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## TOPOLOGICAL OPTIMIZATION HYBRID ALGORITHM FOR THE ADHESIVE JOINT

**The subject** of this study is a topological optimization algorithm for a lapped symmetric adhesive joint. **The purpose** of this research is to create a hybrid optimization algorithm that combines the advantages of a genetic algorithm and a particle swarm algorithm and, at the same time, reduces the time required to solve the problem. **Task:** to create a methodology for solving the optimization problem for a symmetric double-sided lapped adhesive joint, which consists of a main plate and two patches (the main plate has a constant thickness, and the thickness of the patches varies along the length of the joint, this is required to reduce the stress concentration in the joint and reduce its weight) with satisfaction of the optimality criterion, namely, to minimize the mass of the structure with the strength and thickness restrictions for the patch. The optimization problem is that we must find the optimal patch form, namely, the length of the patch and the thickness-on-length dependence for the patch. **Methods:** the modified Goland-Reissner model was used to describe the deflected mode of the joint. The finite difference method was used to solve the direct stress state problem for the structure. For the numerical solution of the optimization problem, a combination of the multi-population model of the genetic optimization algorithm and the particle swarm algorithm was used. To improve the performance of the genetic algorithm, a multi-population model with migration of the best individuals between populations was applied. The introduction of individuals from other populations into the population avoids homogenization of the genotype in a separate population and premature stopping of the optimization process. To describe the shape of the patch, the Fourier series expansion of the patch thickness dependence was used. **Results:** A hybrid algorithm is proposed based on the sequential application of a genetic algorithm and a particle swarm algorithm for three populations of solutions. The particle swarm algorithm makes it possible to improve the value of the objective function achieved at the previous stage by 20%. **Conclusions:** the scientific novelty lies in the improvement of the optimization algorithm compared with the known ones. To reduce the calculation time, a one-dimensional adhesive joint stress state mathematical model was used in this paper. The methods used made it possible to create a combined topological optimization algorithm that combines the advantages of both methods and allows us to find a solution to the problem quite quickly. The Python program run time is only a few minutes.

**Keywords:** adhesive joint; genetic algorithm; particle swarm optimization; finite difference method; topology optimization.

### Introduction

**Motivation.** Adhesive joints have significant advantages compared to classical mechanical joints, such as light weight, tightness, high aerodynamic efficiency, and manufacturability. In addition, gluing does not violate the integrity of the structure of fibrous composite materials and does not reduce their strength, unlike classical mechanical joints. [1]. A well-known disadvantage of lap joints is the stress concentration in the adhesive layer at the edges of the gluing area [2]. To reduce stresses in the joint, symmetrical double overlap joints are often used [3], which allows the exclusion of eccentricity in the transfer of forces between structural elements. The simplest method to reduce the stress concentration at the edge of the joint is to create chamfers on the inner side of the adhesive or on the outer side of the patch [4]. A more general approach to stress reduction in a joint is to use

patches of varying thickness. This makes it possible to ensure a more uniform stress state of the joint compared with that of classical structures [5, 6].

**Sate of the art.** As a rule, the topological optimization problem for overlap joints is posed in a two-dimensional formulation. However, the two-dimensional formulation of the optimization problem, which is based on the use of finite element modeling [5], in a two-dimensional [7, 8] formulation (considering the ability to control the characteristics of the adhesive [9]), and three-dimensional [10] formulation, are correlated with a significant amount of computations. Solving one problem requires approximately 10–20 h of estimated time. This is because the optimization algorithms that are used for the solution require multiple solving of direct problems of finding the stress state of joints for various configurations. The finite element method is used for this purpose.

Although the finite element method is universal, it is expensive in terms of calculation time. In addition, the resulting optimal solution may be difficult to implement in practical applications [5]. One method to increase the computational speed for a problem is the use of one-dimensional mathematical models. Reducing the dimension of the model and ensuring its adequacy and accuracy can significantly reduce the search space and the time required to solve the problem. One-dimensional mathematical models of the joint stress state are used to construct genetic optimization algorithms, for example, in studies [11, 12] and the same for two types of glue, in the study [13]. In papers [11, 12], a similar stressed state mathematical model of a joint was considered; however, the genetic algorithm proposed by the authors as a solution method for the optimization problem is ineffective at the final execution steps. Therefore, an approximate solution is found quite quickly during the initial stage of the algorithm. However, after that further improvement of the solutions found is suspended. In this regard, there is a need to modify the algorithm used. Modifying the algorithm would allow us to avoid premature stopping of the optimization process.

The use of a one-dimensional mathematical model also allows us to develop the optimization problem in a simpler form, replacing the nonparametric optimization problem [14] with a less time-consuming parametric optimization problem. In the last case, the patch shape is described with a set of certain parameters, and the optimization problem is to find the optimal values of these parameters, such as the thickness of the adhesive layer or the adhesive joint length [15]. This approach is also aimed at reducing the time required to solve the problem.

**Objectives.** The authors use a specialized low-dimensional mathematical model (at the same time this model has sufficient accuracy) to reduce computation time. Thus, the topological optimization problem is reduced to a parametric optimization problem. However, using a genetic algorithm that is prone to prematurely stopping is still a bottleneck. The purpose of this research is to create a hybrid optimization algorithm that combines the advantages of a genetic algorithm and a particle swarm algorithm and, at the same time, reduces the time required to solve the problem.

In this paper, to increase the efficiency of execution, we propose using the island model of the genetic algorithm at the initial stage and then improving the resulting solutions using the particle swarm algorithm. The objective function (cross-sectional area of the patch) at various stages of the algorithm is used as a criterion for assessing efficiency. Calculations have shown that using the particle swarm algorithm after the genetic algorithm has lost its effectiveness allows one to quickly improve the value of the objective function by 20%.

## 1. Methodology of the research

To solve the optimization problem, it is proposed to use a one-dimensional stress state model. This model, due to its low dimension, allows one to find the stressed state of the joint in a very short time. This circumstance makes it possible to use iterative methods to solve the optimization problem, based on a consistent solution to several direct problems with subsequent adjustment of the necessary parameters that describe the shape and dimensions of the joint. Moreover, the one-dimensional model allows one to describe the patch shape as a set of certain parameters, despite the reduction in search space. This allows the reduction of the topological optimization problem to a parametric optimization problem, which also simplifies the search for the optimal structural shape.

The classical genetic algorithm method has good convergence only at the initial steps, but then the population becomes homogeneous, and further iterations do not improve the solution, never reaching the neighborhood of the optimum. Therefore, to avoid such premature stopping, various modifications of the genetic algorithm have been proposed, such as the use of several populations. However, this leads to increased computation time, while the tendency to premature stopping still has place, although it is reduced. To speed up the calculation process, the authors propose to use a combination of a genetic algorithm and a particle swarm algorithm. Hybrid algorithms combine the advantages of both methods and represent a modern direction in the development of optimization methods [16, 17]. In the initial steps, the genetic algorithm provides an approximate solution, which can then be improved using the particle swarm algorithm.

In this case, the optimality criterion is the mass of the structure. At the same time, the structure must maintain bearing capacity; therefore, the space of the required parameters is limited by the area where the stress values in the adhesive layer do not exceed the maximum permissible values.

## 2. Problem formulation

### 2.1. Mathematical Model

The differential element of the gluing area and the force factors acting on these elements are shown in Fig. 1.

Equilibrium equations for outer (base) layers have the form

$$\frac{dN_1}{dx} = -\tau; \quad \frac{dN_2}{dx} = \tau; \quad \frac{dQ_1}{dx} = \sigma;$$

$$\frac{dM_1}{dx} - s_1(x)\tau - N_1 \frac{ds_1}{dx} + Q_1 = 0,$$

where  $N_1, N_2$  are longitudinal forces in the base layers;  $Q_1$  are shear force in the patch;  $M_1$  is a bending moment in the patch;  $\tau$  and  $\sigma$  are tangential and normal stresses in the adhesive layer;  $s_1$  is a distance from the neutral axis of the patch to the adhesive layer, in the case of a symmetrical patch structure  $s_1(x) = 0.5 \delta_1(x)$ , where  $\delta_1(x)$  is a patch thickness.

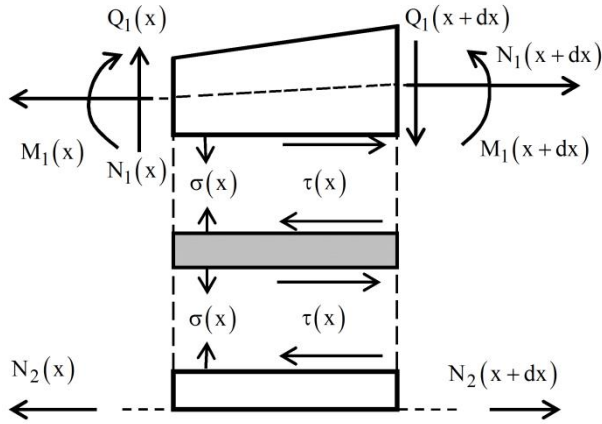


Fig. 1. Stress and force values in the adhesive joint layers

The deformation of the base layer is represented by equations

$$N_1 = B_1(x) \frac{dU_1}{dx}, \quad N_2 = B_2 \frac{dU_2}{dx},$$

$$D_1(x) \frac{d^2 w_1}{dx^2} = M_1,$$

where  $U_1$  and  $U_2$  are longitudinal displacements of the base layers;  $w_1$  are transversal displacements of the patch;  $B_1$  and  $B_2$  are stress-strain rigidity values of the layers, If the layers are homogenous by thickness, then  $B_1(x) = \delta_1(x) E_1$ ,  $B_2 = \delta_2 E_2$ , where  $E_1$  and  $E_2$  is an elastic modulus of the corresponding layer;  $D_1$  is a bending rigidity of the patch,  $D_1(x) = \frac{\delta_1^3(x) E_1}{12}$ .

We consider the stress values in the adhesive layer to be proportional to the difference in displacements of the base layers

$$\sigma = K w_1, \quad \tau = P \left( U_1 - U_2 + s_1(x) \frac{d w_1}{dx} \right),$$

where  $K = E_0 / \delta_0$ ,  $P = G_0 / \delta_0$ , where  $\delta_0$  is a thickness of the adhesive layer,  $E_0$  and  $G_0$  are elastic modulus and shear modulus of an adhesive.

The system of equations can be reduced to a system of three differential equations relative to the displacements of the layer. The boundary conditions are given below:

$$N_2(0) = F, \quad N_2(L) = 0, \quad N_1(0) = 0, \quad U_1(L) = 0,$$

$$Q_1(0) = 0, \quad M_1(0) = 0,$$

$$Q_1(L) = 0, \quad \left. \frac{d w_1}{dx} \right|_{x=L} = 0.$$

The goal of the problem is to find a function  $\delta_1(x)$ , that ensures a minimum mass of the structure, which, up to a constant factor and constant terms, is equal to the cross-sectional area of the patch

$$M = \int_0^L \delta_1(x) dx,$$

and would ensure the bearing capacity of the structure. As a rule, the joint loses its bearing capacity because of the destruction of the adhesive layer. Therefore, the condition for the joint to maintain its bearing capacity is to restrict the stress values in the adhesive layer with certain limiting values, for example

$$\sigma(x) \leq \sigma_0, \quad \tau(x) \leq \tau_0.$$

In addition, other criteria for adhesive strength can be used, these depend on the type of adhesive, gluing technology and other factors.

## 2.2. Numerical solution to the direct problem

We assume, that the function  $\delta_1(x)$  and the length  $L$  of the gluing are given. Hence, functions  $s_1(x)$ ,  $B_1(x)$  and  $D_1(x)$  are also known. To numerically solve the obtained system of equations with the corresponding boundary conditions, we used the finite difference method. The gluing area  $x \in [0; L]$ , is divided into a system of nodal points numbered from zero to  $N$ . Points with numbers 0 and  $N$  are boundary ( $x = 0$  and  $x = L$  correspondingly). Having written in the difference form the system of differential equations for each of these points, as well as the boundary conditions, we obtain a system of linear equations for the displacements of the base layers at these points. This approach makes it possible to determine stresses in the plate, patch, and adhesive layer at the corresponding points.

### 3. Optimization

#### 3.1. Optimization of the genetic algorithm

As noted above, the solution of the optimization problem in an analytical form is very difficult. However, in contrast to the problem of finding the optimal material distribution along the beam [18], if thickness values  $\delta_i^{(1)}$  in neighboring points are significantly different (that can happen due to crossbreeding and mutations), then stress values in the adhesive layer, computed by the finite difference method, have unreal jumps, this is the fact, that the mathematical model becomes inadequate. Therefore, we propose to find an optimal dependence  $\delta_i^{(1)}$  among functions that have smooth apriority. This also follows from intuitive thoughts that the desired function  $\delta_1(x)$  probably is smooth and has no angular points or jumps. In this paper, we propose the use of a cosine Fourier expansion at the interval  $\xi \in [0; 1]$  to describe the function  $\delta_1(x)$

$$\delta_i^{(1)} = y(\xi_i) = \frac{a_0}{2} + \sum_{k=1}^M a_n \cos \pi k \xi_i.$$

If we divide the interval  $\xi \in [0; 1]$ , as well as the interval  $x \in [0; L]$  into  $N + 1$  points  $\xi_i$  numerated from 0 to  $N$ .

A description of a patch geometrical form as a Fourier series allows us to calculate the mass of the patch rather simply

$$M = \int_0^L \delta_1(x) dx = \frac{a_0 L}{2}.$$

To implement a genetic algorithm, it is necessary to create a fitness function that would make it possible to rank different sets of parameters  $L$  and  $a_0, a_1, \dots, a_M$  (i.e. individual) by quality. We can, for example, write the fitness-function in a following form:

$$\Phi = \frac{a_0 L}{2} + \varphi_1 + \varphi_2 + \varphi_3 + \varphi_4,$$

where  $\varphi_1, \dots, \varphi_4$  are penalty functions that are appointed in the case, if a corresponding solution does not satisfy some restriction.

These functions may have, for example, the following form:

$$\varphi_1 = \begin{cases} Z_1 \left( \frac{\max_i(\tau_{\max})}{\tau_0} - 1 \right)^2, & \max_i(\tau_{\max}) > \tau_0 \\ 0, & \max_k(\tau_{\max}) \leq \tau_0 \end{cases}$$

$$\varphi_2 = \begin{cases} Z_2 \left( \frac{\delta_{\min}}{\min_i(\delta_i^{(1)})} - 1 \right)^2, & \min_i(\delta_i^{(1)}) < \delta_{\min} \\ 0, & \min_i(\delta_i^{(1)}) \geq \delta_{\min} \end{cases}$$

where  $Z_1, Z_2$  are some big numbers that define the penalty for leaving the solution out of the available area;  $\tau_{\max}$  are maximal tangential stress values in the adhesive layer in nodal points ( $\tau_{\max} = 0.5\sqrt{\sigma^2 + 4\tau^2}$ );  $\max_i(\tau_{\max})$  is the maximal value of the maximal tangential stress values for all points in the area;  $\min_i(\delta_i^{(1)})$  is a minimal value of the patch thickness.

Similarly, the functions  $\varphi_3$  and  $\varphi_4$ , which are greater than zero if the patch thickness exceeds a maximal restriction and if the derivative modulus of the patch thickness exceeds some given value. This restriction is because the mathematical model of the joint stress state is based on the beam model and is adequate only if the beam height is close to constant.

Therefore, if the solution (i.e. the set of values  $L$  and  $a_0, a_1, \dots, a_M$ ) is available, then the fitness function value is equal to the patch cross-section area  $0.5 a_0 L$ . However, if the imposed restrictions are violated at least in one node, then penalty terms must be added to the area mentioned above, the greater the value of the violation of the corresponding restriction.

When tuning genetic algorithms, it is necessary to maintain a balance between variability and stability. In the case of high variability, convergence is violated, and even appropriate values of the desired parameters are at risk of being lost because of mutations. In the case of low variability, the approximate solution is found quickly, and then convergence slows down and the population degenerates. In the future, even with a large number of iterations, the value of the objective function changes little. One possible way out of this contradiction is to use the island model of the evolutionary algorithm. In this paper, we propose a model with three islands, on which the probability and standard deviation of mutations are higher than those on the other two islands. This combination of two relatively stable islands with one island with a higher mutation rate makes it possible to combine

the speed of the appropriate solution finding with the stability and preservation of the best solutions in the general population.

The general scheme of the evolutionary algorithm for one population has the following form:

1. Creation of the initial population of vectors  $\vec{h}^{(j)}$ , where  $j=1, \dots, n$ , ( $n$  is many individuals in the population). Each vector  $\vec{h}^{(j)}$  (the individual) contains components  $L^{(j)}$  and  $a_0^{(j)}, a_1^{(j)}, \dots, a_M^{(j)}$ .

2. According to these sets of parameters the corresponding values  $\Phi^{(j)} = \Phi(\vec{h}^{(j)})$  are calculated.

3. *Selection*. The vectors available in the population  $\vec{h}^{(j)}$  according to the corresponding values of the fitness function  $\Phi^{(j)}$  are ranked.

4.  $2k$  elements  $\vec{h}^{(j)}$  from the population are selected. It is necessary that the best individuals  $\vec{h}^{(j)}$  from the population be included in the sample, which has fewer fitness functions.

5. *Parents choice*.  $2k$  selected individuals into pairs and obtained  $k$  pairs of “parents” are broken. In the simplest case, we can break it into pairs at random.

6. *Crossover*. Parameters for each new individual  $L^{(j)}$  and  $a_0^{(j)}, a_1^{(j)}, \dots, a_M^{(j)}$  from both parent individuals are randomly selected. Because of such operation, we obtain a population  $k$  of new individuals, “descendants”.

7. *Mutations*. Mutations occur only with a small portion of the vector components  $\vec{h}^{(j)}$  of individuals, which appear as a result of “descendant” breeding.

8. After making changes to the genetic code, the descendants return to the main population. After that, individuals are again ranked according to the values of fitness-function  $\Phi^{(j)}$  and  $k$  the worst individuals are removed from the population.

9. Checking of the stop criterion. If the stop criterion (for example, specified number of reproduction cycles  $M$ ) is not reached, then we return to step 4.

After performing a certain number of algorithm cycles, two islands are selected in each of the three subpopulations, which exchange the best individuals (migrants). Migration provides an influx of new genetic information into each population. The number of migrants is approximately 10% of the population. The criterion for stopping the algorithm is the execution of a given number of migrations.

Computations show [13] that in later iterations of the algorithm, the objective function remains practically unchanged. Therefore, it is proposed to use a combination of

the genetic algorithm and the particle swarm algorithm [19, 20]. In this case, the resulting three subpopulations form the initial state of each of the three swarms of particles. For each of the three swarms, the optimization problem is solved independently. Then, the best solutions are selected, and the values of the desired parameters are averaged over this sample. The scheme of the algorithm is shown in Fig. 2.

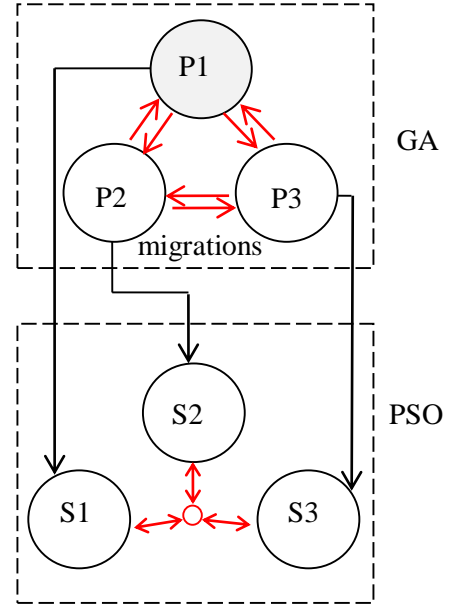


Fig. 2. Thickness of the main plate and patch

Here P1, P2, P3 are populations 1, 2, and 3 in the genetic algorithm (GA), and S1, S2, S3 are populations of particles in the particle swarm optimization algorithm (PSO). The dark color indicates the population with an increased level of mutagenesis.

The upper and lower limits of the available value area for the desired parameters necessary for the particle swarm algorithm are calculated as follows:

$$L_{up} = L^* + \frac{L}{\theta}, \quad L_{lo} = L^* - \frac{L}{\theta},$$

$$a_{k,up}^{(j)} = a_k^{(j)*} + \frac{|a_k^{(j)}|}{\theta}, \quad a_{k,lo}^{(j)} = a_k^{(j)*} - \frac{|a_k^{(j)}|}{\theta}.$$

Here the values denoted by “\*”, are the parameters of the best individual in the subpopulation (swarm). The parameter  $\theta$ , that defines a width of the interval, is appointed when tuning the algorithm. The authors used the value  $\theta = 4$ .

Therefore, vectors of the upper and lower restriction of the required parameters have the form:

$$\begin{aligned}\vec{b}_{lo} &= (L_{lo}, a_{0,lo}, a_{1,lo}, \dots, a_{M,lo}), \\ \vec{b}_{up} &= (L_{up}, a_{0,up}, a_{1,up}, \dots, a_{M,up}).\end{aligned}$$

### 3.2. Algorithm, particle swarm optimization

In this case, the particle swarm algorithm, which acts with the population (swarm) obtained at the previous stage (genetic algorithm), has the following form:

- the initial coordinates of each particle  $\vec{h}^{(j)}$  (individual) are assigned to the vector of the best obtained position of each individual  $\vec{p}^{(j)}$ ;
- the coordinates of the best individual in the swarm (subpopulation) will be assigned to the vector  $\vec{g}$ ;
- to generate velocity vectors for each particle

$$\vec{v}^{(j)} \sim \bar{U}\left(-(\vec{b}_{up} - \vec{b}_{lo}), (\vec{b}_{up} - \vec{b}_{lo})\right),$$

where  $\bar{U}(\bar{l}, \bar{u})$  is a multidimensional uniform distribution that has a lower and upper limitations of the solution space  $\bar{l}$  and  $\bar{u}$ .

If the algorithm stop criterion is not met (for example, the execution of a specified number of iterations or stabilization of the objective function optimal values), perform the following operations:

- to generate vectors

$$\vec{r}_p = \bar{U}(0,1) \text{ and } \vec{r}_g = \bar{U}(0,1),$$

for each particle (individual);

- to renew the velocity of each particle

$$\vec{v}^{(j)} = \omega \vec{v}^{(j)} + \varphi_p \vec{r}_p \odot (\vec{p}^{(j)} - \vec{h}^{(j)}) + \varphi_g \vec{r}_g \odot (\vec{g} - \vec{h}^{(j)}),$$

where the operation  $\odot$  means component multiplication;

- to renew the particle position by transferring from coordinates  $\vec{h}^{(j)}$  into the point with coordinates  $\vec{h}^{(j)} = \vec{h}^{(j)} + \vec{v}^{(j)}$ ;

- if  $\Phi(\vec{h}^{(j)}) < f(\vec{p}^{(j)})$  then renew the best obtained value of the point  $j$ , i.e. to assign to  $\vec{p}^{(j)}$  the actual coordinates of  $\vec{h}^{(j)}$ . If  $\Phi(\vec{p}^{(j)}) < f(\vec{g})$  then renew the best obtained solution for all the swarm, assigning coordinates  $\vec{p}^{(j)}$  to the vector  $\vec{g}$ .

The parameters  $\omega$ ,  $\varphi_p$ ,  $\varphi_g$  are selected by the calculator and determine the behavior and effectiveness of the method as a whole. The following values were assigned in this paper:  $\omega = 0.6$ ,  $\varphi_p = 1.1$ ,  $\varphi_g = 1.1$ . These parameters describe the contribution of the inertial component to the motion of the particle, the influence of information about the history of the particle itself (its best position in the entire history) and the influence of the actual best value of the particle coordinates of the entire swarm.

We have three swarms of particles that were obtained in the previous step of the algorithm. We can use this fact to modify the classical genetic algorithm by increasing its speed. Note that the genetic algorithm and the particle swarm algorithm are the basis for creating more complex methods [21], which also use particle clustering.

In this case, we will supplement the algorithm with the possibility of information exchange between the three swarms. We perform the algorithm described above for each swarm for a given number of iterations. Then, we compare the best solution  $\vec{g}$ , obtained for each swarm, with the others and select the best. We then transfer the best solution to each of the other two swarms and repeat the optimization cycles with each swarm. We perform this exchange of information between swarms a specified number of times.

## 4. Results and Discussion

Let us apply the proposed joint optimization algorithm to solve two problems that differ only in the load applied to the joint. The rest of the parameters are the same in both cases:  $E_1 = 70$  GPa,  $E_2 = 70$  GPa,  $\delta_2 = 3$  mm,  $\delta_0 = 0.1$  mm,  $E_0 = 2.274$  GPa,  $G_0 = 0.54$  GPa,  $\tau_0 = 15$  MPa,  $\delta_{\min} = 0.5$  mm,  $\delta_{\max} = 8$  mm. We consider one case of loading the structure:  $F = 170$  kN/m. We added a restriction on the value of the derivative of the patch thickness  $|\delta'_1(x)| \leq 0.2$ .

The Fourier series term number is taken as  $M = 15$ . Stress state computation of a joint is performed by splitting the area into  $N = 100$  nodal points. Number of individuals in the population is  $n = 120$ . We choose  $2k = 40$  individuals from them for cross-breeding at each iteration.

As a result of the numerical realization of a given algorithm it was obtained an optimal value of the gluing area length  $L = 31.5$  mm. A graph of the change in patch thickness along the joint length is shown in Fig. 3. The main plate thickness graphs  $\delta_2 = 3$  mm is given for a scale.

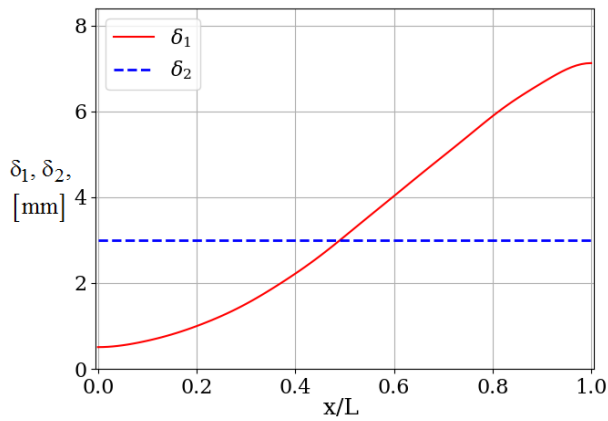


Fig. 3. Main plate thickness and patch profile

Stress graphs  $\tau$ ,  $\sigma$  in the adhesive layer and  $\tau_{max}$  graphs are shown in Fig. 4.

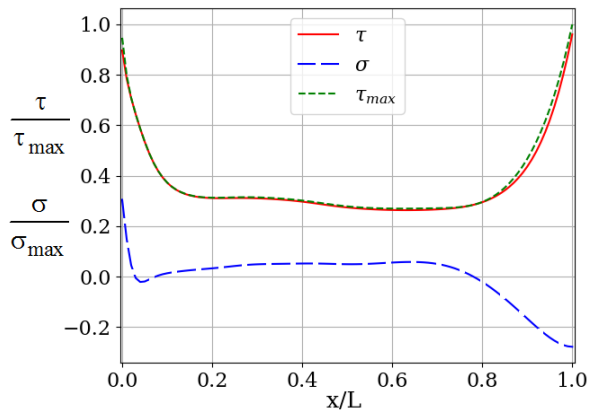


Fig. 4. Stress distribution in the adhesive layer

The change in the average truncated value of the objective function in the population (it is 10% of the maximal values are cast out) during the optimization process is shown in Fig. 5. The number of evolutionary cycles  $M$  is plotted along the horizontal axis.

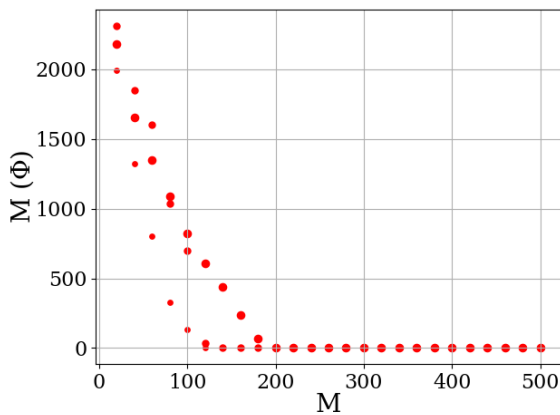


Fig. 5. Objective function values at different steps of the optimization

The objective function minimal value, obtained as a result of the genetic algorithm, is equal to  $\Phi = 0.0001275$ . This value and the corresponding parameters of the individual are the starting point for the performance of the particle swarm algorithm in the next step. A graph of the objective function values at the stages of information exchange between swarms is shown in Fig. 6.

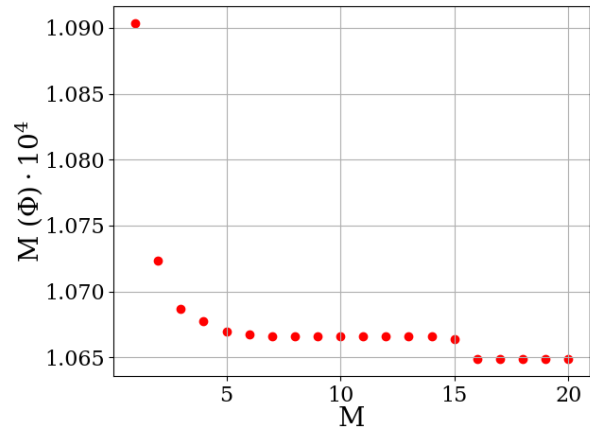


Fig. 6. Objective function values

Therefore, the particle swarm algorithm allows us to decrease the objective function value from  $\Phi = 0.0001275$  to  $\Phi = 0.0001065$ .

Nevertheless, as you can see, a solution close to the optimal can be found fairly quickly. Increasing the number of iterations of the algorithm by several thousand does not significantly affect the solution found above. However, the application of the particle swarm algorithm to the obtained solutions makes it possible to reduce the value of the objective functions by another 20-25% compared with the results achieved using the genetic algorithm.

### Conclusions

To reduce computation time, a one-dimensional mathematical model of the stress state of an adhesive joint is used. To describe the shape of the patch, expansion of the function in a Fourier series is used. This approach made it possible to create a genetic algorithm for topological optimization, which allows one to find a solution to the problem very quickly. The Python program takes only a few minutes to perform the calculations.

Calculations have shown that the use of the particle swarm algorithm at the second stage of optimization allows us to reduce the value of the objective function by approximately 20% compared with that obtained using the genetic algorithm. This suggests that the proposed approach has an advantage over the traditional genetic algorithm, which was previously used to solve this problem [12, 13].

The results obtained can be the basis for the development of several directions, such as the structural optimization of composite patches, the optimization of bi-adhesive joints, and joints of round patches. The mathematical model of the stress state of a round patch [22] is quite close to that considered in this study. The task of optimizing the shape of repair patches is quite relevant. In addition, we plan to develop this approach for solving topological optimization problems in a two-dimensional formulation [23], which is a qualitatively more difficult problem. It is also interesting to use more complex optimization algorithms based on the particle swarm algorithm, where not only velocities but also acceleration of particles [24] and other variants of swarm algorithms are considered [25].

**Contributions of authors:** conceptualization, methodology – **Olexandr Polyakov, Oleksii Vambol**; formulation of tasks, analysis – **Fedir Gagauz, Valeriy Cheranovskiy**; development of model, software, verification – **Olexandr Polyakov, Hanna Barakhova, Kristina Vernadska**; analysis of results, visualization – **Olexandr Polyakov, Oleksii Vambol, Fedir Gagauz, Valeriy Cheranovskiy**; writing – original draft preparation – **Hanna Barakhova, Kristina Vernadska**; writing – review and editing – **Hanna Barakhova, Kristina Vernadska**.

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## References

1. Dveirin, O. Z., Andreev, O. V., Kondrat'ev, A. V., & Haidachuk, V. Y. Stressed State in the Vicinity of a Hole in Mechanical Joint of Composite Parts. *International Applied Mechanics*, 2021, vol. 57, no. 2, pp. 234-247. DOI: 10.1007/s10778-021-01076-4.
2. da Silva, L. F. M., das Neves, P. J. C., Adams, R. D., & Speltz J. K. Analytical models of adhesively bonded joints. Part I: Literature survey. *International Journal of Adhesion and Adhesives*, 2009, vol. 29, iss. 3, pp. 319-330. DOI: 10.1016/j.ijadhadh.2008.0005.
3. Amidi, S., & Wang, J. An analytical model for interfacial stresses in double-lap bonded joints. *The Journal of Adhesion*, 2019, vol. 95, iss. 11, pp. 1031-1055. DOI: 10.1080/00218464.2018.1464917.
4. Barakhov, K. P., & Taranenko, I. M. Influence of Joint Edge Shape on Stress Distribution in Adhesive Film. In: *Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds) Integrated Computer Technologies in Mechanical Engineering - 2021. ICTM 2021. Lecture Notes in Networks and Systems*, 2022, vol. 367, pp. 123-132. Springer, Cham. DOI: 10.1007/978-3-030-94259-5\_12.
5. Ejaz, H., Mubashar, A., Ashcroft, I. A., Uddin, E., & Khan, M. Topology optimisation of adhesive joints using non-parametric methods. *International Journal of Adhesion and Adhesives*, 2018, vol. 81, pp. 1-10. DOI: 10.1016/j.ijadhadh.2017.11.003.
6. Arhore, E. G., Yasaei, M., & Dayyani, I. Comparison of GA and topology optimization of adherend for adhesively bonded metal composite joints. *International Journal of Solids and Structures*, 2021, vol. 226-227, 111078. DOI: 10.1016/j.ijsolstr.2021.111078.
7. Kaye, R., & Heller, M. Through-thickness shape optimisation of typical double lap-joints including effects of differential thermal contraction during curing. *International Journal of Adhesion and Adhesives*, 2005, vol. 25, iss. 3, pp. 227-238. DOI: 10.1016/j.ijadhadh.2004.07.006.
8. Groth, H. L., & Nordlund, P. Shape optimization of bonded joints. *International Journal of Adhesion and Adhesives*, 1991, vol. 11, iss. 4, pp. 204-212. DOI: 10.1016/0143-7496(91)90002-y.
9. Kaufmann, M., Vallée, T. Topology optimization of adhesively bonded double lap joints. *International Journal of Adhesion and Adhesives*, 2022, vol. 118, 103238. DOI: 10.1016/j.ijadhadh.2022.103238.
10. Khadiri, I. E., Zemzami, M., Hmina, N., Lagache, M., & Belhouideg, S. Topology Optimization of Structures Obtained by Additive Manufacturing: Case of 3D beam. *2021 7th International Conference on Optimization and Applications (ICOA)*, 2021, 235307775. DOI: 10.1109/icoa51614.2021.9442628.
11. Kurennov, S., Barakhov, K., Polyakov, O., & Taranenko, I. Application of genetic algorithm for double-lap adhesive joint design. *Archive of Mechanical Engineering*, 2023, vol. 70, no. 1, pp. 27-42. DOI: 10.24425/ame.2022.144074.
12. Kurennov, S., Barakhov, K., & Vambol, O. Topological optimization of a symmetrical adhesive joint. Island model of genetic algorithm. *Radioelectronic and Computer Systems*, 2022, no. 3, pp. 67-83. DOI: 10.32620/reks.2022.3.05.
13. Kurennov, S., Barakhov, K., & Vambol, O. Topological optimization BI-adhesive double lap adhesive joint. One-dimension model. *International Journal of Adhesion and Adhesives*, 2023, vol. 126, 103474. DOI: 10.1016/j.ijadhadh.2023.103474.
14. Wei, N, Ye, H, Wang, W, Li, J, Tian, F, Sui, Y. Topology Optimization for Hybrid Lattice Compliant Mechanisms with Multiple Microstructures. *Materials*, 2022, vol.15, no. 20, 7321. DOI: 10.3390/ma15207321.
15. Arjomandi, M.A., Shishehsaz, M., Ghanbarzadeh, A., Mosallanezhad, B., Akrami, M. Application of Particle Swarm Optimization for Improvement of Peel Strength in a Laminated Double-Lap Composite Joint. *Applied Sciences*, 2022, vol.12, iss.14, 6997. DOI: 10.3390/app12146997.



16. Robinson, J., Sinton, S. and Rahmat-Samii, Y. Particle swarm, genetic algorithm, and their hybrids: optimization of a profiled corrugated horn antenna. *IEEE Antennas and Propagation Society International Symposium (IEEE Cat. No.02CH37313)*, San Antonio, TX, USA, 2002, vol. 1, pp. 314-317. DOI: 10.1109/APS.2002.1016311.
17. Li, J., Gonsalves, T. A Hybrid Approach for Metaheuristic Algorithms Using Island Model. In: *Arai, K. (eds) Proceedings of the Future Technologies Conference (FTC) 2021, Volume 3. FTC 2021. Lecture Notes in Networks and Systems*, 2022, vol 360, pp. 311-322. Springer, Cham. DOI: 10.1007/978-3-030-89912-7\_24.
18. Kurennov, S., Barakhov, K., Taranenko, I., & Stepanenko, V. A genetic algorithm of optimal design of beam at restricted sagging. *Radioelectronic and Computer Systems*, 2022, no. 2, pp. 83-91. DOI: 10.32620/reks.2022.1.06.
19. Wen-Jun Zhang, & Xiao-Feng Xie. DEPSO: hybrid particle swarm with differential evolution operator,” SMC'03 Conference Proceedings. 2003 IEEE International Conference on Systems, Man and Cybernetics. *Conference Theme – System Security and Assurance (Cat. No.03CH37483)*, Washington, DC, USA, 2003, vol.4, pp. 3816-3821. DOI: 10.1109/ICSMC.2003.1244483.
20. Rana, S., Sarwar, Md., & Anwar Shahzad Siddiqui, Prashant. Particle Swarm Optimization: An Overview, Advancements and Hybridization. *Optimization Techniques in Engineering: Advances and Applications*, 2023, pp. 95-113. DOI: 10.1002/9781119906391.ch6.
21. Shami, T. M., El-Saleh, A. A., Alswaitti, M., Al-Tashi, Q., Summakieh, M. A. and Mirjalili, S. Particle Swarm Optimization: A Comprehensive Survey. In *IEEE Access*, 2022, vol. 10, pp. 10031-10061, DOI: 10.1109/ACCESS.2022.3142859.
22. Kurennov, S., & Smetankina, N. Stress-Strain State of a Double Lap Joint of Circular Form. Axisymmetric Model. In: *Nechyporuk, M., Pavlikov, V., Kritskiy, D. (eds) Integrated Computer Technologies in Mechanical Engineering - 2021. ICTM 2021. Lecture Notes in Networks and Systems*, 2022, vol. 367, pp. 36-46. Springer, Cham. DOI: 10.1007/978-3-030-94259-5\_4.
23. Kurennov, S. S., Polyakov, O. G., & Barakhov, K. P. Two-Dimensional Stressed State of an Adhesive Joint. Nonclassical Problem. *Journal of Mathematical Sciences*, 2021, vol. 254, pp. 156-163. DOI: 10.1007/s10958-021-05295-5.
24. Koshevoy, N., Ilina, I., Tokariev, V., Malkova, A., & Muratov, V. Implementation of the Gravity Search Method for Optimization by Cost Expenses of Plans for Multifactorial Experiments. *Radioelectronic and Computer Systems*, 2023, no. 1, pp. 23-32. DOI: 10.32620/reks.2023.1.02.
25. Bodyanskiy, Ye., Shafronenko, A., & Pliss, I. Clusterization of vector and matrix data arrays using the combined evolutionary method of fish schools. *System research and information technologies*, 2022, no. 4, pp. 79-87. DOI: 10.20535/SRIT.2308-8893.2022.4.07.

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## ГІБРИДНИЙ АЛГОРИТМ ТОПОЛОГІЧНОЇ ОПТИМІЗАЦІЇ КЛЕЙОВОГО З'ЄДНАННЯ

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**Предметом** вивчення в статті є алгоритм топологічної оптимізації симетричного клейового з'єднання внапуск. **Метою** є розробка методики розв'язання задачі оптимізації, яка дозволить поєднати високу швидкість та стійкість одержуваних результатів за рахунок поєднання двох алгоритмів оптимізації – генетичного алгоритму на початковому етапі і алгоритму рою частинок на другому етапі оптимізації. **Завдання:** для задачі оптимального проектування симетричного двостороннього клейового з'єднання внапуск, яке складається з основної пластини та двох накладок (основна пластина якого має постійну товщину, а товщина накладок змінюється по довжині з'єднання для зниження концентрації напружень у з'єднанні та зменшення його ваги) створити методику розв'язання оптимізаційної задачі для задоволення критерія оптимальності, а саме, мінімізації маси конструкції за умови обмеження по міцності та по товщині накладки. Задача оптимізації полягає в знаходженні оптимальної форми накладок, а саме – довжини накладки і залежності товщини накладки від її довжини. **Методи:** для описання напружено-деформованого стану з'єднання використана модифікована модель Голанда-Рейсснера. Для розв'язання прямої задачі зі знаходження напруженого стану конструкції використано метод скінченних різниць. Для числового розв'язання задачі оптимізації використано комбінацію багатопопуляційної моделі генетичного алгоритму оптимізації і алгоритму рою частинок. Для покращення роботи генетичного алгоритму застосовано його багатопопуляційну модель із міграцією кращих особин між популяціями. Внесення до популяцій особин із інших популяцій дозволяє уникнути гомогенізації генотипу в окремі популяції та передчасної зупинки процесу оптимізації. Для описання форми накладки використано

розвинення залежності товщини накладки в ряд Фур'є. **Результати:** запропоновано гібридний алгоритм, оснований на послідовному застосуванні генетичного алгоритму і алгоритму рою частинок для трьох популяцій розв'язків. Алгоритм рою частинок дозволяє на 20% поліпшити досягнуте на попередньому етапі значення цільової функції. **Висновки:** Наукова новизна полягає в удосконаленні алгоритму оптимізації у порівнянні з відомими. З метою скорочення часу розрахунку в роботі використовувалась одновимірна математична модель напруженого стану клейового з'єднання. Використовувані методи дозволили створити комбінований алгоритм топологічної оптимізації, який поєднує переваги обох методів та дозволяє дуже швидко знаходити розв'язок досліджуваної задачі. Час реалізації програми на мові Python становить лише декілька хвилин.

**Ключові слова:** клейове з'єднання; генетичний алгоритм; алгоритм рою частинок; метод скінченних різниць; топологічна оптимізація.

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