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Estimation of necessary wing loading of a missile

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Wing loading severely affects the mass of a missile as well as its flight performance. For airplanes this parameter must not exceed allowable values calculated from different requirements especially related to such cases as lift-off, landing, cruise flight and aircraft maneuverability. For missiles wing loading is determined considering launch conditions and providing the necessary maneuverability. Appropriate estimation of wing loading at the initial design stages guarantees the minimal mass of an aircraft with all tactical requirements met.

Review of available literature, related to missile design, has shown that the problem of optimal wing loading estimation contains lengthy and quite approximate analytical expressions.

This article is dedicated to the development of a missile wing loading estimation technique that provides minimal propellant mass fraction and total mass of an aircraft while meeting tactical requirements.

Impact of wing loading onto propellant mass fraction, maximal maneuverability and total mass of a missile is considered. The algorithm of optimal wing loading estimation, which provides necessary tactical characteristics of a missile being designed, is proposed. We define simple polynomial approximations of both the trajectory and the velocity profile. Further analysis is being conducted using two considerations: for an air-based missile the value of wing loading has to provide flight during launch without fall movement as well as the maximal maneuverability at the moment when a missile intercepts the target.

It is shown that for the wing loading in the range from 300 to 1000 kg/m^2 the propellant mass fraction changes rapidly, and inaccurate selection of wing loading may lead to obtaining of an incorrect value of propellant mass fraction. For maximal maneuverability less than 40, inappropriate selection of wing loading may cause significant numerical error. Analysis of relation between wing loading and total mass of a missile revealed that there is a critical value of wing loading which depends on initial data and represents the low limit of an acceptable range.

Keywords*:* propellant mass, wing loading, missile, total mass.

Introduction

Wing loading p_0, which is determined as total mass of an aircraft m_0 divided by the wing area S, significantly affects the missile's mass and its flight performance. For airplanes this parameter must not exceed allowable values calculated according to different requirements especially related to such cases as lift-off, landing, cruise flight as well as airplane's maneuverability [1]. For missiles wing loading is determined considering launch conditions and providing the necessary maneuverability. Appropriate estimation of wing loading at the initial design stages guarantees the minimal mass of a missile with all tactical requirements met; hence the problem of the necessary wing loading estimation of a missile reveals scientific interest.

1. Problem description

The review of available literature related to missile design has shown that the problem of optimal wing loading estimation is considered in the work [2] only, and analytical expressions obtained in that work are lengthy and rather approximate.

This article is dedicated to the development of a missile wing loading estimation technique that provides minimal propellant mass fraction and total mass of a missile while meeting tactical requirements.

2. Research methodology

Missile design starts from the statistical data analysis resulting in a list of average statistical characteristics of prototypes. On the basis of this information there can be selected the appearance of a missile, relative sizes of its lifting surfaces (allowing calculation of aerodynamic coefficients) and wing loading. However, taking into account tactical requirements for a missile being designed which may differ from the ones of prototypes an average wing loading may also be different from the optimal value.

Let us consider the algorithm of optimal wing loading estimation which provides necessary tactical characteristics of a missile being designed.

Having selected the missile appearance we begin the ballistic design whose main goal is the calculation of propellant mass fraction μ_t that will be considered as a function of wing loading: $\mu_t(p_0)$.

When the motor thrust is constant, the propellant mass fraction is calculated as [2]

$$\mu_t(p_0) = \frac{\mu_{tv} + \mu_{th} + \mu_{ta}(p_0)}{1 + \mu_{tvc} + 0{,}5 \cdot \mu_{th}}. \quad (1)$$

The formula above contains following components [2]:

$$\mu_{tv} = \frac{v_k - v_0}{I}; \quad (2)$$

$$\mu_{tvc} = \frac{v_k - v_a}{I}; \quad (3)$$

$$\mu_{th} = \frac{\sin\theta_a}{I} \int_0^{t_k} g(h)dt; \quad (4)$$

$$\mu_{ta}(p_0) = \frac{1}{2 \cdot I \cdot p_0} \int_0^{t_k} c_{xa}\left(\frac{v(\mathrm{t})}{a(h)}\right) \cdot \rho(h) \cdot v(t)^2 dt\,, \quad (5)$$

where t_k is target interception time;

v_0, v_k and v_a is launch, final and average velocity of a missile;

$c_{xa}\left(\frac{v(\mathrm{t})}{a(h)}\right)$ is drag coefficient obtained as a result of aerodynamic analysis of a basic missile [3];

I is propellant specific impulse;

θ_a is average trajectory slope;

$\rho(h)$, $a(h)$ and $g(h)$ are air density, speed of sound and free-fall acceleration correspondingly (these values may be approximated using a standard atmosphere model, e.g. CA-81 [4]).

Considering launch angle θ_0, launch altitude h_0 and coordinates of destination point (x_k, h_k) to be known it is suitable to define the trajectory shape as a second-order polynomial function

$$h(x) = h_0 + a_1 \cdot x + a_2 \cdot x^2, \tag{6}$$

where a_1 and a_2 are obtained from the conditions mentioned above

$$, \; a_1 = \tan\theta_0, \; a_2 = \frac{h_k - h_0 - a_1 \cdot x_k}{x_k^2}. \tag{7}$$

Flight path is calculated as follows

$$s = \int_0^{x_k} \sqrt{1 + \left(\frac{dh}{dx}\right)^2} dx. \tag{8}$$

An average velocity within trajectory

$$v_a = \frac{s}{t_k}. \tag{9}$$

Velocity profile may be defined with any functional dependency, providing necessary launch, final and average velocities [2]. Obviously, the missile’s acceleration at launch must be non-negative. The most real approximation comes with a third-order polynomial dependency

$$v(t) = v_0 + b_1 \cdot t + b_2 \cdot t^2 + b_3 \cdot t^3. \tag{10}$$

Assuming zero acceleration at launch, coefficients b_i are calculated using the following formulas:

$$b_1 = 0, \; b_3 = 4\frac{v_k - 3 \cdot v_a + 2 \cdot v_0}{t_k^3}, \; b_2 = \frac{v_k - v_0 - b_3 \cdot t_k^3}{t_k^2}. \tag{11}$$

To find μ_{th} and μ_{ta} it is necessary to have altitude as a function of time, i.e. to match velocity profile and trajectory shape. This dependency may be written as follows:

$$h(t) = h_0 + \beta_1 \cdot t + \beta_2 \cdot t^2 + \beta_3 \cdot t^3. \tag{12}$$

Unknown coefficients β_i may be found from the following system of equations:

$$h(t_i) = h(x_i), i = \overline{1,3}. \tag{13}$$

The appearance of dependency (1) for one of the considered cases is shown in Fig. 1.

If a missile is air-based the value of wing loading p_{01} has to provide flight during launch without fall movement. The needed value may be found from the equation below

$$p_{01} - \frac{C_y^\alpha\left(\frac{v_0}{a(h_0)}\right) \cdot \alpha_{\max} \cdot \frac{\rho(h_0) \cdot v_0^2}{2}}{g(h_0)} = 0. \tag{14}$$

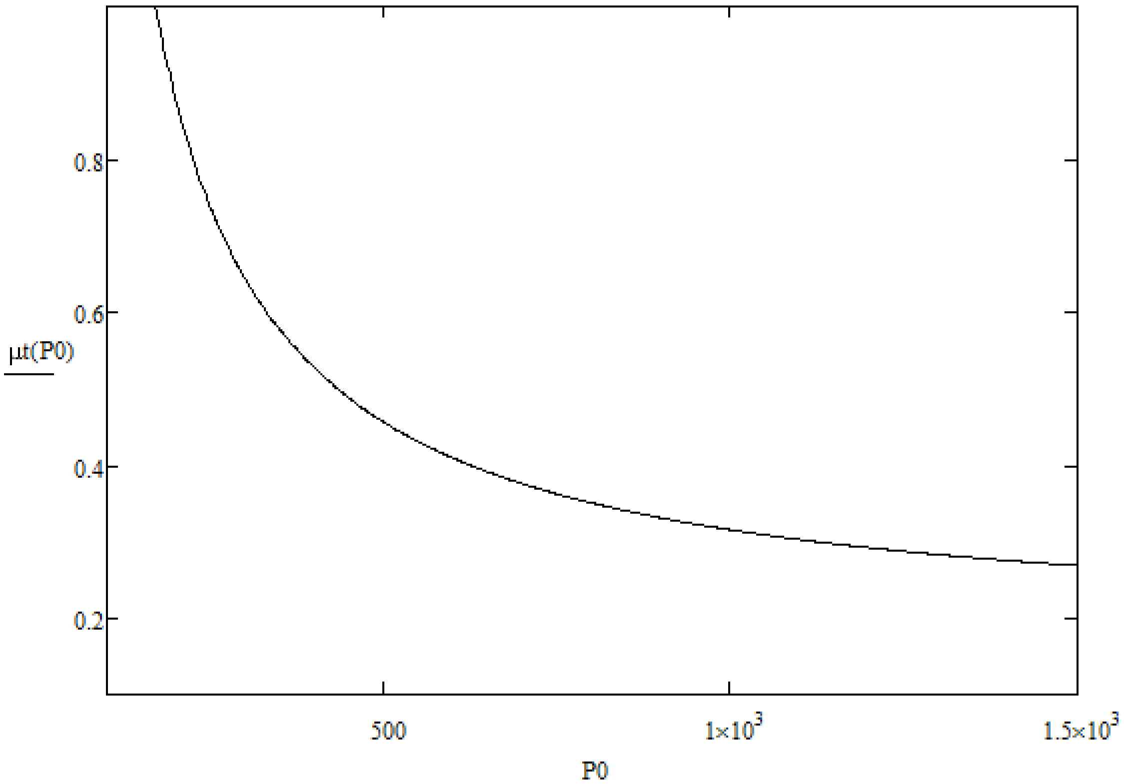

Fig. 1. Appearance of relation between wing loading p_0
and propellant mass fraction μ_t

To provide maximal maneuverability n_{ya} at the moment of interception the value of wing loading p_{02} must comply with the following equation

$$p_{02} - \frac{C_y^{\alpha}\left(\frac{v_k}{a(h_k)}\right)\cdot\alpha_{\max}\cdot\frac{\rho(h_k)\cdot v_k^2}{2}}{g(h_k)\cdot n_{ya}\cdot(1-\mu_t(p_{02})} = 0, \qquad (15)$$

where C_y^{α} is taken from aerodynamic analysis of the basic missile [3];

$\alpha_{\max}$ is maximal angle of attack.

After all calculations are done the minimal value of wing loading should be selected.

3. Results

The following figures show graphical dependencies between maximal maneuverability and wing loading (Fig. 2), wing loading and total mass of a missile (Fig. 3).

The relation between wing loading and propellant mass fraction (Fig. 1) has maximal gradients of μ_t changing in a range of $p_0 = 300...1000$ кг/м^2. Thus inaccurate selection of p_0 within this range may cause obtaining an incorrect value of propellant mass fraction.

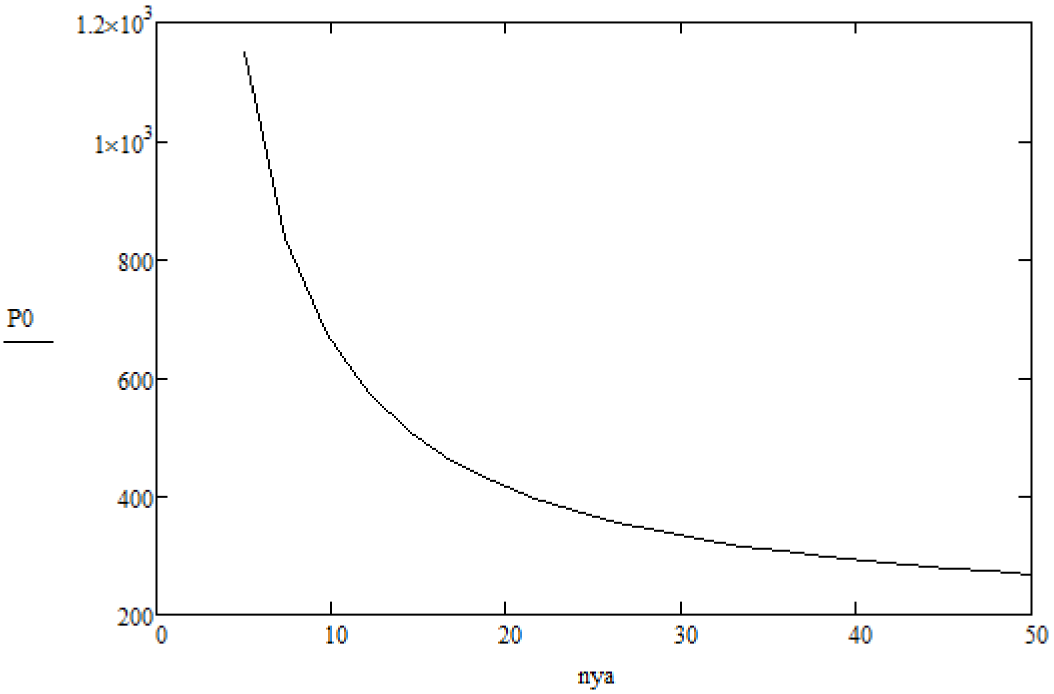

Fig. 2. Relation between maximal maneuverability n_{ya} and wing loading p_0

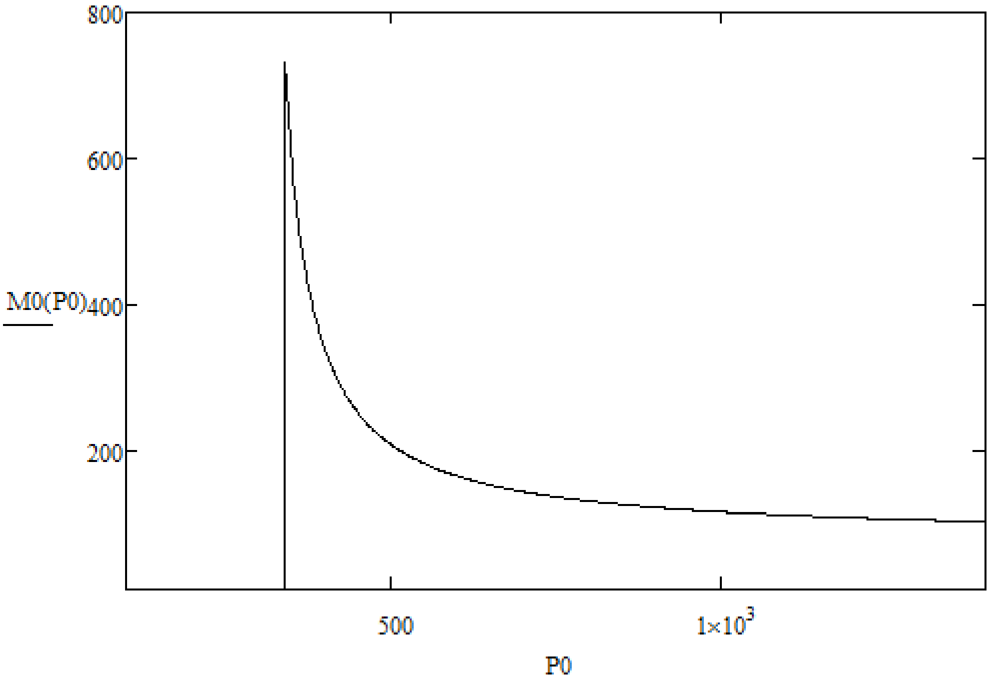

Fig. 3. Relation between wing loading p_0 and total mass of a missile m_0

4. Discussion

The analysis of $p_0(n_{ya})$ dependency (Fig. 2) shows that when $n_{ya} < 40$, the necessary wing loading decreases rapidly and hence wrong value of this parameter may lead to significant numerical error.

As it comes from Fig. 3 there is such a value of p_{0kp} below which a missile cannot be designed. This comes from the fact that when $p_0 < p_{0kp}$, the equation of missile existence is violated (the sum of mass fractions of all units of a missile becomes greater than 1).

Conclusions

Two main results are obtained:

1) wing loading estimation technique has been suggested which provides minimal values of both propellant mass and total mass of a missile while meeting all tactical requirements;

2) it has been shown that inappropriate selection of wing loading may cause a significant numerical error.

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Визначення потрібного питомого навантаження на крило ракети

Величина питомого навантаження на крило істотно впливає на масу літального апарату і його льотні характеристики. Для літаків прийняте значення цього параметра має бути менше або дорівнювати мінімально допустимих значень, отриманих з умови забезпечення необхідних льотних характеристик, особливо злітно-посадкових, крейсерського польоту і маневрених. Для ракет цей параметр вибирається з умови пуску і забезпечення необхідної маневреності. Коректний вибір питомого навантаження на крило на початкових етапах проектування гарантує створення літального апарату мінімальної маси з заданими льотними характеристиками.

У доступній літературі, присвяченій проектуванню ракет, розв’язання задачі вибору оптимального питомого навантаження на крило зводиться до ітераційного застосування громіздких аналітичних формул, що носять вельми наближений характер.

Стаття присвячена розробці методики визначення питомого навантаження на крило ракети, що забезпечує мінімальний запас палива і мінімальну масу літального апарату при заданих його тактичних характеристиках.

Розглянуто вплив питомого навантаження на крило на запас палива, максимальне перевантаження і на стартову масу літального апарату.

Запропоновано алгоритм визначення питомого навантаження на крило, який забезпечує необхідні тактичні характеристики літального апарату, що проектується. Форма траєкторії ті профіль швидкостей апроксимуються поліномами. Подальший аналіз виконується з урахуванням двох умов: для літальних апаратів повітряного базування питоме навантаження має забезпечити старт без просідання, а також максимальне перевантаження в момент перехоплення цілі.

Показано, що в діапазоні зміни питомого навантаження на крило від 300 до 1000 кг/м^2 спостерігається найбільший градієнт зміни відносного запасу палива, тому неточність завдання цього навантаження в цьому діапазоні може привести до помилкового значення відносного запасу палива. Відзначено, що при максимальному перевантаженні менше 40 одиниць, неправильний вибір питомого навантаження на крило може привести до істотних похибок. Аналіз залежності стартової маси ракети від питомого навантаження на крило показав, що існує критичне значення питомого навантаження на крило, яке залежить від вихідних даних, що є мінімально допустимим.

Ключові слова: запас палива, питоме навантаження на крило, ракета, стартова маса.

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