UDK 621.735

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Selection of Optimal Center-of-Gravity of Transport Category Airplane from Minimum Required Thrust Condition

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Expression for optimum center-of-gravity of transport category airplane at cruise flight mode is created from the condition of minimum required thrust (aerodynamic drag). For example, graph for optimum center-of-gravity of transport category airplane vs. relative area of horizontal tail and relative distance between leading edges of mean aerodynamic chords of wing and horizontal tail is given. It is shown, that the highest gain from the center-of-gravity shift takes place near the best range cruise speed.

Key words: center-of-gravity, centering, target center-of-gravity, fuel trim transfer, required thrust, aerodynamic drag, fuel consumption, drag-due-to-lift factor.

Introduction

The problem to minimize fuel consumption created by a transport category airplane is one of the most actual. One of the way to decrease fuel consumption is increase of airplane lift-to-drag ratio at cruising mode. For this purpose, airplane center-of-gravity (CG) control by means of fuel trim transfer (FTT) is already applied in some foreign airliners (A-310, A-330, A-340, A-380, B-747 etc.), but in domestic practice this way is not still used. Airplane CG calculation taking into account its migration, presence of ribs with baffle check valves, fuel burn schedule and FTT is considered in publications [1-5]. One more problem, which must be solved during development of FTT system is selection of optimal airplane CG at cruising mode.

The aim of this publication is generation of recommendations to select optimal CG for transport category airplanes from condition of minimum required thrust, that in the first approximation should correspond to minimal fuel consumption.

1. Optimization of Airplane Center-of-Gravity

Concept of the performed research becomes clear from the following example. Let's consider airplane steady level flight before (Fig. 1, a) and after (Fig. 1, b) CG shift back by means of FTT. To investigate FTT influence and to determine flight performance, airplane should be considered as a solid body (not as a material point).

Writing down equilibrium equations in wind axes, we get:

$$\sum X = P - X_{aWHT} - X_{aHT} = 0; \tag{1}$$

$$\sum Y = Y_{aWHT} - mg - Y_{aHT} = 0; \tag{2}$$

$$\sum M_{za} = mg\,x_{CG} - Y_{aWHT}\,x_{pWHT} + Y_{aHT}\left(\Delta L + x_{pHT}\right) = 0, \tag{3}$$

where P — is total engine thrust; X_{aWHT}, X_{aHT} — is aerodynamic drag of airplane without horizontal tail (HT) and separate HT, correspondingly; Y_{aWHT}, Y_{aHT} — is required lift of airplane without HT and separate HT, correspondingly; x_{CG} — is airplane CG relatively mean aerodynamic chord (MAC) leading edge; x_{pWHT} — is center-of-pressure (CP) of airplane without HT relative to wing MAC leading edge; x_{pHT} — is

CP of separate HT relative to HT MAC leading edge; ΔL — is distance between MAC leading edges of wing and HT.

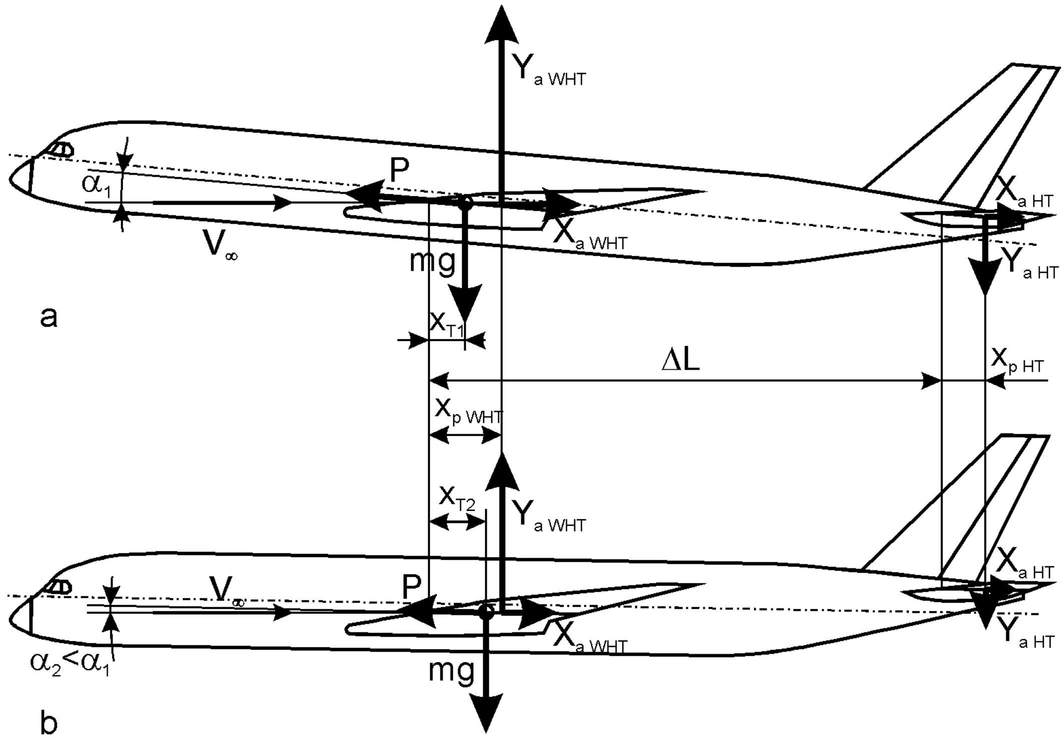

Fig. 1. Forces acting on airplane schematic:
a — in flight with FTT activated; b — in flight without FTT activated

From two last equations, lift of airplane without HT and lift of separate HT can be expressed:

$$Y_{aWHT} = mg\frac{\Delta L + x_{pHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}}; \quad Y_{aHT} = mg\frac{x_{pWHT} - x_{CG}}{\Delta L + x_{pHT} - x_{pWHT}}. \tag{4}$$

Engine thrust required for the steady level flight can be determined from the equation (1)

$$P_{req} = X_{aWHT} + X_{aHT}. \tag{5}$$

Let's use known expressions for the drag

$$X_{aWHT} = 0.7 p_H M^2 S C_{xaWHT}, \qquad X_{aHT} = 0.7 p_H M^2 S_{HT} C_{xaHT}, \tag{6}$$

and its factors

$$C_{xaWHT} = C_{xaWHT_0} + A_{WHT} C_{yaWHT}^2, \qquad C_{xaHT} = C_{xaHT_0} + A_{HT} C_{yaHT}^2, \tag{7}$$

where p_H — is atmospheric pressure at the flight altitude; M — is Mach flight number; S and S_{HT} — is area of wing and HT, correspondingly; C_{xaWHT_0} and C_{xaHT_0} — are drag factors of airplane without HT and separate HT at zero lift; A_{WHT} and A_{HT} — are drag-due-to-lift factors of airplane without HT and separate HT; C_{yaWHT} and C_{yaHT} — are lift factors of airplane without HT and separate HT from equation (4):

$$C_{ya\,WHT} = \frac{Y_{a\,WHT}}{0.7\,p_H M^2 S} = \frac{mg}{0.7\,p_H M^2 S}\left(\frac{\Delta L + x_{p\,HT} - x_{CG}}{\Delta L + x_{p\,HT} - x_{p\,WHT}} \right),$$

$$C_{ya\,HT} = \frac{Y_{a\,HT}}{0.7\,p_H M^2 S_{HT}} = \frac{mg}{0.7\,p_H M^2 S_{HT}}\left(\frac{x_{p\,WHT} - x_{CG}}{\Delta L + x_{p\,HT} - x_{p\,WHT}} \right). \tag{8}$$

Substituting expressions (5-7) to equation (1), formula to calculate required thrust can be obtained:

$$P_{req} = 0.7\,p_H M^2 S \left\{ \left[C_{xa\,WHT_0} + A_{WHT} C^2_{ya\,WHT} \right] + \frac{S_{HT}}{S} \left[C_{xa\,HT_0} + A_{HT} C^2_{ya\,HT} \right] \right\},$$

and taking into account (8)

$$P_{req} = 0.7\,p_H M^2 S \left(C_{xa\,WHT_0} + C_{xa\,HT_0} \frac{S_{HT}}{S} \right) + \frac{(mg)^2}{0.7\,p_H M^2 S} \times$$

$$\times \left[A_{WHT} \left(\frac{\Delta L + x_{p\,HT} - x_{CG}}{\Delta L + x_{p\,HT} - x_{p\,WHT}} \right)^2 + A_{HT} \frac{S}{S_{HT}} \left(\frac{x_{p\,WHT} - x_{CG}}{\Delta L + x_{p\,HT} - x_{p\,WHT}} \right)^2 \right]. \tag{9}$$

Airplane drag at zero lift

$$C_{xa_0} = C_{xa\,WHT_0} + C_{xa\,HT_0} \frac{S_{HT}}{S} \tag{10}$$

does not depend on airplane CG. Thus, expression (9) for engine required thrust differs from the classic one [6]

$$P_{req} = 0.7\,p_H M^2 S\, C_{xa_0} + \frac{(mg)^2}{0.7\,p_H M^2 S} A, \tag{11}$$

only by airplane drag-due-to-lift factor, depending on its CG

$$A(x_{CG}) = A_{WHT} \left(\frac{\Delta L + x_{p\,HT} - x_{CG}}{\Delta L + x_{p\,HT} - x_{p\,WHT}} \right)^2 + A_{HT} \frac{S}{S_{HT}} \left(\frac{x_{p\,WHT} - x_{CG}}{\Delta L + x_{p\,HT} - x_{p\,WHT}} \right)^2 . \tag{12}$$

For transport category airplane, CG providing minimal required thrust can be assumed as optimum CG. To determine it, we differentiate the expression for required thrust (9) by CG coordinate and equate the expression to zero.

$$\frac{dP_{req}}{dx_{CG}} = \frac{(mg)^2}{0.7\,p_H M^2 S} \left[\frac{-2A_{WHT}(\Delta L + x_{p\,HT} - x_{CG})}{(\Delta L + x_{p\,HT} - x_{p\,WHT})^2} - \frac{S}{S_{HT}} \frac{2A_{HT}(x_{p\,WHT} - x_{CG})}{(\Delta L + x_{p\,HT} - x_{p\,WHT})^2} \right] = 0 .$$

Whence, optimum CG is

$$x_{CG} = \frac{A_{WHT} \bar{S}_{HT} (\Delta L + x_{p\,HT}) + A_{HT} x_{p\,WHT}}{A_{WHT} \bar{S}_{HT} + A_{HT}}, \tag{13}$$

where $\bar{S}_{HT} = S_{HT}/S$ — is HT relative area.

To know the type of extremum, let’s consider the required thrust second derivative by CG coordinate.

$$\frac{d^2 P_{req}}{dx_{CG}^2} = \frac{2(mg)^2 \left(A_{WHT}\bar{S}_{HT} + A_{HT}\right)}{0.7\, p_H M^2 S \bar{S}_{HT} \left(\Delta L + x_{p\,HT} - x_{p\,WHT}\right)^2} > 0 \,. \tag{14}$$

It is clear, that it is positive at any parameter real values. It means, that the minimum of required thrust takes place.

2. Analysis of the Optimization Results

Now, let's analyze the obtained expression for the optimum CG. Passing to relative values

$$\Delta L = k_L b_a ;\;\; x_{p\,WHT} = \bar{x}_{p\,WHT} b_a ;\;\; x_{p\,HT} = \bar{x}_{p\,HT} b_a^{HT} \,,$$

we get $$\bar{x}_{CG} = \frac{x_{CG}}{b_a} = \frac{A_{WHT}\bar{S}_{HT}\left(k_L + \bar{x}_{p\,HT}\, b_a^{HT} / b_a\right) + A_{HT}\bar{x}_{p\,WHT}}{A_{WHT}\bar{S}_{HT} + A_{HT}} \,. \tag{15}$$

If we assume for simplification, that wing and HT shapes are similar

$$A_{HT} \approx A_{WHT} ; \qquad b_a^{HT} / b_a \approx \sqrt{\bar{S}_{HT}} \,,$$

then, we get the simplified formula

$$\bar{x}_{CG} = \frac{\bar{S}_{HT}\left(k_L + \bar{x}_{p\,HT}\sqrt{\bar{S}_{HT}}\right) + \bar{x}_{p\,WHT}}{\bar{S}_{HT} + 1} \,. \tag{16}$$

For clearness, it is even possible to assume $\bar{x}_{p\,WHT} = \bar{x}_{p\,HT} = 0.25$. Then

$$\bar{x}_{CG} \approx \frac{\bar{S}_{HT}\left(k_L + 0.25\sqrt{\bar{S}_{HT}}\right) + 0.25}{\bar{S}_{HT} + 1} \,. \tag{17}$$

The last expression can be presented graphically (Fig. 2).

One can see from the graphs, that for actual values $k_L = 2...3.5$ and $\bar{S}_{HT} = 0.2...0.25$, optimal CG lies within the range $\bar{x}_{CG} = 0.56...0.93$. So, optimal CG, that corresponds to the minimum drag (or required thrust) at cruising mode, is placed behind the aerodynamic center of an airplane, which can be implemented only in the statically unstable airplane. Nowadays, there are military airplanes having such CG, which use stability augmentation systems (SAS). It is possible to suggest, that in future with SAS designing methodology improvement and with SAS reliability increase, they will be also applied to transport category airplanes; that will allow both: to decrease fuel expenses, and to decrease harmful emission to the atmosphere.

In existent foreign airliners, the target CG is also limited by stability limits. Practically, aft certified limit CG after deduction of a margin of $\Delta x = 2\,\%$ is assumed as a target CG (Fig. 3) [7-12]:

$$\bar{x}_{CG\,target} = \bar{x}_{Aft\,Limit} - \Delta x \,.$$

It is possible to suggest, that the certified aft CG limit is the aerodynamic center minus a definite margin.

Thus, to decrease the required thrust P_{req} (Fig. 4) and, consequently, fuel consumption at cruising flight mode, it is reasonable to shift CG aft, that can be reached by FTT from wing tanks back to the trim tank, located in stabilizer. It is clear from Fig. 4, that the highest gain from the CG shift is observed near the best range cruise speed, that should correspond to the cruise flight mode. The gain decreases with the flight speed increase.

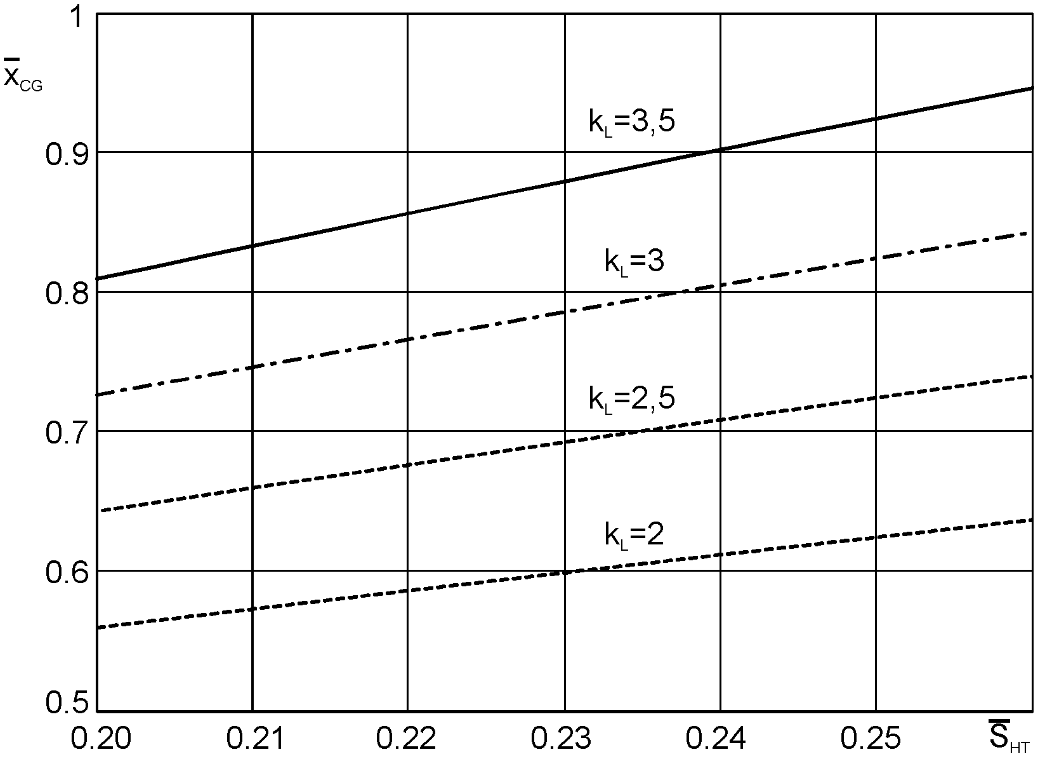

Fig. 2. Optimum CG vs. $\overline{S}_{HT}$ and k_L

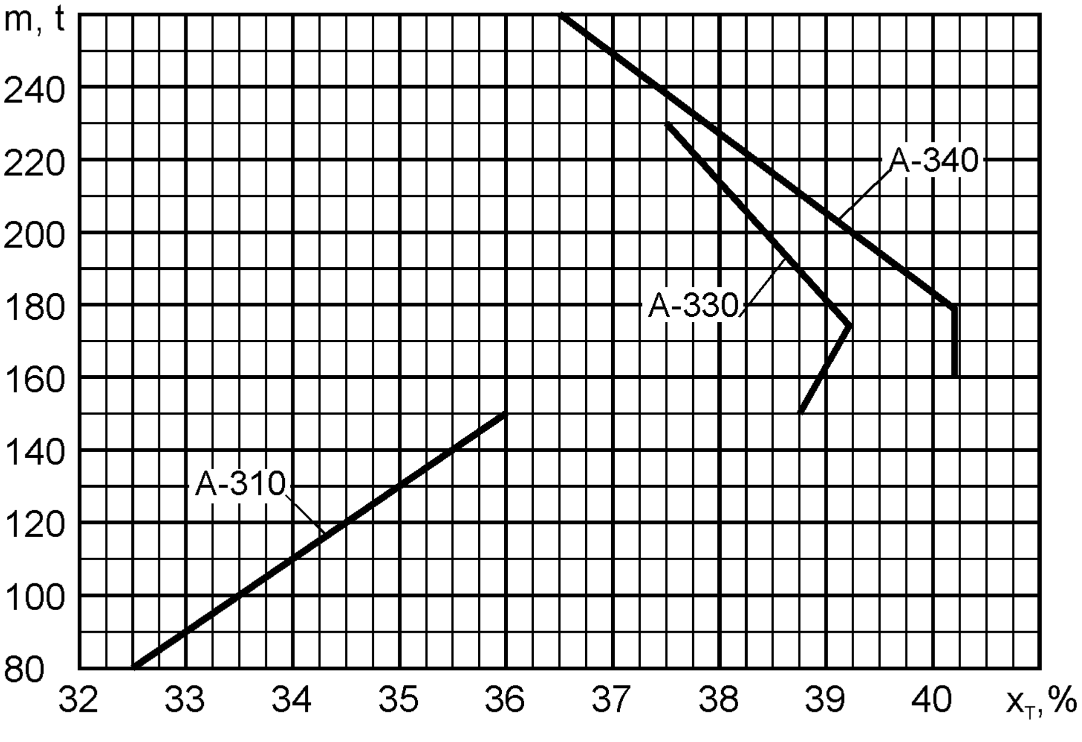

Fig. 3. CG target position

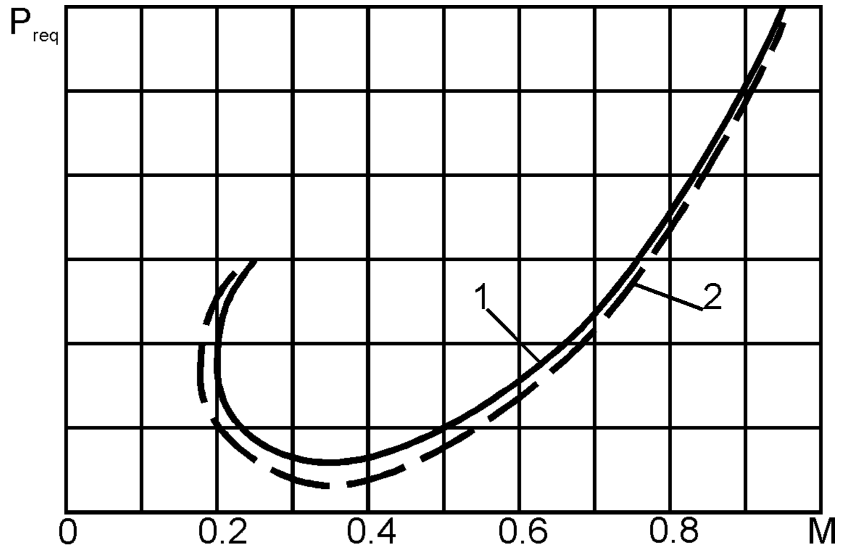

Fig. 4. FTT influence on required thrust graph: 1 — without FTT; 2 — with FTT

As FTT system directly influences airplane trim, stability and controllability, it is considered safety-critical one, and the requirements to reliability it should meet are the same as ones to airplane control system. Practically, it is necessary to provide as minimum dual redundancy of fuel transfer means. To provide safe deceleration and landing, it is necessary to shift CG forward even in case of all engines failed. It can be reached by emergency forward trim transfer (using electro-centrifugal or hydraulic driven fuel pumps powered by emergency ram air turbine) or by fuel jettisoning from tail trim tank.

Thus, solving of the methodological problems of the fuel trim transfer systems in transport category airplanes promises to give considerable economical effect.

Conclusions

1. Dependence of optimum CG for transport category airplane from minimum required thrust condition is created.
2. Comparison of the target CG of existing passenger airplanes with optimum ones is performed and the difference between them is justified.
3. It is shown, that the highest gain from the center-of-gravity shift takes place near the best range cruise speed.

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Came to edition 14.03.2017

Выбор оптимального положения центра масс самолёта транспортной категории из условия минимума потребной тяги

Получено выражение для оптимального положения центра масс самолёта транспортной категории на крейсерском режиме полёта из условия минимума потребной тяги (аэродинамического сопротивления). Для примера получен график оптимальных положений центра масс самолёта транспортной категории в зависимости от относительной площади горизонтального оперения и относительного расстояния между носками средних аэродинамических хорд крыла и горизонтального оперения. Показано, что наибольший выигрыш от смещения центра масс наблюдается вблизи наивыгоднейшей скорости полёта.

Ключевые слова: центр масс, центровка, целевое положение центра масс, балансировочная перекачка топлива, потребная тяга, аэродинамическое сопротивление, расход топлива, коэффициент отвала поляры.

Вибір оптимального положення центру мас літака транспортної категорії із умови мінімуму потрібної тяги

Отримано вираз для оптимального положення центру мас літака транспортної категорії на крейсерському режимі польоту з умови мінімуму потрібної тяги (аеродинамічного опору). Як зразок надано графік оптимального положення центру мас літака транспортної категорії залежно від відносної площі горизонтального оперення та відносної відстані між носками середніх аеродинамічних хорд крила та горизонтального оперення. Показано, що найбільший виграш від зміщення центру мас спостерігається поблизу найвигіднішої швидкості польоту.

Ключові слова: центр мас, центрування, цільове положення центру мас, балансувальне перекачування палива, потрібна тяга, аеродинамічний опір, витрата палива, коефіцієнт відвалу поляри.

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