

МИНИСТЕРСТВО ОБРАЗОВАНИЯ И НАУКИ УКРАИНЫ  
Национальный аэрокосмический университет им. Н.Е. Жуковского  
«Харьковский авиационный институт»

A. Ja. Zimovin, V.N. Postnikov, L.I. Volchanskaya  
А.Я. Зимовин, В.Н. Постников, Л.И. Волчанская

BASIC ELECTRICAL ENGINEERING

Part 1

Manual

ОСНОВЫ ЭЛЕКТРОТЕХНИКИ

Часть 1

Учебное пособие

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Учебное пособие написано в соответствии с программой подготовки бакалавров, специалистов и магистров. В первой части рассмотрены вопросы теории линейных и нелинейных электрических и магнитных цепей.

Каждый раздел включает в себя теоретическую часть, задачи, лабораторные работы, контрольные вопросы и толковый словарь ключевых (базовых) терминов.

Для студентов механических и электротехнических специальностей высших учебных заведений с углубленным изучением технического английского языка.

Ил. 83. Табл. 18. Библиогр.: 6 назв.

This manual has been prepared to suit bachelor, specialist and master curriculum. In the first part, calculation of linear and nonlinear electric and magnetic circuits is discussed.

Each section includes theory, glossary, problems, investigation practice, and test questions.

The book can be useful for everyone in the field of engineering for learning the advanced technical English.

Illustrations 83. Tables 18. Bibliography: 6 sources

Рецензенты: канд. техн. наук, доц. А.Н. Зарубин,  
канд. техн. наук Е.В. Павлюк

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## INTRODUCTION

Electrical engineering is a science dealing with the techniques of utilizing electrical and magnetic phenomena for practical use.

One of the main branches of Electrical Engineering is Electrical Power Engineering. It concerns production of electrical energy from other forms of energy, power transmission over long distances, distribution among consumers and conversion of power into other forms of energy.

Why does electrical energy have such a wide application? This is because of very valuable advantages.

1. Electrical power can be easily produced from other forms of energy (mechanical, chemical, thermal, radiant etc.) and can be effectively transformed into the other forms of energy.

2. The production of relatively big quantities of electrical energy is usually concentrated in a relatively small space forms.

3. Electrical energy can be transmitted at very high speed with low losses for hundreds or even thousands of kilometers.

4. Electrical energy can be easily distributed among consumers.

5. Electrical devices and machines are safe and simple in service.

However there is one major disadvantage – electrical energy cannot be stored in great quantities. Produce and use immediately!

Wide application of electronic devices and instrumentation in all aspects of engineering is one of the manifestations of modern electronic revolution. Every aspect of engineering practice and even of everyday life is affected in some way by electrical and electronic devices and instruments.

Computers and over-all computerization are perhaps the most obvious manifestation of this presence. However, many other areas of electrical engineering are also important to the practicing engineer, from mechanical engineering to chemical, nuclear and civil engineering.

The objective of this manual is to introduce students in English basic principles of electrical engineering they could meet in their future work.

We are going to use the International System of Units (also called SI). SI units are based on six fundamental quantities which are presented in the Table 1.

Table 1

**Fundamental SI units**

<b>Quantity</b>	<b>Unit</b>	<b>Symbol</b>
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	K
Luminous intensity	Candela	cd

All other units may be derived in terms of the fundamental units (Table 2).

Table 2

**Electrical quantities**

Quantity	Unit	Symbol
Charge	Coulomb	C
Voltage	Volt	V
Work	Joule	J
Power	Watt	W
Resistance	Ohm	$\Omega$
Conductance	Siemens	S
Capacitance	Farad	F
Inductance	Henry	H
Frequency	Hertz	Hz
Magnetic inductance (flux density)	Tesla	B
Magnetic flux	Weber	$\Phi$
Magnetic field intensity	Amperes per meter, A/m	H
Electric field intensity	Volts per meter, V/m	E
Flux linkage	Weber	$\Psi$
Magnetic force	Ampere – turns	At

In practice, one often needs to describe quantities that occur in large multiples or small fractions of units. Standard prefixes are used to denote powers of SI and derived units. These prefixes are listed in Table 3 below.

Table 3

**Standard prefixes**

Prefix	Symbol	Power
Atto	a	$10^{-18}$
femio	f	$10^{-15}$
Pico	p	$10^{-12}$
nano	n	$10^{-9}$
micro	$\mu$	$10^{-6}$
milli	m	$10^{-3}$
centi	c	$10^{-2}$
deci	d	$10^{-1}$
deca	da	10
Kilo	k	$10^3$
mega	M	$10^6$
giga	G	$10^9$
tera	T	$10^{12}$
peta	P	$10^{15}$
exa	E	$10^{18}$

The prefixes most often used in Electrical Engineering are micro, milli, kilo, and mega.

Usually, in general engineering units are expressed in powers of 10 that are multiples of 3. For example,  $10^{-4}$ s would be referred to as  $100 \cdot 10^{-6}$  s, i.e. 100  $\mu$ s, or less frequently as  $0,1 \cdot 10^{-3}$ s, i.e. 0.1ms.

# Chapter 1

## SIMPLE DIRECT CURRENT ELECTRIC CIRCUITS

### 1.1 Circuit Elements and Parameters

A typical simple electric circuit (Fig. 1.1) consists of three elements: a **source** of energy, or power-source, a **receiver** of energy, or load, and a pair of **conductors** which connect the receiver to the power-source terminals.

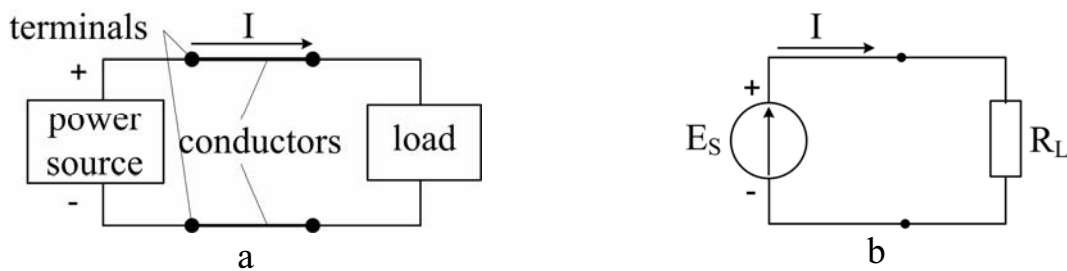


Fig. 1.1 – Simple electrical circuit

The source converts mechanical, chemical, thermal, and other kinds of energy into electromagnetic energy, or simply, into electric energy (generates an electromotive force  $E$ ).

The receiver, on the contrary, absorbs electrical energy and transforms it into mechanical, chemical, thermal, radiant, and other forms of work.

The wire conductors are used to deliver electrical energy from the source to the load.

In Fig. 1.1 two different representations are shown to illustrate the conceptual and symbolic significance of the source-load idea for a simplified electrical circuit. Symbols of ideal circuit elements in Fig. 1.1,b depict the physical reality of Fig. 1.1,a.

#### Electric current

Electric current is defined as the time rate of change of charge passing through predetermined area

$$i = \frac{dq}{dt}. \quad (1.1)$$

In order for the current to flow there must exist a closed circuit (see Fig. 1.1). The arrow in the diagram depicts the direction of the current.

A current that reverses its direction and magnitude periodically is called **alternating current (ac, AC)**.

When the current is able to flow in one direction continuously it is called constant, or **direct current (dc, DC)**. For a direct current, instead of (1.1) we may write

$$I = \frac{q}{t}. \quad (1.2)$$

The character  $I$  is a conventional notation for direct current magnitude as well as for effective value of alternating current.

It is seen from (1.2) that a current strength is equal to the number of electrical charges passed through the cross sectional area of a conducting material per second.

The units of current are called **amperes (A)**:  $IA=IC/Is$ , where C, **the coulomb**, is the unit of electrical charge. The accumulated charge of 6.28 billion electrons is called 1 coulomb.

### **Electromotive force and voltage**

In the circuit a movement of electrons from one point to another (i.e. the electric current) will be possible if a **difference in electric pressure** between the two points is available.

The force that causes electric charge motion can be called one of the following: **electrical pressure, potential difference, voltage, or electromotive force**.

The electromotive force (EMF) is what produces, or generates the current in a closed electrical circuit. The EMF is associated with the capital character  $E$  next to symbol of the source of energy in the circuit diagram as depicted in Fig. 1.1.

The practical unit for the force above is the **volt (V)**.

In general, a **voltage** (potential difference) between the two points is defined as the **work** needed for carrying a unit **charge** from one point of the circuit to another. In other words, if a voltage is applied **across** a part of the circuit, a current will flow in the direction from the point of higher potential towards the point of lower potential. The character for a voltage is the capital  $V$ .

### **Resistance and Ohm's law**

The property of a substance, a circuit element or a device to impede the current is called the electric resistance or simply **resistance (R)**. As a consequence, current flowing through the element will cause energy to be dissipated in the form of heat according to the principle of energy conservation. An ideal **resistive** element (a resistor) is the element that exhibits linear resistance properties. A formal expression of that is known as **Ohm's law**: the current flowing through a resistor is directly proportional to the applied voltage and inversely proportional to its resistance:

$$I = \frac{V}{R}. \quad (1.3)$$

In (1.3)  $R$  is the value of resistance in **ohms** ( $\Omega$ ), where  $1 \Omega = 1V/A$ .

Conventional circuit symbols for both the resistor and the resistance property of any element (followed by the element heating) are presented in Fig. 1.2.

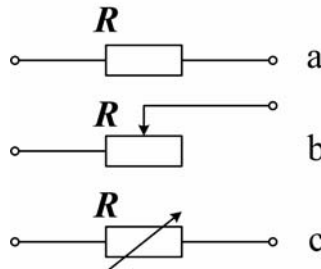


Fig. 1.2 – Resistor symbols: fixed (a), variable (b, c)

### Energy, work and power of electric current

**Energy** is the ability to do **work**.

If you apply voltage  $V$  across a portion of an electrical circuit with the aim to cause current  $I$  in it, the work performed by the electric current in that portion of the circuit during the time interval  $t$  according to (1.2) can be determined from

$$W = qV = VIt . \quad (1.4)$$

If when doing work electric energy is converted into heat only (as in the case of current flowing through a resistor or other circuit element obtaining internal resistance) we can get by substitution of (1.2) into (1.3):

$$W = I^2 R t . \quad (1.5)$$

This equation expresses **Lenz-Joule Law**.

The **joule (J)** is the SI unit of work and energy.

The electric **power P** is the rate of doing work. A source of energy is the element provided with the ability to produce, or generate, electric power. It is the active circuit element. Vice versa, a passive element is designed to absorb the power.

Applying the definition of electric power to the equation in (1.4) we'll have:

$$P = VI . \quad (1.6)$$

Hence, electrical power generated by a source, or that dissipated or stored by a load is equal to the product of the voltage across the element and the current flowing through it. For known resistance  $R$  the absorbed power can be determined through voltage drop or current flow alone:

$$P = I^2 R . \quad (1.7)$$

The practical unit of electric power is **the watt (W)**.



Ammeter, voltmeter, and wattmeter are used to measure the amount of current, voltage, and electric power, respectively.

**Resistors in series**

A practical electrical circuit may contain several loads of different resistances. Often they form rather complicated configuration. Nevertheless, even the most involved circuits can be reduced to combinations of circuit elements **in parallel** and **in series**.

Two or more circuit elements are said to be in series if the identical current flows through each of the elements (Fig. 1.3).

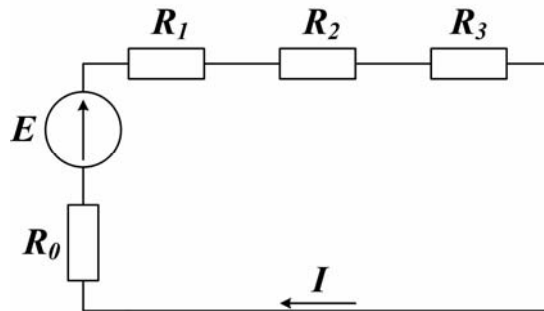


Fig. 1.3 – Resistors in series

It is easy to see that with respect to the source, the three external resistances  $R_1$ ,  $R_2$ , and  $R_3$  together with the internal source resistance  $R_0$  appear as a single equivalent resistance of value  $R_t$ , where

$$R_t = R_1 + R_2 + R_3 + R_0. \tag{1.8}$$

**Resistors in parallel. Kirchhoff’s first law**

Two or more circuit elements, e.g. resistors, are said to be in parallel if the identical voltage drops across each of the elements. Fig. 1.4 illustrates the notion of four parallel resistors located between two junction points, **a** and **b**. Usually these common points are called **nodes**, or nodal point junctions.

A current path between two adjacent nodes of the circuit is known as a circuit **branch**.

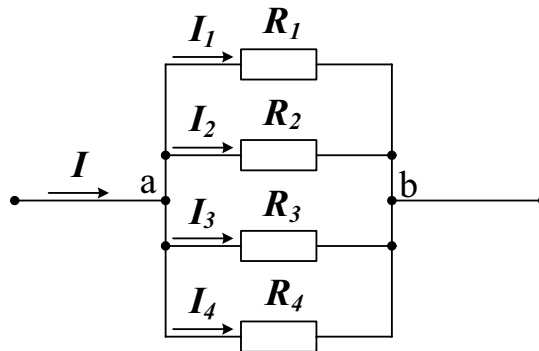


Fig. 1.4 – Resistors in parallel

To the part of a circuit containing only parallel branches **Kirchhoff's first law**, or **Kirchhoff's current law (KCL)** can be applied. It states that the sum of all currents flowing into any node in the circuit is equal to the sum of the currents flowing out from this node. For the circuit of Fig. 1.4 this statement results (look at **a** node) in

$$I = I_1 + I_2 + I_3 + I_4. \quad (1.9)$$

There is another form of KCL: the algebraic sum of all currents in the node equals zero:

$$\sum_{k=1}^n I_k = 0. \quad (1.10)$$

If voltage between two nodes is constant the currents through parallel branches will remain independent from one another, and disconnection of one or a few resistors from the circuit doesn't affect the operation of the rest of the resistors. This explains why lighting lamps, electric motors, and other loads are mainly placed in parallel with the source of voltage.

For the circuit with a group of resistors connected in parallel (see Fig. 1.4) the equivalent resistance  $R_t$  can be determined as

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \sum_{k=1}^4 \frac{1}{R_k}. \quad (1.11)$$

This expression is valid for any number of parallel-connected resistors. In a particular case, when a circuit contains only two connected in parallel resistances  $R_1$  and  $R_2$  the relation (1.11) can be written in the following form:

$$R_t = \frac{R_1 R_2}{R_1 + R_2}. \quad (1.12)$$

The electrical circuit of Fig. 1.5 where two resistors,  $R_2$  and  $R_3$ , connected in parallel are placed in series with the third resistor  $R_1$  is called a series-parallel circuit.

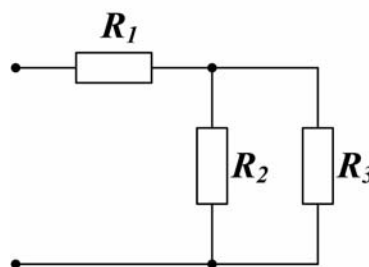


Fig. 1.5 – A series parallel circuit

The equivalent resistance of this circuit can be determined in turn by reducing the group of parallel resistances to the single resistance in form (1.12) and adding the latter to the resistance of the series part of circuit, that is:

$$R = R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3} . \quad (1.13)$$

### Kirchhoff's Second Law

**Kirchhoff's second law**, or **Kirchhoff's voltage law (KVL)** is a statement of the principle of conservation of energy in an electric circuit. KVL says that, in a closed circuit, the algebraic sum of electromotive forces is equal to the algebraic sum of the voltage drops across the elements of the circuit (loop):

$$\sum_{k=1}^n E_k = \sum_{k=1}^m I_k R_k . \quad (1.14)$$

You must use **the signs rule** when you travel around the circuit for taking into account the polarity of the EMF and the voltage drops in (1.14). This convention may be as follows: move counter clockwise around the circuit (as it's shown in Fig. 1.6) starting with the positive terminal of the largest EMF; voltages acting in the same sense are positive (+); voltages acting in the opposite sense are negative (-).

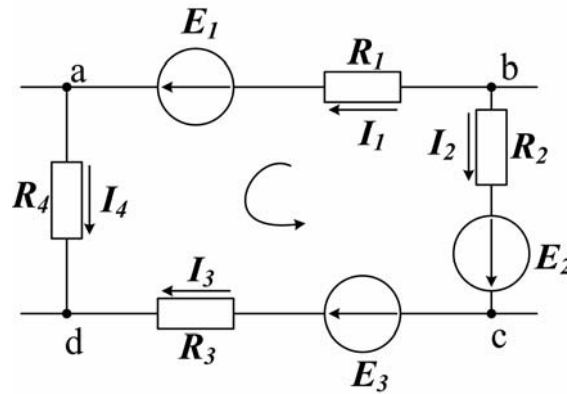


Fig. 1.6 – A closed loop

The closed loop shown in Fig. 1.6 and framed by points **a**, **b**, **c** and **d** is commonly a part of the complex circuit. Since each branch here is connected to the individual pair of nodes, the branch currents  $I_1, I_2, I_3, I_4$  differ in magnitude and direction. Therefore, applying Kirchhoff's second law, we can write for this loop:

$$E_1 - E_2 - E_3 = I_1 R_1 - I_2 R_2 - I_3 R_3 + I_4 R_4, \quad (1.15)$$

where  $I_i R_i, i = 1 \dots 4$  – voltage drops across each of the loop resistors.

## 1.2 Terms and concepts

**Charge** is a fundamental unit of matter responsible for electrical phenomena (the quantity of electricity).

**Circuit** is interconnection of electrical elements in a closed path.

**SI** is the International System of Units.

**Voltage** is work or energy required for moving a positive charge of 1 coulomb through a part of the circuit.

**Branch** is a current path with one or more elements connecting two nodes.

**Node** (*nodal point, junction, junction point*) – common point of three or more branches of the circuit.

**Loop** is closed path consisting of two or more branches.

**Resistance** is physical property of an element to impede current flow.

**Resistor** is a device or an element whose primary purpose will be to impose the resistance into a circuit.

**Parallel connection** is arrangement of resistors with each resistor having the same voltage applied across it.

**Series connection** is a chain of resistors when the same current passes through each of the resistors.

**Conductance** is the inverse of resistance.

## 1.3 Review Questions

1. What are the units for EMF, voltage, and current?
2. Define Ohm's Law for an entire circuit and for a part of the circuit.
4. What is the difference between the EMF of a source and the voltage at the source terminals?
5. What can you say about the current through the voltage source whose terminals are short circuited?
6. State Kirchhoff's current law and Kirchhoff's voltage law.
7. Write the relationships for equivalent resistance of series circuit, parallel circuit, and series-parallel circuit.
8. Define electrical energy, work, and power and denote their units.
9. Formulate Joule-Lenz's Law.

## 1.4 Problems

### Examples

**Problem 1.** A charge of 5 kC has passed through an element. Energy required was 20 MJ. Find the voltage across the element.

Solution

$$V = \frac{W}{q} = \frac{20 \cdot 10^6}{5 \cdot 10^3} = 4 \text{ kV} .$$

**Problem 2.** A constant current of 2A is flowing through an element. Energy required for carrying charges is 10 Joules.

1. What charge is passing through the element every second?
2. Find the voltage across the element.

Solution

$$q = I \cdot t = 2 \cdot 1 = 2 \text{ C} ,$$

$$V = \frac{W}{q} = \frac{10}{2} = 5 \text{ V} .$$

**Problem 3.** Fig. 1.7 presents a series circuit containing three known resistances  $R_1=30 \Omega$ ,  $R_2=60 \Omega$ ,  $R_3=10 \Omega$ , as well as source voltage  $V=150 \text{ V}$ . Using these quantities determine the total resistance  $R_t$ , the current  $I$  through the circuit, voltage drop across each resistor, and the power  $P_S$  supplied by the source and dissipated by the resistances.

Write the equation of the second Kirchhoff's Law and be sure it's true. Check the condition of power balance.

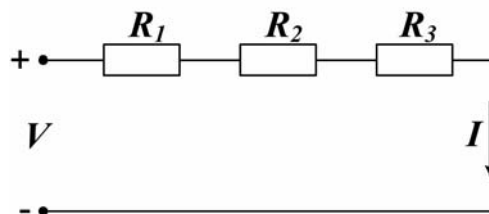


Fig. 1.7

Solution

1. Total resistance of the circuit:  $R_t = R_1 + R_2 + R_3 = 30 + 60 + 10 = 100 \Omega$ .
2. We can determine current  $I$  by using Ohm's Law for the entire circuit:

$$I = \frac{V}{R_t} = \frac{150}{100} = 1,5 \text{ A} .$$

3. Application of Ohm's Law to each circuit resistor will result in voltage drops:

$$V_1 = I \cdot R_1 = 1,53 \times 30 = 45 \text{ V} ,$$

$$V_2 = I \cdot R_2 = 1,53 \times 60 = 90 \text{ V} ,$$

$$V_3 = I \cdot R_3 = 1,53 \times 10 = 15 \text{ V} .$$

4. Source power:

$$P_S = V \cdot I = 150 \times 1,5 = 225 \text{ W} .$$

5. Dissipated power:

$$P_R = I^2 \cdot R_1 + I^2 \cdot R_2 + I^2 \cdot R_3 = 225 \text{ W} .$$

The requirement of power balance states that in the circuit source power and dissipated power must be equal, i.e.  $P_S = P_R$ . In our case this is true.

**Problem 4.** A parallel circuit is depicted in Fig. 1.8.

1. Determine the current flowing through each of three resistors,  $R_1=5\Omega$ ,  $R_2=10\Omega$ , and  $R_3=20\Omega$  and total current drawn from the 100V source.
2. Calculate dissipated power for the entire circuit and check the validity of the first Kirchhoff's Law.

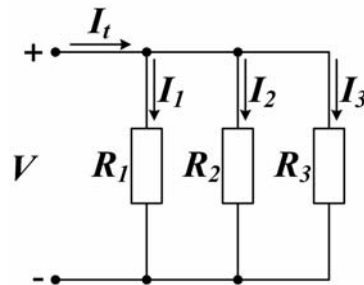


Fig 1.8

### Solution

1. Because of 100V applied across each resistor in the circuit current flows through each of the resistors will be:

$$I_1 = \frac{V}{R_1} = \frac{100}{5} = 20 \text{ A},$$

$$I_2 = \frac{V}{R_2} = \frac{100}{10} = 10 \text{ A},$$

$$I_3 = \frac{V}{R_3} = \frac{100}{20} = 5 \text{ A}.$$

2. Also we can calculate equivalent resistance of three resistors connected in parallel

$$R_t = \frac{R_1 \cdot R_2 \cdot R_3}{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1} = \frac{5 \cdot 10 \cdot 20}{50 + 200 + 100} = \frac{20}{7} \Omega$$

with the purpose to find the source (i.e. total) current:

$$I_t = \frac{V}{R_t} = \frac{100}{20/7} = 35 \text{ A}.$$

3. To determine power dissipated by the total resistance we can use

$$P_t = I_t^2 \cdot R_t = \frac{35^2 \cdot 20}{7} = 3500 \text{ W}.$$

4. Finally, for the circuit in Fig. 1.8 the first Kirchhoff's Law results in

$$I_t = I_1 + I_2 + I_3,$$

and numerically:

$$35 = 5 + 10 + 20 = 35.$$

### Supplement Problems

**Problem 5.** Find the energy required to move 2 coulombs of charge through the 4 volts potential difference.

**Problem 6.** A current of 10 A is flowing through an element for 5 seconds. What energy is required to maintain 10 V voltage drop across that element?

**Problem 7.** Modern technology has produced a small 1,5 volt alkaline battery with a nominal life of 150 joules. For how many days will it power calculator that draws a 2mA current?

**Problem 8.** Find the power absorbed by a 100 Ohm resistor when it is connected directly to the contacts of 10 V source.

**Problem 9.** Simplify the circuit connected to **a, b** points in Fig. 1.9. Find currents flowing through all branches. Determine each voltage drop.

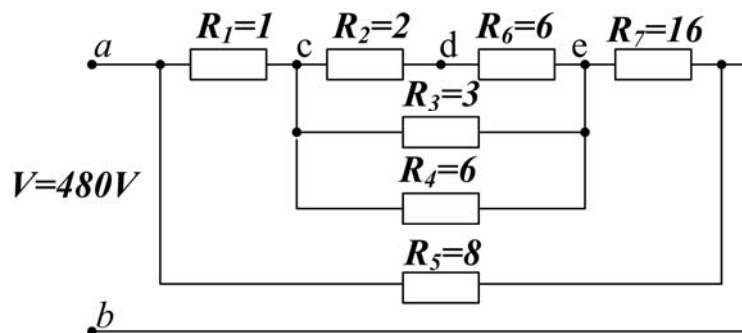


Fig. 1.9

**Problem 10.** Calculate all currents and voltages in the circuit of Fig. 1.10.

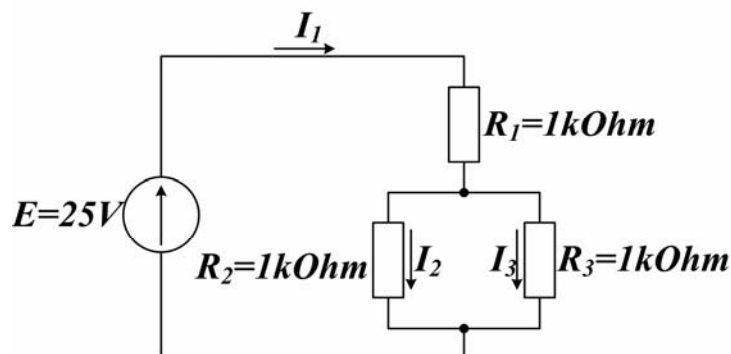


Fig. 1.10

**Problem 11.** Calculate the resistance between terminals **A** and **B** in each of the circuits below (Fig. 1.11). Also determine the voltage dropped across each resistor and the current flowing through it if a 6 volt source supplies the circuit.

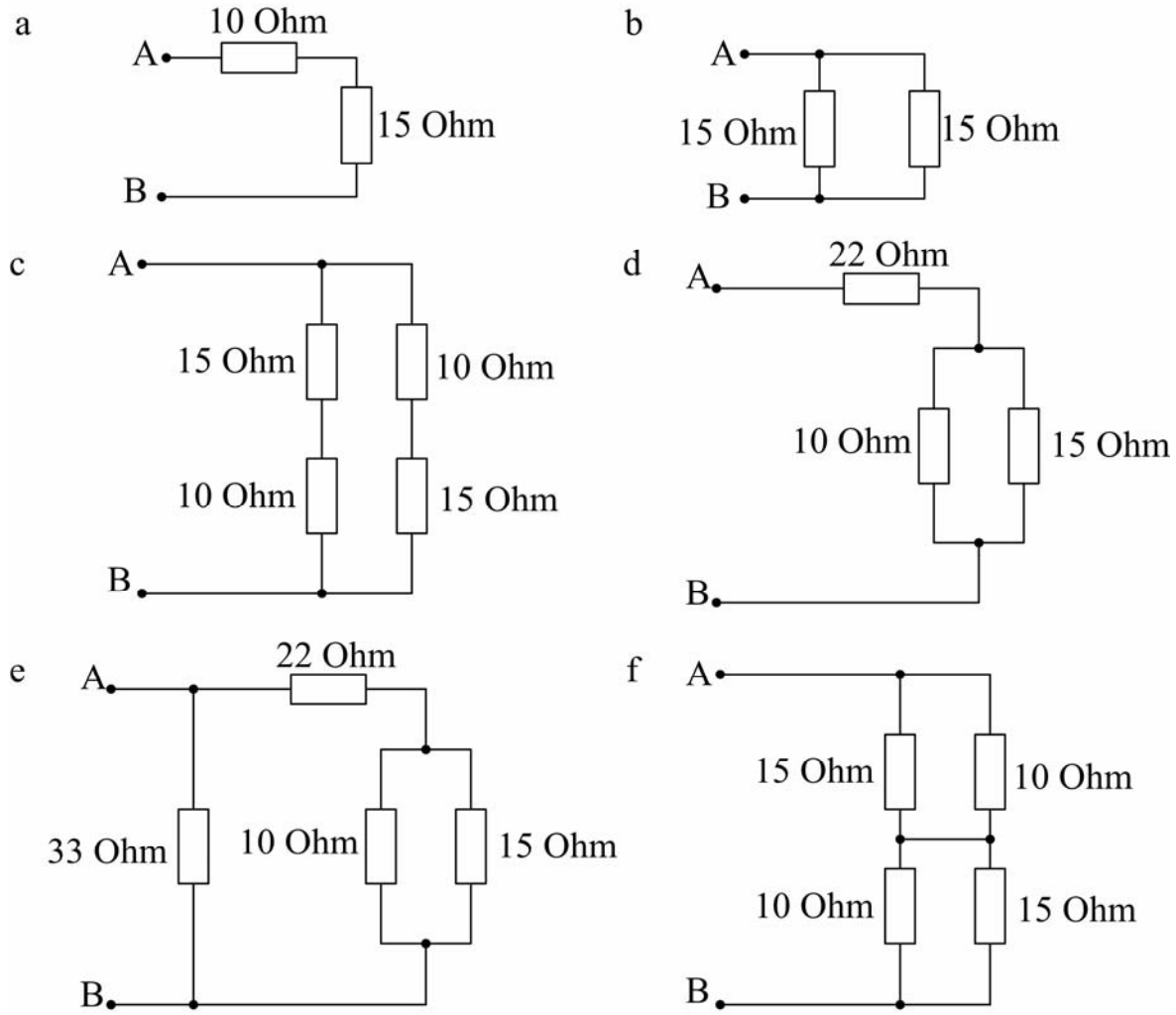


Fig. 1.11

### 1.5 Practice

#### “Measurements in DC and AC Electrical Circuits”

**Objective:** to learn how to use electrical measuring instruments (to investigate characteristics of devices and to carry out measurements with the given accuracy).

#### Work task

1. Studying instrument structure and technical requirements necessary for carrying out electrical measurements with a given accuracy.
2. Studying current, voltage, resistance, and active power measurement techniques; expansion of the meter measurement range.



3. Measuring and calculating electrical parameters used when analyzing and investigating AC and DC circuits.


### Theory and preparation

1. To use electrical instrumentation correctly it is necessary to take into account the type of the current (AC or DC) you deal with; the expected range of the parameter which is to be measured (i.e. voltage, current, or power); the frequency (a rate of change) of the current in the circuit; admissible instrumentation error; absorbed initial device power; internal resistance and inductance of the instrument, etc.


Meters are classified in accordance with:

- 1) purpose ( A – ammeter, V – voltmeter, and W – wattmeter are related to fundamental instruments of electrical engineering);
- 2) work system being used. Widely used device systems are as follows:

 – permanent-magnet measuring system ,

 – electromagnetic measuring system ,

 – electro-dynamic measuring system ,

 – ferromagnetic measuring system ;

- 3) type of operating current (mark  $\text{—}$  is the conventional symbol for DC instrument;  $\sim$  – AC device;  $\approx$  – multifunctional AC/ DC instrument);
- 4) operation frequency for which the device intends (500 Hz, 20...50 Hz);
- 5) class of accuracy of the instrument (0.02, 0.05, etc.);
- 6) operating status (  $\updownarrow$  – vertical,  $\rightarrow \square$  – horizontal);
- 7) isolation test voltage, kV (  $2\text{⚡}$ ,  $\star_2$  ).

#### The ammeter

Two requirements are practically evident for obtaining the correct measurement of current:

1. The ammeter must be placed in series with the element whose current is to be measured.
2. The ammeter should not restrict the flow of current (i.e. cause a voltage drop), or else it will not be measuring the true current flowing in the circuit. An ideal ammeter has zero internal resistance.

#### The voltmeter

A voltmeter must also fulfill two requirements:

1. The voltmeter must be placed in parallel with the element whose voltage it is measuring.

2. The voltmeter should draw no current away from the element whose voltage it is measuring, or else it will not be measuring the true voltage across that element. Thus, an ideal voltmeter has infinite internal resistance.

### The wattmeter

Fig. 1.12 depicts the typical connection of a wattmeter in a circuit with an electrical load  $R_L$ . In effect, the wattmeter measures the current flowing through the load and, simultaneously, the voltage across it and multiplies the two to provide a reading of the power dissipated by the load. In Fig. 1.12,  $I^*$  and  $V^*$  both are the so called generator facing endings of the wattmeter windings.

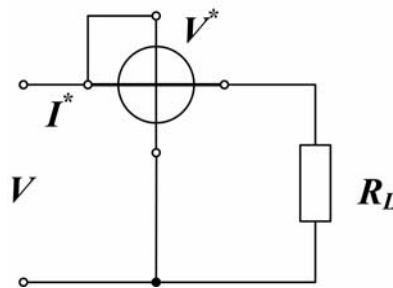


Fig. 1.12 – Connection of the wattmeter in the circuit

Practical investigations during the laboratory work are carried out at the laboratory bench that includes:

- DC and AC power sources,
- special breadboard for circuit assembling, accompanied with a set of both tipped insulated wires and special plugs with mounted electric components being investigated,
- measuring instruments (voltmeter, three ammeters, wattmeter, and oscilloscope).

### Procedure

#### 1. Investigation of the simple series-parallel DC circuit

Before carrying out the investigation, be sure you depict the experiment circuit diagram shown in Fig. 1.13, and a table for necessary measurements recording (Table 1.1).

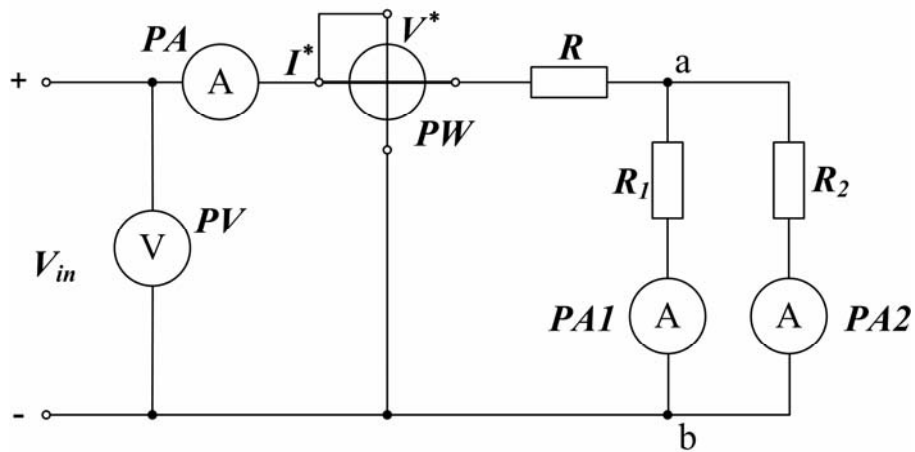


Fig. 1.13

From your instructor, take a recommended value of the input DC voltage and prepare a measuring instrument for each branch, i.e. select the device, study specifications, set the range, and evaluate a price of division of a meter.

Connect the circuit according to the diagram of Fig. 1.13.

Switch the DC power supply on (after instructor's permission) and set recommended input voltage.

Measure each of the necessary circuit parameters and record your results in Table 1.1.

Table 1.1

Parameter	Measured							Calculated		
Symbol	$V_{in}$	$I$	$V_R$	$V_{ab}$	$I_1$	$I_2$	$P$	$R$	$R_1$	$R_2$
Unit	V	A	V	V	A	A	W	$\Omega$	$\Omega$	$\Omega$
Value										

In the table:  $V_{in}$  is the power supply voltage (circuit input voltage);  $V_R$  – voltage drop across the resistor  $R$ ;  $V_{ab}$  – voltage drop across either  $R_1$  or  $R_2$  parallel branch;  $I_1$ ,  $I_2$  – branch currents;  $P$  – entire circuit dissipation power.

**2. Investigation of the simple AC circuit including a resistor and an inductor (capacitor) in series (Fig. 1.14).**

Repeat the preparations like those in the previous section. In addition, study a phasor (complex) form of voltage and current presentation to be able to operate with different concepts of AC power.

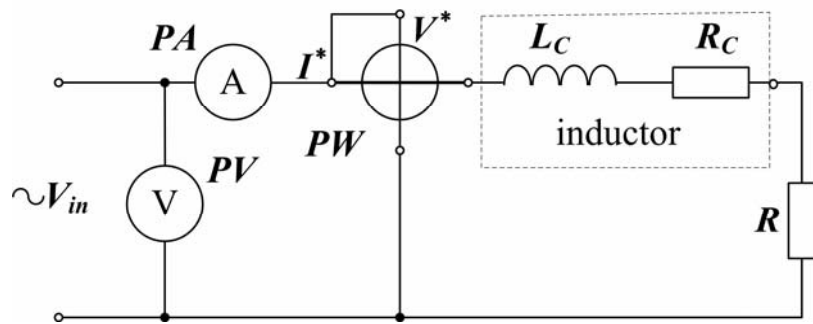


Fig. 1.14

After connecting the circuit, take the value of the input voltage from the instructor and switch the voltage source on. Record your values of circuit parameters in Table 1.2.

Table 1.2

Parameter	Measured				Calculated	
Symbol	$V_{in}$	$V_R$	$V_{C(cap)}$	$P$	$\varphi$	$I$
Unit	V	V	V	W	deg	A
Value						

In the table:  $V_R$  – voltage across the resistor;  $V_C$  – voltage drop across the inductor;  $V_{CAP}$  – voltage drop across the capacitor (when connected),  $\varphi$  – phase shift between the voltage  $V_{in}$  and the current  $I$ .

### Summarizing

Calculate unknown values in Tables 1.1 and 1.2.

Analyze the measured and calculated results to make short conclusions.

### Questions to be answered

1. How do we choose a device to measure the electrical quantity?
2. Describe the main characteristics of your measuring device using symbols on its panel.
3. What is the principle of operation of a device based on a permanent magnet system? Electromagnetic system? Electrodynamical measuring system?
4. In what way should we connect a wattmeter in the electric circuit and obtain the price division for this device?
5. How do we measure the sine wave amplitude and period with the help of oscilloscope?

## Chapter 2

### COMPLEX CIRCUIT ANALYSIS

The electric circuit is considered to be complex if there are several power sources and (or) complicated connection of elements in it.

Following methods for complex circuit calculation are generally used:

1. The method of direct use of Kirchhoff's laws.
2. The mesh current method.
3. The node voltage method.
4. The method of equivalent transformation of delta-connected resistances into wye connection, and vice-versa.
5. Superposition method.
6. Equivalent generator method.

#### 2.1 Method of Kirchhoff's Laws Direct Use

To apply the method one must write both current and voltage Kirchhoff's equations to determine every branch current. The total number of equations is equal to the number of unknown currents.

The rules of equations compilation are as follows:

a) the first part of the set contains equations written in accordance with the first Kirchhoff's law; the number of equations in this part should be equal to the number  $n$  of all circuit nodes less one;

b) remaining equations must be added by using the second Kirchhoff's law applied to independent loops of the circuit. Each of these loops contains at least one branch not used in previous loops.

The following example illustrates the method.

In Fig. 2.1 a network with three ( $n = 3$ ) nodes named  $a, b, c$ , and five branches is shown. EMF of the power sources and circuit resistances are given.

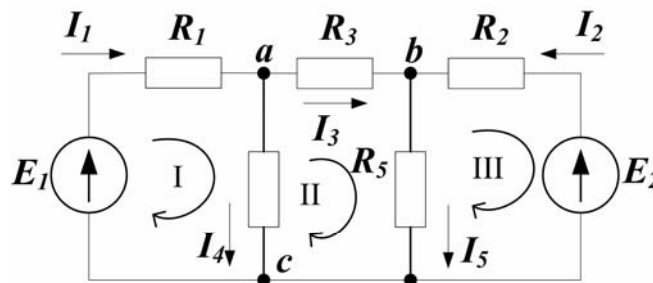


Fig. 2.1 – Complex circuit

There are five unknown currents we have to determine.

To start with the problem we should assume directions of unknown currents (the supposed preposition is depicted in the figure), and write two independent equations using the first Kirchoff's law.

At  $a$  node we have

$$I_1 - I_3 - I_4 = 0; \quad (2.1)$$

and at  $b$  node:

$$I_2 + I_3 - I_5 = 0. \quad (2.2)$$

To complete the set of five necessary equations we'll select three closed loops (meshes I, II, III in the figure) and, taking directions of mesh currents, apply the second Kirchoff's law:

$$E_1 = I_1 R_1 + I_4 R_4, \quad (2.3)$$

$$0 = I_3 R_3 - I_4 R_4 + I_5 R_5, \quad (2.4)$$

$$-E_2 = -I_2 R_2 - I_5 R_5. \quad (2.5)$$

*Note:* a **mesh** is a loop that does not contain other loops.

All five currents can be determined now. If a solution obtained collected minus sign, corresponding current actually flows in the opposite direction.

## 2.2 Node Voltage Method

The direct use of Kirchoff's laws for determining the currents in a complex circuit demands solving considerable number of equations, what makes the computations extremely time-consuming. However, there are a number of methods, based on the same Kirchoff's laws, that obviate the solution of a set of equations or reduce the number of equations and, hence, significantly cut computation time.

One of those is a **node-voltage method**, which may be more effective if the circuit contains only two nodes (Fig. 2.2).

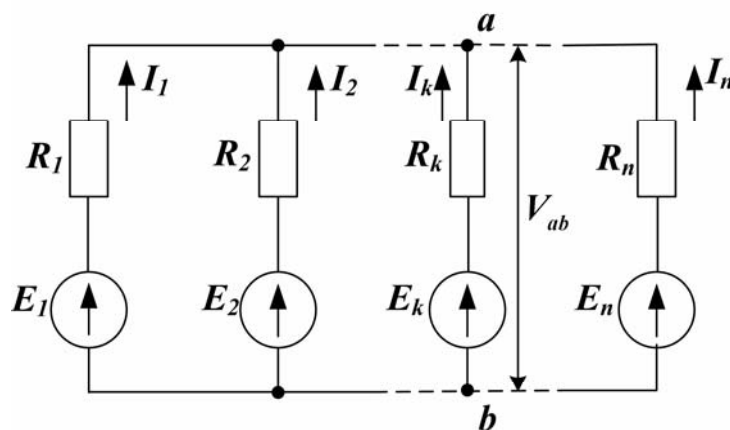


Fig. 2.2 – The circuit with two nodes

In accordance with Kirchhoff's law, the voltage between two nodes (for example node voltage  $V_{ab} = \varphi_a - \varphi_b$  in Fig 2.2) may be defined as the difference between EMF  $E_k$  of definite circuit branch and the voltage drop  $I_k R_k$  across the resistance in that branch:

$$V_{ab} = E_k - I_k R_k .$$

Whence the  $k$  - branch current could be derived from

$$I_k = \frac{E_k - V_{ab}}{R_k} . \quad (2.6)$$

At the same time, according to First Kirchhoff's Law, the algebraic sum of currents at the node  $a$  equals zero:

$$\sum_{k=1}^n I_k = \sum_{k=1}^n \frac{E_k - V_{ab}}{R_k} = 0 ,$$

or

$$\sum_{k=1}^n \frac{E_k}{R_k} = \frac{V_{ab}}{\sum_{k=1}^n R_k} .$$

This result in

$$V_{ab} = \frac{\sum_{k=1}^n \frac{E_k}{R_k}}{\sum_{k=1}^n \frac{1}{R_k}} . \quad (2.7)$$

Whence, for the circuit like the network in Fig. 2.2 the node voltage can be easily found by parameters of all branches on the base of (2.7).

After that one can simply use (2.6) to find any required current.

If one or several EMF have a direction reverse to that shown in Fig. 2.2, then EMF must be included in equations (2.6) and (2.7) with a minus sign.

Often one of the potentials of the node voltage is taken zero (e.g., let in Fig. 2.2 be  $\varphi_b = 0$ ). Corresponding node ( $b$ ) is referred to as a **reference node**. The other node ( $a$ ) is then called a **principal node**.

### 2.3 Loop Current Method

If a circuit with several power sources obtains more than two nodes, the currents in it may be found by using loop currents concept. This is one of the most universal methods for circuit calculation.

In a number of cases, a circuit can be considered as consisting of a set of adjoining loops (see Fig. 2.1).

Let us introduce a notion of the loop current, in addition to the branch current. The loop current is an imaginary (virtual) current that passes through the elements of its own loop only. For example, the circuit shown in Fig. 2.1, and consisting of three meshes, receives three loop currents:  $I_I, I_{II}, I_{III}$ .

It is evident that the actual current in the adjacent branch would result from the currents of two adjacent loops.

From the other hand the current through the external branch of the loop will be equal in magnitude to the corresponding loop current (as the only current in that branch) and its sign will depend upon the fact whether its direction is the same as that of the loop current or opposite:

$$I_1 = I_I, I_2 = -I_{II}, I_3 = I_{II}, I_4 = I_I - I_{II}, I_5 = I_{II} - I_{III}. \quad (2.8)$$

If for all three loops in Fig. 2.1 we write equations based on Kirchoff's second law relative to loop currents, that will result in a set

$$\begin{aligned} I_I(R_1 + R_4) - I_{II}R_4 &= E_1, \\ -I_I R_4 + I_{II}(R_3 + R_4 + R_5) - I_{III}R_5 &= 0, \\ -I_{II}R_5 + I_{III}(R_2 + R_5) &= -E_2. \end{aligned} \quad (2.9)$$

After solving this set of equations and finding all the loop currents one could derive the actual currents with the help of (2.8).

#### 2.4 Conversion of Delta-connected Resistances into Equivalent Wye Connection

Not always circuits with one power source may be simplified by connection of the group of resistances in series, parallel or series-parallel. For example, the resistors  $R_1, R_2$  and  $R_3$  in Fig. 2.3 are **wye** (or **Y**) connected, while the resistors  $R_2, R_3, R_4$  (or  $R_5, R_6$  and  $R_4$ ) form a so-called **delta** (or  $\Delta$ ) configuration.

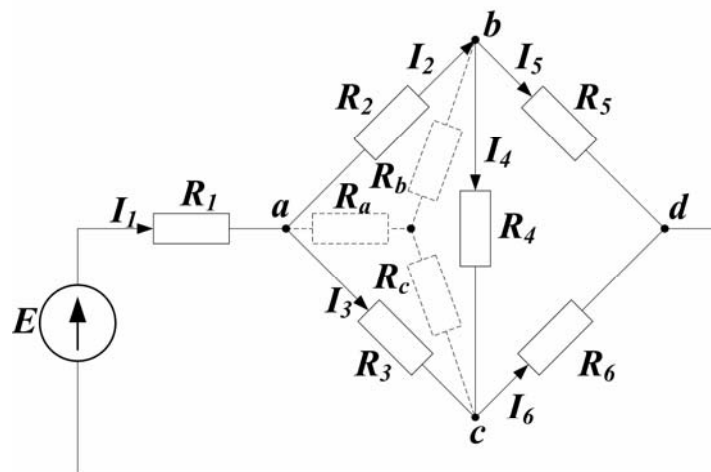


Fig. 2.3 – Complex circuit with one source



This circuit may be converted into a series-parallel circuit if one or several delta-connections of resistances are replaced by their equivalent wye, and vice versa.

Delta-connected and wye-connected resistances will be equivalent if their replacing each other does not change origin node potentials of the circuit.

If *abc* delta configuration of the circuit in Fig. 2.3 is replaced by the equivalent wye form like shown in Fig. 2.4 it will result in the conventional series-parallel connection of resistances.

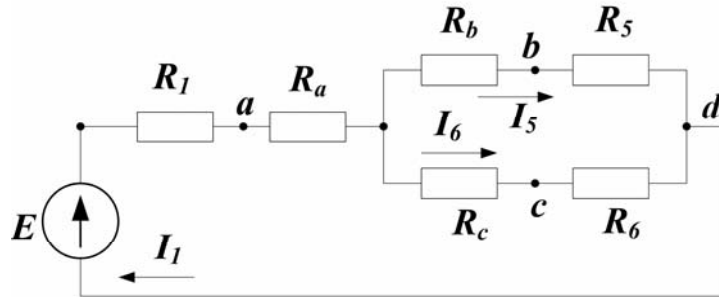


Fig. 2.4 – The converted complex circuit

Typical calculation of the total resistance of this circuit

$$R_t = R_1 + R_a + \frac{(R_b + R_5)(R_c + R_6)}{R_b + R_c + R_5 + R_6},$$

will greatly simplify each of the currents  $I_1, I_5, I_6$  derivation.

In order to determine remaining currents  $I_2, I_3$  and  $I_4$ , first one must find the potentials of the nodes *a, b* and *c* (respectively  $\varphi_d = 0$ ):

$$\varphi_b = I_5 R_5, \varphi_c = I_6 R_6, \varphi_a = I_5 (R_5 + R_6) + I_1 R_a,$$

and then calculate:

$$I_2 = \frac{(\varphi_a - \varphi_b)}{R_2}, I_3 = \frac{(\varphi_a - \varphi_c)}{R_3}, I_4 = \frac{(\varphi_b - \varphi_c)}{R_4}.$$

The two circuits, delta and wye Fig. 2.5, are equivalent if the resistance between any two terminals of one is equal to the resistance between the same two terminals of the other.

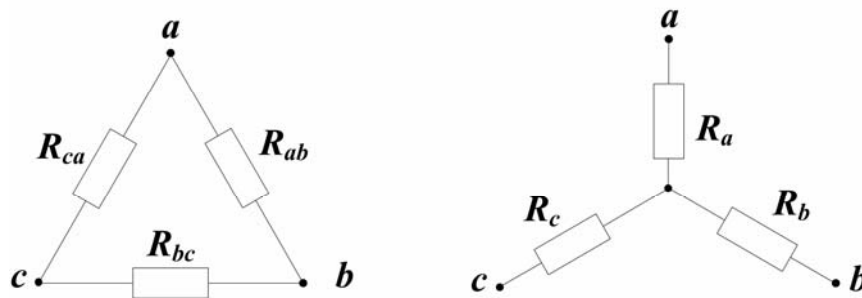


Fig. 2.5 – Equivalent delta and wye connections

The criterion of equivalence could be satisfied if

$$R_a = \frac{R_{ca}R_{ab}}{R_{ab} + R_{bc} + R_{ca}}, R_b = \frac{R_{ab}R_{bc}}{R_{ab} + R_{bc} + R_{ca}}, R_c = \frac{R_{bc}R_{ca}}{R_{ab} + R_{bc} + R_{ca}}, \quad (2.10)$$

and, on the contrary,

$$R_{ab} = R_a + R_b + \frac{R_a R_b}{R_c}, R_{bc} = R_b + R_c + \frac{R_b R_c}{R_a}, R_{ca} = R_c + R_a + \frac{R_c R_a}{R_b}. \quad (2.11)$$

Besides, if each delta resistance is of the same value ( $R_d$ ), the three resistances in the equivalent wye ( $R_w$ ) configuration are identical too:

$$R_w = \frac{1}{3} R_d.$$

## 2.5 Superposition method

One of the properties of a linear element or a device is superposition.

If a device excited by current  $I_1$ , will exhibit response  $V_1$ , then similarly, excitation  $I_2$  will cause response  $V_2$ , and if we use excitation ( $I_1 + I_2$ ) we will find a response ( $V_1 + V_2$ ).

The superposition principle can serve a basis for the method of analysis of electric circuits called the superposition method:

1) in a circuit with many power sources partial currents produced by the action of each EMF are consequently determined (other EMFs each time being put equal to zero);

2) the partial current components for the same branch are then algebraically added (superimposed) to find the resultant current produced by the combined action of all EMFs.

## 2.6 Calculation of Nonlinear DC Circuits

### Nonlinear Circuit Elements

An electric circuit resistance of which depends upon neither current nor voltage is called linear. The relationship between the current flowing through the circuit and the applied voltage, i.e.  $I = f(V)$  is called a **volt-ampere characteristic** ( $I$ - $V$  characteristic) of the circuit.

The  $I$ - $V$  characteristic of a linear element or a linear circuit is a straight line. Say, line  $oa$  in Fig. 2.6 could present an ideal resistor that exhibits linear resistance properties according to Ohm's law. A resistor made of carbon or metal filament is mainly considered to meet these requirements.

Often, on the other hand, the electric circuit contains elements for which Ohm's law does not hold, that is the current is not proportional to the applied voltage directly. Such elements are called nonlinear.

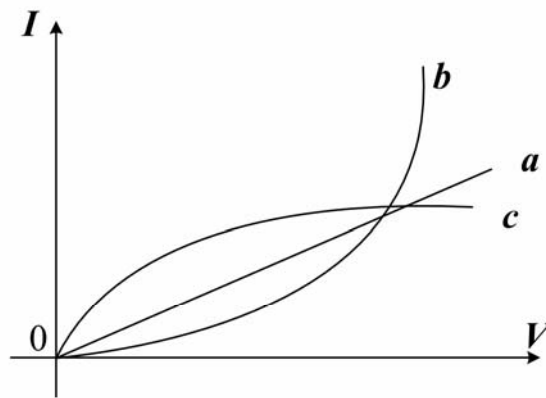


Fig. 2.6 – Instances of  $I$ - $V$  characteristics

For a nonlinear element  $I$ - $V$  characteristic is no longer a straight line, but a curve. The example of a nonlinear element is an ordinary incandescent lamp, as in this case the resistance does not remain constant but varies greatly with the current and temperature of the filament. Corresponding  $I$ - $V$  characteristics are presented in Fig 2.6 by curves  $Ob$  (carbon) and  $Oc$  (metal filament).

Other examples of nonlinear elements are the vacuum tubes, semiconductor devices and so on.

If a circuit contains at least one nonlinear element we can speak of nonlinear circuit. For its analysis we can't use the methods discussed above. The difficulty is that it is not possible, in general, to obtain a closed form analytical solution, even for a very simple circuit.

### Graphical (Load-Line) Analysis of Nonlinear Circuits

There are some specific ways of nonlinear circuit analysis and calculation. One of them is the graphical analysis of a circuit.

With the knowledge of equivalent circuits we have just acquired, one approach to analyzing a circuit containing a nonlinear element might be to treat the nonlinear element as a load and to compute the equivalent of the remaining circuit (an equivalent generator), as shown in Fig. 2.7. Applying KVL, the following equation may then be obtained:

$$I = -\frac{I}{R}V + \frac{E}{R}. \quad (2.12)$$

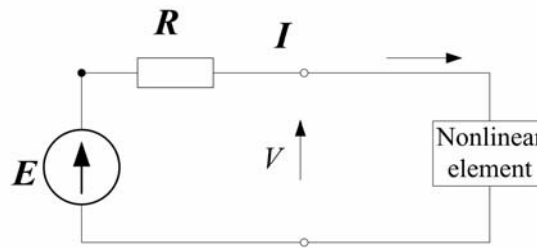


Fig. 2.7 – Representation of nonlinear element in a linear circuit

We notice first that equation 2.12 describes the behaviour of any load, linear or nonlinear, since we have made no assumptions regarding the nature of the load voltage and current. Second, it is the equation of a line in the  $I - V$  plane. This equation is referred to as the load-line equation; its graphical interpretation is very useful and is shown in Fig. 2.8.

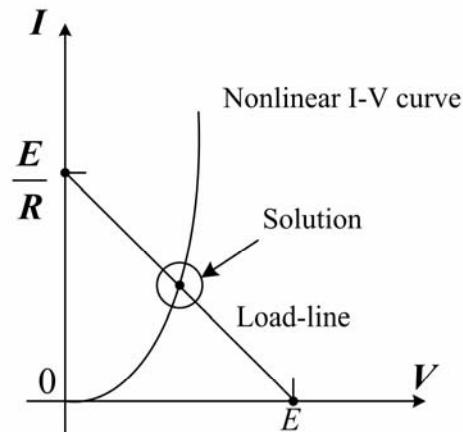


Fig. 2.8 – Graphical solution for a nonlinear circuit

The load-line equation is but one of two  $I - V$  characteristics we have available, the other being the nonlinear device characteristic. The intersection of the two curves yields the solution of our nonlinear system of equations. This result is depicted in Fig. 2.8.

Finally, another important point should be emphasized: the linear network reduction methods introduced in the preceding sections can always be employed

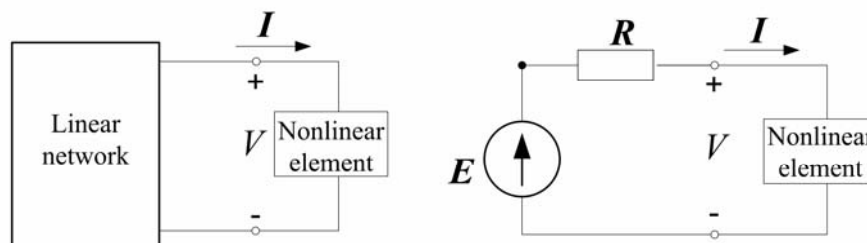


Fig. 2.9 – Transformation of nonlinear circuit into equivalent form

to reduce any circuit containing a single nonlinear element to the Thevenin equivalent form, as illustrated in Fig. 2.9. The key is to identify the nonlinear element and to treat it as a load. Thus, the equivalent-circuit solution method (equivalent generator method) can be very useful in simplifying problems in which a nonlinear load is present.

## 2.7 Terms and Concepts

**Loop current** is the current that flows through the elements constituting the loop.

**Node voltage** is voltage from a reference node to the selected (principal) node.

**Reference node** is a node selected as the reference for all other nodes.

**Linear element** is an element that satisfies the properties of superposition and homogeneity.

**Delta connection** is the closed connection of three receivers, when the ending of previous receiver is connected with the beginning of following one.

**Wye connection** is the connection of three receivers when all their endings are connected in one point.

## 2.8 Review Questions

1. How many Kirchhoff's equations are to be used in order to find values of all currents in a complex circuit?
2. What advantage has the loop-current method as compared to other methods?
3. What equations and how many of them can be made up when loop-current method is used?
4. How can one define the actual current in a circuit branch by means of the loop-currents method?
5. What electric circuit is called a complex circuit?
6. Write down equations for defining node voltage and currents in the case of a two node circuit.
7. When do we use the superposition method for electric circuit calculation?
8. What are the conditions of equivalent conversion of delta-connected resistances into wye-connected?
9. How can you define the value of wye-connected resistances?
10. What elements are called nonlinear?
11. Explain the essence of equivalent conversion method for nonlinear circuits.

## 2.9 Problems

### Examples

**Problem 1.** The resistive circuit is shown in Fig. 2.10.

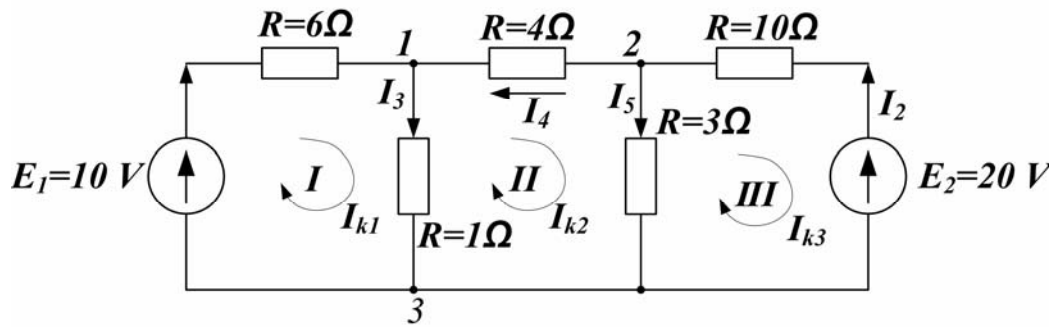


Fig. 2.10

Two voltage sources  $E_1$  and  $E_2$  and all resistances are known.

1. Write the equations required to determine all currents with the direct use of Kirchhoff's Laws;
2. Using mesh current analysis, find all currents.

### Solution

1. There are three nodes  $1, 2, 3$  and three independent loops (meshes)  $I, II, III$  in this circuit. Let us set the directions of unknown currents and traveling direction around the meshes (clockwise).

Using the first Kirchhoff's Law we can make up two equations:  $I_1 + I_4 = I_3$  and  $I_2 = I_4 + I_5$ , and using the second Kirchhoff's law add three equations more for meshes:

$$\begin{aligned} E_1 &= I_1 R_1 + I_3 R_3, \\ 0 &= -I_3 R_3 - I_4 R_4 + I_5 R_5, \\ -E_2 &= -I_2 R_2 - I_5 R_5. \end{aligned}$$

Now all five currents can be found by solving this set of equations.

2. We define each of the  $I, II, III$  mesh currents applying KVL for each of the  $I, II, III$  mesh:

$$\begin{aligned} E_1 &= I_{k1}(R_1 + R_3) + I_{k2} R_3, \\ 0 &= -I_{k1} R_3 + I_{k2}(R_3 + R_4 + R_5) - I_{k3} R_5, \\ -E_2 &= -I_{k2} R_5 + I_{k3}(R_2 + R_5). \end{aligned}$$

Substitution of the circuit parameters

$$\begin{cases} 10 = I_{k1}(6 + 7) - I_{k2} = 13I_{k1} - I_{k2}, \\ 0 = -I_{k1} + I_{k2}(1 + 2 + 3) - I_{k3} = -I_{k1} + 6I_{k2} - 3I_{k3}, \\ -20 = -3I_{k2} + I_{k3}(3 + 10) = -3I_{k2} + 13I_{k3} \end{cases}$$

results in the solution of mesh currents:

$$\begin{aligned} I_{k1} &= \frac{9}{13} \text{ A}; \\ I_{k2} &= -1 \text{ A}; \\ I_{k3} &= -\frac{23}{13} \text{ A}. \end{aligned}$$

The values of actual branch currents will then be as follows:

$$\begin{aligned} I_1 = I_{k1} &= \frac{9}{13} \text{ A}; & I_2 = -I_{k3} &= \frac{23}{13} = 1.77 \text{ A}; & I_3 = I_{k1} - I_{k2} &= \frac{9}{13} - \left(-\frac{23}{13}\right) = \frac{32}{13} \text{ A}; \\ I_4 = -I_{k2} &= 1 \text{ A}; & I_5 = I_{k2} - I_{k3} &= -1 - \left(-\frac{23}{13}\right) = \frac{23}{13} - 1 = \frac{10}{13} \text{ A}. \end{aligned}$$

**Problem 2.** For the resistive circuit (Fig. 2.11) there are known quantities:  $E_1=220\text{V}$ ;  $E_2=110\text{V}$ ;  $E_3=100\text{V}$ ;  $R_1=2\Omega$ ;  $R_2=2\Omega$ ;  $R_3=1\Omega$ ;  $R_L=5\Omega$ .

Calculate the currents  $I_1, I_2, I_3, I_L$  through each resistor.

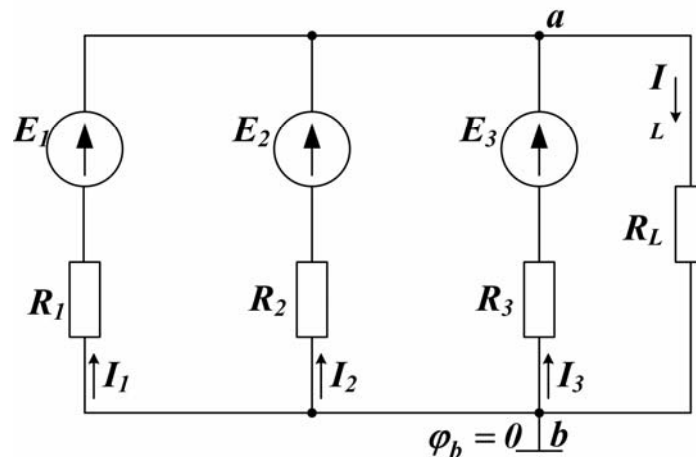


Fig. 2.11

### Solution

1. Let us take the node  $b$  as a reference node having  $\varphi_b=0$ , then the  $a$  node will be the principal node.

Using the equation for the node voltage we can find

$$V_{AB} = \frac{E_1 q_1 + E_2 q_2 + E_3 q_3}{q_1 + q_2 + q_3 + q_L} = \frac{220 \cdot 0.5 + 110 \cdot 0.5 + 100}{0.5 + 0.5 + 1 + 0.2} = \frac{265}{2.2} = 120.46 \text{ V}.$$

2. It's easy then to find the required currents:

$$I_1 = \frac{E_1 - V_{AB}}{R_1} = \frac{220 - 120.46}{2} = 49.77 \text{ A};$$

$$I_2 = \frac{E_2 - V_{AB}}{R_2} = \frac{110 - 120.46}{2} = \frac{-10.46}{2} = -5.23 \text{ A};$$

$$I_3 = \frac{E_3 - V_{AB}}{R_3} = \frac{100 - 120.46}{1} = -20.46 \text{ A};$$

$$I_L = \frac{V_{AB}}{R_L} = \frac{120.46}{5} = 24.09 \text{ A}.$$

### Supplement Problems

**Problem 3.** Find currents  $I_1$ ,  $I_2$ ,  $I_3$  for the circuit of Fig. 2.12, if we have the known quantities:  $E_1=50V$ ;  $E_2=100V$ ;  $R_1=5\Omega$ ;  $R_2=30\Omega$ ;  $R_3=20\Omega$ ;  $R_4=10\Omega$ .

*Note:* Apply the direct use of Kirchhoff's Laws and both node-voltage and loop-current methods.

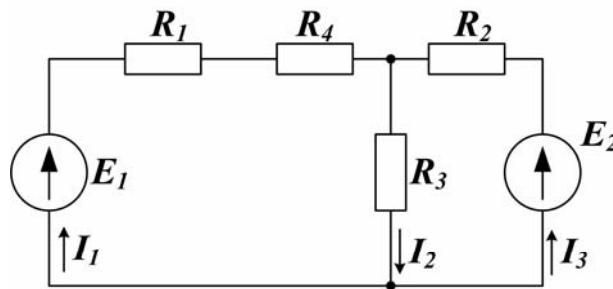


Fig. 2.12

**Problem 4.** For the circuit shown in Fig. 2.13:

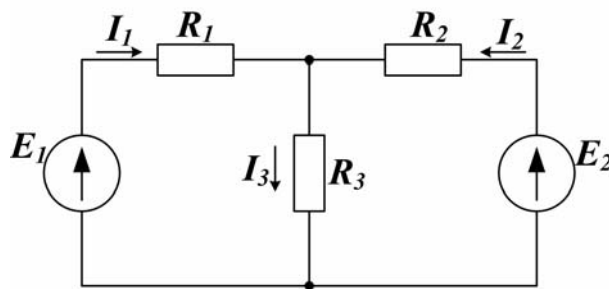


Fig. 2.13

- calculate  $I_1, I_2, I_3$  when  $R_1=R_2=R_3=3\Omega$  and  $E_1=1V$ ;  $E_2=3V$ ;
- convince yourself that the energy is conserved in the circuit.



**Problem 5.** On the base of superposition principle find currents in the circuit (see Fig. 2.13), if:  $R_1=5\Omega$ ,  $R_2=30\Omega$ ,  $R_3=20\Omega$ ,  $E_1=1V$ ;  $E_2=3V$ .

**Problem 6.** Using the method of equivalent generator calculate the current through the resistor  $R$  in the circuit of Fig. 2.14 ( $R_1=5\Omega$ ,  $R_2=4\Omega$ ,  $R_3=20\Omega$ ,  $R=2\Omega$ ,  $E=50V$ ).

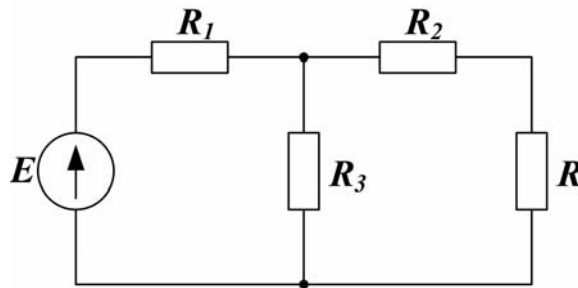


Fig. 2.14

Answer: 4A.

## 2.10 Practice

### “Investigation of DC Circuits Containing Linear and Non-linear Elements”

**Objective:** to study basic methods of analysis and calculation of direct current circuits with linear and non-linear elements, simple and complex DC circuits, and to familiarize with types and properties of non-linear elements.

#### Work task

1. Measurement and calculation of currents. Proceeding in a circuit with linear elements and potentials in the given points of a circuit.
2. Construction of the potential diagram for the circuit under analysis.
3. Defining currents and voltages in the circuit to be investigated by the graphic method.

#### Theory and Preparation

Analysis of electric circuits with linear elements can be done with the direct use of Ohm’s and Kirchhoff’s laws. Given the EMF of energy sources and values of the resistance of resistive elements in a complex electric circuit with several branches, then the analysis of such circuits leads to determination of currents in the branches. Calculation of currents in the complex circuits is carried out by various methods: making use of Ohm’s and Kirchhoff’s laws, the conversion of delta-

connected resistances into equivalent wye-connection and vice-versa, loop-current, node-voltage etc.

Nonlinear circuit calculations don't use Ohm's law because they haven't linear relationship between current and voltage. Analysis of such circuits are realized by graph-analytical methods.

Experimental work is carried out on the laboratory stand with the use of DC power supplies, linear and nonlinear elements specified to the workplace, special plugs and conductors for assembling investigated circuits, measuring devices needed to conduct necessary experimental works.

## Procedure

### 1. DC circuit with series-parallel connection of linear elements

Before carrying out the investigation, be sure you depict an experiment circuit diagram (Fig. 2.15) and a table for the necessary measurements recording (Table 2.1).

From your instructor, take the recommended value of the input DC voltage and prepare a measuring instrument (select the device, study its specifications, set the range).

Connect the circuit (see Fig. 2.15).

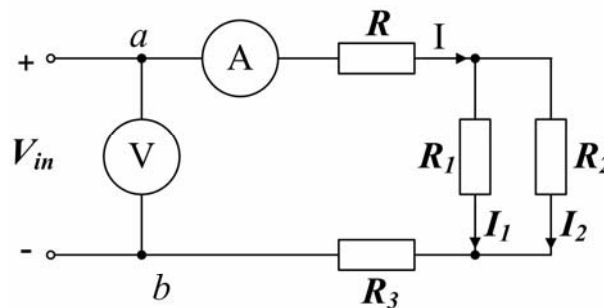


Fig. 2.15

Switch the DC power supply on (after instructor's admission), regulate the input voltage taken and measure each of the necessary parameters. Record your results in Table 2.1.

Table 2.1

Parameter	Readings						Calculations		
Symbol	$V_{in}$	$I$	$\varphi_a$	$\varphi_b$	$\varphi_c$	$\varphi_d$	$R_C$	$I_1$	$I_2$
Unit	V	A	V	V	V	V	$\Omega$	A	A
Value									

In the table  $\varphi_i$  is the potential of the given point fixed when traveling through the investigated circuit;  $I$  is the current in unbranched part of a circuit;  $I_1$ ,  $I_2$  are branch currents.

## 2. DC circuit with non-linear elements

Repeat the preparations like those in the previous section. After connecting the circuit (Fig. 2.16) with nonlinear element NE1, take the value of the input voltage from the instructor and switch the voltage source on. Record your values of the circuit parameters in Table 2.2.

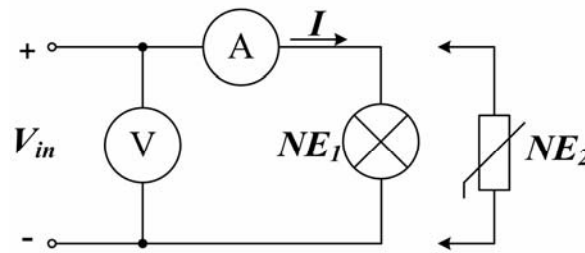


Fig. 2.16

Table 2.2

$V_{in}, V$									
$I_{NE_1}, A$									
$I_{NE_2}, A$									

Repeat the measurements for different values of the input voltage.

Investigate the circuit from the beginning having replaced NE1 by the element NE2.

## 3. Series connection of nonlinear elements

Set up the circuit with series connection of nonlinear elements taken from the previous experiment (Fig. 2.17), and measure quantities specified in Table 2.3 at the given input voltage.

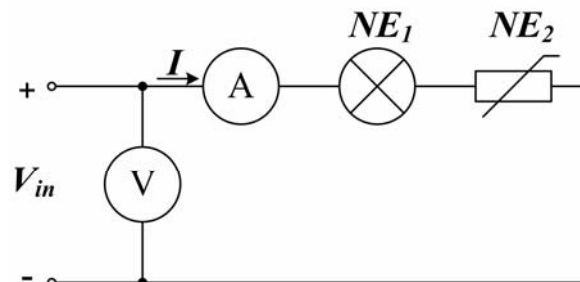


Fig. 2.17

Table 2.3

Parameter	Measured				Calculated		
Symbol	$V_{in}$	$I'$	$V_1$	$V_2$	$I'$	$V'_1$	$V'_2$
Unit	V	A	V	V	A	V	V
Value							

In Table 2.3  $V_1$ ,  $V_2$  are the voltage drops across the elements.

#### 4. Parallel connection of the nonlinear elements

Set up the circuit with parallel connection of the chosen nonlinear elements (Fig. 2.18) and measure quantities specified in Table 2.4, at the given input voltage.

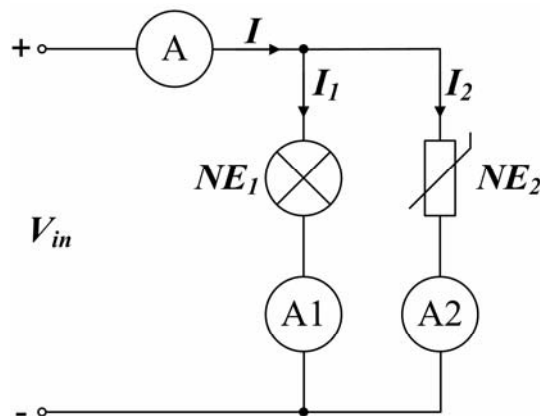


Fig. 2.18

Table 2.4

Parameter	Measured				Calculated		
Symbol	$V_{in}$	$I$	$I_1$	$I_2$	$I'$	$I'_1$	$I'_2$
Unit	V	A	A	A	A	A	A
Value							

#### Summarizing

1. For experiment 2 design the circuit potential diagram using the values of resistances and measurements of Table 2.1.
2. Depict volt-ampere characteristics of two nonlinear elements on the basis of Table 2.2 and find graphically the quantities specified in Tables 2.3 and 2.4.
3. Analyze the results and write brief conclusions.

## Questions to be answered

1. What is an active electric circuit and what elements it consists of?
2. What is an equivalent circuit?
3. What laws of electrical engineering are used in analyzing the electric circuits?
4. How are electric circuits analyzed by the method of converting the delta-connected resistances into equivalent wye-connected and vice-versa?
5. How are electric circuits analyzed by the loop-current method?
6. How are electric circuits analyzed by the node-voltage method?
7. How is a potential diagram constructed?
8. How can the circuit analysis be performed for series- and parallel-connected nonlinear elements?

## Chapter 3

### ALTERNATING CURRENT CIRCUITS

Alternating current (AC) circuits have largely replaced direct current (DC) circuits in commercial power systems for a number of reasons. AC can be transmitted over long distances more readily and more economically than DC, since AC voltages can be increased or decreased by means of transformers.

As more units are operated electrically in airplanes, the power requirements are such that a number of advantages can be realized by using alternating current. Space and weight can be saved, since AC devices, especially motors, are smaller and simpler than DC devices. In most AC motors no brushes are required, and commutation trouble at high altitude is eliminated. Circuit breakers will operate satisfactorily under load at high altitudes in AC system whereas arcing is so excessive in DC systems that circuit breakers must be replaced frequently. Finally, most airplanes using a  $27\text{ V}$  DC system have special equipment, which requires a certain amount of  $400\text{ V}$  AC current.

#### 3.1 Sinusoidal current and its representation

A current (or voltage) is called alternating if it periodically changes its direction and instant value. The time of complete cycle of current variations is usually called the **period**  $T$  of an alternating current. Reversely, the number of the periods per one second is called AC line **natural frequency**  $f = 1/T$ . The unit of frequency is 1/sec; more often it is called the **hertz (Hz)**.

The most important of all periodic curves is the sine waveform (Fig. 3.1), which is the only periodic function having a derivative and an integral like itself.

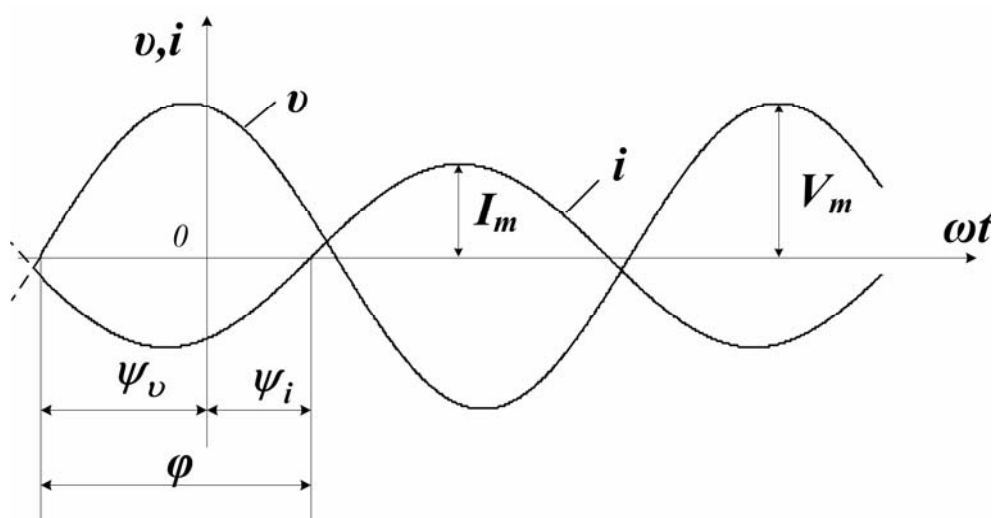


Fig. 3.1 – Graphical representation of sine functions

An EMF produced by commercially available AC generators can be presented in analytical **time-domain** form as sinusoidal time-varying function:

$$e = E_m \cdot \sin(\omega t \pm \psi_e), \quad (3.1)$$

where the **radian** (or, **angular**) frequency  $\omega = 2\pi f$  (radians/s) represents one more relevant parameter of a sinusoidal waveform.

In a closed electric circuit the sinusoidal EMF will excite a sine wave current

$$i(t) = I_m \cdot \sin(\omega t \pm \psi_i), \quad (3.2)$$

and a voltage dropped across the part of the circuit:

$$v(t) = V_m \cdot \sin(\omega t \pm \psi_v). \quad (3.3)$$

Here and routinely in the remainder of the manual we'll use small letters  $v$ ,  $i$ ,  $e$  to denote **instantaneous** (at arbitrary moment  $t$ ) values of different time-varying signals: a voltage, a current, an EMF, etc. The capital letters  $I_m$ ,  $V_m$ ,  $E_m$  denote the **amplitudes**, i.e. the peak values of the signals.

Symbols  $\psi_i, \psi_v, \psi_e$  denote the sine wave **initial phases**, resulting in the values of current (voltage, EMF, etc.) at the moment that was chosen as the time origin ( $\omega t = 0$  reference point is depicted in Fig. 3.1).

In general, the **phase** of a process at any time is defined by the angle ( $\omega t + \psi$ ) of the sine wave and is called the **phase angle** or **instant phase**. The difference between initial phases of two functions, for instance, between  $\psi_v$  and  $\psi_i$  in Fig. 3.1, is called the phase difference or **phase shift**:  $\varphi = \psi_v - \psi_i$ .

### Average and effective values of AC

For quantifying the strength of time-varying electrical signals several suitable measurements are known. The most common types of measurements are the **average** (or **mean**, or **DC**) **value** of a signal waveform, and the **root-mean-square** (or **rms**) **value**.

The average value of current (voltage, EMF, any alternating quantity) corresponds to measuring the mean current or other variable, over a period of time. Formally, for AC current (as a periodic function) it is calculated by integrating the signal wave form over the interval of half a period:

$$I_{mean} = \frac{2}{T} \int_0^{\frac{T}{2}} i(t) dt. \quad (3.4)$$

Substitution of  $i(t) = I_m \sin \omega t$  in (3.4) will give a simple relation between the current amplitude  $I_m$  and mean value  $I_{mean}$ :

$$I_{mean} = \frac{2I_m}{\pi} = 0.637 I_m. \quad (3.5)$$

In alternating current engineering, one often deals with the thermal or mechanical effects of a current. The intensity of heat generation and in many cases the magnitude of mechanical forces in electromechanical devices is proportional to the square of the current. The square root of this quantity defines the **root-mean-square (rms) value** of the current as follows:

$$I = \sqrt{\frac{1}{T} \int_0^T i^2 dt}. \quad (3.6)$$

More often rms value is called the **effective** value. Besides, a capital letter without indexation is commonly used to denote effective value of AC current.

The effective  $I$  value of the alternating current that flows through a resistor  $R$  is equivalent to that value  $I_{\underline{}}$  of the direct current which would cause the liberation of the same quantity of heat by the resistor, i.e.

$$\int_0^T (i^2) R dt = I^2 T \cdot R = I_{\underline{}}^2 RT. \quad (3.7)$$

For a sinusoidal current:

$$I = \sqrt{\frac{1}{T} \int_0^T I_m^2 \sin^2(\omega t) dt} = \frac{I_m}{\sqrt{2}} = 0.707 I_m, \quad (3.8)$$

i.e. the effective value is  $\sqrt{2}$  times less the current peak amplitude.

The effective values of an alternating voltage, EMF, magnetic flux, etc. are derived similarly. These are the most useful measures of an AC voltage. Voltmeters and ammeters, designed for the circuits being operated with the sinusoidal current, are usually graduated to indicate the effective values of voltage or current.

### Vector Diagrams

In various calculations it is often necessary to express some current as the sum of two or more currents (Fig. 3.2).

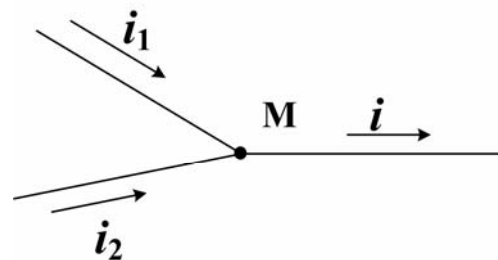


Fig. 3.2 – Sum of two currents



According to KCL, the resulting current  $i$  through the right hand conductor equals  $i_1 + i_2$ . The currents to be added have different amplitude, and besides, may pass through their zeros at different moments, i.e. they may have a phase shift.

We could perform currents adding with the help of mathematical conversions or do that graphically having both the current sketches (oscillograms).

But the same result may be quickly obtained with the help of vector diagrams, when another, **frequency-domain** (or **phasor**) form of sinusoidal signal representation is used. The latter is provided by the use of Euler's identity:

$$e^{j\omega t} = \cos \omega t + j \sin \omega t . \quad (3.9)$$

In this case we use expressions for voltage and current sine waves which look like

$$\begin{aligned} \dot{V} &= V_m e^{j\omega t} , \\ \dot{I} &= I_m e^{j(\omega t + \varphi)} . \end{aligned} \quad (3.10)$$

In Fig. 3.3 a radius vector  $I_m$  being rotated counter clockwise with the angular velocity  $\omega$  is shown. It depicts the amplitude of the alternating current. As a function of  $t$ , the projection of the vector along the  $Im$  axis represents the actual alternating current  $i(t) = I_m \sin(\omega t + \psi_i)$ . The positive initial phase angle  $+\psi_i$  means that at  $t=0$  radius vector  $I_m$  fronts the Re axis. Reversely, it lags behind that when being negative.

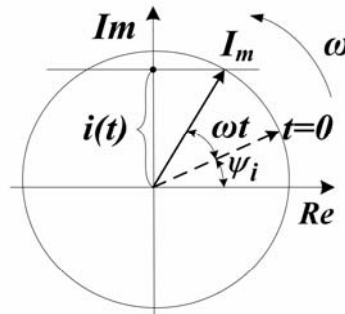


Fig. 3.3 – Radius vector of the current

Suppose we have to add the current  $i_1 = I_{m1} \sin \omega t$  ( $\psi = 0$ ) with the current  $i_2 = I_{m2} \sin(\omega t - 90)$  ( $\psi = -90$ ) which, hence, lags  $i_1$  by  $90^\circ$ . We depict the both radius vectors as shown in Fig. 3.4. After that, in order to find the vector  $I_m$  representing the resultant current, we only need to combine the vectors  $I_{m1}$  and  $I_{m2}$  by the parallelogram rule. That yields:

$$I_m = \sqrt{I_{m1}^2 + I_{m2}^2} , \quad \varphi = \arctg \frac{I_{m2}}{I_{m1}} . \quad (3.11)$$

In equations (3.11) the member  $I_{m2}$  is to be substituted with the minus sign in accordance with the direction of the vector  $I_{m2}$  in the plane of Fig. 3.4.

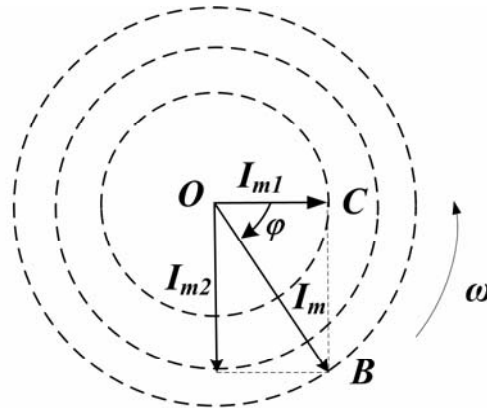


Fig. 3.4 – Vector representation of the resultant current

Since the projection of the rotating vector in time-domain representation is a sine waveform, it follows that the sum of two sinusoidal currents (or voltages) is a sinusoidal waveform too. However, the purpose of depicting vector diagrams is mostly to define not the instantaneous value of the current at any time but the amplitude or, preferably, the effective value, and the phase shift.

In one picture, a vector diagram may represent different AC line parameters, not current or voltage vectors only, to help one to display their interaction in the circuit.

### 3.2 Voltage-Current Relationships for AC Circuit Elements

#### Ideal Circuit Elements

An electrical circuit which provides the flow of electric current is formed by the interconnection of various circuit elements. The basic so-called passive circuit components are a resistor, a capacitor, and an inductor. Fig. 3.5 represents circuit symbols for these elements.

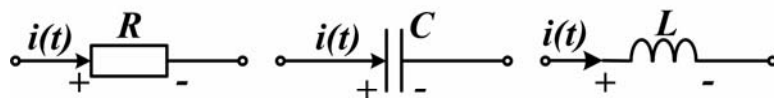


Fig. 3.5 – Symbols of R, L, C elements

Obviously that for a resistor the voltage-current relationship corresponds to Ohm's Law:

$$v_R = Ri \quad \text{or} \quad i = Gv_R, \quad (3.12)$$

where  $R$  and  $G$  are the resistance and the conductance (the inverse of the resistance) of the resistor, correspondingly.

The ideal resistor introduced through Ohm's law is a useful realization of many practical electrical devices. However, in addition to resistance to the flow of electric current, which is purely a dissipative (i.e., an energy-loss) phenomenon, electric devices may also exhibit energy-storage properties. Two important mechanisms for energy storage exist in electric circuits: **capacitance** and **inductance**. Appropriately, two ideal circuit elements will be introduced to represent these properties: the **ideal capacitor** and the **ideal inductor** (see Fig. 3.5).

A physical capacitor is a device that can store energy in the form of a charge separation in an electric field (i.e., by applied voltage), and because of special construction (presence of insulating material between conducting plates) does not allow for the flow of DC current. Thus, a capacitor acts as an open circuit in the presence of DC currents. In a capacitor, the charge separation caused by the polarization of the dielectric is proportional to the external voltage, that is, to the applied electric field:

$$C = \frac{Q}{V}, \quad (3.13)$$

where the parameter  $C$  is called the capacitance of the element. The unit of capacitance is the coulomb/volt and is called the **farad (F)**.

From the equation (3.13) we can realize that if the external voltage applied to the capacitor changes in time, so will the charge that is internally stored by the capacitor. Thus, a time-varying voltage will cause charge to vary in time.

Using the definition of current:

$$i(t) = \frac{dQ}{dt}, \quad (3.14)$$

one can obtain a relationship between the current and voltage in a capacitor:

$$i(t) = C \frac{dv_c(t)}{dt} \quad (3.15)$$

or

$$v_c(t) = \frac{1}{C} \int_{t_0}^t i dt' + V_0. \quad (3.16)$$

The significance of the initial voltage,  $V_0$ , is that at time  $t_0$  some charge is stored in the capacitor, giving rise to a voltage,  $V_c(t_0)$ , according to the relationship  $Q=CV$ .

The third circuit element known as **ideal inductor** is an element that has the ability to store energy in a magnetic field. Inductors are typically made by winding a coil of wire around a core. When a current flows through the coil, a magnetic field is established. In an ideal inductor, the resistance of the wire is zero, so that a constant current through the inductor will flow freely without caus-

ing a voltage drop (the ideal inductor acts as a short circuit in the presence of DC currents).

If a time-varying voltage is established across the inductor a corresponding current will result according the following relationship:

$$v_L(t) = L \frac{di_L(t)}{dt}, \quad (3.17)$$

where  $L$  is called the inductance of the coil and is measured in **henrys** (H).

It is interesting to compare equation (3.17) which defines the behavior of an ideal inductor, with expression relating capacitor current and voltage (3.15). We note that the roles of voltage and current are reversed in the two elements. This duality between inductance and capacitance can be exploited to derive the same basic results for the inductor that we already have for the capacitor simply by replacing the capacitance parameter,  $C$ , with the inductance,  $L$ , and voltage with current.

Thus, the inductor current is found by integrating the voltage across the inductor:

$$i_L(t) = \frac{1}{L} \int_{t_0}^t v_L dt + I_0. \quad (3.18)$$

### Steady-state Response of R, L, C Circuits to Sine Input Voltage

We now analyze the  $i$ - $v$  relationship of the three ideal circuit elements with the help of the phasor (complex) notation. In the context of AC circuits any one of the three ideal circuit elements will be described by a parameter called **impedance**, which may be viewed as a complex resistance and likely depicted in circuit diagrams, e.g. as shown in Fig. 3.6.

Let the source exiting a circuit with the element be defined by  $E(j\omega) = Ee^{j\theta}$  instead of  $v_R(t) = V_{Rm} \sin \omega t$ . This complex voltage appears in the sketch together with the parameter  $Z(j\omega)$ .

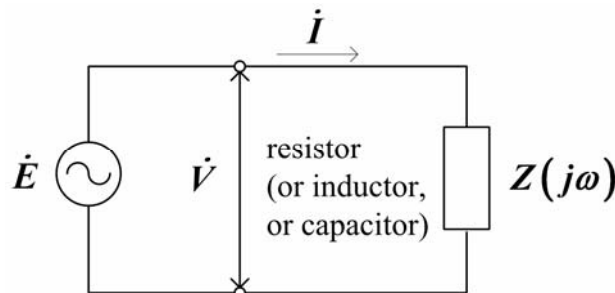


Fig. 3.6 – AC circuit in complex/impedance form

The current  $i(t)$  (or  $I(j\omega)$  for complex representation) is defined by the  $i - v$  relationship for each circuit element.

Let us examine the frequency-dependant properties of the resistor, inductor, and capacitor, first one at a time.

### The resistor

Ohm's law dictates the well-known relationship  $v = iR$ . In the case of sinusoidal sources the current flowing through the resistor may be expressed as

$$i_R(t) = \frac{v_R}{R} = \frac{\dot{V}_{mR}}{R} \sin \omega t = \dot{I}_{mR} \sin \omega t, \quad (3.19)$$

Using complex notations for both the voltage and the current, we obtain

$$\dot{I}_{mR} = \frac{\dot{V}_{mR}}{R}. \quad (3.20)$$

$$Z(j\omega) = V_R(j\omega) / I(j\omega) = R \quad \text{or} \quad \dot{V}_R = R \dot{I}_R. \quad (3.20')$$

Equation (3.20') corresponds to Ohm's law in complex form.

Finally, we can see that the impedance of a resistor is simply its actual resistance  $R$ , which is frequency-independent.

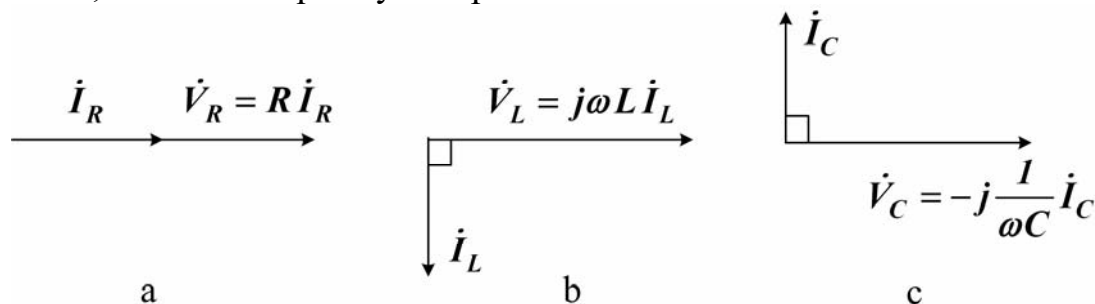


Fig. 3.7 – Vector diagrams: a – resistor; b – inductor; c - capacitor

The vector diagram for the equation (3.20) is depicted in Fig. 3.7,a.

### The inductor

Let the inductor voltage be  $v_L(t) = V_{Lm} \sin \omega t$  (resulted from the source voltage). Then the following expression may be derived for the steady-state inductor current:

$$i_L(t) = \int \frac{1}{L} v_L dt = -\frac{V_m}{\omega L} \cos \omega t = I_m \sin(\omega t - 90^\circ). \quad (3.21)$$

Thus, the current is not just scaled version of the source voltage, as it was for the resistor. The current lags the voltage (shifted in phase, or delayed) by  $90^\circ$

and its magnitude is  $\frac{I}{\omega L}$  times the voltage, i.e. the response of the inductor current depends on the source EMF frequency,  $\omega$ .

In vector form, the relation (3.21) for effective values  $V$  and  $I$  may be rewritten as:

$$\dot{V}_L = j\omega L \dot{I}_L, \quad (3.22)$$

where impedance of the inductor  $Z_L(j\omega)=j\omega L$  now appears to behave like a complex frequency-dependant resistor. The pure imaginary impedance  $jX_L = j\omega L$  is called the **inductive (positive) reactance** and is measured in ohms. Thus, an inductor will impede current flow in proportion to the sinusoidal frequency of the source signal.

The vector diagram for the equation (3.22) is depicted in Fig. 3.7,b.

### The capacitor

Beginning with (3.15), an analogous procedure may be followed to derive the equivalent result for a capacitor:

$$i_c(t) = C \frac{dv_c}{dt} = C \frac{d(V_m \sin \omega t)}{dt} = \omega C V_m \cos \omega t = \omega C V_m \sin(\omega t + 90^\circ), \quad (3.23)$$

so that, in vector form:

$$\dot{V}_C = \frac{1}{j\omega C} \dot{I}_C. \quad (3.24)$$

The impedance of the ideal capacitor,  $Z_C(j\omega)$ , is therefore defined as follows:

$$Z_C(j\omega) = -\left(\frac{j}{\omega C}\right), \quad (3.25)$$

where quantity  $jX_c = j(-1/\omega C)$  is the **capacitive (negative) reactance**. Thus, the impedance (reactance) of a capacitor is also a frequency-dependant quantity, with the impedance of the capacitor varying as an inverse function of frequency.

The vector diagram Fig. 3.7,c depicts the fact that, for a capacitor, the current leads the voltage by  $90^\circ$ .

## 3.3 Series AC Circuits

### Series R–L circuit

We consider AC networks with more than one circuit element.

For instance, the series **R–L** circuit in Fig. 3.8,a can model a practical inductor, where series resistor represents the resistance of the coil wire.

The each voltage and the current are presented in the figure in a complex form. Following complex form of KVL, we have:

$$\dot{V} = \dot{V}_R + \dot{V}_L = R\dot{I} + j\omega L\dot{I} = (R + j\omega L)\dot{I} = \underline{Z}\dot{I},$$

where

$$\underline{Z} = R + j\omega L = R + jX_L \quad (3.26)$$

is the **equivalent impedance** of the circuit compiled of the AC resistance  $R$  and inductive reactance  $X_L$ . Impedance  $\underline{Z}$  is a complex number with the modulus and the phase angle (positive)

$$Z = \sqrt{R^2 + (\omega L)^2}, \quad (3.27)$$

$$\varphi = \arg \underline{Z} = \arctg \frac{X_L}{R}. \quad (3.28)$$

The voltage–current relationship depicted in Fig. 3.8,b, is the vector diagram for the series  $R,L$  circuit.

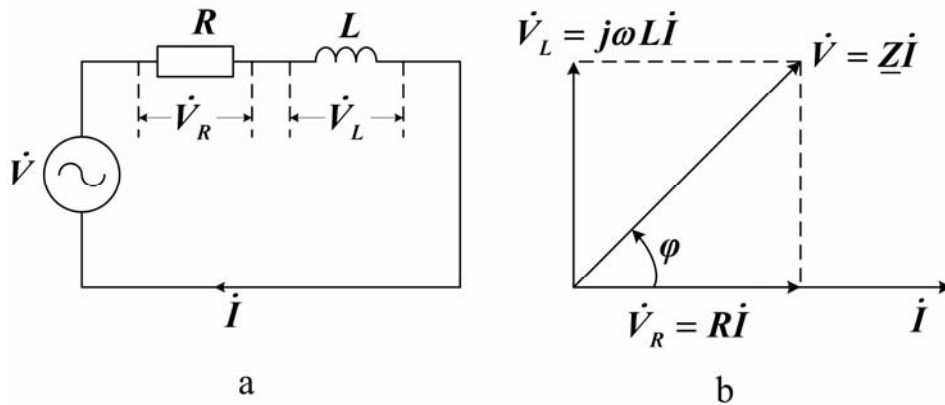


Fig. 3.8 – R and L in series

In constructing this diagram we have chosen  $\dot{I}$  (which is common to the entire circuit) as the **reference vector**. Note that  $\dot{I}$  lags  $\dot{V}$  by the angle  $\varphi$ .

Now, for reason that will become apparent later, we define **power factor** as

$$\cos \varphi = \frac{R}{Z} \quad (3.29)$$

and state that series  $R$ – $L$  circuits have the lagging power factor.

### Series $R$ – $C$ circuit

Now considering the series  $R$ – $C$  circuit of Fig. 3.9,a we'll derive

$$\dot{V} = \dot{V}_R + \dot{V}_C = R\dot{I} - j\frac{1}{\omega C}\dot{I} = (R - j\frac{1}{\omega C})\dot{I} = \underline{Z}\dot{I},$$

where  $\underline{Z}$  is the circuit equivalent impedance:

$$\underline{Z} = R - j\frac{1}{\omega C} = R - jX_C. \quad (3.30)$$

The vector diagram for a series  $R-C$  circuit is shown in Fig 3.9,b. This circuit

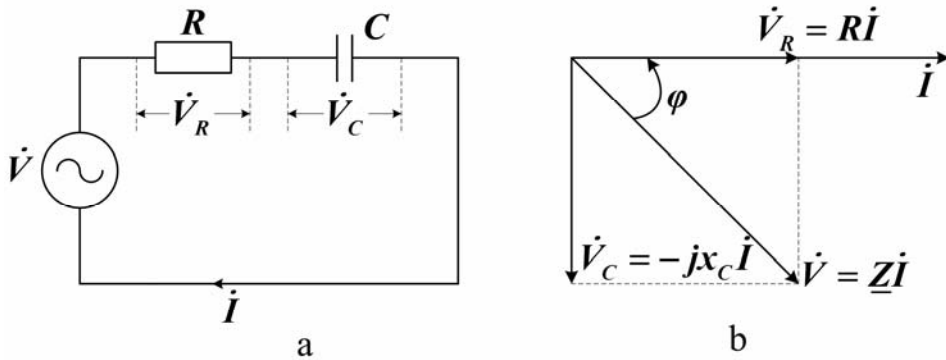


Fig. 3.9 – R and C in series

collects the current  $\dot{I}$  that leads the voltage  $\dot{V}$  because of the negative phase angle  $\varphi$  of the equivalent impedance. So is the power factor  $\cos \varphi$  of the circuit (leading).

#### Series R-L-C circuit and voltage resonance

Finally, for the series  $RLC$  circuit shown in Fig. 3.10,a, we obtain the equivalent impedance

$$\underline{Z} = R + j(\omega L - \frac{1}{\omega C}) = R + j(X_L - X_C). \quad (3.31)$$

The instance vector diagram, Fig. 3.10,b shows the lagging power factor ( $X_L > X_C$ ).

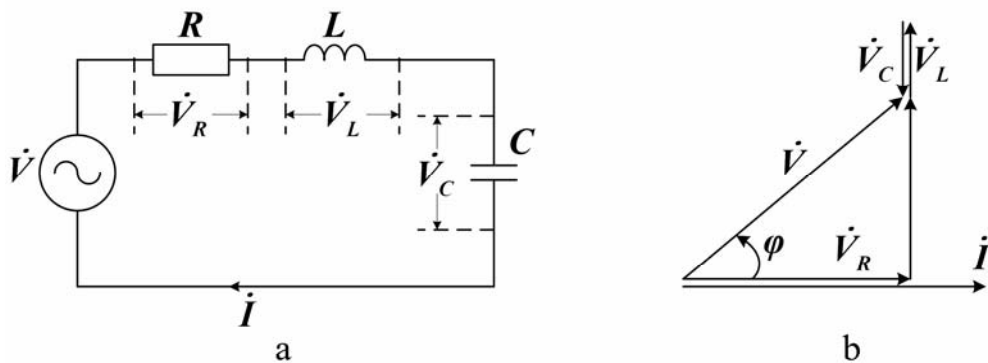


Fig. 3.10 – R, L and C in series



You may have a chance of getting a leading power factor ( $X_L < X_C$ ), and the **unity** factor ( $X_L = X_C$ ). For the series **RLC** circuit represented in Fig. 3.10,a the vector form of Ohm's law is as follows:

$$\dot{I} = \frac{\dot{V}}{R + j(\omega L - \frac{1}{\omega C})}. \quad (3.32)$$

If we have zero equivalent reactance

$$X = \omega L - \frac{1}{\omega C} = 0, \quad (3.33)$$

then:

1) the impedance will obtain the minimum value  $Z = R$ , and the current  $I$  will reach the peak value for a given voltage and be limited by the resistance  $R$  of the circuit only:

$$I_0 = \frac{V}{R};$$

2) the current is in phase with the voltage (since  $X_L = X_C$ , then  $V_L = V_C$  and  $\varphi = 0$ ).

Vector diagrams for this case are shown in Fig. 3.11,a and b.

The phenomenon for a series **RLC** circuit corresponding to (3.33) is called **voltage resonance** or **series resonance**.

The condition (3.33) shows that the resonance can be obtained either by varying the frequency, or by changing  $L$  or  $C$ . The angular frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

derived from the resonance condition is called **resonant frequency**.

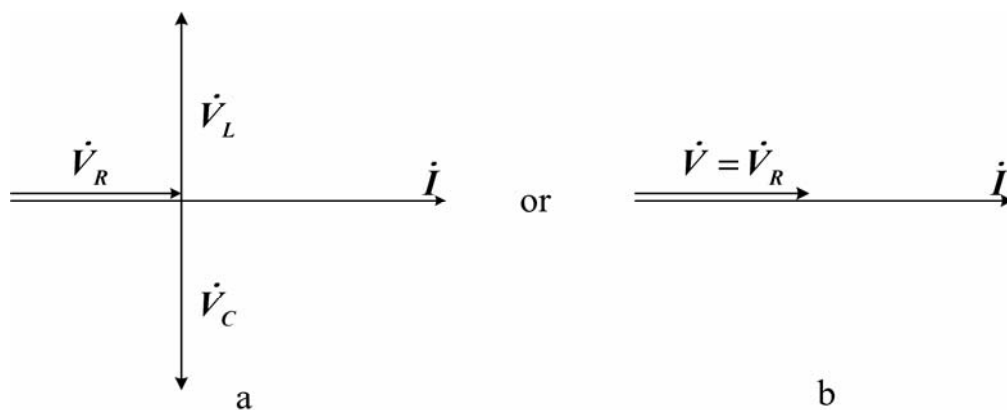


Fig. 3.11 – Vector diagram for a voltage resonance

Voltage resonance may prove to be dangerous when running the electrical installation. This would result from the big values of  $V_L$  and  $V_C$  which might become many times greater an applied voltage from the generator.

Nevertheless, the voltage resonance is very useful in various fields of electrical engineering, electronics, measuring technologies.

### 3.4 Terms and Concepts

**Alternating current** is the current changing its direction and magnitude.

**Angular velocity** is time rate of changes of an angle rotated around an axis in degrees per second or degrees per minute.

**Capacitance(C)** is the ability of an element to accumulate electric energy.

**Capacitor** is a two-terminal device consisting of conducting plates separated by a dielectric and used to introduce capacitance into a circuit.

**Coil** is one or more turns of a conductor designed for use in a circuit to produce inductance or an electromagnetic field.

**Effective value** is a term used to indicate the actual working value of an alternating current based on its heating effect. Also called the root-mean-square (rms) value.

**Effective value of a current** is the steady current that is as effective in transferring power as the average power of the varying current.

**Energy storage** is work performed on moving a charge, resulting in energy storage in a capacitor; or work performed to establish a magnetic field resulting in energy storage in an inductor.

**Equivalent circuit** is arrangement of ideal circuit elements that is equivalent to a more complex arrangement of elements. A circuit equivalent to another circuit exhibits identical characteristics (behavior) at identical terminals.

**Henry (H)** is the unit of inductance. It is the amount of inductance in a coil that will induce an emf of 1 V in the coil when the current flow is changing at the rate of 1 A/s.

**Impedance (Z)** is the combined effect of resistance, capacitive reactance, and inductive reactance in an AC circuit; ratio of the phasor voltage  $V$  to the phasor current  $I$  for a circuit element or set of elements so that  $Z=V/I$ .  $Z$  is measured in ohms.

**Inductance (L)** is property of an electric device by virtue of which a time varying current produces a voltage across the device, or ability of a coil to accumulate the magnetic energy.

**Inductive reactance ( $X_L$ )** is the effect of inductance in an AC circuit. The equation for inductive reactance is  $X_L = 2\pi fL$ .  $X_L$  is measured in ohms.

**Inductor** is two-terminal element consisting of a winding of  $N$  turns for introducing inductance into an electric circuit (an inductance coil in other words).

**Passive element** is element that absorbs energy. The energy delivered to it is always nonnegative (zero or positive).

**Phase shift** is phase angle  $\phi$  associated with a variable  $x$  so that  $x = x_m \sin(\omega t - \phi)$ , or phase angle between a voltage and a current.

**Phasor diagram** is relationship of phasors on a plane.

**Power factor** is equal to  $\cos\theta$ , where  $\theta$  is the phase angle between the sinusoidal steady-state voltage and current; ratio of average power to apparent power.

**Reactance** is imaginary part of impedance, denoted as  $X$ .

**Resonance** is a condition in LC circuit in which capacitive reactance and inductive reactance are equal, also condition in a circuit, occurring at the resonant frequency, when  $H(j\omega)$  becomes a real number (nonreactive).

**Tuning** is the process of adjusting circuits to resonance at a particular frequency.

### 3.5 Review Questions

1. What parameters of a sinusoidal waveform do you know?
2. Define the instantaneous, maximum, average and effective values of AC current.
3. What is impedance?
4. What can be measured by the instrument, impedance or impedance absolute value? What is the difference between them mathematically?
5. Explain the phenomenon of voltage resonance.
6. What electric circuit collects the current that leads the applied voltage?
7. Write down the relations for computing capacitive and inductive reactance.
8. Sketch a vector diagram for the series *RLC* circuit. What is it used for?

### 3.6 Problems

#### Examples

**Problem 1.** For the series *RLC* circuit (see Fig. 3.10) the known quantities are as follows:  $V = 127\text{ V}$ ;  $f = 50\text{ Hz}$ ;  $R = 6\ \Omega$ ;  $L = 25.5\text{ mH}$ ;  $C = 1590\ \mu\text{F}$ .

- 1) Find:  $I$ ,  $V_R$ ,  $V_L$ ,  $V_C$ .
- 2) Draw: a vector diagram corresponding to the circuit.

#### Solution

1) First, we compute inductive and capacitive reactances and total impedance:

$$X_L = \omega L = 2\pi fL = 314 \cdot 25.5 \cdot 10^{-3} = 8 \Omega;$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{314 \cdot 1590 \cdot 10^{-6}} = 2 \Omega;$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 8.5 \Omega;$$

$$Z_{coil} = \sqrt{R^2 + X_L^2} = 10 \Omega,$$

Then, we can calculate the current effective value:

$$I = \frac{V}{Z} = \frac{127}{8.5} = 14.9 A,$$

and voltage drops across each element:

$$V_R = IR = 14.9 \cdot 6 = 89.4 V;$$

$$V_C = IX_C = 14.9 \cdot 2 = 29.8 V;$$

$$V_L = IX_L = 14.9 \cdot 8 = 119.7 V;$$

$$V_{coil} = IZ_{coil} = 14.9 \cdot 10 = 149 V.$$

2) We must follow a series of steps to construct the diagram (Fig. 3.12):

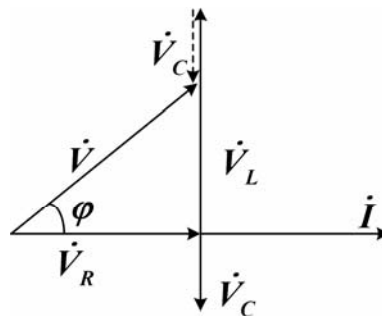


Fig. 3.12

Step 1. Lay out the  $I$  vector which is the same for all elements of the circuit and therefore may be chosen as a reference for drawing each voltage drop. For convenience, we chose the horizontal direction of the reference vector.

Step 2. To draw voltage vector diagram we start with the  $V_R$ , depicting it along the  $I$  vector with the magnitude have been computed (89.4 V). Note that a scale for the voltage is somewhat different to that of the current.

Step 3. At the right angle (upward because of inductive reactance), draw the vector  $\dot{V}_L$  of 119.7 V length starting at the end of the vector  $V_R$ . Similarly, you

could draw voltage vector  $\dot{V}_C$  down from the end of  $V_R$  (capacitive reactance results in  $V_C$  leads  $I$ ).

Step 4. To complete vector addition according to equation  $\dot{V} = \dot{V}_R + \dot{V}_C + \dot{V}_L$  we transport the origin of the vector  $\dot{V}_C$  at the end of  $\dot{V}_L$  vector. The length of  $\dot{V}_C$  corresponds to the magnitude 29.8 V.

Step 5. Resulting vector drawn from the origin of  $V_R$  to the end point of  $\dot{V}_C$  will represent the input voltage  $\dot{V}$ . It is seen that  $\dot{V}$  leads  $\dot{I}$  by the angle  $\varphi$ , i.e. the circuit exposes inductive reactance.

**Problem 2.** Find the equivalent impedance of the circuit (Fig. 3.13) if known quantities equal:  $R_1 = 100\Omega$ ;  $R_2 = 50\Omega$ ;  $L = 10\text{mH}$ ;  $C = 10\mu\text{F}$ ;  $\omega = 10^4 \text{ rad/s}$ .

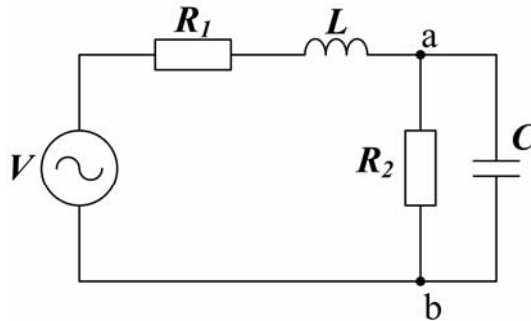


Fig. 3.13

Solution

First, we determine the impedance of parallel  $R_2 - C$  circuit:

$$\begin{aligned} \underline{Z}_{ab} &= \frac{R_2 \cdot \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{1 + j\omega C R_2} = \frac{50}{1 + j \cdot 10^4 \cdot 10 \cdot 10^{-6} \cdot 50} = \frac{50}{1 + j \cdot 5} = \\ &= 1.92 - j \cdot 9.62 = 9.81 \cdot e^{-j \cdot 1.373} (\Omega). \end{aligned}$$

Next we compute the equivalent impedance  $\underline{Z}_{eq}$ :

$$\begin{aligned} \underline{Z}_{eq} &= R_1 + j \cdot L + \underline{Z}_{ab} = 100 + j \cdot 10^4 \cdot 10^{-2} + 1.92 - j \cdot 9.62 = 101.92 + j \cdot 90.38 = \\ &= 136.2 \cdot e^{j \cdot 0.72} (\Omega). \end{aligned}$$

In the end we can see that at the frequency used in the example the circuit has an inductive impedance since the reactance is positive (or alternatively, the phase angle is positive).

### Supplement Problems

**Problem 3.** A circuit carries a current  $\dot{I} = 10 \cdot e^{j0} A$ . The circuit contains three series elements with equivalent parameters  $R = 8 \Omega$ ;  $X_C = 6 \Omega$ ,  $X_L = 12 \Omega$ .

Determine:

- voltages across the three elements;
- voltage across the entire circuit.

**Problem 4.** For the series **RLC** circuit like that in Fig. 3.10, where AC power supply voltage  $V = 100V$ , supply frequency  $f = 50 Hz$ , and circuit parameters  $L = 0.2 H$ ,  $C = 10 \mu F$ ,  $R = 100 \Omega$ , determine the total impedance  $Z$ , the current flow  $I$ , and the phase angle  $\varphi$ .

**Problem 5.** Find  $v_{out}(t)$  for the circuit shown in Fig. 3.14, if known quantities  $I = 10e^{j\pi/4} mA$ ;  $X_L = 1k\Omega$ ;  $X_C = 10k\Omega$ .

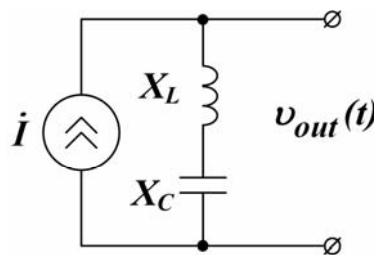


Fig. 3.14

**Problem 6.** Find voltage across capacitance  $C$  of the series **RLC** circuit at resonance, if known quantities are as follows:  $V = 20V$ ;  $R = 10\Omega$ ;  $X_L = 20\Omega$ .

### 3.7 Practice

#### “Investigation of Single-phase AC Circuits with Series Resistor, Inductor and Capacitor”

**Objective:** to gain practical skills in performing AC circuit analysis and carrying out the bench-top investigation of the circuits that contain connected in series energy-loss and energy-storage elements, and are energized by the single-phase sinusoidal power source.

#### Work task

- Studying practical methods of an AC circuit analysis and calculation.

2. Computing the simplest series RLC circuits with the help of complex and vector form of circuit variables representation.

3. Practical investigation of the circuits containing series resistor, inductor, and capacitor when supplied by a single-phase sinusoidal current.

4. Calculation of unknown electric quantities on measured parameters of the circuit.

### Theory and preparation

To provide for analysis of a single-phase sinusoidal AC circuit with real energy dissipating (energy-loss) and energy-storage elements one can use an equivalent circuit containing combination of ideal circuit elements (R,L,C), properly connected. Electrical processes happened to this circuit must follow phenomena in the real circuit as much as possible.

For instance, in Fig. 3.15 the elements of a low-pass filter are shown. These elements are a physical inductor and a resistor connected in series.

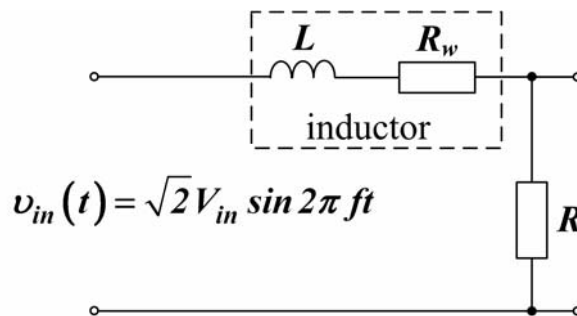


Fig. 3.15 – Equivalent circuit of a low-pass filter

For analytic investigation of the filter and receiving the result close to the physical reality, we have, in turn, to represent the inductor by the series connection of the coil inductance  $L$  and its wire resistance  $R_w$ . The equivalent circuit is then analyzed by one of the known methods (e.g. presented above in this chapter).

To carry out practical experiments with circuits of the objectives you must study the circuits operation carefully and, beforehand, draw the three following circuit diagrams: for series connection of the inductor and resistor, series connection of a capacitor and the resistor, and series connection of the resistor, the inductor, and the capacitor. Besides, you must prepare the three blank tables for the following recording of experiment data, in accordance with the forms shown below for, study necessary measuring instruments, be ready to answer test questions.

*NB:* a variometer is used in the work as an inductive element; the device consists of two interconnected inductor coils which may be located at variable angle to each other for changing the inductance; the natural frequency of a power

source is  $50\text{ Hz}$ ; at the same time it can be measured during the investigation if necessary.

### Procedure

#### 1. Investigation of connected in series resistor and inductor

From your instructor, take a recommended value of the input AC voltage and prepare a measuring instrument for each branch (i.e., select the device, study specifications, set the range, and evaluate a price of division of the meter).

Connect the circuit according to the diagram of Fig. 3.16. Set the variometer coils at the angle  $90^\circ$ .

Switch the AC power supply on (after instructor's admission), and regulate the voltage  $V_{in}$  across the input of the circuit.

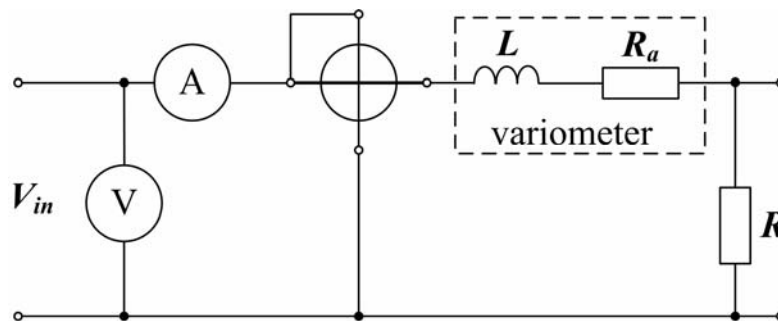


Fig. 3.16

Measure each of the necessary circuit parameters and record your results in Table 3.1, where  $V_R, V_{var}$  are the voltage drops across the resistor and the variometer;  $R_a$  – the variometer ohmic resistance;  $Z_{var}$  – the magnitude of the variometer impedance;  $X_L$  – the variometer reactance;  $\cos \varphi$  – the circuit power factor.

Table 3.1

Parameter	Measured					Calculated						
	$V_{in}$	$I$	$V_R$	$V_{var}$	$P$	$R$	$Z_{var}$	$X_L$	$V_a$	$V_L$	$f$	$\cos \varphi$
Unit	$V$	$A$	$V$	$V$	$W$	$\Omega$	$\Omega$	$\Omega$	$V$	$V$	$Hz$	-
Value												

#### 2. Investigation of connected in series a resistor and a capacitor

Repeat the procedure of previous section for the circuit shown in Fig 3.17.



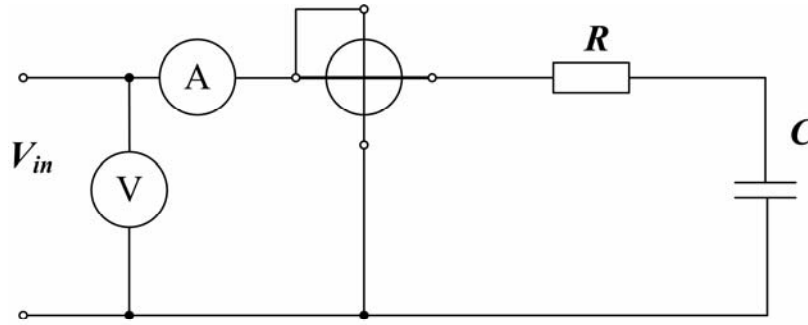


Fig. 3.17

Measure each of the necessary circuit parameters and record your results in Table 3.2, where  $V_{cap}$  is the voltage drop across the capacitor;  $X_C$  – the capacitor

Table 3.2

Parameter	Measured					Calculated				
Symbol	$V_{in}$	$I$	$V_R$	$V_{cap}$	$P$	$X_C$	$C$	$R$	$S$	$\cos \varphi$
Unit	$V$	$A$	$V$	$V$	$W$	$\Omega$	$F$	$\Omega$	$VA$	-
Value										

reactance;  $S$  – the apparent power;  $\cos \varphi$  – the circuit power factor.

### 3. Investigation of R–L–C series circuit. Tuning the circuit in resonance

Repeat the procedure of preceding sections for the circuit shown in Fig. 3.18.

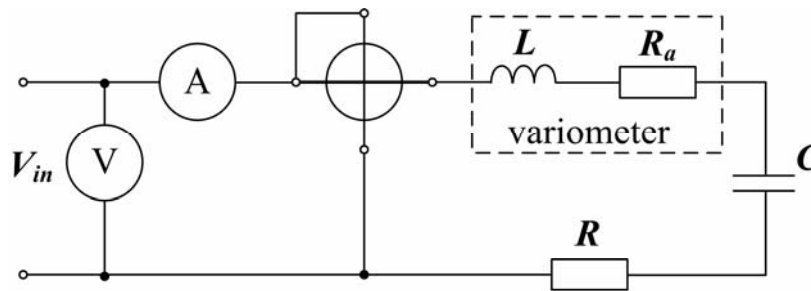


Fig. 3.18

Measure each of the necessary circuit parameters and record your results in Table 3.3 where, in addition to the parameters that have already been introduced,  $\alpha$  is the angle between variometer coils.

The circuit is tuned in resonance by changing variometer inductance, that is, by varying the parameter  $\alpha$  (Table 3.3).

Table 3.3

Readings						Calculations						
$\alpha$	$I$	$V_R$	$V_{var}$	$V_{cap}$	$P$	$R$	$R_a$	$Z_{var}$	$X_L$	$V_L$	$\cos \varphi$	$L$
degrees	$A$	$V$	$V$	$V$	$W$	$\Omega$	$\Omega$	$\Omega$	$\Omega$	$V$	-	$H$
0												
30												
60												
90												
120												
150												
180												

*NB:* try to investigate a fine resonance tuning by the accurate variation of  $L$ .

### Summarizing

Using experimental data (see the Tables 3.1 to 3.3), calculate the unknown parameters of all three AC electric circuits.

1. Using the calculated values (see Table 3.3) construct dependence curves of  $I, V_C, V_{LK}$  and  $\cos \varphi$  as  $f(L)$  in one coordinate system.

2. Construct the equivalent scheme of replacement of the electrical circuits during investigation of voltage resonance for  $\alpha = 0^\circ, 90^\circ$  and  $180^\circ$ , and construct vector diagrams for these cases.

3. Make conclusions according to the results of the completed work.

### Questions to be answered

1. What features affect a phase shift between the voltage and the current in AC circuit?

2. What is an inductive and capacitive reactance?

3. What does resonant frequency depend on?

4. What is the effective value of a current? R.m.s value? Mean value?

5. How can one compute the effective value of the AC circuit current if parameters  $R, L, C$  of the circuit as well as parameters of supply voltage are known?

6. What parameters does a circuit impedance depend on?

7. In what circuit and at what condition will a voltage resonance happen?

8. What features of a voltage resonance do you know?

9. In what way can energy-storage parameters of the inductor and the capacitor be measured?

## Chapter 4

### ALTERNATING-CURRENT NETWORKS

#### 4.1 Powers in AC Circuit

The objective is to introduce the notion of **AC power**, i.e., power absorbed by both resistive and complex loads.

Let  $v(t) = V_m \sin \omega t$  and  $i(t) = I_m \sin(\omega t - \varphi)$  be a steady-state voltage and current delivering by AC power source to an arbitrary load  $\underline{Z} = \mathbf{R} + j\mathbf{X}$ . From here on, let us assume that the reference phase angle of the source voltage is zero, so that a phase shift between the voltage and the current caused by the load impedance  $\underline{Z}$  and resulted in the current waveform  $i(t)$  equals  $\varphi$ .

The instantaneous power  $p$  dissipated by a circuit element is given by the product of the instantaneous voltage and current:

$$p(t) = v(t) \cdot i(t) = V_m I_m \sin(\omega t) \sin(\omega t - \varphi). \quad (4.1)$$

It is customary in AC power analysis to employ the effective (rms) values of the voltage and current:  $V = V_m / \sqrt{2}$ ,  $I = I_m / \sqrt{2}$ . The detail solution resulted from (4.1) may reveal the fact that the instantaneous power is equal to the sum of an average component  $VI \cos \varphi$  and a sinusoidal component  $VI \sin(2\omega t - \varphi)$  oscillating at a frequency doubled in comparison with the original source frequency.

The notion of **average power**  $P$  corresponding to the introduced voltage and current waveforms can be obtained by integrating the instantaneous power over the period of the sinusoidal signal:

$$P = \frac{1}{T} \int_0^T p(t) dt = \frac{1}{2} V_m I_m \cos \varphi = VI \cos \varphi. \quad (4.2)$$

Note that the average power is always positive, even though the instantaneous power (4.1) can be negative for brief periods of time.

The phase angle of the load is of very importance in the absorption of power by the load impedance  $\underline{Z} = \mathbf{R} + j\mathbf{X}$ . As illustrated in equation 4.2 the average power dissipated by an AC load is dependant on the cosine of the angle of the impedance. That's why the term  $\cos \varphi = \frac{\mathbf{R}}{\mathbf{Z}}$  that was introduced in the preceding

chapter is referred to as the **power factor**. The power factor is equal to zero for a purely inductive or capacitive load and equals one (i.e., maximum value) for a pure resistance load. Thus, the main definition of the power factor is given by

$$\cos \varphi = \frac{P}{VI}. \quad (4.3)$$

If one studies expression (4.1) for the instantaneous power in detail, it will be found that the power consists of three components: the mentioned average power represented in a form  $P = I^2R$ , a time-varying component  $P\cos(2\omega t)$  with zero average value (power fluctuations in the resistive component of the load  $R$ ); and a time-varying component  $I^2X\sin(2\omega t) = Q\sin(2\omega t)$  with zero average value (due to the power fluctuation in the reactive component  $X$  of the load). The term  $Q$  is called the *reactive power* in contrary to the active power  $P$ . Due to the fact that  $P$  corresponds to the power absorbed by the load resistive part,  $R = \text{Re}Z$ , the active power  $P$  is also called the *real*, or *true*, power.  $Q$  power is associated with the load reactance  $X = \text{Im}Z$  and therefore represents an exchange of energy between the source and the reactive (energy-storage) part of the load.

The units of  $P$  and  $Q$  are watts (W) and volt-amperes reactive (VAR), respectively.

One more idea related to the AC power is a fictitious quantity called the **complex power**,  $\tilde{S}$ :

$$\tilde{S} = \dot{V}I^* , \quad (4.4)$$

where the asterisk denotes the complex conjugate. You may easily verify that

$$\tilde{S} = VI\cos\varphi + jVI\sin\varphi , \text{ or } \tilde{S} = P + jQ. \quad (4.5)$$

The complex power  $\tilde{S}$  may be interpreted graphically as a vector in the complex plane, as shown in Fig. 4.1.

The magnitude of  $\tilde{S}$ , measured in volt-amperes (VA), is called **apparent power** owing to one's capability to compute this quantity having measurements of the rms values of the load voltage and current directly, without regard for the difference in phase between them.

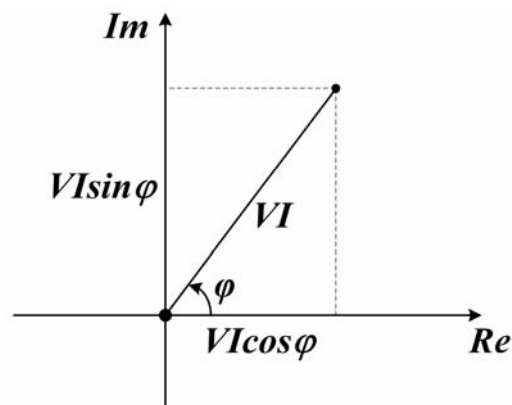


Fig. 4.1 – Powers in AC circuits

Note again that whereas active power is never negative, reactive power following the circuit reactance can have any sign. For instance, in Fig. 4.2 a positive, i.e. inductive reactive power  $Q$  is given.

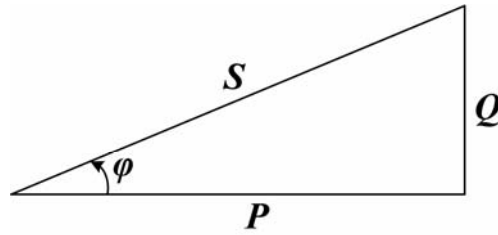


Fig. 4.2 – Power triangle

The complex power and graphical AC power representation shown in Fig. 4.2 simplify calculation of AC load power considerably.

#### 4.2 Parallel AC Circuits. Circuit Admittance

When the current and the voltage in any series part of the AC circuit have a difference in phase, then we consider the voltage across this section consisting of two components: active and reactive. At the same time, in network analysis the current occurs to be splinted into two components, the active component  $I_a$  that is in phase with the voltage, and the reactive component  $I_r$ , that leads or lags the voltage by  $90^\circ$ .

For instance, for the circuit containing a resistance  $R$  and an inductance  $L$  connected in parallel and excited by a source voltage  $V$  the vector diagram can be depicted as it's shown in Fig. 4.3.

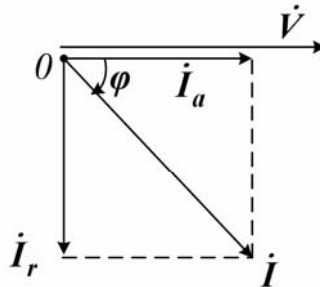


Fig. 4.3 – Vector diagram for R-L circuit

From the vector diagram, it follows that

$$I = \sqrt{I_a^2 + I_r^2} ; \varphi = \arctg(I_r / I_a) ;$$

$$I_a = I \cos\varphi ; I_r = I \sin\varphi.$$

In accordance with the conception of Ohm's law we may write:

$$I = \frac{V}{Z} = Y \cdot V ,$$

where the inverse of impedance  $Y = \frac{1}{Z}$  is called the **admittance**.

Now for the current flow components we have:  $I_a = VY \cos \varphi$  and  $I_r = VY \sin \varphi$ . If we define the **AC conductance**  $G = Y \cos \varphi$  and the **susceptance**  $B = Y \sin \varphi$  we can simplify the component denotations:

$$I_a = GV; I_r = BV. \quad (4.6)$$

Further, if we divide all the sides of the right triangle in Fig. 4.3 by  $V$ , we'll obtain the so-called **admittance triangle** shown in Fig. 4.4. Conductance  $G$  and susceptance  $B$  will be the triangle legs and the hypotenuse will represent the admittance  $Y$ . Hence, the following identities are true:

$$Y = \sqrt{G^2 + B^2}; \quad \operatorname{tg} \varphi = \frac{B}{G}.$$

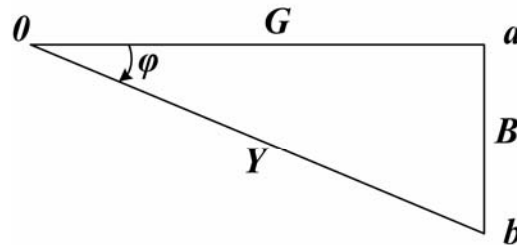


Fig. 4.4 – Admittance triangle

It is easy to derive relationships between the conductance with the susceptance, on one hand, and the resistance with the reactance, on the other:

$$G = \frac{R}{Z^2} \quad \text{and} \quad B = \frac{X}{Z^2}, \quad (4.7)$$

$$R = \frac{G}{Y^2} \quad \text{and} \quad X = \frac{B}{Y^2}. \quad (4.8)$$

In a particular case of pure resistive circuit ( $X = 0$ ) we have  $B = 0$ , and  $Y = G = \frac{1}{R}$ .

Let the **parallel R-L circuit** in Fig. 4.5 be presented in a complex form.

For this circuit  $\dot{V}_R = \dot{V}_L = \dot{V}$  and according to the first Kirchhoff's law

$$\dot{I} = \dot{I}_R + \dot{I}_L = \frac{\dot{V}}{R} + \frac{\dot{V}}{j\omega L} = (G - jB) \cdot \dot{V} = \underline{Y} \cdot \dot{V}, \quad (4.9)$$

where  $\underline{Y} = G - jB_L$  is a complex admittance,  $G = \frac{1}{R}$  is a conductance,  $B_L = \frac{1}{X_L}$  is an inductive susceptance.

Note that inductive susceptance, when related to the conductance, is always negative because of the lagging current.

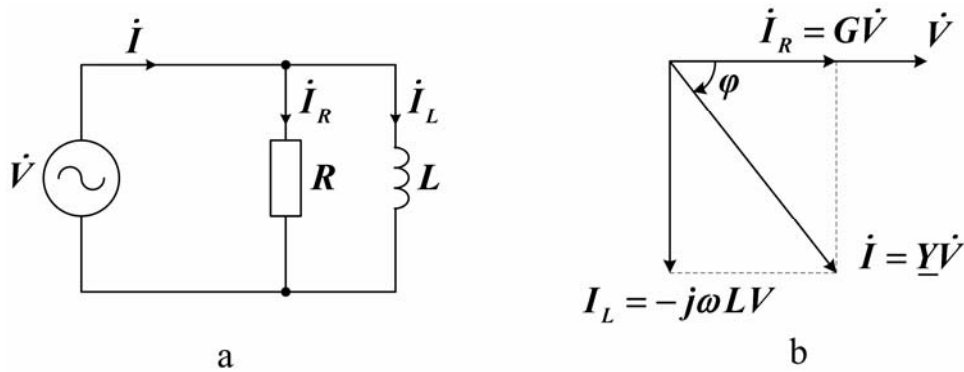


Fig. 4.5 – Parallel RL circuit

By the analogy, for a **parallel R-C circuit** we can get  $\underline{Y} = \frac{\dot{I}}{\dot{V}} = G + jB_C$ , where  $B_C = \omega C = \frac{1}{X_C}$  is a positive **capacitive susceptance**.

Finally, for a generalized **parallel R-L-C circuit** (Fig. 4.6,a) we'll have the current equation  $\dot{I} = \dot{I}_R + \dot{I}_C + \dot{I}_L$  that results in the complex admittance

$$\underline{Y} = \frac{\dot{I}}{\dot{V}} = G + j\left(\omega C - \frac{1}{\omega L}\right) = G + j(B_C - B_L). \quad (4.10)$$

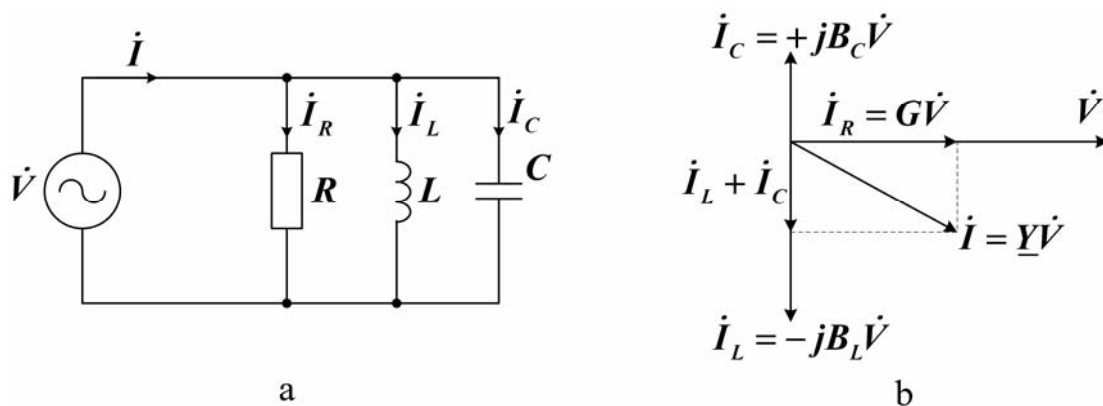


Fig. 4.6 – Parallel RLC circuit

Fig. 4.6,b depicts a vector diagram for the case when  $B_C < B_L$  (lagging power factor). Notice, that when drawing vector diagrams with regard to parallel circuits, we establish  $\dot{V}$  as a reference vector, because it is the same for each element of the circuit.

### 4.3. Current Resonance

If two complementary energy-storage elements of AC circuit (inductor and capacitor) are both connected in parallel with respect to the exciting source, the circuit may meet a **current resonance**, when reactive branch currents exceed a total current drawn from the source considerably.

The circuit diagram of Fig. 4.6,a depicts a practical example of the network we want to analyze now. In the picture, the resistance  $R$  represents possible equivalent losses of the practical inductor and capacitor, while the inductance  $L$  and capacitance  $C$  denote pure energy-storage properties of the reactive elements.

Ohm's law for this parallel circuit is expressed by the following relation:

$$I = V\sqrt{G^2 + (B_C - B_L)^2} . \quad (4.11)$$

The general **condition of current resonance** will be satisfied if the same values of the capacitive and inductive susceptance,  $B_C = B_L$ , occur. This yields the minimum circuit current in accordance with (4.11). The equal values of the opposite circuit susceptances mean

$$\omega C = \frac{1}{\omega L} ,$$

whence we define the **resonant frequency**  $\omega_0 = \frac{1}{\sqrt{LC}}$ ; if parameters  $L$  and  $C$  of the circuit are given the voltage source must be set at this value of the circuit excitation frequency for the resonance to take place.

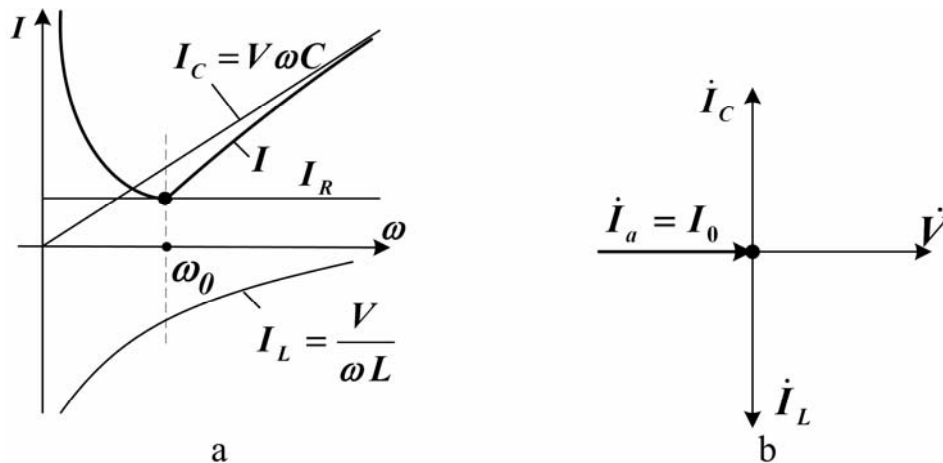


Fig. 4.7 – Currents of parallel RLC circuit.  
Current resonance

Another condition for the resonance to occur is when the source's excitation is fixed. In this case, to tune the resonance we need to vary the capacitance, or the



inductance, or both circuit parameters simultaneously.

Fig. 4.7,a illustrates frequency-dependant curves (frequency responses) demonstrating three branch currents of the parallel  $L$ - $C$ - $R$  circuit as well as the current  $I$  of the entire circuit when frequency  $\omega$  of sinusoidal excitation is being changed.

Fig. 4.7,b explains the current resonance with the help of the vector diagram. At resonance, since the capacitive current equals the inductive one in magnitude but is opposite in direction, the resulting source current will become purely resistive (i.e., co-phased with the applied voltage), thus reaching the least possible magnitude.

#### 4.4. Phase-shift Compensation. Power Factor Correction

It is possible to improve the power factor of a load owing to the procedure called **power factor correction** – that is, by placing a suitable reactance in parallel with the load so that reactive power component generated by the additional reactance is of opposite sign to the original load reactive power. The following example illustrates typical power factor correction for industrial, mostly inductive loads.

The power factor  $\cos \varphi$  of a predominantly inductive element can be raised by connecting a capacitor in parallel with it (Fig. 4.8,a). The explanation can be found in the diagram of Fig. 4.8,b, where  $\dot{I}_1$  denotes the current flowing through the inductive element,  $\dot{I}_2$  is the current through the correcting capacitor,  $\dot{I}_2 = \dot{I}_C$ , and  $\dot{I}$  is the resultant current of the source.

As it can be seen from the diagram, the resultant phase shift  $\varphi_2$  is less than the phase shift  $\varphi_1$  of the origin load. Consequently,  $\cos \varphi_2 > \cos \varphi_1$ .

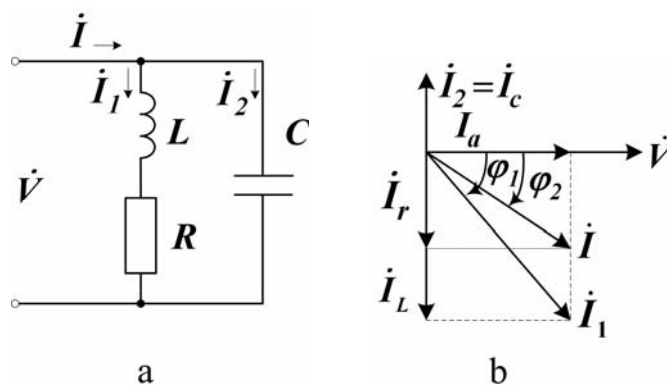


Fig. 4.8 – Phase-shift compensation

It is easy to determine the capacitance necessary to raise the power factor to unity: you need to reduce the phase shift from the given value  $\varphi_1$  to zero.

First you compute the complex power in order to determine the positive re-

active component  $ImS = Q_L$  caused by the origin load and, as a result, depending on  $\cos \varphi_1$ . Then, from an equal and opposite reactive power component  $Q_c = Q_L$ , we compute the reactance of compensating capacitor  $X_C = V^2 / Q_L$ . Finally, substituting  $X_C = 1 / \omega C$ , we determine

$$C = Q_L / \omega V^2, \quad (4.12)$$

or, alternatively:

$$C = \frac{P}{V^2 \omega} (tg \varphi_1 - tg \varphi_2), \quad (4.13)$$

if the true power  $P$  of the load is given and the power factor required ( $\cos \varphi_2$ ) is not equal to 1.

### 4.5 Three-phase Power Circuits

Most of electric power systems use three-phase power, when simultaneously three sinusoidal voltages are generated out of phase with respect to each other. The reasons are as follows. The first reason is higher efficiency due to lower weight of the conductors and other components per unit of power. Further, a three-phase system can provide a steady, constant supply of power with low amount of pulsation.

Three-phase voltages being displaced in phase by  $120^\circ$  are called balanced if they have equal amplitudes and frequency:

$$\begin{aligned} v_A &= V_m \sin \omega t, & \dot{V}_{mA} &= V_m e^{i\omega t}, \\ v_B &= V_m \sin(\omega t - 120^\circ), & \dot{V}_{mB} &= V_m e^{i(\omega t - 120^\circ)}, \\ v_C &= V_m \sin(\omega t + 120^\circ), & \dot{V}_{mC} &= V_m e^{i(\omega t + 120^\circ)}. \end{aligned} \quad \text{or}$$

A three-phase power source can be assembled in a form of wye configuration to produce the three-phase power like that shown in Fig. 4.9.

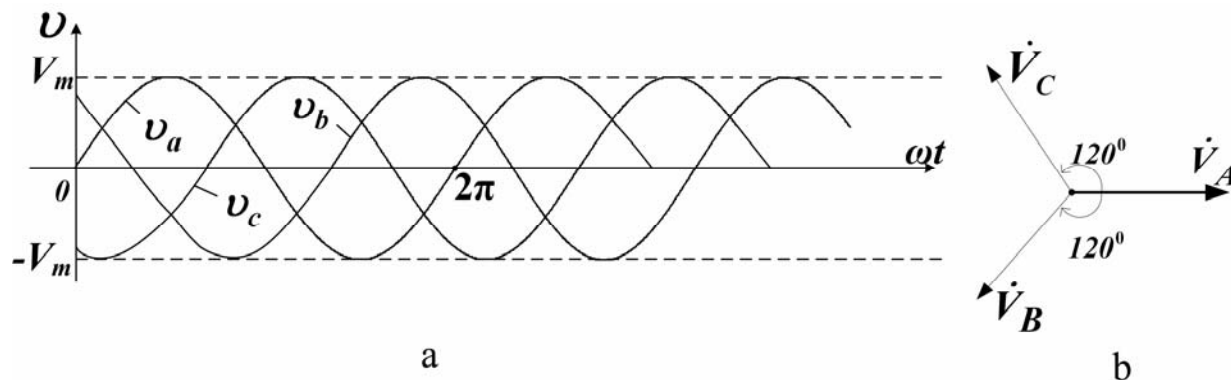


Fig. 4.9 – Representation of three-phase voltages

The construction principle of the wye-connected source is given by Fig. 4.10. From here on we will use conventional effective (not amplitude) values for all voltage and current quantities.

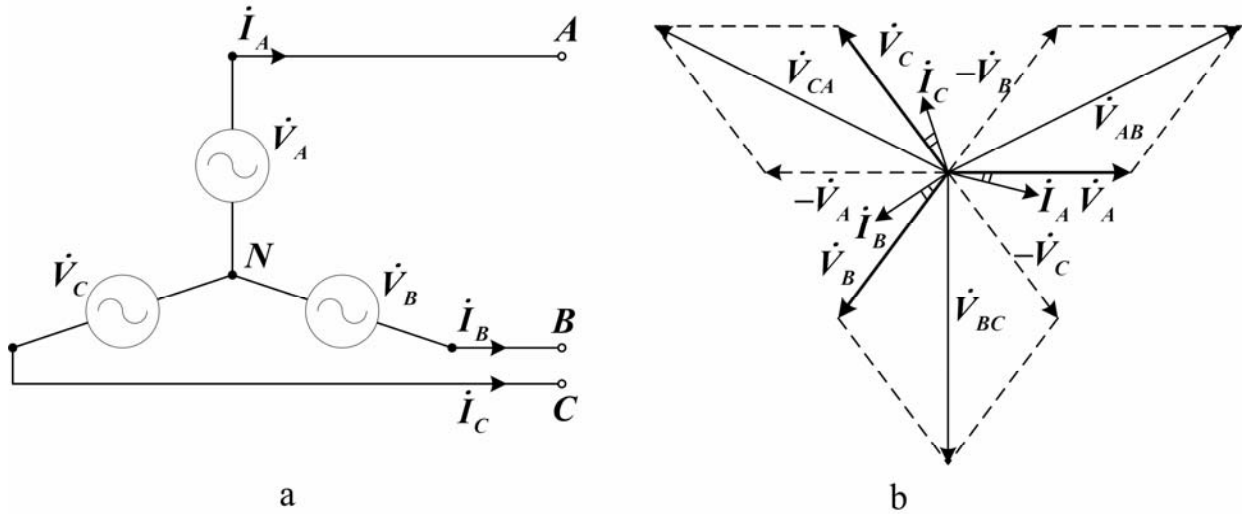


Fig. 4.10 – A three-voltage source connected in wye

Three AC sources in Fig. 4.10,a join with each other at the **neutral node N**. Individual source voltages developed with respect to the  $N$  node,  $\dot{V}_A$ ,  $\dot{V}_B$ ,  $\dot{V}_C$ , are called **phase voltages**. The phase voltages along with a few supplement voltages and currents related to the integrated three-phase source are shown in Fig. 4.10,b in a form of a vector diagram. In the diagram one can see the voltages denoted by  $\dot{V}_{AB}$ ,  $\dot{V}_{BC}$ ,  $\dot{V}_{CA}$  and called **line voltages**. The vector line voltages may be computed relative to the reference phase voltage  $\dot{V}_A = Ve^{j0}$ :

$$\dot{V}_{AB} = \sqrt{3}Ve^{j30}, \quad \dot{V}_{BC} = \sqrt{3}Ve^{-j90}, \quad \dot{V}_{CA} = \sqrt{3}Ve^{j150}.$$

It can be seen that the magnitude of the line voltage is equal to  $\sqrt{3}$  times the magnitude of the phase voltage. Besides, it is obvious for the wye connection that the phase current is the same as the line current. Thus, we have

$$V_{line} = \sqrt{3}V_{phase}, \quad I_{line} = I_{phase}. \quad (4.14)$$

A **delta-connected** three-phase system and related vector diagram are presented in Fig. 4.11 a, and b, respectively. From Fig. 4.11 one can obtain the following relationships between the phase and line voltages and corresponding currents:

$$V_{line} = V_{phase}, \quad I_{line} = \sqrt{3}I_{phase}. \quad (4.15)$$

In Fig. 4.10,b and Fig 4.11,b the currents are arbitrarily depicted as lagging the voltages.

From (4.14) and (4.15) it is clear that, for any connection, the apparent three-phase power is given by

$$S = \sqrt{3}V_{line} \cdot I_{line} = 3V_{phase} \cdot I_{phase}, \quad (4.16)$$

that yields the active and reactive components

$$P = \sqrt{3}V_{line} \cdot I_{line} \cdot \cos \varphi \quad \text{and} \quad Q = \sqrt{3}V_{line} \cdot I_{line} \cdot \sin \varphi, \quad (4.17)$$

where  $\varphi$  is the angle by which  $\dot{V}_{phase}$  leads  $\dot{I}_{phase}$  as it is depicted in Fig. 4.10,b and Fig. 4.11,b.

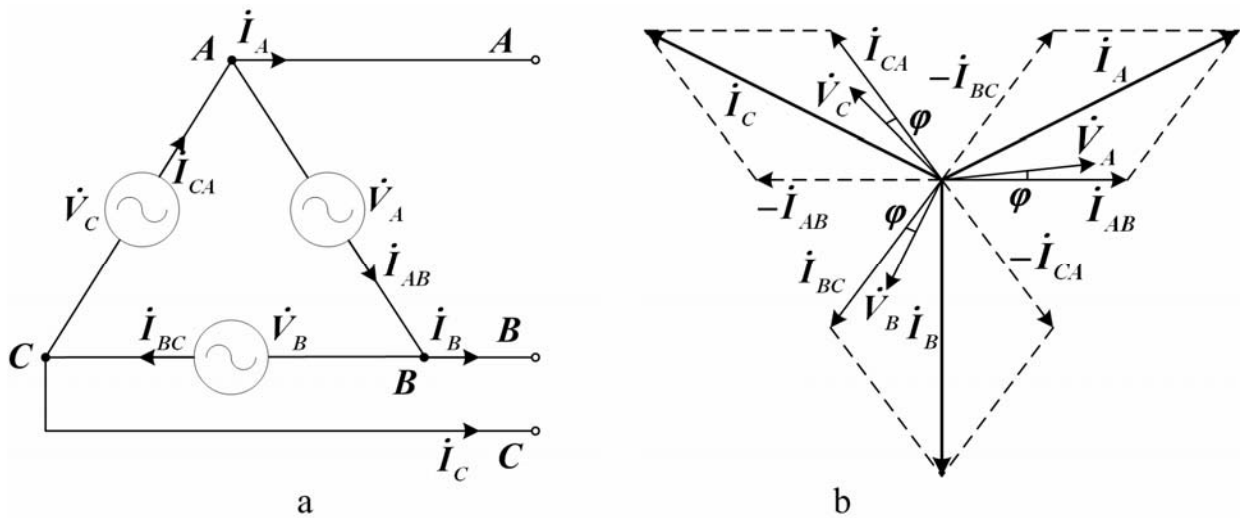


Fig. 4.11 – Three-phase source connected in delta

Just as voltage sources can be connected in wye and delta configurations, so impedances can be connected to constitute three-phase loads. To have its advantages a three-phase power system must employ a three-phase load that consists of the balanced (i.e., equal) parts as well. One of the important features of the balanced system is that it does not require a fourth wire (the neutral connection), since the current through that wire (if installed) is actually zero.

For the balanced three-phase power system one can use per-phase approach consisting in a reduction of the circuit to one phase and in further study of the single-phase circuit in usual manner.

In conclusion, we wish to emphasize the following: as an impedance  $Z$  of the AC circuit is analogous to pure resistance  $R$  of the DC circuit, then most of the rules and laws, e.g. Ohm's law, Kirchhoff's laws etc., developed for DC circuits are equally applicable for AC circuits.

## 4.6 Terms and Concepts

**Apparent power** is the product of the rms voltage the rms current. It is denoted  $VI$  with units of volt-amperes, VA.

**Active (true, real, in-phase) power** is the power dissipated by the resistance in AC circuit.

**Complex power** is the sum of the average power and the reactive power.

**Reactive power** is the power associated with the inductive or capacitive reactance in AC circuit, imaginary part of the complex power with units of volt-amperes reactive, VAR.

**Admittance** is the reverse of the impedance, i.e. the ratio of the complex current to the complex voltage of an element or set of elements.

**Susceptance** (inductive and capacitive) is the imaginary part of the admittance.

**Conductance** is the real part of the admittance, the inverse of the resistance of a resistor in units of siemens, S.

**Current resonance** is a condition in parallel  $LC$  circuit in which capacitive susceptance and inductive susceptance are equal; also a condition in a circuit, occurring at the resonant frequency, when the current through the entire circuit collects a minimum value.

**Resonant frequency** is a frequency at which, in  $LCR$  circuit, the current is in phase with the applied voltage. For a parallel resonant circuit the resonant frequency corresponds to the state when its admittance is not reactive.

**Power factor** is equal to  $\cos\phi$ , where  $\phi$  is the phase angle between the sinusoidal steady-state voltage and current; ratio of the average power to the apparent power.

**Three-phase power source** is the arrangement in which three sinusoidal voltages are generated out of phase with each other ( $0^\circ$ ,  $120^\circ$ ,  $-120^\circ$ ).

**Neutral node** (or simply **the neutral**) is the point of wye configuration where the three AC sources are all connected together.

**Line** is the conductor connecting each source of the three-phase power system to a load.

**Phase voltage** is the potential difference between the line and the neutral.

**Line voltage** is the voltage between two lines of the three-phase power source.

## 4.7 Review Questions

1. What relations can be used to determine branch currents in a parallel AC circuit?
2. Explain the notions of susceptance and admittance.
3. Can you explain what the current resonance is?
4. What is active, reactive and apparent AC power and what are their units?
5. What is phase-shift compensation used for?
6. What types of three-phase circuit connection do you know?
7. What are the relations between the phase and line voltages, the phase and line

currents for delta- and wye-connected loads?

#### 4.8 Problems

##### Examples

**Problem 1.** For an AC circuit  $v = 200\sin 377t$  (V) and  $i = 8\sin(377t - 30^\circ)$  (A) are given. Determine (a) the power factor, (b) true power, (c) apparent power, and (d) reactive power.

##### Solution

(a) The current lags the voltage by  $\phi = 30^\circ$ .

Hence power factor  $\cos\phi = \cos 30^\circ = 0.866$  lagging;

(b) From the given data, first we compute  $V = (200/\sqrt{2})$  V and  $I = 8/\sqrt{2}$  A.

Then, the true power  $P = VI \cos\phi = (200/\sqrt{2})(8/\sqrt{2})0.866 = 692.8$  W;

(c) the apparent power  $S = VI = (200/\sqrt{2})(8/\sqrt{2}) = 800$  VA;

(d) the reactive power  $Q = VI \sin\phi = (200/\sqrt{2})(8/\sqrt{2})0.5 = 400$  VAR.

**Problem 2.** Find the impedance and the current of the parallel AC circuit (Fig. 4.12) if  $E_t = 120$  V;  $X_L = 12 \Omega$ ;  $X_C = 4 \Omega$ ;  $R = 5 \Omega$ .

Determine the circuit phase angle.

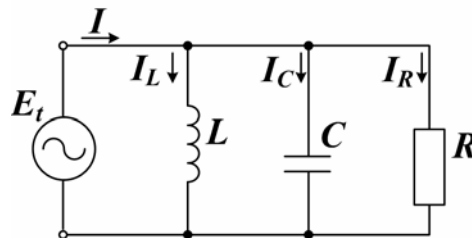


Fig. 4.12

##### Solution

(a) Invert  $R$ ,  $X_L$ , and  $X_C$  to find  $G$ ,  $B_L$ , and  $B_C$ :

$$G = 1/R = 1/5 = 0.2 \text{ S};$$

$$B_L = 1/X_L = 1/12 = 0.083 \text{ S};$$

$$B_C = 1/X_C = 1/4 = 0.25 \text{ S}.$$

(b) From the admittance triangle, determine the admittance:

$$Y = \sqrt{(G^2 + (B_C - B_L)^2)} = \sqrt{(0.2^2 + (0.25 - 0.083)^2)} = 0.2606 \text{ S}.$$

(c) Determine the total current  $I$  of the circuit, applying Ohm's law:

$$I = E \cdot Y = 120 \text{ V} \cdot 0.2606 \text{ S} = 31.25 \text{ A}.$$

(d) To calculate the phase shift between the current and the applied voltage study the admittance triangle and compute:

$$\sin \varphi = (B_C - B_L)/Y = 0.167/0.2606 = 0.641, \text{ whence } \varphi = 39.8^\circ.$$

The total current  $I$  leads the applied voltage  $E$ . Why?

**Problem 3.** Find the steady-state voltage  $V_S$  for the circuit (Fig. 4.13) when  $i_S(t) = 10\cos 1000t \text{ A}$ ,  $R_1 = R_2 = 10 \Omega$ ,  $L = 10 \text{ mH}$ ,  $C = 100 \mu\text{F}$ .

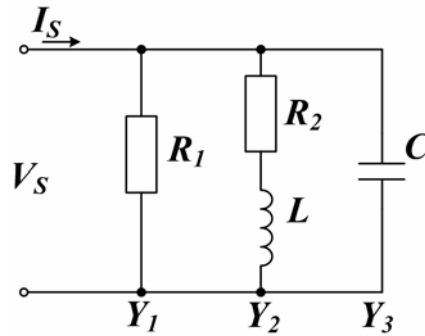


Fig. 4.13

### Solution

Applied complex voltage can easily be obtained from the complex form of Kirchhoff's current law:

$$\dot{V}_S \underline{Y}_1 + \dot{V}_S \underline{Y}_2 + \dot{V}_S \underline{Y}_3 = (\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3) \dot{V}_S = \dot{I}_S,$$

where rms complex current  $\dot{I}_S = (10/\sqrt{2})e^{j0}$ .

Compute the circuit branches admittance, when knowing that the AC current radian frequency  $\omega = 1000 \text{ rad/s}$ :

$$\begin{aligned} \underline{Y}_1 &= 1/R_1 = 1/10, \\ \underline{Y}_2 &= 1/(10 + j\omega L) = 1/(10 + j10), \\ \underline{Y}_3 &= j\omega C = j0.1. \end{aligned}$$

Substituting these quantities into above equation we have

$$\dot{I}_S = (1/10 + 1/(10 + j10) + j/10) \dot{V}_S = (10/\sqrt{2})e^{j0},$$

or

$$(0.15 + j0.05) \dot{V}_S = 10/\sqrt{2},$$

whence complex voltage (amplitude) will be

$$\dot{V}_{mS} = 10e^{j0}/0.158e^{j18.4^\circ} = 63.3e^{-j18.4^\circ}.$$

Finally, we may write the steady-state voltage waveform:

$$v_S(t) = 63.3\cos(1000t - 18.4^\circ).$$

**Problem 4.** A 3-phase power source with  $120\text{ V}$  phase voltage supplies a wye-connected load with impedance  $(36 + j48)\ \Omega$  in each leg.

Calculate (a) the line voltage, (b) the line current, (c) the power factor, and (d) total 3-phase active power dissipated by the load.

Solution

- (a)  $V_L = \sqrt{3}V_P = \sqrt{3} \cdot 120 = 207.8\text{ V};$
- (b)  $I_L = I_P = V_P / Z = 120 / \sqrt{36^2 + 48^2} = 2\text{ A};$
- (c)  $\cos \varphi = R/Z = 36/60 = 0.6$  lagging ;
- (d) dissipated power  $P = \sqrt{3}V_L I_L \cos \varphi = \sqrt{3} \cdot 207.8 \cdot 2 \cdot 0.6 = 432\text{ W}.$

**Problem 5.** Load impedances  $36 + j48$  are connected in delta, and the line voltage across the impedance equals  $207.8\text{ V}.$

Calculate (a) the phase current, (b) the line current, (c) the power factor, and (d) the total (active) power.

Solution

- (a)  $I_P = V_L / Z = 207.8 / 60 = 3.46\text{ A};$
- (b)  $I_L = \sqrt{3}I_P = \sqrt{3} \cdot 3.46 = 6\text{ A};$
- (c)  $\cos \varphi = R/Z = 36/60 = 0.6$  lagging ;
- (d)  $P = \sqrt{3}V_L I_L \cos \varphi = \sqrt{3} \cdot 207.8 \cdot 6 \cdot 0.6 = 1296\text{ W}.$

### Supplement problems

**Problem 6.** Given data:  $v(t) = 70.7 \sin(314t - 20^\circ)\text{ V},$   
 $i(t) = 2.12 \sin(314t + 10^\circ)\text{ A}.$

- (a) What phase shift is there between the voltage and the current?
- (b) What is the waveform frequency, in Hz?
- (c) Write down  $V$  and  $I$  parameters in an exponential form.
- (d) Calculate the instantaneous, average, reactive, and apparent power as well as the power factor.



**Problem 7.**  $75 \Omega$  resistor is connected with  $10 \mu F$  capacitor in parallel. Find a series  $RC$  circuit which is equivalent to the former by the impedance. An excitation angular frequency equals  $1000 \text{ rad/s}$ .

**Problem 8.** An inductor coil possesses a resistance of  $30 \Omega$  but its inductance is unknown. The coil is connected in parallel with  $100 \Omega$  resistor. This set, when applied across a  $100\text{-V}$ ,  $60\text{-Hz}$  source terminals, draws  $400 \text{ W}$ .

Determine the value of the inductance.

Answer:  $L = 26.53 \mu H$ .

**Problem 9.** A parallel  $RL$  circuit consuming  $480 \text{ W}$  at  $120 \text{ V}$  has the lagging power factor  $0.8$ . You must bring the factor close to unity by connecting a capacitor in parallel with the  $R$ - $L$ . Find the value of required capacitance, if the source frequency is  $60 \text{ Hz}$ .

Answer:  $C = 66.3 \mu F$ .

**Problem 10.** For parallel circuit in Fig. 4.14  $V = 120 \text{ V}$ ;  $R_1 = 1 \Omega$ ;  $X_L = 6 \Omega$ ;  $X_C = 9.95 \Omega$ .

Find: currents  $I_1$ ,  $I_2$ ,  $I$ ; current phase angles  $\phi_1$ ,  $\phi_2$ ,  $\phi$ . Draw the vector diagram.

**Problem 11.** For the circuit in Fig. 4.15  $L = 0.5 \text{ H}$ ;  $R = 30 \Omega$ ;  $C = 50 \mu F$ ;  $V = 60 \text{ V}$ . Find:  $Z$ ,  $I$ ,  $I_1$ ,  $I_2$ , resonant frequency  $f_0$ , resonant current  $I(f_0)$  and draw the vector diagram related to resonance.

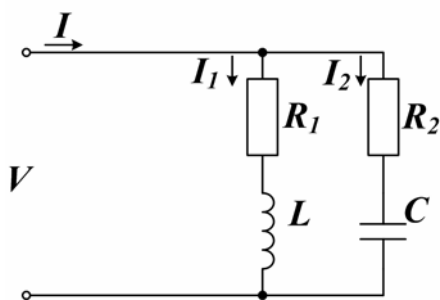


Fig. 4.14

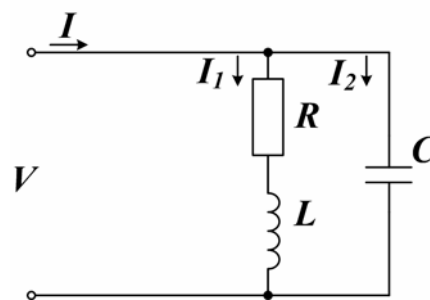


Fig. 4.15

**Problem 12.** A balanced 3-phase network with phase voltage  $V_a = 120e^{j0}$  is shown in Fig. 4.16. Determine: 1)  $I_a$ ; 2)  $V_{an}$ ; 3) the total power dissipated by line resistors  $R_L$ ; 4) the total power entered the circuit.

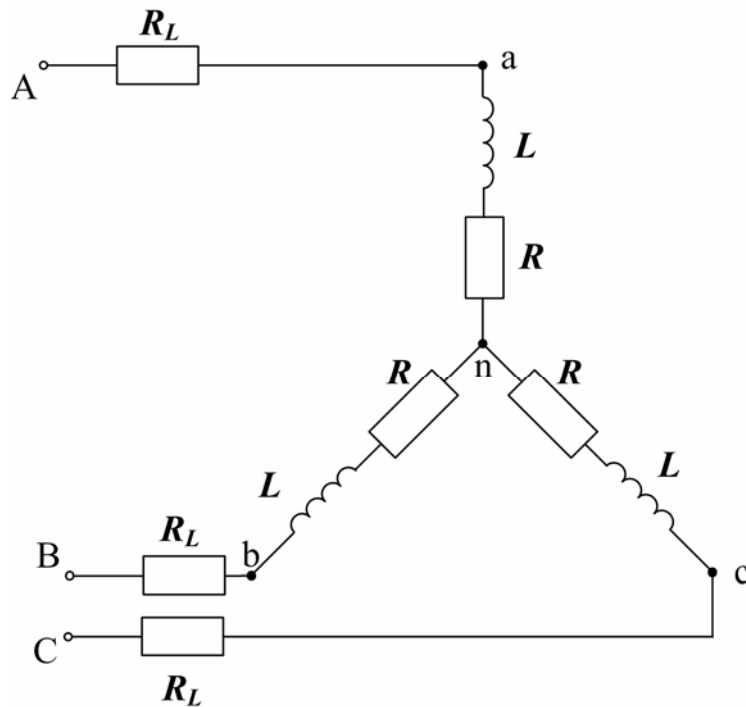


Fig. 4.16

## 4.9 Practice

### 4.9.1 “Investigation of Single-phase AC Circuits with Parallel Resistor, Inductor and Capacitor”

**Objective:** to develop practical skills in performing AC circuit analysis and carrying out the bench-top investigation of the circuits that contain energy-loss and energy-storage elements connected in parallel.

#### Work Task

1. Computing the simplest parallel RLC circuits with the help of complex and vector forms of circuit variables representation.
2. Practical investigation of the circuits containing parallel resistor, inductor, and capacitor supplied by a single-phase sinusoidal current.
3. Calculation of unknown electric quantities based on the circuit parameters collected during the measurements.

#### Theory and Preparation

The elements to be investigated during the work will be the inductor, and sets of capacitors and resistors. Remember that practical inductor coil possesses a

loss resistance together with the inductance and therefore must be viewed in the equivalent form as series connection of  $L_C$  and  $R_C$ .

To carry out practical experiments with the circuits of objectives you must study the circuits operation carefully and, beforehand, draw the following three circuit diagrams: for parallel connection of the inductor and resistor, parallel connection of a capacitor and resistor, parallel connection of the inductor and the capacitor. Besides, following the forms shown below, you must prepare three blank tables for coming measurements and computations recording. Study of the necessary measuring instruments is of special importance, as well as readiness to answer test questions.

### Procedure

#### 1. Investigation of connected in parallel the resistor and inductor (Fig. 4.17)

From your instructor, take a recommended value of the input AC voltage and prepare a measuring instrument for each branch (i.e., select the device, study specifications, set the range, and define the scale spacing of the meter).

In accordance with the diagram in Fig. 4.17, connect the circuit having two parallel branches and applied with the meters.

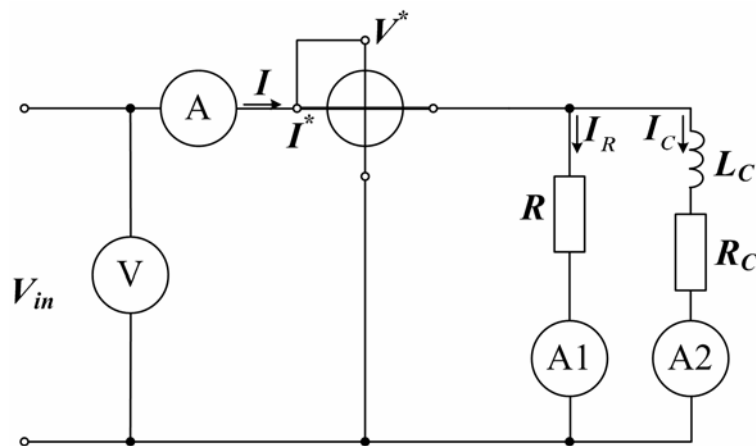


Fig. 4.17

After instructor's admission, switch the AC power supply on and regulate the voltage across the input of the circuit.

Measure each of the necessary circuit parameters and record your results in Table 4.1.

Table 4.1

Parameter	Measured				Calculated						
Symbol	$I$	$I_R$	$I_C$	$P$	$Z_c$	$g_c$	$b_c$	$I_{Ac}$	$I_{Lc}$	$S$	$\cos \varphi$
Unit	A	A	A	W	Sm	Sm	Sm	A	A	VA	-
Value											

## 2. Investigation of connected in parallel the resistor and capacitor

Repeat the procedure of the previous section for the circuit shown in Fig. 4.18.

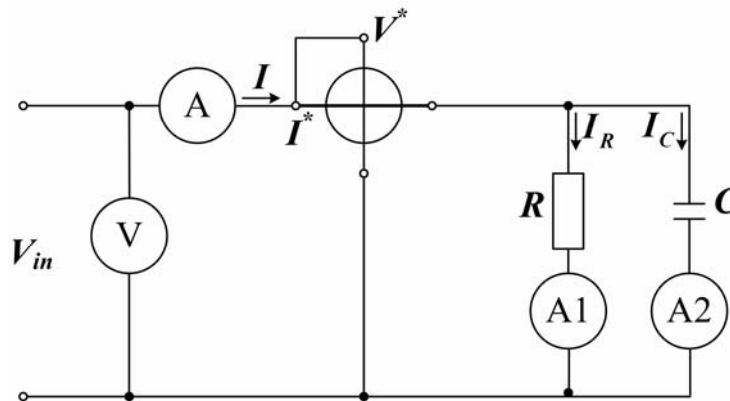


Fig. 4.18

Measure each of the necessary circuit parameters and record your results in Table 4.2.

Table 4.2

Parameter	Measured				Calculated						
Symbol	$I$	$I_R$	$I_C$	$P$	$Y$	$g$	$b$	$C$	$Q$	$S$	$\cos \varphi$
Unit	A	A	A	W	Sm	Sm	Sm	A	VAR	VA	-
Value											

## 3. Investigation of $R-L-C$ parallel circuit. Tuning the circuit at resonance

The experiment circuit diagram is shown in Fig. 4.19.

Beforehand, the capacitance value  $C_0$  that is necessary to reveal the current resonance in this circuit must be computed. The value of supply voltage frequency  $50 \text{ Hz}$  which is the resonant frequency now, and the value of the variometer inductance which one can derive from the computed data of preceding experiments, are used to do that.

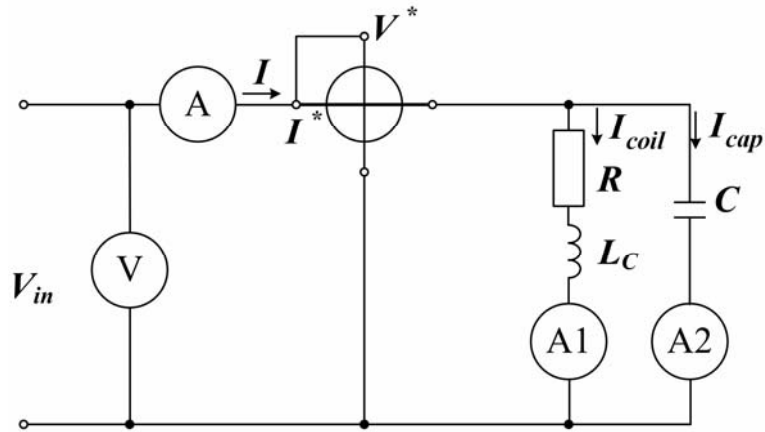


Fig. 4.19

Now, repeat the procedures of preceding sections and measure each of the necessary circuit parameters; record your results in Table 4.3.

*NB:* a fine tuning of the circuit may be needed for clear resonance detection in the circuit with practical elements. It is performed by changing the circuit capacitance within the range  $0.25 C_0$  to  $2 C_0$ , where  $C_0$  is the above calculated theoretical capacitance.

Table 4.3

Parameter	Measured					Calculated				
Symbol	$C$	$I$	$I_{coil}$	$I_{cap}$	$P$	$g_c$	$b_c$	$I_{Ac}$	$I_{Lc}$	$\cos \varphi$
Unit	mkF	A	A	A	W	Sm	Sm	A	A	-
Value										

When carrying out the experiment copy oscillograms relating to the input voltage as well as the input current, those have been investigated for different values of the capacitance, i.e. for  $C$  equal to  $0.25C_0$ ,  $C_0$ ,  $2C_0$ .

### Summarizing

1. Using results of the experiments (see Tables 4.1 to 4.3) calculate the necessary unknown quantities.
2. In the same figure, construct the plots for  $I, I_{Lc}, I_{cap}$ , and  $\cos \varphi$  as functions of circuit capacitance  $C$ .
3. Draw circuit equivalents which feature a behavior of the  $R-L-C$  circuit for different value of the capacitance.
4. Make conclusions.

## Questions to be answered

1. Write the complex Ohm's law form for a part of the circuit with an inductor and a capacitor connected in parallel.
2. What is the admittance of the circuit with an inductor and resistor in parallel?
3. What ways of current resonance revealing do you know?
4. Depict a vector diagram for parallel connection of an inductor and a capacitor.
5. How can active and reactive components of the current be determined in the electrical circuit with parallel connection of inductor and capacitor?
6. What is the way for measuring a waveform period with the help of oscilloscope? What about a phase shift?
7. How can you determine the branch impedance for the electrical circuit of sinusoidal current?
8. How can you calculate an effective value of the current in unbranched part of a linear circuit with sinusoidal current?
9. What is the resonance condition for AC circuit?
10. What factors does a resonant frequency in parallel circuit depend on?

### 4.9.2 "Investigation of Wye-Wye Three-Phase Circuit"

**Objective:** to collect practical experience in analysis and application of three-phase electric power.

#### Introduction

Three-phase power circuits including both balanced and arbitrary (unbalanced) loads are subjects of investigation in this practice.

During the analysis and practical experiments different kinds of the power (active, reactive, apparent) must be recognized and determined, the line and phase currents and voltages as well as the voltage between neutral points of the source and receiver and the current  $I_0$  through neutral conductor are to be defined, measured, evaluated and used correctly in calculations offered.

#### Procedure

Experimental part of the work is carried out on the bench with the applied three-phase power source, the set of conventional measuring instruments, and a number of loads. Those different load elements will be resistors, capacitors and inductors.

For carrying out the work it is necessary:

1) to depict a circuit diagram with necessary instruments for investigating the balanced operation of three-phase circuit (Fig. 4.20);

2) to set up the circuit. Turn on the phase source after instructor's checking and write down the readings of meters into Table 4.4;

3) to make similar experiments for unbalanced wye-connected load both without neutral wire and with it, with the breaking of linear conductor and short circuit in one phase; to write the results of all experiments into the Table 4.4.

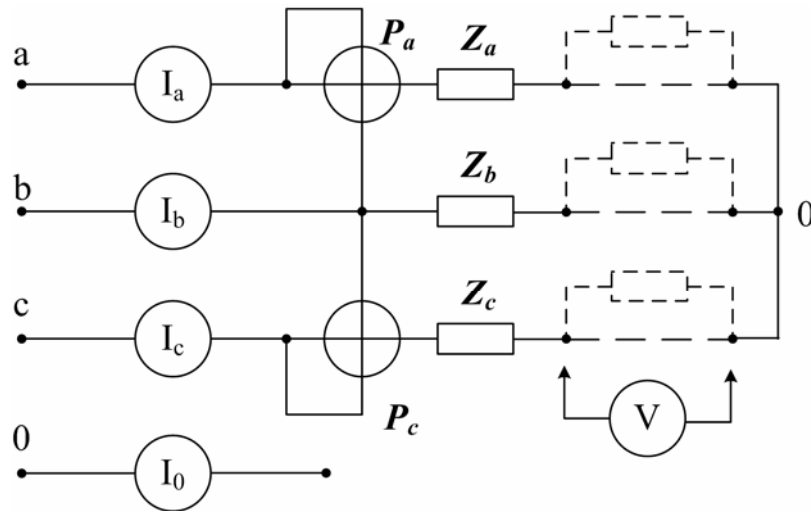


Fig. 4.20

Table 4.4

№	Mode	$V_{ab}$	$V_{bc}$	$V_{ca}$	$V_a$	$V_b$	$V_c$	$I_a$	$I_b$	$I_c$	$V_0$	$I_0$	$P$	$\frac{V_l}{V_p}$
1	Balanced load													
2	Unbalanced three-wire load													
3	Unbalanced four-wire load													
4	Open circuit (one phase)													
5	Short circuit (one phase)													

## Summarizing

1. Using the experimental data (see Table 4.4) it is necessary to calculate the values given in the table.
2. On the experimental data (see Table 4.4) you have to build vector diagrams for all modes of operation.
3. Make conclusions.

## Questions to be answered

1. What is a three-phase balanced system of EMF?
2. How are the receivers of a three-phase system connected in wye?
3. What two kinds of voltages and currents are there in three-phase systems?
4. What relationship is there between line and phase voltages when the receivers are wye-connected?
5. What relationship is there between line and phase currents when receivers are delta-connected?
6. What are the conditions of getting symmetrical system of currents in a three-phase system of EMF?
7. What changes occur in a three-phase unbalanced system with wye-connected load?
8. What is neutral displacement?
9. How can you determine analytically the neutral displacement in an unbalanced three-phase system?
10. What is it necessary to do for restoring the symmetry of phase voltages into unbalanced phase load?
11. Determine the current through neutral wire for balanced and unbalanced receivers?
12. What happens in a three-phase system with wye-connected receivers when one of phases is disconnected?
13. What happens with wye-connected receivers when one of the phases is short circuit?
14. How to build a vector diagram for a three-conducting system using all measurements and calculations for distributed loading, disconnecting and short circuiting one of the phases?
15. Build vector diagram for a three-phase system with three or four wye-connected receivers for three or four wires?



## Chapter 5

### MAGNETIC CIRCUITS

Various electromechanical transducers such as electric machines and apparatus, as well as sensors and measuring instruments operate on the basis of electromechanical energy conversion. The magnetic field forms a necessary connection between electrical and mechanical energy.

#### 5.1 Parameters of Magnetic Field

Magnetic fields are generated by electric charge in motion, and their effect is measured by the force they exert on a moving charge. It is convenient to represent the magnetic field by means of the lines of force (Fig. 5.1) when we visualize the strength of a magnetic field by observing the density of these closed lines in space.

The quantities used to quantify the strength of a magnetic field are the **magnetic flux**  $\Phi$ , in units of webers (Wb) and the **magnetic induction** (often called **magnetic flux density**)  $\mathbf{B}$  in units of webers per square meter, or teslas (T).

First of all the intensity and the direction of a magnetic field at each point are defined by the magnetic induction vector  $\bar{\mathbf{B}}$  shown in Fig. 5.1.

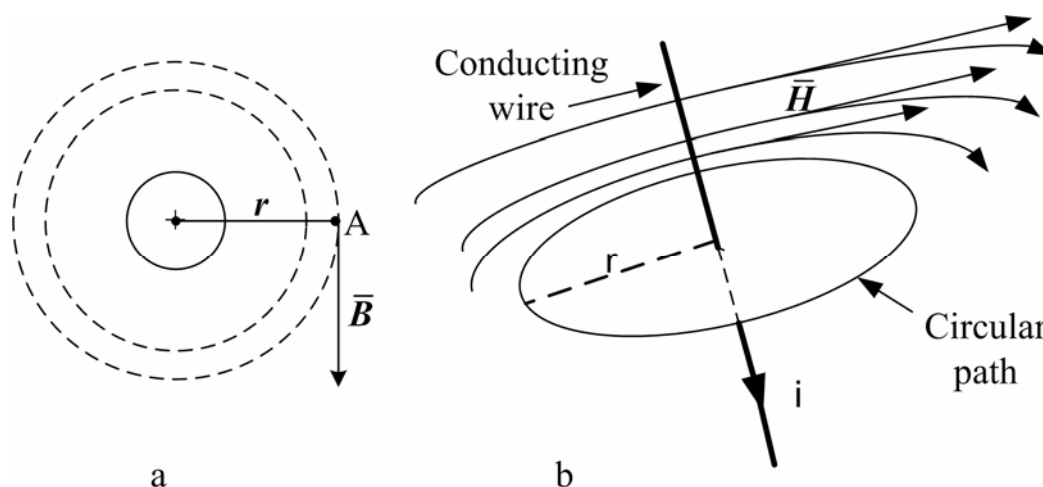


Fig. 5.1 – Current and magnetic field interrelation

The magnetic flux  $\Phi$  is then defined as the integral of the flux density (the magnetic induction) over some surface area  $s$ :

$$\Phi = \oint_s \bar{\mathbf{B}}_n ds. \quad (5.1)$$

If the flux is uniform in bounds of the area  $s$  and perpendicular to the surface, the preceding integral can be approximated by the simpler expression

$$\Phi = \mathbf{B} \cdot \mathbf{s}. \quad (5.2)$$

From here on, in discussing magnetic induction (also, following magnetic field intensity) we shall assume that the field is a scalar field, i.e. let it lie in a single spatial direction. This will simplify many explanations.

The magnetic field always accompanies the electric current, i.e. the current generates a magnetic field. In detail, discussion of the matter will follow later in this chapter. Now we shall restrict our consideration to the problem of determining the surrounding magnetic field produced by the current when we know the magnitude and the direction of the current. For this purpose we must apply the fundamental **law of total current**.

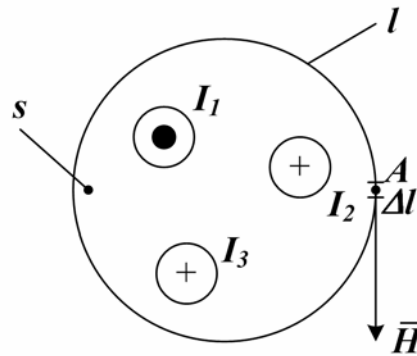


Fig. 5.2 – Illustration of the total current law

To explain the law let us introduce one more vector quantity called **the magnetic field intensity  $\bar{H}$**  which is measured in units of amperes per meter ( $\frac{A}{m}$ ) and also characterizes the strength of the magnetic field (Fig. 5.2). The reason of introducing the magnetic field intensity is that it is independent of the properties of the materials used in the construction of magnetic circuits. Thus, a given magnetic field intensity  $\bar{H}$  will give rise to different magnetic induction (the flux density)  $\mathbf{B}$  in different materials. For example, in vacuum the magnetic field intensity and magnetic induction are related by

$$\mathbf{H}_0 = \frac{\mathbf{B}_0}{\mu_0}, \quad (5.3)$$

where a constant  $\mu_0$  is called the **magnetic permeability of free space**.

In the above context it is useful to define *sources* of magnetic energy in terms of *magnetic field intensity*; in analogy with *electromotive force* this source could be termed *magnetomotive force*.

The law of the total current states that the vector integral of the magnetic field intensity  $\bar{H}$  around a closed path  $l$  (see Fig. 5.2) is equal to the total current  $I$  linked by the closed path:

$$\oint H dl = \sum_{i=1}^n I = I_1 - I_2 - I_3, \quad (5.4)$$

where  $dl$  is an increment in the direction of the closed path.

Consider typical examples.

**Example 1.** For the case of a long wire carrying current  $I$  and of a circular path of radius  $r$  surrounding the wire (see Fig. 5.1), determine the magnetic field intensity  $H$ .

In this simple case we can see that the magnetic field intensity  $H$  following after the induction  $B$ , which is constant due to the constant conductor current, can be determined by the known right-hand rule. This rule states that if the direction of the current (into the page in the sketch) points to the direction of the thumb of one's right hand, the resulting magnetic field encircles the conductor in the direction in which the other four fingers will encircle it. Hence, the closed-path integral in (5.4) results in

$$H \oint dl = 2\pi r H,$$

since the circular path and the magnetic field are in the same direction.

The magnitude of the magnetic field intensity is then given by

$$H = \frac{I}{2\pi r}.$$

**Example 2.** A practical inductor coil, shown in Fig. 5.3 and having  $w$  turns uniformly wound around a toroidal iron core with the cross-sectional area  $S$ , is energized with the current  $I$ . What is the magnetic field intensity inside this structure? The structure geometry is given.

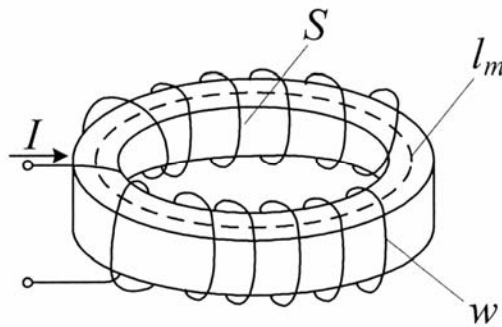


Fig. 5.3 – Practical toroidal inductor

For such a coil, with  $w$  turns, the lines of force associated with the magnetic field ( $l_m$  is the mean path of the magnetic flux lines) link all the turns of the conducting coil, so that we have effectively increased,  $w$  times, the current linked by the flux lines. Therefore the law of total current takes the form:

$$H \cdot l_m = I \cdot w,$$

that implies

$$H = \frac{I \cdot w}{l_m}.$$

Note that the path of the flux is enclosed within the ferromagnetic structure preferentially.

The product  $I \cdot w$  is a very important and useful quantity in electromagnetic circuits. It is this product that defines **magnetizing**, or **magnetomotive force** (often abbreviated **mmf**):

$$F = I \cdot w. \quad (5.5)$$

Although dimensionally equal to amperes, the units of this force are the ampere-turns ( $A \cdot t$ ).

## 5.2 Current and Magnetic Field Interrelation

The operation of the electric motors and generators is based on two phenomena: a) – electromagnetic induction, and b) – electromechanical interaction between a current-carrying conductor and a magnetic field.

### A current-carrying conductor placed in a magnetic field.

The force the magnetic field exerts on a charge of  $q$  moving at velocity  $v$  is given by the equation

$$F = qvB \sin \alpha, \quad (5.6)$$

where  $B$  is the magnetic induction of the field which direction makes the angle  $\alpha$  with the velocity vector  $v$ ; the direction of this force is at right angles with the plane formed by the vectors  $B$  and  $v$ .

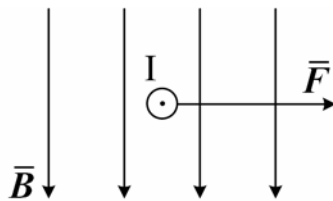


Fig. 5.4 – A current – carrying conductor in a magnetic field

If the charge is moving through a conductor of  $l$  length, for the conductor in the field the force (Fig 5.4) can be written as

$$F = BIl \sin \alpha. \quad (5.7)$$

This relationship is known as **Ampere's law**.

The force will take the greatest value if  $\alpha = 90^\circ$ , i.e. when the conductor is at right angle (as depicted in Fig. 5.4) with the direction of the magnetic lines. In this case we have  $F = BIl$ .

The simplest method to find the pointing of the force results from the **left-hand rule** that states: if the palm of one's left hand is placed to intercept the magnetic lines (see Fig. 5.4) and the four fingers show the direction of the current, the extended thumb will point the direction of the force exerting on the conductor.

Ampere's law plays a very important role in electrical engineering, as it provides for the basis of computing the torque in both power electric machines (those like motors and generators) and numerous measuring instruments.

### Electromagnetic induction

Electromagnetic induction phenomenon lies in the fact that a changing magnetic field induces a voltage, and therefore a current, in a closed conductor bounding the surface with the changing magnetic flux. The quantitative relationship between the magnitude of the induced electromotive force,  $e$ , and the rate of change of the magnetic flux was formulated by Maxwell:

$$e = -\frac{d\Phi}{dt}. \quad (5.8)$$

This is known as **the law of electromagnetic induction**.

Besides, the equation (5.8) defines the direction of the induced emf. The minus sign denotes the principle discovered in 1833 by the Russian academician Lenz and known as **Lenz's Law: a change in the magnetic flux through a closed conducting circuit induces in the latter an emf of such a direction that the current produced by it and the mechanical force associated with it oppose to any change of the magnetic flux**.

In practical applications, the size of the voltages induced by the changing magnetic field can be significantly increased if the conducting wire is coiled many times around, so as to multiply the area crossed by the magnetic flux lines many times over.

The sum of the fluxes linked with all the turns of the coil is called the **flux linkage** and is denoted by  $\psi$ . If all the turns are linked with the same flux  $\Phi$  (see Fig. 5.5, a), then  $\psi = w \cdot \Phi$  and total induced emf:

$$e = -\frac{d\psi}{dt}. \quad (5.9)$$

Note that equation (5.9), relating the derivative of the flux linkage to the induced emf, is analogous to the equation describing current as the derivative of charge i.e.  $i = dq/dt$ . This fact discovers a possibility to view flux linkage as the dual of charge in a circuit analysis sense.

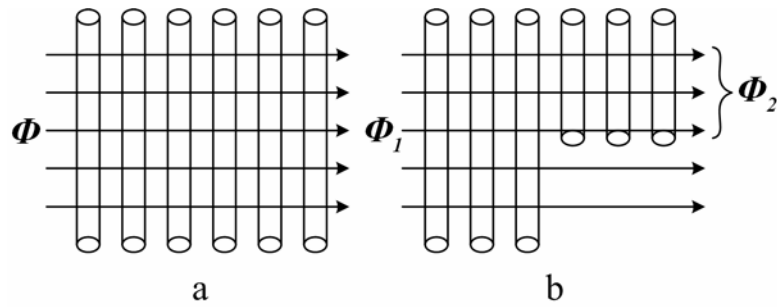


Fig. 5.5 – The flux linkage explanation

If the coil, consisting of  $w$  turns, is intersected by different fluxes  $\Phi_1, \Phi_2, \dots, \Phi_n$  like those shown in Fig. 5.5, b, then the emf of the coil must be represented by the sum of the electromotive forces induced in separate groups of turns, i.e.

$$e = - \left( \frac{d\Phi_1}{dt} + \frac{d\Phi_2}{dt} + \dots + \frac{d\Phi_n}{dt} \right) = - \frac{d(\Phi_1 + \Phi_2 + \dots + \Phi_n)}{dt}.$$

In this case the flux linkage is defined as

$$\psi = \sum_{k=1}^n w_k \cdot \Phi_k. \quad (5.10)$$

What are the physical mechanisms that can cause magnetic flux to change and therefore to induce an electromotive force? Two such mechanisms are possible. The first consists of physically moving a permanent magnet in the vicinity of a coil. The second requires that we first produce a magnetic field by means of electric current and then vary the current, thus varying the associated magnetic field. The latter method is more practical, since it does not require the use of permanent magnets and allows variation of field strength by conveniently varying the applied current. The voltages induced by such a way are often called transformer voltages.

### 5.3 Properties of Ferromagnetic Materials

The note made in previous sections for a concentration of the magnetic field within a core underlines a distinguishing feature of ferromagnetic materials leading to their wide use in numerous electric machines, devices, and instruments: in the presence of a magnetic material the magnetic induction (magnetic flux lines density) would be far greater, for a given field intensity, than if the conductor were surrounded by air or other nonmagnetic material.

Recognizing the relationship (5.3) we can derive the magnetic induction in the material from the given field intensity with the help of the notion **permeability of the medium,  $\mu$** :

$$\mathbf{B} = \mu \mathbf{H} = \mu' \mu_0 \mathbf{H} . \quad (5.11)$$

Note that the permeability of a material is factored as the product of the permeability of free space  $4\pi \cdot 10^{-7} \frac{\text{H}}{\text{m}}$  times the so-called **relative permeability,  $m$** , which varies greatly according to the medium. For various types of steel, for example, the latter takes values in the hundreds or thousands (300 to 80000). Compare: for air and for most electrical conductors and insulators  $\mu'$  is equal to 1.

The magnetic induction in ferromagnetic materials is not linearly proportional to the field intensity, so that their ratio

$$\mu = \frac{B}{H}$$

is not a constant, but depends on the value of  $H$ . This is because all magnetic materials exhibit a phenomenon called **saturation**. Fig 5.4 illustrates the general behavior of all magnetic materials. The curve  $B$ - $H$ , or magnetization curve, is nonlinear, exhibiting how the induction increases in proportion to the field intensity until it cannot do so any longer. A practical conclusion from this fact is following: as the material reaches saturation, further increases in the mmf (or, equivalently, in the driving current) do not cause further increases in the magnetic flux.

Magnetization curves  $B$ - $H$  are useful services for determining the magnetic field in ferromagnetic cores in practice. An instance of  $B$ - $H$  curve depicted in Fig. 5.6 is called the initial magnetization curve.

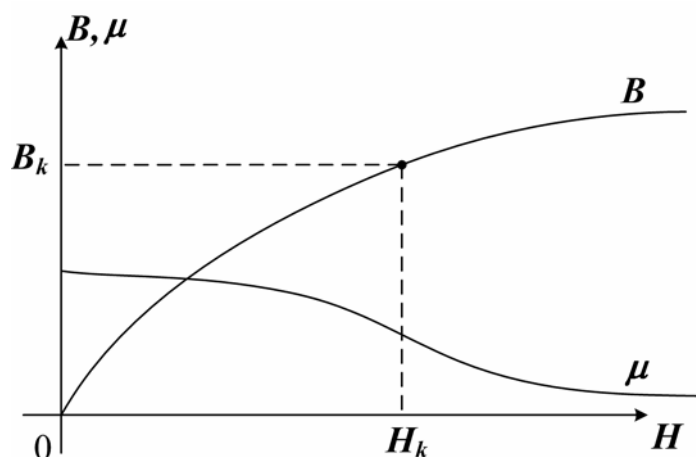


Fig. 5.6 – Magnetization curves

In the case of alternating magnetizing (magnetization followed by demagnetization, and back) the process doesn't follow the initial magnetization curve, but runs along a hysteresis loop in accordance with the picture in Fig. 5.7.

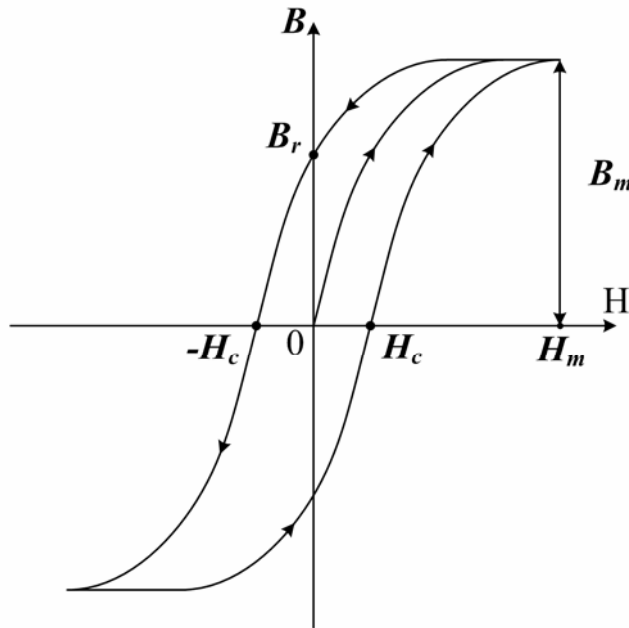


Fig. 5.7 – Hysteresis in magnetization curves

The upper branch of the loop corresponds to the decrease of induction in the material after preceding primary or any other opposite cycle of material forward magnetization (during those the mmf increases). When the mmf is exactly zero the material displays the **residual** (or **remanent**) **induction**  $B_r$ . To bring the flux density to zero we must further decrease the mmf, i.e. produce a negative current. The abscissa  $H_c$  is called **the coercive force**.

The result of this process, by which an excess magnetomotive force is required to magnetize or demagnetize the material, is a net energy loss. It can be shown that this loss is related to the area between the curves of Fig. 5.7.

Materials with a comparatively low coercive force, i.e. with the narrow hysteresis loop, are called “**soft magnetic materials**”. These are various formed irons, as well as permalloy and (silicon) siliferous steel. Soft magnetic materials are most suitable to operate in alternating magnetic fields.

Materials with a relatively high coercive force form the group of “**hard magnetic materials**” which are used in permanent magnets manufacturing.

#### 5.4 Magnetic Circuit Computation

It is possible to analyze the operation of electromagnetic devices such as the one depicted in Fig. 5.3 by means of *magnetic equivalent circuits*, similar in many aspects to the *equivalent electrical circuits*.



### Ohm's law for a magnetic circuit

Let us derive the dependence between the mmf,  $F = I \cdot w$ , and the magnetic flux  $\Phi$ . For the toroidal coil with a ferromagnetic core and similar magnetic structures we relate the mmf of the coil to the product of the magnetic field intensity  $H$  and the length of the magnetic mean path  $l_m$ :

$$H \cdot l_m = I \cdot w.$$

Since  $\mu' \mu_0 H = B$  and  $BS = \Phi$ , the mmf is then equal to the magnetic flux times the length of the magnetic path, divided by the permeability of the material times the cross-sectional area:

$$F = \Phi \cdot l_m / \mu' \mu_0 S. \quad (5.12)$$

A review of this formula reveals that the magnetomotive force  $I \cdot w$  may be viewed as being analogous to the voltage source in a series electrical circuit, and that the flux,  $\Phi$ , is equivalent to the electrical current in a series circuit. The ratio  $l_m$  to  $\mu' \mu_0 S$  is then convenient to apply as the term of **magnetic resistance** (assigned name – **reluctance**):

$$R_{mag} = \frac{l_m}{\mu' \mu_0 S}.$$

After that the equation (5.7) can be written in the form:

$$\Phi = \frac{Iw}{R_{mag}} = \frac{F}{R_{mag}}, \quad (5.13)$$

which is equivalent to Ohm's law for electric circuit.

Fig. 5.8 depicts the equivalent circuit analogy representing the magnetic circuit of Fig. 5.3 which Ohm's law equivalent (5.13) is applicable to.

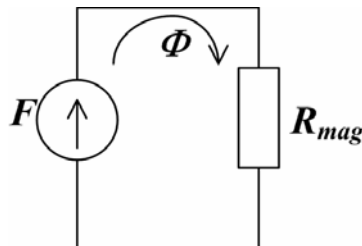


Fig. 5.8 – Analog circuit representation of toroidal inductor

Some assumptions were taken in the example that allowed to simplify the problem of magnetic circuit analysis and made approximate quantitative solution possible in a few simple steps. Those assumptions were: 1) all of the magnetic flux is linked by all of the turns of the coil; 2) the flux is confined exclusively

within the magnetic core; 3) the density of the flux is uniform across the cross-sectional area of the core. The usefulness of this approximate method is that you can quickly calculate the magnetic flux in given structure (a direct problem) or evaluate the necessary coil current and number of turns for the desired flux or flux density in given structure geometry (an inverse problem).

Consider the direct problem analysis for a magnetic structure similar to that has just been analyzed but having an air gap. The structure geometry is shown in Fig. 5.9. In general, air gaps are very common in magnetic structures; in rotating machines, for example, air gaps are necessary to allow for free rotation of the inner core (armature) of the machine; also, air gap serves as a working part in the assembly called electromagnet, etc.

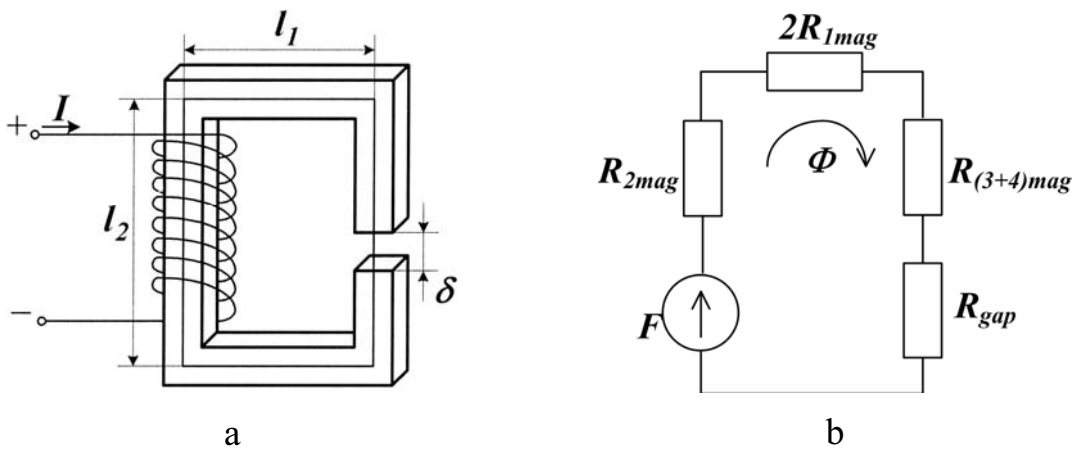


Fig. 5.9 – Magnetic circuit with air gap and its analog equivalent

The equivalent circuit for the structure of Fig. 5.9,a may be drawn as shown in Fig. 5.9,b, where  $R_{mk}$  is the reluctance of path  $l_k$ , for  $k = 1, 2, \dots, 4$ , and  $R_{gap}$  is the reluctance of the gap. In accordance with (5.12) the reluctances can be expressed as follows:

$$R_{mag} = \frac{l_m}{\mu' \mu_0 S}, \quad R_{gap} = \frac{\delta}{\mu_0 S'}$$

where the magnetic structure is considered to have a uniform cross-sectional area;  $S_k = S$ , for  $k = 1, 2, \dots, 4$ , but  $s'$  is different from  $s$  (because of **fringing**  $S'$  is greater than  $S$ ); and the permeability of the gap is given by  $\mu_0$ .

Now, for the given magnetizing force, the magnetic flux as well as the induction established in the structure can be easily found using Ohm's law equivalent.

**The inverse problem** is considerably more complicated. In order to solve it one must find the magnetization forces corresponding to several given values of magnetic flux.

If the cross-sections  $S_k$  of some parts of the core are different, the magnetic induction in them will also differ and can be determined in each cross-section by

$$B_k = \frac{\Phi}{S_k},$$

where  $\Phi$  is the given magnetic flux, which is uniform in all the sections.

For each portion of the ferromagnetic core the magnetic field intensity  $H_k$  is determined by the values of the magnetic induction  $B_k$  evaluated from the appropriate magnetization curve (see Fig. 5.6).

For the nonmagnetic portion of the circuit field intensity  $H_0$  is determined by

$$H_0 = \frac{B_0}{\mu_0}.$$

Since  $H$  remains constant in the bounds of each portion and changes when crossing from one portion to another only, the integral form of the law of total current can be replaced by the summation:

$$\oint H dl = \sum_{k=1}^n H_k l_k.$$

Hence, the total current law will take the form that is analogous to Kirchhoff's voltage law for electric circuit:

$$I \cdot w = \sum_{k=1}^n H_k l_k,$$

In the foregoing example

$$I \cdot w = H_1 l_1 + 2H_2 l_2 + H_1(l_1 + \delta) + H_0 \delta.$$

From this equation it is easy to find the magnetizing force of the coil required to establish a given magnetic flux  $\Phi$ .

If the number of turns is assumed to be known, then one can find the required current of the coil:

$$I = \frac{\sum_{k=1}^n H_k l_k}{w}.$$

In the foregoing example, the magnetic flux was given and the magnetizing force of the coil was determined.

## 5.5 Terms and Concepts

**Ampere-turn** is the magnetizing force produced by a current of 1 A flowing through one turn of a coil. Ampere-turns = amperes  $\times$  number of turns of wire in the coil.

**Coil** is one more turns of a conductor designed for use in a circuit to produce inductance or an electromagnetic field.

**Counter electromotive force (cemf)** is a voltage developed in the armature of a motor that opposes the applied emf. The same principle applied to any inductor through which an alternating current is flowing.

**Eddy currents** is currents induced in the cores of coils, transformers, and armatures by the changing magnetic fields associated with their operation. These currents cause great losses of energy. For this reason such cores are composed of insulated laminations that limit the current paths.

**Electromagnet** is a magnet formed in a way when an iron core is placed in a current-carrying coil.

**Electromagnetic induction** is the transfer of electric energy from one conductor to another by means of a moving electromagnetic field. A voltage is produced in a conductor as the magnetic lines of force intersect or link with the conductor.

**Electromagnetism** is the magnetism produced by the flow of electric current.

**Ferromagnetic materials** is magnetic materials composed largely of iron.

**Flux** is electrostatic or magnetic lines of force.

**Flux linkage ( $\psi$ )** is sum of the fluxes linked with all the turns of the coil.

**Henry (H)** is the unit of inductance. It is the amount of inductance in a coil that will induce an emf of 1 V in the coil when the current flow is changing at the rate of 1 A/s.

**Hysteresis** is the ability of a magnetic material to withstand changes in its magnetic state. When a magnetomotive force (mmf) is applied to such a material, the magnetization lags the mmf because of a resistance to change in orientation of the particles involved.

**Ideal transformer** is model of a transformer with a coupling coefficient equal to unity and with the primary and secondary reactance's very large compared to the impedances connected to the transformer terminals.

**Induction motor** is an ac motor in which the rotating field produced by the stator induces a current and an opposing field in the rotor. The reaction of the fields creates the rotation force.

**Lenz's law** is a law stated by H.F.E. Lenz in 1833 to the effect that an induced current in a conductor is always in such a direction that its field opposes the change in the field causing the induced current.

**Magnetic induction** (magnetic flux density) defines the intensity and direction of magnetic field.

**Magnetic field** is state produced either by current flow or by a permanent magnet that can induce voltage in a conductor when the current changes in the conductor.

**Magnetomotive force (mmf)** is magnetizing force, measured in gilberts or ampere-turns.

**Mutual inductance** is the inductance of a voltage in one coil due to the field produced by an adjacent coil. Inductive coupling is accomplished through the mutual inductance of two adjacent coils.

**Permeability ( $\mu$ )** is the property of a magnetic substance determining the flux density produced in the substance by a magnetic field of a given intensity. The equation is  $\mu = B/H$ , where  $B$  is flow density in gauss and  $H$  is the field intensity in oersteds. The permeability of air is 1.

**Primary winding** is the input winding of a transformer.

**Relay** is an electromagnetic device having a fixed core and a pivoting mechanical linkage. An electric switch operated by an electromagnet.

**Reluctance** is the property of a material that opposes the passage of magnetic flux lines through it.

**Residual magnetism** is the magnetism that remains in a deenergized electromagnet.

**Self-inductance** is the property of a single conductor or a coil that causes it to induce a voltage in itself whenever there is a change of current flow.

**Solenoid** is an electromagnetic device having a movable core. An electrically operated switch.

**Transformer** is a device used to increase or decrease the voltage in an AC circuit. It couples electric energy between circuits by means of mutual inductance.

**Turns ratio** is ratio  $n$  equal to  $w_1/w_2$ , where  $w_1 = \textit{turns}$  in the primary and  $w_2 = \textit{turns}$  in the secondary of an ideal transformer.

**Magnetic induction** (magnetic flux density)  $B$ , ( $T$  – tesla) defines the intensity and direction of magnetic field.

## 5.6 Review Questions

1. What is the magnetic induction?
2. What is the magnetic field intensity?
3. What is the mmf and magnetic resistance?
4. Formulate Ampere's circuit law.
5. Why do we use magnetic relative permeability of a substance?
6. What does the left-hand rule explain?
7. State the right-hand rule.
8. What is the law of total current related to?
9. Set the direct problem of magnetic circuit analysis.
10. State the reverse problem.

## 5.7 Problems

### Examples

**Problem 1.** The rectilinear conductor of  $l=0.5m$  length is placed in an uniform magnetic field with  $B=1T$  induction and is carrying a current  $I=30A$ . The wire conductor is located at the angle  $\alpha_1 = \frac{\pi}{6}$  in one case, and  $\alpha_2 = \frac{\pi}{2}$  in another. Determine the force  $F$  exerting on the conductor.

#### Solution

According to Ampere's law the force acting on the conductor in the magnetic field for the two above cases is as follows:

$$F_1 = BIl \sin \alpha_1 = 1 \cdot 30 \cdot 0.5 \cdot 0.5 = 7.5(H),$$

$$F_2 = BIl \sin \alpha_2 = 1 \cdot 30 \cdot 0.5 \cdot 1 = 15(H).$$

**Problem 2.** The coil with the number of turns  $w=300$  is uniformly wound around a cast steel ring core having the rectangular cross-section. An inner diameter of the ring is  $d=20sm$ , an outer diameter is  $D=24sm$ , the ring is of  $b=6sm$  height and  $a = \frac{D-d}{2} = 2sm$  width.

Determine the coil current  $I$ , for which the magnetic flux in the core will be  $\Phi = 1.0 \cdot 10^{-3}Wb$ , assuming that: 1) the ferromagnetic ring is closed; 2) the ring core has an air gap  $\delta = 5mm$ ; and 3) the ring is manufactured of a nonmagnetic material.

#### Solution

The cross-sectional area of the magnetic core equals

$$S = b \cdot \frac{D-d}{2} = 6 \cdot \frac{24-20}{2} = 12sm^2.$$

1) As the magnetic induction in the core is determined by

$$B_c = \frac{\Phi}{S} = \frac{1.0 \cdot 10^{-3}}{12 \cdot 10^{-4}} = 0.835T,$$

the magnetic field intensity in the closed core can be found, for example, with the help of B-H curve like that shown in Fig. 5.4. Let it be  $H_c = 5 \frac{A}{sm} = 500 \frac{A}{m}$ .

The average length of the field line (the average core length) we have to calculate to find the magnetizing force is as follows:

$$l_{av} = \pi \cdot D_{av} = \pi \cdot \frac{D+d}{2} = 3.14 \cdot \frac{24+20}{2} = 69.2sm = 0.692m.$$

Then, for the closed core, we'll get the magnetomotive force:

$$F_1 = H_{av} \cdot l_{av} = 500 \cdot 0.692 = 346 \text{ At}.$$

Hence, the driving current of the coil is given by:

$$I = \frac{H_c \cdot l_{av}}{w} = \frac{346}{300} = 1.15 \text{ A}.$$

2) If the ring has the air gap we analyze the magnetic circuit as consisting of two parts (magnetic and nonmagnetic) connected in series. Neglecting the flux dissipation, we can assume that magnetic inductions in the air gap and in the magnetic core are the same:

$$B_0 = B_c = 0.835 \text{ T}.$$

The magnetic field intensity in the air gap is then defined by

$$H_0 = \frac{B_0}{\mu_0} = \frac{0.835}{4\pi \cdot 10^{-7}} = 6.68 \cdot 10^5 \frac{\text{A}}{\text{m}}.$$

Again, the magnetic field intensity in the steel core can be determined with the help of B-H curve, for the operation point associated with  $B_c = 0.835 \text{ T}$ :

$$H_c = 500 \frac{\text{A}}{\text{m}}.$$

Thus, for the case when the inductor core got the air gap the magnetomotive force is found on the basis of solving the equation:

$$F_2 = w \cdot I_2 = H_c \cdot l_{av} + \delta \cdot H_0 = 500 \cdot 0.678 + 0.5 \cdot 10^{-3} \cdot 6.68 \cdot 10^5 = 677.5 \text{ A}.$$

Whence, the coil must be energized by the current:

$$I' = \frac{F_2}{w} = \frac{667.5}{300} = 2.25 \text{ A}.$$

3) In the same coil being wound around a completely nonmagnetic core, the magnetic induction  $B_c = B_0 = 0.835 \text{ T}$  might be established because of the amount of the magnetic field intensity

$$H_0 = \frac{B_0}{\mu_0} = \frac{0.835}{4\pi \cdot 10^{-7}} = 668000 \frac{\text{A}}{\text{m}},$$

resulted from the winding current

$$I = \frac{H_c \cdot l_{av}}{w} = \frac{668000 \cdot 0.692}{300} = 1535 \text{ A}.$$

**Problem 3.** An inductor coil with  $w=500$  turns is placed in the uniform field, which magnetic induction grew up from  $0$  to  $0.8\text{ T}$  in time  $0.1\text{ s}$ . There is a resistor  $R=20\Omega$  that is connected to the coil terminals.

Determine the current induced in the coil by the flux changing, also the power dissipated by the resistor, if the inductor coil cross-sectional area  $S=12\text{sm}^2$  and winding wire resistance  $R_c=24\Omega$ .

Solution

The emf induced in the coil:

$$e = -w \frac{d\Phi}{dt} = -wS \frac{dB}{dt} = -500 \cdot 12 \cdot 10^{-4} \cdot \frac{0,8}{0,1} = -4,8V .$$

After computing the total resistance  $R=24\Omega$  of the circuit we determine the coil current:

$$I = \frac{V}{R} = \frac{4,8}{24} = 0,2\text{ A}$$

and the resistor dissipated power

$$P = I^2 R = (0,2)^2 \cdot 20 = 0,8W .$$

**Supplement problems**

**Problem 4.** Two wire conductors of  $l=2\text{m}$  length, each conducting the current  $I_1=I_2=100\text{A}$  in the same direction, are placed in space parallel each other at the distance  $a=5\text{sm}$ .

Determine forces  $F_1$  and  $F_2$  acting on each conductor, and the direction of interaction between them. Point the direction of the magnetic field induction that the conductor currents gave rise to.

Answer:  $F_1=F_2=F=0,08\text{H}$ .

**Problem 5.** Linear conductor with the length  $l=0,3\text{m}$ , through which the current  $I=12\text{A}$  flows, is placed in uniform magnetic field with the magnetic induction  $B=0,5\text{T}$ . Determine the force working on the conductor, if it's spaced:

- a) perpendicularly to the flux lines;
- b) lengthwise to the magnetic field lines.

**Problem 6.** Uniform magnetic field with the magnetic induction  $B=1,0\text{T}$  worked on the linear current-conducting wire with the force  $F=0,5\text{H}$ . The conductor is perpendicular to the magnetic field lines and has the length  $l=20\text{sm}$ .

Determine the current flow through the conductor.



**Problem 7.** Determine the number of turns and the conductor length for cylindrical coil winding, if the coil length  $l=20\text{sm}$  and its diameter  $d=1,6\text{sm}$ . With the current in the winding  $I=3\text{A}$  the magnetic flux collects value  $\Phi = 3 \cdot 10^{-7} \text{Wb}$ . Determine the magnetic field intensity in the inner space of the coil.

**Problem 8.** A cylindrical coil of  $l=15\text{sm}$  length and  $S=6,5\text{sm}^2$  cross-section area contains  $w=600$  turns.

Determine the inductance of such a coil without any ferromagnetic core and with the core with  $\mu = 800$  permeability.

How much will the coil inductance change, if the length and magnetic core permeability are increased three and four times, respectively, and the number of turns is decreased by two?

**Problem 9.** Upon the ring made of electro-technical steel (the outer diameter  $D=42\text{mm}$ , the inner diameter  $d=30\text{mm}$ , the height  $b=8\text{mm}$ ), 200 turns of wire was uniformly wound.

Determine the magnetic flux in the ring if the winding carries the current  $I=0.5\text{A}$ ; find the coil inductance.

**Problem 10.** Taking conditions of the previous problem determine the current flow through the coil for creating the same core magnetic flux, if the core ring collected the air gap  $\delta = 0.5\text{mm}$ .

**Problem 11.** Under what conditions is the force worked on the conductor with the current in the magnetic field absent?

**Problem 12.** How does the intensity of the magnetic field depend upon the substance surrounding the source of the field?

## 5.8 Practice

### «Investigation of Inductive-Coupled Circuits»

**Objective:** to gain skills of analysis and practical investigation of AC circuits that experience the magnetic coupling between each other (inductive-coupled circuits).

#### Work task

1. Determining the value of mutual inductance of two coils.
2. Conducting the experiments needed for variometer winding parameters determination, as well as the magnetic coupling factor calculation.

### Theory and preparation

In addition to self-inductance  $L$ , it is always important to see the **magnetic coupling** that can occur between neighboring circuits each containing an inductor.

In the inductive-coupled circuits the magnetic flux linkage of each circuit is created both by the current flowing in the same circuit and the current flowing through the other one:

$$\psi_{1\text{ full}} = L_1 \cdot i_1 \pm M \cdot i_2,$$

$$\psi_{2\text{ full}} = L_2 \cdot i_2 \pm M \cdot i_1,$$

where  $M$  is the quantity called **mutual inductance**. The mutual inductance is defined by the equation  $v_2 = M di_1/dt$  that is analogous to the expression  $v = L di/dt$  defining the self-induction of an inductor coil.

In the expressions above the “plus” sign corresponds to the same (called agreed) directions of self-induction flux and mutual induction flux, and “minus” corresponds to the counter (opposite) direction of the fluxes.

To emphasize the problem of the matched or mismatched interaction of self-induction and mutual induction fluxes and to indicate the polarity of the coupling, the concept of the same terminal names in neighboring coils is introduced: if the dots shown in Fig. 5.10 for this purpose are at the same ends of the coils, the voltage induced in coil 2 by a current in coil 1 has the same polarity as the voltage induced by the same current in coil 1; otherwise the voltages are in opposition.

The electromotive forces induced in the two coils are defined by the law of electromagnetic induction:

$$e_1 = \frac{d\psi_{1\text{ full}}}{dt} = -L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt};$$

$$e_2 = \frac{d\psi_{2\text{ full}}}{dt} = -L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}.$$

Or in a complex form:

$$\dot{E}_1 = -j \cdot \omega \cdot L_1 \cdot I_1 \pm j \cdot \omega \cdot M \cdot I_2;$$

$$\dot{E}_2 = -j \cdot \omega \cdot L_2 \cdot I_2 \pm j \cdot \omega \cdot M \cdot I_1.$$

If needed, mutual inductance can be determined experimentally (explaining schematics one can find in Fig. 5.10 and 5.11).

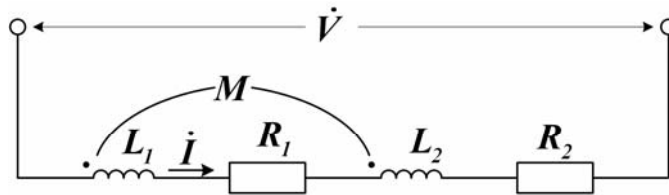


Fig. 5.10

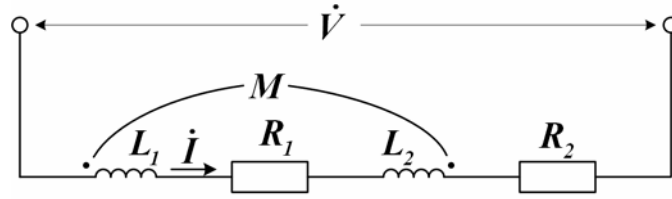


Fig. 5.11

An approach to do that is as follows.

First, connect the both coils in series with the agreed position as shown in Fig. 5.10, and energize the circuit with a voltage  $V$ . In this case we may write

$$\dot{V} = [R_1 + R_2 + j \cdot \omega \cdot (L_1 + L_2 + 2M)] \cdot \dot{I}.$$

Then, connect the coils in series-opposite direction (see Fig. 5.11) and write down the following equation:

$$\dot{V} = [R_1 + R_2 + j \cdot \omega \cdot (L_1 + L_2 - 2M)] \cdot \dot{I}.$$

It is easy to see from these relations that equivalent inductive reactance of the circuit for the agreed and opposite connections will be as follows:

$$\begin{aligned} X_{agree} &= \omega \cdot (L_1 + L_2 + 2M), \\ X_{opposite} &= \omega \cdot (L_1 + L_2 - 2M). \end{aligned}$$

If we measure the circuit reactance for both those connections, the mutual inductance may be calculated as

$$M = \frac{X_{agree} - X_{opposite}}{4 \cdot \omega},$$

where  $\omega$  is the current supply radian frequency.

The value of mutual inductance can also be determined in a different way.

When we have the open circuit operation of the secondary winding in the circuit called the air transformer (Fig. 5.12) with the sine voltage  $V_1$  applied across the primary winding, the electromotive force induced in the secondary winding due to the primary current,  $\dot{E}_2 = -j\omega M \cdot I_{10}$ , appears at the output in the form of the open-circuit secondary voltage  $V_{20}$ . Hence, the mutual inductance of two coupled inductors of the arrangement like that in Fig. 5.12 can be found from

$$M = \frac{V_{20}}{\omega \cdot I_{10}},$$

where the term  $I_{10}$  is called the open-circuit transformer supply current. A transformer consumes this current when operating in the open circuit mode.

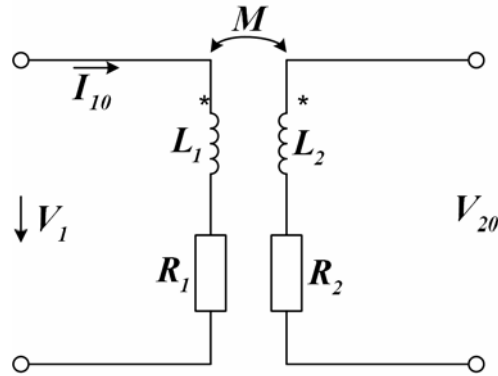


Fig. 5.12

The power of magnetic coupling of two coils is characterized by a **coefficient of coupling** (magnet coupling factor)

$$k = \frac{M}{\sqrt{L_1 L_2}}.$$

Also, with the help of the relation

$$\dot{I}_2 = -\frac{jX_M \cdot \dot{I}_1}{R_{2L} + jX_{2L}} \dots$$

derived from the transformer equations that corresponded to transformer operation with a load, one can evaluate the transformer **turns ratio** (transformation factor):

$$k_T = \frac{I_2}{I_1} = \frac{X_M}{X_{2L}},$$

where  $X_{2L}$  is the total secondary reactance, including both reactance of the secondary coil and the load.

### Procedure

Experimental work is carried out on the laboratory stand with the use of the block of single-phase AC voltage sources, the variometer, special plugs with mounted elements and conductors for circuit assembling, measuring devices.

#### 1. Investigation of Two Inductive-coupled Series Coils

In order to carry out the work it is necessary:

- to examine the stand, the object of investigation, as well as measuring instruments with the related characteristics written down in the test report.

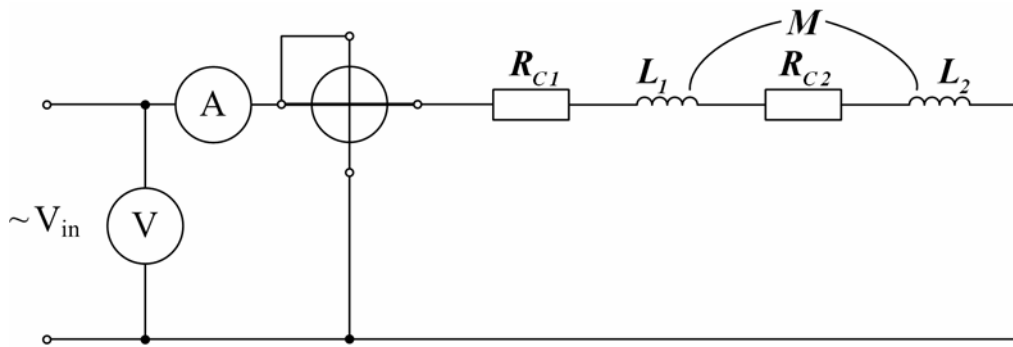


Fig. 5.13

The object to investigate the mutual inductance of two inductors will be the variometer known from preceding sections of this handbook (this concerns the necessary instruments too). The two variometer coils should be connected in series for both agreed and opposite interaction of the inductors;

- to draw the circuit diagram of the experiment shown in Fig. 5.13;
- to prepare the blank for measurement and following calculation recordings (Table 5.1).

Table 5.1

Coils connection	Measured			Calculated				
	$V_{in}$	$I$	$P$	$R_{ke}$	$Z_{ke}$	$X_{ke}$	$L_{ke}$	$M$
	$V$	$V$	$W$	$\Omega$	$\Omega$	$\Omega$	$H$	$H$
agreed								
opposite								

After that set up the circuit. Taking the admission of the instructor you may switch the stand on and write down your readings in the table.

## 2. Open circuit test of the transformer without the ferromagnetic core

Draw the scheme corresponding to open circuited of the air transformer, including the devices of the measurement of the quantities appointed in the Table 5.2 (see Fig. 5.12).

*The note.* Active resistance of the secondary winding should be measured by the digital device, and its inductance should be calculated using the data of table 5.1.

Set up the circuit. To switch on the stand after the correctness of connections was checked by the teacher, to write down readings of all meters in the Table 5.2 for all positions of the mobile coil.

Table 5.2

$\alpha$	$I_1$	$P_0$	$V_{20}$	$Z_{k1}$	$R_{k1}$	$X_{k1}$	$L_{k1}$	$L_{k2}$	$R_{k2}$	$M$	$K$
degrees	A	W	V	$\Omega$	$\Omega$	$\Omega$	H	H	$\Omega$	H	-
0											
30											
60											
90											
120											
150											
180											

### Summarizing

1. Calculate the values specified in the Table 5.1 and 5.2 using experimental data.
2. Construct depends  $M = f(\alpha)$  and  $k = f(\alpha)$  according to the Table 5.2.
3. Make the conclusions according to results of experimental work.

### Questions to be answered

1. What circuits are called inductive – coupled?
2. How is the full magnetic linkage determined?
3. How are expressed the electromotive forces of self-induction and mutual inductance in the coils?
4. In which way and according to what scheme can we obtain the value of mutual induction of two coils?
5. How is the complex the current determined in inductive-coupled coils, connected in series?
6. Make the system of equations in complex form for balance of voltages for primary and secondary circuits of the transformer.
7. How are the coefficients of coupling of two coils determined?

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Зимовин Анатолий Яковлевич, Постников Владимир Николаевич,  
Волчанская Любовь Ивановна

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Национальный аэрокосмический университет им. Н.Е. Жуковского  
«Харьковский авиационный институт»  
61070, Харьков-70, ул. Чкалова, 17  
<http://www.khai.edu>  
Издательский центр «ХАИ»  
61070, Харьков-70, ул. Чкалова, 17  
[izdat@khai.edu](mailto:izdat@khai.edu)