T44

THEORETICAL MECHANICS.

KINEMATICS

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THEORETICAL MECHANICS. KINEMATICS

Tutorial for self-education

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T33

Розглянуто кінематику точки, найпростіші види руху – поступальний та обертовий, а також плоскопаралельний і складний рухи. Наведено короткі відомості з теоретичного курсу, основні формули і пояснення до них. Подано розв'язання задач різної складності.

Для студентів механічних та інших спеціальностей (з повною та скороченою програмою з теоретичної механіки).

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The workbook considers kinematics of a particle, the simplest types of motion – translational and rotational, and also plane and compound motions. The information from the theoretical course, the basic formulas and their explanations are given in the workbook. The problems of different complexity are presented.

For college students studying theoretical mechanics (for full-time and for reduced courses of study)

Fig. 62. Table 1. Bibliography: 3 names

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1. KINEMATICS OF PARTICLES

1.1. Main information from the theoretical course

Kinematics is the branch of mechanics, which treats of particle motion as such, without regard to its cause that is Kinematics deals only with geometrical aspect of the motion.

Motion is changing in the time domain of a particle's location with respect to other point. This point is the origin (datum) of a coordinate system. Position of a particle is determined with respect to chosen coordinate system by coordinates that are functions of time, so time reference point must be specified too.

Motion of a particle is considered as defined if the following characteristics are specified

- particle trajectory;
- particle velocity and acceleration;
- type of motion (accelerated, decelerated, special points of the trajectory).

Trajectory (path) is the continuous line along which particle travels. The motion is called **rectilinear** if path is a straight line; if path is curved line the motion is **curvilinear.**

Particle motion can be describes by three methods:

1. **Vectorial.** The position of a particle in three-dimensional space is specified by its **vector-position** $\vec{r}$ connecting the origin of reference, the point O, with a point M, where the particle is situated. Vector position $\vec{r}$ is determined by its magnitude (module) $|\vec{r}|$ and direction. The motion is prescribed if vector $\vec{r}(t)$ is known as a function of the time

$$\vec{r} = \vec{r}(t). \tag{1.1}$$

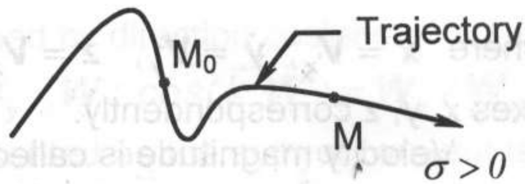
2. **Coordinate.** Motion of a particle is prescribed if particle coordinates are known as a function of the time. For Cartesian coordinate system axes it means that the following functions are known

$$\begin{cases} x = f_1(t), \\ y = f_2(t), \\ z = f_3(t). \end{cases} \tag{1.2}$$

3. **Natural.** It can be realized only if the particle trajectory is given. Natural method supposes that motion is prescribed if the position of a particle on its trajectory is known as function of the time.

To realize the natural method of the particle motion representation it is necessary to introduce:

- the reference point (the position of the particle on the trajectory at the moment when time t equals zero),
- the positive direction of curvilinear coordinate reading,
- time dependence of curvilinear coordinate $\sigma = M_0M = f(t)$, the last expression is called motion law.



The methods of particle motion representation are interconnected. For vectorial method we have

$$\vec{r} = \vec{r}(t) = r_x(t)\vec{i} + r_y(t)\vec{j} + r_z(t)\vec{k},$$

where

$$x = r_x(t), \quad y = r_y(t), \quad z = r_z(t).$$

So

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}. \qquad (1.3)$$

For vector position magnitude we get

$$r = \sqrt{x^2 + y^2 + z^2}, \qquad (1.4)$$

Orientation of the vector position is determined by direction cosines:

$$cos(\vec{i}, \vec{r}) = x / r, \quad cos(\vec{j}, \vec{r}) = y / r, \quad cos(\vec{k}, \vec{r}) = z / r. \qquad (1.5)$$

The relation between coordinate and natural methods may be expressed as

$$\sigma = \pm \int_0^t \sqrt{(\dot{x})^2 + (\dot{y})^2 + (\dot{z})^2}\, dt. \qquad (1.6)$$

Signs «+» and «-» specify the direction of a particle motion. If a particle moves in direction of chosen positive arc reading, sign «+» should be used, otherwise – sign «-».

Particle velocity.

Particle instantaneous velocity is limiting value of particle displacement $\Delta\vec{r}$ divided by time interval Δt as the **time interval appropriates zero.

$$\vec{V} = \lim_{t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}. \qquad (1.7)$$

So for vectorial method particle velocity is derivative of vector position with respect to time. Velocity is always a vector tangent to the path, velocity points to the side of particle motion.

For coordinate method particle velocity is determined as

$$\vec{V} = \dot{\vec{r}} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}, \qquad (1.8)$$

where $\dot{x} = V_x$, $\dot{y} = V_y$, $\dot{z} = V_z$ are velocity projections on the coordinate axes x, y, z correspondently.

Velocity magnitude is called speed and is determined as

$$V = \sqrt{V_x^2 + V_y^2 + V_z^2} = \sqrt{(\dot{x})^2 + (\dot{y})^2 + (\dot{z})^2}. \qquad (1.9)$$

Orientation of the velocity is determined by direction cosines:

$$cos(\vec{i},\vec{V}) = V_x / V, \; cos(\vec{j},\vec{V}) = V_y / V, \; cos(\vec{k},\vec{V}) = V_z / V . \qquad (1.10)$$

Motion is called uniform if speed does not vary with time.

For natural method particle velocity is

$$\vec{V} = V_\tau \vec{\tau} = \frac{d\sigma}{dt}\vec{\tau}, \qquad (1.11)$$

where $\sigma = \dot{\sigma}(t)$ is the law of motion along the trajectory, $\vec{\tau}$ is unit vector of tangent to the trajectory.

For speed we have

$$V = |\dot{\sigma}(t)|. \qquad (1.12)$$

Velocity direction is determined by the sign of the derivative $\frac{d\sigma}{dt}$. If the derivative is positive (it means that the velocity projection on the tangent to the trajectory is positive $V_\tau = V$), than velocity points to the side of positive direction of curvilinear coordinate σ. If the derivative is negative ($V_\tau = -V$), than velocity points to the side of negative direction of curvilinear coordinate σ.

Particle acceleration.

Particle instantaneous acceleration is derivative of velocity or the second time derivative of particle vector position.

$$\vec{W} = \dot{\vec{V}} = \ddot{\vec{r}}. \qquad (1.13)$$

For coordinate method we may write

$$\vec{W} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}, \qquad (1.14)$$

where $\ddot{x} = W_x$, $\ddot{y} = W_y$, $\ddot{z} = W_z$ are velocity projections on the coordinate axes x, y, z correspondently.

For acceleration magnitude we may write

$$W = \sqrt{W_x^{\,2} + W_y^{\,2} + W_z^{\,2}} = \sqrt{(\ddot{x})^2 + (\ddot{y})^2 + (\ddot{z})^2}. \qquad (1.15)$$

Orientation of the acceleration is determined by direction cosines:

$$cos(\vec{i}, \vec{W}) = W_x / W;\ cos(\vec{j}, \vec{W}) = W_y / W;\ cos(\vec{k}, \vec{W}) = W_z / W.$$

For natural method acceleration is represented as the sum of two mutual perpendicular vectors

$$\vec{W} = \vec{W}_\tau + \vec{W}_n. \qquad (1.16)$$

The first term $\vec{W}_\tau$ is called tangent acceleration and may be determined as

$$\vec{W}_\tau = \frac{dV_\tau}{dt}\vec{\tau}. \qquad (1.17)$$

The vector $\vec{W}_\tau$ is always tangent to the trajectory, magnitude of the acceleration $\vec{W}_\tau$ is magnitude of the derivative of the velocity projection on the tangent.

$$\left|\vec{W}_\tau\right| = \left|\frac{dV_\tau}{dt}\right| = \left|\frac{d^2\sigma}{dt^2}\right|. \qquad (1.18)$$

Tangent acceleration direction is determined by the sign of the derivative $\frac{d^2\sigma}{dt^2}$. If the derivative positive, than the tangent acceleration points to the side of positive direction of curvilinear coordinate σ. If the derivative negative, than the tangent acceleration points to the side of negative direction of curvilinear coordinate σ.

Comparing signs of the $\frac{d^2\sigma}{dt^2}$ and $\frac{d\sigma}{dt}$ we can characterize **the type of motion**:

- if the derivatives have the same sign motion is **accelerated** (in this case velocity and tangent acceleration point to the same side),
- if the derivatives have the opposite signs motion is **decelerated** (velocity and tangent acceleration are opposite). So the tangent acceleration characterizes the variation of velocity magnitude.

The second term $\vec{W}_n$ in the equation (1.16) is called normal acceleration. For its magnitude we have

$$W_n = \frac{V^2}{\rho}, \qquad (1.19)$$

where V is velocity magnitude, ρ is radius of curvature of the trajectory at the given position of a particle.

The normal acceleration is always along the normal to the trajectory and points toward the center of curvature of the trajectory. The normal acceleration characterizes the variation of the velocity direction.

The total acceleration magnitude may be determined as

$$W = \sqrt{W_n^2 + W_\tau^2}. \qquad (1.20)$$

1.2. Problems solving

Problem 1.1

A law of particle motion along a trajectory is given: $s = t^3 - 3t$ (m). Find the displacement which is passed by the particle during the period of time [0; 3].

Solution

In the given case the law of motion is given in natural form. If *s(t)* is a monotonic function then the past displacement on the instant of time [t_1; t_2] is calculated as $l = s(t_2) - s(t_1)$. If the function on the considered segment is not monotonic then it must be resolved into segments of monotony and we must summarize displacements on each segment.

Let's examine the function *s(t)*. Let's differentiate it by time and equate to zero:

$$\dot{s} = 3t^2 - 3 = 0.$$

This quadratic equation has two roots: $t_1 = 1$, $t_2 = -1$, but the second one is out of the interval [0; 3]. Therefore, there are two segments of monotony of our function and the displacement, which is passed by the particle during the time [0; 3],

$$l_{03} = l_{01} + l_{13} = |s(1) - s(0)| + |s(3) - s(1)| =$$

$$= |1 - 3| + |27 - 9 - 1 + 3| = 2 + 20 = 22 \ (m).$$

Problem 1.2

A particle moves on a plane according to the equations $x = 8\cos(24t)$, $y = 7\sin^2(12t)$. Find the equation of the particle's trajectory, the law of its motion along this trajectory, calculating distance from the initial position, the velocity and the acceleration when $t_1 = 7\pi/48$ (**x** and **y** are given in meters, **t** – in seconds).

Solution

The motion of particle is given by coordinate way.

Let's find the equation of trajectory, excluding parameter **t** from the equations of motion:

$$x = 8\cos(24t) = 8(\cos^2(12t) - \sin^2(12t)) = 8(1 - 2\sin^2(12t)).$$

It follows that

$$\sin^2(12t) = (8 - x)/16.$$

Substitute this value to the dependence **y(t)**, we get

$$y = 3,5 - 7x/16. \qquad (1.21)$$

This is an equation of line, but the trajectory of the particle is only the segment M_0M_1 (Fig. 1.1), because according to the given equations of motion the coordinate **x** is on the interval **(-8; 8)**, and the coordinate **y** – on the interval **(0; 7)**.

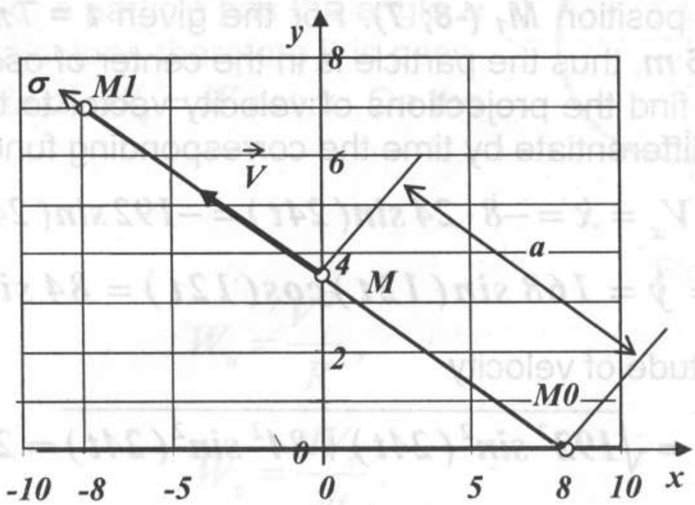


Fig. 1.1

Let's find the law of particle's motion along the trajectory. To do this let's differentiate the dependencies **x(t)** и **y(t)**:

$$dx = -8\cdot 24\sin(24t)dt = -192\sin(24t)dt;$$

$$dy = 168\sin(12t)\cos(12t)dt = 84\sin(24t)dt.$$

Then

$$d\sigma = \sqrt{dx^2 + dy^2} = \sqrt{192^2 + 84^2}\, sin(24t)dt = 209{,}57\, sin(24t)dt\,. \quad (1.22)$$

In order to obtain the equation $\sigma(t)$, let's integrate the expression (1.22):

$$\sigma(t) = -8{,}73cos(24t) + C. \quad (1.23)$$

The coordinate σ is counted from the initial position of the particle, that is why when $t = 0$ $\sigma = 0$ and equation (1.23) gets a look like

$$0 = -8{,}73 + C,$$

From this $C = 8{,}73$.

Finally, the law of motion will look like

$$\sigma(t) = 8{,}73(1 - cos(24t)).$$

These are harmonic oscillations with an amplitude $a = 8{,}73$ m and a period of oscillations $\tau = \pi/48$ s.

In the beginning of motion, when $t = 0$, the particle is in the extreme position M_0 (8; 0), and in the moment of time $t = \pi/48$ s, when $cos(24t) = 0$, – in the center of oscillations M (0; 3,5). When $t = \pi/24$ s the particle reaches the second extreme position M_1 (-8; 7). For the given $t = 7\pi/48$ s the coordinate are $x = 0$, $y = 3{,}5$ m, thus the particle is in the center of oscillations.

In order to find the projections of velocity vector to the coordinate axes it is necessary to differentiate by time the corresponding functions:

$$V_x = \dot{x} = -8 \cdot 24\, sin(24t) = -192\, sin(24t)\,;$$

$$V_y = \dot{y} = 168\, sin(12t)cos(12t) = 84\, sin(24t).$$

The magnitude of velocity

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{192^2 sin^2(24t) + 84^2 sin^2(24t)} = 209{,}57 \cdot |sin(24t)|\,.$$

In the instant of time $t_1 = {}^{7\pi}\!/_{48}$

$$V_x = 192 \ m/s, V_y = -84 \ m/s,$$

$$V = 209{,}57 \cdot \left| sin(\frac{24}{48} \cdot 7\pi) \right| = 209{,}57 \ m/s.$$

The direction of $\vec{V}$, with taking into account the signs of the projections V_X, V_Y, is shown on the Fig. 1.1.

Let's find the acceleration of the particle. Projections of the acceleration to the coordinate axes are

$$W_x = \ddot{x} = -192 \cdot 24\cos(24t) = -4608\cos(24t),$$

$$W_y = \ddot{y} = 84 \cdot 24\cos(24t) = 2016\cos(24t).$$

At $t_1 = {}^{7\pi}\!/_{48}$ we get

$$W_x = -4608\cos(\frac{7\pi}{2}) = 0,\; W_y = 2016\cos(\frac{7\pi}{2}) = 0.$$

Therefore, the total acceleration of the particle is $W = 0$. Note that for straight linear trajectory the radius of its curvature is $\rho = \infty$.

Problem 1.3

A particle moves along a circle with radius R. The initial velocity is V_0 (Fig.1.2). The acceleration of particle has a constant angle α with the velocity of it and this angle is constant.Find the value of particle's velocity as a function of time.

Solution

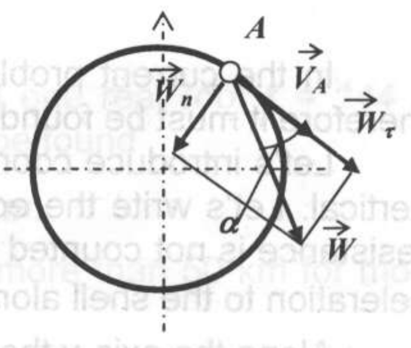


Fig. 1.2

The acceleration of a particle has the angle α with a tangent to the trajectory, therefore it is possible to write $W_n = W\sin\alpha$, $W_\tau = W\cos\alpha$. On the other hand,

$$W_n = \frac{V^2}{R},$$

$$W_\tau = \frac{dV_\tau}{dt}.$$

It is known that $\alpha = const$. Therefore,

$$ctg\alpha = \frac{W_\tau}{W_n} = \frac{\dot{V}_\tau R}{V^2} = const.$$

This is a regular differential equation with separable variables. Let's integrate it and find dependence between the velocity of a particle and time:

$$\int\frac{dV_\tau}{V^2} = \int\frac{ctg\alpha}{R}dt,\; -\frac{1}{V} = \frac{ctg\alpha}{R}t + C. \qquad (1.24)$$

A constant of integration can be found from the initial condition. It is known that when $t_0 = 0$ the velocity of a particle is $V = V_0$. Substituting these values to the equation (1.24) it is found that $C = -\frac{1}{V_0}$.

Then the dependence ***V(t)*** will look like

$$V = -\frac{1}{\frac{ctg\alpha}{R}t - \frac{1}{V_0}} = \frac{V_0 R}{R - V_0 t \cdot ctg\alpha}.$$

Problem 1.4

A cannon of coast guard shoots from a height of ***h = 30 m*** with an initial velocity ***V₀ = 1000 м/c*** on the angle $\alpha = 45°$ to the horizon. What distance does the shell fall from the cannon if the air resistance is neglected?

Solution

In the current problem the law of particle's motion is not given directly, therefore it must be found with the help of known conditions.

Let's introduce coordinate axes, the axis ***x*** is horizontal and the axis ***y*** is vertical. Let's write the equation of shell's motion in coordinate form. If the air resistance is not counted then there are no forces, which would induce and acceleration to the shell along the axis ***x***, thus $W_x = \ddot{x} = 0$.

Along the axis ***y*** there is a gravity force which acts to the shell and induce the acceleration of free falling, thus $W_y = \ddot{y} = -g$.

Integrating twice these equations we get

$$V_x = \int \ddot{x}\,dt = \int 0\,dt = C_1$$

and

$$x = \int V_x dt = \int C_1 dt = C_1 t + C_3,$$

$$V_y = \int \ddot{y}\,dt = \int (-g)\,dt = -gt + C_2,$$

$$y = \int V_y dt = \int (-gt + C_2)dt = \frac{-gt^2}{2} + C_2 t + C_4.$$

The constant of integration is found from the initial conditions. It is known that when ***t = 0***

$$x(0) = 0, y(0) = h, V_X = V_0 \cos\alpha, V_Y = V_0 \sin\alpha.$$

Then

$$V_X = V_0 \cos\alpha = C_1, \qquad 0 = V_0 \cos\alpha \cdot 0 + C_3 \Rightarrow C_3 = 0,$$
$$V_Y = V_0 \sin\alpha = -g \cdot 0 + C_2, \qquad h = -g0/2 + V_0 \sin\alpha \cdot 0 + C_4 \Rightarrow C_4 = h.$$

The law of the shell's motion with taking the values of constants into account is:

$$x = (V_0 \cos\alpha)t; \tag{1.25}$$
$$y = -gt^2/2 + (V_0 \sin\alpha)t + h. \tag{1.26}$$

The sought-for distance can be found from the equation (1.25), if the time of the shell's flight is substituted there. The time is found from the equation (1.26), taking into consideration that in the moment of fall to the ground the coordinate $y = 0$.

Substituting the known values we get

$$-4{,}9t^2 + 707t + 30 = 0.$$

Having solved this quadratic equation we find one real root $t = 144$ **c**. Then from the equation (a) the desired distance can be found

$$x = 1000 \cdot 0{,}71 \cdot 144 = 101{,}8 \text{ km}.$$

Note: the maximum distance of firing is a bit more than 50 km for modern cannons with the given initial velocity.

Problem 1.5

Find the trajectory and examine the character of motion of the point **M**, which is placed on the rocker with length ***l***. The rocker is a part of a crank mechanism. The distance between the particle and the place where the rocker is joined to the crank **OA** is ***a***. The length of the crank is also ***l*** (Fig. 1.3). The angle φ changes according to the law $\varphi(t) = \pi t$, **AB = OA**.

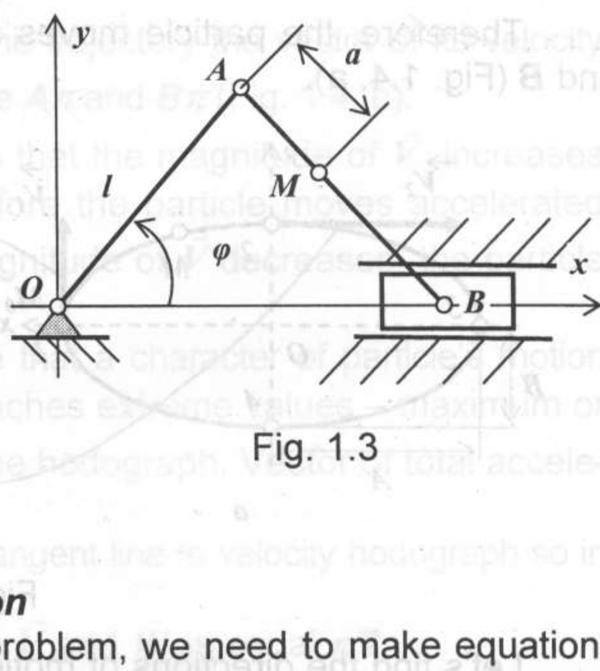


Fig. 1.3

Solution

Here, the same as in the previous problem, we need to make equations of motion. In the current case it is rational to use a coordinate way. In order to do this let's direct the abscissa horizontally and the ordinate – vertically and let's define the coordinates of the point ***M***:

$$x = (l+a)cos\varphi,$$
$$y = (l-a)sin\varphi.$$

Let's substitute the given dependence $\varphi = \pi t$ into these expressions, we get the law of the point motion

$$\begin{cases} x = (l+a)cos(\pi t), \\ y = (l-a)sin(\pi t). \end{cases} \qquad (1.27)$$

The equation of trajectory is a dependence between the point coordinates ***y = f(x).*** It can be obtained by excluding the parameter ***t*** from the system (1.27). In the current case it is optimal to use the following approach: to square left and right parts of each equation and then summarize them. Then we have

$$\frac{x^2}{(l+a)^2} + \frac{y^2}{(l-a)^2} = cos^2(\pi t) + sin^2(\pi t).$$

Signing ***(l+a) = A*** and ***(l-a) = B*** and taking into account that

$$cos^2(\pi t) + sin^2(\pi t) = 1,$$

the equation of trajectory can be found

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1,$$

Therefore, the particle moves along an ellipse in which semiaxes are ***A*** and ***B*** (Fig. 1.4, a).

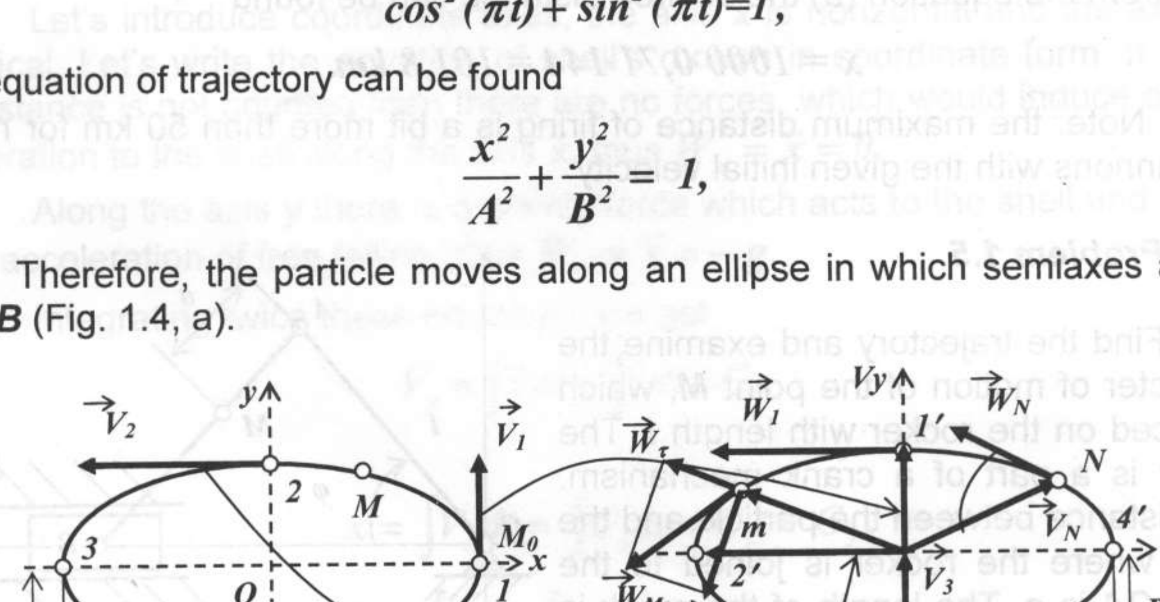


Fig. 1.4

Let's find the directions of motion. In order to do this let's suppose that in (a) ***t = 0***. Then in the initial instant the coordinates of the particle M (A; 0), thus the particle is in the position 1. When t increases the coordinate x decreases and y increases. Therefore, the particle moves along the ellipse anticlockwise. When $t = 0{,}5$ s the coordinates of the article M (0; B), it is in the position 2, etc.

Note, that direction of motion can be also found with the help of direction of velocity vector. Let's find its projections:

$$V_x = \dot{x} = -A\pi \sin(\pi t); \qquad V_y = \dot{y} = B\pi \cos(\pi t). \qquad (1.28)$$

When $t = 0$ we get $V_X = 0$, therefore, $V = V_Y = B\pi$ and the vector is directed up in the initial position, which also shows that the particle moves anticlockwise.

To prove monotony of the motion there are several ways. Analytical methods of functions' examination are red in the course of High Mathematics. In the current case let's use the graph-analytical method, for which we need to make a hodograph of velocity.

Let's remind that hodograph is a curve which is drawn by a vector tip if its beginning is fixed in some unmovable point. Hodograph of velocity is drawn on coordinates V_X, V_Y. It means that equations of velocity hodograph in parametric form that look like (1.28). The equation of velocity hodograph can be obtained by the same way as was used for finding the trajectory of the particle. Let's exclude the parameter t from the equations (1.28) and get the equation of the velocity hodograph

$$\frac{V_x^2}{(A\pi)^2} + \frac{V_y^2}{(B\pi)^2} = 1.$$

Thus when the particle moves along the trajectory the vector of its velocity moves along an ellipse in which semiaxes are $A\pi$ and $B\pi$ (Fig. 1.4, b).

It is seen from the velocity hodograph that the magnitude of $\vec{V}$ increases on the intervals 1 - 2 and 3 - 4 and therefore the particle moves accelerated and on the intervals 2 - 3 and 4 - 1 the magnitude of $\vec{V}$ decreases, the particle moves decelerated.

It is known from the theoretical course that a character of particle's motion changes when $W_\tau = 0$, thus the velocity reaches extreme values – maximum or minimum. These point are ***1', 2', 3', 4'*** on the hodograph. Vector of total acceleration $\vec{W} = \dfrac{d\vec{V}}{dt}$ is always directed along a tangent line to velocity hodograph so in these points $\vec{W} = \vec{W}_n$ and the angle between $\vec{V}$ and $\vec{W}$ is equal $\pi/2$.

Let's consider an arbitrary position of the particle M on the segment of trajectory 1 - 2. A point ***m*** on the velocity hodograph corresponds to this position (Fig. 1.4, a, б). Let's resolve the total acceleration $\vec{W}$, which is directed along a tangent line to the hodograph $\vec{V}$, into components $\vec{W}_n$ and $\vec{W}_\tau$. As it is known, $\vec{W}_\tau$ is directed along the same line as $\vec{V}$, and $\vec{W}_n$ is perpendicular to $\vec{W}_\tau$, then $\vec{W} = \vec{W}_n + \vec{W}_\tau$. Let's note that this is a segment of accelerated motion and that is why $\vec{V}$ and $\vec{W}_\tau$ are directed to the same side.

1.3. Self-control questions

1. Formulate the ways of particle motion defining.
2. Formulate the definition of velocity and acceleration of particle when the vector way of defining motion is used.
3. Formulate the definition of velocity and acceleration of particle when the coordinate way of defining motion is used.
4. How to resolve vectors of velocity and acceleration of particle to the axes of natural trihedral?
5. Write the formulas for velocity, tangential and normal accelerations of particle through angular position.
6. What conditions does the vector of total acceleration coincide with the vector of: a) normal acceleration; б) tangential acceleration in?
7. How is the character of motion defined when the vector way is used to do this?
8. How is the character of motion defined when the coordinate way is used?
9. What are the conditions of accelerated and decelerated motion of the particle when the natural way is used to define its motion?
10. Write and show on the drawing the relation between the vectors of total, normal and tangential accelerations.
11. What is the scalar product of velocity and normal acceleration vectors equal to?

1.4. Solving problems on your own

According to the given equations of motion of the particle M determine the type of the trajectory and for a moment of time $t=t_1$ find its position on the trajectory, its velocity, total, tangential and normal acceleration and a radius of the trajectory curvature.

All necessary data is given in the Table.

Variant number	Equations of motion		t_1, s
	x = x (t), cm	y= y (t), cm	
1	$-2t^2+3$	$-5t$	1/2
2	$4\cos^2(\pi t/3)+2$	$4\sin^2(\pi t/3)$	1
3	$-\cos(\pi t^2/3)+3$	$\sin(\pi t^2/3)-1$	1
4	$4t+4$	$-4/(t+1)$	2
5	$2\sin(\pi t/3)$	$-3\cos(\pi t/3)+4$	1
6	$3t^2+2$	$-4t$	1/2
7	$3t^2-t+1$	$5t^2-\frac{5}{3}t-2$	1
8	$7\sin(\pi t^2/6)+3$	$2-7\cos(\pi t^2/6)$	1
9	$-3/(t+2)$	$3t+6$	2
10	$-4\cos(\pi t/3)$	$-2\sin(\pi t/3)-3$	1
11	$-4t^2+1$	$-3t$	1/2
12	$5\sin^2(\pi t/6)$	$-5\cos^2(\pi t/6)-3$	1
13	$5\cos(\pi t^2/3)$	$5\sin(\pi t^2/3)$	1
14	$-2t-2$	$-2/(t+1)$	2
15	$4\cos(\pi t/3)$	$-3\sin(\pi t/3)$	1
16	$3t$	$4t^2+1$	1/2
17	$7\sin^2(\pi t/6)-5$	$-7\cos^2(\pi t/6)$	1
18	$1+3\cos(\pi t^2/3)$	$3\sin(\pi t^2/3)+3$	1
19	$-5t^2-4$	$3t$	1
20	$2-3t-6t^2$	$3-\frac{3}{2}t-3t^2$	0
21	$6\sin(\pi t^2/6)-2$	$6\cos(\pi t^2/6)+3$	1
22	$7t^2-3$	$5t$	1/4
23	$3-3t^2+t$	$4-5t^2+5t/3$	1
4	$-4\cos(\pi t/3)-1$	$-4\sin(\pi t/3)$	1
25	$-6t$	$-2t^2-4$	1
26	$8\cos^2(\pi t/6)+2$	$-8\sin^2(\pi t/6)-7$	1
27	$-3-9\sin(\pi t^2/6)$	$-9\cos(\pi t^2/6)+5$	1
28	$-4t^2+1$	$-3t$	1
29	$5t^2+\frac{5}{3}t-3$	$3t^2+t+3$	1
30	$2\cos(\pi t^2/3)-2$	$-2\sin(\pi t^2/3)+3$	1

2. THE SIMPLEST MOTIONS OF A RIGID BODY

2.1. Main information from the theoretical course

The simplest motions of a rigid body are translation and rotation about ixed axis.

2.1.1. Translation

Translation is defined as motion in which a straight line between two oints of the body always retains an orientation parallel to its original direction at all time.

In translation the trajectories of points of rigid body are equidistant curves, all points have the same velocities and accelerations relative to some reference

$$\vec{V}_1 = \vec{V}_2 = \dots = \vec{V}_n$$

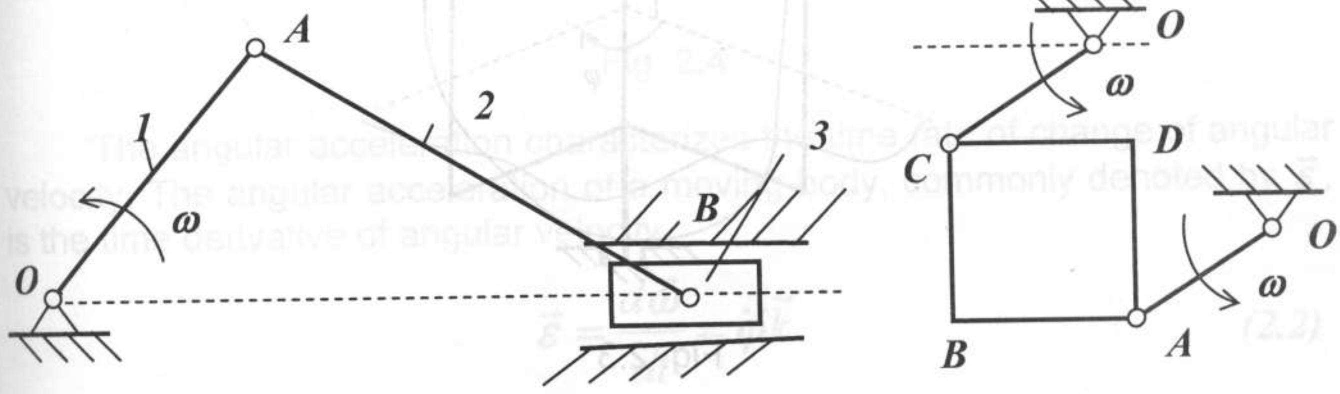
and

$$\vec{W}_1 = \vec{W}_2 = \dots = \vec{W}_n.$$

Therefore motion of one point completely describes the motion of the entire rigid body. We generally choose the motion of the center of mass to denote the motion of the rigid body.

In the following sketches the samples of translation of rigid bodies are represented.

In the Fig. 2.1, a slider 3 has translational motion, in the Fig. 2.1, b framework ***ABCD (O₁C = O₂A)*** is in translation, in the Fig. 2.2 slotted link ***3*** constrained by the supports ***K*** and ***L*** moves translationally.



a b

Fig. 2.1

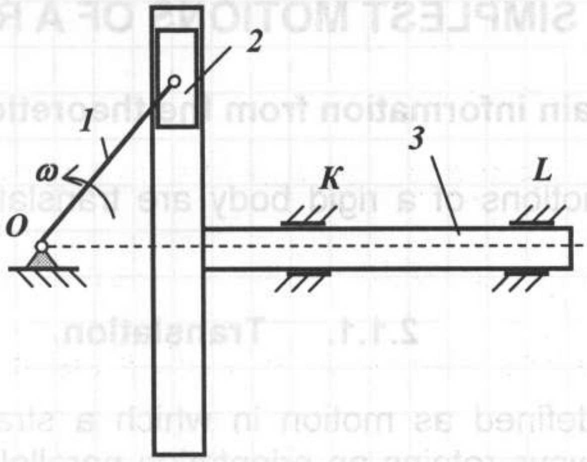


Fig. 2.2

2.1.2. Rotation about fixed axis

A rotation is such a motion of a rigid body that one line of the body or of an extension of the body remains fixed. The fixed line is called the axis of rotation.

Points of the body move in the planes that is perpendicular to the axis of rotation. Each point moves in a circle of radius equal the shortest distance from the point to the axis of rotation.

Rigid body position in rotation about fixed axis depends only on its angular coordinate φ . Let us consider two planes passing through the axis of rotation: fixed σ_1 and rigidly connected with rotating body σ_2 (Fig.2.3).

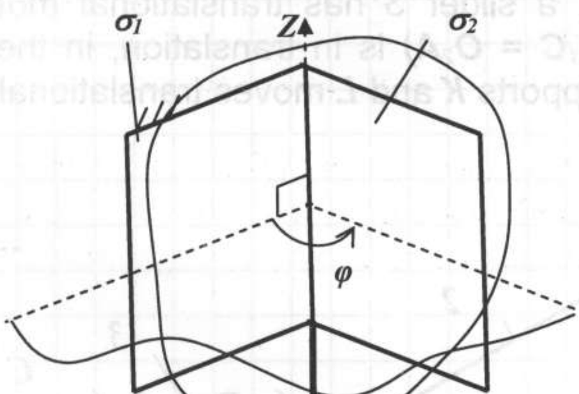


Fig. 2.3

The angle φ that these planes form is measure of rotation. The angle φ is regarded as positive if from the positive direction of the axis Oz we can see rotation of the moving plane contraclockwise. A single revolution is defined as the amount of rotation in either a clockwise or a counterclockwise direction about the axis of rotation that brings the body back to its original position. If the angle φ is not divisible to 2π we will say about partial revolutions.

Angular velocity $\vec{\omega}$ and angular acceleration ε are kinematical characteristics of a rotating body. These notions may be used for rigid body only (it is impossible to use these notion for particle).

The angular velocity characterizes the time rate of change of angular coordinate. The angular velocity of a rotating body is vector. It is directed along the axis of rotation of body such that from the end of the vector $\vec{\omega}$ rotation of the body about the axis of rotation is viewed anti clockwise (Fig. 2.4). Its magnitude is module of the time derivative of the angle φ

$$\vec{\omega} = \frac{d\varphi}{dt}\vec{k} = \dot{\varphi}\vec{k}. \tag{2.1}$$

The angular velocity is measured by rad/s or 1/s.

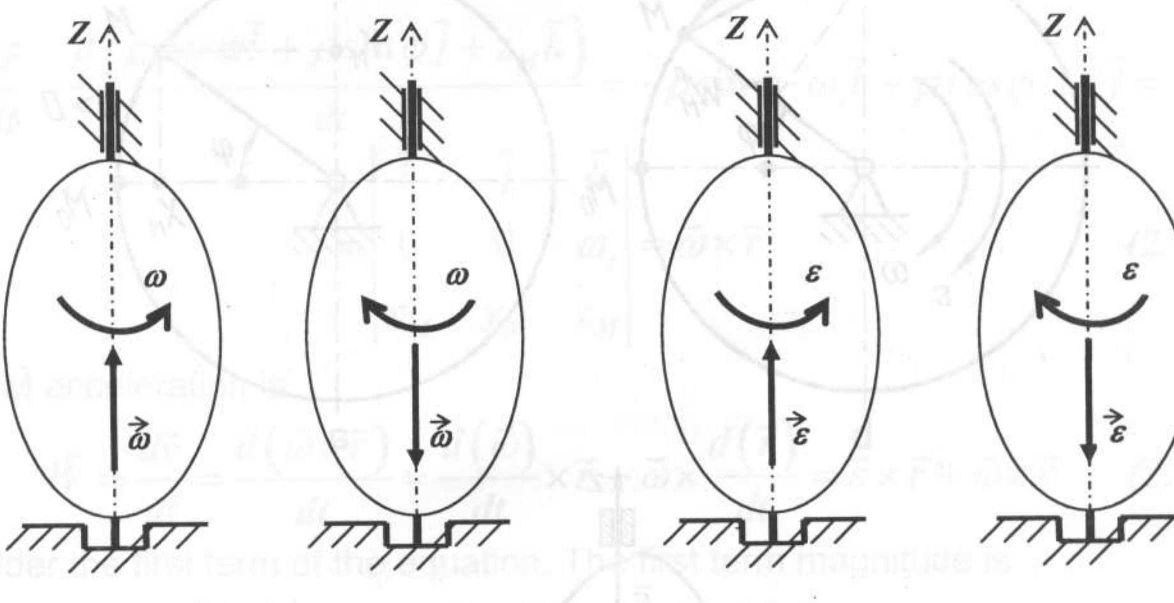


Fig. 2.4

The angular acceleration characterizes the time rate of change of angular velocity. The angular acceleration of a moving body, commonly denoted by $\vec{\varepsilon}$, is the time derivative of angular velocity

$$\vec{\varepsilon} = \frac{d\vec{\omega}}{dt} = \ddot{\varphi}\vec{k}. \tag{2.2}$$

Therefore it is evident from this equation that $\vec{\varepsilon}$ is positive or negative according to the sign of $d\omega$, that is, according to whether positive or negative angular velocity is being taken on. As with angular velocity, we associate sign with direction: positive angular acceleration is anti clockwise, and negative angular acceleration is clockwise.

If the angular velocity and angular acceleration have the same direction, the rotation is accelerated, in opposite case – rotation is decelerated.

There are simple relationships between the angular velocity and angular acceleration of a rotating body and the velocity and acceleration of any point of the body.

Let M be any point of the rotating body, ρ is the shortest distance between the point M and the axis of rotation O, σ the curvilinear coordinate of M measured from any arbitrarily selected origin M_0 on its path, and φ the angle between OM_0 and OM, measured in radians (Fig. 2.5, a). Then for natural method of the point motion description we have

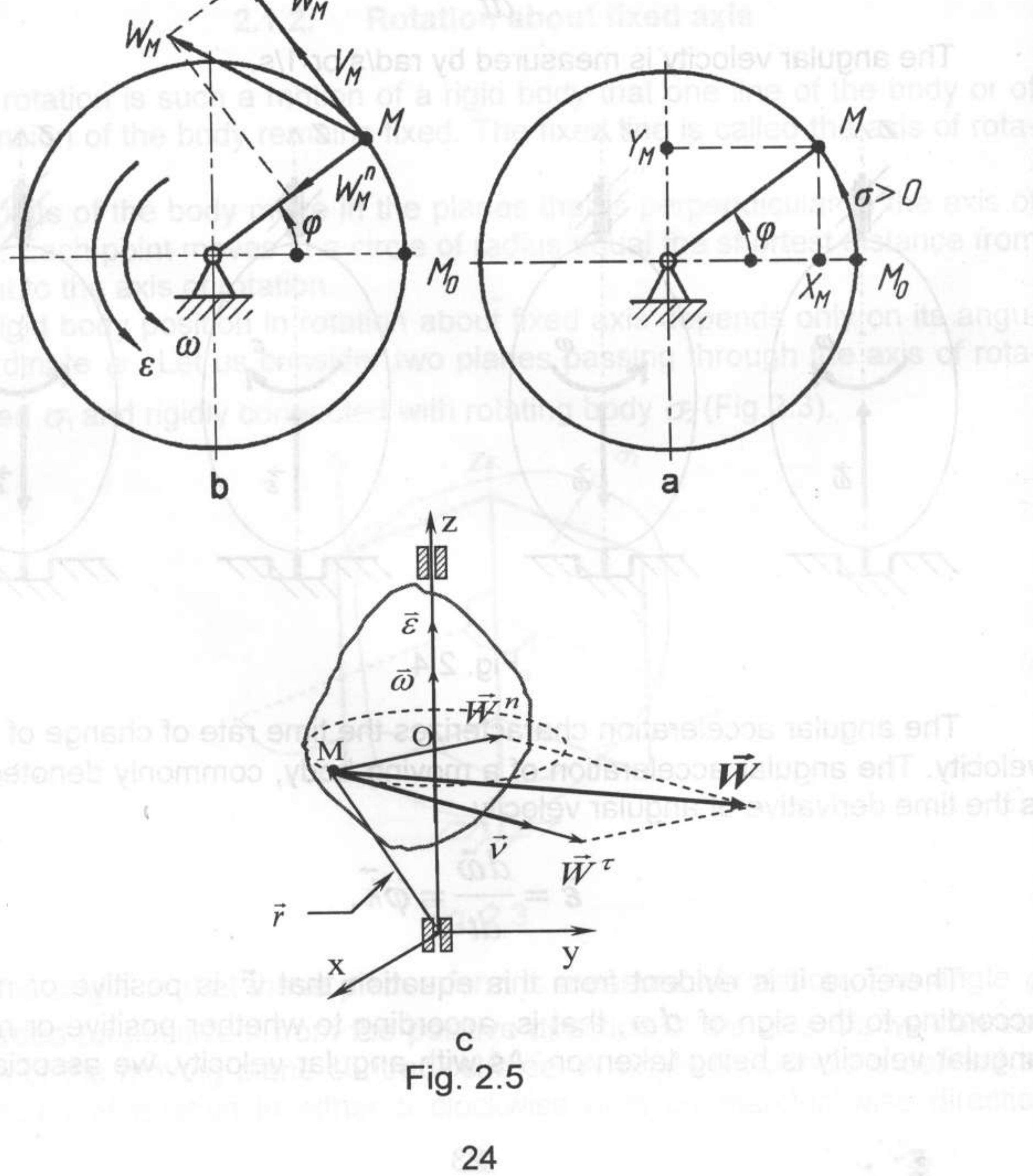


Fig. 2.5

$$\vec{v} = \frac{d\sigma}{dt}\vec{\tau} = \frac{d(\rho\varphi)}{dt}\vec{\tau} = \rho\frac{d\varphi}{dt}\vec{\tau} = \rho\omega\vec{\tau}, \tag{2.3}$$

$$\vec{W}_n = \frac{d^2\sigma}{dt^2}\vec{\tau} = \frac{d^2(\rho\varphi)}{dt^2}\vec{\tau} = \rho\frac{d^2\varphi}{dt^2}\vec{\tau} = \varepsilon\rho\vec{\tau}, \tag{2.4}$$

$$\vec{W}_n = \frac{v^2}{\rho}\vec{n} = \omega^2\rho\vec{n}. \tag{2.5}$$

For vector method (Fig. 2.5, b, c) point M coordinates are

$$x_M = \rho\cos\varphi,\ y_M = \rho\sin\varphi,\ z_M = const.$$

Point M position vector is

$$\vec{r}\left(x_M, y_M, z_M\right) = x_M\vec{i} + y_M\vec{j} + z_M\vec{k} = \rho\cos\varphi\vec{i} + \rho\sin\varphi\vec{j} + z_M\vec{k}.$$

Point M velocity is

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\left(\rho\cos\varphi\vec{i} + \rho\sin\varphi\vec{j} + z_M\vec{k}\right)}{dt} = -\rho\sin\varphi\cdot\omega_z\vec{i} + \rho\cos\varphi\cdot\omega_z\vec{j} =$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \omega_z \\ x_M & y_M & z_M \end{vmatrix} = \vec{\omega}\times\vec{r}. \tag{2.6}$$

Point M acceleration is

$$\vec{W} = \frac{d\vec{v}}{dt} = \frac{d\left(\vec{\omega}\times\vec{r}\right)}{dt} = \frac{d\left(\vec{\omega}\right)}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\left(\vec{r}\right)}{dt} = \vec{\varepsilon}\times\vec{r} + \vec{\omega}\times\vec{v}. \tag{2.7}$$

Consider the first term of the equation. The first term magnitude is

$$\left|\vec{\varepsilon}\times\vec{r}\right| = \varepsilon\cdot r\cdot Sin(\vec{\omega},\vec{r}),\ r\cdot Sin(\vec{\omega},\vec{r}) = \rho,$$

$$\left|\vec{\varepsilon}\times\vec{r}\right| = \varepsilon\cdot\rho. \tag{2.8}$$

Vector $\vec{\varepsilon}\times\vec{r}$ is at the same time directed perpendicular to the vectors $\vec{\varepsilon}$ and $\vec{r}$, in according with the right hand rule vector $\vec{\varepsilon}\times\vec{r}$ is collinear with unit vector of tangent for $\varepsilon_z > 0$, or is opposite to the unit vector of tangent for $\varepsilon_z < 0$, so we get that $\vec{\varepsilon}\times\vec{r}$ is tangent acceleration of the point in rotating body:

$$\vec{\varepsilon}\times\vec{r} = \vec{W}_\tau.$$

The second term magnitude is

$$|\vec{\omega}\times\vec{v}| = |\vec{\omega}\times(\vec{\omega}\times\vec{r})| = \omega\cdot v\cdot Sin(\vec{\omega},\vec{v}),\ Sin(\vec{\omega},\vec{v}) = Sin(\pi/2) = 1,$$

$$|\vec{\omega}\times(\vec{\omega}\times\vec{r})| = \omega^2\rho. \qquad (2.9)$$

Vector $\vec{\omega}\times\vec{v}$ is directed perpendicular to the vectors $\vec{\omega}$ and $\vec{v}$ at the same time and in according with the right hand rule points to the axis of rotation, so we get that $\vec{\omega}\times\vec{v}$ is normal acceleration of the point in rotating body:

$$\vec{\omega}\times(\vec{\omega}\times\vec{r}) = \vec{W}^n. \qquad (2.10)$$

So for total acceleration we have

$$\vec{W} = \vec{W}^\tau + \vec{W}^n, \qquad (2.11)$$

$$W = \rho\sqrt{\omega^4 + \varepsilon^2}. \qquad (2.12)$$

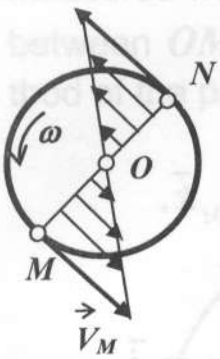


Fig. 2.6

These formulas give the magnitudes and direction of the velocity, the tangential acceleration, and the normal acceleration (centripetal acceleration) of M, and they show that for different particles of a rotating body these magnitudes are directly proportional to the distances of the particles from the axis of rotation (Fig. 2.6).

2.1.3. Conversation about the simplest motions of a rigid body

There are several variants of conversations of the simplest motions of rigid body. They are

- conversation of translational motion into rotational one or inverse;
- conversation of rotation about some fixed axis into rotation about another fixed axis;
- conversation of some body translation into translation of another body.

In the Fig. 2.7 the bodies 1 and 5 are in translation, the bodies 2, 3 и 4 rotate.

Transmission of a rotating motion is realized by gear trains, friction gearing (Fig. 2.8 a, b), or by belt transmission (Fig. 2.9, a, b).

For internal-gear train (see Fig. 2.8, a) and for non-crossing belt transmission (see Fig. 2.9, a) driving and driven wheels have the same direction of rotation.

For external rearing (see Fig. 2.8, b) and for crossing belt transmission (see Fig. 2.9, b) driving and driven wheels have opposite direction of rotation.

For contact without slipping the velocities of the contacting points $\vec{V}_K$ (see Fig. 2.8) are the same. If belt does not slip on the drum the velocities of the points A and B on the rims of the drums (see Fig. 2.9) are the same.

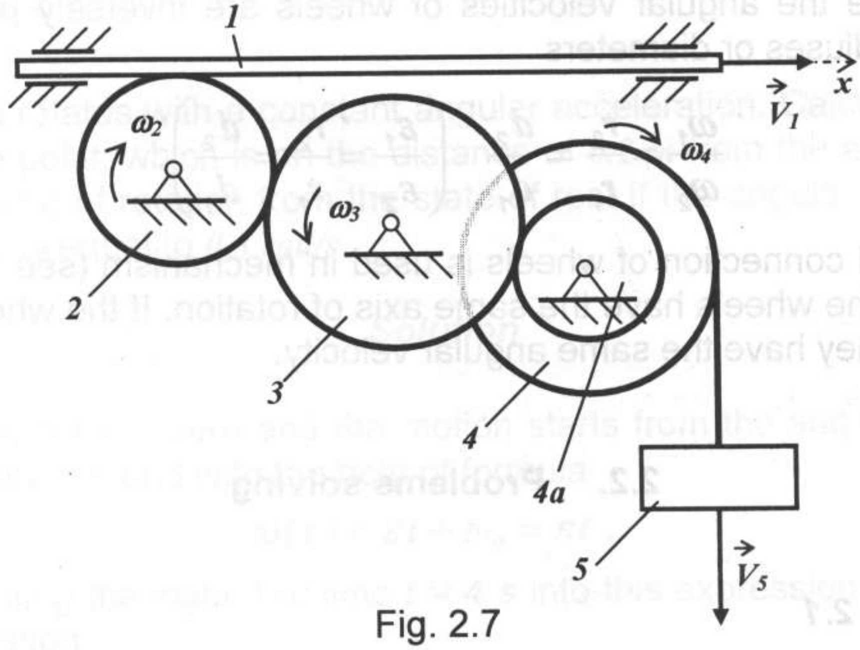


Fig. 2.7

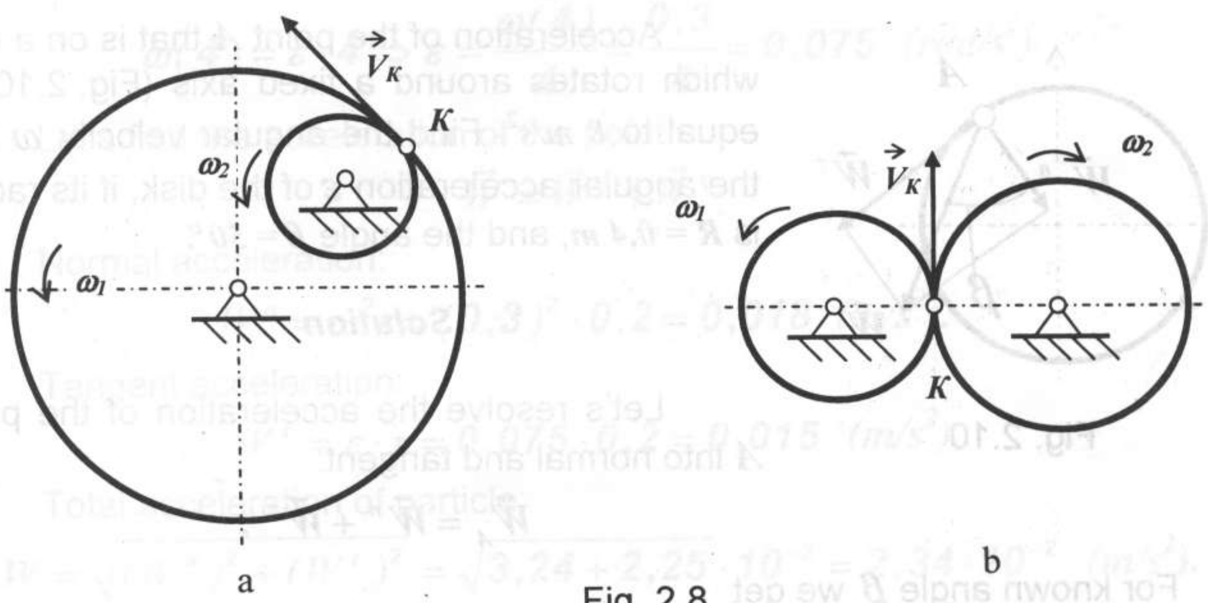


Fig. 2.8

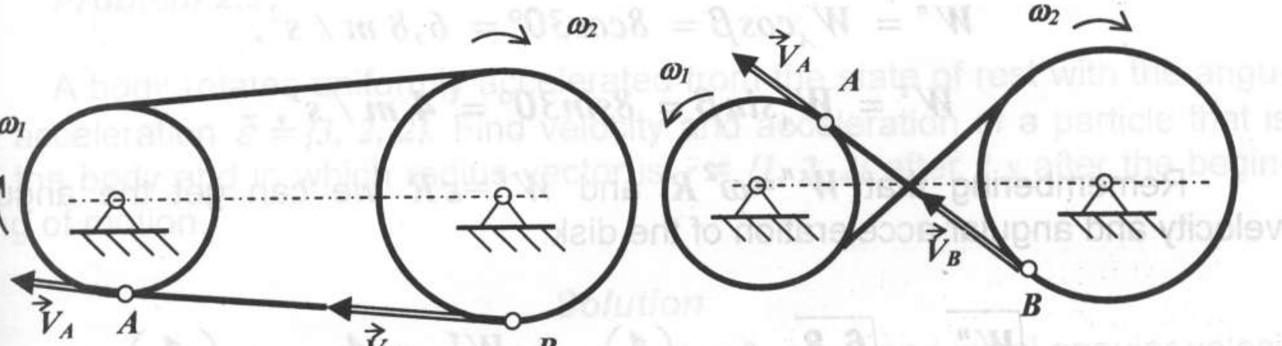


Fig. 2.9

Therefore the angular velocities of wheels are inversely proportional to the wheels' radiuses or diameters

$$\frac{\omega_1}{\omega_2}=\frac{r_2}{r_1}=\frac{d_2}{d_1} \quad \left(\frac{\varepsilon_1}{\varepsilon_2}=\frac{r_2}{r_1}=\frac{d_2}{d_1}\right). \qquad (2.13)$$

If parallel connection of wheels is used in mechanism (see Fig. 2.7, bodies 4 and 4a) the wheels have the same axis of rotation. If the wheels are rigidly connected they have the same angular velocity.

2.2. Problems solving

Problem 2.1

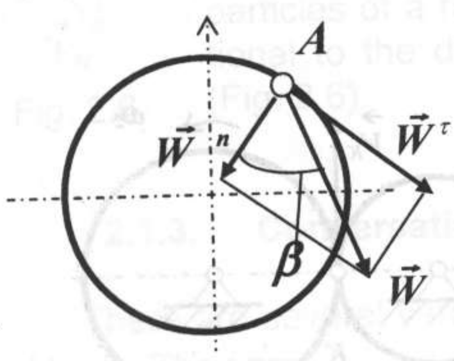


Fig. 2.10

Acceleration of the point A that is on a disk which rotates around a fixed axis (Fig. 2.10) is equal to $8\ m/s^2$. Find the angular velocity ω and the angular acceleration ε of the disk, if its radius is $R = 0{,}4\ m$, and the angle $\beta = 30°$.

Solution

Let's resolve the acceleration of the point A into normal and tangent:

$$\vec{W}_A = \vec{W}^n + \vec{W}^\tau .$$

For known angle β we get

$$W^n = W_A cos\beta = 8cos30° = 6{,}8\ m/s^2,$$

$$W^\tau = W_A sin\beta = 8sin30° = 4\ m/s^2.$$

Remembering that $W^n=\omega^2 R$ and $W^\tau=\varepsilon R$ we can get the angular velocity and angular acceleration of the disk:

$$\omega=\sqrt{\frac{W^n}{R}}=\sqrt{\frac{6{,}8}{0{,}4}}=3{,}8\left(\frac{1}{s}\right),\ \varepsilon=\frac{W^\tau}{R}=\frac{4}{0{,}4}=10\left(\frac{1}{s^2}\right).$$

Problem 2.2

A wheel rotates with a constant angular acceleration. Calculate the acceleration of the point, which is on the distance of *0.2 m* from the axis, at *t=4 s* after the beginning of rotation from the state of rest if the angular velocity at this instant of time is equal to *0,3 rad/s*.

Solution

As we know $\varepsilon = const$ and the motion starts from the state of rest so the angular velocity is found with the help of formula

$$\omega(t) = \varepsilon t + \omega_0 = \varepsilon t .$$

Substituting the instant of time $t = 4$ s into this expression we get the angular acceleration

$$\omega(4) = \varepsilon \cdot 4 \Rightarrow \varepsilon = \frac{\omega(4)}{4} = \frac{0,3}{4} = 0,075 \ \ (rad/s^2).$$

Let's find the acceleration of the point:

$$\vec{W} = \vec{W}^n + \vec{W}^\tau.$$

Normal acceleration:

$$W^n = \omega^2 r = (0,3)^2 \cdot 0,2 = 0,018 \ \ (m/s^2).$$

Tangent acceleration:

$$W^\tau = \varepsilon \cdot r = 0,075 \cdot 0,2 = 0,015 \ \ (m/s^2).$$

Total acceleration of particle:

$$W = \sqrt{(W^n)^2 + (W^\tau)^2} = \sqrt{3,24 + 2,25} \cdot 10^{-2} = 2,34 \cdot 10^{-2} \ \ (m/s^2).$$

Problem 2.3

A body rotates uniformly accelerated from the state of rest with the angular acceleration $\vec{\varepsilon} = \{3, 2, 2\}$. Find velocity and acceleration of a particle that is on the body and in which radius-vector is $\vec{r} = \{1, 2, 2\}$ after *2 s* after the beginning of motion.

Solution

It is known from the task that $\varepsilon_x = 3$, $\varepsilon_y = 2$, $\varepsilon_z = 2$ and initial angular velocity is $\omega_0 = 0$. Let's find the projections of angular velocity vector to the coordinate axes:

$$\omega_x = \int \varepsilon_x dt = \int 3dt = 3t + \omega_{x0} = 3t;$$

$$\omega_y = \int \varepsilon_y dt = \int 2dt = 2t + \omega_{y0} = 2t;$$

$$\omega_z = \int \varepsilon_z dt = \int 2dt = 2t + \omega_{z0} = 2t.$$

When $t = 2\ s - \omega_x = 3 \cdot 2 = 6,\ \omega_y = 2 \cdot 2 = 4,\ \omega_z = 2 \cdot 2 = 4.$

Let's find the projections of particle velocity to the axes of Cartesian coordinate system with the help of formulas

$$V_x = \omega_y z - \omega_z y = 4 \cdot 2 - 4 \cdot 2 = 0,$$

$$V_y = \omega_z x - \omega_x z = 4 \cdot 1 - 6 \cdot 2 = -8,$$

$$V_z = \omega_x y - \omega_y x = 6 \cdot 2 - 4 \cdot 1 = 8.$$

Then the value of particle velocity is

$$V = \sqrt{V_x^2 + V_y^2 + V_z^2} = \sqrt{0^2 + (-8)^2 + 8^2} = 11{,}31\ (m/s).$$

Acceleration of particle of body, which rotates around a fixed axis, can be found as the vector sum of normal and tangent accelerations. The first one of them

$$\vec{W}^n = \vec{\omega} \times (\vec{\omega} \times \vec{r}) = \vec{\omega} \times \vec{v} = \vec{i}(\omega_y v_z - \omega_z v_y) - \vec{j}(\omega_z v_x - \omega_x v_z) + \vec{k}(\omega_x v_y -$$

$$-\omega_y v_x) = \vec{i}(4 \cdot 8 + 4 \cdot 8) - \vec{j}(6 \cdot 8 - 0) + \vec{k}(-6 \cdot 8 - 0) =$$

$$= 64\vec{i} - 48\vec{j} - 48\vec{k}.$$

The magnitude of normal acceleration

$$W^n = \sqrt{64^2 + 48^2 + (-48)^2} = \sqrt{8704} = 93{,}3\ (m/s^2).$$

By analogy let's find the tangent acceleration:

$$\vec{W}^\tau = \vec{\varepsilon} \times \vec{r} = \vec{i}(\varepsilon_y r_z - \varepsilon_z r_y) - \vec{j}(\varepsilon_z r_x - \varepsilon_x r_z) + \vec{k}(\varepsilon_x r_y - \varepsilon_y r_x) =$$

$$= \vec{i}(2 \cdot 2 - 2 \cdot 2) - \vec{j}(3 \cdot 2 - 2 \cdot 1) + \vec{k}(3 \cdot 2 - 2 \cdot 1) = -4\vec{j} + 4\vec{k}.$$

The magnitude of tangent acceleration

$$W^\tau = \sqrt{16 + 16} = \sqrt{32} = 5{,}66\ (m/s^2).$$

The total acceleration of the particle

$$W = \sqrt{(W^n)^2 + (W^\tau)^2} = \sqrt{8704 + 32} = 93{,}47\ (m/s^2).$$

Problem 2.4

A rotor of electric engine starts its rotation from the state of rest and during *5 s* and run *100* rotations. Find the angular acceleration of the rotor with the condition that the motion is uniformly accelerated.

Solution

The motion is uniformly accelerated so we can use the formula

$$\varphi = \varphi_0 + \omega_{0z} t + \frac{\varepsilon_z t^2}{2}.$$

In the current case the motion started form the state of rest so ω_0 and φ_0 are equal to zero so

$$\varepsilon_z = \frac{2\varphi}{t^2}.$$

The shaft rotates on an angle of $2\pi\, rad$ per one rotation therefore for the time of $t = 5\ s$ the shaft will rotate on an angle of $200\pi\, rad$. Thereby,

$$\varepsilon_z = \frac{2 \cdot 200\pi}{5^2} = 50{,}3\,(rad / s^2).$$

Problem 2.5

The weight *1* moves vertically according to the law

$$y = \sqrt{2}\, sin\left(\frac{\pi}{4} t\right)(m)$$

and actuates the pulleys 2 and 3 of radii R_2 = 0,5 m, r_2 = 0,2 m, R_3 = 0,4 m (Fig. 2.11).

Find the acceleration of the point *M* when $t = 1\ s$.

Solution

Translation motion of the body *1* is transformed into rotational motion of the two-stage pulley *2*. The pulleys *2* and *3* are in an external engagement so they rotate with different directions.

Let's apply the next designations: ω_2 and ε_2 – angular velocity and angular acceleration of the pulley *2*; ω_3 and ε_3 – angular velocity and angular acceleration of the pulley *3*.

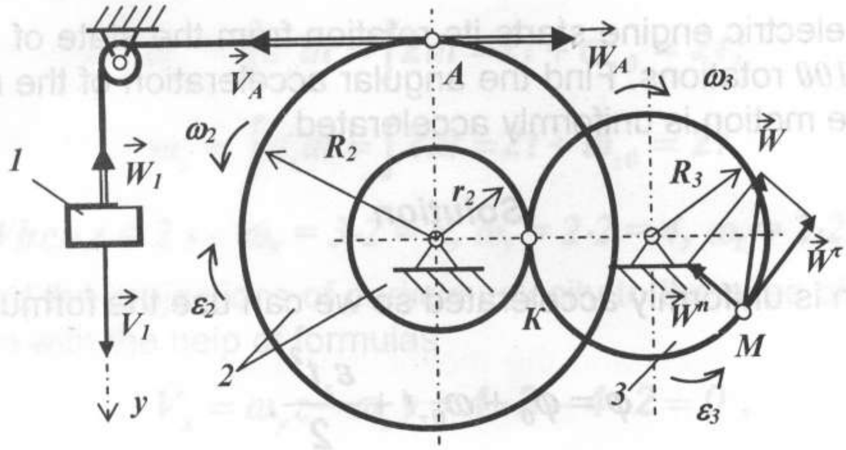


Fig. 2.11

Let's find the velocity and acceleration of the weight *1* from the law of its motion:

$$V_{1y} = \dot{y} = \left(\sqrt{2}\sin\left(\frac{\pi}{4}t\right)\right)' = \sqrt{2}\frac{\pi}{4}\cos\left(\frac{\pi}{4}t\right);$$

$$W_{1y} = \ddot{y} = \dot{V}_{1y} = \left(\sqrt{2}\frac{\pi}{4}\cos\left(\frac{\pi}{4}t\right)\right)' = -\sqrt{2}\frac{\pi^2}{16}\sin\left(\frac{\pi}{4}t\right).$$

When $t = 1\ s$ we get $V_{1y} = \frac{\pi}{4}(m/s)$, $W_{1y} = -\frac{\pi^2}{16}(m/s^2)$.

The point A is on the rope and the pulley simultaneously thus

$$V_A = V_1 = \frac{\pi}{4}\ (m/s)\ \ and\ \ W_A^\tau = W_1 = \frac{\pi^2}{16}\left(m/s^2\right).$$

Let's find angular velocity ω_2 and angular acceleration ε_2 of the pulley *2*:

$$V_A = \omega_2 R_2,\ therefore,\ \omega_2 = \frac{V_A}{R_2} = \frac{\pi}{4\cdot 0{,}5} = \frac{\pi}{2}\left(\frac{1}{s}\right);$$

$$W_A^\tau = \varepsilon_2 R\ ,\ therefore,\ \varepsilon_2 = \frac{W_A^\tau}{R_2} = \frac{\pi^2}{16\cdot 0{,}5} = \frac{\pi^2}{8}\left(\frac{1}{s^2}\right).$$

Taking into account the directions of the vectors $\vec{V}_A$ и $\vec{W}_A^{rot}$ we have $\omega_{2z} > 0$; and $\varepsilon_{2z} < 0$ (see Fig. 2.11).

The point ***K*** is the common one for the pulleys **2** and **3**, so

$$V_K = \omega_2 r_2 = \omega_3 R_3,$$

$$W_K^{\tau} = \varepsilon_2 r_2 = \varepsilon_3 R_3.$$

After this we get

$$\omega_3 = \omega_2 \frac{r_2}{R_3} = \frac{\pi}{2} \cdot \frac{0,2}{0,4} = \frac{\pi}{4}\left(\frac{1}{s}\right) (\omega_{3z} < 0),$$

$$\varepsilon_3 = \varepsilon_2 \frac{r_2}{R_3} = \frac{\pi^2}{8} \cdot \frac{0,2}{0,4} = \frac{\pi^2}{16}\left(\frac{1}{s^2}\right) \quad \left(\varepsilon_{3z} > 0\right).$$

Let's find velocity and acceleration of the point M:

$$V_M = \omega_3 R_3 = \frac{\pi}{4} \cdot 0,4 = 0,1\pi \ (m/s);$$

$$\vec{W}_M = \vec{W}^n + \vec{W}^{\tau};$$

$$W^n = \omega_3^2 R_3 = \left(\frac{\pi}{4}\right)^2 \cdot 0,4 = \frac{\pi^2}{40}\left(m/s^2\right);$$

$$W^{\tau} = \varepsilon_3 R_3 = \frac{\pi^2}{16} \cdot 0,4 = \frac{\pi^2}{40}\left(m/s^2\right);$$

$$W_M = \sqrt{\left(W^n\right)^2 + \left(W^{\tau}\right)^2} = \sqrt{\left(\frac{\pi^2}{40}\right)^2 + \left(\frac{\pi^2}{40}\right)^2} = \frac{\pi^2}{40}\sqrt{2}\left(m/s^2\right).$$

The directions of the vectors $\vec{V}_M, \vec{W}^n, \vec{W}^{\tau}, \vec{W}$ are shown on the Fig. 2.11.

Problem 2.6

A mechanism, which consists of the drum **6** and a gear, is moved by the wheel ***1*** (Fig. 2.12). Find the magnitude and direction of velocity $\vec{V}_p$ of the weight ***P***, if $\omega_1 = 4,5$ ***(1/s)***, and radiuses of wheels and drum are $R_1 = 0,2$ ***м***, $R_2 = 0,4$ ***m***, $R_3 = 0,5$ ***m***, $R_4 = 0,25$ ***m***, $R_5 = 0,6$ ***m***. The wheels ***4*** and ***3,*** and the wheel *5* and the drum *6* are rigidly joined ($\omega_3 = \omega_4$ и $\omega_5 = \omega_6$).

Solution

To find the velocity of the weight ***P*** let's use the known relations:

$$\frac{\omega_1}{\omega_2} = \frac{R_2}{R_1}, \text{ from which } \omega_2 = \omega_1 \frac{R_1}{R_2}; \qquad (2.14)$$

$$\frac{\omega_2}{\omega_3} = \frac{R_3}{R_2}, \text{ from which } \omega_3 = \omega_4 = \omega_2 \frac{R_2}{R_3}; \qquad (2.15)$$

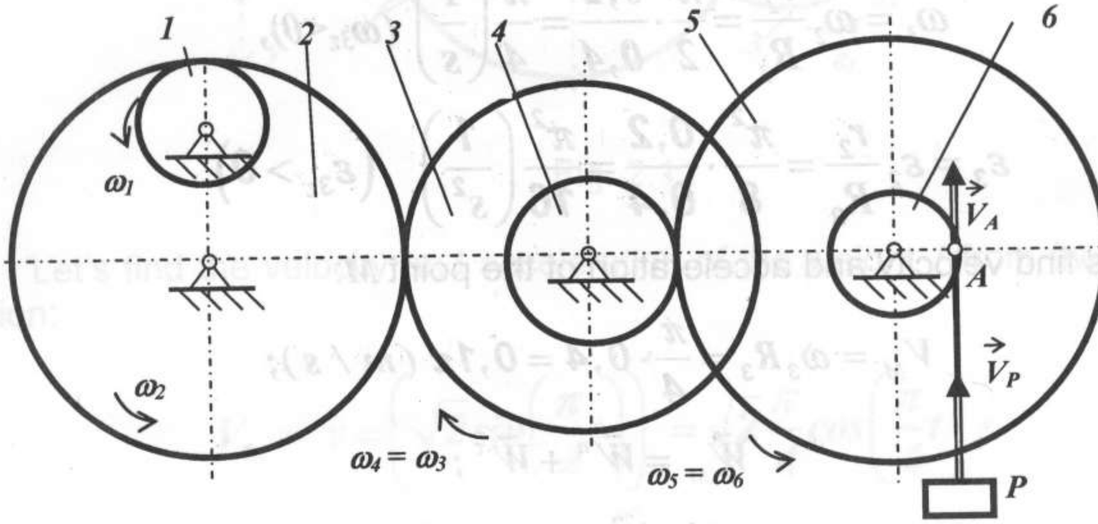


Fig. 2.12

$$\frac{\omega_4}{\omega_5} = \frac{R_5}{R_4}, \text{ from which } \omega_5 = \omega_6 = \omega_4 \frac{R_4}{R_5}. \qquad (2.16)$$

After this substitute one-by-one the expression (2.14) into (2.15) and into the dependence (2.16), we get the angular velocity of the drum **6**

$$\omega_6 = \omega_5 = \omega_1 \frac{R_1 R_4}{R_3 R_5}.$$

The velocity of weight ***P*** is found by the formula

$$V_P = V_A = \omega_6 R_6 = \omega_1 \frac{R_1 R_4 R_6}{R_3 R_5} = 4{,}5 \frac{0{,}2 \cdot 0{,}25 \cdot 0{,}1}{0{,}5 \cdot 0{,}6} = 0{,}076\,(m/s).$$

The directions of wheels rotation are shown on the Fig. 2.12.
Thereby the weight ***P*** will be moving up with the velocity of ***0,076 m/s***.

Problem 2.7

Find the velocity ***V*** of jack cup ***Q*** lifting. The scheme of jack is shown on he Fig. 2.13, if the handle ***P*** performs $n = 10$ rotations per minute.
It is known that $R_1 = 2\ m$, $r_2 = 3\ m$, $R_2 = 4\ m$, $r_3 = 1\ m$, $R_3 = 5\ m$.

Solution

First let's analyze the motion of all mechanism links. The wheel *1*, and the ulleys *2* and *3* rotate around fixed axes O_1, O_2 and O_3 respectively. The wheel $_3$ is engaged with the toothed bar *4,* which performs linear motion together with he cup ***Q*** that is attached to it. Thereby, to find the velocity of cup it is enough o find the velocity of toothed bar.

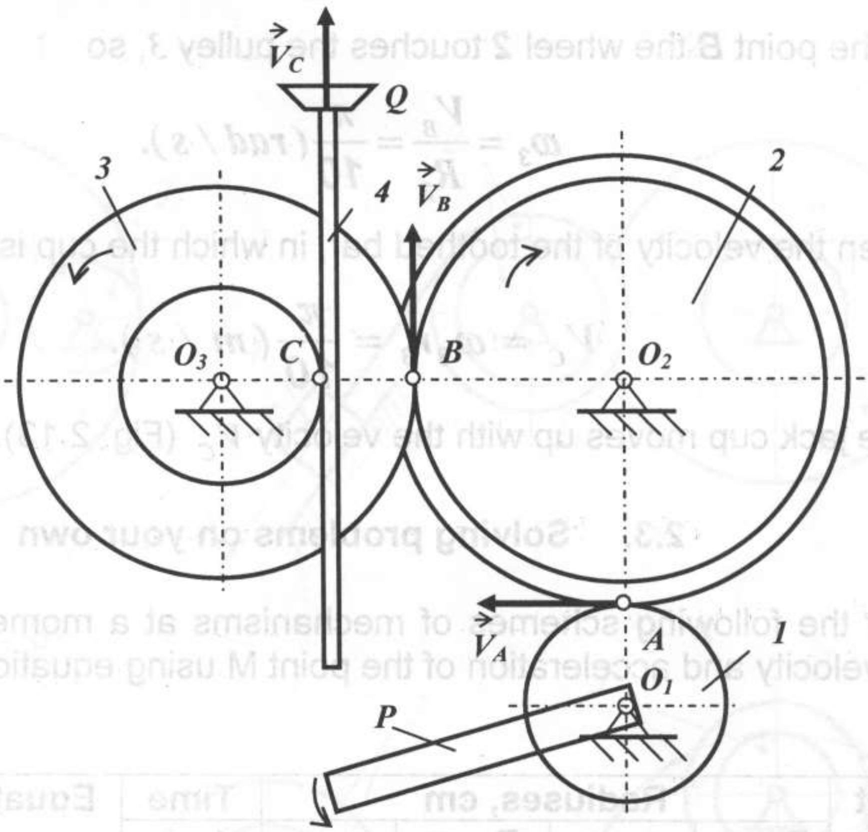


Fig. 2.13

Knowing how many rotations per minute are performed by the body *1*, let's calculate its angular velocity

$$\omega_1 = \frac{\pi n}{30} = \frac{\pi}{3}\ \ (rad/s).$$

The velocity of point on the edge of the first wheel

$$V_A = \omega_1 R_1 = \frac{\pi}{3} \cdot 2(m/s).$$

If there is no slipping the same velocity will be on the edge of the wheel 2 edge. Then its angular velocity

$$\omega_2 = \frac{v_1}{R_2} = \frac{\pi}{6}(rad/s).$$

Let's find the velocity of the point ***B*** of the wheel **2**. The point is placed on the distance r_2 from the axis of rotation:

$$V_B = \omega_2 r_2 = \frac{\pi}{2}(m/s).$$

In the point ***B*** the wheel **2** touches the pulley *3*, so

$$\omega_3 = \frac{V_B}{R_3} = \frac{\pi}{10}(rad/s).$$

Then the velocity of the toothed bar, in which the cup is attached, is

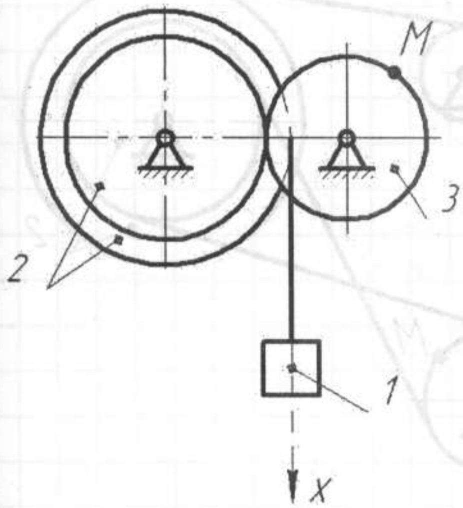
$$V_C = \omega_3 r_3 = \frac{\pi}{10}(m/s).$$

The jack cup moves up with the velocity V_C (Fig. 2.13).

2.3. Solving problems on your own

For the following schemes of mechanisms at a moment of time $t=t_1$ determine velocity and acceleration of the point M using equation of motion for the 1st body.

Variant number	Radiuses, cm				Time	Equations of motion
	R_2, m	r_2, m	R_3, m	r_3, m	t_1, s	x(t), m
1	60	45	36	-	1	$-5t^2-3t-2$
2	32	16	32	16	2	$7t^2+4t-1$
3	35	10	10	-	1	$4t^2-3t+8$
4	25	20	50	25	3	$-10t^2-8t+6$
5	20	15	15	-	3	$5t^2+6t-2$
6	80	-	45	30	1	$2t^2-4t-2$
7	30	15	20	-	3/2	$3t^2+8t-12$
8	25	15	10	-	1/2	$7t^2-3t-8$
9	40	20	35	-	2	$-7t^2+4t-1$
10	15	10	20	-	1	$9t^2-2t-8$



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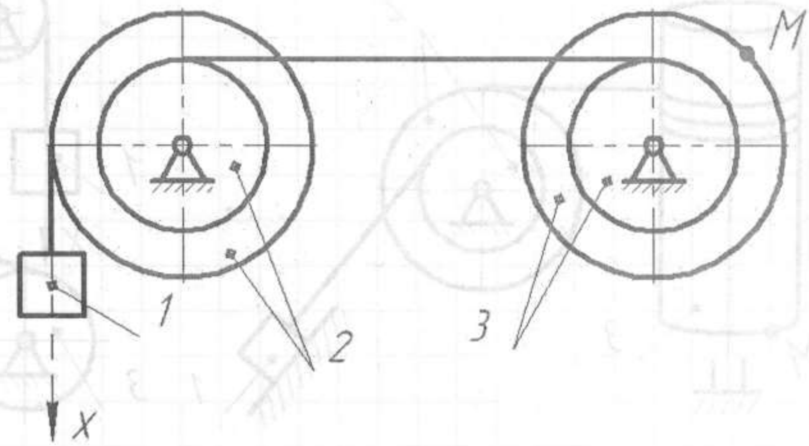

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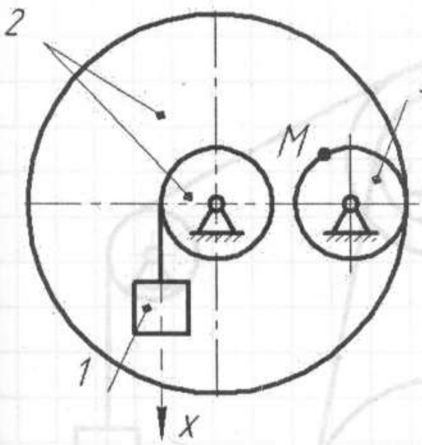

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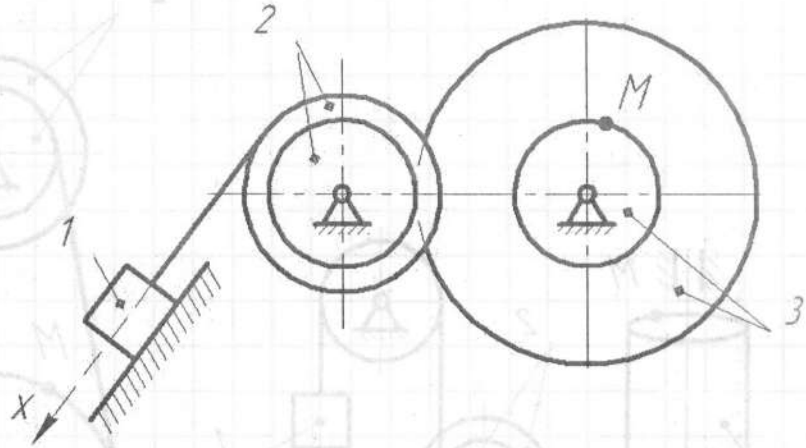

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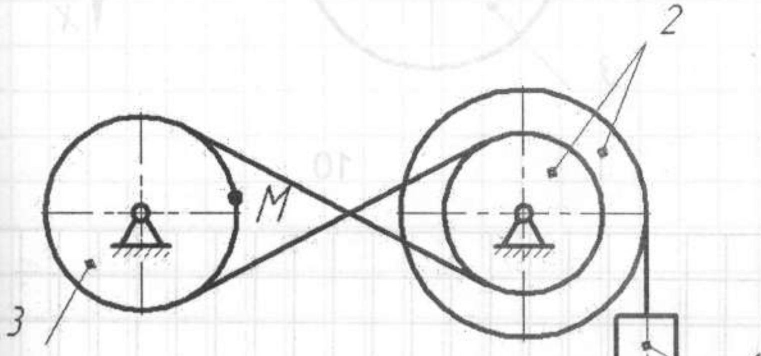

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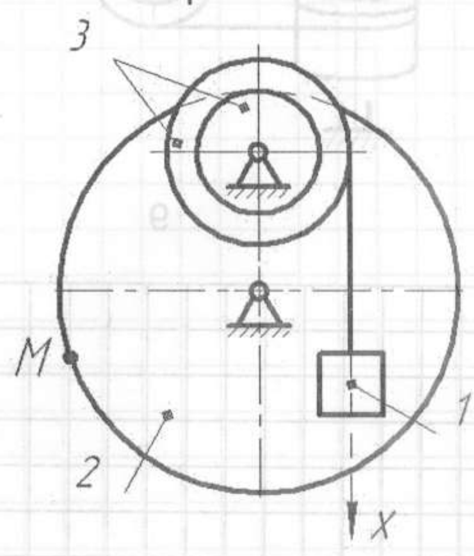

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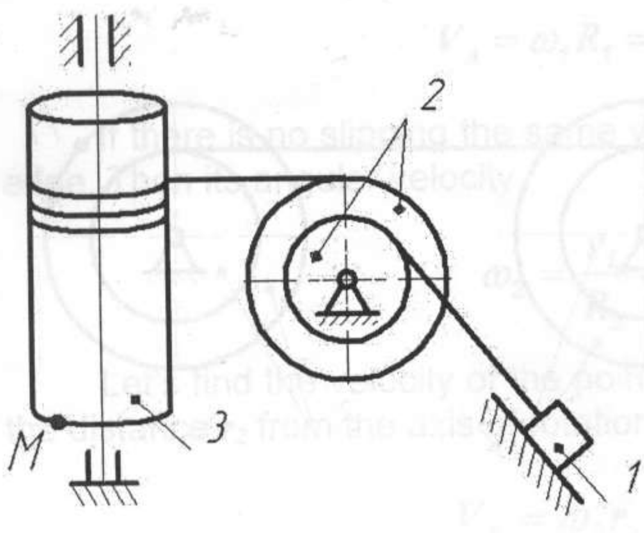

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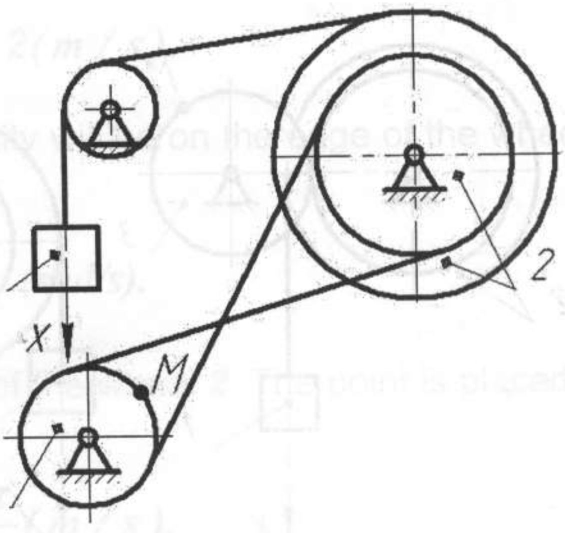

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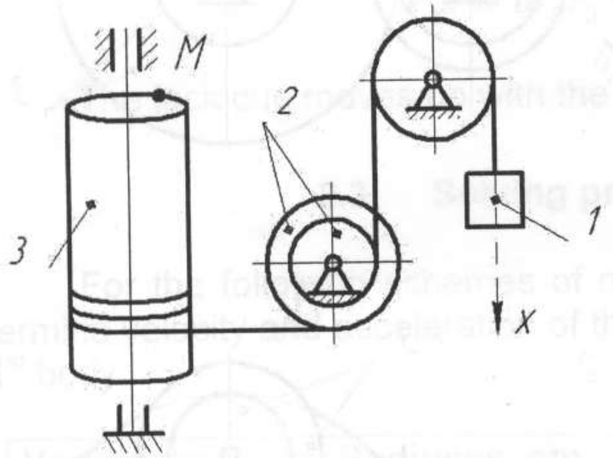

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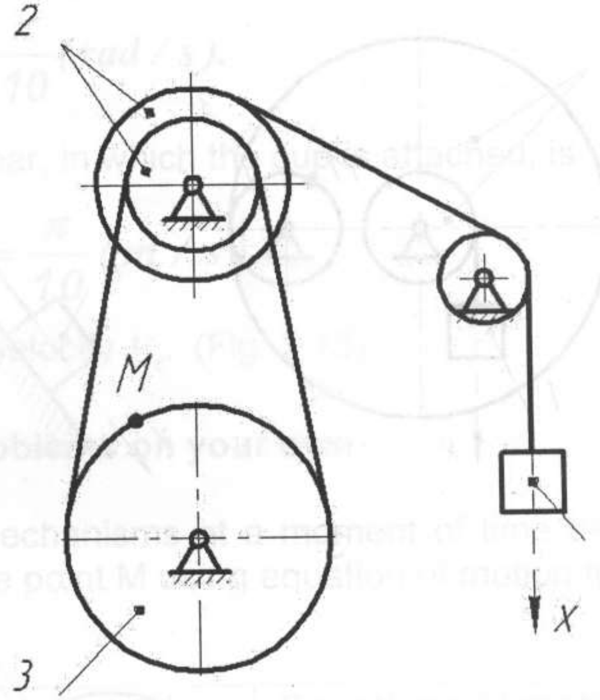

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2.4. Self-control questions

1. How many independent parameters are necessary to be defined in order to find a position of rigid body: a) a free one; b) with a fixed point; c) with two fixed points?
2. Define translation motion of rigid body. What are the properties of this motion?
3. Define rotation motion of body around a fixed axis. How to determine the law of motion?
4. How to find the velocity of arbitrary point of body, which rotates around an axis?
5. What are the magnitude and direction of particle velocity of body, which rotates around an axis?
6. How to determine the magnitude and direction of normal acceleration vector of particle, when body rotates around an axis?
7. How to find the magnitude and direction of tangent acceleration vector of particle when body rotates around an axis?
8. How are the vectors of angular velocity $\bar{\omega}$ and angular acceleration $\bar{\varepsilon}$ directed when a body rotates around an axis?
9. How can the character of body rotation around an axis be found if the law of its motion is given $\varphi = \varphi(t)$?

3. PLANE MOTION OF A RIGID BODY

3.1. Main information from the theoretical course

3.1.1. Definition and equations of plane motion

Plane motion is motion in which each point of the moving body remains at a constant distance from a fixed plane. Each point of the body moves in a plane that is called the **plane of the motion**. The axis of rotation of a body remains perpendicular to the plane of the motion during the time of motion. We know that the distances between points in rigid body are constant so to prescribe the motion of any point in the body it is enough to know the motion of the **plane figure** of the body. This plane figure is obtained by dissecting the body by the plane of motion of any point. In our following consideration we will speak about motion of the plane figure instead of the rigid body motion.

In the previous chapters two simple motions of a body (translation and rotation) were considered. We shall now demonstrate that at each instant, the plane motion of any rigid body can be thought of as the superposition of both a translational motion and a rotational motion.

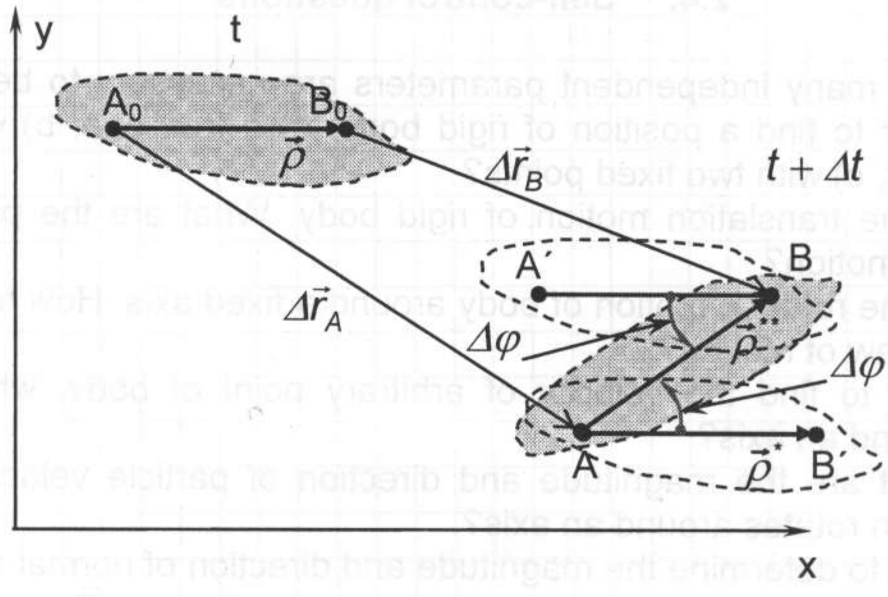


Fig. 3.1. Plane motion of a rigid body

Positions of the body are shown tinted at times t and $(t+\Delta t)$ in Fig. 3.1. Let us select any point A of the body. Imagine that the body is displaced without rotation from its position at time t to the position at time $(t+\Delta t)$ so that point A reaches its correct final position. The displacement vector for this translation is shown at $\Delta\vec{r}_A$. To reach the correct orientation for $(t+\Delta t)$), we must now rotate the body an angle $\Delta\varphi$ about an axis of rotation which is normal to the plane and which passes through point *A*.

What changes would occur had we chosen some other point B for such a procedure? Consider Fig. 3.1 where we have included an alternative procedure by translating the body so that point B reaches the correct final position. Next, we must rotate the body an amount $\Delta\varphi$ about an axis of rotation which is normal to the plane and which passes through B in order to get to the final orientation of the body. Thus, we have indicated two routes. We conclude from the diagram that the displacement $\Delta\vec{r}_B$ differs from $\Delta\vec{r}_A$, but there is no difference in the amount of rotation $\Delta\varphi$. Thus, in general, $\Delta\vec{r}$ and the axis of rotation will depend on the point chosen, while the amount of rotation $\Delta\varphi$ will be the same for all such points.

Conclusion: Any displacement of a body in plane motion can be accomplished by means of a translation of the body followed by a rotation, or vice versa. By means of a translation, one point of the body could be put into its final position, and by a rotation about that point the body could be put into its final position. The point of the body chosen for the translation and for center of rotation is called **the pole or the base point**.

To characterize the translational component of the plane motion it is enough to prescribe the position of the pole as function of time. To characterize the rotational component of the plane motion it is enough to prescribe the angle of rotation about the axis crossing the pole as function of time

$$\begin{cases} x_A = f_1(t), \\ y_A = f_2(t), \\ \varphi = f_3(t). \end{cases}$$

This system of equations is called **the equations of plane motion of a body.**

3.1.2. Point position

Let us consider the point A of the plane figure as the pole (Fig.3.2). Let us choose two coordinate systems:

- the first Oxy is fixed,
- the second Ax_1y_1 is moving, Ax_1y_1 has motion of translation with respect to the fixed coordinate system.

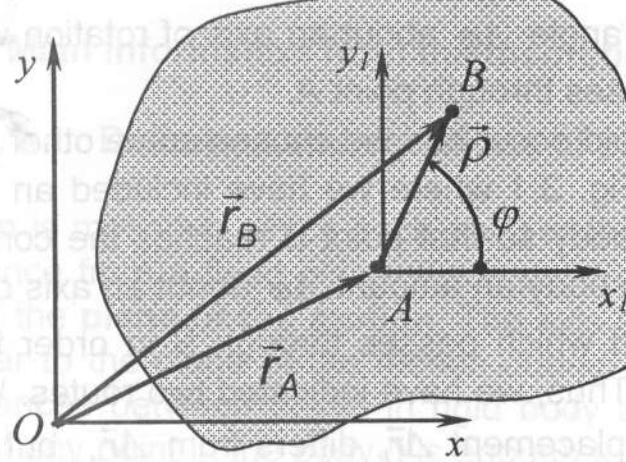


Fig. 3.2. Determination of a point B position in plane motion of a body

Position vector of point B is

$$\vec{r}_B = \vec{r}_A + \vec{\rho}, \qquad (3.1)$$

wherein $\vec{r}_A = x_A\vec{i} + y_A\vec{j}$ is the vector position of the pole A in the fixed frame of reference Oxy, $\vec{\rho}$ is vector position of the point B in the frames of reference Ax_1y_1, the magnitude of $\vec{\rho}$ is constant, $\vec{\rho} = \overrightarrow{AB}$, the orientation changes with time.

3.1.3. Point velocity

Determine velocity of point B

$$\vec{v}_B = \frac{d\vec{r}_B}{dt} = \frac{d\left(\vec{r}_A + \vec{\rho}\right)}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{\rho}}{dt}. \qquad (3.2)$$

The first term $\frac{d\vec{r}_A}{dt}$ is the velocity $\vec{v}_A$ of the pole A.

Let us analyze the second term. From the point of view of observe at the origin O (see Fig. 3.1) in fixed reference Oxy the position and orientation of the vector $\vec{\rho}$ alter. Let us resolve the motion of the vector $\vec{\rho}$ on the translation with the base point A and rotation about A. In the step of translation the position of $\vec{\rho}$ line of action alters but the orientation is fixed, therefore the $\vec{\rho}$ and the $\vec{\rho}^*$ are the equivalent vectors. During the second step the vector $\vec{\rho}^*$ rotates about the point A through an angle $d\varphi$, therefore the time rate of change of $\vec{\rho}$ as seen from fixed reference Oxy characterizes rotation of $\vec{\rho}$ about the base point A:

$$\frac{d\vec{\rho}}{dt} = \vec{\omega} \times \vec{\rho} = \vec{v}_{BA}, \qquad (3.3)$$

where $\vec{\omega} = \frac{d\varphi}{dt}\vec{k}_1$ is vector of angular velocity of the body rotation about the pole A. It points along the axis perpendicular to the plane of figure in direction where rotation is viewed anti clockwise. The magnitude of angular velocity does not depend on the choosing of the pole.

The vector $\vec{V}_{BA}$ is directed perpendicular to the $\vec{\rho}$ in accordance with the direction of body rotation (Fig. 3.3).

As a result we get

$$\vec{v}_B = \frac{d\vec{r}_A}{dt} + \frac{d\vec{\rho}}{dt} = \vec{v}_A + \vec{v}_{BA} = \vec{v}_A + \vec{\omega} \times \vec{\rho}. \qquad (3.4)$$

The velocity of a point is equal to the vector sum of the velocity base point and the velocity of the point relative to the chosen pole.

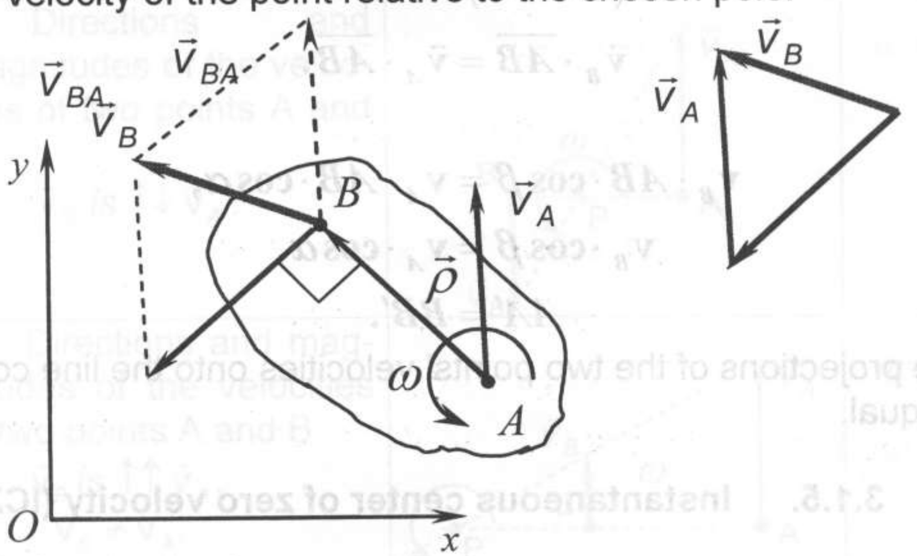


Fig. 3.3. Determination of a point velocity for plane motion of a body

3.1.4. Equiprojectivity

For two points A and B of a rigid body in plane motion (Fig. 3.4) we can write

$$\vec{v}_B = \vec{v}_A + \vec{v}_{BA} = \vec{v}_A + \left(\vec{\omega} \times \overrightarrow{AB}\right),$$

$$\vec{\mathbf{v}}_B \cdot \overrightarrow{AB} = \vec{\mathbf{v}}_A \cdot \overrightarrow{AB} + \left(\vec{\omega} \times \overrightarrow{AB}\right) \cdot \overrightarrow{AB}.$$

But the vector $\vec{V}_{BA} = \vec{\omega} \times \overrightarrow{AB}$ is perpendicular to the vector $\overrightarrow{AB}$, so

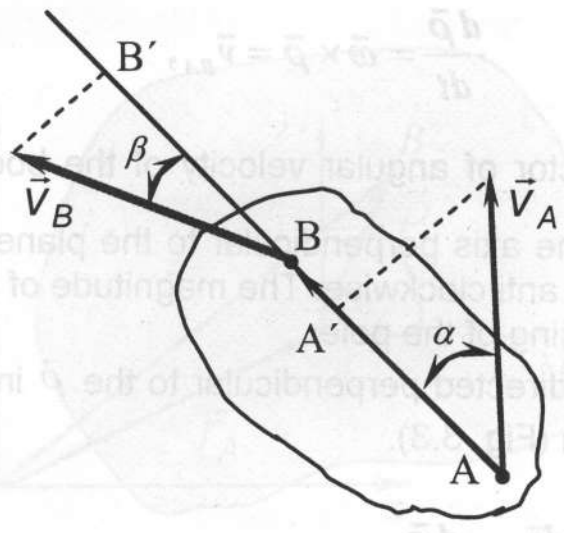


Fig. 3.4. Equiprojectivity

$$\left(\vec{\omega}\times\overrightarrow{AB}\right)\cdot\overrightarrow{AB}=0,$$

$$\vec{\mathbf{v}}_B\cdot\overrightarrow{AB}=\vec{\mathbf{v}}_A\cdot\overrightarrow{AB}. \qquad (3.5)$$

Or

$$\mathbf{v}_B\cdot AB\cdot\cos\beta=\mathbf{v}_A\cdot AB\cdot\cos\alpha, \qquad (3.6)$$

$$\mathbf{v}_B\cdot\cos\beta=\mathbf{v}_A\cdot\cos\alpha,$$

$$AA'=BB'. \qquad (3.7)$$

So the projections of the two points' velocities onto the line connecting the points are equal.

3.1.5. Instantaneous center of zero velocity (ICZV)

If angular velocity of a rigid body in plane motion is non-zero, in the plane figure moving with the body there is unique point that momentarily has zero velocity. This point is called instantaneous center of zero velocity (ICZV) and usually denoted as P. Choosing the ICZV as pole the velocity of any point may be determined as

$$\vec{v}_B=\vec{v}_P+\vec{v}_{BP}=\vec{\omega}\times\overrightarrow{PB}. \qquad (3.6)$$

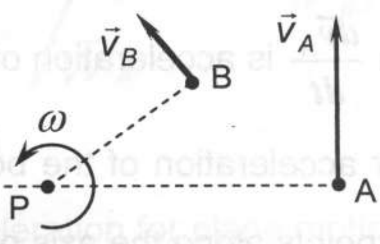
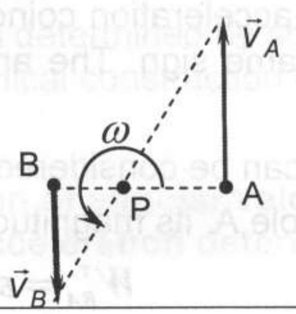
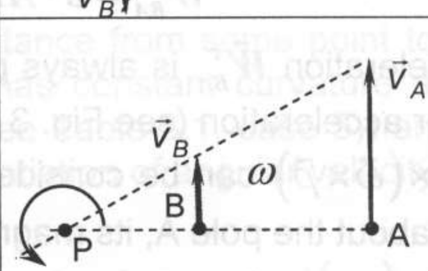
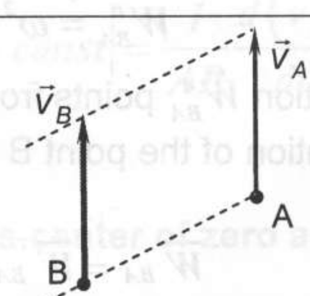
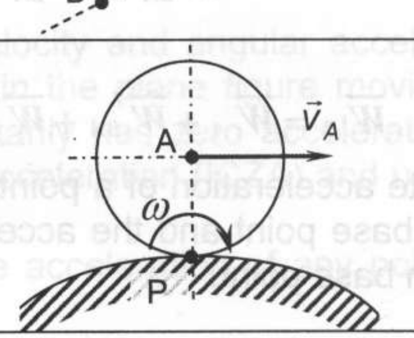
The last equation means that for any point of rigid body in plane motion:

1) the magnitude of the velocity of is directly proportional to the distance between the point and ICZV,
2) the vector of velocity is perpendicular to the segment connecting the point and ICZV and points in direction corresponding to the body's angular velocity.

The foregoing equation and equation for velocity of a point in the rotating ody are the same so we can consider the plane motion of a body as rotation bout axis passing throw the ICZV. The position of ICZV varies with time.

Methods of ICZV position determinations are presented in the Table 3.1.

Table 3.1

Cases	What are given	Position of ICZV	Determination of angular velocity
1	1. Directions of the velocities of two points A and B. 2. Magnitude of point A velocity. 3. $\vec{v}_B$ *is not* $\parallel \vec{v}_A$.		$\omega = \dfrac{V_A}{AP}$
2	1. Directions and magnitudes of the velocities of two points A and B. 2. $\vec{v}_B$ *is* $\uparrow\downarrow \vec{v}_A$.		$\omega = \dfrac{V_A + V_B}{AB}$
3	1. Directions and magnitudes of the velocities of two points A and B. 2. $\vec{v}_B$ *is* $\uparrow\uparrow \vec{v}_A$, $v_B \neq v_A$.		$\omega = \dfrac{V_A - V_B}{AB}$
4	1. Directions and magnitudes of the velocities of two points A and B. 2. $\vec{v}_B$ *is* $\uparrow\uparrow \vec{v}_A$, $v_B = v_A$.		$\omega = 0$ - instantaneous translation
5	1. Direction and magnitude of the velocity of point A. 2. Body rolls without slipping on the fixed surface.		$\omega = \dfrac{V_A}{AP}$

3.1.6. Acceleration of a point

To determine an acceleration of a point in the rigid body that has plane motion let us differentiate the equation (3.4) with respect to time

$$\overrightarrow{W}_B = \frac{d\vec{v}_B}{dt} = \frac{d\left(\vec{v}_A + (\vec{\omega}\times\vec{\rho})\right)}{dt} =$$

$$= \frac{d\vec{v}_A}{dt} + \left(\frac{d\vec{\omega}}{dt}\times\vec{\rho}\right) + \left(\vec{\omega}\times\frac{d\vec{\rho}}{dt}\right).$$

The first term $\frac{d\vec{v}_A}{dt}$ is acceleration of the base point $\overrightarrow{W}_A$. The time derivative $\frac{d\vec{\omega}}{dt}$ is angular acceleration of the body $\vec{\varepsilon}$ rotation about the pole A (Fig. 3.5). The vector $\vec{\varepsilon}$ points along the axis perpendicular to the plane of figure. Direction of the angular acceleration coincides with direction of angular velocity if $\dot{\varphi}$ and $\ddot{\varphi}$ have the same sign. The angular velocity does not depend on the choosing of the pole.

The term $\vec{\varepsilon}\times\vec{\rho}$ can be considered as tangent acceleration $\vec{W}^{\tau}_{BA}$ of point B in rotation about the pole A, its magnitude is

$$W^{\tau}_{BA} = \varepsilon \cdot AB. \tag{3.7}$$

Tangent acceleration $\vec{W}^{\tau}_{BA}$ is always perpendicular with AB and points in direction of angular acceleration (see Fig. 3.5).

The term $\vec{\omega}\times(\vec{\omega}\times\vec{\rho})$ can be considered as normal acceleration $\vec{W}^{n}_{BA}$ of point B in rotation about the pole A, its magnitude is

$$W^{n}_{BA} = \omega^2 \cdot AB. \tag{3.8}$$

Normal acceleration $\vec{W}^{n}_{BA}$ points from point B to the pole A (see Fig. 3.5).

So total acceleration of the point B in rotational motion about chosen pole A is

$$\overrightarrow{W}_{BA} = \overrightarrow{W}^{\tau}_{BA} + \overrightarrow{W}^{n}_{BA} \tag{3.9}$$

and

$$\overrightarrow{W}_B = \overrightarrow{W}_A + \overrightarrow{W}^{\tau}_{BA} + \overrightarrow{W}^{n}_{BA} = \overrightarrow{W}_A + \overrightarrow{W}_{BA}. \tag{3.10}$$

The absolute acceleration of a point is equal to the vector sum of the acceleration of the base point and the acceleration of the point in rotational motion about chosen base point.

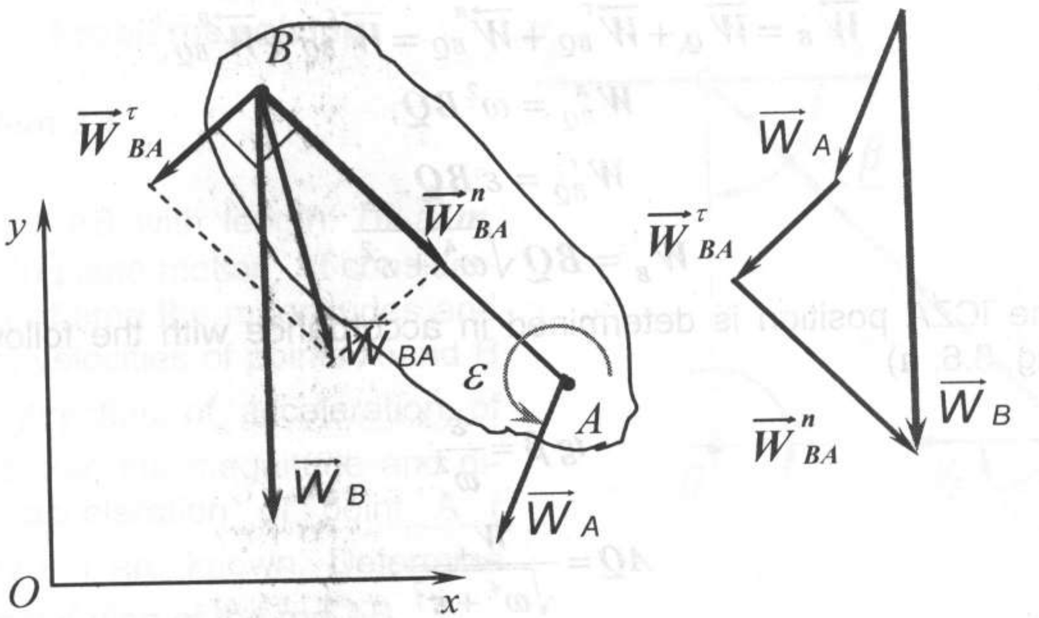


Fig. 3.5. Determination of a point acceleration for plane motion of a body

In practice a point's acceleration is determined by scalar algebra and geometry, by vector algebra, or by graphical construction (see samples).

3.1.7. Formal differentiation of angular velocity expression as methods of angular acceleration determination

If body moves so that the distance from some point to the ICZV is constant (for example, body surface has constant curvature and the body rolls without slipping on fixed surface (see Table 3.1, case 5)) angular acceleration can be determined by formal differentiation of angular velocity expression

$$\varepsilon = \frac{d\omega}{dt} = \frac{d\left(\frac{v_A}{AP}\right)}{dt} = \left| if\ AP = const \right| = \frac{1}{AP}\frac{d\left(v_A\right)}{dt} = \frac{1}{AP}W_A^{\tau} \quad (3.11)$$

3.1.8. Instantaneous center of zero acceleration

If at the same time angular velocity and angular acceleration of a rigid body in plane motion are non-zero, in the plane figure moving with the body there is a unique point that momentarily has zero acceleration. This point is called instantaneous center of zero acceleration (ICZA) and usually denoted as Q.

Choosing the ICZA as pole the acceleration of any point may be determined as

$$\overrightarrow{W}_B = \overrightarrow{W}_Q + \overrightarrow{W}^{\tau}_{BQ} + \overrightarrow{W}^{n}_{BQ} = \overrightarrow{W}^{\tau}_{BQ} + \overrightarrow{W}^{n}_{BQ}, \tag{3.12}$$

$$W^{n}_{BQ} = \omega^2 BQ, \tag{3.13}$$

$$W^{\tau}_{BQ} = \varepsilon BQ, \tag{3.14}$$

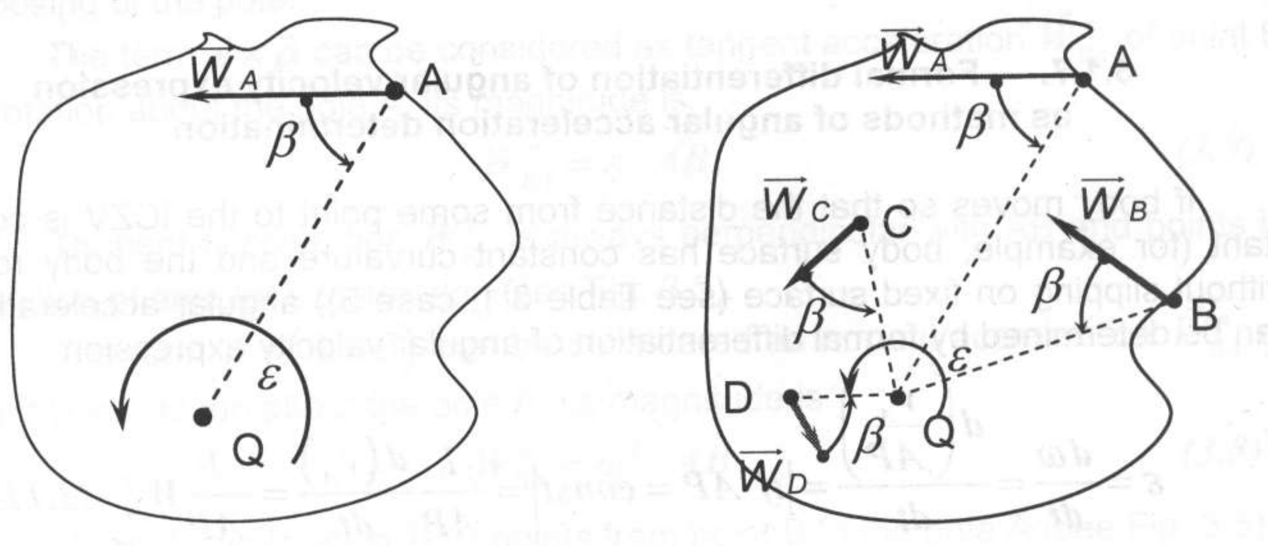
$$W_B = BQ\sqrt{\omega^4 + \varepsilon^2}. \tag{3.15}$$

The ICZA position is determined in accordance with the following relations (Fig. 3.6, a)

$$tg\,\beta = \frac{\varepsilon}{\omega^2}, \tag{3.16}$$

$$AQ = \frac{W_A}{\sqrt{\omega^4 + \varepsilon^2}}. \tag{3.17}$$

where β is angle formed by the point's acceleration and the semiline in which Q is situated, the angle β is drawn from the vector of the point's acceleration in direction of the angular acceleration, AQ is the distance between the point and ICZA along the semiline.



a b

Fig. 3.6

An acceleration of any body' point makes the same angle β with a segment connecting this point with ICZA (Fig. 3.6, b).

On the Fig. 3.6, b it is shown that the acceleration distribution is such as though at the given moment of time plane body rotates about axis passing through ICZA.

3.2. Problems solving

Problem 3.1

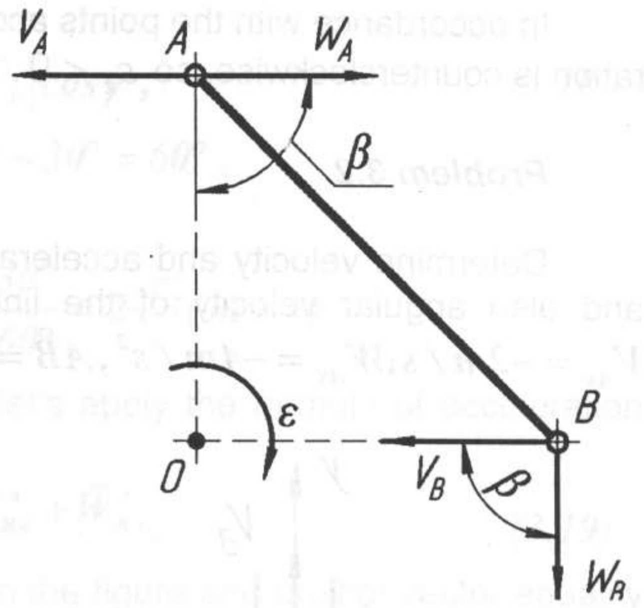


Fig. 3.7

The rod AB with length $10\sqrt{2}m$ (Fig. 3.7) is in plane motion. At considering moment of time the magnitudes and directions of velocities of points A and B ($\vec{V}_A = \vec{V}_B$), direction of acceleration of point B and also the magnitude and direction of acceleration of point A ($W_A = 40 m/s^2$) are known. Determine angular acceleration of the rod AB.

Solution

Instantaneous center of zero velocity of the rod AB at given moment of time does not exist, as velocities of points A and B are parallel and equal by magnitude, i. e. $AP \to \infty$, therefore ω=0.

The ICZA position is determined in accordance with the following relations

$$tg\,\beta = \frac{\varepsilon}{\omega^2} = \infty,$$

it means that

$$\beta = \frac{\pi}{2}.$$

Two semilines are drawn at the angle β from the points A and B accelerations. The ICZA is at the point of intersection of semilines, therefore

$$AQ = AB \sin 45° = 10\sqrt{2}\frac{\sqrt{2}}{2} = 10\ (m).$$

The angular acceleration can be determined using the relation

$$AQ = \frac{W_A}{\sqrt{\omega^4 + \varepsilon^2}},$$

We have $\omega = 0$, so $W_A = AQ \cdot \varepsilon$ and

$$\varepsilon = W_A/(AQ) = 40/10 = 4\ \ (1/s^2).$$

In accordance with the points accelerations' directions the angular acceleration is counterclockwise, so $\varepsilon_z < 0$.

Problem 3.2

Determine velocity and acceleration of point B of ellipsograph (Fig. 3.8), and also angular velocity of the link AB at the given moment of time, if $V_{Ax} = -2 m/s, W_{Ax} = -4 m/s^2, AB = 0,8 m.$

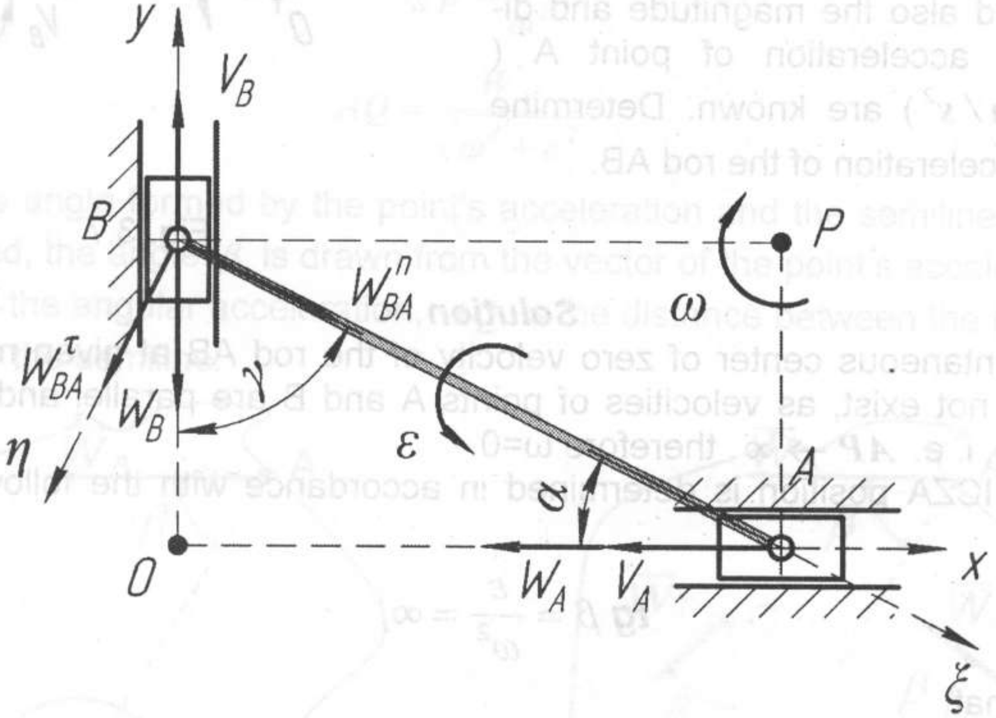


Fig. 3.8

Solution

Velocity of point B is directed upwards along the OY-axis. Therefore instantaneous center of zero velocity (ICZV) is on intersection of perpendiculars to velocities of points A and B, i. e. at point P. Angular velocity of the rod AB at given moment of time is

$$\omega = \frac{V_A}{AP},$$

where $AP = AB \cdot \sin\alpha = 0,8 \cdot \sin 30° = 0,4 (m)$, then $\omega = 5(\frac{1}{s}) \; (\omega_z < 0).$

Let's determine velocity of point B according the Equiprojectivity theorem

$$\left|\vec{V}_A\right| \cos\alpha = \left|\vec{V}_B\right| \cos\gamma , \qquad (3.18)$$

$$\gamma = 90° - \alpha = 90° - 30° = 60° .$$

From (3.18) we get

$$V_B = \frac{V_A \cos\alpha}{\cos\gamma} = 2\frac{\cos 30°}{\cos 60°} = 2\sqrt{3}(m/s).$$

Having chosen point A as a pole, let's apply the formula of acceleration distribution for point B:

$$\vec{W}_B = \vec{W}_A + \vec{W}^n_{BA} + \vec{W}^\tau_{BA}. \qquad (3.19)$$

Let's show vectors $\vec{W}_B, \vec{W}^n_{BA}, \vec{W}^\tau_{BA}$ on the figure and project vector equality (3.19) on axes Bξ and Bη:

$$\begin{cases} W_B \cos\gamma = -W_A \cos\alpha + W^n_{BA}; \\ W_B \sin\gamma = -W_A \sin\alpha + W^\tau_{BA}, \end{cases} \qquad (3.20)$$

where $W^n_{BA} = \omega^2 \cdot AB = 25 \cdot 0,8 = 20(m/s^2)$.

Having solved the system of equation (3.20), we find

$$W_B = \frac{W^n_{BA} - W_A \cos\alpha}{\cos\gamma} = \frac{20 - 4 \cdot 0,85}{0,5} = 33,2(m/s^2),$$

$$W^\tau_{BA} = W_B \sin\gamma - W_A \sin\alpha = 33,2 \cdot 0,85 - 4 \cdot 0,5 = 28,2(m/s^2).$$

Since

$$W^\tau_{BA} = \varepsilon \cdot AB,$$

angular acceleration of the link AB

$$\varepsilon = \frac{W^\tau_{BA}}{AB} = \frac{28,2}{0,8} = 35,2(1/s^2).$$

According to the direction of the vector $\vec{W}^\tau_{BA}$ we'll find $\varepsilon_z > 0$.

Problem 3.3

Crank 1 with length $OA = 6m$ oscillates in vertical plane and actuates the wheel 2 with radius $r = 2m$, that rolls without slipping on the concave circular arc (Fig. 3.9).

At given moment of time $\omega_1 = 2rad/s$ and $\varepsilon_1 = 1rad/s^2$.

Determine accelerations of points M and P of the wheel for this particular instant.

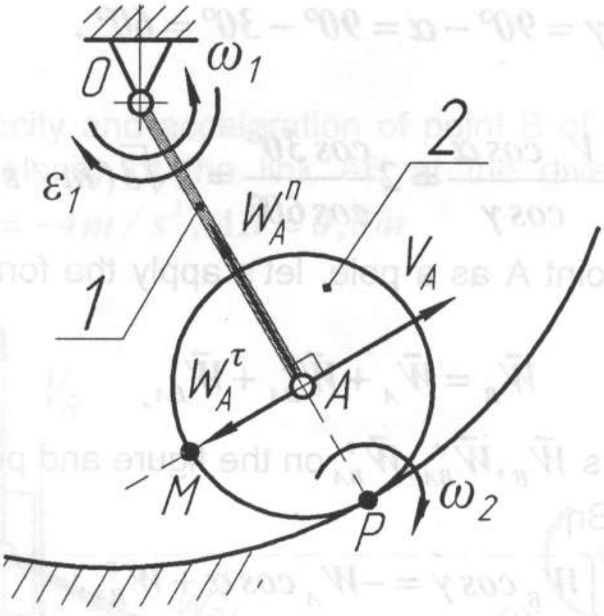


Fig. 3.9

Solution

Let's determine velocity of the point A:

$$V_A = \omega_1 \cdot OA = 2 \cdot 6 = 12(m/s).$$

As ICZV of the wheel 2 is in point P, the angular velocity of the wheel 2:

$$\omega_{2z} = -\frac{V_A}{AP} = -\frac{12}{2} = -6(s^{-1}).$$

Let's determine the acceleration of point A:

$$\vec{W}_A = \vec{W}^n_{AO} + \vec{W}^\tau_{AO},$$

where $W^n_{AO} = \omega_1^2 OA = 4 \cdot 6 = 24(m/s^2)$, vector $\vec{W}^n_{AO}$ is directed toward the axis O.

$$W^\tau_{AO} = \varepsilon_1 OA = 1 \cdot 6 = 6(m/s^2), \vec{W}^\tau_{AO} \perp OA.$$

Then

$$W_A = \sqrt{\left(W_A^n\right)^2 + \left(W_A^\tau\right)^2} = \sqrt{24^2 + 6^2} = 24{,}74(m/s^2).$$

Let's consider the wheel 2 (Fig. 3.10). Since at wheel motion along cylindrical surface the distance from the point A to ICZV does not change, so for determining ε of the wheel the method of formal differentiation is used:

$$\varepsilon_{2z} = \frac{d\omega_{2z}}{dt} = \frac{d}{dt}\left(-\frac{V_A}{AP}\right) = -\frac{1}{AP}\frac{dV_A}{dt} = \frac{W_A^\tau}{AP} = \frac{6}{2} = 3(1/s^2)\ .$$

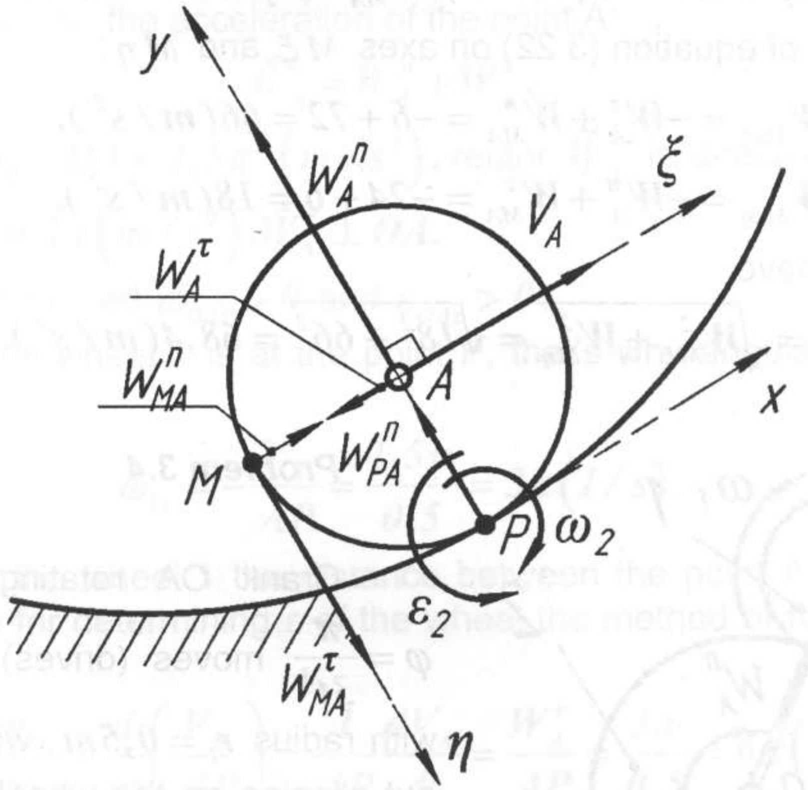


Fig. 3.10

When determining ε_{2z} we take into account, that point A motion is decelerated: vectors $\vec{V}_A$ and $\vec{W}_A^\tau$ are directed into opposite sides.

For determining the acceleration of point P (ICZV), the formula of acceleration distribution is used, choosing point A as a pole:

$$\vec{W}_P = \vec{W}_A^n + \vec{W}_A^\tau + \vec{W}_{PA}^n + \vec{W}_{PA}^\tau, \tag{3.21}$$

$$W_{PA}^n = \omega_2^2 \cdot AP = 6^2 \cdot 2 = 72(m/s^2),$$

$$W_{PA}^\tau = \varepsilon_2 \cdot AP = 3 \cdot 2 = 6(m/s^2).$$

Projecting equation (3.21) on axes Px and Py, we obtain

$$W_{Px} = -W_A^\tau + W_{PA}^\tau = -6 + 6 = 0,$$

$$W_{Py} = -W_A^n + W_{PA}^n = 24 + 72 = 96(m/s^2).$$

Thus the acceleration of point P (ICZV) is directed to the wheel center and equals 96 m/s^2.

For determining the acceleration of point M the point A is chosen as a pole and according to the formula of velocities distribution we'll get:

$$\vec{W}_M = \vec{W}_A^n + \vec{W}_A^\tau + \vec{W}_{MA}^n + \vec{W}_{MA}^\tau, \qquad (3.22)$$

$$W_{MA}^n = \omega_2^2 \cdot AM = 72(m/s^2), W_{MA}^\tau = \varepsilon_2 \cdot AM = 6(m/s^2).$$

Projections of equation (3.22) on axes $M\xi$ and $M\eta$:

$$W_{Mx\xi} = -W_A^\tau + W_{MA}^n = -6 + 72 = 66(m/s^2),$$

$$W_{Mx\eta} = -W_A^n + W_{MA}^\tau = -24 + 6 = 18(m/s^2).$$

Finally we have:

$$W_M = \sqrt{W_{M\xi}^2 + W_{M\eta}^2} = \sqrt{18^2 + 66^2} = 68,4(m/s^2).$$

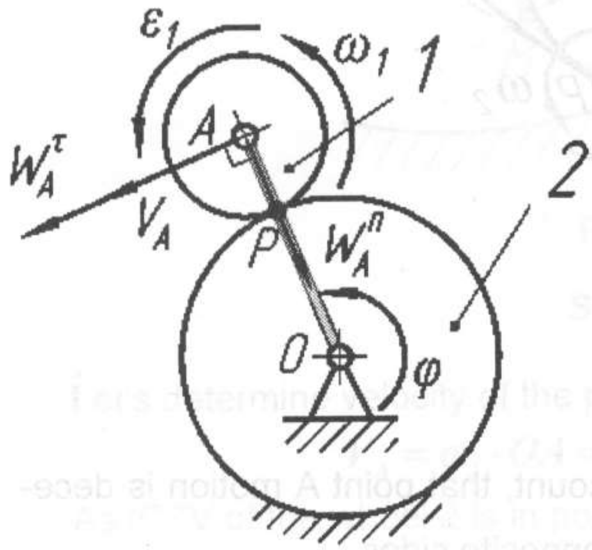


Fig. 3.11

Problem 3.4

Crank OA rotating by the law $\varphi = \frac{\pi}{3t^3}$ moves (drives) the wheel 1 with radius $r_1 = 0,5m$, which rolls without slipping on the wheel 2 with radius $r_2 = 1m$ (Fig. 3.11). At the moment $t = 1s$ find the acceleration of the point coinciding with instantaneous center of zero velocity of the wheel 1.

Solution

Instantaneous velocity center of the wheel 1 rolling without slipping on the fixed wheel 2 is at the contact point of these wheels (point P).

At first we determine the velocity of the point A. Angular velocity of the crank OA at the moment $t = 1s$:

$$\omega_{OAz} = \dot{\varphi} = \left(\frac{\pi}{3}t^3\right)' = \pi t^2 = \pi(rad/s),$$

and the angular acceleration:

$$\varepsilon_{OAz} = \ddot{\varphi} = \left(\pi t^2\right)' = 2\pi t = 2\pi(rad/s^2),$$

and point A velocity

$$OA = r_1 + r_2 = 1,5(m),$$
$$V_A = \omega_{OA} \cdot OA = 1,5\pi(m/s).$$

Let's determine the acceleration of the point A:

$$\vec{W}_A = \vec{W}_A^n + \vec{W}_A^\tau,$$

where $W_A^n = \omega_{OA}^2 \cdot OA = 1,5\pi^2\left(m/s^2\right)$, vector $\vec{W}_A^n$ is directed to the O-axis, $W_A^\tau = \varepsilon_{OA} \cdot OA = 3\pi\left(m/s^2\right), \vec{W}_A^\tau \perp OA.$

It should be noted $\omega_{OAz} > 0$ and $\varepsilon_{OAz} > 0$.

ICZV of the wheel 1 is at the point P, that's why angular velocity of the wheel 1 is

$$\omega_{1z} = -\frac{V_A}{AP} = \frac{1,5\pi}{0,5} = 3\pi\left(1/s\right).$$

Since for the wheel 1 the distance between the point A and ICZV does not change, so for determining ε of the wheel the method of formal differentiation is used:

$$\varepsilon_{1z} = \frac{d\omega_{1z}}{dt} = \frac{d}{dt}\left(\frac{V_A}{AP}\right) = \frac{1}{AP}\frac{dV_A}{dt} = \frac{W_A^\tau}{AP} = \frac{3\pi}{0,5} = 6\pi\left(1/s^2\right).$$

For determining the acceleration of the point P (ICZV) the formula of acceleration distribution is used, point A is chosen as a pole(Fig. 3.12):

$$\vec{W}_P = \vec{W}_A^n + \vec{W}_A^\tau + \vec{W}_{PA}^n + \vec{W}_{PA}^\tau, \qquad (3.23)$$

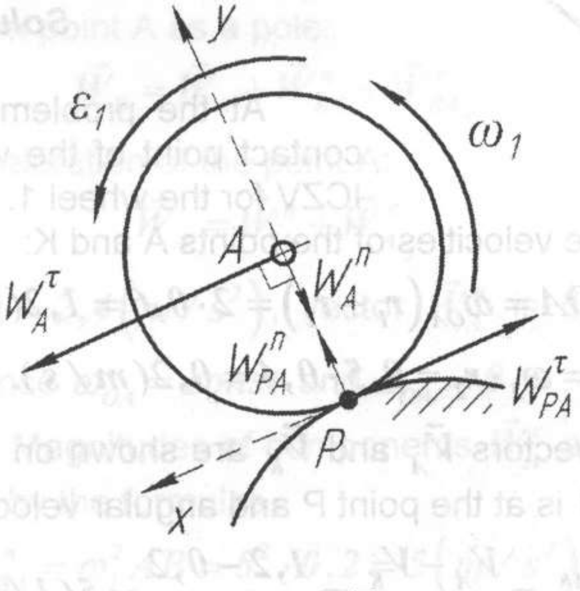


Fig. 3.12

$$W_{PA}^{n} = \omega_1^2 \cdot AP = (3\pi)^2 \cdot 0,5 = 4,5\pi^2 \left(m/s^2\right),$$

$$W_{PA}^{\tau} = \varepsilon_1 \cdot AP = 6\pi \cdot 0,5 = 3\pi\left(m/s^2\right).$$

Having projected (3.23) on the axes Px and Py we obtain:

$$W_{Px} = W_A^{\tau} - W_{PA}^{\tau} = 3\pi - 3\pi = 0,$$

$$W_{Py} = -W_A^{n} + W_{PA}^{n} = 4,5\pi^2 - 1,5\pi^2 = 3\pi^2\left(m/s^2\right).$$

Therefore acceleration of the point P (ICZV) is directed to the center of the wheel 1.

Problem 3.5

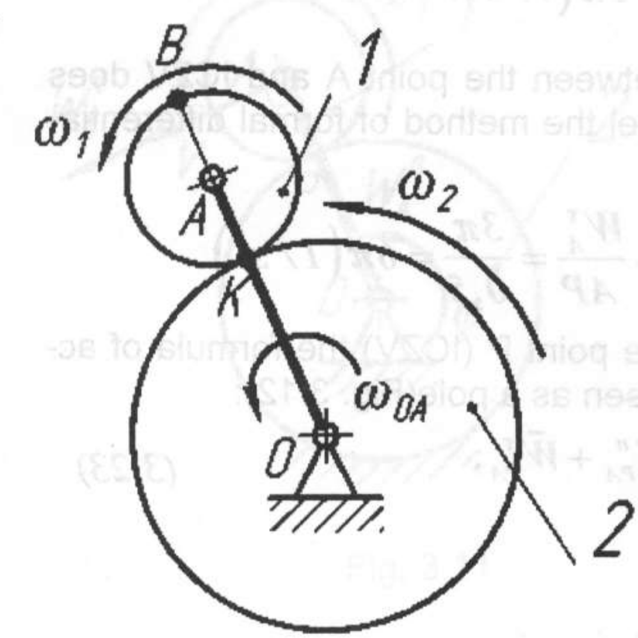


Fig. 3.13

The crank OA rotates about center O with the constant angular velocity $\omega_{OA} = 2rad/s$ and actuates the wheel 1 with radius $r_1 = 0,2m$ that rolls on the wheel 2 with radius $r_2 = 0,4m$ (Fig. 3.13). The wheel 2 is rotated about point O with the constant angular velocity $\omega_2 = 0,5\left(s^{-1}\right)$.

Determine velocity and acceleration of the point B of the wheel 1.

Solution

At the problem the point K that is contact point of the wheel 1 and 2 is not ICZV for the wheel 1.

Let's determine the velocities of the points A and K:

$$V_A = \omega_{OA} \cdot OA = \omega_{OA}\left(r_1 + r_2\right) = 2 \cdot 0,6 = 1,2(m/s);$$

$$V_K = \omega_2 \cdot r_2 = 0,5 \cdot 0,4 = 0,2(m/s).$$

Directions of the vectors $\vec{V}_A$ and $\vec{V}_K$ are shown on the Fig. 3.14. Therefore ICZV of the wheel 1 is at the point P and angular velocity of the wheel 1 is:

$$\omega_1 = \frac{V_{AK}}{AK} = \frac{V_A - V_K}{r_1} = \frac{1,2 - 0,2}{0,2} = 5(1/s).$$

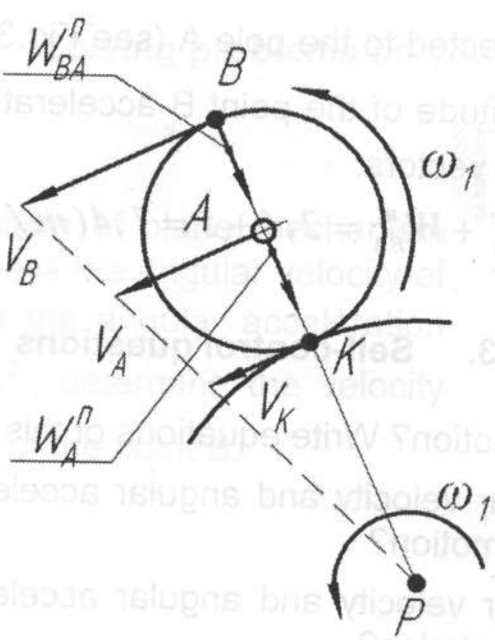


Fig. 3.14

So long as a velocity of a point of a plane figure is proportional to the distance to ICZV, it can be written:

$$\frac{V_A}{V_K}=\frac{r_1+PK}{PK}.$$

The velocity of the point B is determined by the formula $V_B=\omega_1 BP$, substituting in it ω_1 and $BP=2r_1+PK$. Then we obtain:

$$V_B=2V_A-V_K=2\cdot 1,2-0,2=2,2(m/s).$$

For finding the acceleration of the point B the formula of acceleration distribution is used, chosen point A as a pole:

$$\vec{W}_B=\vec{W}_A+\vec{W}_{BA}^n+\vec{W}_{BA}^\tau. \qquad (3.24)$$

Let's find the acceleration of the point A:

$$\vec{W}_A=\vec{W}_A^n+\vec{W}_A^\tau,$$

where $W_A^n=\omega_{OA}^2\cdot OA=2,4\left(m/s^2\right)$, vector $\vec{W}_A^n$ is directed to the O-axis; $W_A^\tau=\varepsilon_{OA}\cdot OA=0$, since $\omega_{OA}=const$ and $\varepsilon_{OA}=0$.

Then $\vec{W}_A=\vec{W}_A^n$. Magnitudes of components $\vec{W}_A^\tau$ and $\vec{W}_{BA}^n$ in the expression (a) are calculated by the formulas

$$W_{BA}^n=\omega_1^2 AB=5^2\cdot 0,2=5\left(m/s^2\right),$$

$$W_{BA}^\tau=\varepsilon_1 AB=0, \text{ since } \omega_1=const \text{ and } \varepsilon_1=0.$$

The vector $\vec{W}_A^n$ is directed to the pole A (see Fig.3.8) and codirected with $\vec{W}_{BA}^n$, that's why the magnitude of the point B acceleration is equal to the sum of the magnitudes of these vectors:

$$W_B = W_A^n + W_{BA}^n = 2,4 + 5 = 7,4(m/s^2).$$

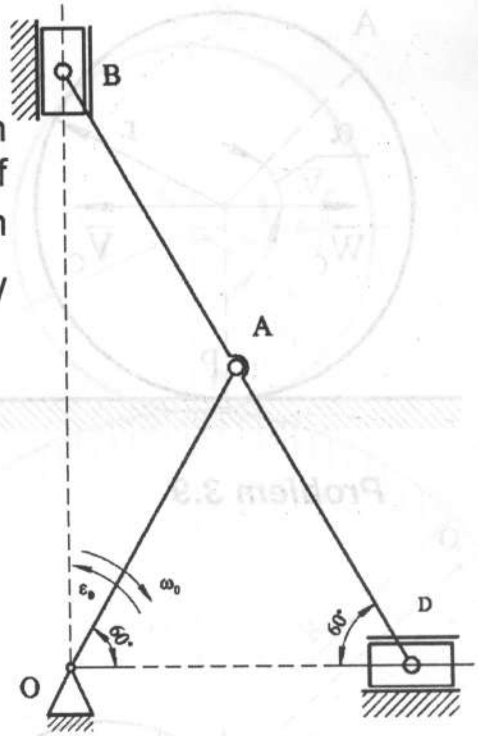
3.3. Self-control questions

1. What is plane motion? Write equations of this motion.
2. How are angular velocity and angular acceleration of a body determined at plane motion?
3. How do angular velocity and angular acceleration of a plane body depend on pole choice?
4. Write a formula of velocities distribution of points at plane motion of a rigid body? Show all vectors on a figure.
5. Formulate the Equiprojectivity theorem of a plane body. Give graphical (geometrical) illustration.
6. Give the definition of ICZV of a plane body. What are the conditions of its existence?
7. What is the figure of the velocities distribution of the plane bogy's points if the pole coincides with ICZV?
8. Name the methods of ICZV finding.
9. For the plane body at time t $\vec{V}_A = \vec{V}_B = \vec{V}_C = ...$ Where is ICZV? What is angular velocity equal to?
10. Where is ICZV of the movable curve at its rolling without slipping on a fixed curve?
11. Write the formula of accelerations distribution of plane body points?
12. Points A, B belong to a plane body. At a given moment $\varepsilon_z < 0, \omega_z < 0.$ Show on a figure the accelerations $\vec{W}_{AB}^n, \vec{W}_{AB}^\tau$.
13. Points A, B belong to a plane body. At a given moment $\varepsilon_z > 0, \omega_z < 0.$ Show on a figure the accelerations $\vec{W}_{BA}^n, \vec{W}_{BA}^\tau$.
14. Points A, B belong to a plane body. What are the angles $tan\,\alpha_A, tan\,\alpha_B$, if $\alpha_A = \vec{W}_{BA}^n \wedge \vec{W}_{BA}, \alpha_B = \vec{W}_{AB}^n \wedge \vec{W}_{BA}$?
15. What is ICZA? What are the conditions of its existence?

3.4. Solving problems on your own

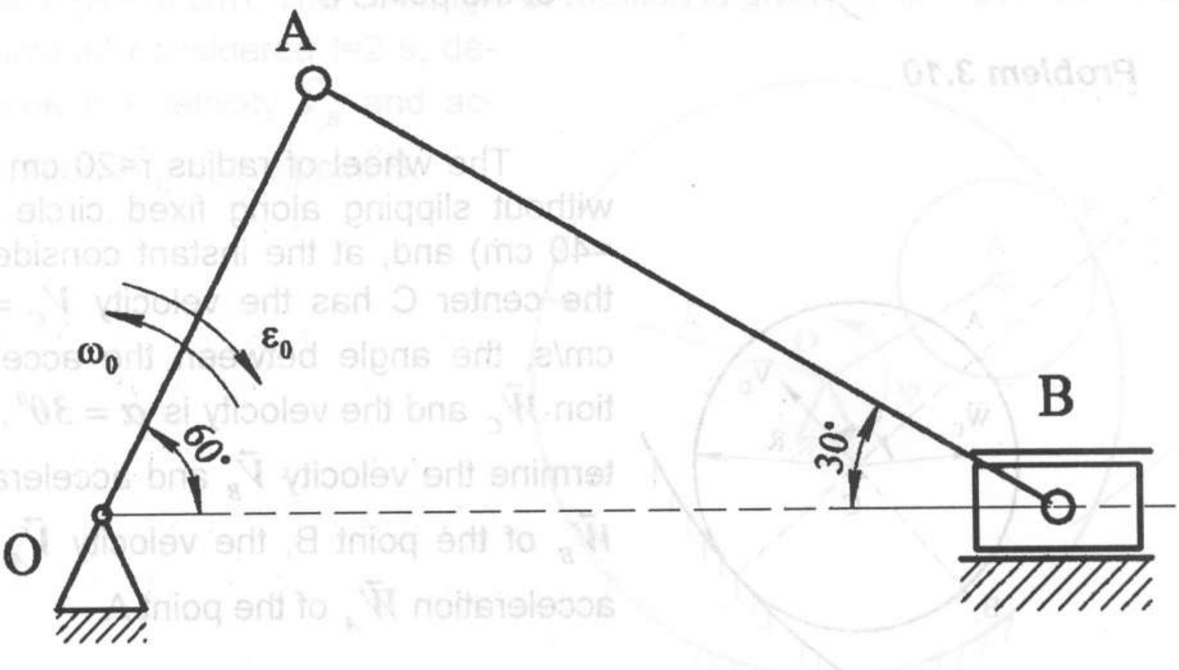
Problem 3.6

For the given position of plane mechanism (AB=AD=OA=10 cm) where the angular velocity of OA is $\omega_0 = 1\ rad/s$ and the angular acceleration of OA is $\varepsilon_0 = 2\ rad/s^2$, determine the velocity $\vec{V}_B$ and acceleration $\vec{W}_B$ of the point B.

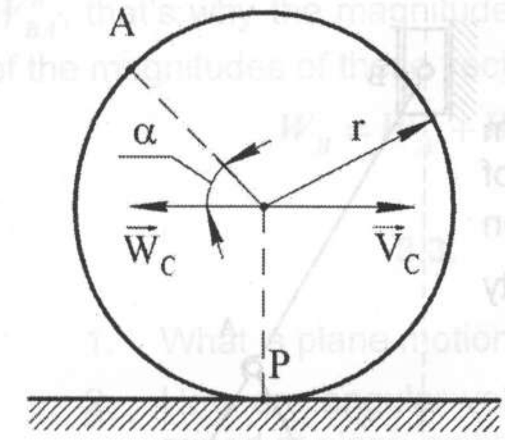


Problem 3.7

For the given position of plane mechanism (OA=10 cm) where the angular velocity of OA is $\omega_0 = 2\ rad/s$ and the angular acceleration of OA is $\varepsilon_0 = 1\ rad/s^2$, determine the velocity $\vec{V}_B$ and acceleration $\vec{W}_B$ of the point B.

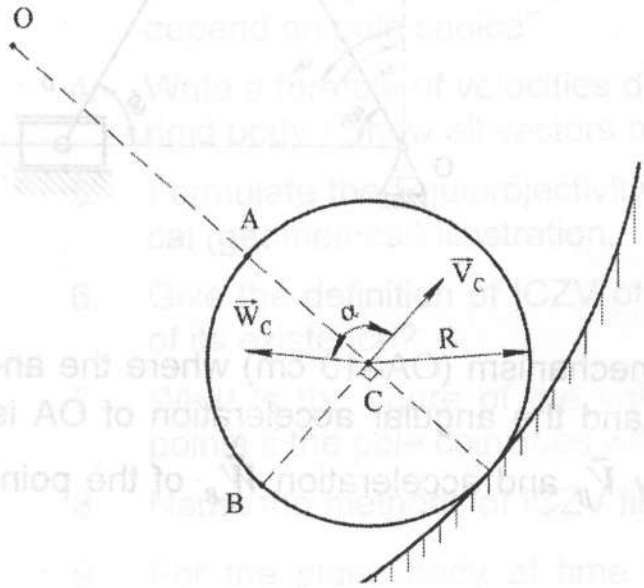


Problem 3.8



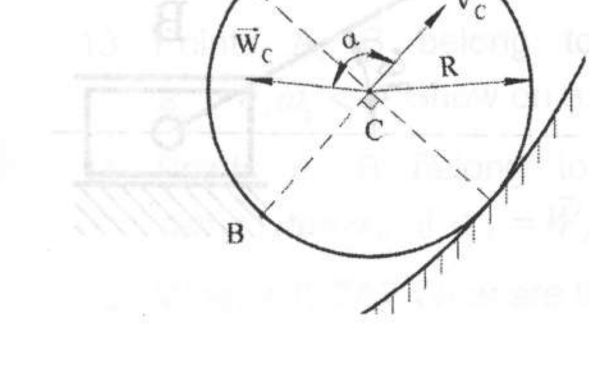
The wheel of radius r=20 cm rolls to the right without slipping and, at the instant considered, the center C has the velocity $\vec{V}_C = 10$ cm/s and the acceleration $\vec{W}_C = 10$ cm/s^2 to the left. Determine the velocity $\vec{V}_P$ and acceleration $\vec{W}_P$ of the point P, the velocity $\vec{V}_A$ and acceleration $\vec{W}_A$ of the point A ($\alpha = 45^0$).

Problem 3.9



The wheel of radius r=10 cm rolls without slipping along fixed circle (OC =30 cm) and, at the instant considered, the center C has the velocity $V_C = 10$ cm/s, the angle between the acceleration $\vec{W}_C$ and the velocity is $\alpha = 135^0$. Determine the velocity $\vec{V}_B$ and acceleration $\vec{W}_B$ of the point B, the velocity $\vec{V}_A$ and acceleration $\vec{W}_A$ of the point A.

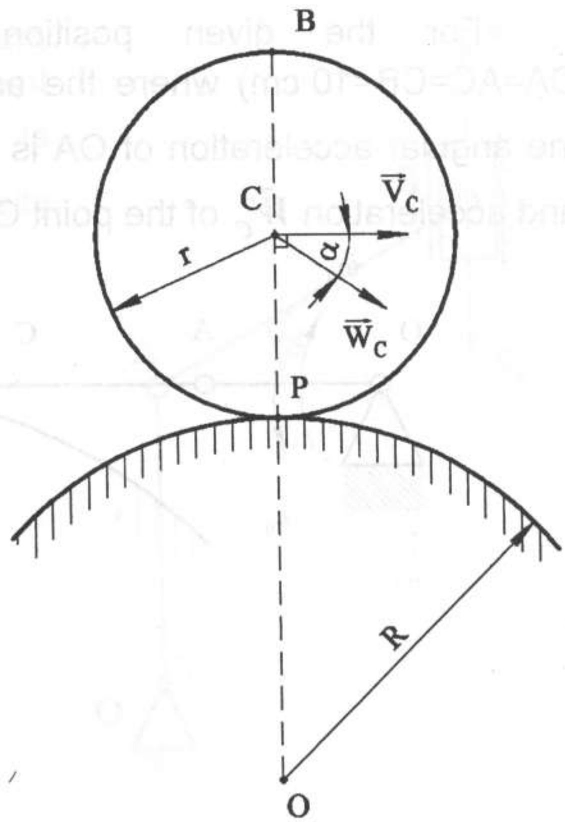
Problem 3.10



The wheel of radius r=20 cm rolls without slipping along fixed circle (OC =40 cm) and, at the instant considered, the center C has the velocity $V_C = 10$ cm/s, the angle between the acceleration $\vec{W}_C$ and the velocity is $\alpha = 30^0$. Determine the velocity $\vec{V}_B$ and acceleration $\vec{W}_B$ of the point B, the velocity $\vec{V}_A$ and acceleration $\vec{W}_A$ of the point A.

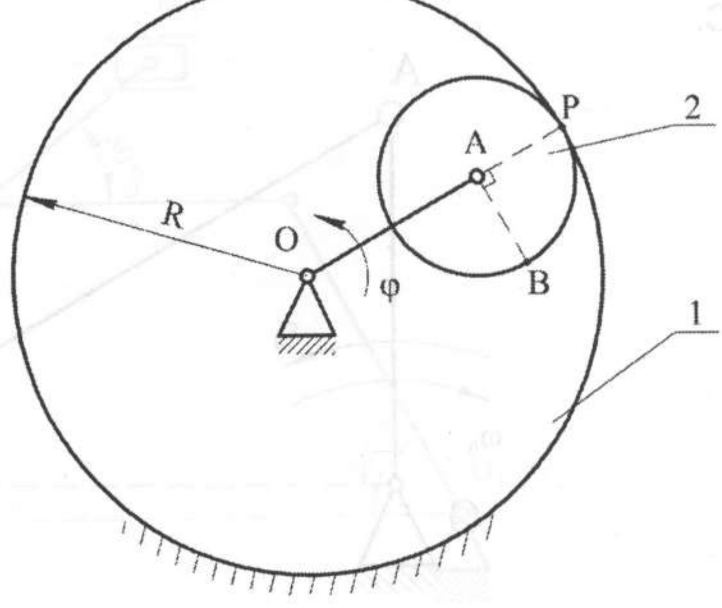
Problem 3.11

The wheel of radius r=10 cm rolls without slipping along fixed circle (R=20 cm) and, at the instant considered, the center C has the velocity $\vec{V}_C = 10$ cm/s, the angle between the acceleration $\vec{W}_{\tilde{N}}$ and the velocity is $\alpha = 30^0$. Determine the velocity $\vec{V}_P$ and acceleration $\vec{W}_P$ of the point P, the velocity $\vec{V}_B$ and acceleration $\vec{W}_B$ of the point B.



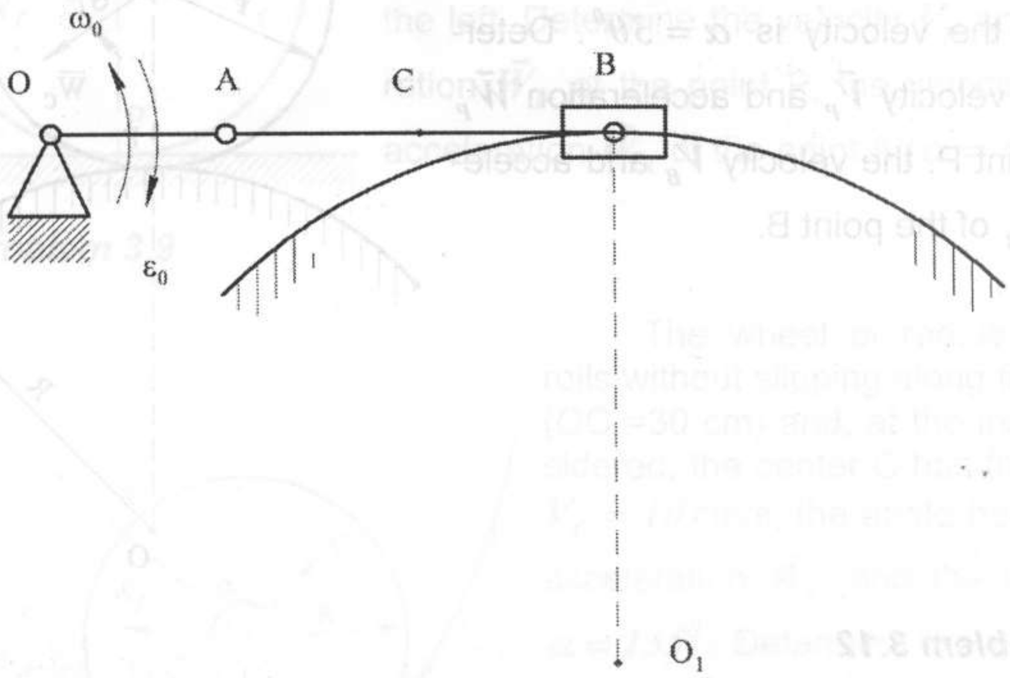
Problem 3.12

Crank OA=40 cm rotates about fixed axis Oz (Oz is perpendicular to the plane of sketch) causing the second disk to roll without slipping along fixed surface (R=30 cm). The OA angle of rotation is given by $\varphi = 2t - 2t^2$, rad. At the instant considered t=2 s, determine the velocity $\vec{V}_B$ and acceleration $\vec{W}_B$ of the point B.



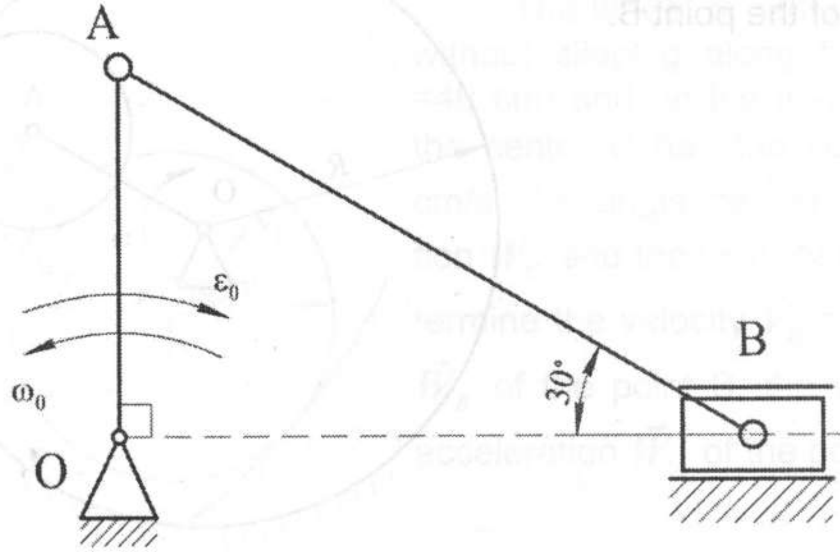
Problem 3.13

For the given position of plane mechanism (O_1B=20 cm, OA=AC=CB=10 cm) where the angular velocity of OA is $\omega_0 = 1\ rad/s$ and the angular acceleration of OA is $\varepsilon_0 = 1\ rad/s^2$, determine the velocity $\vec{V}_C$ and acceleration $\vec{W}_C$ of the point C.



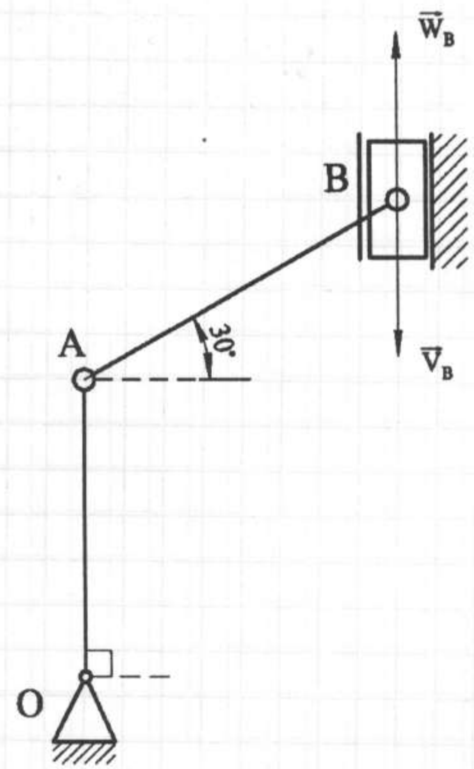
Problem 3.14

For the given position of plane mechanism (OA=AC=10 cm) where the angular velocity of OA is $\omega_0 = 1\ rad/s$ and the angular acceleration of OA is $\varepsilon_0 = 1\ rad/s^2$, determine the velocity $\vec{V}_C$ and acceleration $\vec{W}_C$ of the point C.



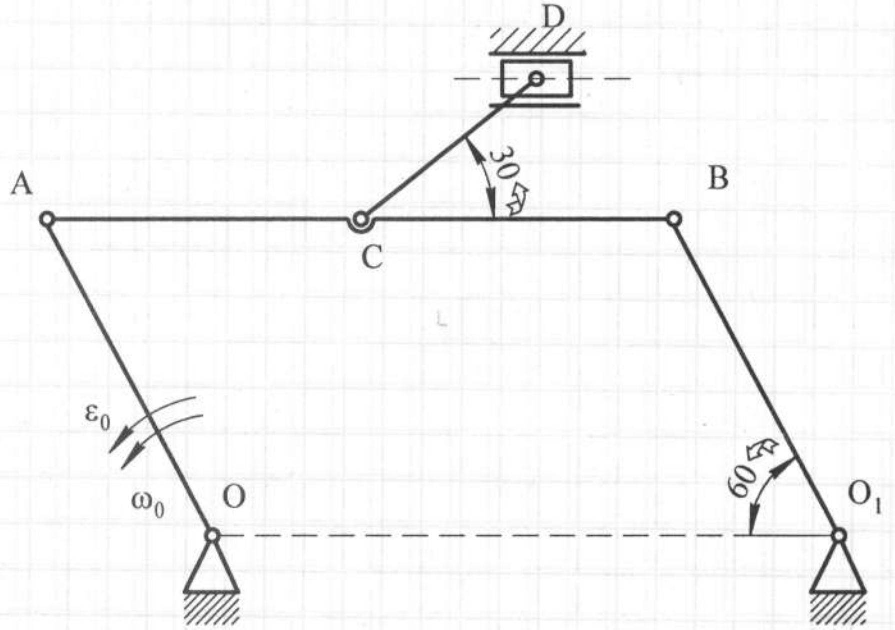
Problem 3.15

For the given position of plane mechanism (OA=AB=10 cm) where the velocity of the point B is $\vec{V}_B = 10$ cm/s and the acceleration is $\vec{W}_B = 3$ cm/s^2, determine the angular velocity ω_{OA} and acceleration ε_{OA} of the rod OA.



Problem 3.16

For the given position of plane mechanism (O_1B=OA= =5 cm, O_1O=AB, CD=10 cm) where the angular velocity of OA is $\omega_0 = 1\ rad/s$ and the angular acceleration of OA is $\varepsilon_0 = 1\ rad/s^2$, determine the angular velocity ω_{CD} and acceleration ε_{CD} of the rod CD.



4. COMPOUND MOTION OF POINT

4.1. Main information from the theoretical course

4.1.1. Definitions

If motion of a particle is at the same time considered in two frames of references fixed (main) and moving (additional) such particle motion is called compound.

Motion of the particle determined with respect to the fixed reference is called **absolute**. Motion of the particle determined with respect to the moving reference is called **relative**. Motion of the moving reference with respect to the fixed reference is called **bulk (transport)**.

Let us consider special point that belongs to moving reference and at some time this point coincides with the particle. Motion of the special point is called bulk motion of the particle.

Consider the sample. There is an observer on a platform. Train moves past the observer, a goes along the train corridor. The motion of the passenger along the corridor is relative motion. Point of the train where passenger is at this time moves with respect to the observer, this motion is bulk one, motion of the moving passenger with respect to the observer on the platform is absolute.

Let us denote as $\vec{V}_a, \vec{W}_a$ absolute velocity and acceleration of the particle correspondently, $\vec{V}_r, \vec{W}_r$ relative velocity and acceleration of the particle correspondently and $\vec{V}_e, \vec{W}_e$ bulk (transport) velocity and acceleration of the particle.

Main aim of problems on particle compound motion is determination of relations between kinematical characteristics in absolute, relative and bulk motion.

4.1.2. Determination of a particle position

Let us consider a body that is in plane motion. Two coordinate systems are chosen (Fig. 4.1):

- the first $Oxyz$ is fixed;
- the second $Ax_1y_1z_1$ is moving, $Ax_1y_1z_1$ has motion of translation with respect to the fixed coordinate system, the translation is characterized by coordinates of the origin A in the fixed reference $\vec{r}_A = x_A\vec{i} + y_A\vec{j}$;
- the third Ax_2y_2 is moving and rigidly connected with the body so Ax_2y_2 is in plane motion with respect to the fixed coordinate system and at the same time rotates about axis Az_1, the rotation is characterized by angle of rotation φ.

There is a particle M moving on the surface of the body. Position of the particle in the moving reference is given by vector position $\overrightarrow{AM} = \vec{\rho}$, the magnitude and direction of $\vec{\rho}$ change with time. Position of the particle in the fixed reference is given by vector position $\overrightarrow{OM} = \vec{r}$:

$$\vec{r} = \vec{r}_A + \vec{\rho}.$$

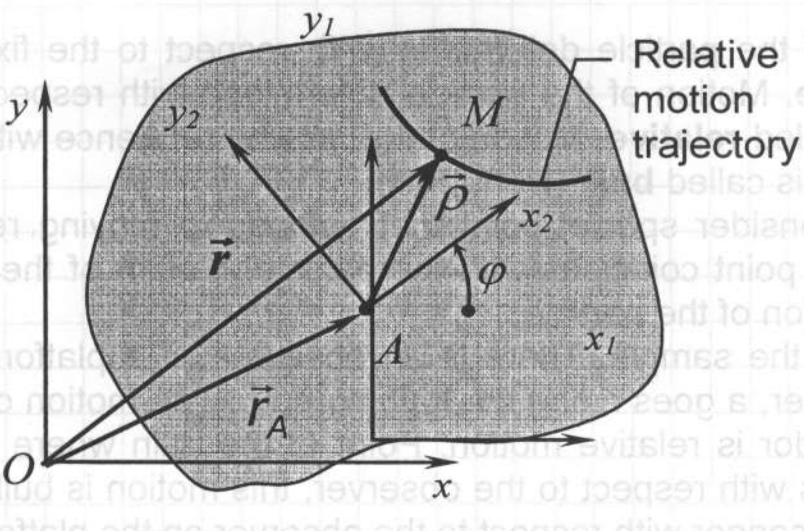


Fig. 4.1. Determination of a point position in compound motion of a body

4.1.3. Velocity of a particle in compound motion

Let us consider the problem of particle velocity determination. In accordance with the velocity definition we have

$$\vec{v}_a = \frac{d\vec{r}}{dt} = \frac{d\left(\vec{r}_A + \vec{\rho}\right)}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{\rho}}{dt}.$$

The first term $\frac{d\vec{r}_A}{dt}$ is the velocity of the pole A.

Let us analyze the second term. From the point of view of an observer at the origin O (in fixed reference) (Fig. 4.2) the orientation of the $\vec{\rho}$ changes due to the body (and reference Ax_2y_2) rotation with angular velocity $\vec{\omega}_e$ and at the same time the magnitude and orientation of the $\vec{\rho}$ change due to relative motion of the particle in the moving reference. Therefore

$$\frac{d\vec{\rho}}{dt} = \vec{\omega}_e \times \vec{\rho} + \frac{\tilde{d}\vec{\rho}}{dt} = \vec{\omega}_e \times \vec{\rho} + \vec{v}_r.$$

The term $\vec{\omega}_e \times \vec{\rho}$ gives us the velocity in rotation about origin A of the point in the body where the particle M is, the term $\frac{\tilde{d}\vec{\rho}}{dt}$ characterizes the relative velocity of the particle and is called first local time derivative.

As a result we get

$$\vec{v}_a = \vec{v}_A + \vec{\omega}_e \times \vec{\rho} + \vec{v}_r.$$

The sum $\vec{v}_A + \vec{\omega}_e \times \vec{\rho}$ gives us the velocity of the point that belongs to the moving reference (or the body) and coincides with the particle, so

$$\vec{v}_A + \vec{\omega}_e \times \vec{\rho} = \vec{v}_e$$

and

$$\vec{v}_a = \vec{v}_e + \vec{v}_r.$$

The absolute velocity of a particle in compound motion is equal to the vector sum of the bulk and relative velocities.

The last equation in Russian school of mechanics is known as **theorem of velocities addition**.

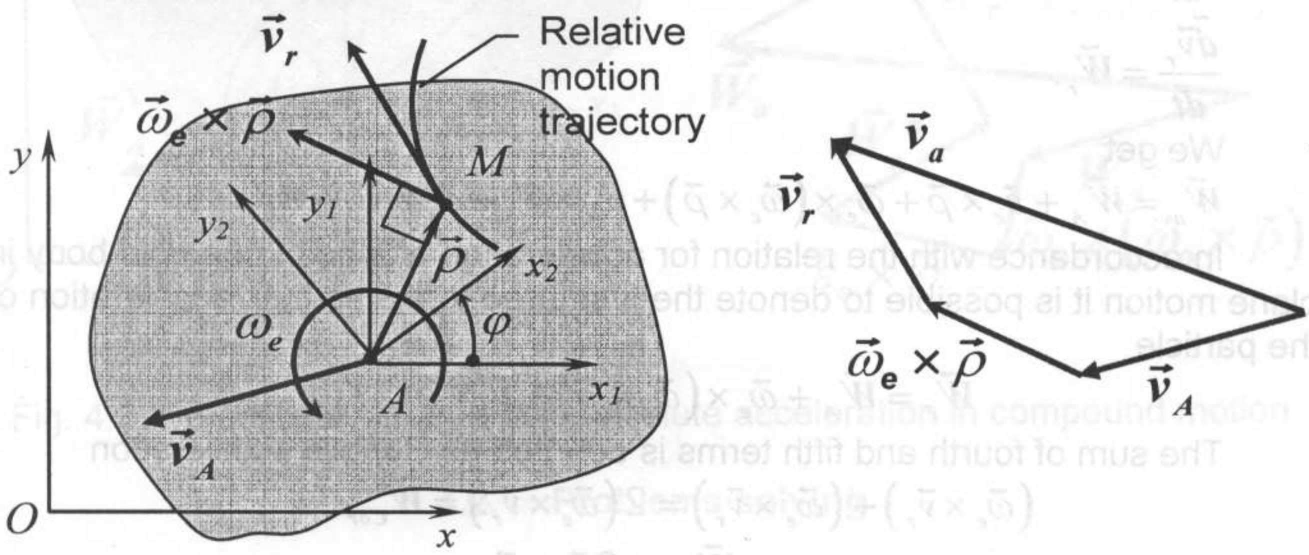


Fig. 4.2. Determination of a particle absolute velocity in compound motion

4.1.4. Acceleration of a particle in compound motion

Let us consider the problem of particle acceleration determination. In accordance with the acceleration definition we have

$$W_a = \frac{d\vec{v}_a}{dt} = \frac{d\vec{v}_e}{dt} + \frac{d\vec{v}_r}{dt} =$$

$$= \frac{d}{dt}\left(\vec{v}_A + \vec{\omega}_e \times \vec{\rho}\right) + \frac{d\vec{v}_r}{dt} = \frac{d\vec{v}_A}{dt} + \frac{d\vec{\omega}_e}{dt} \times \vec{\rho} + \vec{\omega}_e \times \frac{d\vec{\rho}}{dt} + \frac{d\vec{v}_r}{dt}.$$

Vectors $\vec{\rho}$ and $\vec{v}^r$ are given in moving reference so

$$\frac{d\vec{v}_r}{dt} = \vec{\omega}_e \times \vec{v}_r + \frac{\tilde{d}\vec{v}_r}{dt}$$

and

$$\frac{d\vec{\rho}}{dt} = \vec{\omega}_e \times \vec{\rho} + \frac{\tilde{d}\vec{\rho}}{dt} = \vec{\omega}_e \times \vec{\rho} + \vec{v}^r,$$

therefore

$$\vec{W}^a = \frac{d\vec{v}_A}{dt} + \frac{d\vec{\omega}_e}{dt} \times \vec{\rho} + \vec{\omega}_e \times \left(\vec{\omega}_e \times \vec{\rho} + \frac{\tilde{d}\vec{\rho}}{dt} \right) + \vec{\omega}_e \times \vec{v}_r + \frac{\tilde{d}\vec{v}_r}{dt}.$$

Let us denote as

$\frac{d\vec{v}_A}{dt} = \vec{W}_A$ the origin A acceleration;

$\frac{d\vec{\omega}_e}{dt} = \vec{\varepsilon}_e$ angular acceleration of the moving reference (or the body);

$\frac{\tilde{d}\vec{v}_r}{dt} = \vec{W}_r$.

We get

$\vec{W}_a = \vec{W}_A + \vec{\varepsilon}_e \times \vec{\rho} + \vec{\omega}_e \times \left(\vec{\omega}_e \times \vec{\rho} \right) + \vec{\omega}_e \times \vec{v}^r + \vec{\omega}_e \times \vec{v}^r + \vec{W}^r$.

In accordance with the relation for acceleration of a point of a rigid body in plane motion it is possible to denote the first three terms as bulk acceleration of the particle

$$\vec{W}_e = \vec{W}_A + \vec{\omega}_e \times \left(\vec{\omega}_e \times \vec{\rho} \right) + \vec{\varepsilon}_e \times \vec{\rho}.$$

The sum of fourth and fifth terms is denoted as Coriolis acceleration

$$\left(\vec{\omega}_e \times \vec{v}_r \right) + \left(\vec{\omega}_e \times \vec{v}_r \right) = 2\left(\vec{\omega}_e \times \vec{v}_r \right) = \vec{W}_{Cor},$$

$$\vec{W}_{cor} = 2\vec{\omega}_e \times \vec{v}_r.$$

Coriolis acceleration magnitude is

$$W_{cor} = 2\omega_e \cdot v_r \cdot sin\,\alpha,$$

wherein α is angle between the vectors $\vec{\omega}_e$ and $\vec{v}_r$.

Coriolis acceleration characterizes the change in relative velocity due to the bulk motion and the change in bulk velocity due to the relative motion.

Coriolis acceleration is zero if:

1) $\omega_e = 0$, it means bulk motion is translation;

2) $\vec{v}_r = 0$, it means the particle does not move in moving reference;

3) $sin\,\varphi = 0$, it means $\vec{\omega}_e \parallel \vec{v}_r$.

Coriolis acceleration sense is established by the right-hand rule for the cross product.

As a result we get

$$\vec{W}_a = \vec{W}_e + \vec{W}_r + \vec{W}_{Cor}.$$

The absolute acceleration of an article in compound motion is vector sum of bulk relative and Coriolis accelerations (Fig. 4.3)). This relation is known as Coriolis' law.

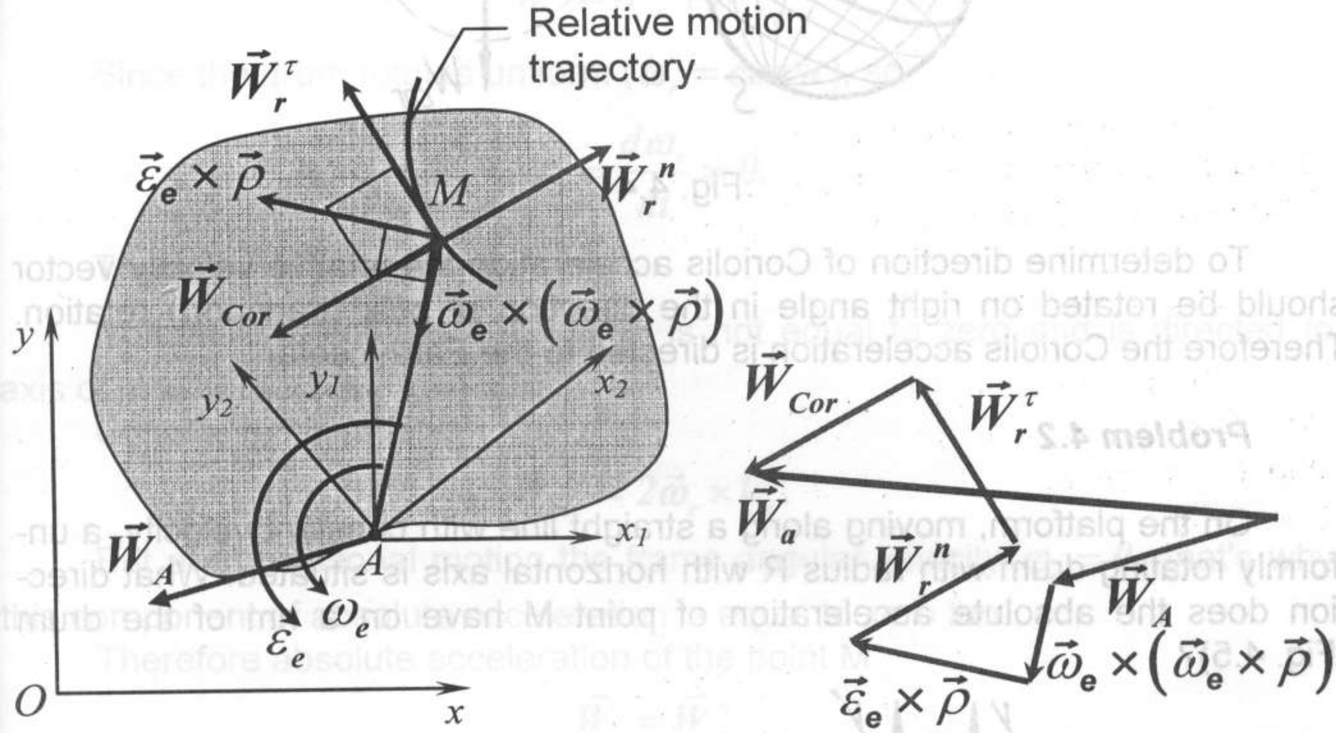


Fig. 4.3. Determination of particle absolute acceleration in compound motion

4.2. Problems solving

Problem 4.1

A car moves along equator from the west to the east (Fig. 4.4). What direction does the Coriolis acceleration have?

Solution

In comparison with the Earth a car can be considered as a point. Its movement along a road is relative motion and a rotation together with the Earth is bulk (transport) one. Vector of the Earth angular velocity is directed along its axis from the South pole to the North one. Direction of relative velocity vector is shown on the Fig.4.4.

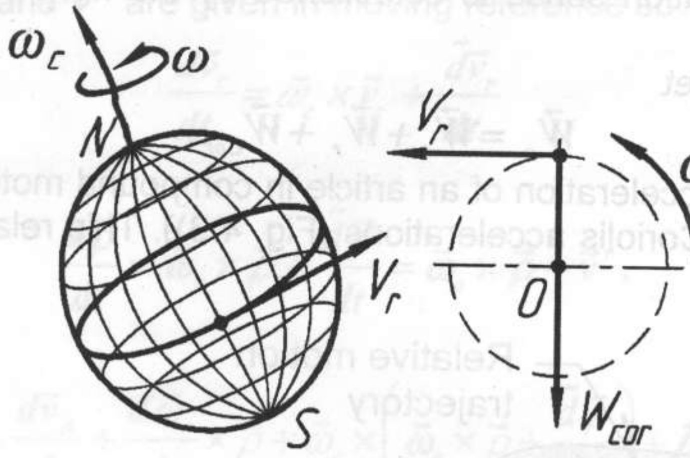


Fig. 4.4

To determine direction of Coriolis acceleration the relative velocity vector should be rotated on right angle in the direction of bulk (transport) rotation. Therefore the Coriolis acceleration is directed to the Earth center.

Problem 4.2

On the platform, moving along a straight line with constant velocity, a uniformly rotating drum with radius R with horizontal axis is situated. What direction does the absolute acceleration of point M have on a rim of the drum (Fig. 4.5)?

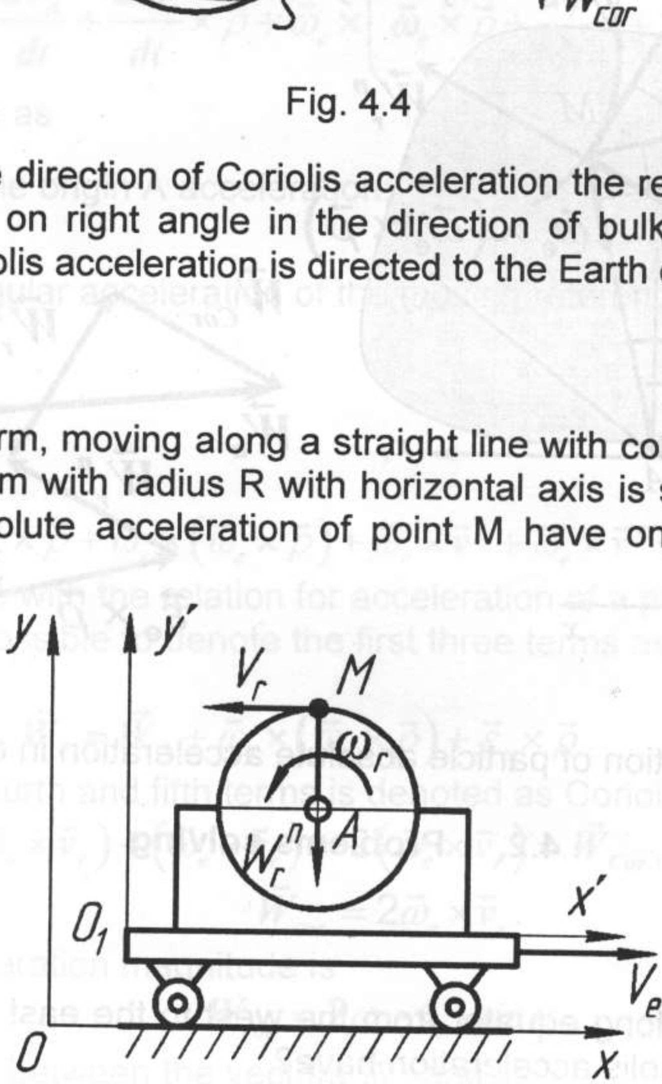


Fig. 4.5

Solution

Main reference system (xOy) is connected with fixed plane and auxiliary one ($x'O_1y'$) is connected with moving platform. We note, that in this case bulk (transport) motion is rectilinear translation and relative motion is rotation.

Absolute acceleration of the point M is determined according Coriolis theorem:

$$\vec{W}_a = \vec{W}_r + \vec{W}_e + \vec{W}_{cor}.$$

Frame acceleration $\vec{W}_e$ is equal to zero, because the platform moves along a straight line with constant velocity. Relative acceleration is equal to the sum of normal and tangent accelerations:

$$\vec{W}_r = \vec{W}_r^n + \vec{W}_r^\tau.$$

Since the drum rotates uniform ($\omega_r = const$), so

$$\varepsilon_r = \frac{d\omega_r}{dt} = 0.$$

Therefore $W_e^\tau = \varepsilon R = 0$.

Normal acceleration $W_e^n = \omega^2 R$ is not equal to zero and is directed to axis of rotation A (see Fig. 4.5).

Coriolis acceleration

$$\vec{W}_{cor} = 2\vec{\omega}_e \times \vec{V}_r.$$

But at translational motion the frame angular velocity $\omega_e = 0$, that's why this component of absolute acceleration is equal to zero too.

Therefore absolute acceleration of the point M

$$\vec{W}_a = \vec{W}_r^n$$

and is directed to the center of the drum.

Problem 4.3

Link AC rotates in indicated direction (Fig. 4.6) and moves slider C and the crankshaft BC. What is the angle between the vectors of absolute velocity and Coriolis acceleration?

Solution

Auxiliary reference system is connected with rotating oscillating arm and main one with the fixed plane. Relative motion of the slider is its moment along AC, and transport one is rotation together with AC about the axis, perpendicular to the drawing plane (vector of the angular velocity is directed to us). Since point C is rigidly connected with the link BC, so its absolute velocity is directed perpendicular BC in the direction of rotation, and relative one – from C to A. Then Coriolis acceleration will be directed vertically downwards (vector $\vec{V}_r$ is ro-

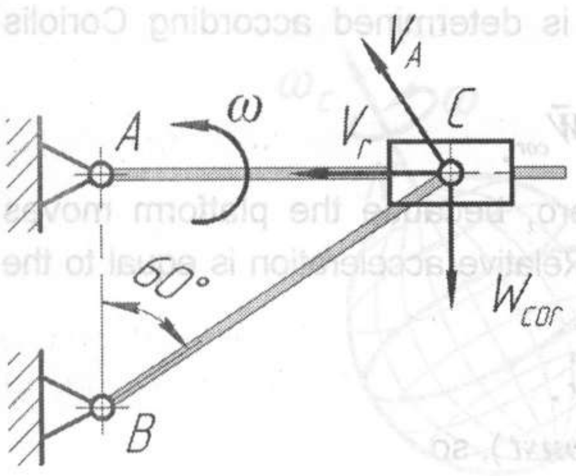


Fig. 4.6

tated on 90° in direction of the frame rotation, e. i. counterclockwise), and required angle is 150°.

Problem 4.4

Ring with radius $R=0,3\ m$ rotates about axis O_1, which is perpendicular to the plane of the ring, according to the law $\varphi=(t^2-6t)rad$ (Fig. 4.7). Along the ring from the point O counterclockwise the point M moves according to the law $\sigma(t)=\breve{OM}=0,3\pi\cos\left(\frac{\pi t}{3}\right)(m)$, where $\sigma(t)$ is arc coordinate. The distance OO_1=0,1 m.

Find the absolute velocity and acceleration of the point at t=1s.

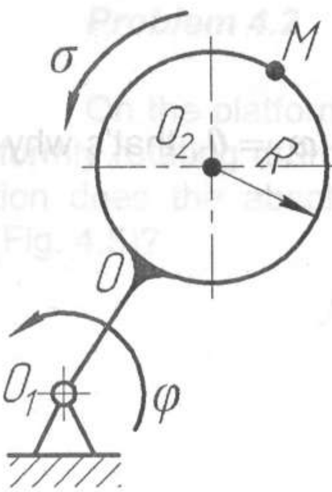


Fig. 4.7

Solution

For investigation of the point compound motion the reference systems are chosen. Main (fixed) O_1xyz is connected with the fixed point O_1, and auxiliary (movable) O_2 – with rotating ring.

Let's mark out relative and bulk motion of the point.

For it the motion of the whole body about point O_1 is mentally stop, then the point will move curvilinearly along the circle with radius R=0,3 m. This motion is relative one. For determining the bulk motion the point motion along the circle is mentally stopped. Then the point together with the body will rotate about fixed axis, i. e. auxiliary reference system is in rotational motion about point O_1 according to the known law $\varphi(t)$. This is its bulk motion.

Let's determine the point position at the given moment of time

$$\sigma(1)=0,3\pi\cos\left(\frac{\pi\cdot 1}{3}\right)=\frac{0,3\pi}{2}(m).$$

At point motion along a circle a central angle, arc length and circle radius are connected by the equation $\sigma = \alpha R$ (Fig. 4.8).

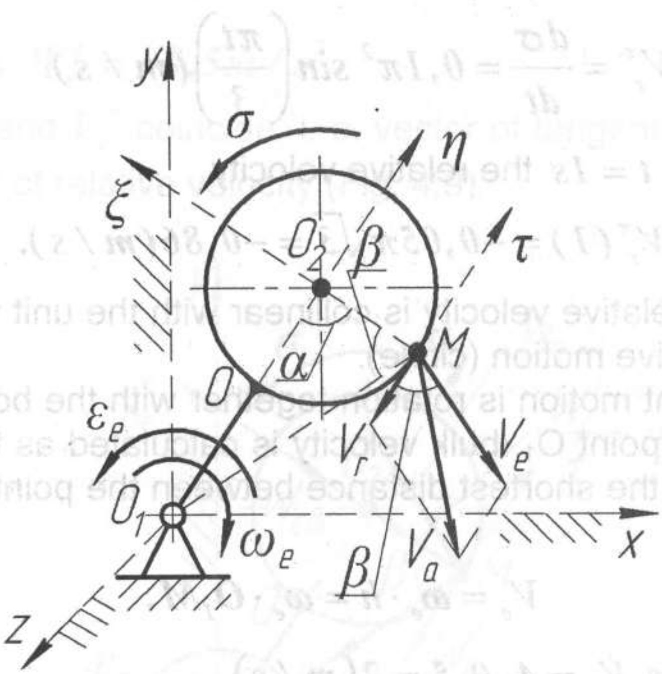


Fig. 4.8

From here

$$\alpha = \frac{\sigma}{R} = \frac{0,3\pi}{2\cdot 0,3} = \frac{\pi}{2}.$$

The distance between the point M and axis of rotation:

$$O_1M = \sqrt{(O_1O_2)^2 + (O_2M)^2} = \sqrt{(O_1O + R)^2 + R^2}.$$

Substituting known values we'll get $O_1M = 0,5m$.

Knowing the law of ring rotation let's determine bulk angular velocity and angular acceleration: $\omega_{ez} = \dot{\varphi} = 2t - 6$, i. e. at $t = 1s$ $\omega_{ez} = -4(rad/s)$, $\varepsilon_{ez} = \ddot{\varphi} = 2(rad/s)$ $(\varepsilon_{ez} = const)$.

Thus at the given moment of time the ring rotation is decelerated $(\varepsilon_{ez} > 0, \omega_{ez} < 0)$ clockwise $(\omega_{ez} < 0)$.

Absolute velocity of the point is equal to the geometric sum of relative and bulk velocities:

$$\vec{V}_a = \vec{V}_e + \vec{V}_r.$$

Let's determine the projection of the vector $\vec{V}_r$ on the unit vector of the circle tangent $\vec{\tau}$ (the circle is trajectory of relative motion, $\sigma > 0$ for counterclockwise direction, so $\vec{\tau}$ points in this direction) . For it the law of relative motion specified in a natural form is differentiated by the time:

$$V_r^{\tau} = \frac{d\sigma}{dt} = 0,1\pi^2 \sin\left(\frac{\pi t}{3}\right)(m/s).$$

At the moment $t = 1s$ the relative velocity

$$V_r^{\tau}(1) = -0,05\pi\sqrt{3} = -0,86(m/s).$$

Vector of the relative velocity is collinear with the unit vector of tangent to the trajectory of relative motion (circle).

Since bulk point motion is rotation together with the body about fixed axis passing through the point O_1, bulk velocity is calculated as the product of body angular velocity and the shortest distance between the point and the axis of rotation:

$$V_e = \omega_e \cdot h = \omega_e \cdot O_1M.$$

Hence at $t = 1s$ $V_e = 4 \cdot 0,5 = 2(m/s)$.

The vector of bulk velocity is directed perpendicular to O_1M to the side of body rotation.

Since the angle between relative and bulk velocities is not equal to 90°, the absolute point velocity can be found by the cosine theorem:

$$V_a = \sqrt{V_e^2 + V_r^2 + 2V_eV_r\cos\beta}.$$

cos β is determined from the triangle O_1O_2M:

$$\cos\beta = \frac{O_2M}{O_1M} = 0,6 \Rightarrow \beta \approx 53°.$$

Then absolute velocity $V_a = 2,6 m/s$.

The point acceleration is determined according to the Coriolis theorem:

$$\vec{W}_a = \vec{W}_r + \vec{W}_e + \vec{W}_{cor}.$$

The point in relative motion moves along the ring (curvilinear trajectory), that's why

$$\vec{W}_r = \vec{W}_r^n + \vec{W}_r^{\tau}.$$

The law of point relative motion is given in the natural form. Tangent acceleration is determined as derivative by the time of the relative velocity projection on the $\vec{\tau}$:

$$W_r^\tau = \frac{dV_r^\tau}{dt} = -0,033\pi^3 \cos\left(\frac{\pi}{3}t\right).$$

Thus at $t = 1s$ $W_r^\tau = -0,5 m/s^2$.

Signs of W_r^τ and V_r^τ coincide, i. e. vector of tangent acceleration is collinear with the vector of relative velocity (Fig. 4.9).

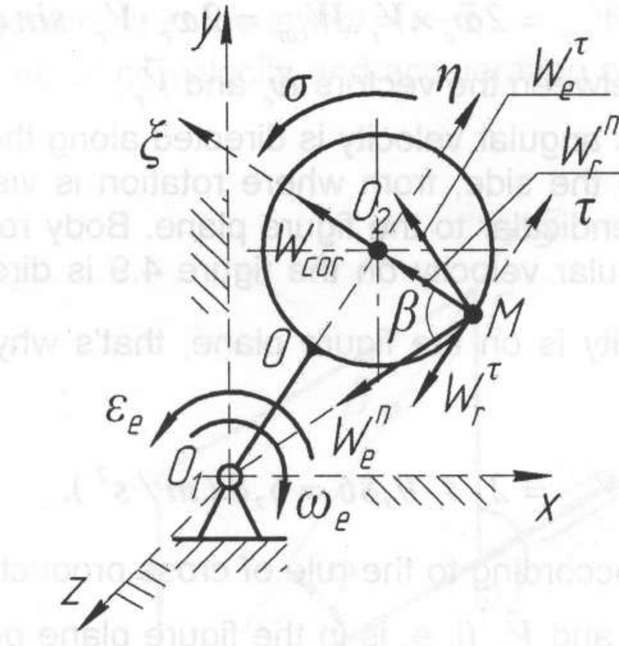


Fig. 4.9

Normal acceleration is $W_r^n = \frac{V_r^2}{\rho}$, where ρ – radius of curvature of point trajectory.

Trajectory of the relative motion is circle with the radius R, that's why radius of curvature is equal to R.

Then

$$W_r^n = \frac{V_r^2}{\rho} = \frac{(-0,86)^2}{0,3} = 2,46\left(m/s^2\right).$$

Normal acceleration is directed along the principal normal to the trajectory of relative motion to the side of trajectory concavity, i. e. to the circle center.

As it was mentioned above, bulk motion is rotation about fixed axis O_1. The acceleration of the point M is equal to the sum of normal and tangent accelerations.

Normal acceleration is

$$W_e^n = \omega_e^2 \cdot O_1M = 4^2 \cdot 0,5 = 8\,(m/s^2)$$

and is directed to the axis of rotation along O_1M.

Tangent acceleration is

$$W_e^\tau = \varepsilon_e \cdot O_1M = 2 \cdot 0,5 = 1\,(m/s^2).$$

Vectors of tangent and normal accelerations are orthogonally related (on the Fig. 4.9 it is taken into account, that $\varepsilon_{ez} < 0$).

Coriolis acceleration is

$$\vec{W}_{cor} = 2\vec{\omega}_e \times \vec{V}_r, W_{cor} = 2\omega_e \cdot V_r \cdot \sin\varphi,$$

where φ is the angle between the vectors $\vec{\omega}_e$ and $\vec{V}_r$.

The vector of bulk angular velocity is directed along the axis of rotation of the body and points to the side, from where rotation is viewed counterclockwise. This axis is perpendicular to the figure plane. Body rotation is clockwise. Thus the vector of angular velocity on the figure 4.9 is directed from us. The vector of relative velocity is on the figure plane, that's why the angle $\varphi = \frac{\pi}{2}$ and

$$W_{cor} = 2 \cdot 4 \cdot 0,86 = 6,8\,(m/s^2).$$

The vector $\vec{W}_{cor}$ according to the rule of cross product must be perpendicular to the vectors $\vec{\omega}_e$ and $\vec{V}_r$ (i. e. is in the figure plane perpendicular to $\vec{V}_r$) and directed to the side, from where the shortest rotation from $\vec{\omega}_e$ to $\vec{V}_r$ is counterclockwise, i. e. to the circle center. We note, that for determining the direction of $\vec{W}_{cor}$ Zhukovsky rule can be used.

To find the magnitude of absolute acceleration let's rewrite expression (a) in detail:

$$\vec{W} = \vec{W}_r^n + \vec{W}_r^\tau + \vec{W}_e^n + \vec{W}_e^\tau + \vec{W}_{cor}.$$

Let's project this vector equality on axes $O_2\xi$ and $O_2\eta$ (see Fig. 4.9) and find the projections $W_{a\xi}$ and $W_{a\eta}$:

$$W_{a\xi} = W_r^n + W_e^\tau \sin\beta + W_e^n \cos\beta + W_{cor} =$$
$$= 2,46 + 1 \cdot 0,8 + 8 \cdot 0,6 + 6,8 = 14,86\,(m/s^2);$$

$$W_{a\eta} = W_e^\tau \cos\beta - W_r^\tau - W_e^n \sin\beta =$$
$$= 1 \cdot 0,6 - 0,5 - 8 \cdot 0,8 = -6,3\,(m/s^2).$$

Then

$$W=\sqrt{W_{\xi}^{2}+W_{\eta}^{2}}.$$

Problem 4.5

The point B (Fig. 4.10) moves along the diagonal of the rectangular frame according to the law $S=OB=0,8t^{2}(m)$. The frame rotates in indicated on the figure direction according to the law $\varphi=t-t^{3}$.

Determine the absolute velocity and acceleration of the point B at $t=2s$ $(\alpha=30^{\circ})$.

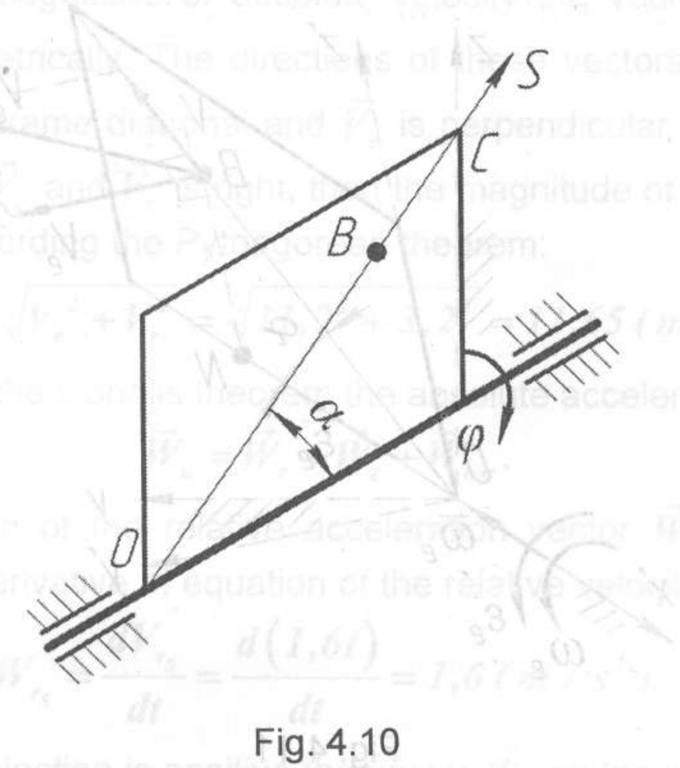


Fig. 4.10

Solution

The main reference system – Cartesian coordinates x, y, z – is connected with a fixed axis of rotation, and auxiliary one – coordinates x′, y′, z′ - with the rotating frame (axis Oy' is perpendicular to the frame plane). Then absolute motion is the motion of the point B relative to the main reference system, bulk motion is the rotation of auxiliary system (frame), relative one is moment of the point B along the S-axis which is directed along the frame diagonal (Fig. 4.11).

The shortest distance BN from the point B to the axis of rotation at $t=2s$:

$OB=0,8\cdot 2^{2}=3,2\,(m)$ and $BN=OB\sin\alpha=3,2\cdot 0,5=1,6\,(m).$

The kinematic characteristics of bulk motion at $t=2s$ is calculated:

– angular velocity

$$\omega_{ez} = \dot{\varphi} = \frac{d}{dt}\left(t - t^3\right) = 1 - 2t^2 = 1 - 2 \cdot 2^2 = -7\ (rad / s);$$

– angular acceleration

$$\varepsilon_{ez} = \frac{d\omega_{ez}}{dt} = -4t = -8\ (rad / s^2).$$

The vectors $\vec{\omega}_e$ and $\vec{\varepsilon}_e$ are shown on the Fig. 4.11.

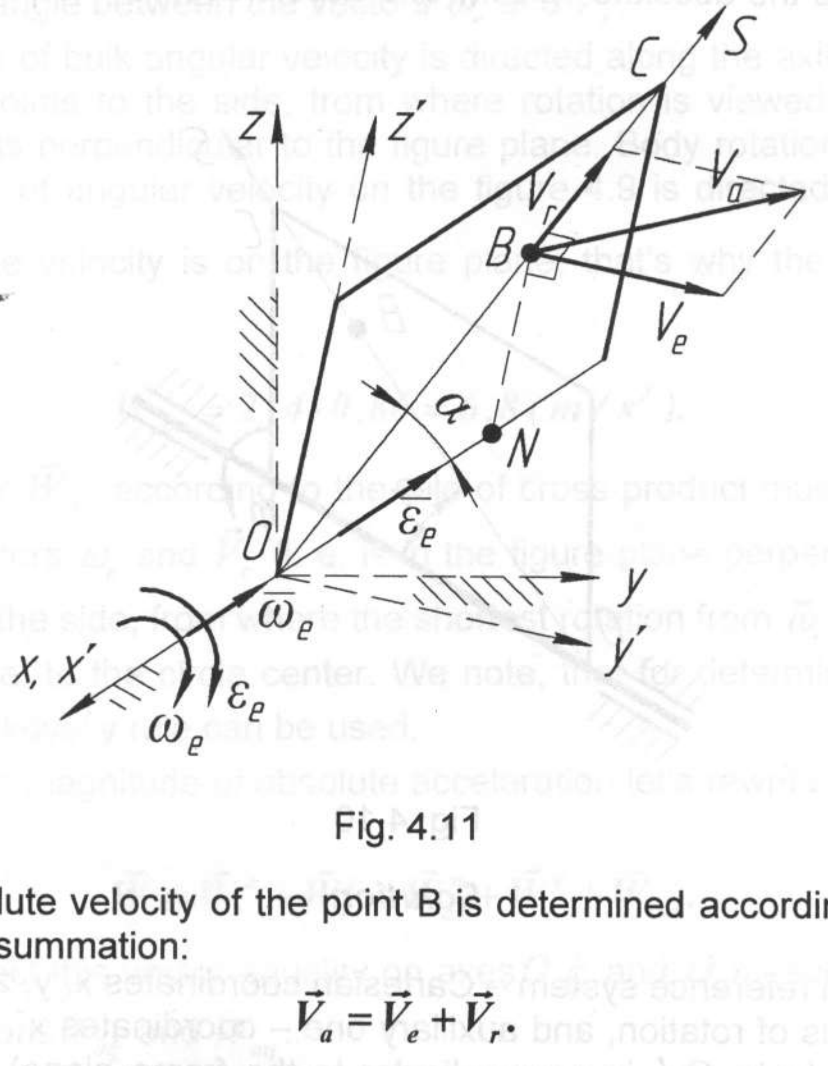


Fig. 4.11

The absolute velocity of the point B is determined according to the theorem of velocity summation:

$$\vec{V}_a = \vec{V}_e + \vec{V}_r.$$

The projection of the relative velocity on the S-axis is found. Since the motion is given in the natural form this projection is

$$V_{r_s} = \frac{dS}{dt} = 0,8 \cdot 2t = 1,6t.$$

At $t = 2s$ we obtain $V_{r_s} = 1,6 \cdot 2 = 3,2\ (m / s)$. Since the velocity projection is positive the vector $\vec{V}_r$ points along the S-axis in direction of OB increasing (see Fig.4.11).

When determining bulk velocity of the point B it is taken into account that auxiliary reference system rotates about fixed axis. Then in accordance with the definition of bulk point motion:

$$\vec{V}_e = \vec{\omega}_e \times \overrightarrow{OB}.$$

From here it follows that vector $\vec{V}_e$ is directed perpendicularly to the frame plane (see Fig. 4.11). The magnitude of bulk velocity

$$V_e = \omega_e \cdot BN.$$

At $t = 2s$ the velocity will be $V_e = 1{,}6 \cdot 7 = 11{,}2\ (m/s)$.

To find the magnitude of absolute velocity the vectors $\vec{V}_e$ and $\vec{V}_r$ are summarized geometrically. The directions of these vectors are defined ($\vec{V}_r$ is directed along the frame diagonal and $\vec{V}_e$ is perpendicular to it). We note, that an angle between $\vec{V}_e$ and $\vec{V}_r$ is right, then the magnitude of the absolute velocity is calculated according the Pythagorean theorem:

$$V_a = \sqrt{V_e^2 + V_r^2} = \sqrt{11{,}2^2 + 3{,}2^2} = 11{,}65\ (m/s).$$

According to the Coriolis theorem the absolute acceleration of the point:

$$\vec{W}_a = \vec{W}_r + \vec{W}_e + \vec{W}_{cor}.$$

The projection of the relative acceleration vector $\vec{W}_r$ on the S-axis is equal to the time derivative of equation of the relative velocity motion:

$$W_{r_s} = \frac{dV_{r_s}}{dt} = \frac{d(1{,}6t)}{dt} = 1{,}6\ (m/s^2).$$

Since this projection is positive the vector $\vec{W}_r$ as the vector $\vec{V}_r$ is directed in the side of OB increasing (Fig. 4.12).

We take notice that in relative motion the point moves along the straight line ($W_r^n = 0$) and its motion is uniformly accelerated ($W_r = const$).

Since the bulk point motion is rotational, $\vec{W}_e$ is equal to the sum of normal and tangent accelerations:

$$\vec{W}_e = \vec{W}_e^n + \vec{W}_e^\tau.$$

We take notice that in relative motion the point moves along the straight line ($W_r^n = 0$) and its motion is uniformly accelerated ($W_r = const$).

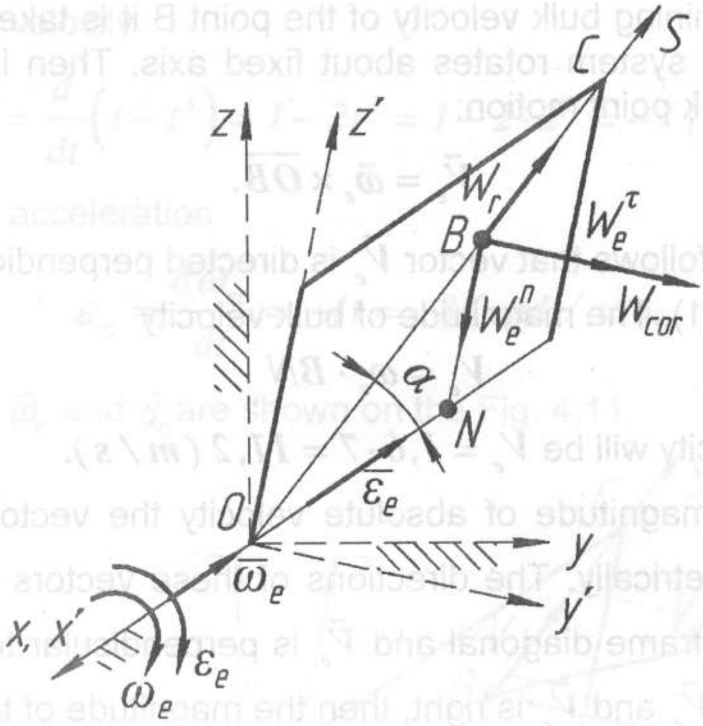


Fig. 4.12

Since the bulk point motion is rotational, $\vec{W}_e$ is equal to the sum of normal and tangent accelerations:

$$\vec{W}_e = \vec{W}_e^n + \vec{W}_e^\tau.$$

Magnitudes and directions of these components are determined. It is known that $\vec{W}_e = \vec{\omega}_e^n \times \vec{V}_e$. Therefore the magnitude of normal acceleration

$$W_e^n = \omega V_e \sin 90° = \omega_e^2 BN = 7^2 \cdot 1{,}6 = 78{,}4\ (m/s^2).$$

The vector $\vec{W}_e^n$ is directed to the axis of rotation along BN.

It is known that tangent acceleration

$$\vec{W}_e^\tau = \vec{\varepsilon}_e \times \overline{OB}.$$

Its magnitude

$$W_e^\tau = \varepsilon_e BN = 8 \cdot 1{,}6 = 12{,}8\ (m/s^2).$$

The vector of tangent acceleration is collinear with the vector of bulk velocity $\vec{V}_e$ (see Fig. 4.12).

Coriolis acceleration is determined by the formula $\vec{W}_{cor} = 2\vec{\omega}_e \times \vec{V}_r$, from which it follows that

$$W_{cor} = 2\omega_e \cdot V_r \cdot sin\varphi,$$

where φ is angle between the vectors $\vec{\omega}_e$ and $\vec{V}_r$, φ=α=30° (see Fig. 4.12). Substituting known values we'll obtain

$$W_{cor} = 2 \cdot 7 \cdot 3,2 \cdot sin\,30° = 22,4\;(m/s^2).$$

The direction of Coriolis acceleration is found according Zhukovsky's rule. For it vector $\vec{V}_r$ is projected on a plane which is perpendicular to the axis of rotation and rotate the projection in the direction of rotation – counterclockwise – on the right angle. Thus the direction of Coriolis acceleration coincides with the direction of bulk tangent acceleration vector.

For finding absolute acceleration it is reasonable to project the vector equality

$$\vec{W}_a = \vec{W}_r + \vec{W}_e^n + \vec{W}_e^\tau + \vec{W}_{cor}$$

on axes of movable coordinate system:

$$W_{a_{x'}} = -W_r\,cos\,\alpha = -1,6 \cdot 0,85 = -1,36\;(m/s^2);$$

$$W_{a_{y'}} = W_e^\tau + W_{cor} = 12,8 + 22,4 = 35,2\;(m/s^2);$$

$$W_{a_{z'}} = W_r\,sin\,\alpha - W_e^n = 1,6 \cdot 0,5 - 78,4 = -77,6\;(m/s^2).$$

Then the absolute acceleration is

$$W_a = \sqrt{W_{a_{x'}}^2 + W_{a_{y'}}^2 + W_{a_{z'}}^2} = \sqrt{1,36^2 + 35,2^2 + 77,6^2} = 85,2\;(m/s^2).$$

Problem 4.6

Determine the absolute velocity of the point M at $t = 1\;s$ if its law of motion along the diagonal of square plate is $AM = \sigma = 0,5t^2\;m$.

The links OA and O_1B rotates according to the law $\varphi = 0,25\pi t$, the distance $OA = O_1B = 0,5\;m$ (Fig. 4.13).

Solution

In this problem you should pay attention on rational choice of an auxiliary reference system. It is reasonable to connect it with the plate ABCD which has translation motion. At this fact the bulk point motion is rectilinear.

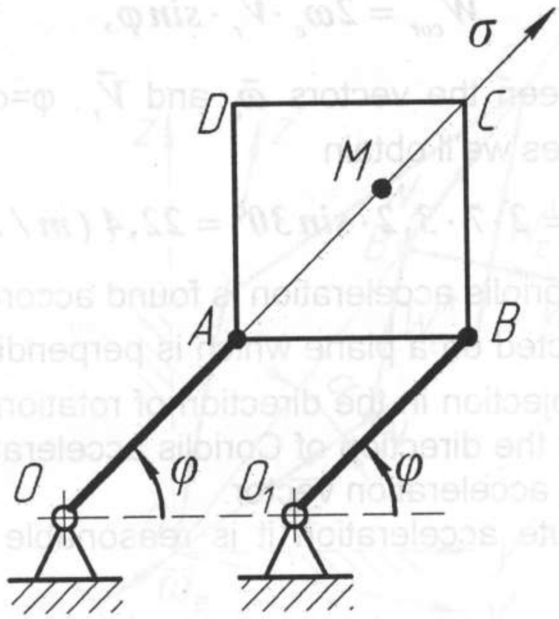


Fig. 4.13

The main reference system (xOy) is connected with the fixed rotational axis of the one of the link. (Fig. .4.14).

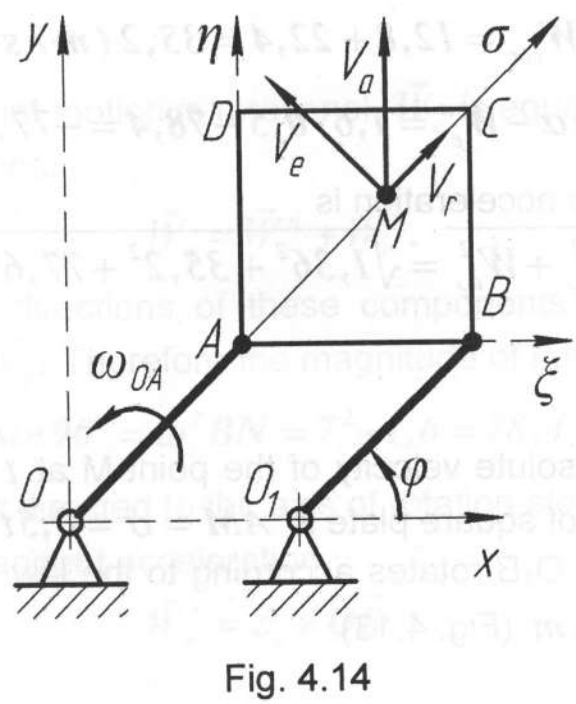


Fig. 4.14

From the equation of rotation we'll find that at $t = 1\ s$ $\varphi = \pi / 4$, therefore the point O, A and M is on the one straight line.

According to the theorem of velocities summing

$$\vec{V}_a = \vec{V}_e + \vec{V}_r.$$

From the definition of a point bulk velocity and features of a translational motion we'll obtain

$$\vec{V}_e = \vec{V}_A, \text{ i. e. } V_e = V_A = \omega_{OA} OA.$$

The angular velocity of the link

$$\omega_{OAz} = \dot{\varphi} = 0,25\pi \ (rad/s),$$

therefore the link rotates counterclockwise.

Then

$$V_e = 0,25 \cdot \pi \cdot 0,5 = 0,125\pi \ (m/s) \text{ and } \vec{V}_e \perp \overline{OM}.$$

At rectilinear motion by the known law

$$V_{r\sigma} = \dot{\sigma} = t$$

at the moment of time $t = 1\ s$ the relative point velocity $V_{r\sigma} = 1 m/s.$

Since $V_{r\sigma} > 0$, the vector $\vec{V}_r$ is directed to the side of σ increasing.

The angle between $\vec{V}_e$ and $\vec{V}_r$ is right, that's why the magnitude of the absolute velocity is found by the Pythagorean theorem:

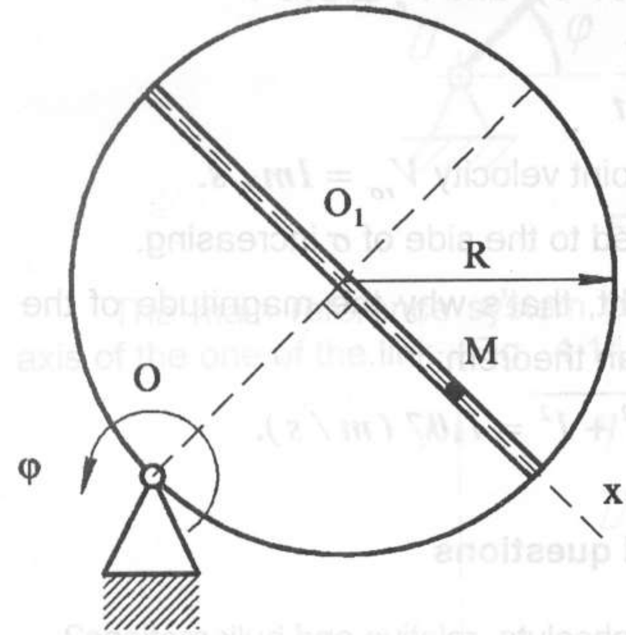
$$V_a = \sqrt{V_e^{\,2} + V_r^{\,2}} = \sqrt{(0,125\pi)^2 + 1^2} = 1,07 \ (m/s).$$

4.3. Self-control questions

1. A point is in compound motion. What are absolute, relative and bulk motions?
2. Write the formula expressing the relationship between absolute and relative derivatives of the vector given at auxiliary reference system.
3. Give the definitions of absolute, relative and bulk velocities of a point that is in compound motion.
4. Formulate the theorem of velocities summing for a point that is in compound motion.
5. A point is in compound motion. Bulk motion is rotation about axis. How is bulk velocity of a point determined?
6. A point is in compound motion. Bulk motion is translation. How is bulk velocity of a point determined?
7. Formulate the theorem of velocities summing at compound motion of a point.
8. Formulate the theorem of accelerations summing at compound motion of a point.
9. A point is in compound motion. Bulk motion is rotation about axis. How is bulk acceleration of a point determined?
10. How should an auxiliary reference system move in order that bulk acceleration of a point (at its compound motion) is equal to zero?

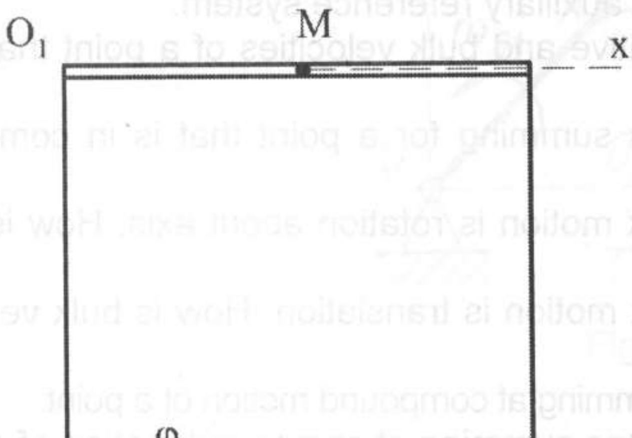
11. Give the definition of Coriolis acceleration. How are its magnitude and direction determined?
12. What is Zhukovsky method for calculation and construction of the Coriolis acceleration vector?
13. When is Coriolis acceleration equal to zero?
14. What should relative motion of a point be in order that relative acceleration is equal to zero?

4.4. Solving problems on your own



Problem 4.7

The disc (R=20 cm) rotates about axis Oz. The disc angle of rotation is given by $\varphi = 2t - 2t^2$, rad. The point M moves along the diameter of the disc. Point position is given by $O_1M = S = 20\sin\frac{\pi t}{6}$, cm. At t=1 s determine absolute velocity $\vec{V}_M^a$ and absolute acceleration of the point $\vec{W}_M^a$.

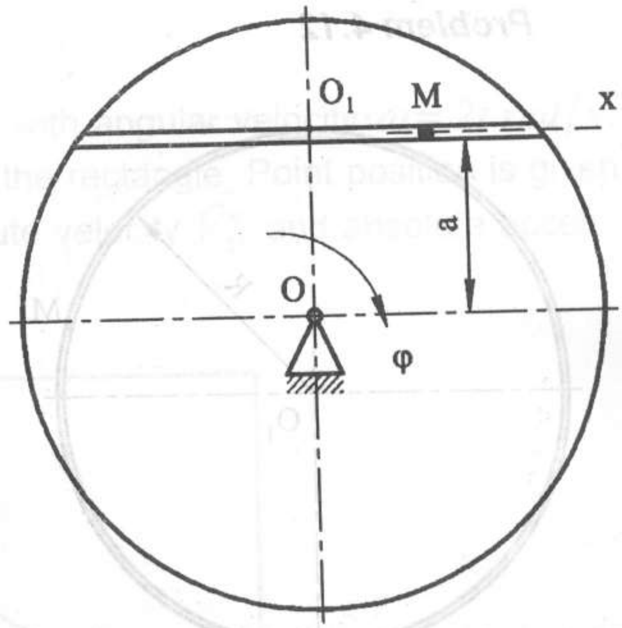


Problem 4.8

The square with side a=20 cm rotates about axis Oz perpendicular to the plane of sketch. The square angle of rotation is given by $\varphi = 2t^2$, rad. The point M moves along the side of the square. Point position is given by $OM = X_M = 5t^3$ cm.

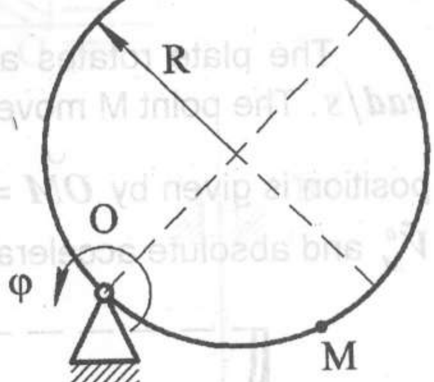
At t=1s determine absolute velocity $\vec{V}_M^a$ and absolute acceleration of the point $\vec{W}_M^a$.

Problem 4.9



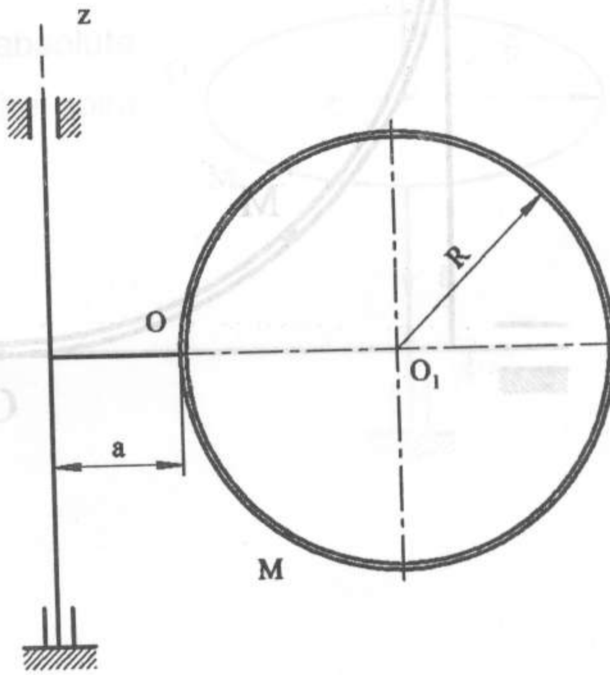
The point M moves in the slot at the same time that the dick rotates about its center O (the distance between the slot and the center is a=10 cm). The disc angle of rotation is given by $\varphi = 3t^2$, rad. Point position is given by $O_1M = S = 20\sin\dfrac{\pi t}{6}$, cm. At t=1s determine absolute velocity $\vec{V}_M^a$ and absolute acceleration of the point $\vec{W}_M^a$.

Problem 4.10



The disc (R=20 cm) rotates about axis Oz. The disc angle of rotation is given by $\varphi = t^3$, rad. The point M moves along the rim of the disc. Point position is given by $\overset{\cup}{OM} = S = 5\pi t^2$, cm. At t=1 s determine absolute velocity $\vec{V}_M^a$ and absolute acceleration of the point $\vec{W}_M^a$.

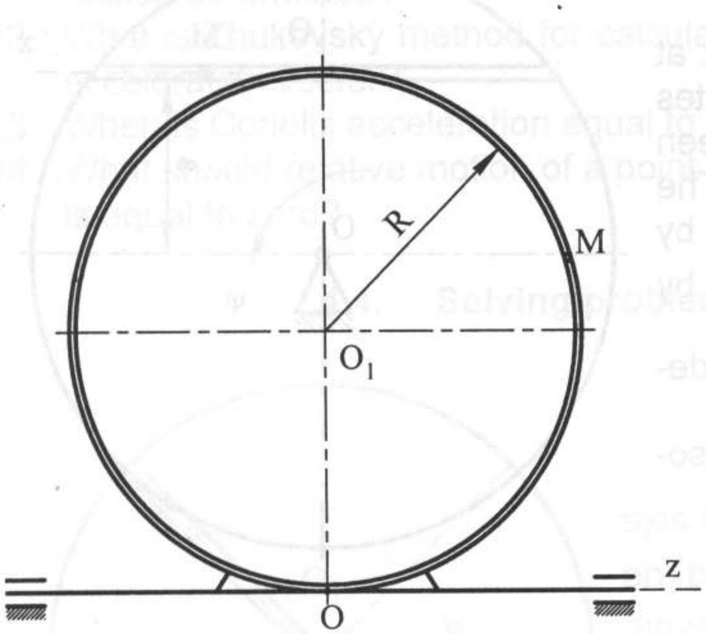
Problem 4.11



The point M moves along the tube of the radius R=30 cm, rotating with the angular velocity $\omega = 2t$ s^{-1}, accoding to the law $\overset{\cup}{OM} = S = 10\pi t^2$ cm.

Determine $\vec{V}_M^a$ and $\vec{W}_M^a$ at t_1=1s, if a=10 cm.

Problem 4.12

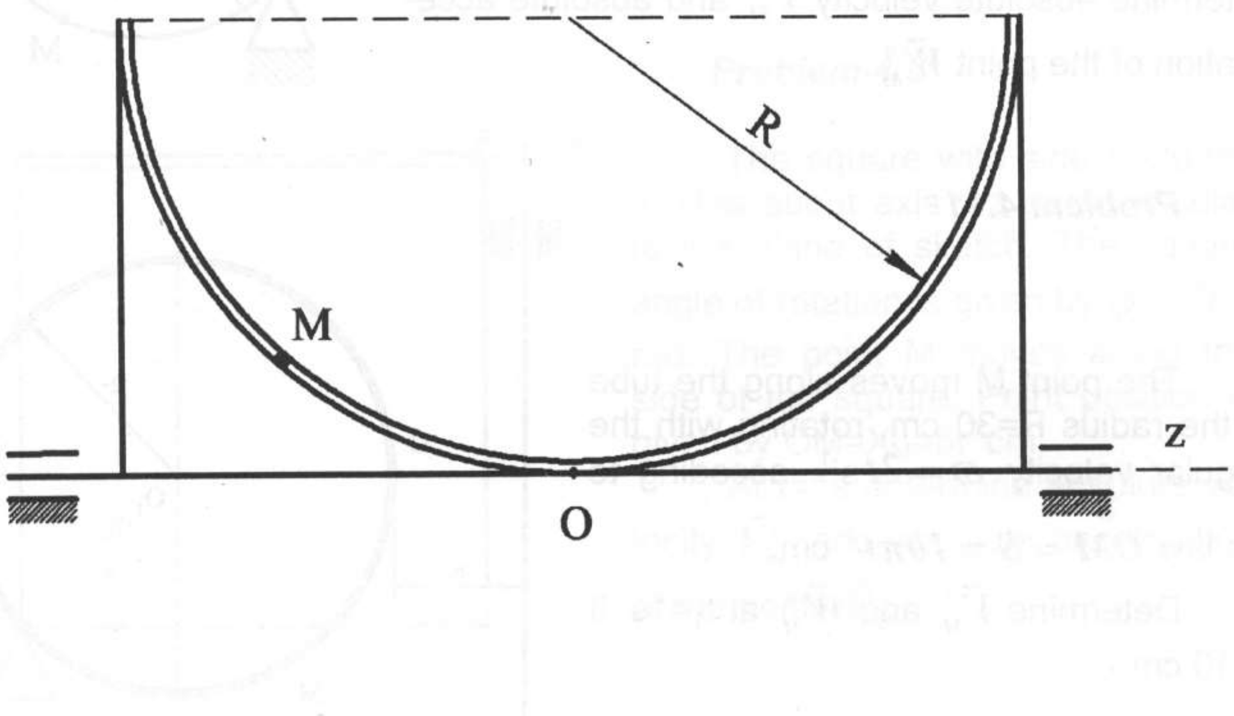


The point M moves along the tube of the radius R=30 cm, rotating with the angular velocity $\omega = 2t\,s^{-1}$, according to the law $OM = S = 30\pi t^2$ cm.

Determine $\vec{V}_M^a$ and $\vec{W}_M^a$ at t_1=1s, if a=10 cm.

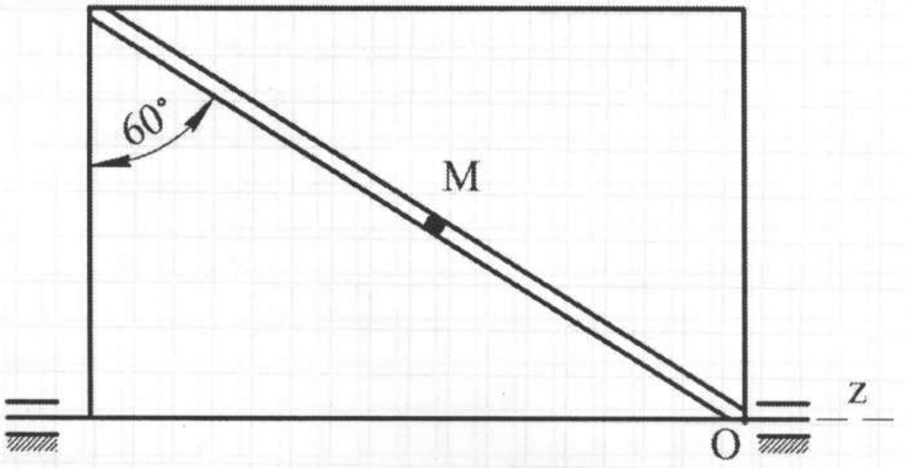
Problem 4.13

The plate rotates about axis Oz with constant angular velocity $\omega = 2t$ rad/s. The point M moves along the circular slot with radiuses R=20 cm. Point position is given by $\overset{\smile}{OM} = S = 5\pi t^2$, cm. At t=1 s determine absolute velocity $\vec{V}_M^a$ and absolute acceleration of the point $\vec{W}_M^a$.



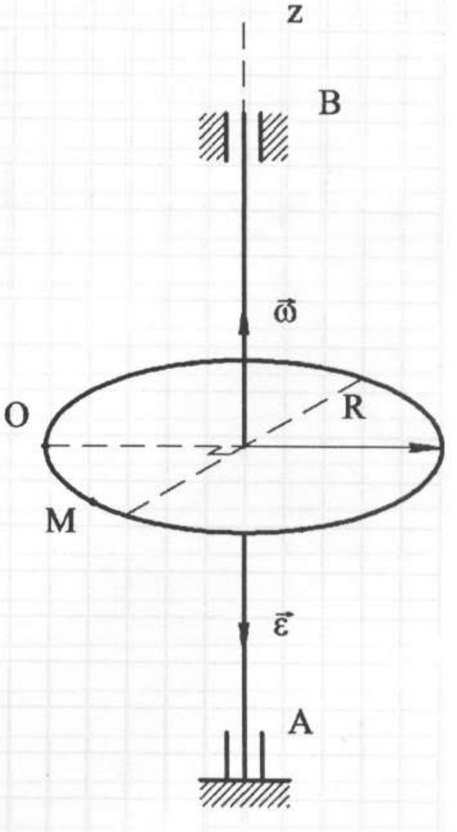
Problem 4.14

The rectangle rotates about axis Oz with angular velocity $\omega = 2t\ rad/s$. The point M moves along the diagonal of the rectangle. Point position is given by OM=2t^2, cm. At t=1 s determine absolute velocity $\vec{V}_M^a$ and absolute acceleration of the point $\vec{W}_M^a$.



Problem 4.15

Disc (R=20 cm) rotates about axis Oz and at t_1=1s has angular velocity $\omega_z = 2\ rad/s$ and angular acceleration $\varepsilon_z = 2\ rad/s^2$. Point M moves along the disc rim. The point position is given by $\breve{OM} = S = 10\pi t^2$ cm. At t=1 s determine absolute velocity $\vec{V}_M^a$ and absolute acceleration of the point $\vec{W}_M^a$.



4.5. List of exam questions

1. Basic conceptions: rigid body, force, force system, rigid body equilibrium. Statics problems. Statics axioms.
2. Moment of force about a center: magnitude, direction.
3. Moment of force about an axis: magnitude, methods of calculation.
4. Couple. Couple vector moment: direction, magnitude. The features of couple.
5. Total vector and total moment of a force system. Methods of total vecto and total moment determination. Statics invariants.
6. Reducing of the general force system. Total vector and total moment. Dependence of the total vector and total moment on the particular cente of reduction selected.
7. Force system resultant. Varignon's theorem.
8. Special force systems. Conditions of equilibrium.
9. Free and constrained body. Constraints and their reactions. Commo types of constraints and their reactions (2-D and 3-D). Constructing o free body diagram.
10. Statically determinate and indeterminate rigid body. The crucial steps i solving equilibrium problem for a single body.
11. Equilibrium of system of rigid bodies. External and internal forces, the fea tures of internal forces. Method of section (method of isolating of con nected system members).
12. Center of gravity and its coordinates. Ways of definition of the center o gravity position.
13. The ways of particle motion representation. Trajectory, velocity and acce leration of a particle in terms of rectangular coordinates x,y,z.
14. The ways of particle motion representation. Velocity and acceleration of a particle in terms of path variables.
15. The ways of particle motion representation. Normal W^n and tangent W accelerations of a particle in terms of rectangular coordinates x,y,z.
16. Conditions of accelerated, decelerated, or uniform motion of a particle. Researching of particle motion (type of motion determination) for differen methods of particle motion representation.
17. The simplest types of rigid body motion: translation, rotation about fixed axis. The features of these motions.
18. Velocity of point in rigid body in plane motion with respect to any base point. Angular velocity of rigid body in plane motion. Angular velocity in-dependence of particular base point selected.
19. Plane motion of rigid body. Ways of particle velocity determination: with respect to a base point (pole), according to the equiprojectivity principle.
20. Plane motion of rigid body. Instantaneous center of zero velocity: defini-tion, ICZV existence condition, the ways of definition of ICZV position.

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21. Plane motion of rigid body. Ways of particle acceleration determination: with respect to a base point (pole), with respect to instantaneous center of zero acceleration
22. A particle motion observed from a system which itself is in plane motion. Basic conceptions: absolute, relative and transport motion. Absolute velocity of a particle.
23. A particle motion observed from a system which itself is in plane *motion*. Basic conceptions: absolute, relative and transport motion. Absolute acceleration of a particle. Coriolis acceleration: magnitude, direction, conditions under which Coriolis acceleration is equal to zero.

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