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STRENGTH ANALYSIS
OF ROTOR BLADE

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Tutorial

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Розглянуто аналіз статичної міцності робочих лопаток компресорів і турбін, який полягає у розрахунку напружень і запасів міцності у найбільш напружених точках за висотою пера. Наведено інформацію про програмне забезпечення для виконання розрахунків на ПЕОМ, а також послідовність і приклади розрахунків. Зміну геометричних характеристик пера за висотою задано за допомогою степеневих функцій, параметри яких визначаються у трьох перерізах – кореневому, середньому та периферійному. Аналіз лопатки турбіни виконано із урахуванням її температури.

Для студентів, які навчаються за напрямком «Авіаційна і ракетно-космічна техніка», при виконанні практичних робіт, підготовці до екзаменів з дисциплін «Конструкція авіаційних двигунів і енергетичних установок», «Двигуни літаків і вертольотів», а також курсових і дипломних проектів.

Reviewers: Doctor of Engineering, Prof. V. Pylyov,
PhD, Associate Prof. A. Bratchenko

Yepifanov, S.
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Analysis of compressor and turbine rotor blade static strength is considered which consists of stresses and safety factors calculation in the most loaded points on length of the blade body. Information about software, recommendations for calculation and examples are given. Variation of blade body geometric parameters on the blade length is determined by exponential functions which parameters are defined in three cross-sections – root, meaning and peripheral. Turbine blade is analyzed taking into account its temperature.

This book is profitable for students studying "Aerospace Engineering" to prepare for practical activities and examinations on disciplines "Construction of Aero Engines and Power Plants", "Engines of Airplanes and Helicopters", and to make course and diploma projects.

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CONTENTS

1 GENERAL INFORMATION

Rotor blades of compressor and turbine are very important parts of gas turbine engine; their reliable operation determines reliability of the engine as a whole.

The compressor rotor blades are of airfoil section and usually designed to give a pressure gradient along their length to ensure that the air maintains a reasonably uniform axial velocity. The higher pressure towards the tip balances out the centrifugal action of the rotor on the airstream. To obtain these conditions, it is necessary to 'twist' the b lade from root to tip to give the correct angle of accidence at each point.

The turbine blades are also of an airfoil shape, designed to provide passages between adjacent blades that give a steady acceleration of the flow up to the ‘throat’, where the area is smallest and the velocity reaches that required at exit to produce the required degree of reaction. The actual area of each blade cross-section is fixed by the permitted stress of material used and by the size of any holes which may be required for cooling purposes. High efficiency demands thin trailing edges to the sections, but a compromise has to be made so as to prevent the blades cracking due to the temperature changes during engine operation. The turbine blade is usually binded using lashing wires, bush or rim shroud to improve its oscillation properties. Peripheral shroud constitute a ring structure which prevents leakage losses; turbine efficiency is increased. Tips of shrouds have ribs of labyrinth seals which operate in combination with abradable metal-ceramic (made of iron-nickel-graphite) or honeycomb insertions in turbine case.

During operation of gas turbine engine, the rotor blades are affected by static, dynamic and temperature loads causing very complex picture of stresses.

In this manual only strength analysis on the blade faces when they are under static load is considered. This load consist of centrifugal forces caused by the blade mass during rotation and gas forces which appear when the blade profile is streamlined by air or gas.

Centrifugal forces cause tensile, bending and torsion strains, gas forces cause bending and torsion strains.

Torsion stresses caused by centrifugal and gas forces in weakly twisted compressor and turbine blades are small and usually are neglected.

Tensile stresses caused by centrifugal forces are the most significant, especially for turbine blades.

Bending stresses are usually lower than tensile ones. If it is necessary to decrease bending caused by gas forces, the blade is designed and installed in such a way that bending moments caused by centrifugal forces are opposite in sign to the moments caused by gas forces and consequently, decrease (balance) the gas forces.

During strength analysis the following assumption are considered:

– blade is considered as a cantilever beam rigidly mounted in the rim of the disk;

– stresses are determined separately by each kind of strains (for strongly twisted blades this assumption is not correct);

– temperature in considered blade section face is set constant; it means that thermal stresses are absent;

– blade is considered rigid and its deformation (blade axis deviation) caused by forces and moments is neglected;

– blade strains supposed to be in the zone of elasticity, it means that stresses in the blade face are lower than the limit of proportionality;

– turbine blade temperature changes only along a length of the blade face (increasing of temperature results in decreasing of material mechanical stress-strain properties).

– goal of blade strength analysis is to determine stresses and safety margin in different cross-sections along a length of the blade; comparison of minimal safety margin with recommended values is a reason for conclusion about blade strength quality.

Usually analysis is carried out in the following order: in blade design section the tensile stresses caused by centrifugal forces and bending stresses caused by gas and centrifugal forces are determined. Maximum stresses in each cross-section are determined by summing up tensile and bending stresses in points maximally distant from the neutral axis of blade’s cross-sections. Then safety margins along the blade length are determined. These safety margins are to be not less than the set values specified by Strength Regulations.

As a rule, design mode is a mode with conditions of maximum rotor rotational speed and maximum air consumption through the engine. These conditions correspond to taking-off and flying with high speed at low altitude in winter conditions. Sometimes, if blade cross-sections gravity center is displaced, such blade needs strength analysis in flight conditions with maximum rotation speed, maximum altitude and minimum flight velocity, as at these conditions the bending moment of centrifugal forces is maximum, but the bending moment of gas forces is minimum.

2 METHOD OF ANALYSIS

2.1 Initial data

For strength analysis of compressor or turbine blade the initial data are taken from gas-dynamic design:

R_r – radius at root section;
R_p – radius at tip (peripheral) cross-section;
L – blade length
p_1, p_2 – gas pressure before and after the blade;
ρ_1, ρ_2 – gas density before and after the blade;
c_{1a}, c_{2a} – axial component of the gas velocity before and after the blade;
c_{1u}, c_{2u} – circumferential component of the gas speed before and after the blade;
z – number of blades in the impeller;
n – rotational speed.

Besides, it is also necessary to have profiles of blade face in three cross-sections (root, mean and peripheral sections). Their parameters are (Table 2.1):

b_r, b_m, b_p – chord of profile in root, mean and peripheral sections;
δ_r, δ_m, δ_p – maximum thickness of blade in root, mean and peripheral sections;
a_r, a_m, a_p – maximum bending deflection of profile centerline in respective sections;
α_r, α_m, α_p – pitch angle in respective sections.

Table 2.1

Blade geometric parameters

Parameter	Root section	Mean section	Peripheral section
b, mm			
δ, mm			
a, mm			
α, radian			

If the blade has a shroud it is necessary to know its volume V_s and coordinates of its center of gravity.

It is necessary to have data concerning the material used in the construction of the blade:

ρ – density;
σ_T^t –- long-term strength for time T and temperature t.

During turbine blade strength analysis one should take into consideration temperature distribution along the blade length.

2.2 Determining geometric characteristic of design blade face cross-sections

At tensile and bending analyses of the blade face it is necessary to have the following geometric characteristics of section:

F – cross-section area;

x_0, x_0 – coordinates of the center of gravity;

I_ξ, I_η, – main central moments of inertia;

W_ξ, W_η – section modulus of bending, determining stresses in three dangerous points A, B, C (points located the outermost from the neutral line, which can be set with enough accuracy from the axis of minimal moment of inertia ξ).

With accurate calculation, the cross-sectional area and area moment of inertia are determined by these profile sections using well known methods [1]. Calculations by approximate formulas are allowed [2]:

$$F = 0{,}693\, B\delta;$$

$$x_0 = 0{,}429\, b;\ y_0 = 0{,}762\, a;$$

$$I_\xi = I_{\min} = 0{,}041\, b\delta^3\left(1+\left(\frac{a}{\delta}\right)^2\right);$$

$$I_\eta = I_{\max} = 0.038 b^3\delta.$$

Section moduli of bending are determined by the moment of inertia and coordinates of points. So, for example, for point A (fig. 2.1) with coordinates ξ_A, and η_A we have:

$$W_{\xi A} = \frac{I_\xi}{\eta_A};\quad W_{\eta A} = \frac{I_\eta}{\xi_A}.$$

Coordinates of points η_A, ξ_A, η_B, ξ_B, η_C, ξ_C are determined by direct measurements on the blade drawing.

Using computed values of inertia moments, section modulus of bending can be determined by formulas [3] quite accurately:

$$W_{\xi A} = W_{\xi B} = \frac{I_\xi}{y_m} = \frac{I_\xi}{0{,}762a};\ W_{\xi C} = \frac{I_\xi}{0{,}218a + 0{,}486\delta};$$

$$W_{\eta A} = 0{,}088 b^2\delta;\quad W_{\eta B} = 0{,}066 b^2\delta;\quad W_{\eta C} = \infty.$$

Besides, variation of F, $W_{\xi A}$, $W_{\xi C}$, $W_{\eta A}$, $W_{\eta B}$ and pitch angle of the profile α along the airfoil face is approximated by a power functions as follows:

$$f = f_0 - Ax^B,$$

where x is current distance to the root section.

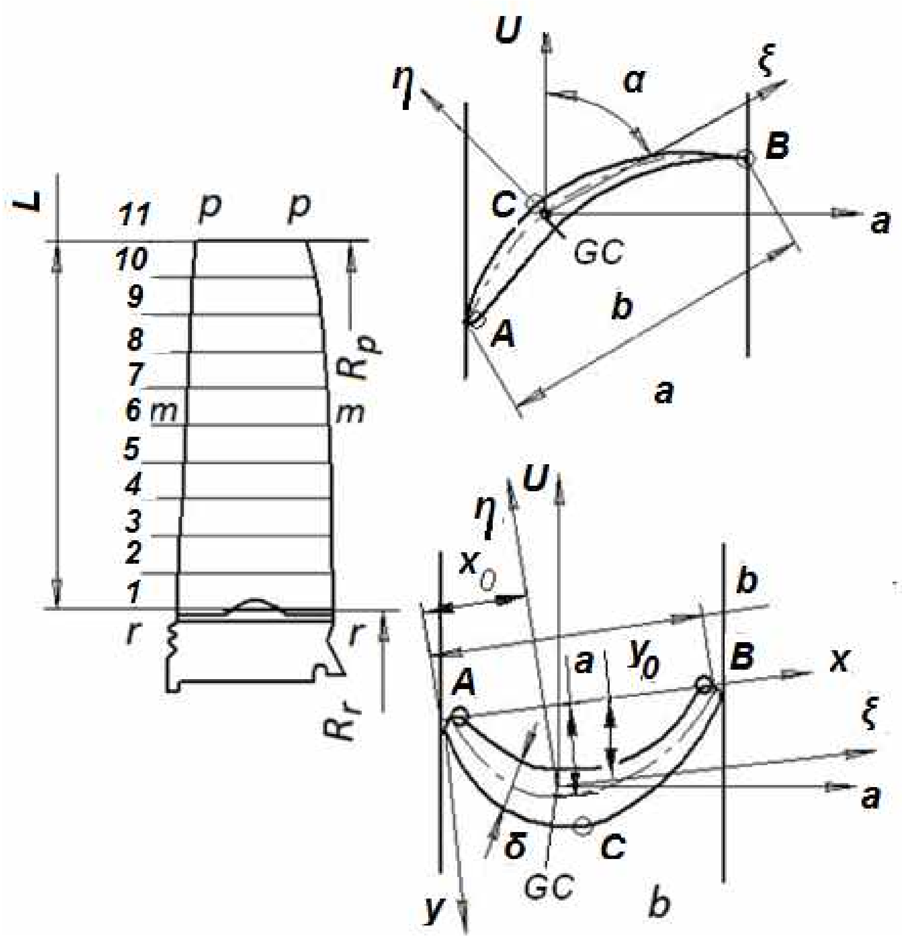

Figure 2.1 – For determining geometric parameters of design sections:
a − compressor blade; b – turbine blade

Coefficients f_C, A and B are determined from the condition that the considered function goes through its three known values (in root, mid and circumferential sections). Then:

$$\boldsymbol{A} = \frac{\boldsymbol{f_r} - \boldsymbol{f_p}}{\boldsymbol{L^B}}; \quad \boldsymbol{B} = \frac{\left(\lg \frac{\boldsymbol{f_r} - \boldsymbol{f_p}}{\boldsymbol{f_r} - \boldsymbol{f_m}}\right)}{\lg 2},$$

where $\boldsymbol{L}$ is the blade length;

$\boldsymbol{f_r}$, $\boldsymbol{f_p}$, $\boldsymbol{f_m}$ – values of the function in root, mean and peripheral sections.

Hence, cross-sectional area along the blade length can be determined by this formula:

$$\boldsymbol{F} = \boldsymbol{F_r} - \boldsymbol{A_F}\boldsymbol{x}^{\boldsymbol{B_F}};$$

$$\boldsymbol{A_F} = \frac{\boldsymbol{F_r} - \boldsymbol{F_p}}{\boldsymbol{L}^{\boldsymbol{B_F}}};$$

$$\boldsymbol{B_F} = \frac{\left(\lg \frac{\boldsymbol{F_r} - \boldsymbol{F_p}}{\boldsymbol{F_r} - \boldsymbol{F_m}}\right)}{\lg 2},$$

where F_r, F_p, F_m − cross-sectional area in root, mid and peripheral sections.

In the same way one can get formulas to determine section modulus of bending and pitch angles.

In 'manual' blade strength analysis usually 3-5 design sections are used, but if a computer is used, this number may be more. It depends on the set goal. In educational and senior thesis it will be enough to divide the airfoil blade face length into 10 equal parts and then there will be 11 sections, the first one in the root and the eleventh one in the periphery.

2.3 Analysis of tension caused by centrifugal forces

In general case the centrifugal force acting in the designed section i is generated by rotated mass of a part of the blade face situated above this section:

$$P_{c\,i} = \rho\omega^2 \int_{R_i}^{R_p} F \cdot r \cdot dr \,, \qquad (2.1)$$

where ρ – blade material density;

$\omega = \dfrac{\pi n}{30}$ – rotor angular velocity;

$F = F_r - A_F x^{B_F}$ - area on radius $r = R_r + x$.

Substituting the values of F and r into (2.1), taking the integral and re-arranging it, is possible to get a design formula for computing:

$$P_{c\,i} = \rho\omega^2 \int_{R_i}^{R_p} \left(F_k - A_F x^{B_F}\right) \cdot \left(R_r + x\right) dx = C_1\left[C_2 - C_3 x_i + C_4 x_i^{B_F+1} - C_5 x_i^2 + C_6 x_i^{B_F+2}\right], \qquad (2.2)$$

where

$$C_1 = \rho\omega^2;$$

$$C_2 = F_r L\left(R_r + \frac{L}{2}\right) - A_F L^{B_F+1}\left(\frac{R_K}{B_F+1} + \frac{L}{B_F+2}\right);$$

$$C_3 = F_r R_r;$$

$$C_4 = \frac{A_F R_r}{B_F+1};$$

$$C_5 = \frac{F_r}{2};$$

$$C_6 = \frac{A_F}{B_F+2}$$

If the blade has a shroud, it is necessary to take into account the centrifugal force caused by its mass:

$$P_s = m_s R_s \omega^2 = \rho V_s R_s \omega^2,$$

where V_s – shroud volume;

$\boldsymbol{R_s}$ – radius of shroud center of gravity (assuming equal to peripheral radius of the blade face).

Tensile strain in the designed section ***i*** of the blade face:

$$\sigma_{t\ i} = \frac{P_{c\ i} + P_s}{F_i}.$$

2.4 Analysis of bending caused by gas forces

For analysis of rotor blades bending we shall use the "***aur***" coordinate system (Fig. 2.2) , where ***"a"*** is axial direction, "***u***" – circumferential and ***"r"*** – radial. Axis ***"a"*** coincides with the axis of rotation, axis ***"r"*** is directed along the radius and axis ***"u"*** lies in the plane of rotation.

The "u_i" and "a_i" axes parallel to axes "***u***" and "***a***" go through the center of gravity (CG) of ***i*** blade section. The main central axes of inertia are ξ and η, whereas ξ is the axis with lower blade rigidity (is drawn parallel to the chord). ξ and ***u*** axes form α angle, which determines the blade profile position.

2.4.1 Bending moments caused by the gas forces

Analysis of rotor blade bending caused by gas forces is performed in the following way: one finds location of the bending moments acting on the designed blade section in the plane of rotation M_a and in the axial plane M_u. Knowing the moments and pitch angle of the blade airfoil, one defines the bending moments relative to the main central axes of inertia M_ξ and M_η, and also bending stresses caused by the gas forces in the points A, B, C located most far from the neutral section line which virtually coincides with ξ axis.

The formula below defines the gas forces acting on the rotor blade per length unit (load rate):

– in axial plane:

$$p_a = \frac{2\pi r}{z}\left(p_2 - p_1 + c_{2a}^2\rho_2 - c_{1a}^2\rho_1\right); \qquad (2.3)$$

– in plane of rotation:

$$p_u = \frac{2\pi r}{z}\left(c_{2a}\rho_2 w_{2u} \pm c_{1a}\rho_1 w_{1u}\right), \qquad (2.4)$$

where r – radius of section;

ρ_1, ρ_2 – gas density before and after the blade;

c_{1a}, c_{2a} – axial gas speed before and after the blade;

w_{1u}, w_{2u} – linear components of gas velocity while it is moving before and after the blade;

p_1, p_2 – gas pressure before and after the blade.

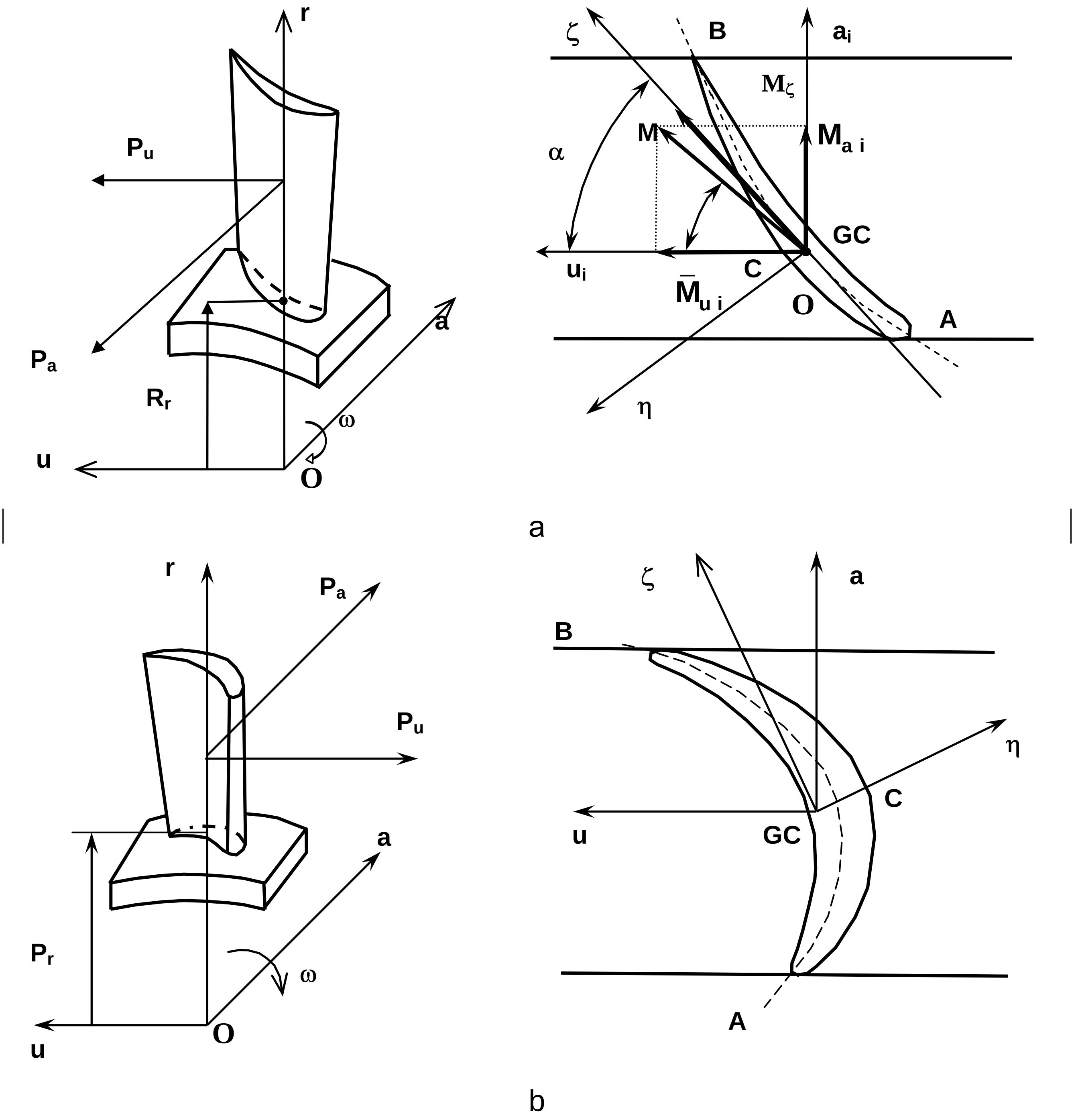

Figure 2.2 – Rotor blade bending analysis:
a – compressor blade; b – turbine blade

All values necessary for determination of $\boldsymbol{p_a}$ and $\boldsymbol{p_U}$ are known from gas-dynamic design.

For turbine blade we put “+” in formula (2.4) and “-” for compressor blade.

In general case, value of parameter $\boldsymbol{p_u}$ is varied on the blade radius, it depends on the blade profiling law. When profiling is carried out according to the law of circulation constancy, the value of $\boldsymbol{p_u}$ is constant over the height of the blade face; if it’s carried out according to other laws – the value is variable.

Nevertheless, for practical blade design the load rate in the plane of rotation can be considered constant along the blade length and equal to the load rate present in mean radius of the blade, i.e. $\boldsymbol{p_u} = \boldsymbol{p_{u\,m}} = \text{const}$.

Load rate in axial plane:

$$\boldsymbol{p_a} \approx \frac{2\pi \boldsymbol{r}}{\boldsymbol{z}}\left(\boldsymbol{p}_1 - \boldsymbol{p}_2\right), \qquad (2.5)$$

where $(\boldsymbol{p_1}$-$\boldsymbol{p_2})$ – pressure difference on mean radius, depends on engine operation mode.

It is clear from formula (2.5) that $\boldsymbol{p_a}$ increases linearly from the blade root section to the tip section:

$$\boldsymbol{p_a} = \boldsymbol{p_{a\,\mathrm{r}}} + (\boldsymbol{p_{a\,\mathrm{p}}} - \boldsymbol{p_{a\,\mathrm{r}}})\frac{\boldsymbol{x}}{\boldsymbol{L}},$$

where $\boldsymbol{p_{a\,r}}, \boldsymbol{p_{a\,p}}$ – load rates in root and tip sections, accordingly;
$\boldsymbol{X}$ – current distance to root section;
$\boldsymbol{L}$ – blade length.

Bending moments that arise due to gas forces in i-section and act in circumferential and axial directions are as follows:

$$\boldsymbol{M_{u\,g\,i}} = \boldsymbol{p_u}\int_{\boldsymbol{x_i}}^{\boldsymbol{L}} (\boldsymbol{x} - \boldsymbol{x_i})\, d\boldsymbol{x} = \frac{\boldsymbol{p_u}}{2}(\boldsymbol{L} - \boldsymbol{x_i})^2;$$

$$\boldsymbol{M_{a\,\mathrm{g}\,i}} = \int_{\boldsymbol{x_i}}^{\boldsymbol{L}}\left[\boldsymbol{p_{a\,\mathrm{r}}} + (\boldsymbol{p_{a\,\mathrm{p}}} - \boldsymbol{p_{a\,\mathrm{r}}})\frac{\boldsymbol{x}}{\boldsymbol{L}}\right](\boldsymbol{x} - \boldsymbol{x_i})\, d\boldsymbol{x} = \boldsymbol{C}_{13} - \boldsymbol{C}_{14}\boldsymbol{x} + \boldsymbol{C}_{15}\boldsymbol{x}^2 - \boldsymbol{C}_{16}\boldsymbol{x}^3, \qquad (2.6)$$

where

$$\boldsymbol{C}_{13} = \frac{\boldsymbol{L}^2(2\boldsymbol{p_{a\,\mathrm{p}}} + \boldsymbol{p_{a\,\mathrm{r}}})}{6}; \quad \boldsymbol{C}_{15} = \frac{\boldsymbol{p_{a\,\mathrm{r}}}}{2};$$

$$\boldsymbol{C}_{14} = \frac{\boldsymbol{L}(\boldsymbol{p_{a\,\mathrm{p}}} + \boldsymbol{p_{a\,\mathrm{r}}})}{2}; \quad \boldsymbol{C}_{16} = \frac{\boldsymbol{p_{a\,\mathrm{r}}} - \boldsymbol{p_{a\,\mathrm{p}}}}{6\boldsymbol{L}}.$$

2.4.2 Bending moments caused by centrifugal forces

The bending stresses in the blade caused by gas forces may be some compensated by displacement of the blade axis in circumferential direction. The blade axis is geometrical place of centers of gravity of all cross-sections. Usually this axis is radial and coincides with axis “r” in fig. 2.2. For providing compensation the blade axis is deflected from radial direction aside action of gas forces. Deflection of the projection of this axis from radial direction on the ***“uOr”*** and *“**aOr**”* planes is called offset. The offset of the GC produce bending moment caused by centrifugal forces.

Let ***u*** represents offset of GC in circumferential direction and ***a*** –offset in axial direction. Then bending moments in i-section in circumferential and axial directions are as

$$M_{u\,\mathrm{c}\,i} = \rho\omega^2 \int_{R_i}^{R_p} F \cdot r \cdot \left(u \frac{r_i}{r} - u_i \right) dr \;. \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (2.7)$$

$$M_{a\,\mathrm{c}\,i} = \rho\omega^2 \int_{R_i}^{R_p} F \cdot r \cdot \;\; a - a_i \;\; dr \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (2.8)$$

The gravity centers of the blade sections may be quite precisely assumed to be located on segment of the straight line. Then GC offsets in circumferential and axial directions are represented as:

$$u = A_u x ;\; a = A_a x \,,$$

where A_u, A_a are relative GC offsets in circumferential and axial directions.

Then $u_p = A_u L$ and $a_p = A_a L$ are circumferential and axial offsets in peripheral cross-section.

Let's substitute these expressions into formulas (2.7), (2.8) for bending moments and transform them.

$$M_{u\,\mathrm{c}\,i} = \rho\omega^2 A_u \int_{x_i}^{L} \;\; F_r - A_F x^{B_F} \;\;\; R_r + x \;\; \cdot \left(\frac{R_r + x_i}{R_r + x} x - x_i \right) dx =$$

$$= C_1 A_u \left(C_7 R_r + (C_7 - C_2) x_i + \frac{C_3 x_i^2}{2} + C_8 x_i^{B_F + 2} \right), \qquad (2.9)$$

where coefficients C_1, C_2, C_3, C_4 and C_5 keep their previous values;

$$C_7 = L^2 \left(C_5 - \frac{A_F L^{B_F}}{B_F + 2} \right);$$

$$C_8 = A_F R_r \left(\frac{1}{B_F + 2} - \frac{1}{B_F + 1} \right).$$

$$M_{a\,\mathrm{c}\,i} = \rho\omega^2 A_a \int_{x_i}^{L} \;\; F_r - A_F x^{B_F} \;\;\; R_r + x \;\; \cdot \;\; x - x_i \;\; dx =$$

$$= C_1 A_a \left(C_9 - C_2 x_i + \frac{C_3 x_i^2}{2} + C_8 x_i^{B_F + 2} + C_{10} x_i^3 + C_{11} x_i^{B_F + 3} \right), \qquad (2.10)$$

where the new coefficients are equal to

$$C_9 = L^2 \left(F_r \left(\frac{L}{3} + \frac{R_r}{2} \right) - A_F L^{B_F} \left(\frac{L}{B_F + 3} + \frac{R_r}{B_F + 2} \right) \right);$$

$$C_{10} = \frac{F_r}{6};$$

$$C_{11} = A_F \left(\frac{1}{B_F + 3} - \frac{1}{B_F + 2} \right).$$

If the blade has a shroud, then its mass produce centrifugal forces and these forces create bending moments acting in circumferential and axial directions in the designed section as

$$\boldsymbol{M}_{\boldsymbol{u}\,\mathrm{s}} = \boldsymbol{P_s}\left(\frac{\boldsymbol{R_r}+\boldsymbol{x}}{\boldsymbol{R_r}+\boldsymbol{L}}\boldsymbol{u_p} - \boldsymbol{u_s}\frac{\boldsymbol{x}}{\boldsymbol{L}}\right);$$

$$\boldsymbol{M}_{\boldsymbol{a}\,\mathrm{s}} = \boldsymbol{P_s}\left(\boldsymbol{a_p} - \boldsymbol{a_s}\frac{\boldsymbol{x}}{\boldsymbol{L}}\right),$$

where $\boldsymbol{P_s}$ – centrifugal force produced by the mass of shrouded blade;
$\boldsymbol{u_s}$ – offset of the shroud CG in circumferential direction;
$\boldsymbol{a_s}$ – offset of the shroud CG in axial direction.

2.4.3 Determination of total bending moment and bending moments in relation to main central axis of inertia

Since the moments caused by centrifugal forces acting on the blade face are directed oppositely to gas forces, the total bending moment in *i*-section can be calculated as

$$\boldsymbol{M_i} = \sqrt{\left(\boldsymbol{M}_{\boldsymbol{a}\,\mathrm{g}\,\boldsymbol{i}} - \boldsymbol{M}_{\boldsymbol{a}\,\mathrm{c}\,\boldsymbol{i}} - \boldsymbol{M}_{\boldsymbol{a}\,\mathrm{s}}\right)^2 + \left(\boldsymbol{M}_{\boldsymbol{u}\,\mathrm{g}\,\boldsymbol{i}} - \boldsymbol{M}_{\boldsymbol{u}\,\mathrm{c}\,\boldsymbol{i}} - \boldsymbol{M}_{\boldsymbol{u}\,\mathrm{s}}\right)^2}.$$

Vector of bending moment acting in the designed section forms an angle $\boldsymbol{\gamma_i}$ with the plane of rotation:

$$\gamma_i = \boldsymbol{arctg}\,\frac{\boldsymbol{M}_{\boldsymbol{a}\,\mathrm{g}\,\boldsymbol{i}} - \boldsymbol{M}_{\boldsymbol{a}\,\mathrm{c}\,\boldsymbol{i}} - \boldsymbol{M}_{\boldsymbol{a}\,\mathrm{s}}}{\boldsymbol{M}_{\boldsymbol{u}\,\mathrm{g}\,\boldsymbol{i}} - \boldsymbol{M}_{\boldsymbol{u}\,\mathrm{c}\,\boldsymbol{i}} - \boldsymbol{M}_{\boldsymbol{u}\,\mathrm{s}}}.$$

Bending moments in relation to the main central axis of inertia of the designed section is determinate as the projection of the discovered vector of total bending moment in the direction of these axes in *i*-section:

$$\boldsymbol{M}_{\xi\,\boldsymbol{i}} = \boldsymbol{M}_{\boldsymbol{i}}\cdot\mathbf{cos}\left(\alpha_i - \gamma_i\right);$$

$$\boldsymbol{M}_{\eta\,\mathrm{i}} = \boldsymbol{M}_{\boldsymbol{i}}\cdot\mathbf{sin}\left(\alpha_i - \gamma_i\right).$$

2.4.4 Bending stress determination

Bending stresses in each point of the design cross-section are determined as

$$\boldsymbol{\sigma_{b\,i}} = \pm\frac{|\boldsymbol{M}_\xi|}{|\boldsymbol{W}_\xi|} \pm \frac{|\boldsymbol{M}_\eta|}{|\boldsymbol{W}_\eta|}.$$

To simplify calculations values of bending moments and support moments are taken on modulus (without sign).

So, in point A:

$$\boldsymbol{\sigma_{b\,A}} = \pm\frac{|\boldsymbol{M}_{\xi}|}{|\boldsymbol{W}_{\xi\,A}|} \pm \frac{|\boldsymbol{M}_{\eta}|}{|\boldsymbol{W}_{\eta\,A}|},$$

in point B:

$$\sigma_{b\ B} = \pm \frac{|M_{\xi}|}{|W_{\xi\ B}|} \pm \frac{|M_{\eta}|}{|W_{\eta\ B}|},$$

in point C:

$$\sigma_{b\ C} = \pm \frac{|M_{\xi}|}{|W_{\xi\ C}|} \pm \frac{|M_{\eta}|}{|W_{\eta\ C}|}.$$

At calculation of bending stress the sign determines a kind of deformation of blade filaments. So, if the blade filaments are spread, bending stresses have a sign "+", if they are compressed, a sign is "-". Let's note, that gas loads initiate on edges of the blade profile (in points A and B) tension stress and on a back of the profile (in a point C) - compressive stress. Thus, at definition of bending stress using the above formulas the signs should be taken according to Table 2.2.

Table 2.2

Blade	Point				
	A		B		C
	M_{ξ}/W_{ξ}	M_{η}/W_{η}	M_{ξ}/W_{ξ}	M_{η}/W_{η}	M_{ξ}/W_{ξ}
compressor	+	-	+	+	-
turbine	+	+	+	-	-

2.5 Determination of safety factors

When calculating the blade safety factors, it is necessary to take into account both tensile-compressive and bending stress in each point of design cross-section:

$$\sigma_{\Sigma\ i} = \sigma_{t\ i} + \sigma_{b\ i}.$$

For example, in a point A of i-th cross-section

$$\sigma_{\Sigma\ A\ i} = \sigma_{t\ i} + \sigma_{b\ A\ i}.$$

Analogical expressions may be written for points B and C.

For compressor (cold) blades safety factor in each point of design cross-section

$$K_i = \frac{\sigma_{\lim}}{\sigma_{\Sigma\ i}},$$

where $\sigma_{\lim}$ - strength limit.

For turbine and last stages compressor blades safety factor in each point of design cross-section

$$K_{Ti} = \frac{\sigma^{t}_{T.i}}{\sigma_{\Sigma.i}},$$

where $\sigma^{t}_{T\ i}$ - long-term strength limit of material taking into account temperature t in a given cross-section and operation time T.

According to Norms of Strength minimal safety factor of static strength in a blade body is to be not less than 1,25…1,30, and for compressor rotor blades – not less than 1,5.

2.5.1 Turbine blade temperature determination

It is necessary to know temperature of turbine blade in its different cross-sections for setting strength limit.

For non-cooled blades a temperature on mean radius may be determined with precision enough as

$$\boldsymbol{t_{b\,m}} = \left(\boldsymbol{t_m^*} - \frac{\boldsymbol{c_1^2}}{2300}\right) + 0{,}8\frac{\boldsymbol{w_1^2}}{2300},$$

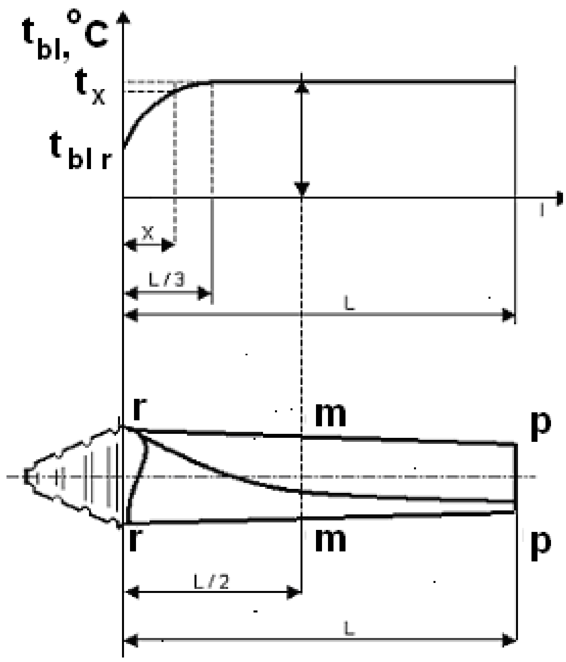

Figure 2.3 – Variation of a turbine blade temperature on length

where $\boldsymbol{t_m^*}$ – gas stagnation temperature on mean radius at the inlet to turbine rotor, $^\circ \boldsymbol{C}$;

$\boldsymbol{c_1}$, $\boldsymbol{w_1}$ – absolute and relative velocities on a mean radius at the inlet to turbine rotor, m/s.

Because of a heat transmission from blade to disk by heat conductivity, blade temperature approximately at one third of length near root significantly decreases (Fig. 2.3). Usually blade temperature in root section is

$$\boldsymbol{t_{b\,r}} = \boldsymbol{t_{b\,m}} - 100\ldots150^\circ\boldsymbol{C}\ .$$

Approximately it is possible to consider that at two thirds of blade length (from peripheral cross-section) temperature is constant, and at one third (near the root) varies under the law of cubic parabola:

$$\boldsymbol{t_b} = \boldsymbol{t_{b.m}} - \frac{\boldsymbol{t_{b.m}} - \boldsymbol{t_{b.r}}}{\left(\boldsymbol{L}/3\right)^3}\left(\frac{\boldsymbol{L}}{3} - \boldsymbol{x}\right)^3,$$

where $\boldsymbol{L}$ – length of a blade body;

$\boldsymbol{x}$ – distance from root section to design $\left(\boldsymbol{x} \le \frac{\boldsymbol{L}}{3}\right)$.

Temperature of cooled blade on mean diameter taking into account cooling is to be assumed as $\boldsymbol{t_{bl}} \le 850\ldots900\ ^\circ\boldsymbol{C}$. Then temperature is distributed along blade in accordance with Fig. 2.3.

3 EXECUTION ORDER

Described method of blade strength analysis is convenient for computing as geometric properties of design sections (areas, moments of inertia etc.), forces and bending moments are represented as power relations.

Recommended order of analysis:

1. Determine design mode for blade strength analysis and design value of rotation speed ***n.*** Choose blade material, find its density ρ and strength limit $\sigma_{\mathbf{lim}}$ or long-term strength limit $\sigma_T^t = f(r)$. To determine the long-term strength limit there is necessary to calculate preliminary the temperature distribution on length of blade. Values of long-term strength limit for different materials are to be find in material properties manual.
2. Using drawings of three blade sections (root, mean and peripheral) there is necessary to determine geometric parameters – values of chord $\boldsymbol{b}$, maximal thickness δ, sag of profile $\boldsymbol{a}$ and pitch angle α, and write them in table 2.1.
3. If the blade is equipped with shroud, its volume V and center of gravity coordinates u_s and a_s.
4. For first approximation (first attempt of calculation) there is reasonable not to displace center of gravity and to set u_s=0, a_s=0. If obtained bending stresses will exceed tolerant then second calculation is to be executed after setting displacements of sections gravity centers. Usually these displacements are set as relative fractions of the blade length in limits 0,01…0,035.
5. Using results of thermodynamic design, calculate values of gas forces intensity (load rate) on plane of rotation and on axial plane (p_u, $p_{a\,r}$, $p_{a\,p}$) using formulas (2.3), (2.4).

   These data are initial ones for computations.

Order of computing:

1. Find areas and modulus of bending in three sections (root, mean and peripheral): F, $W_{\xi A}$, $W_{\xi B}$, $W_{\xi C}$, $W_{\eta A}$, $W_{\eta B}$.
2. Calculate coefficients:
   - C_2, C_3, C_4, C_5 using formula (2.2) for $P_{c\,i}$;

– $\boldsymbol{C_7, C_8}$ using formula (2.9) for $\boldsymbol{M_{u\,c\,i}}$;

– $\boldsymbol{C_9, C_{10}, C_{11}}$ using formula (2.10) for $\boldsymbol{M_{a\,c\,i}}$;

– $\boldsymbol{C_{13}, C_{14}, C_{15}, C_{16}}$ using formula (2.6) for $\boldsymbol{M_{a\,g\,i}}$;

3. For ten set design sections of the blade body determine geometric parameters and acting moments:
   – F, $\boldsymbol{W_{\xi\,A}}$, $\boldsymbol{W_{\xi\,C}}$, $\boldsymbol{W_{\eta\,A}}$, $\boldsymbol{W_{\eta\,B}}$, α and gravity center displacement $\boldsymbol{u}$ and $\boldsymbol{a}$;
   – bending moments acting in circumferential and axial directions – from gas forces $\boldsymbol{M}_{a\,g}$, $\boldsymbol{M}_{u\,g}$, from centrifugal force of the blade body $\boldsymbol{M}_{a\,c}$, $\boldsymbol{M}_{u\,c}$; total bending moment M, its direction – angle γ and projections of total moment $\boldsymbol{M_\xi}$ and $\boldsymbol{M_\eta}$, acting on planes of minimal and maximal rigidity of each design section.
4. In each design section calculate stresses: tensile $\boldsymbol{\sigma_t}$ and bending (in points A, B, C) $\boldsymbol{\sigma_{b\,A}}$, $\boldsymbol{\sigma_{b\,B}}$, $\boldsymbol{\sigma_{b\,C}}$, total stresses $\boldsymbol{\sigma_{\Sigma\,A}}$, $\boldsymbol{\sigma_{\Sigma\,B}}$, $\boldsymbol{\sigma_{\Sigma\,C}}$ and safety factors $\boldsymbol{K_A, K_B, K_C}$.
5. Draw graphic representing variation of stresses and safety factors in points A, B, C on the blade length (fig. 3.1) and make conclusion about static strength of blade body (weather the obtained safety factors satisfy or not the Strength Requirements).

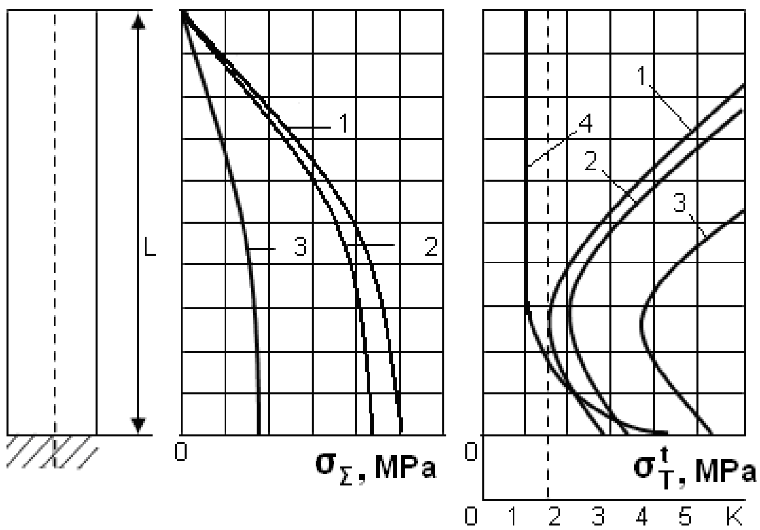

Figure 3.1 − Variation of stress, strength limit and safety factor on the blade length: 1 – total stress and safety factor in point A, 2 – total stress and safety factor in point B, 3 – total stress and safety factor in point C, 4 – long-term strength limit

4 SOFTWARE FOR BLADE STRENGTH ANALYSIS

4.1 Software structure

The software STABLADE realizing this task consists of interface and functional program:

1) interface program serves data input and output;

2) functional program for calculation the blade strength parameters.

Interface program is made using Delphi tools, functional program is realized using Fortran language.

The above mentioned programs communicate through one data file of digital format. Name of this file is set by User in initial dialog. First part of this file contains initial data. After first execution of the program second part is added to this file containing results.

4.2 List of identificators

Parameter	Designation in formulas	Identificator
Long-term strength limit, MPa	σ_T^t	SPT
Blade material density, kg\m^3	ρ	RO
Volume of shroud, m^3	V_s	VP
Shroud gravity center circumferential offset, m	u_s	UPP
Shroud gravity center axial offset, m	a_s	APP
Blade cross-section gravity center circumferential relative offset (in relation to the blade length)	A_u	AU
Blade cross-section gravity center axial relative offset (in relation to the blade length)	A_a	AA
Root radius, m	R_r	RK
Blade length, m	L	CL
Rotor rotation speed, rpm	n	EN
Gas forces intensity, N\m: – circumferential – axial in root section – axial in peripheral section	 p_u $p_{a\,r}$ $p_{a\,p}$	 PU PAK PAP
Blade chord, m	b	B
Maximal thickness of blade profile in section	d	D
Maximal sag of profile centerline in section	a	AP
Blade profile pitch angle in section	α	AL

5 RECOMMENDATIONS TO CALCULATIONS

5.1 Order of calculations

1. Switch PC on.
2. Find folder with executive files of Chair #203, enter and find the file STABLADE.EXE.
3. If the above mentioned folder is stored in a disc which is closed for writing then copy the executive file to other disc opened for Users.
4. Start STABLADE. EXE.
5. Enter initial data taking special attention to engineering units of values to be input (see chapter 5.2). After portion of data is input, the program checks them and asks for repeating input if wrong data are detected.
6. When all initial data are input, the data processing begins automatically. Results are displayed and stored in file which name is displayed also.
7. Analyze correctness of results and save them.
8. Print results or make electronic copy to use them in further preparing final report.

5.2 Initial data input

Initial data are input in dialog mode. List of input parameters and examples are represented in Appendixes 1, 4: Appendix 1 – for compressor blade, Appendix 4 – for turbine blade.

5.3 Calculation results

Results are stored in file and displayed. Examples of output data are represented in Appendixes 2, 3, 5, 6: Appendixes 2,3 – for compressor blade, Appendixes 5, 6 – for turbine blade.

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Appendix 1

STRENGTH ANALYSIS OF ROTOR BLADE BODY

Name of executor:	Jameson J., 232 gr.
Engine unit:	compressor
Material:	VT-3
BLADE GEOMETRIC PARAMETERS:	
Type of blade	solid
Blade length, m:	0.1183
Root radius, m:	0.2013
Shroud radius, m:	0.0
Shroud volume, m^3:	0.0
Blade chord, m:	
root section:	0.04
mean section:	0.04
peripheral section:	0.04
Maximal thickness of blade profile, m:	
root section:	3.2e-3
mean section:	2.4e-3
peripheral section:	1.6e-3
Maximal sag of profile centerline, m:	
root section:	0.0035
mean section:	0.00275
peripheral section:	0.002
Blade profile pitch angle (rad):	
root section:	1.171
mean section:	0.93
peripheral section:	0.696
Shroud gravity center circumferential offset, m:	0
Shroud gravity center axial offset, m:	0
Blade cross-section gravity center circumferential relative offset (in relation to the blade length):	5.0e-3
Blade cross-section gravity center axial relative offset (in relation to the blade length):	2.5e-3
LOADINGS	
Gas forces intensity, N\m:	
circumferential:	554
axial in root section:	692
axial in peripheral section:	1093
Rotor rotation speed, rpm:	10457
PROPERTIES OF MATERIAL	
Density, kg\m^3	4530

Long-term strength (array of 11 values):

N	1	2	3	4	5	6	7	8	9	10	11
σ_{lg}	950	950	950	950	950	950	950	950	950	950	950

Appendix 2

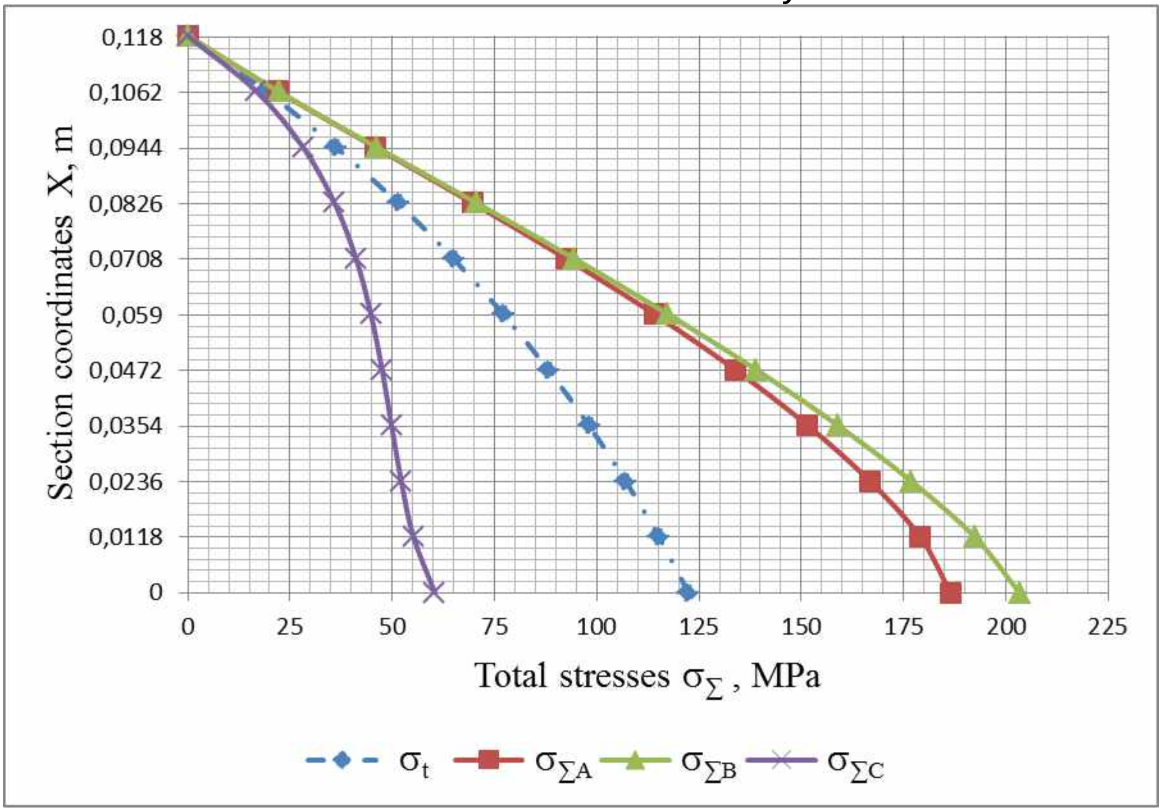
Results of calculations

Section N	X, m	F, m^2	J_{min}, m^4	σ_t, MPa	Bending stresses σ_b, MPa		
					σ_{bA}	σ_{bB}	σ_{bC}
1	0	0,0000887	1,18E-10	122,36	64,0445	80,9735	-61,9748
2	0,01183	0,0000843	9,39E-11	115,1369	63,9319	77,2276	-59,9889
3	0,02366	0,0000798	8,09E-11	107,0697	59,7239	69,8538	-54,8922
4	0,03549	0,0000754	7,02E-11	98,0988	53,4765	60,8982	-48,2924
5	0,04732	0,000071	6,09E-11	88,1498	45,739	50,9069	-40,6528
6	0,0592	0,0000665	5,24E-11	77,1286	36,9564	40,3156	-32,3567
7	0,071	0,0000621	4,459E-11	64,9138	27,5989	29,5781	-23,8091
8	0,0828	0,0000577	3,722E-11	51,3472	18,2345	19,2359	-15,4938
9	0,0946	0,0000532	3,024E-11	36,2172	9,6254	10,0122	-8,0461
10	0,1065	4,879E-05	2,359E-11	19,236	2,908	2,9874	-2,3859
11	0,1183	4,435E-05	1,721E-11	0	0	0	0

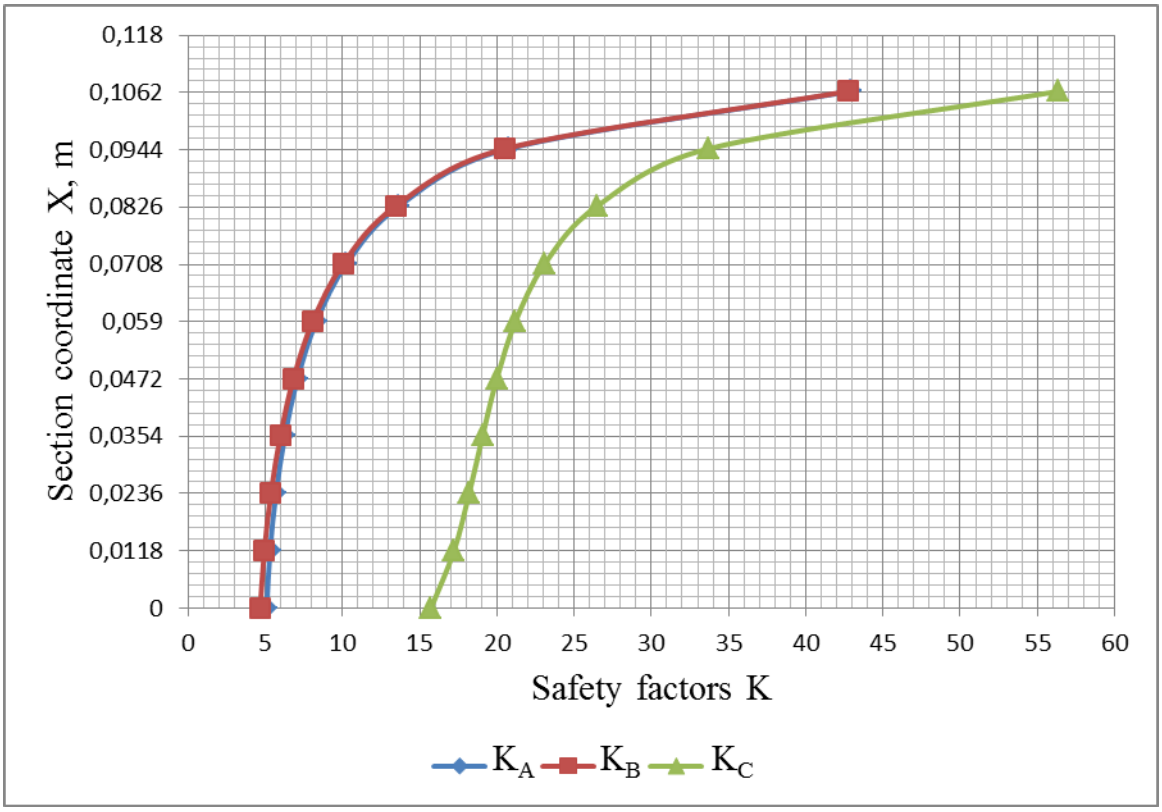
Section N	Total stresses σ_Σ, MPa			Safety factors K		
	$\sigma_{\Sigma A}$	$\sigma_{\Sigma B}$	$\sigma_{\Sigma C}$	K_A	K_B	K_C
1	186,4046	203,3335	60,3851	5,0964	4,6721	15,7324
2	179,0688	192,3645	55,1479	5,3052	4,9385	17,2264
3	166,7936	176,9235	52,1774	5,6957	5,3696	18,2071
4	151,5752	158,997	49,8062	6,2675	5,975	19,0739
5	133,8888	139,0567	47,4969	7,0954	6,8317	20,0013
6	114,085	117,4442	44,7718	8,3271	8,0889	21,2187
7	92,5127	94,4919	41,1046	10,2689	10,0538	23,1118
8	69,5817	70,583	35,8533	13,653	13,4593	26,4969
9	45,8427	46,2294	28,171	20,7231	20,5497	33,7226
10	22,144	22,2234	16,85	42,9011	42,7477	56,3799
11	0	0	0	∞	∞	∞

Appendix 3

Distribution of stresses and safety factors



Total stresses dependent on section coordinates



Safety factors dependent on section coordinates

Appendix 4

STRENGTH ANALYSIS OF ROTOR BLADE BODY

Name of executor:	Jameson J., 232 gr.
Engine unit:	turbine
Material:	GS-6K
BLADE GEOMETRIC PARAMETERS:	
Type of blade	solid
Blade length, m:	0.041
Root radius, m:	0.2515
Shroud radius, m:	0.2925
Shroud volume, m^3:	3.0e-7
Blade chord, m:	
root section:	0.0247
mean section:	0.0247
peripheral section:	0.0247
Maximal thickness of blade profile, m:	
root section:	6.43e-3
mean section:	5.93e-3
peripheral section:	5.44e-3
Maximal sag of profile centerline, m:	
root section:	0.0038
mean section:	0.0035
peripheral section:	0.0034
Blade profile pitch angle (rad):	
root section:	0.837
mean section:	0.767
peripheral section:	0.733
Shroud gravity center circumferential offset, m:	0
Shroud gravity center axial offset, m:	0
Blade cross-section gravity center circumferential relative offset (in relation to the blade length):	0
Blade cross-section gravity center axial relative offset (in relation to the blade length):	0
LOADINGS	
Gas forces intensity, N\m:	
circumferential:	2643
axial in root section:	7762
axial in peripheral section:	9027
Rotor rotation speed, rpm:	13970
PROPERTIES OF MATERIAL	
Density, kg\m^3	8200

Long-term strength (array of 11 values):

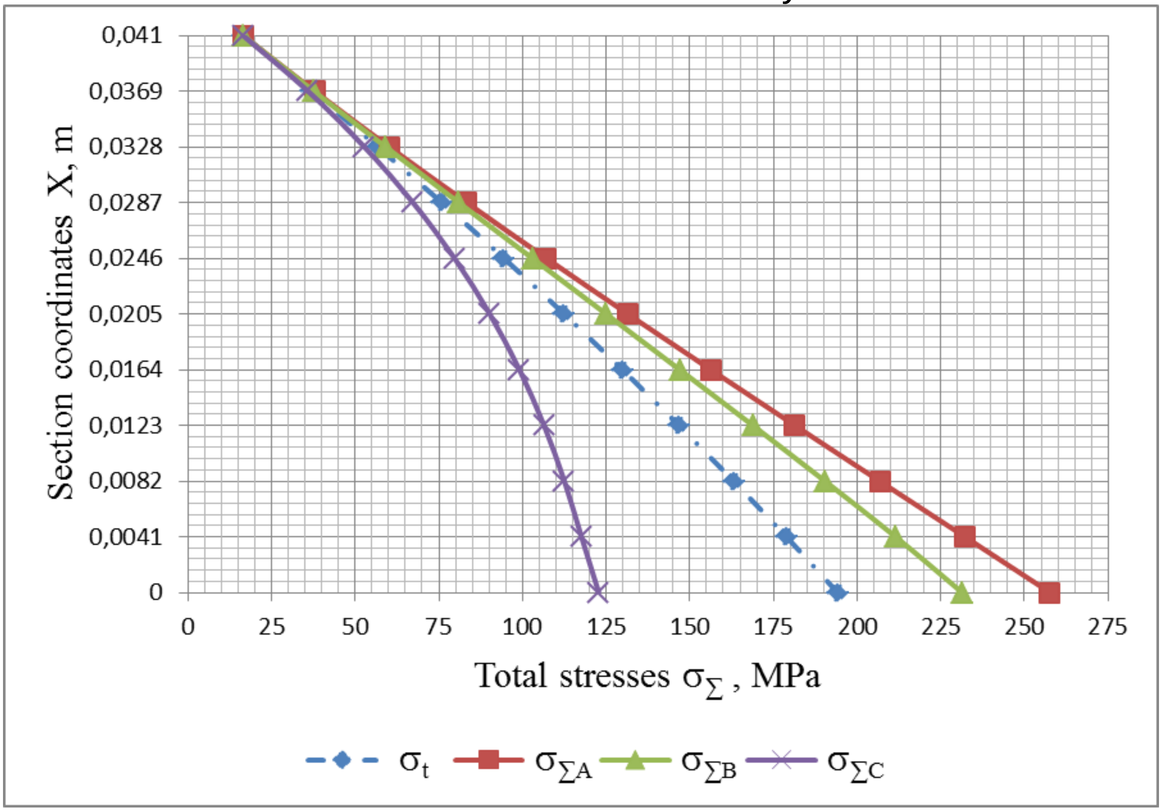
N	1	2	3	4	5	6	7	8	9	10	11
σ_{lg}	450	385	370	360	360	360	360	360	360	360	360

Results of calculations

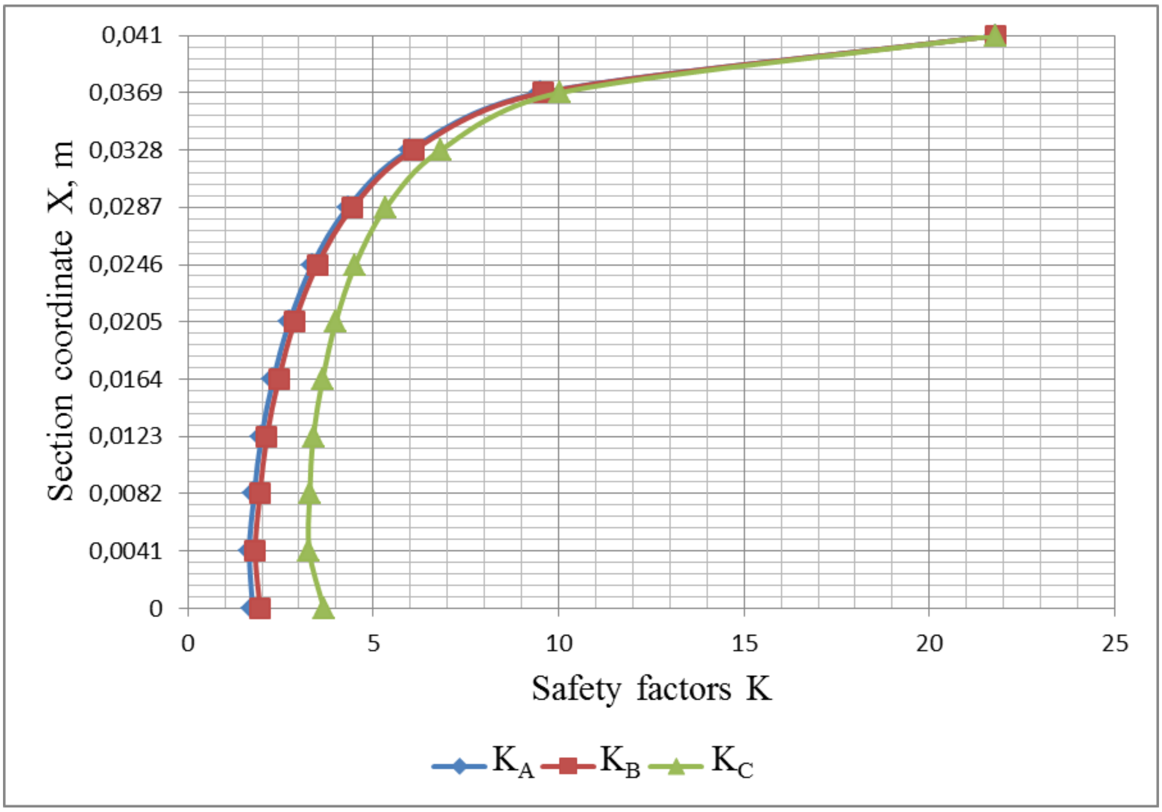
Section N	X, m	F, m^2	J_{min}, m^4	σ_t, MPa	Bending stresses σ_b, MPa		
					σ_{bA}	σ_{bB}	σ_{bC}
1	0	0,0001101	3,633E-10	194,149	63,517	37,267	-71,3603
2	0,0041	0,0001083	3,415E-10	178,9	53,3517	32,6194	-61,2822
3	0,0082	0,0001066	3,255E-10	163,0945	43,6584	27,3031	-50,5754
4	0,0123	0,0001049	3,11E-10	146,7582	34,6429	22,0828	-40,3205
5	0,0164	0,0001032	2,976E-10	129,8845	26,4036	17,124	-30,8009
6	0,0205	0,0001015	2,847E-10	112,4616	19,0421	12,5505	-22,2224
7	0,0246	0,0000998	2,724E-10	94,475	12,6717	8,481	-14,7702
8	0,0287	0,0000981	2,605E-10	75,9083	7,4212	5,0409	-8,627
9	0,0328	0,0000965	2,49E-10	56,7433	3,439	2,3698	-3,9814
10	0,0369	0,0000948	2,377E-10	36,9603	0,8979	0,6275	-1,0337
11	0,041	0,0000931	2,267E-10	16,5379	0	0	0

Section N	Total stresses σ_Σ, MPa			Safety factors K		
	$\sigma_{\Sigma A}$	$\sigma_{\Sigma B}$	$\sigma_{\Sigma C}$	K_A	K_B	K_C
1	257,6659	231,4159	122,7886	1,7464	1,9446	3,6648
2	232,2517	211,5194	117,6177	1,6577	1,8202	3,2733
3	206,753	190,3976	112,5191	1,7896	1,9433	3,2883
4	181,4012	168,841	106,4376	1,9846	2,1322	3,3823
5	156,2881	147,0085	99,0835	2,3034	2,4488	3,6333
6	131,5037	125,0122	90,2391	2,7376	2,8797	3,9894
7	107,1467	102,956	79,7047	3,3599	3,4966	4,5167
8	83,3295	80,9492	67,2812	4,3202	4,4472	5,3507
9	60,1823	59,1131	52,7618	5,9818	6,09	6,8231
10	37,8581	37,5878	35,9265	9,5092	9,5776	10,0205
11	16,5379	16,5379	16,5379	21,7681	21,7681	21,7682

Distribution of stresses and safety factors



Total stresses dependent on section coordinates



Safety factors dependent on section coordinates

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