступательного и углового движений в нелинейной и линеаризованной формах как объекта автоматического управления; разработаны структурные схемы и проанализированы функциональные свойства двухколесного экспериментального образца как объекта автоматического управления, необходимые для решения задач синтеза алгоритмов управления. **Выводы.** Научная новизна заключается в получении новых моделей, описывающих поступательное и угловое движения двухколёсного экспериментального образца как объекта автоматического управления. Полученные модели отличаются от известных учётом динамических свойств датчиков и электроприводов, а также учётом взаимосвязей движений.

Ключевые слова: двухколесный экспериментальный образец; модели движений; объект автоматического управления; математические модели; пространство состояний; управляемость; наблюдаемость; диагностируемость.

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