MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

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STRENGTH ANALYSIS OF DISC

Tutorial

Kharkov «KhAI» 2014

UDC 629.7.036:621.438-226 (075.8)
LBC 39.55:31.363я73
Y-44

Розглянуто аналіз статичної міцності дисків компресорів і турбін, який полягає у розрахунку напружень і запасів міцності у найбільш напружених точках за профілем диска. Наведено інформацію про програмне забезпечення для виконання розрахунків на ПЕОМ, а також описано послідовність і приклади розрахунків. Використано метод скінченних різниць. Диск вважається симетричним, а його профіль задається значеннями ширини на заданих радіусах. Аналіз міцності диска турбіни виконується із урахуванням зміни температури за радіусом.

Для студентів, що навчаються за напрямком «Авіаційна і ракетно-космічна техніка», при виконанні практичних робіт, курсових і дипломних проектів, підготовці до екзаменів з дисциплін «Конструкція авіаційних двигунів і енергетичних установок», «Двигуни літаків і вертольотів».

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Yepifanov, S.
Y-44 Strength analysis of disc [Text]: Tutorial / S. Yepifanov, Y. Shoshin, R. Zelenskyi. – Kharkiv: National Aerospace University «Kharkiv Aviation Institute», 2014. – 28 p.
ISBN 978-966-662-345-7

Tutorial addresses the static analysis of compressor and turbine discs. Analysis aims calculating stresses and safety factors distribution on disk radius. Student can find information about software, recommendations for calculation and examples. The analysis is carried out according to method of finite differences. Disc is considered symmetrical; its profile is set by values of width on given radiuses. The temperature of turbine disc is considered to vary with the radius.

This book will be useful for students studying "Aerospace Engineering" to prepare for practical activities and examinations on disciplines "Construction of Aero Engines and Power Plants", "Engines of Airplanes and Helicopters", and to make course and diploma projects.

Il. 4. Tabl. 3. Bibliogr.: 3 names

UDC 621.452.3 (075.8)
LBC 39.55:31.363я73

ISBN 978-966-662-345-7

CONTENT

1 GENERAL INFORMATION

Discs of compressors and turbines are very important parts of gas turbine engine; their reliable operation determines reliability of the engine as a whole. They are the principal parts of engine which reliability and life time must be proven during engine certification by strength and life time calculation and testing analysis.

During operation of gas turbine engine, the discs undergo inertia centrifugal forces. They are actuated by a mass of rotor blades and own mass of disc. These forces cause **tensile stresses**. Non-uniform heating of turbine disc causes **thermal stresses**, thus disc elements may be as tensed as compressed.

Besides tension and compression discs may be also under torsion and bending. **Torsion** appears only in case disc is engaged into torque transmission. **Bending** appears at conditions, when temperature and pressure acting side surfaces of disc are not equal. Besides, the sources of bending are axial gas forces acting rotor blades, vibrations of blades and disc and gyroscopic moments at aircraft maneuvers.

Stresses actuated by centrifugal forces acting blades and disc bodies and thermal stresses take the lion's share from the considered above stresses (if disc is heated non-uniformly). Torsion stresses are negligibly low to be considered. Bending intensity depends on disc width and method of discs coupling together and to a shaft. Considering these stresses makes sense only for thin discs.

For analyzing, disc is assumed to be warped within the elastic limits. In some instances, stresses may exceed the elastic and yield limits of material, thus the most loaded regions of disc operate outside the elastic limits in plasticity region. When disc operates for a long time under high temperatures there a tendency appears when solid material slowly moves or permanently deforms under the influence of stresses. This tendency is termed creep. It this case the strength analysis must consider the plastic effects and creep. In this tutorial these cases are not considered.

During strength analysis the following assumption are considered:

– disc is symmetrical relative to a middle plane, which is perpendicular to axis of rotation;

– the disc is considered to be in plane stress state;

– the metal temperature varies only on disc radius and is uniform on width;

– the stresses are width independent at any radius;

– presence of holes, bosses and fillets is not taken into account.

The goal of disc strength analysis is determination of stresses and safety factors distribution on a disc radius.

2 BASIC EQUATIONS FOR DETERMINATION OF THE ELASTIC STRESS IN DISC UNDER ACTION OF CENTRIFUGAL FORCES AND NON-UNIFORM HEATING

Basic equations for the strength analysis were deduced at the lecture. The first equation was deduced by considering the disc element which is formed by two meridional planes and two concentric surfaces. The second equation was deduced by considering deformations of the disc element. Finally we obtain two differential equations which are basic for the strength analysis:

$$d\sigma_R = -\sigma_R\left(\frac{db}{b}+\frac{dR}{R}\right)+\sigma_T\frac{dR}{R}-\rho\omega^2 R^2\frac{dR}{R}, \tag{2.1}$$

$$d\sigma_T = \sigma_T\left(\frac{dR}{R}+\frac{dE}{E}\right)+\sigma_R\left(\frac{dR}{R}-\mu\frac{db}{b}-\mu\frac{dE}{E}\right)-\mu\rho\omega^2 R^2\frac{dR}{R}-Ed\left(\alpha t\right), \tag{2.2}$$

where σ_R and σ_T – radial and circumferential normal stresses;

b, R – current width and radius of the disc;

ω – angular velocity;

ρ – material density;

E – elasticity modulus;

μ – Poisson ratio;

α – thermal expansion factor;

T – disc temperature at radius ***R***.

Precise solution of these basic differential equations can be obtained only for limited number of disc profiles [1]. Therefore approximate methods are used to analyze the disc of arbitrary shape. One of them is the finite differences method [2].

3 CALCULATION ORDER AND BASIC FORMULAS

3.1 Finite differences method

Disc analysis using the finite differences method is based on approximate solution of differential equations set (2.1), (2.2) by substituting differentials with finite differences. To make analysis, the disc must be split into sections. These sections have numbers from ***0*** to ***k*** (Figure 3.1). Differentials are substituted with finite differences according to the following equations:

$$d\sigma_R \approx \Delta\sigma_R = \sigma_{Rn} - \sigma_{R(n-1)};\ d\sigma_T \approx \Delta\sigma_T = \sigma_{Tn} - \sigma_{T(n-1)};$$
$$dR \approx \Delta R = R_n - R_{n-1};\ db \approx \Delta b = b_n - b_{n-1},$$

where indexes ***n*** show numbers of annular sections, hence they can take values from ***0*** to ***k***.

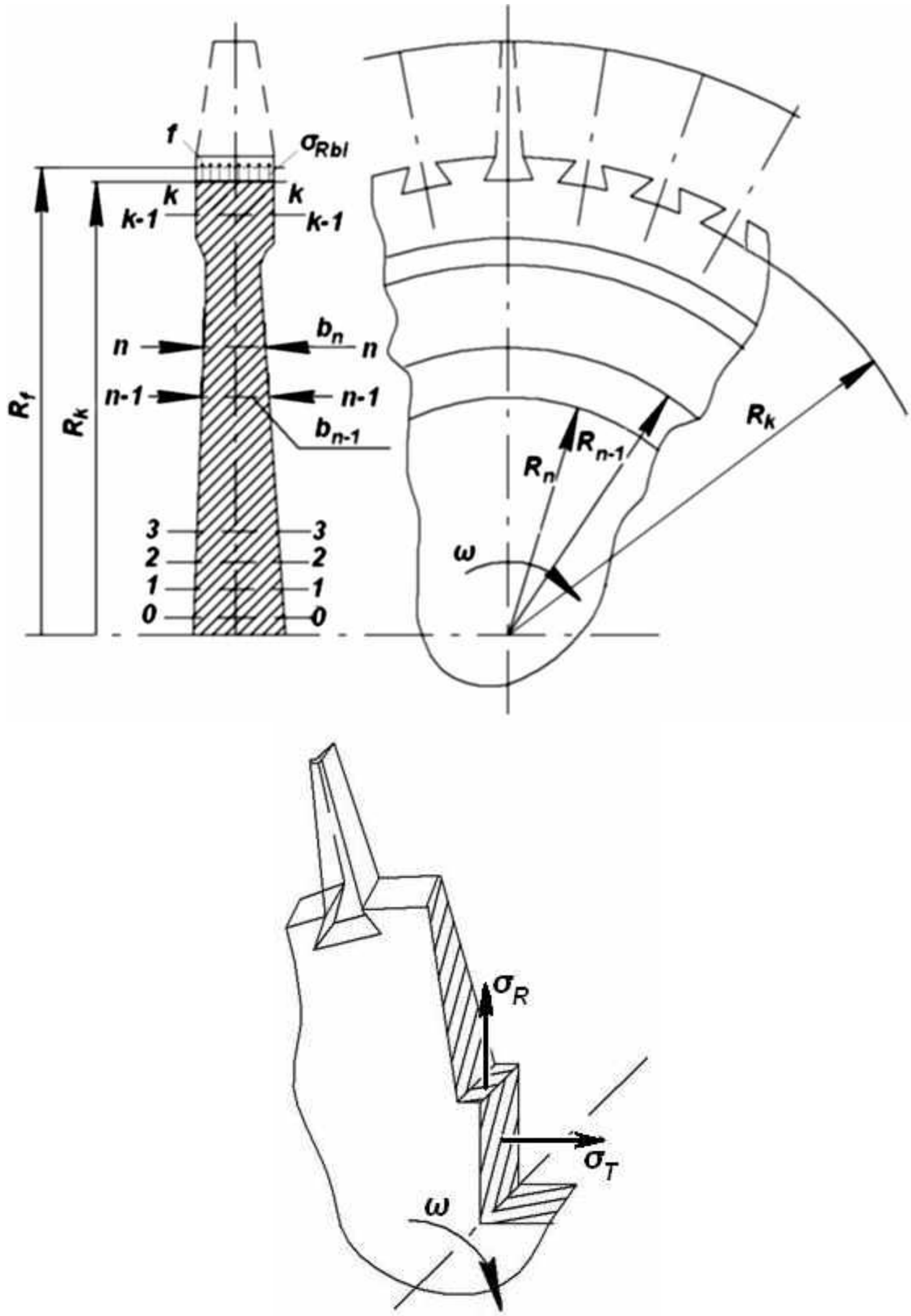

Figure 3.1 – Designing disc analytical model

Substituting the differentials by finite differences in (2.1) and (2.2) and making necessary transformations, it is possible to deduce equations for calculating stresses in ***n**-th* cross-section according to known stresses in ***(n-1)**-th* cross-section, and further – by stress in zero cross-section σ_o.

Finally equations for the strength analysis are:

$$\sigma_{Rn} = A_n \sigma_0 + B_n; \tag{3.1}$$

$$\sigma_{Tn} = N_n \sigma_0 + Q_n, \tag{3.2}$$

where

$$\begin{gathered} A_n = A_{n-1}\xi_n + N_{n-1}\vartheta_n; \\ B_n = B_{n-1}\xi_n + \left(Q_{n-1} - C_n\right)\vartheta_n; \\ N_n = N_{n-1}\varphi_n + A_{n-1}\lambda_n; \\ Q_n = Q_{n-1}\varphi_n + B_{n-1}\lambda_n - \mu C_n \vartheta_n - \psi_n. \end{gathered} \tag{3.3}$$

Coefficients A_n, B_n depend on disc geometry and material properties. Coefficients B_n, Q_n depend on both geometry and material as well as on centrifugal and thermal loads acting the disc.

Coefficients $\xi_n, \vartheta_n, \varphi_n, C_n, \lambda_n$ and ψ_n are determined as

$$\begin{gathered} \xi_n = 3 - \frac{R_n}{R_{n-1}} - \frac{b_n}{b_{n-1}}; \quad \vartheta_n = \frac{R_n}{R_{n-1}} - 1; \\ \varphi_n = 1 - \frac{R_n}{R_{n-1}} + \frac{E_n}{E_{n-1}}; \quad C_n = \rho\omega^2 R^2{}_{n-1}; \\ \lambda_n = \frac{R_n}{R_{n-1}} - 1 - \mu\left(\frac{b_n}{b_{n-1}} + \frac{E_n}{E_{n-1}} - 2\right); \quad \psi_n = \frac{E_n + E_{n-1}}{2}\left[(\alpha t)_n - (\alpha t)_{n-1}\right]. \end{gathered} \tag{3.4}$$

Formulas (3.1) and (3.2) for ***"0"*** cross-section are the following:

a) for solid disc – $\sigma_0 = A_0\sigma_0 + B_0$, $\sigma_0 = N_0\sigma_0 + Q_0$;

b) for disc with free central hole – $0 = A_0\sigma_0 + B_0$; $\sigma_0 = N_0\sigma_0 + Q_0$;

c) for tightly fitted disc – $\sigma_{R\,fit} = A_0\sigma_0 + B_0$; $\sigma_0 = N_0\sigma_0 + Q_0$.

Coefficients A_0, B_0, N_0, Q_0 are to be chosen to meet the condition when these formulas transform into identities at any stress σ_0. This is valid if:

- $A_0 = 1$, $B_0 = 0$, $N_0 = 1$, $Q_0 = 0$ (solid disc);
- $A_0 = 0$, $B_0 = 0$, $N_0 = 1$, $Q_0 = 0$ (disc with free central hole);
- $A_0 = 0$, $B_0 = \sigma_{R\,fit}$, $N_0 = 1$, $Q_0 = 0$ (tightly fitted disc).

The unknown stress in zero cross-section σ_0 is calculated by known radial stress in ***k**-th* cross-section. The latest is equal to stress $\sigma_{R\ bl}$ which is actuated by centrifugal forces acting rotor blades.

Then $\sigma_{R\ k} = \sigma_{R\ bl} = A_k \sigma_0 + B_k$, whence

$$\sigma_0 = \frac{\sigma_{R\ bl} - B_k}{A_k}. \tag{3.5}$$

If blades and disc are made of materials with the same densities, stress $\sigma_{R\ bl}$ due to centrifugal forces of blades and locking part of disc rim are determined as

$$\sigma_{R\ bl} = \frac{z \sigma_{t\ k} F_k + \rho f\, 2\pi R_f^2 \omega^2}{2\pi R_k b_k}, \tag{3.6}$$

where *z* – number of rotor blades;

$\sigma_{t\ k}$ – tensile stresses in a blade root due to centrifugal forces;

F_k – area of a blade root;

ρ – density;

f – area of radial cross-section of grooved disc part;

R_f – radius of a gravity center of a radial cross-section of a grooved disc part;

R_k – outer radius of un grooved disc part);

b_k – disc width at a radius R_k.

3.2 Design modes for safety factor determination

Designed disc is checked at the mode with the highest rotational speed. Centrifugal forces at this mode are the highest. Namely they determine the disc strength.

In some instances, thermal strains considerably affect the disc strength. Thermal strains mainly depend on temperature gradient on a disc radius. Temperature gradient is maximum during engine starting, when peripheral part of the disc is intensively heated by hot gas, and central part remains cold. Thus, thermal stresses become maximum during engine starting, and the checking analysis of disc strength at starting conditions is also expedient.

Disc is considered plane-stressed. So equivalent stresses are needed to estimate strength. These stresses are determined using energy fracture criterion. Hence equivalent stresses are:

$$\sigma_{eq} = \sqrt{\sigma_R^2 + \sigma_T^2 - \sigma_R \sigma_T}\ . \tag{3.7}$$

Safety factor is

$$K = \frac{\sigma_{lt}}{\sigma_{eq}}, \tag{3.8}$$

where σ_{lt} – long-term strength of material taken for the material temperature at the particular disc cross-section.

According to the Norms of Strength the safety factor for the disc must be more than 1,3… 1,5, i. e. $K \geq 1{,}3 \ldots 1{,}5$.

3.3 Calculation order

1. Draw the upper part of the disc meridional cross-section (disc profile). This drawing is usually full-scale.

2. The disc profile is split into design cross-sections from “0” to ***k****-th* (Figure 3.2). A number of design cross-sections must be more than 8…9. Zero section of the solid disc is selected at the distance $(0{,}05 \ldots 0{,}1)R_k$, and at the disc with a central hole is to be equal to the radius of a hole.

To get a good precision at the analysis you must split the disc according to the following rules:

–**solid disc*:*** ratio of radiuses $R_n / R_{n-1} \leq 1{,}4 \ldots 1{,}5$, and the ratio of disc widths $0{,}8 \leq b_n / b_{n-1} \leq 1{,}2$.

–**disc with a central hole:** the ratio of radiuses $R_n / R_{n-1} \leq 1{,}1 \ldots 1{,}2$ (first two-three cross-sections), and $R_n / R_{n-1} \leq 1{,}4 \ldots 1{,}5$ for the rest cross-sections. Ratio of disc width for all cross-sections $0{,}8 \leq b_n / b_{n-1} \leq 1{,}2$.

3. Coefficients $\xi_n, \sigma_n, \varphi_n, \lambda_n, C_n$ and ψ_n are calculated for each cross-section using equations (3.4). These coefficients depend on a disc shape, material properties, rotational speed n and temperature distribution.

4. Coefficients A_n, B_n, Q_n, N_n are determined set by set for each cross-section(from the center to the periphery) by formulas (3.3).

5. Contour load acting outer un grooved radius $\sigma_{R\,bl}$ is obtained by formula (3.6).

6. Stresses in zero cross-section σ_0 are obtained by formula (3.5) using known coefficients $\boldsymbol{A_k}$, $\boldsymbol{B_k}$ and contour load $\sigma_{R\,bl}$.

7. Stresses $\sigma_{R\,n}$, $\sigma_{T\,n}$ are obtained at each section by the formulas (3.1) and (3.2).

8. Equivalent stresses σ_{eq} and safety factors $\boldsymbol{K}$ are determined at each cross-section by the formulas (3.7), (3.8).

9. Visualizing calculation results. Draw the diagrams with variation of stresses σ_R, σ_T and safety factor $\boldsymbol{K}$ on the disc radius. Make conclusions about a disc static strength.

10. Hand calculations are not time efficient, hence we recommend you to use the programs that represent data in a tabular format, eq. Microsoft Excel (see Table 4.1).

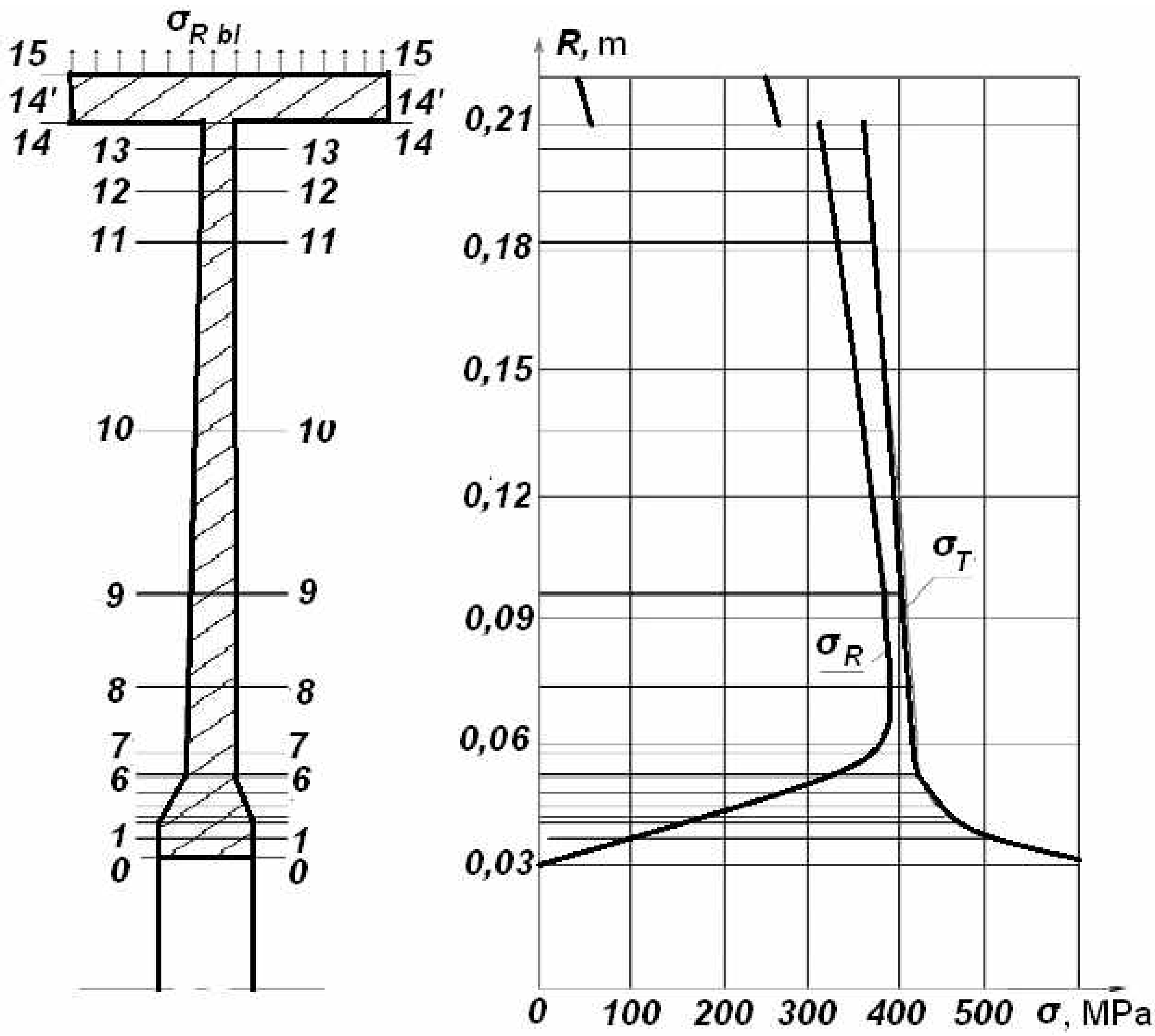

Figure 3.2 - Compressor disc with a central hole and stepped variation of width. Distribution of stresses

3.4 Initial data for the analysis

Disc strength analysis needs the following data:

1) design rotational speed $\boldsymbol{n}$, rpm;

2) radius of each cross-section and disc width at every cross-section (use engine drawing for references): radius and width, m;

3) material properties: density ρ, kg/m^3, long-term strength σ_{lt}, MPa, elasticity modulus $\boldsymbol{E}$, MPa, thermal expansion coefficient α, 1/K, Poisson's ratio μ (set 0,3 for all metals); values σ_{lt}, $\boldsymbol{E}$ and α depend on temperature which in turn is different for different disc cross-sections (method to estimate temperature state of the disc is presented in the chapter 3.4.1);

4) tensile stresses at the blade root cross-section $\sigma_{t\ k}$, MPa, that appear under centrifugal forces at design mode, and area of blade root section $\boldsymbol{F_k}$, m^3 (these values are outputs of blade strength analysis);

5) number of rotor blades $\boldsymbol{z}$ (is known from gas dynamic analysis);

6) area of radial cross-section of grooved part of the disc rim $\boldsymbol{f}$, m^2 and radius $\boldsymbol{R_f}$, m of the gravity center of this cross-section (are determined using a drawing).

3.5 Estimating of disc temperature state

Heat flows from blades to the disc. So the maximum temperature is at the periphery (on outer diameter) and minimum one is in the central area.

Temperature distribution on a disc radius depends on intensity of cooling, heat conductivity of material and disc constructive features.

The disc temperature at outer diameter $\boldsymbol{t_k}$ is determined knowing the temperature of the blade in the root cross-section $\boldsymbol{t_{bl\ k}}$ [1] and thermal resistance of blade lock Δt:

$$t_k = t_{bl\ k} - \Delta t. \tag{3.10}$$

Value Δt depends on lock construction. For fir-tree locks it is50…100 ^{0}C.

Temperature gradient on radius may be approximately considered as parabolic [1]:

$$t_k = t_0 + \left(t_k - t_0\right)\left(\frac{R}{R_k}\right)^2, \tag{3.11}$$

where $\boldsymbol{t_0}$ – temperature at the center of disc;

$\boldsymbol{R_k}$, $\boldsymbol{R}$ – outer and inner disc radiuses.

The temperature difference between disc rim and center $\left(t_k - t_0\right)$ at maximum steady operational mode is 100…250 ^{0}C. It depends on disc cooling. So, to calculate temperature at the center of the disc t_0 you need to assume the difference between temperature at the center of the disc and temperature at the periphery t_k. The temperature at the disc periphery t_k is known.

Temperature at a design cross-section of the disc with central hole is determined as

$$t_k = t_0 + \left(t_k - t_0\right)\left(\frac{R}{R_k}\right)^2, \tag{3.12}$$

where t_0 – disc temperature at the radius R_0;

R_0 – radius of central hole.

3.6 Analysis of disc with abrupt width changing

If disk width changes stepwise, then stresses are also supposed to change stepwise. To calculate the stresses at the cross-section with abrupt width change you must consider the condition when the cross-section is acted by equal radial forces, and the condition when circumferential strains of isolated disk are equal.

The analysis of disc with abrupt width change differs from general analysis by splitting the disc into cross-sections. The disc has two different cross-sections at same radius but with different width. Respectively two lines corresponding to these cross-sections in the calculation table appear: first line is parameters before width change, second line is after width change.

Formulas to calculate the stresses at the cross-section after width change are:

$$\sigma_{R'n} = A'_n\sigma_0 + B'_n;\;\; \sigma_{T'n} = N'_n\sigma_0 + Q'_n, \tag{3.13}$$

where $\sigma_{R'n}$, $\sigma_{T'n}$ – radial and circumferential stresses in disc at radius R_n after width change;

σ_0 –stress in disc center.

Coefficients A'_n, B'_n, N'_n, Q' are obtained as

$$A'_n = A_n\frac{b_n}{b_{n'}};\; B'_n = B_n\frac{b_n}{b_{n'}};\; N'_n = N_n + \mu A_n\frac{b_n}{b_{n'}} - \mu A_n;\; Q'_n = Q_n + \mu B_n\frac{b_n}{b_{n'}} - \mu B_n, \tag{3.14}$$

where $b_n, b_{n'}$ – disc width at the radius R_n before and after width change.

3.7 Analysis of disc with blades on side surface

Disc of centrifugal compressors and centripetal turbines have blades attached to their side surfaces.

To make the correct strength analysis of such disk it is necessary to consider joint elastic deformation of blade sand disc acted by centrifugal forces and thermal strains. Generally, assessed analysis considers only loading due to attached blades neglecting ability to take loading their bearing capacity. Following made assumption, blades are simulated as virtual mass joined to disc loading it. Apparently, in analysis disk is acted by enlarged loads; therefore real disc will have more strength than the one from analysis.

Virtual mass that simulates blades, result in additional centrifugal force. This increases stresses in disc. The above mentioned virtual mass is simulated by increasing density at each design section. This method allows using the method of disc strength analysis represented in subchapter 3.1.

Virtual density of two-face impeller at radius ***R*** cam be calculated as [2]

$$\rho_{virt} = \rho \cdot \left(1 + \frac{f_{bl} \cdot z}{\pi R b} \right), \tag{3.15}$$

where f_{bl} – cross-sectional area of one blade at radius ***R*** (Figure 3.3);

z – number of blades from one side of disc;

b – disc wide that ***R***;

ρ – density of material.

Virtual density of one-face impeller at radius ***R*** can be calculated as

$$\rho_{virt} = \rho \cdot \left(1 + \frac{f_{bl} \cdot z}{2\pi R b} \right). \tag{3.16}$$

Formulas (3.15), (3.16) show that virtual density ρ_{virt} varies with radius.

Applying the virtual density ρ_{virt} approach makes the application of finite element method passible for strength analysis of disc with blades at side surface. The proposed method requires ρ_{in} formula (3.4) to be substituted with ρ_{virt} .

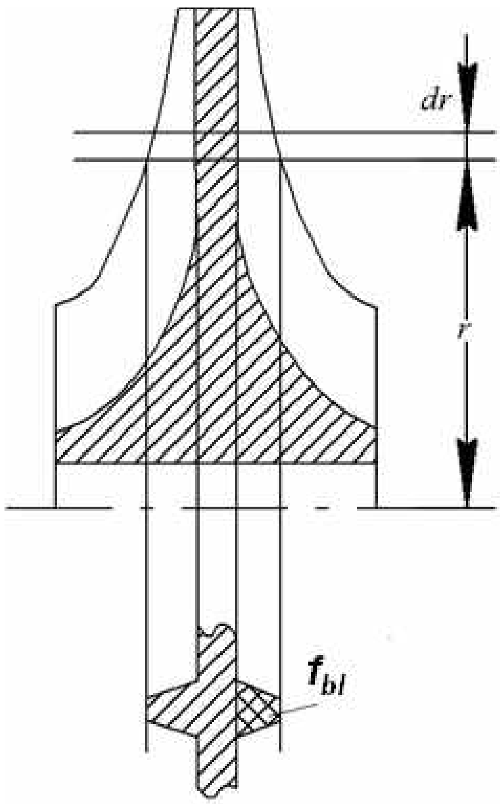

Figure 3.3 – Analytical model of disc with blades on side surface

Then the value of coefficient C_n is determined as

$$C_n = \rho_{virt\ (n-1)} \cdot \omega^2 \cdot R^2_{n-1}. \tag{3.17}$$

4 EXAMPLE OF HAND CALCULATION

Determine stresses and safety factors in compressor disc with central hole and with abrupt width change.

Initial data:

- rotational speed $\boldsymbol{n} = 11350\ rpm$;
- disk geometry and chosen design cross-sections are shown in Table 4.1 and in Figure 3.2;
- disc material – titanium alloyBT3-1;
- density $\rho = 4500\ kg/m^3$;
- Poisson's ratio $\mu = 0,3$;
- disc temperature is constant for all radiuses: $\boldsymbol{T_{disc}} = 100\ C$;
- long-term strength of titanium alloys at temperature 100^0Cis equal to ultimate limit σ_u, so set $\sigma_{lt} = \sigma_u = 900\ MPa$ [1];
- contour distributed force $\sigma_{R\ bl} = 30,5\ MPa$.

Set 16 design radiuses. Disc width abruptly changes at radius $\boldsymbol{R} = 0,211\,m$, so at this radius the disc has two design cross-sections: 14-14 and 14'-14'.Thus, total number of cross-section is17.

Results of calculations are presented in a Table 4.1. Order of calculation is given in paragraph 3.3. Figure 3.2 represents the diagram of stress variation. As it is seen from Table 4.1, obtained safety factors at all disc cross-sections satisfy the Norms of Strength.

Table 4.1

Results of disc hand calculation

Section	R_n, m	$\frac{R_n}{R_{n-1}}$	$b_n 10^2$, m	$\frac{b_n}{b_{n-1}}$	$R_n^2 10^4$, m^2	$C_n/10^5$	ξ_n	σ_n	φ_n
0-0	0.03	-	0.014	-	9	-	-	-	-
1-1	0.0335	1.115	0.014	1	11.2	57.9	0.885	0.115	0.885
2-2	0.0385	1.15	0.014	4	14.8	71.9	0.85	0.15	0.85
3-3	0.0405	1.05	0.012	0.856	16.4	95.1	1.092	0.05	0.95
4-4	0.0428	1.057	0.010	0.834	18.4	105.3	1.109	0.057	0.943
5-5	0.0468	1.09	0.0085	0.85	21.9	118.2	1.06	0.09	0.91
6-6	0.0510	1.09	0.0073	0.858	26	141.0	1.052	0.09	0.91
7-7	0.0558	1.09	0.0065	0.89	31.2	165.7	1.02	0.09	0.91
8-8	0.0712	1.28	0.0065	1	50.69	200.6	0.72	0.28	0.72
9-9	0.0962	1.35	0.0065	1	92.54	326	0.65	0.35	0.65
10-10	0.1348	1.40	0.0065	1	181.8	562	0.6	0.40	0.60
11-11	0.182	1.35	0.0065	1	331.2	1169	0.65	0.30	0.65
12-12	0.195	1.071	0.0065	1	380.3	2133	0.929	0.071	0.929
13-13	0.205	1.05	0.0065	1	420.3	2450	0.95	0.05	0.85
14-14	0.211	1.03	0.0065	1	445.2	2709	0.97	0.03	0.97
14\`-14\`	0.211	1.00	0.0557	8.8	445.2	2709	-	-	-
15-15	0.222	1.05	0.0572	1	492.8	3175	0.95	0.05	0.95

Table 4.1 (completion)

Section	λ_n	A_n	$B_n / 10^5$	N_n	$Q_n / 10^5$	σ_R, MPa	σ_T, MPa	σ_{eq}, MPa	K
0-0	-	0	0	1	0	0	613.1	613	1.47
1-1	0.115	0.115	-6.658	0.885	-1.997	69.84	542.3	510	1.76
2-2	0.15	0.23	-16.74	0.77	-5.932	139.1	471.5	419	2.15
3-3	0.093	0.289	-19.06	0.753	-8.612	175.6	461.0	403	2.23
4-4	0.107	0.036	-27.63	0.741	-11.95	220.4	453.1	392	2.29
5-5	0.135	0.453	-40.99	0.742	-17.8	273.5	442.2	386	2.33
6-6	0.133	0.537	-57.42	0.719	-25.44	323.5	438.4	393	2.28
7-7	0.123	0.612	-75.77	0.72	-34.69	367.8	437.9	407	2.21
8-8	0.28	0.643	-90.86	0.689	-63.04	385.1	416.1	401	2.22
9-9	0.35	0.658	-195.2	0.673	-107.0	383.9	401.9	393	2.28
10-10	0.40	0.663	-384.7	0.667	-209.7	368.0	387.9	378	2.38
11-11	0.35	0.665	-732.6	0.665	-393.7	334.4	368.3	352	2.56
12-12	0.071	0.665	-860.0	0.665	-463.2	321.7	361.4	343	2.62
13-13	0.05	0.665	-962.7	0.665	-473.5	311.4	360.3	338	2.66
14-14	0.03	0.665	-1029.3	0.665	-511.5	304.8	356.6	333	2.7
14\`-14\`	-	0.075	-116.96	0.488	-277.8	34.53	275.4	259	3.48
15-15	0.05	0.096	-281.74	0.468	-279.4	30.49	258.8	245	3.67

5 EXAMPLES OF COMPUTER CALCULATIONS

Example 1. Let's calculate the compressor solid disc with abrupt width change. The disc geometry is presented in Appendix A.

Rotational speed – $n = 11350\,rpm$, contour distributed force – $\sigma_{R\,bl} = 30{,}5\ MPa$, disc material–titanium alloy BT3-1,which density $\rho = 4500\ kg/m^3$, Poisson's ratio $\mu = 0{,}3$ and long-term strength $\sigma_{lt} = 900\ MPa$.

Calculation results are presented in Appendix A.

Safety factors satisfy the Norms of Strength at all disc cross-sections (K ≥ 1.5). They essentially exceed recommended value, so there is possible to decrease disc width and mass. But analyzed disc is already too thick, and its width decreasing may decrease its bending rigidity. Therefore final decision about disc width changing cannot be made without analysis of disc bending and oscillation properties.

Example 2. Let's calculate the disc of the first stage of axial turbine with central hole. The disc geometry is presented in Appendix B.

Rotational speed – $n = 14500\,rpm$; distributed load on outer contour $\sigma_{R\,bl} = 152\ MPa$; disc material – nickel-chromium alloy XH70BMЮT. Blade temperature in root section is $t_{bl\,k} = 730\ °C$.

Thermal state of the disc cannot be neglected, because temperature affects elastic properties and strength of material essentially. Let's determine temperature distribution using a method mentioned in chapter 3.4.1. Considering specifics of the blade root part cooling, let's set $\Delta t = 100\ °C$ = 100 °C. Then according to formula (3.10) we obtain temperature at disc periphery:

$$t_k = t_{bl\,k} - \Delta t = 730 - 100 = 630\ °C.$$

Set a disc temperature difference $t_k - t_0$ equal 130 °C (t_0 is temperature in the zone of central hole). Then, substituting values of disc parameters into (3.12), we receive formula for calculation temperature at given radius R:

$$t_R = t_0 + (t_k - t_0)\left(\frac{R - R_0}{R_k - R_0}\right)^2 = 500 + 300\left(\frac{R - 0{,}055}{0{,}127}\right)^2.$$

Divide a disc profile into 15 different sections (Figure 5.1) according to limitations on variation of radius and width, presented in chapter 3.3. Substitute values of radius in this formula and calculate the disc temperature. Then, using characteristics $E(t)$, $\alpha(t)$, $\sigma_{lt}(t)$ of the material XH70BMЮT, determine values of elasticity modulus, thermal expansion factor and long-term strength σ_{lt} for each radius (Table 5.1).

Set Poisson's ratio equal 0,3.

Calculation results are presented in Appendix 2. Safety factors satisfy the Norms of Strength at all disc cross-sections $(K \geq 1{,}2)$.

Table 5.1

Distribution of disc properties on radius

Section	$\boldsymbol{R}$, m	$\boldsymbol{b}$, m	$\boldsymbol{t}$, ^{0}C	$\boldsymbol{E}/10^3$, MPa	$\alpha/10^{-6}$, 1/K	σ_{lt}, MPa
1	0,055	0,096	500	1,77	14,8	880
2	0,063	0,096	500	1,77	14,8	880
3	0,072	0,096	502	1,77	14,95	875
4	0,081	0,096	505	1,765	15,05	868
5	0,096	0,072	514	1,758	15,18	854
6	0,104	0,058	519	1,775	15,27	841
7	0,11	0,048	524	1,75	15,32	827
8	0,115	0,04	529	1,746	15,36	812
9	0,124	0,032	538	1,74	15,45	788
10	0,137	0,026	554	1,73	15,58	756
11	0,154	0,022	579	1,731	15,74	718
12	0,172	0,02	610	1,693	15,91	682
13	0,174	0,022	614	1,69	15,93	673
14	0,176	0,025	618	1,687	15,95	664
15	0,182	0,025	630	1,68	16,0	610

Example 3. Let's calculate the disc of two-face centrifugal compressor. Disc geometrical model is presented in Appendix 3. The prototype is compressor disc of VK-1 turbojet.

Rotational speed is $\boldsymbol{n} = 11560\ rpm$; number of blades from each side $\boldsymbol{z}_{bl} = 29$. The disc is manufactured from aluminum based thermal resistant wrought alloy which properties are: density $\rho = 2850\ kg/m^3$, Poisson's ratio $\mu = 0{,}3$ and long-term strength at given temperature $\sigma_{lt} = 380\ MPa$.

The dick is not loaded at periphery, hence contour distributed load $\sigma_{R\ bl}$ is set zero.

The disc profile (see Figure 5.1) is split into 24 design sections according to splitting rules mentioned in subchapter 3.3. Radius $\boldsymbol{R}$, disc width b and blade area $\boldsymbol{f}_{bl}$ were determined for all sections using drawing of impeller. These parameters are presented in Table 5.2.

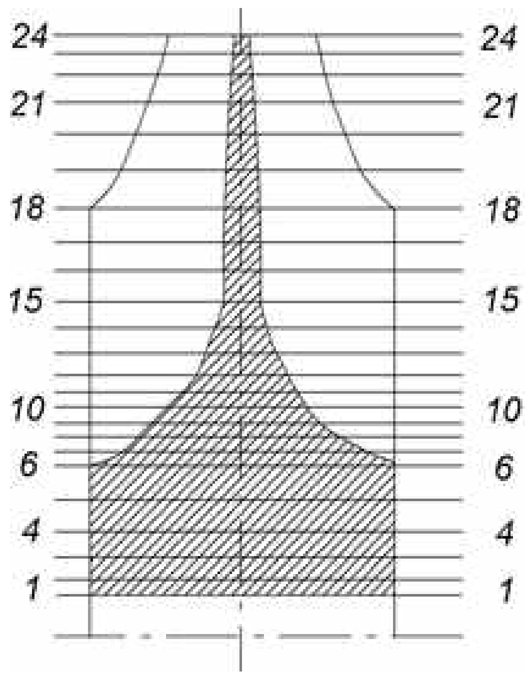

Figure 5.1 – Disc with blades at side surfaces

Table 5.2

Initial data for disc analysis

Section	R, m	*b*, m	$f_{bl} \cdot 10^6,\ m^2$	$\rho_{virt},\ kg/m^3$
1	0,027	0,2	0	2850
2	0,032	0,2	0	2850
3	0,038	0,2	0	2850
4	0,056	0,2	0	2850
5	0,08	0,2	0	2850
6	0,105	0,2	0	2850
7	0,112	0,174	126	3020
8	0,120	0,152	260	3225
9	0,130	0,122	385	3489
10	0,140	0,096	514	3860
11	0,150	0,08	596	4157
12	0,160	0,064	678	4590
13	0,170	0,056	712	4818
14	0,180	0,048	746	5110
15	0,190	0,038	794	5745
16	0,200	0,030	842	6544
17	0,220	0,024	756	6618
18	0,240	0,02	704	6670
19	0,280	0,016	561	6120
20	0,310	0,013	446	5763
21	0,340	0,01	332	5420
22	0,360	0,008	280	5409
23	0,380	0,006	233	5540
24	0,391	0,0044	209	6020

Calculation results are presented in Appendix C. Safety factors satisfy the Norms of Strength for all disc cross-sections $(K \geq 1{,}5)$.

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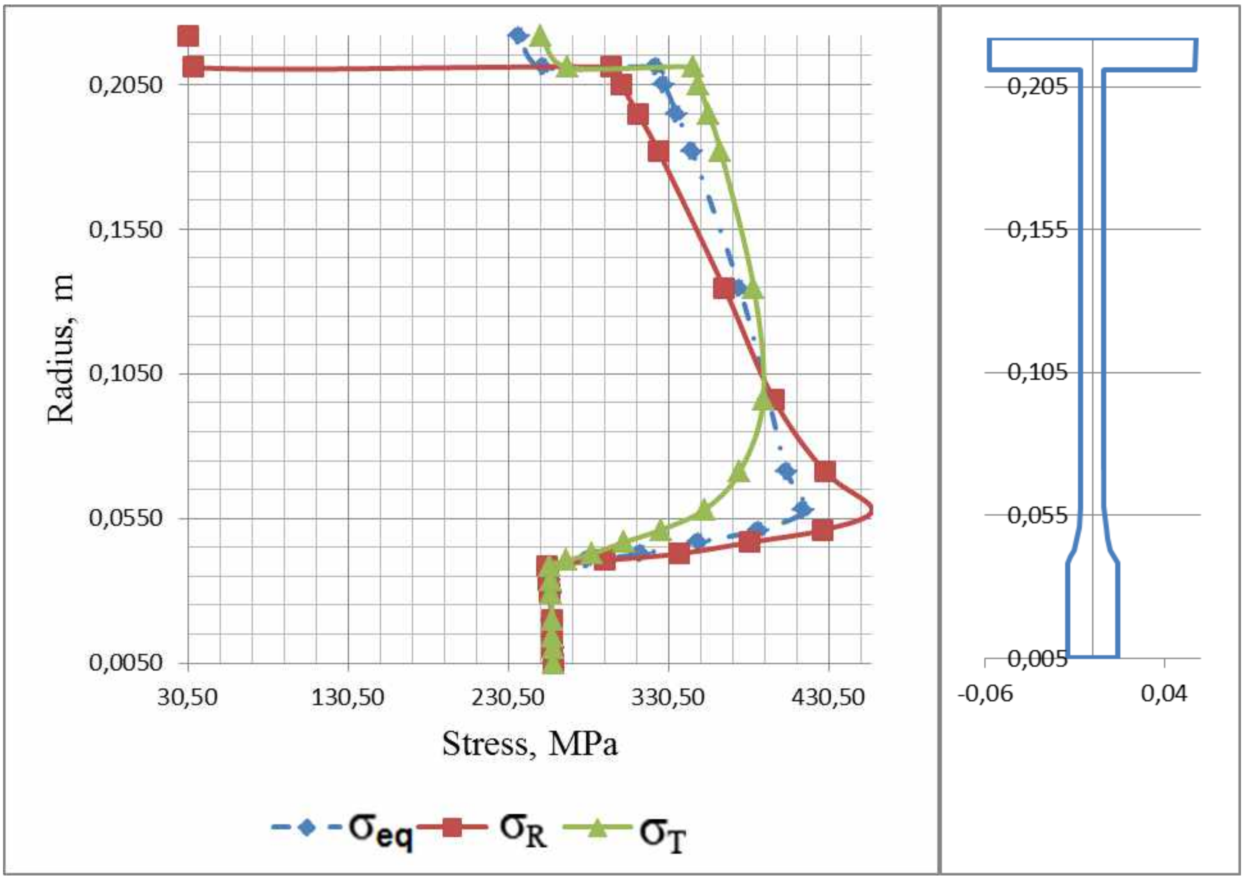
Appendix A

STRENGTH ANALYSIS OF DISC

Name of executor:		Jameson J., 232 gr.
Engine unit:		turbine
Material:		BT-3-1
DISK GEOMETRIC PARAMETERS:		
Type of disc		central hole
Sign of temperature variation at radius		not varied
Sign of corrected density variation at radius		not varied
LOADINGS		
Rotor rotation speed, rpm:		11350
Radial stress on disc rim, MPa		30.5
PROPERTIES OF MATERIAL		
Density, kg\m^3		4500
Long-term strength, MPa		900

DISC PROFILE AND REZULTS OF STRENGTH CALCULATION

Section	Radius, m	Width, m	σ_{lt}, MPa	σ_R, MPa	σ_T, MPa	σ_{eq}, MPa	K
1	0.005	0.014	900	258.5206	258.5206	258.5206	3.4813
2	0.01	0.014	900	258.2821	258.449	258.3656	3.4834
3	0.014	0.014	900	258.0437	258.2907	258.1673	3.4861
4	0.02	0.014	900	257.5012	257.9903	257.7461	3.4918
5	0.029	0.014	900	256.3196	257.3497	256.8362	3.5042
6	0.0335	0.014	900	255.5854	256.9216	256.2561	3.5121
7	0.0385	0.014	900	254.6406	256.3789	255.5142	3.5223
8	0.0405	0.012	900	290.6059	267.0511	279.5737	3.2192
9	0.0428	0.01	900	337.0935	282.7364	313.4698	2.8711
10	0.0468	0.0085	900	381.4382	302.6439	348.7814	2.5804
11	0.051	0.0073	900	426.9114	325.4786	386.3148	2.3297
12	0.058	0.0065	900	457.3488	352.7086	415.0427	2.1685
13	0.0712	0.0065	900	428.1132	374.897	404.1415	2.2269
14	0.0962	0.0065	900	396.1254	389.5918	392.8993	2.2907
15	0.1348	0.0065	900	365.1617	383.7107	374.7806	2.4014
16	0.182	0.0065	900	324.1274	362.957	345.1841	2.6073
17	0.195	0.0065	900	311.3227	355.51	335.6052	2.6817
18	0.205	0.0065	900	300.8743	349.4297	327.8598	2.7451
19	0.211	0.0065	900	294.3618	345.6284	323.0605	2.7859
20	0.211	0.057	900	33.5676	267.3901	252.2868	3.5674
21	0.222	0.0572	900	30.5	250.6232	236.8506	3.7999



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Stress, MPa
σeq
σR
σT
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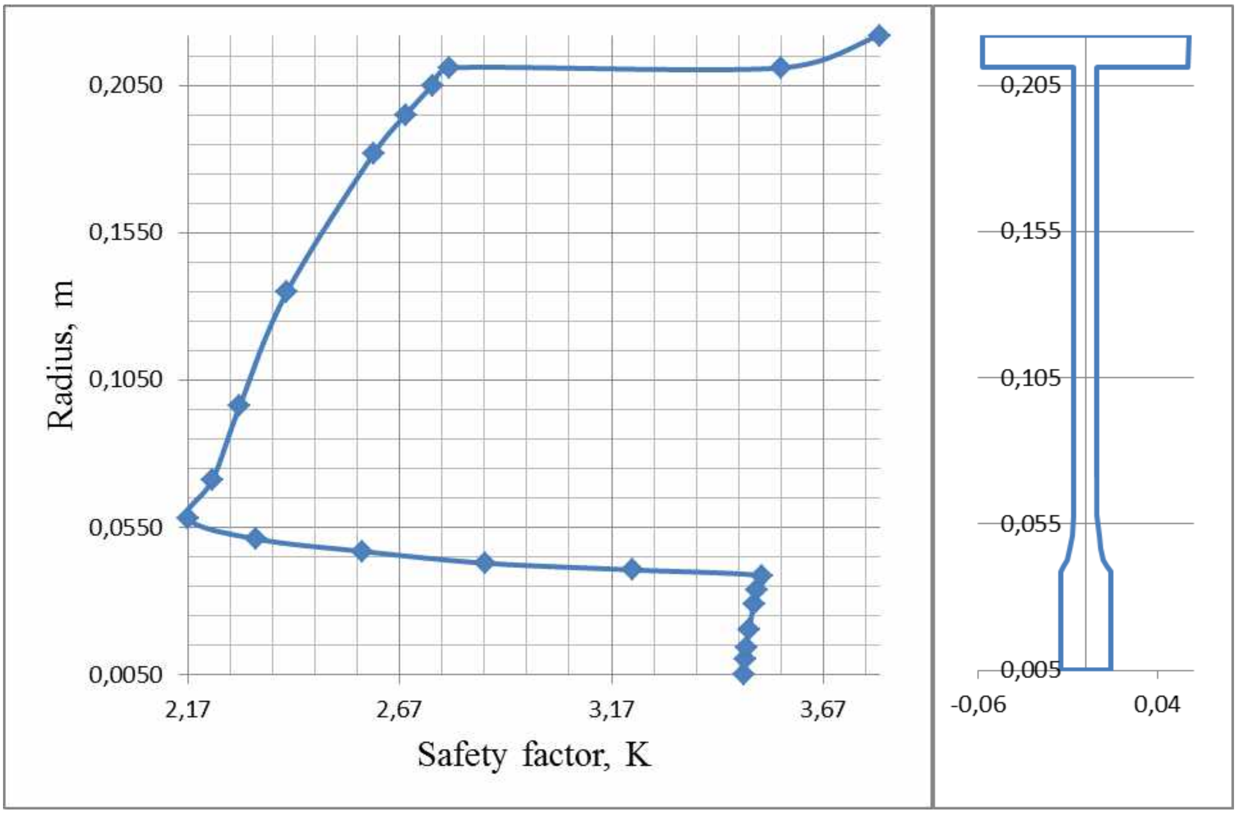
0,055
0,005
-0,06
0,04



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0,1050
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Radius, m
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Safety factor, K
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0,105
0,055
0,005
-0,06
0,04

Appendix B

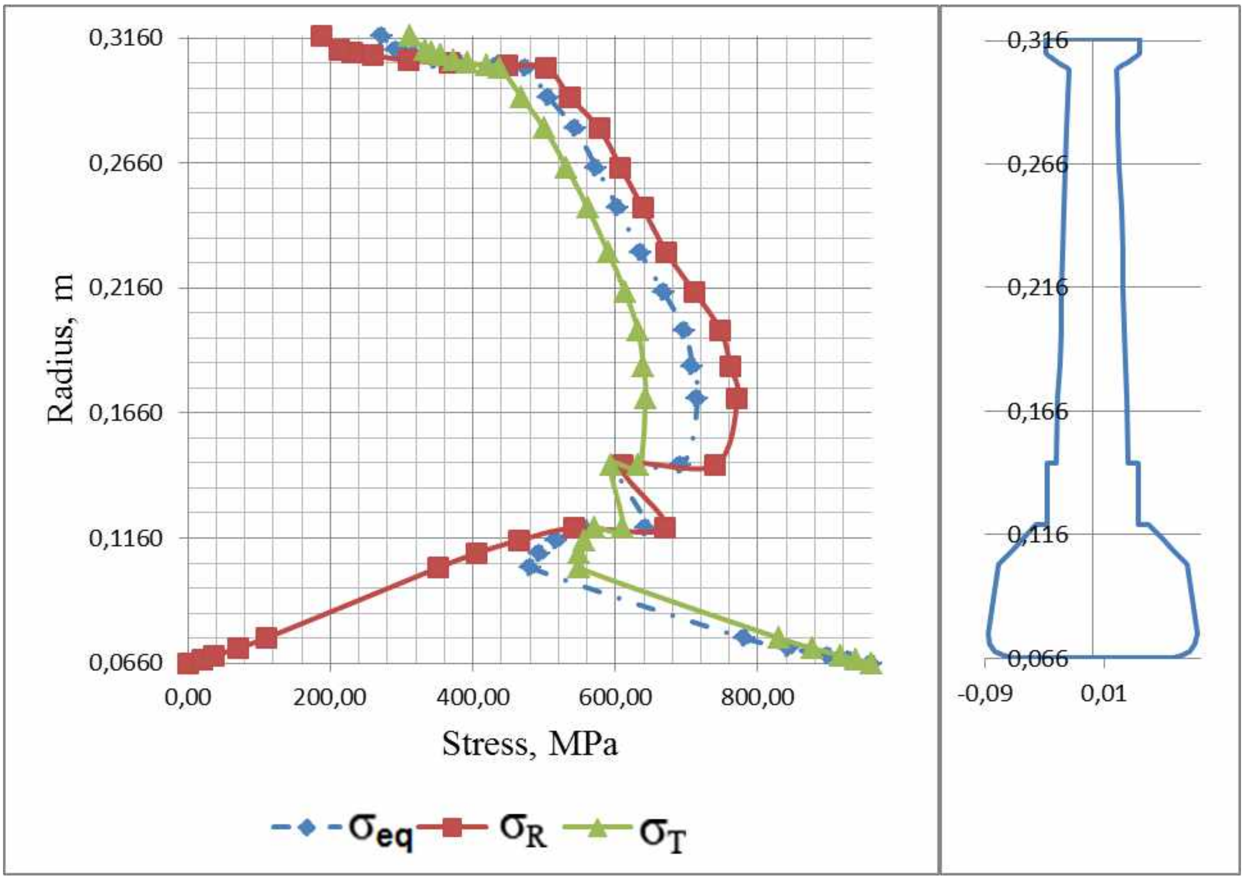
STRENGTH ANALYSIS OF DISC

Name of executor:	Jameson J., 232 gr.	
Engine unit:	turbine	
Material:	ХН70ВМЮТ	
DISK GEOMETRIC PARAMETERS:		
Type of disc		central hole
Sign of temperature variation at radius		varied
Sign of corrected density variation on radius		not varied
LOADINGS		
Rotor rotation speed, rpm:		12456
Radial stress on disc rim, MPa		187.9
DISC PROFILE AND PROPERTIES OF MATERIAL		
Density, kg\m^3		8350

Section	Radius, m	Width, m	***t,*** ^oC	***E,*** MPa	***α,*** 1/K	σ_{lt}, MPa
1	0.066	0.067	527	171491	1.295E-05	1151.42
2	0.0675	0.0741	527	171490.9	1.295E-05	1151.41
3	0.069	0.0813	527.01	171490.4	1.295E-05	1151.39
4	0.072	0.0853	527.06	171488.6	1.295E-05	1151.31
5	0.076	0.087	527.16	171484.3	1.295E-05	1151.11
6	0.1042	0.0785	529.32	171392.3	1.296E-05	1146.9
7	0.11	0.067	530.07	171360.3	1.297E-05	1145.43
8	0.1151	0.0569	530.82	171328.4	1.297E-05	1143.96
9	0.12	0.047	531.62	171294	1.297E-05	1142.38
10	0.12	0.038	531.62	171294	1.297E-05	1142.38
11	0.145	0.038	536.9	171069.4	0.000013	1131.96
12	0.145	0.03	536.9	171069.4	0.000013	1131.96
13	0.1716	0.02875	544.67	170737.9	1.303E-05	1116.36
14	0.1848	0.0275	549.37	170537.8	1.305E-05	1106.82
15	0.199	0.02625	555.16	170290.9	1.307E-05	1094.92
16	0.2146	0.02563	562.02	169998.6	0.0000131	1080.65
17	0.2301	0.025	569.69	169671.9	1.314E-05	1064.46
18	0.248	0.02375	579.52	169252.7	1.318E-05	1043.32
19	0.2643	0.0225	589.37	168832.6	1.322E-05	1021.72
20	0.2797	0.02125	599.39	168405.3	1.327E-05	999.32
21	0.2922	0.02076	608.1	168034.2	0.0000133	979.52
22	0.3041	0.02	616.92	167658	1.334E-05	959.12
23	0.3049	0.02204	617.52	167632.2	1.334E-05	957.71
24	0.306	0.02576	618.63	167584.9	1.335E-05	955.11
25	0.307	0.0295	619.76	167536.6	1.335E-05	952.46
26	0.309	0.03344	620.85	167490	1.336E-05	949.9
27	0.31	0.0366	621.92	167444.5	1.336E-05	947.39
28	0.311	0.039	622.67	167412.7	1.337E-05	945.63
29	0.317	0.039	627	167227.9	1.339E-05	935.38

REZULTS OF DISC STRENGTH CALCULATION

Section	σ_R, MPa	σ_T, MPa	σ_{eq}, MPa	K
1	0	959.7566	959.7566	1.1997
2	20.3902	937.5166	927.4896	1.2414
3	37.3526	916.0767	897.9832	1.2822
4	70.692	876.2909	843.1703	1.3655
5	109.8587	829.6153	780.5063	1.4748
6	351.8565	548.4936	481.3077	2.3829
7	405.5786	548.4636	492.8095	2.3243
8	465.2013	555.6966	516.4302	2.2151
9	541.7782	571.4845	557.2255	2.0501
10	670.094	609.9793	642.1505	1.779
11	610.5094	594.0102	602.4293	1.879
12	739.0377	632.5687	691.9738	1.6358
13	771.1698	642.2083	715.4597	1.5603
14	761.363	639.3583	708.2859	1.5627
15	747.815	631.6641	697.0359	1.5708
16	710.4668	613.4839	667.2823	1.6195
17	672.1073	590.026	635.0576	1.6762
18	638.4626	561.2753	603.582	1.7285
19	607.5122	531.4558	573.2804	1.7822
20	577.5685	500.5403	543.1663	1.8398
21	536.7548	468.8213	506.2184	1.935
22	502.8741	437.9606	473.7645	2.0245
23	447.949	420.0083	434.6527	2.2034
24	367.6449	392.9618	380.9348	2.5073
25	309.9961	372.4313	345.4713	2.757
26	260.2485	354.0464	317.7076	2.9899
27	231.5621	342.0564	302.3513	3.1334
28	212.3229	333.7788	292.6096	3.2317
29	187.9	311.4306	271.6204	3.4437



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200,00
400,00
600,00
800,00
Stress, MPa
σ_{eq}
σ_R
σ_T
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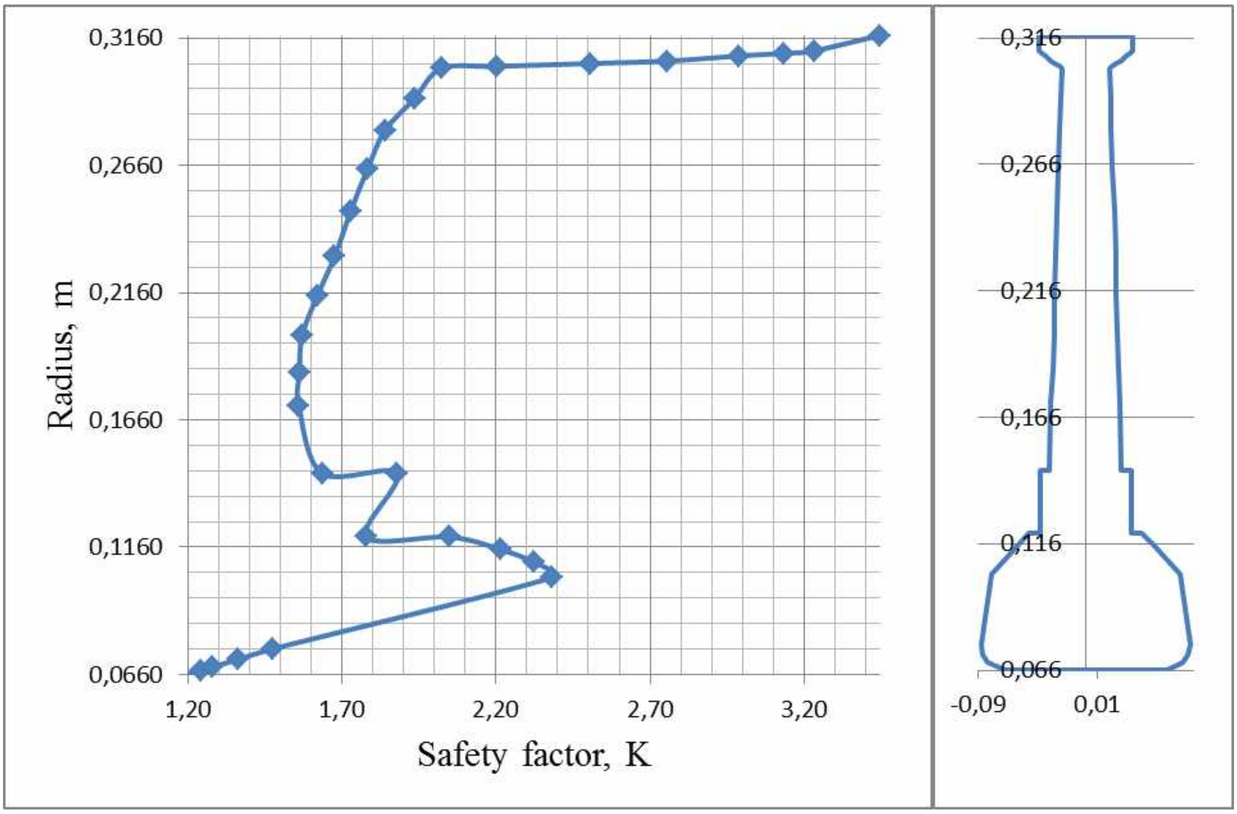
0,216
0,166
0,116
0,066
-0,09
0,01



Radius, m
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3,20
Safety factor, K
0,316
0,266
0,216
0,166
0,116
0,066
-0,09
0,01

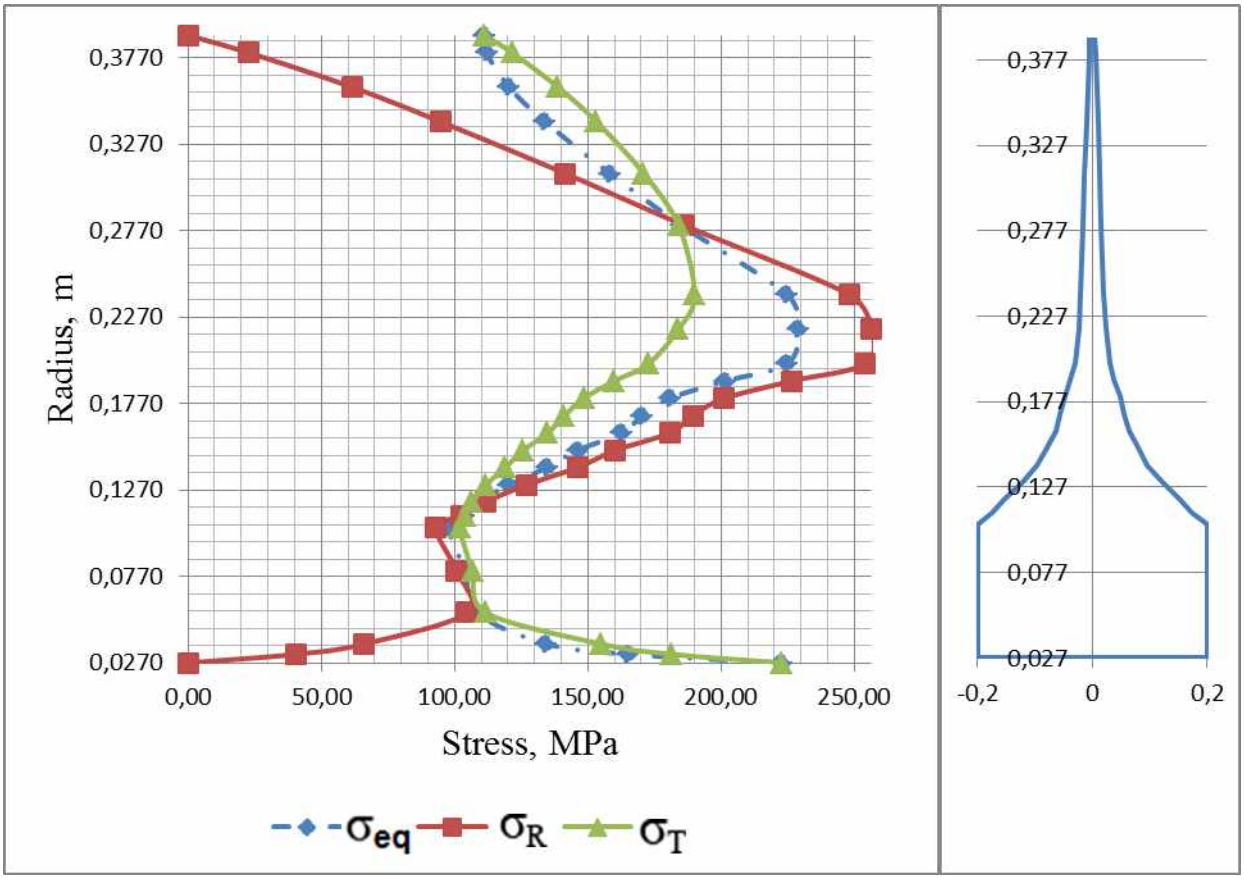
Appendix C

STRENGTH ANALYSIS OF DISC

Name of executor:	Jameson J., 232 gr.
Engine unit:	compressor
Material:	AK-6
DISK PARAMETERS:	
Type of disc	bladed
Sign of temperature variation on radius	not varied
Sign of corrected density variation on radius	varied
LOADINGS	
Rotor rotation speed, rpm:	11560
Radial stress on disc rim, MPa	0
PROPERTIES OF MATERIAL	
Long-term strength, MPa	380

DISK PROFILE AND REZULTS OF STRENGTH CALCULATION

Section	Radius, m	Width, m	***ρ, kg\m***^{3}	σ_R, MPa	σ_T, MPa	σ_{eq}, MPa	***K***
1	0.027	0.2	2850	0	222.8745	222.8745	1.70
2	0.032	0.2	2850	40.657	181.4166	164.8912	2.30
3	0.038	0.2	2850	66.1723	154.7611	134.4948	2.83
4	0.056	0.2	2850	104.6021	111.738	108.3464	3.51
5	0.08	0.2	2850	100.8441	106.6349	103.8607	3.66
6	0.105	0.2	2850	92.9954	101.9278	97.7681	3.89
7	0.112	0.174	3020	102.5082	104.0075	103.266	3.68
8	0.12	0.152	3225	111.4691	106.5565	109.0958	3.48
9	0.13	0.122	3489	127.1525	111.7937	120.2113	3.16
10	0.14	0.096	3860	146.1666	119.0338	134.6661	2.8218
11	0.15	0.08	4157	160.3874	125.8196	146.2013	2.5992
12	0.16	0.064	4590	180.7179	134.9146	162.725	2.3352
13	0.17	0.056	4818	189.3462	141.2246	170.4583	2.2293
14	0.18	0.048	5110	201.209	148.4633	180.7049	2.1029
15	0.19	0.038	5745	226.3435	159.813	201.4918	1.8859
16	0.2	0.03	6544	254.0759	172.6849	224.7211	1.691
17	0.22	0.024	6618	256.4741	183.9851	229.0023	1.6594
18	0.24	0.02	6670	248.017	190.0149	224.7024	1.6911
19	0.28	0.016	6120	186.2971	184.0661	185.1917	2.0519
20	0.31	0.013	5763	141.6164	170.9726	158.3487	2.3998
21	0.34	0.01	5420	94.7948	153.2329	133.9429	2.837
22	0.36	0.008	5409	61.5915	138.8031	120.4605	3.1546
23	0.38	0.006	5540	22.6215	121.5357	111.9524	3.3943
24	0.39	0.0044	6020	0.0000583	111.3654	111.3654	3.4122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0,0770
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50,00
100,00
150,00
200,00
250,00
Stress, MPa
σ_{eq} σ_R σ_T
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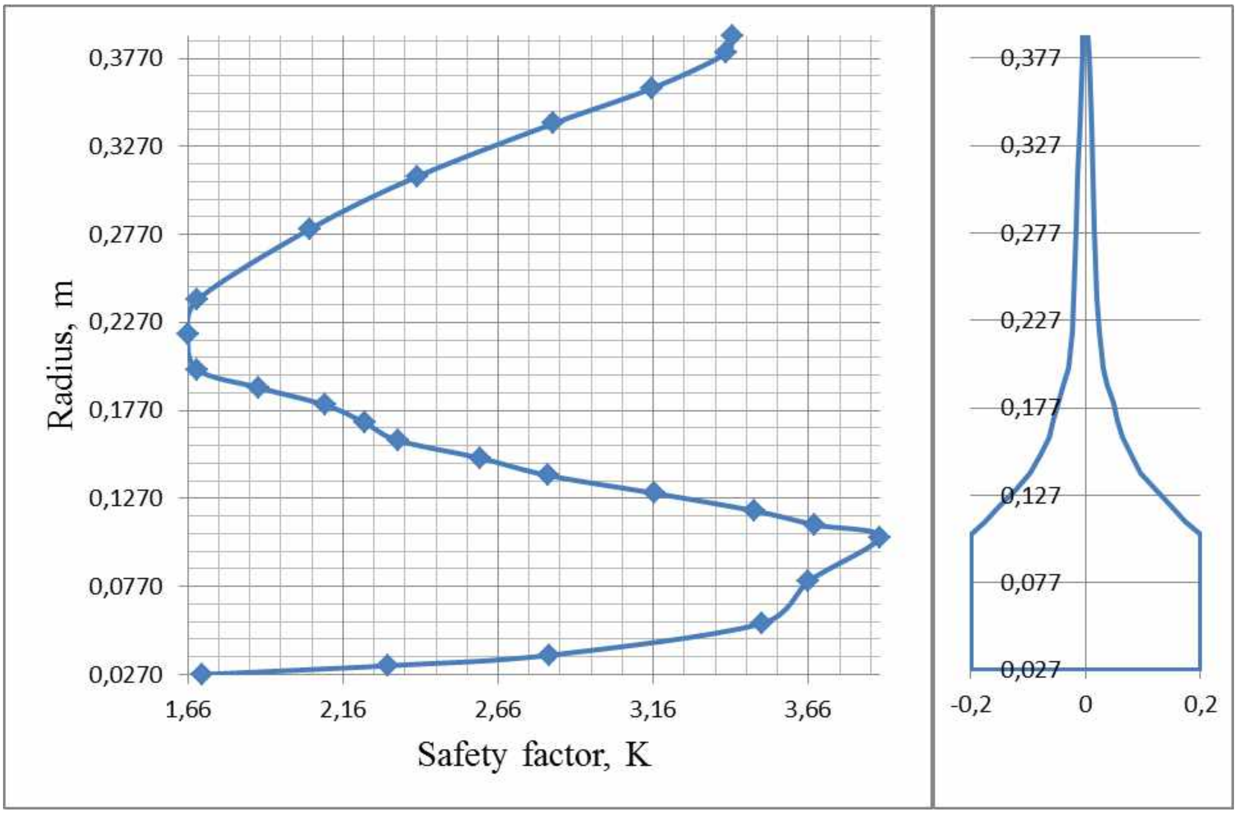
0,2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0,1770
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0,0770
0,0270
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2,66
3,16
3,66
Safety factor, K
0,377
0,327
0,277
0,227
0,177
0,127
0,077
0,027
-0,2
0
0,2

Навчальне видання

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АНАЛІЗ МІЦНОСТІ ДИСКА
(Англійською мовою)

Редактор Н. Б. Зюбанова

Технічний редактор Л. О. Кузьменко

Зв. план, 2014
Підписано до друку 14.07.2014
Формат 60x84 1/16. Папір офс. № 2. Офс. друк
Ум. друк. арк. 1,4. Обл.-вид. арк. 1,62. Наклад 80 пр. Замовлення 276.
Ціна вільна

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Свідоцтво про внесення суб'єкта видавничої справи
до Державного реєстру видавців, виготовлювачів і розповсюджувачів
видавничої продукції сер. ДК № 391 від 30.03.2001