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Quasi-one-dimensional mathematical model of processes in Hall effect and plasma-ion thrusters

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Plasma-ion thrusters with a radial magnetic field in ionization chamber and Hall effect thrusters are electrostatic electric propulsion thrusters with closed electron drift. Axial symmetry in the dynamics of the components of propellant in these thrusters allows to write the equations of plasma-dynamics for electrons, ions and neutral atoms in two-dimensional axial-radial form. Attempts to reduce the equations to simpler one-dimensional form by simply removing the components with radius differentiation lead to the loss in the description of important effects, responsible for values of thruster performance. At the same time, a significant disadvantage of gas dynamics equation set is its fundamental openness – the correspondence between the number of unknown variables and equations is achieved approximately basing on some assumptions. In traditional form of gas dynamics, such closeness is made under the assumption of thermodynamic equilibrium with velocity distribution functions of components close to Maxwell one, which is the limit result of collisions. The use of such approximation to plasma components dynamics in electric propulsion thrusters is impossible due to the rarefaction of the substance in them. A mathematical model of two-component plasma-dynamics is represented in stationary form to describe the processes in the Hall effect thruster channel and the ionization chamber of plasma-ion thruster with radial magnetic field. Due to the impossibility of using the method of local thermodynamic equilibrium to describe the rarefied substance in electric propulsion thruster, a more advanced form of equations is used. A more reliable means of approximate closure of the set of equations is proposed in the description of the rarified gas. An approach to the description of the specifics of electrons energy transfer from the plasma to the walls of the channel, as well as the non-mirror reflection of electrons from the potential barrier within the Langmuir layer is shown. A method of averaging the parameters over the cross section of the channel is proposed, which allows to convert the equation into a quasi-one-dimensional form with the preservation of charge, momentum and energy losses on the channel walls.

Keywords: Hall effect thruster; plasma-ion thruster; Langmuir layer; pressure tensor

Introduction

Hall effect thruster (HET) relates to electrostatic thrusters where acceleration of ions is made by electrostatic component of electromagnetic field produced by electrode system and with direction of electric field tension almost in parallel to necessary direction of thrust. It is enough for thrust produce but without any other factor it would mean acceleration of electrons in opposite direction with extreme electron current and the most their part in power consumption.

To prevent this effect the magnetic field is used in HET. Similar processes take place in ionization chamber of plasma-ion thruster (PIT) with radial magnetic field.

The radial magnetic field in the HET created by a special magnetic system (Fig. 1) ensures the confinement of electrons in the region of the most intense ionization and eliminates unacceptably high values of the electron current to the anode so that the main part of the electric energy is spent on the ion component of the current directly involved in creating thrust.

The HET chamber is an annular channel bounded in the radial direction by the

dielectric walls of the chamber and in the axial direction by the gas distribution anode. The working fluid (usually an inert gas – most often Xe) is fed into the thruster chamber through the holes in the anode.

To compensate the current of the external beam and for ionization of atoms in the thruster chamber, one electron source is used, called a cathode-neutralizer. The total current from the cathode I_d (discharge current HET) is related to the ion current of the external beam I_i as a rule as follows: $I_d = (1.2 - 1.3)I_i$. Thus, an electron current equal to the ionic current goes into the external beam $|I_e^+| = I_i$, and the rest of electrons enter the thruster chamber $|I_e^-| = (0.2 - 0.3)I_i$.

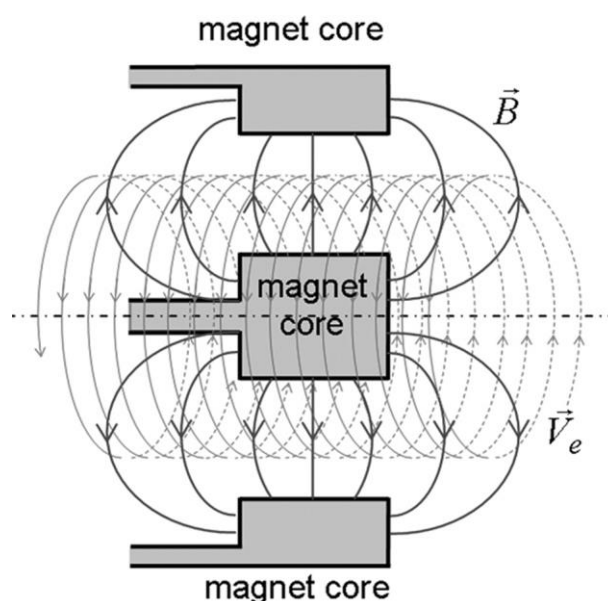


Fig. 1. “Closed” electron drift

Electrons enter the thruster chamber with moderate energy. While moving into thruster chamber the electron energy and the ionization cross-section increase. Directly near the anode the sign of the axial projection of the mass flow velocity and ion current density changes – ions born in this region “fall” onto the walls of the chamber and anode. Thus, the electron current to the anode must exceed the discharge current by the amount of ion current to the anode.

1. Formulation of the problem

Gas dynamics equation set is fundamentally opened, and the number of equations is brought into correspondence with the number of unknowns only approximately – as a result of certain assumptions.

Due to the rarefaction of the substance in the electric propulsion thrusters the local thermodynamic equilibrium method is insufficient to describe the processes there, and there is a need to use more extended form of equations as it was done in the article [1] but here – in stationary form:

- substance equation of the particle of α sort:

$$\nabla \cdot \vec{\Gamma}_\alpha = \frac{\delta n_\alpha}{\delta t}, \quad (1)$$

- motion equation:

$$\nabla \cdot \mathbf{\Pi}_\alpha - q_\alpha (n_\alpha \vec{E} + \vec{\Gamma}_\alpha \times \vec{B}) = \frac{\delta \vec{p}_\alpha^{(V)}}{\delta t}, \quad (2)$$

- pressure tensor equation:

$$\nabla \cdot \left(\vec{V}_\alpha \mathbf{P}_\alpha + \mathbf{G}_\alpha \right) - 2 \frac{q_\alpha}{m_\alpha} [\mathbf{P}_\alpha \times \vec{B}] + 2 [\mathbf{P}_\alpha \cdot \nabla \vec{V}_\alpha] = \frac{\delta \mathbf{P}_\alpha}{\delta t}, \quad (3)$$

- "pressure flow" equation:

$$\nabla \cdot \left(\vec{V}_\alpha \mathbf{G}_\alpha + \mathbf{W}_\alpha \right) - 3 \frac{q_\alpha}{m_\alpha} [\mathbf{G}_\alpha \times \vec{B}] + 3 \left[\mathbf{G}_\alpha \cdot \nabla \vec{V}_\alpha + \frac{\mathbf{P}_\alpha \nabla \cdot \mathbf{P}_\alpha}{m_\alpha n_\alpha} \right] = \frac{\delta \mathbf{G}_\alpha}{\delta t}, \quad (4)$$

- energy equation:

$$\nabla \cdot \left(\vec{V}_\alpha \left(\frac{m_\alpha n_\alpha V_\alpha^2}{2} + \frac{3}{2} P_\alpha \right) + \vec{V}_\alpha \cdot \mathbf{P}_\alpha + \vec{q}_\alpha^{(cond)} \right) - q_\alpha \vec{\Gamma}_\alpha \cdot \vec{E} = \frac{\delta \varepsilon_\alpha^{(V)}}{\delta t}, \quad (5)$$

where n_α – population (number of particles in unit volume), $1/m^3$;

$\vec{\Gamma}_\alpha = n_\alpha \langle \vec{v}_\alpha \rangle = n_\alpha \vec{V}_\alpha$ – particles flow density, $1/(m^2s)$;

$\frac{\delta}{\delta t}$ – parameter change in time because of collisions;

$\mathbf{\Pi}_\alpha = m_\alpha n_\alpha \langle \vec{v}_\alpha \vec{v}_\alpha \rangle = m_\alpha n_\alpha \vec{V}_\alpha \vec{V}_\alpha + \mathbf{P}_\alpha$ – momentum flow density 2nd rank tensor, Pa;

q_α – charge, C;

\vec{E} – electric field tension, V/m;

\vec{B} – magnetic induction, T;

$\vec{p}_\alpha^{(V)}$ – momentum density, $kg/(m^2s)$;

$\mathbf{P}_\alpha = m_\alpha n_\alpha \langle \vec{v}_\alpha \vec{v}_\alpha \rangle$ – pressure tensor, Pa;

$\mathbf{G}_\alpha = m_\alpha n_\alpha \langle \vec{v}_\alpha \vec{v}_\alpha \vec{v}_\alpha \rangle$ – "pressure flow" 3rd rank tensor, W/m^2 ;

m_α – mass, kg;

$\vec{V}_\alpha = \langle \vec{v}_\alpha \rangle$ – mass flow velocity m/s;

\vec{v}_α – particle's velocity m/s;

$\vec{v}_\alpha = \vec{v}_\alpha - \vec{V}_\alpha$ – velocity linear deviation (chaotic velocity), m/s;

$\mathbf{W}_\alpha = m_\alpha n_\alpha \langle \vec{v}_\alpha \vec{v}_\alpha \vec{v}_\alpha \vec{v}_\alpha \rangle$ – nameless 4th rank tensor, $kg \cdot m/s^4$;

$\varepsilon_\alpha^{(V)}$ – energy density, J/m^3 ;

$\vec{q}_\alpha^{(cond)}$ – thermal conductivity, W/m^2 ;

$\langle \rangle, \lfloor \rfloor$ – averaging and tensor symmetrization operations.

A half of pressure tensor trace is the density of particles thermal (chaotic) movement:

$$\frac{1}{2} Tr \mathbf{P}_\alpha = \frac{1}{2} \sum_m P_\alpha^{(mm)} = \frac{3}{2} P_\alpha = \frac{3}{2} n_\alpha k T_\alpha. \quad (6)$$

In fact, the 3rd rank tensor "pressure flow" \mathbf{G}_α has not own name but a half of it's vector trace is thermal conductivity:

$$\frac{1}{2} Tr \mathbf{G}_\alpha = \frac{1}{2} \sum_{mn} i_n G_\alpha^{(mm,n)} = \vec{q}_\alpha^{(cond)}. \quad (7)$$

Equation similar to (3) is used, for example, in the book [2] in components but without the discussion about "pressure flow" \mathbf{G}_α as well as without equation (4).

Energy equation (5) here is not independent one but can be obtained like linear combination of equations (1) – (3). Equation set (1) – (4) is opened and can be approximately closed using the generalization of Maxwell form of tensor made in [1]:

$$\mathbf{W}_\alpha = 3 \frac{\lfloor \mathbf{P}_\alpha \mathbf{P}_\alpha \rfloor}{m_\alpha m_\alpha}, \quad (8)$$

which gives:

$$\nabla \cdot (\vec{V}_\alpha \mathbf{G}_\alpha) - 3 \frac{q_\alpha}{m_\alpha} \lfloor \mathbf{G}_\alpha \times \vec{B} \rfloor + 3 \left[\mathbf{G}_\alpha \cdot \nabla \vec{V}_\alpha + \frac{\mathbf{P}_\alpha}{m_\alpha} \cdot \nabla \left(\frac{\mathbf{P}_\alpha}{n_\alpha} \right) \right] = \frac{\delta \mathbf{G}_\alpha}{\delta t}. \quad (9)$$

The radial magnetic field is a strong factor in the isotropy of the electron distribution along the axial and azimuth velocity projections – the axial projection of the velocity of each individual electron after a quarter of the cyclotron period (at a small bias) becomes to be azimuth one and vice versa with small change of absolute value of velocity and position on small displacement equal to cyclotron radius.

In turn, elastic but non-mirror reflection of electrons from a potential barrier at the plasma boundary [3] means that there is a tendency to isotropy in the distribution of velocity projections along the transverse (radial) and parallel (axial and azimuth) projections to the boundary. Non-mirror reflection also means the presence of the azimuth projection of the momentum flow in the radial direction in the absence of mass flow in this direction. As a result, it is possible not to distinguish the diagonal components of the electron pressure tensor but one off-diagonal components of the electron pressure must be taken into consideration:

$$P_e^{(xx)} \approx P_e^{(rr)} \approx P_e^{(\varphi\varphi)} \approx P_e. \quad (10)$$

$$P_e^{(r\varphi)} \neq 0. \quad (11)$$

The recombination of ions on the surface (or their escape into a vacuum surrounding the external beam) means the flow of their momentum and energy together with the mass, that is, the absence of dissipative transfer of momentum and energy – the off-diagonal components of the pressure tensor can be neglected:

$$P_i^{(r\varphi)} = 0, \quad P_i^{(xr)} = 0, \quad P_i^{(x\varphi)} = 0. \quad (12)$$

On the other hand, the appearance of new slow ions in a stream of already

accelerated ions decreases the mass flow velocity and increases the velocity dispersion. In each direction, the variance of each ion velocity projection is determined by the value of this particular projection. In this case, the ion acceleration in the axial direction (thrust direction) is noticeably greater than the radial acceleration (in the flow to the walls). The acceleration of ions in the azimuth direction does not occur at all. Thus, the values of the diagonal components of the pressure tensor must be distinguished – the dispersion factors in different directions are different, and the mechanism of pressure degeneration into a scalar (ion-ion collisions) is practically absent:

$$P_i^{(xx)} \neq P_i^{(rr)}, \quad P_i^{(\varphi\varphi)} = 0. \quad (13)$$

The electron flow in the entire channel volume is substantially subsonic:

$$\mathbf{\Pi}_e \approx \mathbf{P}_e. \quad (14)$$

In a rarefied electric propulsion thruster medium, the characteristic times of momentum and energy exchange between atoms and ions are large compared with the residence time of atoms and ions in the volume of the thruster chamber – a change in the densities of momentum and energy of atoms and ions as a result of elastic collisions can be neglected. In this case, the right-hand sides of (1) – (5) for different components can be written as follows:

$$\nabla \cdot \vec{\Gamma}_e = \nabla \cdot \vec{\Gamma}_i = -\nabla \cdot \vec{\Gamma}_a = n_e n_a v_e \sigma_i, \quad (15)$$

$$\nabla \cdot \mathbf{P}_e + e(n_e \vec{E} + \vec{\Gamma}_e \times \vec{B}) = -v_e^{(p)} m_e n_e \vec{V}_e, \quad (16)$$

$$\nabla \cdot \mathbf{\Pi}_i - e n_e \vec{E} = 0, \quad (17)$$

$$\nabla \cdot \left(\vec{V}_e \mathbf{P}_e + \mathbf{G}_e \right) + 2 \frac{e}{m_e} [\mathbf{P}_e \times \vec{B}] + 2 [\mathbf{P}_e \cdot \nabla \vec{V}_e] = -\frac{2}{3} \delta e \varphi_i \nabla \cdot \vec{\Gamma}_i, \quad (18)$$

$$\nabla \cdot (\vec{V}_i \mathbf{P}_i) + 2 [\mathbf{P}_i \cdot \nabla \vec{V}_i] = m_i \vec{V}_i \vec{V}_i \nabla \cdot \vec{\Gamma}_i, \quad (19)$$

$$\nabla \cdot \left(\frac{3}{2} \vec{V}_e P_e + \vec{V}_e \cdot \mathbf{P}_e + \vec{q}_e^{(cond)} \right) + e \vec{\Gamma}_e \cdot \vec{E} = -e \varphi_i \nabla \cdot \vec{\Gamma}_i, \quad (20)$$

$$\nabla \cdot \left(\vec{V}_i \left(\frac{m_i n_e V_i^2}{2} + \frac{3}{2} P_i \right) + \vec{V}_i \cdot \mathbf{P}_i \right) - e \vec{\Gamma}_i \cdot \vec{E} = 0, \quad (21)$$

where $v_e^{(p)} = v_e \left(n_a \left(\sigma_{ea} + \sigma_i \frac{\Delta_i P_e}{m_e V_e} \right) + n_e \sigma_{ei} \right)$ – electrons momentum transfer frequency, 1/s;

v_e – the average modulus of electron velocity, m/s;

σ_{ea} , σ_{ei} , σ_i – average cross-sections of electron-atom and electron-ion elastic collisions and average cross-section of ionization, m²;

φ_i – ionization potential, V;

$\Delta_i P_e$ – the average loss of momentum of an electron as a result of ionization collision, kg·m/s;

δ – unit tensor.

Equation set (15) – (21) is written neglecting magnetic field action on ions and difference between electrons and ion population in plasma. Electric field tension \vec{E} is present in four equations (17), (18), (20) and (21). As it was mentioned already, equations (20) and (21) are not independent. It is possible to build the equation set where \vec{E} is present only in one of them. The first step is to make total plasma motion equation as the sum of electrons and ions ones:

$$\nabla \cdot \mathbf{\Pi} + e n_e \vec{V}_e \times \vec{B} = -v_e^{(p)} m_e n_e \vec{V}_e, \quad (22)$$

where $\mathbf{\Pi}$ – summary plasma momentum flow density, Pa:

$$\mathbf{\Pi} \approx \mathbf{P}_e + \mathbf{\Pi}_i. \quad (23)$$

Electric field tension can be found from any of (17), (18), (20) or (21) equations.

Cylindrical form of equation set is:

- substance equations (15):

$$\frac{\partial \Gamma_{ex}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (\Gamma_{ir} r) = \frac{\partial \Gamma_{ix}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (\Gamma_{ir} r) = -\frac{\partial \Gamma_{ax}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (\Gamma_{ir} r) = n_e n_a v_e \sigma_i; \quad (24)$$

- projections of motion equations (16) – (18) and (22):

$$\frac{\partial P_e}{\partial x} - e n_e \frac{\partial \phi}{\partial x} - e \Gamma_{e\phi} B = 0, \quad (25)$$

$$\frac{\partial \Pi_i^{(xx)}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (\Pi_i^{(xr)} r) + e n_e \frac{\partial \phi}{\partial x} = 0, \quad (26)$$

$$\frac{\partial \Pi^{(xx)}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (\Pi_i^{(xr)} r) - e \Gamma_{e\phi} B = 0, \quad (27)$$

$$\frac{\partial P_e}{\partial r} - e n_e \frac{\partial \phi}{\partial r} = 0, \quad (28)$$

$$\frac{\partial \Pi_i^{(xr)}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (\Pi_i^{(rr)} r) + e n_e \frac{\partial \phi}{\partial r} = 0, \quad (29)$$

$$\frac{\partial \Pi_i^{(xr)}}{\partial x} + m_i \frac{1}{r} \frac{\partial}{\partial r} (\Pi_i^{(rr)} r) + \frac{\partial P_e}{\partial r} = 0, \quad (30)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left((P_e^{(r\phi)} r) r \right) + e \Gamma_{ex} (B r) = -v_e^{(p)} m_e (\Gamma_{e\phi} r); \quad (31)$$

- equations of the diagonal components of the ion pressure tensor (19):

$$\frac{\partial}{\partial x} (V_{ix} P_i^{(xx)}) + \frac{1}{r} \frac{\partial}{\partial r} (V_{ir} P_i^{(xx)} r) + 2 P_i^{(xx)} \frac{\partial V_{ix}}{\partial x} = \left(\frac{\partial \Gamma_{ix}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (\Gamma_{ir} r) \right) m_i V_{ix}^2, \quad (32)$$

$$\frac{\partial}{\partial x} (V_{ix} P_i^{(rr)}) + \frac{1}{r} \frac{\partial}{\partial r} (V_{ir} P_i^{(rr)} r) + 2 P_i^{(rr)} \frac{\partial V_{ir}}{\partial r} = \left(\frac{\partial \Gamma_{ix}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (\Gamma_{ir} r) \right) m_i V_{ir}^2;$$

(33)

- energy equations of electrons (20) and ions (21):

$$\frac{\partial}{\partial x} \left(q_{ex} + \Gamma_{ex} e \varphi_i \right) - e \Gamma_{ex} \frac{\partial \varphi}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left(\left(q_{er} + \Gamma_{ir} e \varphi_i \right) r \right) - e \Gamma_{ir} \frac{\partial \varphi}{\partial r} = 0, \quad (34)$$

$$\frac{\partial q_{ix}}{\partial x} + e \Gamma_{ix} \frac{\partial \varphi}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (q_{ir} r) + e \Gamma_{ir} \frac{\partial \varphi}{\partial r} = 0. \quad (35)$$

Total current density on the surface of the insulator is equal to zero:

$$\Gamma_{er} = \Gamma_{ir}. \quad (36)$$

Equation (24) also takes into account the equality of the atomic flux density from the surface of the insulator and the ion flux density to the surface (surface recombination):

$$\Gamma_{ar} = -\Gamma_{ir}. \quad (37)$$

From (24), the laws of charge and mass conservation follow:

$$\Gamma_d = \Gamma_{ix} - \Gamma_{ex} = \frac{I_d}{e S} = const, \quad (38)$$

$$\Gamma_m = \Gamma_{ix} + \Gamma_{ax} = \frac{\dot{m}}{m_i S} = const, \quad (39)$$

where I_d – discharge current, A;

\dot{m} – mass flow rate through the thruster (without flow through the cathode-neutralizer), kg/s;

S – channel cross-section, m².

Expressing the derivative of the potential with respect to the radius from (28) and the axial coordinate from (29), the electron energy equation (34) can be rewritten:

$$\begin{aligned} \frac{\partial}{\partial x} \left(q_{ex} + \Gamma_{ex} e \varphi_i \right) + V_{ex} \left(\frac{\partial \Pi_i^{(xx)}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left(\Pi_i^{(xr)} r \right) \right) + \\ + \frac{1}{r} \frac{\partial}{\partial r} \left(\left(q_{er} + \Gamma_{ir} e \varphi_i \right) r \right) - V_{ir} \frac{\partial P_e}{\partial r} = 0 \end{aligned} \quad (40)$$

The sum of the equations of energy of electrons (34) and ions (35) taking into account (38) gives the equation of plasma energy:

$$\frac{\partial}{\partial x} \left(q_{ex} + \Gamma_{ex} e \varphi_i + q_{ix} + \Gamma_d e \varphi \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(\left(q_{er} + \Gamma_{ir} e \varphi_i + q_{ir} \right) r \right) = 0. \quad (41)$$

Expression (31) is a record of the angular momentum conservation law. The first term on the left side (divergence of the momentum flux density, where radial projection is equal to $P_e^{(r\varphi)} r$), corresponds to the moment exchange with neighboring volumes; the second term of the left side is the moment of force acting on a unit volume; the right side is the loss of the angular momentum of the electrons

as a result of collisions in the volume.

In accordance with (12), (13), the projections of the ion energy flux density in equations (35) can be written as follows:

$$q_{ix} = V_{ix} \frac{m_i n_e (V_{ix}^2 + V_{ir}^2) + 3P_i^{(xx)} + P_i^{(rr)}}{2}, \quad (42)$$

$$q_{ir} = V_{ir} \frac{m_i n_e (V_{ix}^2 + V_{ir}^2) + P_i^{(xx)} + 3P_i^{(rr)}}{2}. \quad (43)$$

Equations shown here can be used in simulating soft creation for analysis of the processes inside HET channel. But the problem remains because of couple-dimension form of equations and large number of parameters.

2. Method for solving the problem

Due to axial symmetry in Plasma-ion and Hall effect thrusters it is possible to write the equations in two-dimension axial-radial form. Transformation of these equations into one-dimension axial form would mean great facilitation in making of a calculation method. But simple elimination of all the compounds with radius differentiation, as it was done in the papers [4 – 6] would mean the loss in description of the most important effects responsible for engine performance: the loss of ions and electrons on the walls, the loss of the momentum of the ions and the energy of the electrons on the walls - which is simply not the case in a one-dimensional description. However, it is possible to transform the equations into one-dimensional form without the named losses.

The flow of electrons in the axial and azimuth directions is restrained by a radial magnetic field. The flow in the radial direction is not limited by the magnetic field. Thus, the distribution of all plasma characteristics over the radius is smooth. This allows you to go to a simplified form for writing equations in full derivatives – only along the axial coordinate.

Such a transition is possible when solving the problem in quantities averaged over the channel cross section so that for any quantity \mathbf{A} the average value over the cross section is as follows:

$$\overline{\mathbf{A}} = \frac{2}{r_+^2 - r_-^2} \int_{r_-}^{r_+} \mathbf{A} r dr, \quad (44)$$

where r_- , r_+ – internal and external radius of channel.

Here the terms with derivatives with respect to radius are reduced to the form:

$$\frac{2}{r_+^2 - r_-^2} \int_{r_-}^{r_+} \frac{1}{r} \frac{\partial}{\partial r} (i_r \cdot \mathbf{A} r) r dr = i_r \cdot \frac{2(\mathbf{A}(r_+)r_+ - \mathbf{A}(r_-)r_-)}{r_+^2 - r_-^2}. \quad (45)$$

Thus, the quasi-one-dimensional approximation retains a description of the loss of particles, momentum, energy and angular momentum on the lateral surface of the channel, allowing us to simultaneously write equations as one-dimensional. The only uncertainty that remains in this case is the uncertainty in the ratio of plasma concentration at the Langmuir boundaries to the average in channel cross section:

$$\frac{n_e(r_+)}{\bar{n}_e} \text{ and } \frac{n_e(r_-)}{\bar{n}_e}. \quad (46)$$

In future expressions it will be used the following notation:

$$\xi_n = \frac{n_e(r_+)r_+ + n_e(r_-)r_-}{(r_+ + r_-)\bar{n}_e}, \quad (47)$$

omitting the averaging sign "̄" for simplicity.

Using averaging it can be obtained from expressions (24), (27), (31) – (33), (41), (45):

$$\frac{\partial \Gamma_{ex}}{\partial x} = \frac{\partial \Gamma_{ix}}{\partial x} = -\frac{\partial \Gamma_{ax}}{\partial x} = n_e \left(n_a v_e \sigma_i - \frac{2\xi_n}{\Delta r} V_{is} \right), \quad (48)$$

$$\frac{\partial \Pi^{(xx)}}{\partial x} + \frac{2\xi_n}{\Delta r} m_i n_e V_{ix} V_{is} = -\frac{e^2 B^2}{m_e \left(v_e^{(p)} + \frac{\xi_n \eta_e^{(p)} \xi_e^{(M)}}{2\Delta r} v_e \right)} \Gamma_{ex}, \quad (49)$$

$$\frac{\partial}{\partial x} \left(V_{ix} P_i^{(xx)} \right) + 2P_i^{(xx)} \frac{\partial V_{ix}}{\partial x} + \frac{2\xi_n}{\Delta r} P_i^{(xx)} V_{is} = n_e n_a v_e \sigma_i m_i V_{ix}^2, \quad (50)$$

$$\begin{aligned} & \frac{\partial}{\partial x} \left(q_{ex} + \Gamma_{ex} e \varphi_i \right) - V_{ex} \frac{\partial P_e}{\partial x} = \\ & = \frac{e^2 n_e B^2}{m_e \left(v_e^{(p)} + \frac{\xi_n \eta_e^{(p)} \xi_e^{(M)}}{2\Delta r} v_e \right)} V_{ex}^2 - \frac{2\xi_n}{\Delta r} n_e V_{is} (\varepsilon_e - \Delta \varepsilon_e), \end{aligned} \quad (51)$$

$$E_x = -\frac{\partial \varphi}{\partial x} = \frac{1}{e \Gamma_d} \left(\frac{\partial}{\partial x} \left(q_{ex} + q_{ix} + \Gamma_{ex} e \varphi_i \right) + \frac{2\xi_n}{\Delta r} n_e V_{is} \left(\varepsilon_e + \varepsilon_i + e \varphi_i \right) \right), \quad (52)$$

where $\Delta r = r_+ - r_-$ – channel thickness, m;

V_{is} – ion-sound velocity, m/s;

$\xi_e^{(M)} = \frac{V_{e\varphi\pm} r_{\pm}}{V_{e\varphi} r}$ – relation of rotation moment boundary value to average one;

$1 - \eta_e^{(p)}$ – relative electrons rotation moment lost because of non-mirror reflection from potential barrier near the plasma bound;

$$\varepsilon_e = \frac{q_{er\pm}}{\Gamma_{er\pm}}, \quad \varepsilon_i = \frac{q_{ir\pm}}{\Gamma_{ir\pm}}, \quad \Delta\varepsilon_e = \frac{\int_{r_-}^{r_+} V_{ir} \frac{\partial P_e}{\partial r} dr}{\int_{r_-}^{r_+} \frac{1}{r} \frac{\partial}{\partial r} (\Gamma_{er} r) r dr}. \quad (53)$$

Equations set (48) – (52) with consideration of (38), (39) and dependence of ε_e , $\Delta\varepsilon_e$ and q_{ex} on P_e and n_e is closed but only one of them (52) includes the electric field tension E_x . Thus subsystem of four equations (48) – (51) is also closed with four parameters n_e , Γ_{ix} , P_e and $P_i^{(xx)}$ can be used independently to find all parameters except E_x and φ using iteration method. The last step must be the calculation of potential distribution among the channel.

General form of subsystem (48) – (51) can be written as

$$\sum_{m=1}^4 a_m^{(n)} Z_m = b_n, \quad 1 \leq n \leq 4, \quad (54)$$

where $Z_1 \dots Z_4 - n_e$, Γ_{ix} , P_e and $P_i^{(xx)}$.

The problem remains as for factors ξ_n , $\xi_e^{(M)}$, ε_e , ε_i and $\Delta\varepsilon_e$, which can be solved either with some suppositions or through approximate solution of task about radial distribution of plasma parameters with use of equations (9), (28) – (30) and (33).

Conclusions

As the initial data of the task with fixed sizes and configuration of thruster can be used mass flow rate and discharge current. One of the most complicated questions in the task is that only one boundary condition can be written directly in numeric form – electrons temperature in the entrance of cathode beam into thruster plume ($x=x_e$) and potential difference between this entrance and cathode emitter, given from cathode calculation or testing results.

As for anode section ($x=0$) it is known that main determinant of system (54) is equal to zero – critic condition for bound between plasma and Langmuir layer:

$$\Delta = 0. \quad (55)$$

Also it is known that the section exists ($x=x_0$) where two conditions take place at the same time:

$$\Gamma_{ix} = 0 \quad \cup \quad P_i^{(xx)} = 0, \quad (56)$$

but the coordinate x_0 is unknown.

The last condition means that the critic section exists ($x=x_c$) where both main and all partial determinants are equal to zero at the same time:

$$\Delta = 0 \quad \cup \quad \Delta_1 \dots \Delta_4 = 0, \quad (57)$$

but also with unknown coordinate x_c .

So the algorithm of the task solution is as follows (Fig. 2):

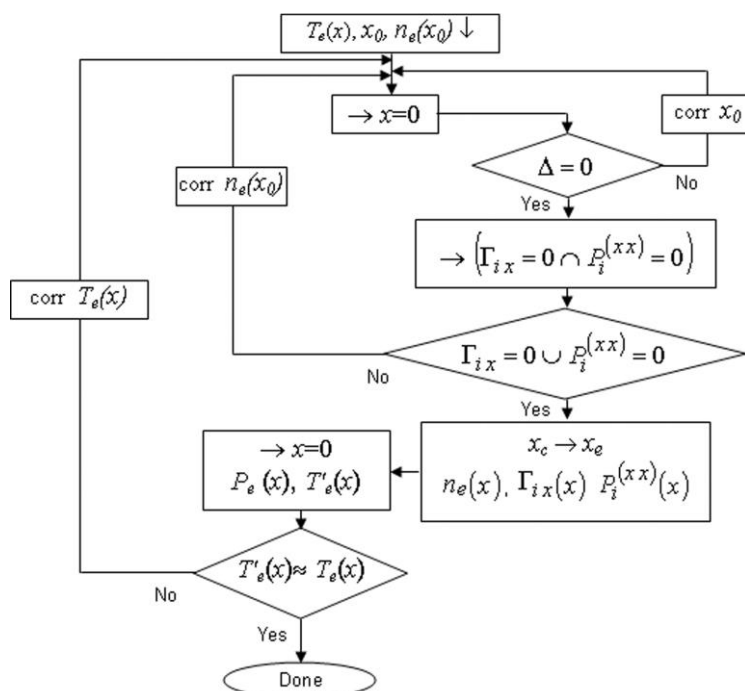


Fig. 2. Iteration method

1. Electrons temperature distribution is set.
2. The coordinate x_0 and plasma concentration here $n_e(x_0)$ are set.
3. Move to anode coordinate $x=0$.
4. Correction of x_0 and reiteration pp. 1 – 3 until condition (55) satisfaction.
5. Move forward from x_0 until one of condition (56) satisfaction.
6. Correction of $n_e(x_0)$ and reiteration pp. 1 – 5 until both of conditions (56) satisfaction.
7. Move forward from x_c to x_e to find n_e , Γ_{ix} and $P_i^{(xx)}$ distribution in channel.
8. Move back to $x=0$ to find P_e and T_e distribution in channel.
9. Correction of T_e distribution and reiteration of pp. 1 – 9 to reach appropriate accuracy.

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Квazіодновимірна математична модель процесів в холлівському та плазмово-іонному двигуні

Плазмово-іонні двигуни з радіальним магнітним полем в іонізаційній камері та холлівські двигуни відносяться до електростатичних двигунів із замкненим дрейфом електронів. Осьова симетрія в динаміці компонент робочої речовини в цих двигунах дозволяє записати рівняння плазмодинаміки для електронів, іонів і нейтральних атомів у двовимірній аксіально-радіальній формі. Спроби зведення рівнянь до більш простої одновимірної форми шляхом простого видалення складових з диференціюванням по радіусу призводять до втрати в описанні важливих ефектів, відповідальних за величини експлуатаційних характеристик двигуна.

При цьому також істотним недоліком системи рівнянь газодинаміки є принципова її незамкненість – відповідність кількості невідомих і рівнянь досягається приблизно на основі певних припущень. У традиційній формі газодинаміки таке замикання здійснюється в припущенні про термодинамічну рівновагу з функціями розподілу компонент за швидкостями, близькими до

максвеллівського розподілу, який є граничним результатом зіткнень. Використання такого наближення до динаміки компонент плазми в електроракетних двигунах є неможливим через розрідженість середовища у них. У стаціонарній формі представлено математичну модель двокомпонентної плазмодинаміки для опису процесів в каналі холлівського двигуна і іонізаційній камері плазмово-іонного двигуна з радіальним магнітним полем.

З урахуванням неможливості використання методу локальної термодинамічної рівноваги для опису розрідженого середовища в електроракетному двигуні використовується більш розширена форма рівнянь. Запропоновано більш надійний в описі розрідженого газу засіб приблизного замикання системи рівнянь. Показано підхід до опису специфіки перенесення енергії електронів від плазми до стінок каналу, а також недзеркального відбиття електронів від потенційного бар'єру всередині ленгмюрівського прошарку. Запропоновано метод усереднення параметрів по перетину каналу, що дозволяє перетворити рівняння у квазіодновимірний вид зі збереженням втрат заряду, імпульсу і енергії на стінках каналу.

Ключові слова: холлівський двигун; плазмово-іонний двигун; ленгмюрівський прошарок; тензор тиску.

Quasi-one-dimensional mathematical model of processes in Hall effect and plasma-ion thrusters

Plasma-ion thrusters with a radial magnetic field in ionization chamber and Hall effect thrusters are electrostatic electric propulsion thrusters with closed electron drift. Axial symmetry in the dynamics of the components of propellant in these thrusters allows to write the equations of plasma-dynamics for electrons, ions and neutral atoms in two-dimensional axial-radial form. Attempts to reduce the equations to simpler one-dimensional form by simply removing the components with radius differentiation lead to the loss in the description of important effects, responsible for values of thruster performance.

At the same time, a significant disadvantage of gas dynamics equation set is its fundamental openness – the correspondence between the number of unknown variables and equations is achieved approximately basing on some assumptions. In traditional form of gas dynamics, such closeness is made under the assumption of thermodynamic equilibrium with velocity distribution functions of components close to Maxwell one, which is the limit result of collisions. The use of such approximation to plasma components dynamics in electric propulsion thrusters is impossible due to the rarefaction of the substance in them. A mathematical model of two-component plasma-dynamics is represented in stationary form to describe the processes in the Hall effect thruster channel and the ionization chamber of plasma-ion thruster with radial magnetic field.

Due to the impossibility of using the method of local thermodynamic equilibrium to describe the rarefied substance in electric propulsion thruster, a more advanced form of equations is used. A more reliable means of approximate closure of the set of equations is proposed in the description of the rarified gas. An approach to the description of the specifics of electrons energy transfer from the plasma to the walls of the channel, as well as the non-mirror reflection of electrons from the potential barrier within the Langmuir layer is shown. A method of averaging the parameters over the cross section of the channel is proposed, which allows to

convert the equation into a quasi-one-dimensional form with the preservation of charge, momentum and energy losses on the channel walls.

Keywords: Hall effect thruster; plasma-ion thruster; Langmuir layer; pressure tensor

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