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National Aerospace University “Kharkiv Aviation Institute”

Andriy Okhrimovskyy, Oksana Podshyvalova, Daniel Voronovich

WAVE OPTICS AND MODERN PHYSICS

Guidance Manual for Recitation

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О-39

Подано варіанти задач для дев'яти практичних занять з фізики, що охоплюють теми "Хвильова оптика", "Спеціальна теорія відносності", "Теплове випромінювання", "Фотони", "Основи квантової механіки" і "Вступ до ядерної фізики". До кожної теми наведено таблицю з формулами.

Для англомовних студентів.

Reviewers: Dr.of Science in Physics & Mathematics, prof. A. Ermolaev,
Dr.of Technical Science, prof. O. Chernikov

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Okhrimovskyy, A.M.

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Manual provide a set of problems to nine recitation classes offered by the Physics department of the National Aerospace University "Kharkiv Aviation Institute". Guidance embraces the following topics: "Wave Optics", "Special Theory of Relativity", "Thermal Radiation", "Photons", "Principles of Quantum Mechanics" and "Introduction to Nuclear Physics". Each chapter is supplied by a table with basic equations.

For english-speaking students.

Figs. 4. Tables 9. Bibl.: 8 items

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Podshyvalova O.V.,
Voronovich D.O., 2014

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INTRODUCTION

Problem solving is an important part of studying physics. It is evident that there are better chances of a student's understanding a topic if he or she can apply his or her knowledge in more than one way. The course is arranged so that a student meets a given topic in a variety of ways: in reading assignments in the text [1–4], in demonstration lectures, in supplementary notes issued to students, in recitation and problem drill in problem sessions [5, 6], in both the study and the performance of laboratory experiments [7], in homework problem sets, and in quizzes and examinations. These various types of presentation are synchronized so that, it is hoped, their impact on the student will have a maximum effectiveness.

Recitation can be an exciting part of the course, or it can be drudgery, depending upon your attitude toward it. If you regard it merely as an impediment to your getting through the course, probably, you will not enjoy it and, furthermore, derive very little benefit from it. On the other hand, if you approach the class with the thought that it is an opportunity to learn, and with a desire to make the most out of it, then it is almost certain you will find the time you spend on it both profitable and interesting.

This manual is a logical continuation of the guidance manuals for the recitations "Mechanics and Thermodynamics" [5] and "Electricity and Magnetism" [6] and it concludes the cycle of manuals for the course "Experimental and Theoretical Physics". It offers a wide range of problems covering the fields of *wave optics*, *special theory of relativity*, *thermal radiation*, *photons*, *principles of quantum mechanics* and *introduction to nuclear physics*. We chose to use once proved structure, with a table containing main definitions and physical laws at the beginning of every chapter and references to corresponding chapters in the textbooks [1–3]. At the end of the manual the appendix material represents fundamental constants and tables of physical data as well as the mathematical reviews of calculus and vector algebra. Most of the problems were taken from the textbooks [1–3], and students can find examples of how to solve typical problems there. Some exercises were taken from the famous problem book [8]. Those who want to be sure that their solutions are correct can compare obtained results with numerical answers given near the end of the manual.

We would like to thank our reviewers and colleagues at Physics department, especially Professor Anatoliy Taran, for stimulating discussions about physics pedagogy and friendly criticism of our work.

We welcome suggestions and comments from our readers and wish our students great success in studying physics.

Chapter 1
ELECTROMAGNETIC WAVES

Equation number	Equation	Equation title	Comments
1	2	3	4
1.1	$\mathrm{div}\vec{\boldsymbol{D}} = \rho_{free}$ $\mathrm{div}\vec{\boldsymbol{B}} = 0$ $\mathrm{rot}\vec{\boldsymbol{E}} = -\dfrac{\partial\vec{\boldsymbol{B}}}{\partial t}$ $\mathrm{rot}\vec{\boldsymbol{H}} = \vec{\boldsymbol{j}}_{cond} + \dfrac{\partial\vec{\boldsymbol{D}}}{\partial t}$	Maxwell's equations in differential form	$\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ are electric and magnetic fields, respectively; ρ_{free} is a volume density of free charge; $\vec{\boldsymbol{j}}_{cond}$ is a conduction current density; see details for div and rot in Appendix
1.2	$\vec{\boldsymbol{D}} = \varepsilon_r\varepsilon_0\vec{\boldsymbol{E}}$ $\vec{\boldsymbol{B}} = \mu_r\mu_0\vec{\boldsymbol{H}}$ $\vec{\boldsymbol{j}}_{cond} = \sigma\vec{\boldsymbol{E}}$	Constitutive relations	ε_r and μ_r are relative permittivity and permeability of the substance, respectively; σ is a conductivity
1.3	$\dfrac{\partial^2\vec{\boldsymbol{E}}}{\partial t^2} = \dfrac{1}{\varepsilon_r\varepsilon_0\mu_r\mu_0}\Delta\vec{\boldsymbol{E}}$ $\dfrac{\partial^2\vec{\boldsymbol{H}}}{\partial t^2} = \dfrac{1}{\varepsilon_r\varepsilon_0\mu_r\mu_0}\Delta\vec{\boldsymbol{H}}$	Wave equations for $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{H}}$ fields in 3D case	$\Delta = \nabla^2 = \vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{\nabla}}$ is a Laplace operator (see Appendix for details)

1	2	3	4
1.4	$\dfrac{\partial^2\vec{\boldsymbol{E}}}{\partial t^2} = v^2\dfrac{\partial^2\vec{\boldsymbol{E}}}{\partial x^2}$	Wave equation of a plane wave propagating along x axis	$v = \dfrac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r}}$ is a propagation speed of the electromagnetic wave
1.5	$c = \dfrac{1}{\sqrt{\varepsilon_0\mu_0}}$	Speed of electromagnetic wave in vacuum	Speed of light in vacuum; ε_0 and μ_0 are electric end magnetic constants, respectively
1.6	$n = \dfrac{c}{v} = \sqrt{\varepsilon_r\mu_r}$	Refractive index of a substance	v is a speed of electromagnetic wave in the substance
1.7	$\vec{\boldsymbol{E}} = \vec{\boldsymbol{B}} \times \vec{\boldsymbol{v}}$ $\vec{\boldsymbol{H}} = \vec{\boldsymbol{v}} \times \vec{\boldsymbol{D}}$	Instantaneous electric and magnetic fields for an electromagnetic wave	$\vec{\boldsymbol{v}}$ is a propagation velocity of the electromagnetic wave
1.8	$u_{EMW} = \vec{\boldsymbol{E}}\cdot\vec{\boldsymbol{D}} = \vec{\boldsymbol{B}}\cdot\vec{\boldsymbol{H}}$	Energy density for a propagating electromagnetic wave	Energy stored by the electromagnetic wave in a unit volume
1.9	$\vec{\boldsymbol{S}} = u_{EMW}\vec{\boldsymbol{v}} = \vec{\boldsymbol{E}} \times \vec{\boldsymbol{H}}$	Energy flux density of an electromagnetic wave (Poynting vector)	Power delivered by the wave per unit area perpendicular to the direction of propagation

1	2	3	4
1.10	$E = E_0 \cos(kx - \omega t)$ $B = B_0 \cos(kx - \omega t)$	Electric and magnetic fields for a plane harmonic wave propagating in $+x$ direction	k and ω are wave number and angular frequency, respectively
1.11	$v = \dfrac{c}{n} = \dfrac{\omega}{k} = \lambda f$	Propagation speed of a harmonic wave	$\lambda = \frac{2\pi}{k}$ is a wavelength; $f = \frac{\omega}{2\pi}$ is a frequency of the wave
1.12	$I = \langle \lvert\vec{\boldsymbol{S}}\rvert \rangle \sim \lvert\vec{\boldsymbol{E}}_0\rvert^2$	Intensity of an electromagnetic wave	Average power delivered by the wave per unit area

Pre-Class Reading: [1], *chap.*29.7&32;[2], *chap.*29.5&34;[3], *chap.*34.

Case 1.1

1.1.1. If an electric field for a plane wave (propagating in vacuum) is given by $E_x = 0$, $E_z = E_0 \cos(ky + \omega t)$, what are $\vec{\boldsymbol{B}}$ and the direction of propagation of the wave?

1.1.2. An FM radio station announcer identifies the station as “R99”, the 99 standing for frequency in some units. What is (a) the wavelength and (b) the frequency of the waves emitted by the radio station?

1.1.3. The electric field of a sinusoidal electromagnetic wave obeys the equation $E = -(375 \text{ V/m}) \sin\left[(5.97 \cdot 10^{15} \text{ rad/s})\ t + +(1.99 \cdot 10^{7} \text{ rad/m})\ x\right]$. (a) What are the amplitudes of the electric and magnetic fields for this wave? (b) What are the frequency, wavelength, and period of the wave? Is this light visible to humans? (c) What is the speed of the wave?

1.1.4. A sinusoidal electromagnetic wave propagates in a vacuum in the $+z$-direction. If at a particular instant and at a certain point in space the electric field is in the $+x$-direction and has a magnitude of 4.00 V/m, what are the magnitude and direction of the magnetic field of the wave at this same point in space and instant in time?

1.1.5. In a region of free space, the electric field at an instant of time is $\vec{\boldsymbol{E}} = (80.0\vec{\boldsymbol{i}} + 32.0\vec{\boldsymbol{j}} - 64.0\vec{\boldsymbol{k}})$ V/m and the magnetic field is $\vec{\boldsymbol{B}} = (0.200\vec{\boldsymbol{i}} + 0.080\vec{\boldsymbol{j}} + 0.290\vec{\boldsymbol{k}})$ μT. (a) Show that the two fields are perpendicular to each other. (b) Determine the Poynting vector for these fields.

Case 1.2

1.2.1. What is the relation between the amplitudes of the electric and magnetic fields for an electromagnetic wave propagating in a medium whose relative dielectric permittivity constant is ε_r? Assume the magnetic permeability of the medium is that of the vacuum.

1.2.2. A plane harmonic wave of electromagnetic radiation with the wavelength λ propagates in the $-x$-direction. The z-component of the electric field has an amplitude E_0, and there is no y-component. (a) Write an expression for the electric field. (b) Use this expression to calculate magnetic field.

1.2.3. At a certain location on the Earth, the rms value of the magnetic field caused by solar radiation is 1.80 μT. Using this value, calculate (a) the rms electric field due to solar radiation, (b) the average energy density of the solar component of electromagnetic radiation in this place, and (c) the average magnitude of the Poynting vector for the Sun radiation.

1.2.4. An electromagnetic wave has an electric field given by $\vec{\boldsymbol{E}}(y,t) = -(3.10 \cdot 10^5 \text{ V/m})\,\vec{\boldsymbol{k}} \sin\left[ky - (4\pi \cdot 10^{12} \text{ rad/s})t\right]$. (a) In which direction is the wave traveling? (b) What is the wavelength of the wave? (c) Write the vector equation for $\vec{\boldsymbol{B}}(y,t)$.

1.2.5. In SI units, the electric field for an electromagnetic wave is described by $E_y = 100 \sin(1.00 \cdot 10^7 x - \omega t)$. Find (a) the amplitude of corresponding magnetic field oscillations, (b) the wavelength λ, and

(c) the frequency f.

Case 1.3

1.3.1. A sinusoidal electromagnetic wave having a magnetic field of amplitude 1.25 μT and a wavelength of 432 nm is traveling in the $+x$-direction through the empty space. (a) What is the frequency of this wave? (b) What is the amplitude of the associated electric field? (c) Write the equations for the electric and magnetic fields as functions of x and t in the form of Eqs. (1.10).

1.3.2. An electromagnetic wave has a magnetic field given by $\vec{\boldsymbol{B}}(x,t) = (8.25 \cdot 10^{-9}\ \mathrm{T})\,\vec{\boldsymbol{j}}\sin\big[(1.38 \cdot 10^4\ \mathrm{rad/m})x + \omega t\big]$. (a) In which direction is the wave traveling? (b) What is the frequency f of the wave? (c) Write the vector equation for $\vec{\boldsymbol{E}}(x,t)$.

1.3.3. A plane electromagnetic wave propagates in vacuum along the direction in xy-plane that makes an angle θ with the y-axis. The amplitude of the electric field of this wave is E_0. Find the space-time dependence of the electric field for this wave if the wave number is k.

1.3.4. Why is the following situation impossible? An electromagnetic wave travels through empty space with electric and magnetic fields described by

$$E = 9.00 \cdot 10^3 \cos\big[(9.00 \cdot 10^6)x - (3.00 \cdot 10^{15})t\big],$$
$$B = 3.00 \cdot 10^{-5} \cos\big[(9.00 \cdot 10^6)x - (3.00 \cdot 10^{15})t\big],$$

where all numerical values and variables are in SI units.

1.3.5. Important news are transmitted by radio waves to people sitting next to their radios 100 km from the station and by sound waves to people sitting across the newsroom 3.00 m from the newscaster. Taking the speed of sound in air to be 343 m/s, who receives the news first? Explain.

Case 1.4

1.4.1. An electromagnetic wave of wavelength 600 nm propagates in the z-direction. The magnetic field points in the y-direction, and has the amplitude of 10^{-8} T. Write an expression for the electric field.

Assume that the magnetic field is maximum at $z = 0$ m, $t = 0$ s.

1.4.2. An electromagnetic wave of wavelength 435 nm travels in vacuum in the $-z$-direction. The electric field has the amplitude of $2.70 \cdot 10^{-3}$ V/m and is parallel to the x-axis. What are (a) the frequency and (b) the amplitude of the magnetic-field ? (c) Write the vector equations for $\vec{\boldsymbol{E}}(z,t)$ and $\vec{\boldsymbol{B}}(z,t)$.

1.4.3. An electromagnetic wave with frequency 65.0 Hz travels in an insulating magnetic material that has relative permittivity constant 3.64 and relative permeability constant 5.18 at this frequency. The electric field has the amplitude of $7.20 \cdot 10^{-3}$ V/m. (a) What is the speed of propagation of the wave? (b) What is the wavelength of the wave? (c) What is the amplitude of the magnetic field? (d) What is the intensity of the wave?

1.4.4. If the intensity of sunlight is 1000 W/m^2 at the Earth's surface under a fairly clear sky, how much electromagnetic energy per cubic meter does the sunlight contain?

1.4.5. In 1965, Arno Penzias and Robert Wilson discovered cosmic microwave radiation left over from the big bang expansion of the Universe. Suppose the energy density of this background radiation to be $4.00 \cdot 10^{-14}$ J/m^3. Determine the corresponding electric field amplitude.

Case 1.5

1.5.1. An electromagnetic wave of frequency $f = 3.0$ MHz passes from vacuum into a non-magnetic medium with relative permittivity $\varepsilon_r = 4.0$. Find the increment of its wavelength.

1.5.2. A sinusoidal electromagnetic wave of frequency $3.00 \cdot 10^{14}$ Hz travels in vacuum in the $+z$-direction. The $\vec{\boldsymbol{B}}$-field is parallel to the y-axis and has the amplitude $6.00 \cdot 10^{-4}$ T. Write the vector equations for $\vec{\boldsymbol{E}}(z,t)$ and $\vec{\boldsymbol{B}}(z,t)$.

1.5.3. Consider each of the following electric and magnetic-field orientations. In each case, what is the direction of propagation of the wave? (a) $\vec{\boldsymbol{E}} = E\vec{\boldsymbol{i}}$, $\vec{\boldsymbol{B}} = -B\vec{\boldsymbol{j}}$; (b) $\vec{\boldsymbol{E}} = E\vec{\boldsymbol{j}}$, $\vec{\boldsymbol{B}} = B\vec{\boldsymbol{i}}$; (c) $\vec{\boldsymbol{E}} = -E\vec{\boldsymbol{k}}$,

$\vec{\boldsymbol{B}} = -B\vec{\boldsymbol{i}}$; (d) $\vec{\boldsymbol{E}} = E\vec{\boldsymbol{i}}, \vec{\boldsymbol{B}} = -B\vec{\boldsymbol{k}}$.

1.5.4. What is the average magnitude of the Poynting vector 5.0 km from a radio transmitter broadcasting isotropically (equally in all directions) with an average power of 314 kW?

1.5.5. Assume the intensity of solar radiation incident onto the cloud tops of the Earth to be 1370 $\mathrm{W/m^2}$. (a) Taking the average Earth–Sun separation to be $1.496 \cdot 10^{11}$ m, calculate the total power radiated by the Sun. Determine the maximum values of (b) the electric field and (c) the magnetic field for the sunlight at the Earth's location.

Chapter 2
INTERFERENCE

Equation number	Equation	Equation title	Comments
1	2	3	4
2.1	$n = \frac{c}{v}$	Refractive index of a substance	c and v are speed of light in vacuum and substance, respectively
2.2	$\lambda = \frac{\lambda_0}{n}$	Wavelength in a substance	λ_0 is a wavelength in vacuum
2.3	$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$	Intensity under interference from two sources	I_1 and I_2 are intensities from individual sources
2.4	$\delta = \frac{2\pi}{\lambda_0}\Delta = k_0\Delta$	Phase difference	k_0 is a wavenumber
2.5	$\Delta = n_2 r_2 - n_1 r_1$	Optical path difference	r_1 and r_2 are paths traveled by waves in substances with refractive indexes n_1 and n_2, respectively
2.6	$\Delta_{\max} = \lambda_0 m = 2m\frac{\lambda_0}{2}$	Condition for constructive interference	m is an integer; waves are in phase with each other

1	2	3	4
2.7	$\Delta_{\min} = (2m+1)\dfrac{\lambda_0}{2}$	Condition for destructive interference	waves are completely out of phase
2.8	$\Delta = d\sin\theta$	Path difference under double slit interference (Young's experiment)	d is a distance between slits; θ is an angular position of a point on the screen with respect to a normal to the screen
2.9	$\Delta = 2nd\cos\beta \pm \dfrac{\lambda_0}{2}$	Optical path difference under interference on a thin film in air	d is a thickness of the film; β is an angle of refraction; n is a refractive index
2.10	$r_m = \sqrt{R\lambda m}$	Radius of an m-th dark Newton's ring in reflected light	R is a radius of a curvature of the convex lens

Pre-Class Reading: [1], *chap.* 35;[2], *chap.* 37;[3], *chap.* 37.

Case 2.1

2.1.1. Two coherent sources, A and B, of radio waves are 40.0 cm apart. Each source emits waves with wavelength 50.0 cm. Consider points along the line between the two sources. At what distances from A, if any, is the interference (a) constructive, (b) destructive?

2.1.2. A coherent source of monochromatic light of unknown wavelength shines on double slits separated by 0.40 mm. Bright spots separated by 0.80 cm appear on a screen 5.0 m away. What is the wavelength of the light?

2.1.3. In a two-slit interference pattern, the intensity at the peak of the central maximum is I_0. (a) What is the intensity at a point in the pattern where the phase difference between the waves from the two slits is 90.0°? (b) What is the path difference for 640-nm light from the two slits at a point where the phase angle is 90.0°?

2.1.4. What is the thinnest film of a coating with $n = 1.40$ on glass ($n = 1.50$) for which destructive interference of the 560 nm component of an incident white light beam in air can take place by reflection?

2.1.5. Laser light with $\lambda = 694$ nm enters a Michelson interferometer. How many fringes will pass through the field of view if one of the mirrors is moved for 0.35 mm?

Case 2.2

2.2.1. Two radio antennas, A and B, radiate in phase. Antenna B is 105 m to the right of antenna A. Consider point P along the extension of the line connecting the antennas, a horizontal distance of 49 m to the right of antenna B. The frequency and, hence, the wavelength of the emitted waves can be varied. (a) What is the longest wavelength for which there will occur destructive interference at point P? (b) What is the longest wavelength for which there will occur constructive interference at point P?

2.2.2. Red light ($\lambda = 0.64$ μm) shines on a double slit separation $d = 0.2$ mm. How far away from the central axis will the first minimum be on a screen situated 3.0 m from the double slit?

2.2.3. Coherent sources A and B emit electromagnetic waves with wavelength 2.20 mm. Point Q is 3.65 cm from A and 4.20 cm from B. What is the phase difference between these two waves at Q?

2.2.4. Two rectangular pieces of plane glass are laid one upon the other on a table. A thin strip of paper is placed between them at one edge so that a very thin wedge of air is formed. The plates are illuminated at normal incidence by 0.546-μm light from a mercury-vapor lamp. Interference fringes are formed, with 2.0 fringes per milliliter. Find the angle of the wedge.

2.2.5. A very sharp wedge of glass of index of refraction 1.50 is introduced perpendicularly in the path of one of the interfering beams of Michelson interferometer illuminated by a narrow beam of light of wavelength 0.500 μm. This causes 600 dark fringes to sweep across the field of view. Calculate the thickness of the glass wedge at the point where the beam passes through it.

Case 2.3

2.3.1. A radio transmitting station operating at a frequency of 100 MHz has two identical antennas that radiate in phase. Antenna B is 12.0 m to the right of antenna A. Consider point Q between the antennas and along the line connecting them, a horizontal distance x to the right of antenna A. For what values of x will constructive interference occur at point Q?

2.3.2. Light of wavelength 600 nm falls onto a wall with two slits 0.1 mm apart. A photographic plate is placed at a distance L from the wall. The $m = 4$ maximum appears 2.4 cm from the central maximum on the photographic plate. How far is the plate from the wall?

2.3.3. Two slits spaced 0.250 mm apart are placed 0.800 m from a screen and illuminated by coherent light with a wavelength of 750 nm. The intensity at the center of the central maximum ($\theta = 0°$) is I_0. (a) What is the distance from the center of the central maximum to the first minimum on the screen? (b) What is the distance from the center of the central maximum to the point where the intensity has fallen to $I_0/4$ on the screen?

2.3.4. A plastic film with index of refraction 1.75 is put onto the surface of a car window to increase the reflectivity and thus to keep the interior of the car cooler. The window glass has index of refraction 1.50. (a) What minimum thickness is required if light with wavelength 0.560 μm in air reflected from the two sides of the film is to interfere constructively? (b) It is found to be difficult to manufacture and install coatings as thin as calculated in part (a). What is the next greatest thickness for which there will be constructive interference?

2.3.5. Identical glass tubes, each 5.0 cm long, are placed in the two optical path of Michelson interferometer. He-Ne laser ($\lambda = 6328$ Å) is used as a source of coherent light. Both tubes are evacuated, then one is slowly filled with gas until it reaches atmospheric pressure. During the filling time, the pattern has moved by 100 fringes. What is the index of refraction of the gas?

Case 2.4

2.4.1. Two light sources can be adjusted to emit monochromatic light of any visible wavelength. The sources are coherent, 1.98 μm apart, and in line with an observer, so that one source is 1.98 μm farther from the observer than the other. (a) For what visible wavelengths (400 to 700 nm) will the observer see the brightest light, owing to constructive interference? (b) How would your answer to part (a) be affected if the two sources were not in line with the observer, but were still arranged so that one source is 1.98 μm farther away from the observer than the other? (c) For what visible wavelengths will there be destructive interference at the place of the observer?

2.4.2. A double-slit experiment produces fringes on a distant screen. How does the linear separation between the bright maxima on the screen change when (a) the wavelength of the light triples? (b) The separation between the slits triples? (c) The distance between the slits and the screen triples? (d) The intensity of the light triples?

2.4.3. The angular width of a maximum of the intensity due to double-slit interference is defined to be the angular separation $\Delta\theta$ of the points where the intensity is half its maximum value. (a) Express the width of the central maximum in terms of the wavelength λ and the slit separation d. (b) Are the widths of all the maxima the same?

2.4.4. The walls of a soap bubble have about the same index of refraction as that of plain water, $n = 1.333$. There is air both inside and outside the bubble. (a) What wavelength (in air) of visible light is most strongly reflected from a point on a soap bubble where its wall is 0.270 μm thick? To what color does this correspond (see Table in

Appendix) (b) Repeat part (a) for a wall thickness of 0.330 μm.

2.4.5. A thin glass plate with index of refraction 1.50 is introduced into one of the beams of a Michelson interferometer. This causes a displacement of 40 fringes for light with $\lambda = 0.550$ μm. Determine the thickness of the plate.

Case 2.5

2.5.1. Two speakers, 3.50 m apart, are driven by the same audio oscillator so that each produces a sound consisting of two distinct frequencies, 750 Hz and 500 Hz. The speed of sound is 350 m/s. Find all the angles relative to the usual center line in front of (and far from) the speakers at which both frequencies interfere constructively.

2.5.2. Light of two different wavelengths, λ_1 and λ_2, is incident on a double slit. On a distant screen, the twentieth maximum of λ_1 overlies the nineteenth minimum of λ_2. Show that the relative difference $(\lambda_2 - \lambda_1)/\lambda_1$ is small, and find a numerical value for this ratio.

2.5.3. Two point sources of radio waves, 10 m apart, radiate in phase with a frequency of $4.0 \cdot 10^7$ Hz. (a) If the average intensity of each single source is $5.0 \cdot 10^{-4}$ W/m^2 at a certain distance, what is the direction in which the combined intensity is maximum? (b) What is the magnitude of the maximum intensity? (c) At what angle will the intensity fall to half its maximum value?

2.5.4. At normal incidence white light reflects from the top and bottom surfaces of a sapphire plate ($n = 1.77$). There is air both above and below the plate. Constructive interference is observed for light which wavelength in air is 550 nm. How thick is the plate if the next longer wavelength for which there is constructive interference is 650 nm?

2.5.5. One leg of a Michelson interferometer contains an evacuated cylinder of length L, with glass plates on each end. Gas is slowly leaked into the cylinder until a pressure of 1 atm is reached. If N bright fringes pass on the screen during this process when light of wavelength λ is used, what is the index of refraction of the gas?

Chapter 3
DIFFRACTION

Equation number	Equation	Equation title	Comments
1	2	3	4
3.1	$r_m = \sqrt{\dfrac{ab}{a+b}\lambda m}$	Radius of m-th Fresnel's zone	a and b are distances from source and screen to the wave surface, respectively
3.2	$w \sin\theta_m = m\lambda$	Condition for minima under diffraction on a single slit	w is a width of the slit; θ_m is an angular position of the m-th minimum
3.3	$I_1 = I_0 \dfrac{\sin^2 \dfrac{\beta}{2}}{\left(\dfrac{\beta}{2}\right)^2}$	Intensity under a single slit diffraction	$\beta = 2\pi \dfrac{w \sin\theta}{\lambda}$ is a phase difference between the waves from edges of the slit; θ is a diffraction angle; I_0 is an intensity at the center of the diffraction pattern

1	2	3	4
3.4	$d\sin\theta_m = m\lambda$	Condition for principal maxima under diffraction on a diffraction grating	d is a diffraction grating spacing; θ_m is an angular position of the m-th maximum
3.5	$I_N = I_1 \dfrac{\sin^2\left(\dfrac{N\delta}{2}\right)}{\sin^2\dfrac{\delta}{2}}$	Intensity under diffraction on a diffraction grating with N slits	$\delta = 2\pi\dfrac{d\sin\theta}{\lambda}$ is a phase difference between waves from adjacent slits; for I_1 see Eq. (3.3)
3.6	$2d\sin\theta_m = m\lambda$	Condition for maxima at X-rays diffraction from a crystal (Bragg's law)	d is a distance between adjacent atomic planes; θ_m is a takeoff angle of reflected X-rays

Pre-Class Reading: [1], *chap.* 36;[2], *chap.* 38;[3], *chap.* 38.

Case 3.1

3.1.1. Monochromatic light from a distant source is incident on a slit 0.60 mm wide. On a screen 3.00 m away, the distance from the central maximum of the diffraction pattern to the first minimum is measured to be 2.40 mm. Calculate the wavelength of the light.

3.1.2. Monochromatic light of wavelength $\lambda = 0.60$ μm from a distant source passes through a slit 0.50 mm wide. The diffraction pattern is observed on a screen 5.00 m from the slit. In terms of the intensity I_0 at the peak of the central maximum, what is the intensity of the light on the screen at the following distances from the center of the central maximum: (a) 1.00 mm; (b) 3.00 mm; (c) 10.00 mm?

3.1.3. An interference pattern is produced by eight parallel and

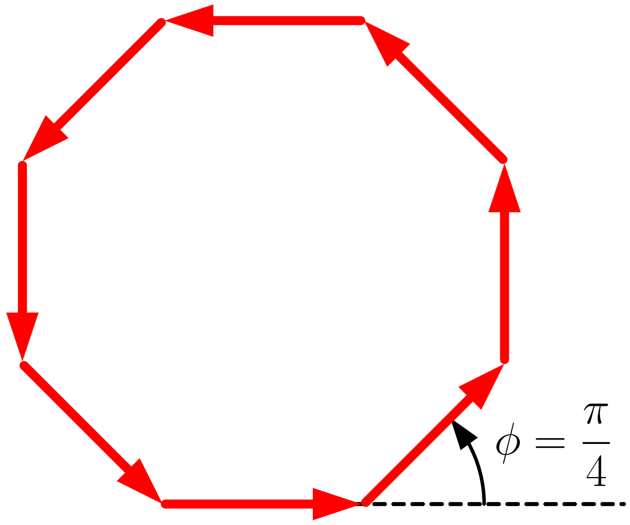

Figure 3.1. Problem 3.1.3

equally spaced narrow slits. There is an interference minimum when the phase difference ϕ between light from adjacent slits is $\pi/4$. The phasor diagram is given in Fig. 3.1. For which pairs of slits does totally destructive interference occur?

3.1.4. Laser light of wavelength 0.600 μm illuminates two identical slits, producing an interference pattern onto a screen 1.00 m from the slits. Bright bands are 2.00 mm apart, and the fourth bright bands on either side of the central maximum are missing in the pattern. Find (a) the separation of two slits and (b) the width of the slit.

3.1.5. The distance between neighboring pairs of Bragg planes in calcite ($CaCO_3$) is 0.3 nm. At what angles to these planes will the first- and second-order diffraction peaks occur for X-rays of wavelength 4.50 Å? (1 Å $= 10^{-10}$ m = 1 Ångstrom unit.)

Case 3.2

3.2.1. A single slit of width $2.5 \cdot 10^{-5}$ m diffracts light of wavelength 500 nm to a screen. The distance between the minima on either side of the central maximum is 4.0 cm. How far is the screen?

3.2.2. A slit 0.260 mm wide is illuminated by parallel light rays of wavelength 520 nm. The diffraction pattern is observed on a screen that is 1.50 m from the slit. The intensity at the center of the central maximum ($\theta = 0°$) is $9.87 \cdot 10^{-6}$ W/m^2. (a) What is the distance between the center of the central maximum and the first minimum on the screen? (b) What is the intensity at a point on the screen midway between the center of the central maximum and the first minimum?

3.2.3. An interference pattern is produced by light of wavelength 0.64 μm from a distant source incident on two identical parallel slits separated by a distance (between centers) of 0.80 mm. (a) If the slits are very narrow, what would be the angular positions of the first-order and second-order, two-slit, interference maxima? (b) Let the slits be 0.600 mm wide. In terms of the intensity I_0 at the center of the central maximum, what is the intensity at each of the angular positions in part (a)?

3.2.4. A laser beam of wavelength $\lambda = 6328$ Å shines onto the reflective side of a compact disc at normal incidence. The tracks of tiny pits in which information is coded onto the CD are 1.60 μm apart. For what angles of reflection (measured from the normal) will the intensity of light be maximum?

3.2.5. Consider a NaCl crystal consisting of identical cubes with atoms at the vertices. The spacing between adjacent atoms is 2.81 Å. X-rays of wavelength 0.166 nm scatter elastically from a set of planes parallel to the face of the cubes. At what angle will the first-order Brag diffraction be observed?

Case 3.3

3.3.1. Microwave of wavelength 1.73 cm are perpendicularly incident on a single slit of 3.00 cm wide, and then onto a screen. (a) At what angles are maxima and minima found on the screen? (b) If the screen is 1.0 m away from the slit, what is the distance between the secondary and the central maximums?

3.3.2. Laser light of wavelength 6328 Å falls normally onto a slit that is 25.0 μm wide. Transmitted light is viewed on a distant screen so that the intensity is 8.0 W/m^2 at the center of the central bright fringe. (a) Find the maximum number of totally dark fringes on the screen, assuming the screen is large enough to show them all. (b) At what angle does the dark fringe that is most distant from the center occur? (c) What is the maximum intensity of the bright fringe that occurs immediately before the dark fringe in part (b)? Approximate the phase at which this fringe occurs by assuming it is midway between the phases of the dark fringes on either side of it.

3.3.3. Monochromatic light illuminates a pair of thin parallel slits at normal incidence, producing an interference pattern on a distant screen. The width of each slit is 1/6 of the center-to-center distance between the slits. (a) Which interference maxima are missing in the pattern on the screen? (b) Does the answer to part (a) depend on the wavelength of the light used? Does the location of the missing maxima depend on the wavelength?

3.3.4. Light of wavelength 550 nm is perpendicularly incident on a diffraction grating. Two adjacent maxima occur at $\sin\theta_1 = 0.33$ and $\sin\theta_2 = 0.385$, respectively. The fifth order is missing. (a) Find the separation distance between adjacent slits. (b) What is the smallest possible individual slit width? (c) Name all orders that appear on the screen, consistent with answer to parts (a) and (b).

3.3.5. A wavelength of 1.29 Å characterizes K_α X-rays from zinc. When a beam of these X-rays is incident on the surface of a crystal with a simple cubic lattice, a second-order maximum is observed at 30.0°. Calculate the interplanar spacing based on this information.

Case 3.4

3.4.1. Light of wavelength 633 nm falls onto a slit 33.3 μm wide. (a) On a very large distant screen, how many totally dark fringes (indicating complete cancellation) will there be, including both sides of the central bright spot? Solve this problem without calculating all the angles! (*Hint*: What is the largest that $\sin\theta$ can be? What does this tell you is the largest that m can be?) (b) At what angle will the dark fringe occur that is most distant from the central bright fringe?

3.4.2. A single-slit diffraction pattern is formed by monochromatic electromagnetic radiation from a distant source passing through a slit 0.0725 mm wide. The total phase difference between wavelets from the top and bottom of the slit is $14.5\pi = 45.55$ rad at the point $3.00°$ from the center of the central maximum. (a) What is the wavelength of the radiation? (b) What is the intensity at this point, if the intensity at the center of the central maximum is I_0?

3.4.3. An interference pattern is produced by four parallel and equally spaced, narrow slits. By drawing appropriate phasor diagrams, show that there is an interference minimum when the phase difference ϕ from adjacent slits is (a) $\pi/2$; (b) π; (c) $3\pi/2$. In each case, for which pairs of slits does totally destructive interference occur?

3.4.4. A slit width of a grating with 100 slits/mm is equal to one-quarter of the slit spacing. What is the ratio of the intensities of the second-order and first-order principal maxima of the grating?

3.4.5. If the planes of a crystal are 2.50 Å apart, (a) what wavelength of electromagnetic waves is needed so that the first strong interference maximum in the Bragg reflection occurs when the waves strike the planes at an angle of $20.0°$, and in what part of the electromagnetic spectrum do these waves lie? (b) At what other angle(s), if any, will strong interference maxima occur?

Case 3.5

3.5.1. Light waves pass through a slit and produce the first dark bands at $\pm 30.0°$ from the center of the diffraction pattern. For this waves the electric field is given by $E_y(x,t) = E_0 \sin[(1.256 \cdot 10^7\,\mathrm{m}^{-1})x - \omega t)]$, (a) What is the frequency of this light? (b) How wide is the slit? (c) At which angles will other dark bands occur?

3.5.2. In Kharkiv public radio station M-FM broadcasts at

90.0 MHz. The radio waves pass between two tall buildings that are 15.0 m apart along their closest walls. (a) At what horizontal angles, relative to the original direction of the waves, will a distant antenna not receive any signal from this station? (b) If the maximum intensity is 3.50 W/m^2 at the antenna, what is the intensity $\pm 30.0°$ from the center of the central maximum at the distant antenna?

3.5.3. Laser light of wavelength 600.0 nm illuminates two identical slits, producing an interference pattern on a screen 1.0 m from the slits. The bright bands are 1.00 cm apart, and the third bright bands on either side of the central maximum are missing in the pattern. Find the width and the separation of the two slits.

3.5.4. Light from an mercury (arc) lamp includes many different wavelengths. Two of these are $\lambda_1 = 579.07$ nm and $\lambda_2 = 576.96$ nm. You wish to resolve these spectral lines in first order using a grating 2.0 cm in length. What minimum number of slits per centimeter must the grating have?

3.5.5. Cu K_α radiation ($\lambda = 1.54056$ Å) is incident on a lanthanum hexaboride (LaB_6) crystal surface. The spacing between planes of atoms in LaB_6 is 4.156 Å. At what angle (relative to the crystal surface) should the beam be directed for a second-order maximum to be observed?

Chapter 4

ABSORPTION, POLARIZATION, AND DISPERSION

Equation number	Equation	Equation title	Comments
1	2	3	4
4.1	$I = I_0 e^{-\varkappa l}$	Intensity of light transmitted through a layer of substance (Bugger's law)	I_0 is an intensity of an incident light; $\varkappa$ is an absorption coefficient of the substance; l is a thickness of the layer
4.2	$I = I_0 \cos^2 \phi$	Intensity of polarized light transmitted through an ideal analyzer (Malus's law)	I_0 is an intensity of the incident polarized light; ϕ is an angle between the polarization direction of the incident light and the polarizing axis of the analyzer
4.3	$\tan \theta_P = \dfrac{n_2}{n_1} = n_{21}$	Condition for total polarization under reflection (Brewster's law)	θ_P is an angle of total polarization; n_{21} is a relative refractive index

1	2	3	4
4.4	$P = \dfrac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$	Degree of polarization	$I_{\max}$ and $I_{\min}$ are the maximum and minimum intensities of a partially polarized light propagated through an ideal polarizer, respectively
4.5	$n = \sqrt{1 + \dfrac{\omega_p^2}{\omega_0^2 - \omega^2}}$	Refractive index	ω is a frequency of an electromagnetic wave; ω_0 is a natural oscillation frequency of electrons
4.6	$\omega_p = e\sqrt{\dfrac{N_e}{m_e\varepsilon_0}}$	Plasma frequency	N_e is a number density of free electrons; m_e is a mass of an electron; e is an elementary charge

Pre-Class Reading: [1], *chap.* 33.5;[2], *chap.* 34.5;[3], *chap.* 38.6.

Case 4.1

4.1.1. What kind of polarization has a plane electromagnetic wave if the projections of the vector $\vec{\boldsymbol{E}}$ on the x and y axes are perpendicular to the propagation direction and are determined by the following equations: $E_x = E\cos(\omega t - kz)$, $E_y = E\cos(\omega t - kz)$.

4.1.2. At what angle should the axes of two ideal Polaroid sheets be placed to reduce the intensity of a given source of unpolarized light to (a) 3/5; (b) 3/8; (c) 1/4; (d) 1/8?

4.1.3. Polarized light of intensity $2.0 \cdot 10^6$ W/m^2 is incident on a Polaroid sheet placed perpendicular to the light beam with the polarizing axis of the sheet at an angle of 60° to the polarization vector of the light. What is the intensity of the beam after passing through the polarized sheet?

4.1.4. Light traveling in water strikes a plate made of transparent substance. Part of the beam is reflected and part is refracted. If the reflected portion is totally polarized and angle of refraction is 30.0°, what is the index of refraction of the substance the plate is made of?

4.1.5. Two plates, one of thickness $d_1 = 3.8$ mm and the other of thickness $d_2 = 9.0$ mm, are manufactured of a certain substance. When placed alternatively in the way of monochromatic light, the first plate transmits $\tau_1 = 0.84$ fraction of luminous flux and the second one, $\tau_2 = 0.70$. Find the coefficient of linear absorption of that substance. Light falls at right angles to the plates. Neglected secondary reflections.

Case 4.2

4.2.1. What kind of polarization has a plane electromagnetic wave if the projections of the vector $\vec{\boldsymbol{E}}$ on the x and y axes are perpendicular to the propagation direction and are determined by the following equations: $E_x = E\sin(\omega t - kz)$, $E_y = E\cos(\omega t - kz)$.

4.2.2. The axes of four ideal Polaroid sheets are stacked, each at 30° with respect to the previous one. What fraction of initially unpolarized light passes through all four sheets?

4.2.3. The beam from Problem 4.1.3, after passing through the Polaroid sheet described, passes through another Polaroid sheet. The polarizing axis of this sheet is at an angle of 90° to the original polarization vector. What is the final intensity of the beam?

4.2.4. The refractive index of an ice is 1.31. At what incident angle is light reflected from the surface of an ice slab completely polarized if the slab is immersed in (a) air and (b) benzene?

4.2.5. A beam of monochromatic light falls normally onto the surface of a plane-parallel plate of thickness l. The absorption coefficient of the substance the plate is made of varies linearly along the normal to its surface from $\varkappa_1$ to $\varkappa_2$. The coefficient of reflection at each surface is equal to ρ. Neglecting secondary reflection find the transmission coefficient of such plate.

Case 4.3

4.3.1. What kind of polarization has a plane electromagnetic wave if the projections of the vector $\vec{\boldsymbol{E}}$ on the x and y axes are perpendicular to the propagation direction and are determined by the following equations: $E_x = E\cos(\omega t - kz)$, $E_y = E\cos(\omega t - kz + \pi)$.

4.3.2. An electromagnetic wave passes through a sheet of Polaroid, and 1/4 of incident intensity gets through. Through what angle should one rotate the sheet of Polaroid to let all the radiation get through?

4.3.3. Unpolarized light of intensity 10.0 W/cm^2 is incident onto two polarizing filters. The axis of the first filter is at an angle of 77.0° counterclockwise from the vertical (viewed in the direction of the light traveling), and the axis of the second filter is at 32.0° counterclockwise from the vertical. What is the intensity of the light after it has passed through the second polarizer?

4.3.4. You use a sequence of ideal polarizing filters, each with its axis making the same angle with the axis of the previous filter, to rotate the plane of polarization of a polarized light beam by a total of 90°. You wish to have an intensity reduction no larger than 5.0%. (a) How many polarizers do you need to achieve your goal? (b) What is the angle between adjacent polarizers? (*Hint:* Use small angle expansion for cos function: $\cos x \approx 1 - \frac{1}{2}x^2$, where x is in radians.)

4.3.5. Find the free electron concentration in the ionosphere if its refractive index is equal to $n = 0.90$ for radio-waves of frequency

$f = 100$ MHz. Neglect the natural oscillation frequency of electrons.

Case 4.4

4.4.1. What kind of polarization has a plane electromagnetic wave if the projections of the vector $\vec{\boldsymbol{E}}$ on the x and y axes are perpendicular to the propagation direction and are determined by the following equations: $E_x = E\cos(\omega t - kz)$, $E_y = E\cos(\omega t - kz + \pi/4)$.

4.4.2. A beam of polarized light passes through a polarizing filter. When the angle between the polarizing axis of the filter and the direction of polarization of the light is 45°, the intensity of the emerging beam is I. If you now want the intensity to be $1.5I$, what should be the angle between the polarizing axis of the filter and the original direction of polarization of the light?

4.4.3. Three polarizing filters are stacked with polarizing axes of the second and third filters at 30.0° and 90.0°, respectively, with that of the first one. (a) If unpolarized light of intensity I_0 is incident on the stack, find the intensity of light emerging from each filter. (b) If the second filter is removed, what is the intensity of the light emerging from each remaining filter?

4.4.4. The critical angle for total internal reflection for sapphire surrounded by air is 34.4°. Calculate the polarizing angle for sapphire.

4.4.5. A sounding of dilute plasma by radiowaves of various frequencies reveals that radiowaves with wavelengths exceeding $\lambda = 0.75$ m experience total internal reflection. Find free electron concentration in that plasma neglecting their natural oscillation frequency.

Case 4.5

4.5.1. The degree of polarization of partially polarized light is $P = 0.25$. Find the ratio of intensities of both polarized and the natural components of this light.

4.5.2. A polarizer and an analyzer are oriented so that the maximum amount of light is transmitted. To what fraction of its maximum value is the intensity of the transmitted light reduced when the analyzer is rotated through (a) 30.0°; (b) 45.0°; (c) 60.0°?

4.5.3. Three polarizing filters are stacked, with the polarizing axis of the second and third filters at 30.0° and 60.0°, respectively, to that of the first one. If unpolarized light is incident on the stack, the light has intensity of 90.0 W/cm^2 after it passes through the stack. If the incident intensity is kept constant, what is the intensity of the light after it has passed through the stack if the second polarizer is removed?

4.5.4. For a particular transparent medium surrounded by air, find the polarizing angle θ_p in terms of the critical angle for total internal reflection θ_{cr}.

4.5.5. A certain birefringent material has indexes of refraction n_o and n_e for the two perpendicular components of linearly polarized light passing through it. The corresponding wavelengths are $\lambda_1 = \dfrac{\lambda_0}{n_o}$ and $\lambda_2 = \dfrac{\lambda_0}{n_e}$, where λ_0 is the wavelength in vacuum. (a) If the crystal is to function as a quarter-wave plate, the number of wavelengths of each component within the material must differ by $\dfrac{1}{4}$. Show that the minimum thickness for a quarter-wave plate is

$$d = \frac{\lambda_0}{4(n_o - n_e)}.$$

(b) Find the minimum thickness of a quarter-wave plate made of tourmaline if the indexes of refraction are $n_o = 1.669$ and $n_e = 1.638$, and the wavelength in vacuum is $\lambda_0 = 589.3$ nm.

Chapter 5
SPECIAL THEORY OF RELATIVITY

Equation number	Equation	Equation title	Comments
1	2	3	4
5.1	$x = \dfrac{x' + Vt'}{\sqrt{1-\beta^2}}$; $y = y'$; $z = z'$; $t = \dfrac{t' + \dfrac{V}{c^2}x'}{\sqrt{1-\beta^2}}$	Lorentz transformations of coordinates (x, y, z) and time t	V is a speed of the motion along x-axis; c is a speed of light in vacuum; (x', y', z', t') are coordinates and time in a moving reference frame; $\beta = V/c$
5.2	$v_x = \dfrac{v'_x + V}{1 + \dfrac{v'_x V}{c^2}}$	Relativistic velocity addition law	v_x and v'_x are particle velocities in different frames of reference; V is a relative speed of the frames of reference with respect to each other

1	2	3	4
5.3	$l = l_0\sqrt{1-\beta^2}$	Relativistic length contraction	l_0 is a proper length of the rod; l is a length of the rod measured in a reference frame moving with velocity V
5.4	$\Delta t = \dfrac{\Delta t_0}{\sqrt{1-\beta^2}}$	Relativistic time dilation	Δt_0 is a proper time interval between two events that occurs at the same point in space; Δt is a time interval observed in a reference frame moving with velocity V
5.5	$\dfrac{\lambda_e}{\lambda_o} = \dfrac{f_o}{f_e} = \dfrac{1+\dfrac{\vec{\boldsymbol{V}}\cdot\vec{\boldsymbol{n}}}{c}}{\sqrt{1-\beta^2}}$	Relativistic Doppler effect due to motion of emitter with velocity $\vec{\boldsymbol{V}}$ with respect to observer	λ_e (f_e) and λ_o (f_o) are emitted and observed wavelengths (frequencies) of light, respectively; $\vec{\boldsymbol{n}}$ is a unit vector from emitter to observer
5.6	$m = \dfrac{m_0}{\sqrt{1-\beta^2}}$	Relativistic mass of a particle that moves with velocity v	m_0 is a rest mass of the particle; $\beta = v/c$
5.7	$\vec{\boldsymbol{p}} = \dfrac{m_0\vec{\boldsymbol{v}}}{\sqrt{1-\beta^2}}$	Relativistic momentum	$\vec{\boldsymbol{v}}$ is a velocity of the particle

1	2	3	4
5.8	$E = mc^2 = \dfrac{m_0c^2}{\sqrt{1-\beta^2}}$	Relativistic energy of a particle	m is a relativistic mass of the particle
5.9	$E_K = E - E_0$	Kinetic energy of a particle	$E_0 = m_0c^2$ is a rest energy of the particle
5.10	$E^2 - p^2c^2 = m_0^2c^4$	Relation between the total relativistic energy and relativistic momentum of a particle	Left side of this relation is independent of the choice of the inertial reference frame

Pre-Class Reading: [1], *chap.* 37;[2], *chap.* 39;[3], *chap.* 39.

Case 5.1

5.1.1. The positive muon (μ^+), an unstable particle, lives on average $2.20 \cdot 10^{-6}$ s before decaying (measured in its own frame of reference). Such a particle moves with a speed of $0.900\,c$, with respect to the laboratory. (a) Determine its average lifetime in the laboratory? (b) What average distance, measured in the laboratory, does the particle move before decaying?

5.1.2. The diameter of our galaxy is about 10^5 l.y. or 10^{21} m. Suppose that a proton moves at a speed such that $\sqrt{1-(v^2/c^2)} \approx$ $\approx 10^{-7}$. Such speed corresponds to most energetic cosmic rays known. How long does it take the proton to cross the galaxy in (a) the galaxy's rest frame, (b) the proton's rest frame?

5.1.3. In a frame S' an observer moves to the right ($+x$-direction) at a speed $u = 0.600\,c$ away from a stationary observer in a frame S. The observer in S' measures the speed v' of a particle moving to the right away from her. What speed v does the observer in S measure for the particle if (a) $v' = 0.400\,c$; (b) $v' = 0.900\,c$; (c) $v' = 0.990\,c$?

5.1.4. (a) How fast must you be approaching a red traffic light ($\lambda_r = 675$ nm) for it to appear green ($\lambda_g = 500$ nm)? Express your answer in terms of the speed of light. (b) If you used this as a reason not to get a ticket for running a red light, how much of a fine would you get for speeding? Assume that the fine is \$ 1.00 for each kilometer per hour that your speed exceeds the posted limit of 90 km/h.

5.1.5. Assume here that the neutrino has a rest mass of zero. What is the momentum, in SI units, of a neutrino with energy 4 MeV? (The neutrino mass is actually small but nonzero.)

Case 5.2

5.2.1. (a) How fast must a rocket travel relative to the Earth so that the time in the rocket "slows down" to half its rate as measured by the Earth-based observers? (b) Do present-day jet planes approach such speeds?

5.2.2. A spaceship passes the control tower of a spaceport at a speed of $0.80\,c$. Automatic instruments in the control tower measure the length of the moving spaceship to be 360 m. What is the length of the spaceship once it lands (assuming you can wait long enough to take measurements).

5.2.3. A pursuit spacecraft from the planet Tatooine attempts to catch up with a Trade Federation cruiser. As measured by an observer on Tatooine, the cruiser travels away from the planet with a speed of $0.600\,c$. The pursuit ship travels at a speed of $0.800\,c$ relative to Tatooine, in the same direction as the cruiser. (a) For the pursuit ship to catch the cruiser, should the speed of the cruiser be positive or negative relative to the pursuit ship? (b) What is the speed of the cruiser relative to the pursuit ship?

5.2.4. Show that when the source of electromagnetic waves moves away from us at $0.600\,c$, the frequency we measure is half the value measured in the rest frame of the source.

5.2.5. A particle, the π^0, has the rest mass of 135 MeV/c^2. It decays at rest into two identical massless particles. What is the momentum of each of two decay products of π^0?

Case 5.3

5.3.1. A spaceship flies past Mars with a speed of $0.985\,c$ relative to the surface of the planet. When the spaceship is directly overhead, a signal light on the Martian surface blinks on and then off. An observer on Mars measures that the signal light was on for 75.0 μs. (a) Does the observer on Mars or the pilot on the spaceship measure the proper time? (b) What is the duration of the light pulse measured by the pilot of the spaceship?

5.3.2. A researcher has a device that can measure length to an accuracy of one part in 10^{12}. What is the minimum speed for which he could measure Lorentz contraction?

5.3.3. Two particles in a high-energy accelerator experiment are approaching each other head-on, each with a speed of $0.9520\,c$ as measured in the laboratory. What is the magnitude of the velocity of one particle relative to the other?

5.3.4. The sodium doublet refers to light waves emitted by sodium in a closely spaced pair of frequencies. The wavelength of this doublet are 589.0 nm and 589.6 nm. Suppose that the lower-wavelength member of the doublet is Doppler redshifted to a wavelength of 593.2 nm in the light emitted by a certain star. What happens to the wavelength of the second member of the doublet?

5.3.5. An electron that is accelerated in the Stanford Linear Accelerator in California has a total energy of 56 GeV. (a) How much of this is kinetic energy? (b) What is the momentum of the electron? (c) What is its speed?

Case 5.4

5.4.1. The negative pion (π^-) is an unstable particle with an average lifetime of $2.60 \cdot 10^{-8}$ s (measured in the rest frame of the pion). (a) If the pion is made to travel at a very high speed relative to a laboratory, its average lifetime is measured in the laboratory to be $4.20 \cdot 10^{-7}$ s. Calculate the speed of the pion expressed as a fraction of c. (b) What distance, measured in the laboratory, does the pion travel during its average lifetime?

5.4.2. A meter stick is tilted to make an angle of $60°$ with the x-axis. How will an observer, at rest in a frame F' that moves at velocity $v = 0.80\,c$ in $+x$-direction relative to the meter stick, describe the stick?

5.4.3. An enemy spaceship moves toward your starfighter with a

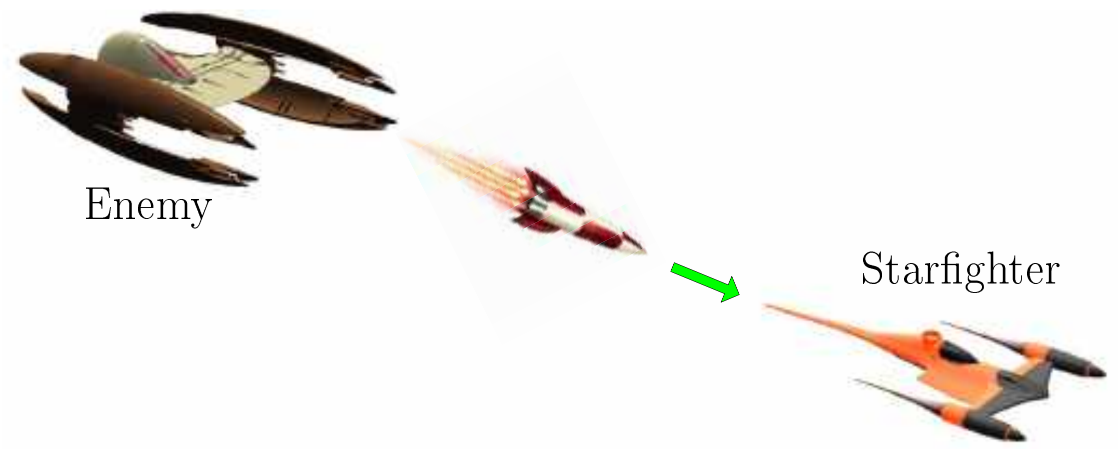

Figure 5.1. Problem 5.4.3

speed, as measured in your frame, of $0.400\,c$. The enemy ship fires a missile toward you at a speed of $0.700\,c$ relative to the enemy ship (Fig. 5.1). (a) What is the speed of the missile relative to you? Express your answer in terms of the speed of light. (b) If you measure that the enemy ship is $8.00 \cdot 10^6$ km away from you when the missile is fired, how much time, measured in your frame, will it take the missile to reach you?

5.4.4. The wavelength of a spectral line in the laboratory is measured to be 108 nm. The same line is observed in light coming from a distant galaxy: in this observation, the wavelength is found to be

124 nm. What is the speed of motion of the galaxy relative to Earth?

5.4.5. A proton accelerated at Fermi National Laboratory in Illinois has a momentum of 746 GeV/c. (a) What is the proton's velocity? (b) What is the proton's kinetic energy?

Case 5.5

5.5.1. An alien spacecraft flies overhead at a great distance as you stand in your backyard. You see its searchlight blink on for 0.190 s. The first officer on the spacecraft measures that the searchlight is on for 12.0 ms. (a) Which of these two measured times is the proper time? (b) What is the speed of the spacecraft relative to the Earth expressed as a fraction of the speed of light c?

5.5.2. A relativistic sprinter running at speed v, near the speed of the light, passes beneath a victory arch a height h above his eyes. Show that he will continue to see the arch, even though his eyes face forward, until he has run a distance $hv/[c\sqrt{1-(v^2/c^2)}] = \gamma hv/c$ beyond the arch. (*Hint:* Work in the rest frame of the sprinter, and think of the top of the arch as emitting pulses of light, the last of which can be seen when it travels vertically downward toward the sprinter.)

5.5.3. An imperial spaceship, moving at high speed relative to the planet Arrakis, fires a rocket toward the planet with a speed of $0.920c$ relative to the spaceship. An observer on Arrakis measures that the rocket is approaching with a speed of $0.360\,c$. (a) What is the speed of the spaceship relative to Arrakis? (b) Does the spaceship move toward or away from Arrakis?

5.5.4. A source radiates light with a frequency of $3.0 \cdot 10^{15}$ Hz. The signal is reflected by a mirror that is moving at speed 1 km/s away from the source. What is the shift of the frequency of the reflected radiation, as observed at the source?

5.5.5. Calculate an expression for the force as defined by $m_0 \dfrac{d(\gamma \vec{\boldsymbol{u}})}{dt}$, and show that the force and the acceleration $\dfrac{d\vec{\boldsymbol{u}}}{dt}$ do not necessarily in the same direction.

Chapter 6
HEAT RADIATION

Equation number	Equation	Equation title	Comments
1	2	3	4
6.1	$P = e_s IA$	Total power of radiation emitted by the surface of a hot body	A is a surface area of the body; I is the intensity of radiation; e_s is the emissivity of the surface
6.2	$I = \sigma T^4$	Stefan-Boltzmann law for a black body radiation	σ is Stefan-Boltzmann constant; T is an absolute temperature
6.3	$\lambda_{\text{peak}} T = b$	Wien's displacement law	b is Wien's constant; λ_{peak} is a wavelength corresponding to the peak value of spectral energy density
6.4	$b = \dfrac{hc}{k_B \beta} = 2.898 \cdot 10^{-3}$ K·m	Wien's law displacement constant	h is Plank constant; k_B is Boltzmann constant; $\beta \approx 4.9651142$

1	2	3	4
6.5	$u_\omega(\omega, T) = = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\hbar\omega/k_B T} - 1}$	Spectral energy density in terms of angular frequency for a black body (Plank's law)	$\hbar$ is reduced Plank's constant; ω is an angular oscillation frequency of radiation
6.6	$u_\lambda(\lambda, T) = = \frac{1}{\lambda^5} \frac{8\pi hc}{e^{hc/\lambda k_B T} - 1}$	Spectral energy density in terms of wavelength	λ is a wavelength of emitted electromagnetic waves
6.7	$U(T) = \int\limits_0^\infty u_\omega \, d\omega = = \int\limits_0^\infty u_\lambda \, d\lambda$	Total energy density of emitted radiation	All wavelengths (frequencies) are taken into account
6.8	$I = \frac{c}{4} U(T)$	Power emitted per unit area by a surface of a black body	Intensity of radiation (energy emitted per unit area per unit time)

Pre-Class Reading: [1], *chap.*17,38.8;[2], *chap.*17.5,40.1;[3], *chap.*40.1.

Case 6.1

6.1.1. What is the rate of energy radiation per unit area of a blackbody at a temperature of (a) 300 K and (b) 3000 K?

6.1.2. The dominant frequency of radiation emitted by an object is related to its temperature by $hf \approx k_B T$. Find the dominant radiation frequency emitted by (a) an object in interstellar space at 3.3 K; (b) a body of water at 58 °C; (c) an electric stove heating unit at 390 °C.

6.1.3. Two stars, both of which behave like ideal blackbodies, radiate the same total energy per second. The cooler one has a surface temperature T and 4.0 times the radius of the hotter star. (a) What is the temperature of the hotter star in terms of T? (b) What is the ratio of the peak-intensity wavelength of the hot star to the peak-intensity wavelength of the cool star?

6.1.4. The brightest star in the sky is Sirius, the Dog Star. It is actually a binary system of two stars, the smaller one (Sirius B) being a white dwarf. Spectral analysis of Sirius B indicates that its surface temperature is 24 000 K and that it radiates energy at a total rate of $1.0 \cdot 10^{25}$ W. Assume that it behaves like an ideal blackbody. (a) What is the radius of Sirius B? Express your answer in kilometers and as a fraction of our Sun's radius ($R_{\odot} = 6.96 \cdot 10^{8}$ m). (b) Which star radiates more total energy per second, the hot Sirius B or the (relatively) cool Sun with a surface temperature of $T_{\odot} = 5800$ K? To find out, calculate the ratio of the total power radiated by our Sun to the power radiated by Sirius B.

6.1.5. Radiation has been detected from space that is characteristic of an ideal radiator at $T = 2.728$ K. (This radiation is a relic of the Big Bang at the beginning of the universe.) For this temperature, at what wavelength does the Planck distribution peak? In what part of the electromagnetic spectrum is this wavelength?

Case 6.2

6.2.1. The emissivity of some surface material is 0.250. A sphere of that material with radius 2.00 cm is suspended within a large evacuated enclosure whose walls are at 300.0 K. What power input is required to maintain the sphere at a temperature of 1000.0 K if heat conduction along the supports is neglected?

6.2.2. Human eye is most sensitive to green light with wavelength of approximately 0.55 μm. What is the temperature of an incandescent bulb filament that radiates most of its energy as a blackbody at this wavelength?

6.2.3. Spectral analysis of Sirius B star indicates that its peak-intensity wavelength is about 120 nm. (a) Is this wavelength visible to humans? Assume that the star behaves like an ideal blackbody. Estimate (b) the surface temperature of Sirius B and (c) the total radiated intensity.

6.2.4. Consider a black body of surface area 1.0 cm^2 and temperature 10 000 K. (a) How much power does it radiate? (b) At what wavelength does it radiate most intensely? Find the spectral power per wavelength interval ($r_\lambda = \frac{c}{4} u_\lambda$) at (c) this wavelength and at wavelengths of (d) 1.00 nm (an X- or gamma ray), (e) 10.0 nm (strong ultraviolet light or an X-ray), (f) 100 nm (UV, near the short wavelength boundary of visible light), (g) 1.00 μm (infrared, near the long wavelength boundary of visible light), (h) 1.00 mm (infrared light or a microwave), and (i) 10.0 cm (a microwave or radio wave).

6.2.5. (a) Write the Planck distribution law in terms of the wavelength λ rather than the angular frequency ω, to obtain $u_\lambda(\lambda)$. (b) Show that

$$U(T) = \int_0^\infty u_\lambda(\lambda)\, d\lambda = \frac{8\pi^5 k_B^4}{15c^2h^3} T^4,$$

where $u_\lambda(\lambda)$ is the Plank distribution formula of Eq. (6.6). (*Hint:* Change integration from λ to ω.) You will need to use following tabulated integral:

$$\int_0^\infty \frac{x^3\, dx}{e^x - 1} = \frac{\pi^4}{15}.$$

(c) The result of (b) has the form of the Stefan-Boltzmann law, $U = \alpha T^4$ (Eq.6.2). Evaluate numerically constant α in (b).

Case 6.3

6.3.1. The operating temperature of a tungsten filament in an incandescent light bulb is 2500 K, and its emissivity is 0.333. Find the surface area of the filament of a 75-W bulb if all the electrical energy consumed by the bulb is radiated by the filament as electromagnetic waves. (Only a fraction of the radiation appears as visible light.)

6.3.2. The temperature of your skin is about 35 °C. Calculate the wavelength at which Planck radiation curve has maximum at this temperature, and therefore the wavelength at which your body radiates the most energy.

6.3.3. The surface of the Sun has a temperature of 6 000 K. At what rate is energy radiated from the whole surface of the Sun, given that the radius of the Sun is $R_{\odot} = 6.96 \cdot 10^{8}$ m?

6.3.4. A typical blue supergiant star (the type that explode and leave behind black holes) has a surface temperature of 30 000 K and a visual luminosity 100 000 times that of our Sun. Our Sun radiates at the rate of $3.86 \cdot 16^{26}$ W. (Visual luminosity is the total power radiated at visible wavelengths.) (a) Assuming that this star behaves like an ideal blackbody, what is the principal wavelength it radiates? Is this light visible? Use your answer to explain why these stars are blue. (b) If we assume that the power radiated by the star is also 100 000 times that of our Sun, what is the radius of this star? Compare its size to that of our Sun, which has a radius of $6.96 \cdot 10^{5}$ km. (c) Is it really correct to say that the visual luminosity is proportional to the total power radiated? Explain.

6.3.5. Show that the integral over all angular frequencies of the Plank law given by

$$U(T) = \int_0^{\infty} \frac{\hbar}{\pi^2 c^3} \frac{\omega^3 \, d\omega}{e^{\hbar\omega/k_B T} - 1}$$

gives a result that is of the from (a constant)$\times T^4$. (*Hint:* Change variables from ω to $\hbar\omega/k_B T$.) The energy emitted per unit area per unit time, $I(T)$, is proportional to total energy density $U(T)$, and thus $I(T)$ is also proportional to T^4, as in the Stefan-Boltzmann formula Eq.(6.2).

Case 6.4

6.4.1. The hot glowing surfaces of stars emit energy in the form of electromagnetic radiation. It is a good approximation to assume $e_s = 1$ for these surfaces. Find the radii of the following stars (assumed to be spherical): (a) Rigel, the bright blue star in the constellation Orion, which radiates energy at a rate of $2.7 \cdot 10^{32}$ W and has a surface temperature of 11 000 K; (b) Procyon B (visible only using a telescope), which radiates energy at a rate of $2.1 \cdot 10^{23}$ W and has a surface temperature of 10 000 K. (c) Compare your answers to the radius of the Earth ($R_\oplus = 6.38 \cdot 10^6$ m), the radius of the Sun ($R_\odot = 6.96 \cdot 10^8$ m), and the distance between the Earth and the Sun ($d = 1.5 \cdot 10^{11}$ m). (Rigel is an example of a supergiant star, and Procyon B is an example of a white dwarf star.)

6.4.2. Determine λ_{peak}, the wavelength at the peak of the Planck distribution, and the corresponding frequency f, at these temperatures: (a) 2.898 K; (b) 289.8 K; (c) 2898 K.

6.4.3. Assume that the radiation emitted from the Sun propagates radially outward from the Sun, and that no radiation is absorbed between the Sun and the Earth. How much energy in the form of radiation will fall per second on an area of 1.0 m^2 on the Earth, if that area is perpendicular to the straight-line path of the radiation? The distance from the Sun to the Earth is $d = 1.5 \cdot 10^{11}$ m. The surface temperature of the Sun is $T_\odot = 5800$ K.

6.4.4. Design an incandescent lamp filament. A tungsten wire radiates electromagnetic waves with power 100.0 W when its ends are connected across a 200-V power supply. Assume its constant operating temperature is 2 900 K and its emissivity is 0.50. Also assume it takes in energy only by electric transmission and emits energy only by electromagnetic radiation. You may take the resistivity of tungsten at 2 900 K as $7.13 \cdot 10^{-7}$ Ω·m. Specify the radius of the filament.

6.4.5. Energy density per unit angular frequency in the angular frequency range from ω to $\omega + d\omega$ in blackbody radiation is $u_\omega(\omega, T)\, d\omega$, where $u_\omega(\omega, T)$ is given by Eq.(6.5). An alternative way to express the blackbody radiation is to give $u_\lambda(\lambda, T)\, d\lambda$, the energy density per unit wavelength in the wavelength range from λ to $\lambda + d\lambda$. (a) Use the fundamental wave relation $c = \lambda f$, where c is the speed of light in vacuum and $f = \dfrac{\omega}{2\pi}$ is a frequency, to show that

$$u_\omega(\omega, T)\, d\omega = u_\omega\left[\left(2\pi\frac{c}{\lambda}\right), T\right] \frac{2\pi c\, d\lambda}{\lambda^2},$$

so $u_\lambda = u_\omega[(2\pi c/\lambda), T]\, 2\pi c/\lambda^2$. (b) Assuming that the temperature is fixed, use the result of part (a) to find an equation for the λ_{peak} for which $u_\lambda(\lambda, T)$ has a maximum.

Case 6.5

6.5.1. The intensity of radiation from a small source decreases with the square of the distance from the source. Consider the energy of radiation that reaches your face after being emitted by a solid aluminum at room temperature (27 °C). At what distance will this energy of radiation be the same as the energy emitted by the same amount of molten aluminum (927 °C) that reaches when you are 20 m from the aluminum?

6.5.2. A 100-W incandescent light bulb has a cylindrical tungsten filament 30.0 cm long, 0.40 mm in diameter, and with an emissivity of 0.26. (a) What is the temperature of the filament? (b) For what wavelength does the spectral emittance of the bulb peak? (c) Incandescent light bulbs are not very efficient sources of visible light. Explain why.

6.5.3. The average surface temperature of the Earth is 290 K. How much energy per second is radiated by the Earth's surface, assuming that it simulates blackbody. (The Earth radius is $6.38 \cdot 10^3$ km.)

6.5.4. Design an incandescent lamp filament. A tungsten wire radiates electromagnetic waves with power of 75.0 W when its ends are connected across a 120-V power supply. Assume its constant operating temperature is 2 900 K and its emissivity is 0.450. Also assume it takes in energy only by electric transmission and emits energy only by electromagnetic radiation. You may take the resistivity of tungsten at 2 900 K as $7.13 \cdot 10^{-7}$ Ω·m. Specify the length of the filament.

6.5.5. Find the constant coefficient of T^4 in the Stefan-Boltzmann formula, Eq. (6.2), given that the relation between $I(T)$ (the energy emitted per unit area per unit time) and the total energy density $U(T)$ calculated in Problem 6.3.5 is $I(T) = U(T)\dfrac{c}{4}$ where c is the speed of light in vacuum. (*Hint:* An appropriate entry in a table integral is $\int\limits_0^\infty \dfrac{x^3\,dx}{e^x - 1} = \dfrac{\pi^4}{15}$.)

Chapter 7
PHOTONS

Equation number	Equation	Equation title	Comments
1	2	3	4
7.1	$\mathcal{E}_{ph} = \hbar\,\omega = h\,f$	Energy of a photon	h is Plank constant; f is a radiation frequency
7.2	$p_{ph} = \dfrac{\mathcal{E}_{ph}}{c} = \dfrac{h}{\lambda}$	Momentum of a photon	λ is a wavelength of radiation
7.3	$\mathcal{E}_{ph} = \phi + K_{\max} = \phi + eV_{st}$	Einstein equation for photoelectric effect	ϕ is a work function; $K_{\max}$ is a maximum kinetic energy of an ejected electron; V_{st} is a stopping potential
7.4	$\lambda_{th} = \dfrac{hc}{\phi}$	Photoelectric effect threshold wavelength	The longest wavelength at which photoelectric effect can still occur

1	2	3	4
7.5	$\lambda_s - \lambda_i = 2\lambda_C \sin^2\dfrac{\theta}{2} = $ $= 2\dfrac{h}{mc}\sin^2\dfrac{\theta}{2}$	Wavelength shift at Compton scattering; λ_i (λ_s) is a wavelength of an incident (scattered) photon	λ_C is Compton wavelength of a target particle with mass m; θ is a scattering angle
7.6	$\lambda_{\min} = \dfrac{hc}{eV}$	Short-wave limit of an X-ray continuous spectrum (bremsstrahlung)	V is a potential difference applied to an X-ray tube for electrons acceleration
7.7	$P_{\text{rad}} = \dfrac{I}{c}(\rho + 1)\cos^2\theta$	Radiation pressure of light with intensity I	ρ is a reflection coefficient; θ is an angle of incidence

Pre-Class Reading: [1], *chap.* 38;[2], *chap.* 40;[3], *chap.* 40.

Case 7.1

7.1.1. How many photons per second are emitted by a 10.0-mW CO_2 laser that has a wavelength of 10.6 μm?

7.1.2. The photoelectric threshold wavelength of a tungsten surface is 272 nm. Calculate the maximum kinetic energy of the electrons ejected from this tungsten surface by ultraviolet radiation of frequency $2.1 \cdot 10^{15}$ Hz. Express the answer in electron-volts.

7.1.3. X-rays with the initial wavelength of 36 pm undergo Compton scattering. (a) What is the longest wavelength found in the scattered X-rays? (b) At which scattering angle is this wavelength observed?

7.1.4. Protons are accelerated from rest by a potential difference of 5.00 kV and strike a metal target. If a proton produces one photon on impact, what is the minimum wavelength of the resulting X-rays? How does your answer compare to the minimum wavelength if 5.00-keV electrons are used instead? Why do X-ray tubes use electrons rather than protons to produce X-rays?

7.1.5. In the 25-ft Space Simulator facility at NASA's Jet Propulsion Laboratory, a bank: of overhead arc lamps can produce light of intensity 2 500 W/m^2 at the floor of the facility. (This simulates the intensity of sunlight near the Venus.) Find the average radiation pressure on (a) a totally absorbing section of the floor, and (b) a totally reflecting section of the floor.

Case 7.2

7.2.1. Photorefractive keratectomy (PRK) is a laser-based surgical procedure that corrects near- and farsightedness by removing part on the lens of the eye to change its curvature and hence focal length. This procedure can remove layers 0.25 μm thick using pulses lasting 10.0 ns from a laser beam of wavelength 193 nm. Low-intensity beams can be used because each individual photon has enough energy to break the covalent bonds of the tissue. (a) In what part of the electromagnetic spectrum does this light lie? (b) What is the energy of a single photon? (c) If a 2.00-mW beam is used, how many photons are delivered to the lens in each pulse?

7.2.2. The photoelectric work function of potassium is 2.3 eV. If light having a wavelength of 200 nm falls on potassium, find (a) the stopping potential in volts; (b) the kinetic energy in electron volts of the most energetic electrons ejected; (c) the speed of these electrons.

7.2.3. A beam of X-rays with wavelength 0.3000 Å is Compton-scattered by the electrons in a sample. At what angle from the incident beam should you look to find X-rays with a wavelength of (a) 0.3364 Å; (b) 0.3243 Å; (c) 0.3121 Å?

7.2.4. (a) What is the minimum potential difference between the filament and the target of an X-ray tube if the tube is to produce X-rays with a wavelength of 0.50 Å? (b) What is the shortest wavelength produced in an X-ray tube operated at 40.0 kV?

7.2.5. A laser emits a beam with an intensity of $3.0 \cdot 10^{12}$ W/m^2 across an area of 0.5 mm^2. What force would the laser beam exert on a perfectly reflecting object?

Case 7.3

7.3.1. What are (a) the energy and (b) momentum of a photon in He-Ne laser light of wavelength 6328 Å?

7.3.2. When ultraviolet light with a wavelength of 0.200 μm falls on a clean copper surface, the stopping potential necessary to stop emission of photoelectrons is 1.5 V. (a) What is the work function for this surface? (b) What is the photoelectric threshold wavelength for this copper surface?

7.3.3. A photon of wavelength 0.486 Å strikes a free electron and is scattered at an angle of 60.0° from its original direction. Find (a) the change in the wavelength of this photon; (b) the wavelength of the scattered light; (c) the change in energy of the photon (is it a loss or a gain?); (d) the energy gained by the electron.

7.3.4. Accelerating voltages in cathode-ray-tube (CRT) TVs are about 30.0 kV. What are (a) the highest frequency and (b) the shortest wavelength (in run) of the X-rays that such a TV screen could produce? (c) What assumptions did you need to make? (CRT televisions contain shielding to absorb these X-rays.)

7.3.5. The total electromagnetic power emitted by the Sun is $3.8 \cdot 10^{26}$ W. What is the radiation pressure exerted on a totally absorbing surface at a distance $r = 1.5 \cdot 10^{11}$ m from the Sun?

Case 7.4

7.4.1. Find the energy of a photon for each of the following cases: (a) a microwave of wavelength 0.5 cm; (b) violet light of wavelength 400 nm; (c) a radio wave of frequency 100 MHz; (d) an X-ray of wavelength 2.0 Å.

7.4.2. The maximum energy of photoelectrons from aluminum is

2.3 eV for radiation of wavelength 200 nm and 1.80 eV for radiation of 216 nm. Use these data to calculate (a) Planck's constant and (b) the work function of aluminum.

7.4.3. A photon scatters in the perpendicular direction ($\theta = 90°$) from a free proton that is initially at rest. What must the wavelength of the incident photon be if it is to undergo a 5.0% change in wavelength as a result of the scattering?

7.4.4. When a voltage applied to an X-ray tube is increased $\eta = 2.5$ times, the short-wave limit of an X-ray continuous spectrum shifts by $\Delta\lambda = 60$ pm. Find the initial voltage applied to the tube.

7.4.5. Tiny flakes of mica are kept aloft by a beam of light projected vertically upward. If the mass of a typical flake is $3.33 \cdot 10^{-9}$ kg, and if on the average the area presented to the beam by a flake is 0.05 mm^2, what is the intensity of the beam? Assume that all the light is reflected.

Case 7.5

7.5.1. A laser used to weld detached retinas emits light with a wavelength of 652 nm in pulses that are 20.0 ms in duration. The average power during each pulse is 0.600 W. (a) How much energy is in each pulse in joules? In electron-volts? (b) What is the energy of one photon in joules? In electron-volts? (c) How many photons are in each pulse?

7.5.2. Threshold wavelength for the photoelectric effect in tungsten is 270 nm. Calculate (a) the work function of tungsten (in electron-volts), and (b) the maximum kinetic energy (in electron-volts) that a photoelectron can have when radiation of 120 nm falls on tungsten.

7.5.3. The wavelength of the incoming X-ray in a Compton scattering experiment is $7.078 \cdot 10^{-2}$ nm, and the wavelength of the outgoing X-rays is $7.314 \cdot 10^{-2}$ nm. At what angle was the scattered radiation measured?

7.5.4. Find the wavelength of the short-wave limit of an continuous spectrum if electrons approach the anticathode tube with velocity

$v = 0.60\,c$ where c is the speed of light.

7.5.5. The short side of a thin, stiff rectangle 3.0 cm $\times$ 1.0 cm

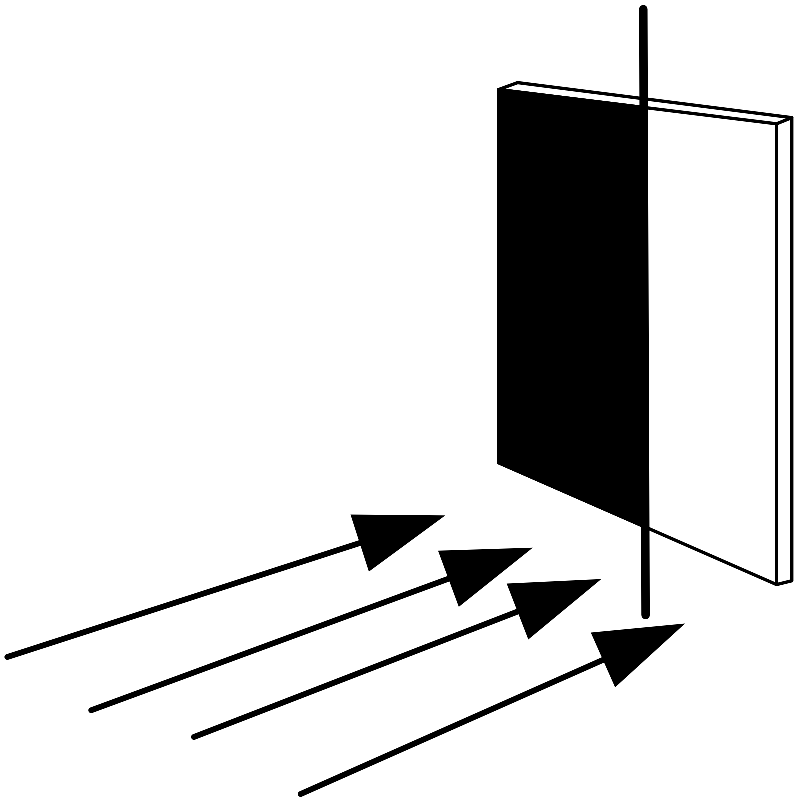

Figure 7.1. Problem 7.5.5

is attached to a vertical axis. Half of each side is painted black and fully absorbent; the other half is shiny, reflecting metal (Fig. 7.1). The back of each half is different from the front. There is no friction at the axis. The apparatus is bathed in a well-collimated (nonspreading) beam of light whose Poynting vector has the magnitude of 0.5 W/m^2 and travels perpendicular to the vertical axis. Is there a net torque on the rectangle's surface? If so, what is its average value due to the light over a full uniform rotation of the rectangle about the axis?

Chapter 8
PRINCIPLES OF QUANTUM MECHANICS

Equation number	Equation	Equation title	Comments
1	2	3	4
8.1	$\lambda_B = \dfrac{h}{p}$	de Broglie wavelength of a particle	p is a momentum of the particle
8.2	$\Delta x \Delta p_x \geq \hbar/2$ $\Delta y \Delta p_y \geq \hbar/2$ $\Delta z \Delta p_z \geq \hbar/2$	Heisenberg uncertainty principle for simultaneous measurements of particle position and momentum	Δx, Δy, Δz are uncertainties of a particle position; Δp_x Δp_y, Δp_z are uncertainties of a particle momentum
8.3	$\Delta \mathcal{E} \Delta t \geq \hbar/2$	Heisenberg uncertainty principle for for simultaneous measurements of particle energy and time	$\Delta \mathcal{E}$ is an uncertainty in the energy of a state that is occupied for a time Δt

1	2	3	4
8.4	$i\hbar\dfrac{\partial\Psi}{\partial t} = -\dfrac{\hbar^2}{2m}\Delta\Psi + U(\vec{\boldsymbol{r}}, t)\Psi$	Time-dependent non-relativistic single particle Schrödinger equation	$\Psi(\vec{\boldsymbol{r}}, t)$ is a wave function of a particle with mass m; $U(\vec{\boldsymbol{r}}, t)$ is its potential energy; Δ is the Laplace operator (see Appendix for details)
8.5	$\dfrac{dP}{dV} = \|\Psi(\vec{\boldsymbol{r}}, t)\|^2$	Probability density of a particle	$\vec{\boldsymbol{r}}$ is a position vector of infinitesimal volume dV of the particle location
8.6	$\iiint \|\Psi(\vec{\boldsymbol{r}}, t)\|^2 dV = 1$	Normalization condition	Particle exists somewhere in space
8.7	$\Psi(\vec{\boldsymbol{r}}, t) = \psi(\vec{\boldsymbol{r}})e^{-i\mathcal{E}t/\hbar}$	Time-dependent wave function of stationary states	$\mathcal{E}$ is an energy of the stationary state; i is an imaginary unit
8.8	$-\dfrac{\hbar^2}{2m}\Delta\psi + U\psi = \mathcal{E}\psi$	Time-independent non-relativistic single particle Schrödinger equation	$\psi = \psi(\vec{\boldsymbol{r}})$ is a spatial wave function of a particle and $U = U(\vec{\boldsymbol{r}})$ is its potential energy
8.9	$-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} + U\psi = \mathcal{E}\psi$	One-dimensional non-relativistic Schrödinger equation	$\psi = \psi(x)$ is an 1D spatial wave function; $U = U(x)$ is a potential energy of a particle

1	2	3	4
8.10	$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right);$ $\mathcal{E}_n = \frac{\pi^2\hbar^2}{2mL^2}n^2$	Wave-function and energy of a particle in an infinite 1D potential well	L is a width of the well; $n = 1, 2, 3, \ldots$ is a principal quantum number; m is a mass of the particle
8.11	$\mathcal{E}_n = \left(n + \frac{1}{2}\right)\hbar\omega$	Energy of a quantum harmonic oscillator	ω is an angular frequency of the oscillator; $n = 0, 1, 2, \ldots$
8.12	$T \approx \exp\left\{-\frac{2}{\hbar}\times \sqrt{2m(U_0 - \mathcal{E})}L\right\}$	Transmission coefficient of a particle tunneling through a rectangular potential barrier	U_0 is a barrier height; L is a width of the barrier; $\mathcal{E}$ is an energy of the particle
8.13	$T \approx \exp\left\{-\frac{2}{\hbar}\times \int_a^b\sqrt{2m(U(x)-\mathcal{E})}dx\right\}$	Transmission coefficient of a particle tunneling through a potential barrier	a and b are positions where the particle enters to and exits the potential barrier $U(x)$
8.14	$\mathcal{E}_n = -\frac{R_E}{n^2}$	Energy levels of an electron in hydrogen atom	R_E is Rydberg energy; $n = 1, 2, 3, \ldots$ is a principal quantum number
8.15	$r_n = a_0 n^2$	Allowed values of orbit radii in Bohr model	a_0 is Bohr radius

1	2	3	4
8.16	$\frac{1}{\lambda_{if}}=R_\infty\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$	Rydberg formula for hydrogen emission (absorption) of radiation	R_∞ is Rydberg constant; n_i and n_f are principal quantum numbers of initial and final states

Pre-Class Reading: [1], *chap.* 39–41;[2], *chap.* 40&41;[3], *chap.* 41&42.

Case 8.1

8.1.1. An electron has a de Broglie wavelength of 3.31 Å. Determine (a) the magnitude of its momentum, and (b) its kinetic energy (in joules and in electron-volts).

8.1.2. A beam of neutrons that all have the same energy scatters

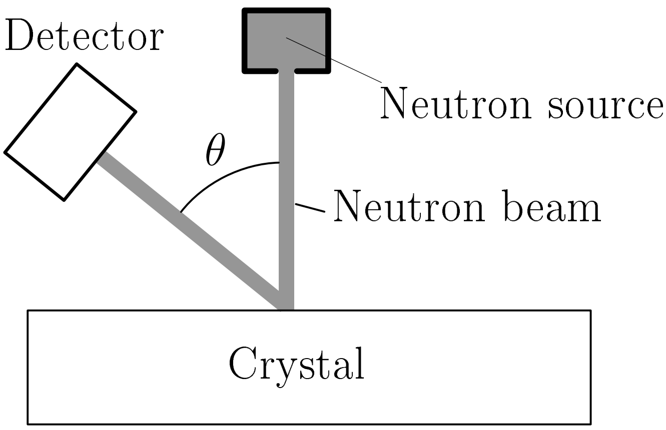

Figure 8.1. Problem 8.1.2

from the atoms that have a spacing of 2.21 Å in the surface plane of a crystal. The second order intensity maximum occurs when the angle θ in Fig. 8.1 is 45°. What is the kinetic energy (in electron-volts) of each neutron in the beam?

8.1.3. A 5.0-g marble is gently placed on a horizontal tabletop that is 2.00 m wide. (a) What is the maximum uncertainty in the horizontal position of the marble? (b) According to the Heisenberg uncertainty principle, what is the minimum uncertainty in the horizontal velocity of the marble? (c) Considering your answer to part (b), what is the longest time the marble could remain on the table? Compare this time to the age of the universe, which is approximately 14 billion years. (*Hint:* Can you know that the horizontal velocity of the marble is exactly zero?)

8.1.4. Consider a wave function given by $\psi(x) = A\cos kx$, where $k = 2\pi/\lambda$ and A is areal constant. (a) For what values of x is there the highest probability of finding the particle described by this wave function? Explain. (b) For which values of x is the probability zero? Explain.

8.1.5. An electron is in a box 1.8 Å wide. What are the de Broglie wavelength and the magnitude of the momentum of the electron if it is in (a) the $n = 1$ level; (b) the $n = 2$ level; (c) the $n = 3$ level? In each case how does the wavelength compare to the width of the box?

Case 8.2

8.2.1. In the Bohr model of the hydrogen atom, what is the de Broglie wavelength for the electron when it is in (a) the $n = 2$ level and (b) the $n = 6$ level? In each case, compare the de Broglie wavelength to the circumference $2\pi r_n$ of the orbit.

8.2.2. The spacing between scattering planes in a crystal is 5.0 Å. What is the scattering angle from such a crystal with electrons of energy 220 eV for which a second maximum observed?

8.2.3. (a) The x-coordinate of an electron is measured with an uncertainty of 0.10 mm. What is the x-component of the electron's velocity, v_x if the minimum percentage uncertainty in a simultaneous measurement of v_x is 2.0%? (b) Repeat part (a) for a proton.

8.2.4. Consider a particle moving in one dimension which we call the x-axis. (a) What does it mean for the wave function of this particle to be normalized? (b) Is the wave function $\psi(x) = e^{-ax}$, where a is a negative real number, normalized? Could this be a valid wave function?

(c) If the particle described by the wave function $\psi(x) = Ae^{-\alpha x^2}$, where A and α are positive real numbers, is confined to the range $x \leq 0$, determine A (including its units) so that the wave function is normalized. (*Hint:* You will need to use following tabulated integral: $\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$.)

8.2.5. An electron is bound in an infinite well. The longest-wavelength photon that is absorbed by the electron has a wavelength of 600.0 nm. Determine the width of the well.

Case 8.3

8.3.1. (a) A non-relativistic free particle with mass m has kinetic energy E_K. Derive an expression for the de Broglie wavelength of the particle in terms of m and E_K. (b) What is the de Broglie wavelength of an 500-eV electron?

8.3.2. Consider a crystal with a planar spacing of 1.0 Å. (a) What energies would reflected electrons need for you to be able to observe up to five interference maxima? (b) Repeat part (a) for neutrons.

8.3.3. (a) The uncertainty in the z-component of a proton's position is $1.0 \cdot 10^{-12}$ m. What is the minimum uncertainty in a simultaneous measurement of the z-component of the proton's velocity? (b) The uncertainty in the x-component of an electron's velocity is 0.50 m/s. What is the minimum uncertainty in a simultaneous measurement of the x-coordinate of the electron?

8.3.4. Calculate $|f|^2$ for the complex-valued function $f(x, y) = = (y^2 + ix^2)/(x^2 - iy^2)$.

8.3.5. (a) An electron with the initial kinetic energy of 32 eV encounters a square barrier 41 eV high and 0.25 nm wide. What is the probability that the electron will tunnel through the barrier? (b) A proton with the same kinetic energy encounters the same barrier. What is the probability that the proton will tunnel through the barrier?

Case 8.4

8.4.1. What is the de Broglie wavelength for an electron with speed

(a) $v = 0.60\,c$ and (b) $v = 0.80\,c$? (*Hint:* Use the correct relativistic expression for linear momentum if necessary.)

8.4.2. In a neutron two-slit diffraction experiment, the slits are 0.1 mm apart. If the second diffraction maximum is located at an angle of $1.0 \cdot 10^{-7}$ rad, what is the kinetic energy of the neutrons?

8.4.3. A ψ ("psi") particle has the rest energy of 3 097 MeV (1 MeV $= 10^6$ eV). It is unstable with a lifetime of $7.6 \cdot 10^{-21}$ s. Estimate the uncertainty in rest energy of the ψ particle. Express your answer in MeV and as a fraction of the rest energy of the particle.

8.4.4. Particle A is described by the wave function $\psi(x, y, z)$. Particle B is described by the wave function $\psi(x, y, z)e^{i\phi}$, where ϕ is a real constant. How does the probability of finding particle A within a volume dV around a certain point in space compare with the probability of finding particle B within this same volume?

8.4.5. The ground-state energy of a harmonic oscillator is 2.21 eV. If the oscillator undergoes a transition from its $n = 5$ to $n = 2$ level by emitting a photon, what is the wavelength of the photon?

Case 8.5

8.5.1. Hydrogen gas (H_2) is at 0°C. The mass of a hydrogen atom is $1.67 \cdot 10^{-27}$ kg. (a) What is the average de Broglie wavelength of the hydrogen molecules? (b) How fast would an electron have to move to have the same de Broglie wavelength as the hydrogen? Do we need to consider relativity for this electron? (c) What would be the energy of a photon having the same wavelength as the H_2 molecules and the electrons?

8.5.2. Although the operating of electron microscope does not depend on the wave the nature of matter, waves associated with electrons do set a limit on the resolving power of such instruments. (a) If the electrons in an electron microscope have the kinetic energy of $2.5{\cdot}10^4$ eV and the aperture of microscope is $3.5 \cdot 10^{-4}$ m, estimate the smallest angle that can be resolved. (b) How much energy would electrons need so that two objects separated by 5 nm could be resolved? The angular aperture is 10^{-3}. Give your answer in electron-volts.

8.5.3. The unstable W^+ particle has a rest energy of 80.41 GeV (1 GeV $= 10^9$ eV) and an uncertainty in rest energy of 2.06 GeV. Estimate the lifetime of the W^+ particle.

8.5.4. A particle moving in one dimension (the x-axis) is described by the wave function

$$\psi(x) = \begin{cases} Ae^{-bx}, & \text{for } x \geq 0; \\ Ae^{+bx}, & \text{for } x < 0, \end{cases}$$

where $b = 2.00\ \text{m}^{-1}$, $A > 0$, and the $+x$-axis points toward the right. (a) Determine A so that the wave function is normalized. (b) Sketch the graph of the wave function. (c) Find the probability of finding this particle in each of the following regions: (i) within 50.0 cm of the origin, (ii) on the left side of the origin (can you first guess the answer by looking at the graph of the wave function?), (iii) between $x = 0.500$ m and $x = 1.00$ m.

8.5.5. Let ΔE_n be energy difference between the adjacent energy levels E_n and E_{n+1} for a particle in a box. The ratio $R_n = \Delta E_n / E_n$ compares the energy of a level to the energy separation of the next higher energy level. a) For what value of n is R_n the largest, and what is this largest R_n? (b) What does R_n approach as n becomes very large? How does this result compare to the classical value for this quantity?

Chapter 9

INTRODUCTION TO NUCLEAR PHYSICS

Equation number	Equation	Equation title	Comments
1	2	3	4
9.1	$R = R_0 A^{1/3}$, $R_0 = 1.2 \cdot 10^{-15}$ m	Radius of a nucleus	$A = Z + N$ is a mass number of the nucleus; Z is a charge number (a number of protons in the nucleus), N is a number of neutrons
9.2	$E_B = (ZM_{\mathrm{H}} + Nm_{\mathrm{n}} - {}^{A}_{Z}M)c^2$	Binding energy for the nucleus	M_{H} is the mass of hydrogen atom; ${}^{A}_{Z}M$ is an atomic mass of the atom with the nucleus ${}^{A}_{Z}\mathrm{X}$
9.3	$N = N_0 e^{-\lambda t}$	Number of nuclei remaining after a time t (radioactive decay law)	N_0 is an initial number of radioactive nuclei; λ is a decay constant

1	2	3	4
9.4	$T_{1/2} = \frac{\ln 2}{\lambda}$	Half-life	Time required for a number of radioactive nuclei to decrease the original number N_0 to one-half
9.5	$Q = [(M_\mathrm{a} + M_\mathrm{X}) - -(M_\mathrm{Y} + M_\mathrm{b})]c^2$	Energy equation of nuclear reaction $\mathrm{a} + \mathrm{X} \to \mathrm{Y} + \mathrm{b}$	M_X is a mass of the target nucleus; M_a is a mass of the bombarding particle; M_Y is a mass of a daughter nuclear and M_b is a mass of an outgoing particle

Pre-Class Reading: [1], *chap.* 43;[2], *chap.* 44;[3], *chap.* 44.

Case 9.1

9.1.1. How many protons and how many neutrons are there in a nucleus of the most common isotope of (a) silicon, $^{28}_{14}\mathrm{Si}$; (b) rubidium, $^{85}_{37}\mathrm{Rb}$; (c) thallium, $^{205}_{81}\mathrm{Tl}$?

9.1.2. What nuclide is produced in the following radioactive decays? (a) α decay of $^{239}_{94}\mathrm{Pu}$; (b) β^- decay of $^{24}_{11}\mathrm{Na}$; (c) β^+ decay of $^{15}_{8}\mathrm{O}$.

9.1.3. Radioactive isotopes used in cancer therapy have a "shelf-life", like pharmaceuticals used in chemotherapy. Just after it has been manufactured in a nuclear reactor, the activity of a sample of $^{60}_{27}\mathrm{Co}$ is 5000 Ci. When its activity falls below 3500 Ci, it is considered too weak a source to use in treatment. You work in the radiology department of a large hospital. One of these $^{60}_{27}\mathrm{Co}$ sources in your inventory was manufactured on June 4, 2010. It is now December 4, 2013. Is the source still usable? The half-life of $^{60}_{27}\mathrm{Co}$ is 5.271 years.

9.1.4. Identify the unknown nuclide or particle for the nuclear reaction: (a) $\mathrm{X} \to {}^{65}_{28}\mathrm{Ni} + \gamma$; (b) $^{215}_{84}\mathrm{Po} \to \mathrm{X} + \alpha$; (c) $\mathrm{X} \to {}^{55}_{26}\mathrm{Fe} + \beta^{+} + \nu$.

9.1.5. A person exposed to fast neutrons receives a radiation dose of 200 rem on part of his hand, affecting 25 g of tissue. The RBE of these neutrons is 10. (a) How many rad did he receive? (b) How many joules of energy did this person receive? (c) Suppose the person received the same rad dosage, but from beta rays with a RBE of 1.0 instead of neutrons. How many rem would he have received?

Case 9.2

9.2.1. Consider the three nuclei of problem 9.1.1. Estimate (a) the radius, (b) the surface area, and (c) the volume of each nucleus. Determine (d) the mass density (in $\mathrm{kg/m^3}$) and (e) the nucleon density (in nucleons per cubic meter) for each nucleus. Assume that the mass of each nucleus is A atomic mass units.

9.2.2. $^{238}_{92}\mathrm{U}$ decays spontaneously by α emission to $^{234}_{90}\mathrm{Th}$. Calculate (a) the total energy released by this process and (b) the recoil velocity of the $^{234}_{90}\mathrm{Th}$ nucleus. The atomic masses are 238.050788 u for $^{238}_{92}\mathrm{U}$ and 234.043601 u for $^{234}_{90}\mathrm{Th}$.

9.2.3. Radioactive isotopes are often introduced into the body through the bloodstream. Their spread through the body can then be monitored by detecting the appearance of radiation in different organs. $^{131}_{53}\mathrm{I}$, a β^{-} emitter with a half-life of 8.0 d is one such tracer. Suppose a scientist introduces a sample with an activity of 375 Bq and watches it spread to the organs. (a) Assuming that the sample all went to the thyroid gland, what will be the decay rate in that gland 24 d (about $3\frac{1}{2}$ weeks) later? (b) If the decay rate in the thyroid 24 d later is actually measured to be 17.0 Bq, what percentage of the tracer went to that gland? (c) What isotope remains after the I-131 decays?

9.2.4. Consider the nuclear reaction

$$^{2}_{1}\mathrm{H} + {}^{14}_{7}\mathrm{N} \to \mathrm{X} + {}^{10}_{5}\mathrm{B},$$

where X is a nuclide. (a) What are Z and A for the nuclide X? (b) Calculate the reaction energy Q (in MeV). (c) If the ${}^{2}_{1}$H nucleus is incident on a stationary ${}^{14}_{7}$N nucleus, what minimum kinetic energy must it have for the reaction to occur?

9.2.5. It has become popular for some people to have yearly whole-body scans (CT scans, formerly called CAT scans) using X-rays, just to see if they detect anything suspicious. A number of medical people have recently questioned the advisability of such scans, due in part to the radiation they impart. Typically, one such scan gives a dose of 12 mSv, applied to the whole body. By contrast, a chest X-ray typically administers 0.20 mSv to only 5.0 kg of tissue. How many chest X-rays would deliver the same total amount of energy to the body of 75-kg person as one whole-body scan?

Case 9.3

9.3.1. The most common isotope of uranium, ${}^{238}_{92}$U, has atomic mass 238.050783 u. Calculate (a) the mass defect; (b) the binding energy (in MeV); (c) the binding energy per nucleon.

9.3.2. The atomic mass of ${}^{14}_{6}$C is 14.003242 u. Show that the β^{-} decay of ${}^{14}_{6}$C is energetically possible, and calculate the energy released in the decay.

9.3.3. The half-life time of ${}^{14}_{6}$C is 5 730 y, and the tissues of organisms accumulate this isotope from the atmosphere while the organisms are living. The skeleton of mammoth is found to have 5% as much ${}^{14}_{6}$C as the atmosphere has. When did the mammoth live? Assume that the concentration of ${}^{14}_{6}$C does not change.

9.3.4. Consider the nuclear reaction

$${}^{2}_{1}\mathrm{H} + {}^{9}_{4}\mathrm{Be} \rightarrow \mathrm{X} + {}^{4}_{2}\mathrm{He},$$

where X is a nuclide. (a) What are the values of Z and A for the nuclide X? (b) How much energy is liberated? (c) Estimate the threshold energy for this reaction.

9.3.5. Food is often irradiated with either X-rays or electron beams to help prevent spoilage. A low dose of 5 – 75 kilorads (krad) helps to reduce and kill inactive parasites, a medium dose of 100 – 400 krad kills microorganisms and pathogens such as salmonella, and a high dose of 2300 – 5700 krad sterilizes food so that it can be stored without refrigeration. (a) A dose of 200 krad kills spoilage microorganisms in fish. If X-rays of RBE 1.0 are used, what would be the dose in Gy, Sv, and rem, and how much energy would a 100-g portion of fish absorb? (b) Repeat part (a) if electrons of RBE 1.50 are used instead of X-rays.

Case 9.4

9.4.1. Calculate (a) the total binding energy and (b) the binding energy per nucleon of ${}^{12}_{6}\mathrm{C}$ (c) What percent of the rest mass of this nucleus is its total binding energy?

9.4.2. What particle (α-particle, electron, or positron) is emitted in the following radioactive decays? (a) ${}^{27}_{14}\mathrm{Si} \rightarrow {}^{27}_{13}\mathrm{Al}$; (b) ${}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th}$; (c) ${}^{74}_{33}\mathrm{As} \rightarrow {}^{74}_{34}\mathrm{Se}$.

9.4.3. The age of marine sediments can be determined by their thorium-230 content. ${}^{230}_{90}\mathrm{Th}$ is the decay product of ${}^{234}_{92}\mathrm{U}$ and it decays to ${}^{226}_{88}\mathrm{Ra}$ with a half-life of 75 200 y. Measurements indicate that the ${}^{230}_{90}\mathrm{Th}$ content of a sediment is 6 times larger at the surface than at the depth of 40 m. What is the deposition rate at the location in mm/y?

9.4.4. Consider the nuclear reaction

$${}^{28}_{14}\mathrm{Si} + \gamma \rightarrow {}^{24}_{12}\mathrm{Mg} + \mathrm{X},$$

where X is a nuclide. (a) What are Z and A for the nuclide X? (b) Ignoring the effects of recoil, what minimum energy must the photon have for this reaction to occur? The mass of a ${}^{28}_{14}\mathrm{Si}$ atom is 27.976927 u, and the mass of a ${}^{24}_{12}\mathrm{Mg}$ atom is 23.985042 u.

9.4.5. A person ingests an amount of a radioactive source with a very long lifetime and activity 0.72 μCi. The radioactive material lodges in the lungs where all the 4.0-MeV α-particles (RBE is 20) emitted are absorbed within a 0.50-kg mass of tissue. Calculate a) the absorbed dose and b) the equivalent dose for one year.

Case 9.5

9.5.1. Calculate the mass defect, the binding energy (in MeV), and the binding energy per nucleon of (a) the nitrogen nucleus, ${}^{14}_{7}\mathrm{N}$, and (b) the helium nucleus, ${}^{4}_{2}\mathrm{He}$. (c) How do the results of parts (a) and (b) compare?

9.5.2. (a) Calculate the energy released by the electron-capture decay of ${}^{57}_{27}\mathrm{Co}$. (b) A negligible amount of this energy goes to the resulting ${}^{57}_{26}\mathrm{Fe}$ atom as kinetic energy. About 90% of the time, the ${}^{57}_{26}\mathrm{Fe}$ nucleus emits two successive gamma-ray photons after the electron-capture process, of energies 0.122 MeV and 0.014 MeV, respectively, in decaying to its ground state. What is the energy of the neutrino emitted in this case? ($M({}^{57}_{27}\mathrm{Co}) = 56.9362914$ u, $M({}^{57}_{26}\mathrm{Fe}) = 56.935934$ u.)

9.5.3. The unstable isotope ${}^{40}_{19}\mathrm{K}$ is used for dating rock samples. Its half-life is $1.28 \cdot 10^{9}$ y. (a) How many decays occur per second in a sample containing $1.63 \cdot 10^{-6}$ g of ${}^{40}_{19}\mathrm{K}$? (b) What is the activity of the sample in curies? ($M({}^{40}_{19}\mathrm{K}) = 39.964$ u.)

9.5.4. Consider the nuclear reaction

$$ {}^{2}_{4}\mathrm{He} + {}^{7}_{3}\mathrm{Li} \rightarrow {}^{1}_{0}\mathrm{n} + \mathrm{X}, $$

where X is a nuclide. (a) What are Z and A for the nuclide X? (b) Is the energy absorbed or liberated? How much?

9.5.5. In the 1986 disaster at the Chernobyl reactor in the Soviet Union (now Ukraine), about 1/8 of the ${}^{137}_{55}\mathrm{Cs}$ present in the reactor was released. The isotope ${}^{137}_{55}\mathrm{Cs}$ has a half-life for β decay of 30.07 y and decays with the emission of a total of 1.17 MeV of energy per decay. Of this, 0.51 MeV goes to the emitted electron, and the remaining 0.66 MeV to γ rays. The radioactive ${}^{137}_{55}\mathrm{Cs}$ is absorbed by plants which are eaten by livestock and humans. How many ${}^{137}_{55}\mathrm{Cs}$ atoms would need to be present in each kilogram of a body tissue if an equivalent dose for one week is 3.5 Sv? Assume that all the energy from the decay is deposited in that 1.0 kg of tissue and that the RBE of the electrons is 1.5.

ANSWERS

Chapter 1

1.1.1 $\vec{\boldsymbol{B}} = \dfrac{E_0}{c}\cos(ky+\omega t)\vec{\boldsymbol{i}}$, in the $-y$-direction

1.1.2 a) 99 MHz b) 3.03 m

1.1.3 a) 375 V/m, 12.5 μT
b) 10^{15} Hz, 10^{-15} s, $3.0 \cdot 10^{-7}$ m
c) $3.0 \cdot 10^{8}$ m/s

1.1.4 $1.50 \cdot 10^{-8}$ T, in the $-x$-direction

1.1.5 b) $(-32\vec{\boldsymbol{i}} + 8\vec{\boldsymbol{j}} + 2\vec{\boldsymbol{k}})$ W/m^2

1.2.1 $E_0\sqrt{\varepsilon_r} = B_0\, c$

1.2.2 a) $\vec{\boldsymbol{E}} = E_0 \cos\left[\dfrac{2\pi}{\lambda}(x+ct)\right]\vec{\boldsymbol{k}}$
b) $\vec{\boldsymbol{B}} = \dfrac{E_0}{c}\cos\left[\dfrac{2\pi}{\lambda}(x+ct)\right]\vec{\boldsymbol{j}}$

1.2.3 a) 360 V/m
b) $1.14 \cdot 10^{-6}$ J/m^3 c) 342 W/m^2

1.2.4 a) $+x$–direction b) 0.5 μm
c) $(10^{-3}\,\mathrm{T})\vec{\boldsymbol{k}}\sin\left[(4\pi\cdot 10^{6}\ \mathrm{rad/m})x - (12\pi \cdot 10^{14}\ \mathrm{rad/s})t\right]$

1.2.5 a) 1.0 μT b) $2.0 \cdot 10^{-7}$ m
c) $1.5 \cdot 10^{15}$ Hz

1.3.1 a) $6.0 \cdot 10^{14}$ Hz
b) 1.5 kV/m
c) $1.5\,\mathrm{kV/m}\ \sin\left[4\pi\cdot 10^{6}(x-3\cdot 10^{8}t)\right]$,
$5.0\,\mu\mathrm{T}\ \sin\left[4\pi\cdot 10^{6}(x-3\cdot 10^{8}t)\right]$

1.3.2 a) $-z$–direction
b) $7.5 \cdot 10^{11}$ Hz c) $(-1.8\,\mathrm{V/m})\vec{\boldsymbol{i}} \times \sin\left[1.57 \cdot 10^{4}x + 4.71 \cdot 10^{12}t\right]$,

1.3.3 $E_0\cos\left[k(x\sin\theta + y\cos\theta - ct)\right]$

1.3.4 $4 \cdot 10^{8}$ m/s $> c$

1.3.5 $5.0 \cdot 10^{-4}$ s $< 10^{-2}$ s, radio audience first

1.4.1 $3.0\cos\left[(1.05 \cdot 10^{7}\ \mathrm{m}^{-1})z - (3.15 \cdot 10^{15}\ \mathrm{rad/s})t\right]\vec{\boldsymbol{i}}$ V/m

1.4.2 a) $7.50 \cdot 10^{14}$ Hz
b) 5.0 pT c) $(1.5 \cdot 10^{-3}\,\mathrm{V/m})\vec{\boldsymbol{k}} \times \sin\left[5\pi \cdot 10^{6}y + 15\pi \cdot 10^{14}t\right]$,
$(5.0\,\mathrm{pT})\vec{\boldsymbol{i}}\sin\left[5\pi\cdot 10^{6}y + 15\pi\cdot 10^{14}t\right]$

1.4.3 a) $6.0 \cdot 10^{7}$ m/s
b) $6.0 \cdot 10^{5}$ m c) $5.0 \cdot 10^{-11}$ T
d) $23.9 \cdot 10^{-9}$ W/m^2

1.4.4 $3.33 \cdot 10^{-6}$ J/m^3

1.4.5 $2.24 \cdot 10^{-10}$ T

1.5.1 -50 m

1.5.2 $\vec{\boldsymbol{B}} = 0.6\sin\left[1.884 \cdot 10^{15}t - 6.28 \cdot 10^{6}z\right]\vec{\boldsymbol{j}}$ mT,
$\vec{\boldsymbol{E}} = 0.18\sin\left[1.884 \cdot 10^{15}t - 6.28 \cdot 10^{6}z\right]\vec{\boldsymbol{i}}$ MV/m

1.5.3 a) $-z$-direction
b) $-z$-direction
c) $-y$-direction
d) $+y$-direction

1.5.4 $\langle|\vec{\boldsymbol{S}}|\rangle = 1.0 \cdot 10^{-3}$ W/m^2

1.5.5 a) $P = 3.85 \cdot 10^{26}$ W

b) $E_{\text{max}} = 1.02$ kV/m
c) $B_{\text{max}} = 3.39$ μT

Chapter 2

2.1.1 a) 20.0 cm b) 7.5 cm, 32.5 cm
2.1.2 640 nm
2.1.3 a) $I_0/2$ b) 160 nm
2.1.4 0.1 μm
2.1.5 1008
2.2.1 a) 210 m b) 105 m
2.2.2 4.8 mm
2.2.3 $5.0 \cdot \pi$ rad $= 15.71$ rad
2.2.4 0.546 mrad $= 0°1'53''$
2.2.5 0.30 mm
2.3.1 11.5 m, 9.0 m, 7.5 m, 6.0 m, 4.5 m, 3.0 m, 1.5 m
2.3.2 1.00 m
2.3.3 a) 2.40 mm b) 0.80 mm
2.3.4 a) 80.0 nm b) 0.240 μm
2.3.5 1.00063
2.4.1 a) 660 nm, 495 nm b) no change c) 566 nm, 440 nm
2.4.2 a) triples b) reduced by $\frac{1}{3}$ c) triples d) no change
2.4.3 a) $\Delta\theta_0 = 2 \arcsin\left(\dfrac{\lambda}{4d}\right)$ b) no
2.4.4 a) 480 nm; indigo (blue) b) 587 nm; yellow (orange)
2.4.5 22.0 μm
2.5.1 23.6°, 53.1°
2.5.2 $\dfrac{\lambda_2 - \lambda_1}{\lambda_1} = 0.08$
2.5.3 a) along the perpendicular bisector and in direction 48.6° relatively to the one
b) $2 \cdot 10^{-3}$ W/m^2 c) 10.8°
2.5.4 1.01 μm
2.5.5 $1 + \dfrac{N\lambda}{2L}$

Chapter 3

3.1.1 480 nm
3.1.2 a) $0.911 I_0$ b) $0.405 I_0$
c) $0.027 I_0$
3.1.3 Every fourth slit cancels each other
3.1.4 a) 0.30 mm b) 75.0 μm
3.1.5 48.6°; no second order diffraction maximum exists
3.2.1 1.00 m
3.2.2 a) 3.00 mm
b) $4.00 \cdot 10^{-6}$ W/m^2
3.2.3 a) ± 0.8 mrad, ± 1.6 mrad
b) $0.09 I_0$, $0.045 I_0$
3.2.4 23.3°, 52.3°
3.2.5 17.2°
3.3.1 a) Minima: $\pm 35.3°$, maxima: 0°, $\pm 60°$ b) 1.73 m
3.3.2 a) 78 b) 80.8°
c) $5.5 \cdot 10^{-4}$ W/m^2
3.3.3 a) Every 6$^{\text{th}}$ bright fringe
b) no c) yes
3.3.4 a) 10.0 μm b) 2.0 μm
3.3.5 2.58 Å

3.4.1 a) 104 b) $\pm 81.3°$
3.4.2 a) 523 nm b) $9.6 \cdot 10^{-4} I_0$
3.4.3 a) and c) odd slits and even slits b) adjacent slits
3.4.4 2
3.4.5 a) 1.71 Å, an X-ray
b) 43.16°
3.5.1 a) $6.0 \cdot 10^{14}$ Hz b) 11.0 μm
c) dark bands occurs at $\pm 30.0°$ only
3.5.2 a) $\pm 12.8°$, $\pm 26.4°$, $\pm 41.8°$, $\pm 62.7°$ b) 35 mW/m^2
3.5.3 20 μm in width and 60 μm in separation
3.5.4 137 cm^{-1}
3.5.5 21.758°

Chapter 4

4.1.1 linear
4.1.2 a) impossible b) 30°
c) 45° d) 60°
4.1.3 $5.0 \cdot 10^5$ W/m^2
4.1.4 2.3
4.1.5 $\varkappa = \dfrac{\ln(\tau_1/\tau_2)}{d_2 - d_1} = 35.0 \text{ m}^{-1}$
4.2.1 circular
4.2.2 27/128
4.2.3 $3.75 \cdot 10^5$ W/m^2
4.2.4 a) 52.6° b) 41.1°
4.2.5 $\tau = (1-\rho)^2 \cdot e^{-\frac{\varkappa_1+\varkappa_2}{2} l}$
4.3.1 linear
4.3.2 60°
4.3.3 $2.5 \cdot 10^6$ W/m^2
4.3.4 a) 50 b) 1.8°
4.3.5 $2.4 \cdot 10^{13}$ m^{-3}
4.4.1 elliptical
4.4.2 30°
4.4.3 a) first: $\frac{1}{2} I_0$, second: $\frac{3}{8} I_0$, third: $\frac{3}{32} I_0$ b) first: $\frac{1}{2} I_0$, last: 0
4.4.4 60.5°
4.4.5 $2.0 \cdot 10^{15}$ m^{-3}
4.5.1 $I_{polarized}/I_{natural} = 1/3$
4.5.2 a) $0.75\, I_0$ b) $0.50\, I_0$
c) $0.25\, I_0$
4.5.3 40.0 W/cm^2
4.5.4 $\theta_P = \arctan\left(\dfrac{1}{\sin\theta_{cr}}\right)$
4.5.5 b) 4.75 μm

Chapter 5

5.1.1 a) $5.05 \cdot 10^{-6}$ s
b) 1.363 km
5.1.2 a) $3.2 \cdot 10^{12}$ s b) 3.7 days
5.1.3 a) $0.806\, c$ b) $0.974\, c$
c) $0.977\, c$
5.1.4 a) $0.291\, c$ b) $3.14 \cdot 10^8$ \$
5.1.5 $2.1 \cdot 10^{-21}$ kg/(m·s)
5.2.1 a) $2.60 \cdot 10^8$ m/s b) no, $v_{\max} = 980.43$ m/s
(Lockheed SR–71)
5.2.2 1.0 km
5.2.3 a) negative b) $-0.385\, c$
5.2.4 use Eq. (5.5)
5.2.5 $3.6 \cdot 10^{20}$ kg/(m·s)
5.3.1 a) on Mars b) 435 μs
5.3.2 424 m/s

5.3.3 $-0.9988\,c$
5.3.4 593.8 nm
5.3.5 a) 55.9995 GeV
b) $2.99 \cdot 10^{-17}(1 - 4 \cdot 10^{-11})$
c) $(1 - 4 \cdot 10^{-11})\,c$
5.4.1 a) $0.998\,c$ b) 126 m
5.4.2 stick in length 0.92 m is tilted at an angle of 71° with the x-axis
5.4.3 a) $0.859\,c$ b) 31 s
5.4.4 $4.1 \cdot 10^7$ m/s
5.4.5 a) $(1 - 7.9 \cdot 10^{-7})\,c$
b) 745 GeV
5.5.1 a) $0.998\,c$ b) on ship
5.5.2 use Eq. (5.4)
5.5.3 a) $0.837\,c$ b) away
5.5.4 $-2.0 \cdot 10^{10}$ Hz
5.5.5 $m_0\gamma\left(\frac{d\vec{u}}{dt} + \frac{u\vec{u}}{c^2 - u^2}\frac{du}{dt}\right)$

Chapter 6

6.1.1 a) 459 W/m^2
b) $4.59 \cdot 10^6$ W/m^2
6.1.2 a) 69 GHz b) $6.9 \cdot 10^{12}$ Hz
c) $1.38 \cdot 10^{13}$ Hz
6.1.3 a) $2.0T$ b) 0.5
6.1.4 a) 6500 km $\approx 1\% R_\odot$
b) 39, sun
6.1.5 1.06 mm, microwave
6.2.1 71.3 W
6.2.2 5270 K
6.2.3 a) no (UV-radiation)
b) $2.5 \cdot 10^4$ K c) 20 GW/m^2
6.2.4 a) 56.7 kW b) 298 nm
c) 1287 MW/(m^2·μm)
d) $2.6 \cdot 10^{-594}$ W/m^3
e) $1.2 \cdot 10^{-38}$ W/m^3
f) 21 MW/(m^2·μm)
g) 116 MW/(m^2·μm)
h) 260 W/m^3
i) $2.60 \cdot 10^{-6}$ W/m^3
6.2.5 c) $7.56 \cdot 10^{-16}$ J/(m^3·K^4)
6.3.1 1.02 cm^2
6.3.2 9.41 μm
6.3.3 $4.46 \cdot 10^{26}$ W
6.3.4 a) 96.6 nm, no (UV)
b) $8.2 \cdot 10^9$ m $\approx 12\,R_\odot$ c) no
6.3.5 $U(T) = \frac{4\sigma}{c}T^4$
6.4.1 a) $1.61 \cdot 10^{11}$ m $\sim d$
b) $5.43 \cdot 10^6$ m $\sim R_\oplus$
6.4.2 a) 1.0 mm, $3.0 \cdot 10^{11}$ Hz
b) 10.0 μm, $3.0 \cdot 10^{13}$ Hz
c) 1.0 μm, $3.0 \cdot 10^{14}$ Hz
6.4.3 1377 W/m^2
6.4.4 16.5 μm
6.4.5 b) $e^{-\beta} + \beta/5 = 1$, where $\beta = \frac{hc}{\lambda k_B T}$
6.5.1 10.0 m
6.5.2 a) 2913 K b) 0.99 μm
6.5.3 $2.05 \cdot 10^{17}$ J
6.5.4 0.33 m
6.5.5 $\frac{2\pi^5 k_B^4}{15h^3c^2}$

Chapter 7

7.1.1 $5.3 \cdot 10^{17}$

7.1.2 4.14 eV

7.1.3 a) 40.86 pm b) 180°

7.1.4 2.48 Å, the same

7.1.5 a) $8.33 \cdot 10^{-6}$ Pa
b) $1.67 \cdot 10^{-5}$ Pa

7.2.1 a) Ultraviolet
b) $1.03 \cdot 10^{-18}$ J = 6.44 eV
c) $1.94 \cdot 10^{7}$

7.2.2 a) 3.9 V b) 3.9 eV
c) $1.17 \cdot 10^{6}$ m/s

7.2.3 a) 120° b) 90° c) 60°

7.2.4 a) 24.8 kV b) 0.31 Å

7.2.5 10.0 mN

7.3.1 a) $3.1 \cdot 10^{-19}$ J = 1.96 eV
b) $1.05 \cdot 10^{-27}$ kg·m/s

7.3.2 a) 4.7 eV = $7.5 \cdot 10^{-19}$ J
b) 0.264 μm

7.3.3 a) 1.2 pm b) 0.498 Å
c) −622 eV, lost d) 622 eV

7.3.4 a) $7.26 \cdot 10^{19}$ Hz
b) 0.413 Å

7.3.5 $4.5 \cdot 10^{-6}$ Pa

7.4.1 a) $2.48 \cdot 10^{-4}$ eV b) 3.1 eV
c) $4.1 \cdot 10^{-7}$ eV d) 6.2 keV

7.4.2 a) $7.2 \cdot 10^{-34}$ J·s
b) 4.45 eV

7.4.3 $2.64 \cdot 10^{-14}$ m

7.4.4 12.4 kV

7.4.5 10^{7} W/m^2

7.5.5 $1.25 \cdot 10^{-15}$ N·m

7.5.1 a) 0.012 J = $7.5 \cdot 10^{16}$ eV
b) $3.05 \cdot 10^{-19}$ J = 1.91 eV
c) $3.93 \cdot 10^{16}$

7.5.2 a) 4.60 eV b) 5.76 eV

7.5.3 88.4°

7.5.4 9.7 pm

Chapter 8

8.1.1 a) $2.0 \cdot 10^{-24}$ kg·m/s
b) $2.2 \cdot 10^{-18}$ J = 13.72 eV

8.1.2 0.13 eV

8.1.3 a) 2.00 m
b) $5.0 \cdot 10^{-33}$ m/s
c) $4.0 \cdot 10^{32}$ s $\approx 10^{25}$ years

8.1.4 a) $x = m\lambda$, $m \in Z$
b) $x = (2m+1)\dfrac{\lambda}{2}$, $m \in Z$

8.1.5 a) 3.6 Å, $1.8 \cdot 10^{-24}$ kg·m/s
b) 1.8 Å, $3.7 \cdot 10^{-24}$ kg·m/s
c) 1.2 Å, $5.5 \cdot 10^{-24}$ kg·m/s

8.2.1 a) 9.97 Å b) 29.91 Å

8.2.2 19.3°

8.2.3 a) 28.9 m/s
b) 1.57 cm/s

8.2.4 a) $\int\limits_{-\infty}^{+\infty} |\psi(x)|^2\, dx = 1$

b) no, no c) $A = \sqrt[4]{\dfrac{8\alpha}{\pi}}$ m$^{-1/2}$

8.2.5 7.39 Å

8.3.1 a) $\lambda_B = \dfrac{h}{\sqrt{2mE_K}}$
b) 0.549 Å

8.3.2 a) $1.51 \cdot 10^{-16}$ J = 0.94 keV

b) $8.21 \cdot 10^{-20}$ J $= 0.51$ eV
8.3.3 a) $3.2 \cdot 10^4$ m/s
b) $1.16 \cdot 10^{-4}$ m
8.3.4 1
8.3.5 a) $4.86 \cdot 10^{-4}$ b) 10^{-143}
8.4.1 a) 3.23 pm b) 1.82 pm
8.4.2 $1.05 \cdot 10^{-17}$ J $= 65.6$ eV
8.4.3 0.043 MeV $= 1.4 \cdot 10^{-5} E_0$
8.4.4 the same
8.4.5 937.5 Å
8.5.1 a) 1.87 Å b) $3.9 \cdot 10^6$ m/s, no c) $1.06 \cdot 10^{-15}$ J
8.5.2 90 keV
8.5.3 $3.0 \cdot 10^{-25}$ s
8.5.4 a) 1.41 $m^{-1/2}$ c) (i) 0.865, (ii) 0.500, (iii) 0.059
8.5.5 a) $n = 1$, $R = 3$ b) 0

Chapter 9

9.1.1 a) $Z = 14$, $N = 14$
b) $Z = 37$, $N = 48$
c) $Z = 81$, N= 124
9.1.2 a) ${}^{235}_{92}$U b) ${}^{24}_{12}$Mg c) ${}^{15}_{7}$N
9.1.3 barely usable
9.1.4 a) ${}^{65}_{28}$Ni b) ${}^{211}_{82}$Pb c) ${}^{55}_{27}$Co
9.1.5 a) 20 rad b) $5.0 \cdot 10^{-3}$ J
c) 20 rem
9.2.1 a) 3.6 fm, 5.3 fm, 7.1 fm
b) 163 fm^2, 353 fm^2, 633 fm^2
c) 195 fm^3, 624 fm^3, 1499 fm^3
d) $2.30 \cdot 10^{17}$ kg/m^3
e) $1.38 \cdot 10^{44}$ nucleon/m^3
9.2.2 a) 4.270 MeV
b) 71.76 keV c) $2.43 \cdot 10^5$ m/s
9.2.3 a) 46.9 Bq b) 36.2 %
c) ${}^{131}_{54}$Xe
9.2.4 a) $Z = 3$, $A = 6$, ${}^{6}_{3}$Li
b) −10.14 MeV c) 11.58 MeV
9.2.5 900 times
9.3.1 a) 1.93 u b) 1.8 GeV
c) 7.57 MeV/nucleon
9.3.2 156 keV
9.3.3 24 760 y ago
9.3.4 a) $Z = 3$, $A = 7$
b) 7.152 MeV c) 1.44 MeV
9.3.5 a) $2.0 \cdot 10^5$ rem $=$
$= 2.0 \cdot 10^5$ Gy $= 2.0 \cdot 10^5$ Sv,
200 J b) 200 krad = 200 kGy =
= 300 krem = 300 kSv, 200 J
9.4.1 a) 92.16 MeV
b) 7.680 MeV/nucleon c) 0.8245%
9.4.2 a) β^+ b) α-particle c) β^-
9.4.3 0.2 mm/y
9.4.4 a) $A = 4$, $Z = 2$
b) 9.984 MeV
9.4.5 a) 1.08 Gy = 108 rad
b) 2160 rem
9.5.1 a) 0.1124 u, 104.7 MeV, 7.48 MeV/nucleon b) 0.0304 u, 28.3 MeV, 7.08 MeV/nucleon
9.5.2 a) 836 keV b) 700 keV
9.5.3 0.422 Bq $= 1.44 \cdot 10^{-11}$ Ci
9.5.4 a) 5 and 10 b) absorbed 2.79 MeV
9.5.5 $3.5 \cdot 10^{16}$ kg

APPENDIX

Universal physical constants

e	$= 1.602\,176\,565 \cdot 10^{-19}$ C	Elementary charge	$e \approx 1.60 \cdot 10^{-19}$ C
c	$= 299\,792\,458$ m/s	Speed of light in vacuum	$c \approx 3.00 \cdot 10^{8}$ m/s
h	$= 6.626\,069\,57 \cdot 10^{-34}$ J·s	Plank's constant	$h \approx 6.63 \cdot 10^{-34}$ J·s
k_B	$= 1.380\,648\,8 \cdot 10^{-23}$ J/K	Boltzmann constant	$k_B \approx 1.38 \cdot 10^{-23}$ J/K
μ_0	$= 4\pi \cdot 10^{-7}$ H/m	Magnetic constant	$\mu_0 \approx 1.26 \cdot 10^{-6}$ H/m
ε_0	$= (c^2\mu_0)^{-1}$	Electric constant	$\varepsilon_0 \approx 8.85 \cdot 10^{-12}$ F/m
k_C	$= (4\pi\varepsilon_0)^{-1}$	Coulomb's constant	$k_C \approx 8.99 \cdot 10^{9}$ Nm^{2}C^{-2}
$\hbar$	$= \dfrac{h}{2\pi}$	Reduced Plank's constant	$\hbar \approx 1.05 \cdot 10^{-34}$ J·s
m_e	$= 9.109\,382\,91 \cdot 10^{-31}$ kg	Rest mass of electron	$m_e \approx 9.11 \cdot 10^{-31}$ kg
m_p	$= 1.672\,621\,777 \cdot 10^{-27}$ kg	Rest mass of proton	$m_p \approx 1.67 \cdot 10^{-27}$ kg
m_n	$= 1.674\,927\,351 \cdot 10^{-27}$ kg	Rest mass of neutron	$m_n \approx 1.67 \cdot 10^{-27}$ kg
1 u	$= \frac{1}{12}M(^{12}\mathrm{C})$	Atomic mass unit	1 u $\approx 1.660539 \cdot 10^{-27}$ kg
μ_B	$= e\hbar(2m_e)^{-1}$	Bohr magneton	$\mu_B \approx 9.27 \cdot 10^{-24}$ A·m^2
μ_N	$= e\hbar(2m_p)^{-1}$	Nuclear magneton	$\mu_N \approx 5.05 \cdot 10^{-27}$ A·m^2
σ	$= \dfrac{\pi^2 k_B^4}{60c^2\hbar^3}$	Stefan-Boltzmann constant	$\sigma \approx 5.67 \cdot 10^{-8} \times$ $\times$W·m^{-2}·K^{-4}
λ_C	$= \dfrac{h}{m_e c}$	Compton wave-length of electron	$\lambda_C \approx 2.43 \cdot 10^{-12}$ m
R_E	$= m_e k_C^2 e^4 (2\hbar^2)^{-1}$	Rydberg energy	$R_E \approx 13.6$ eV
a_0	$= \dfrac{\hbar^2}{k_C e^2 m_e}$	Bohr radius	$a_0 \approx 0.529$ Å
R_∞	$= k_C^2 m_e e^4 (4\pi\hbar^3 c)^{-1}$	Rydberg constant	$R_\infty \approx 1.10 \cdot 10^{7}$ m^{-1}

Scale of electromagnetic waves

Type	Radio	MW	IR	Visible	UV	X-rays	γ-rays
λ, m	10^3	10^{-2}	10^{-5}	$5\cdot10^{-7}$	10^{-8}	10^{-10}	10^{-12}
f, Hz	$3\cdot10^5$	$3\cdot10^{10}$	$3\cdot10^{13}$	$6\cdot10^{14}$	$3\cdot10^{16}$	$3\cdot10^{18}$	$3\cdot10^{20}$

Spectrum of visible light

Color	λ-range, nm	f-range, THz	Color	λ-range, nm	f-range, THz
V	380 – 450	789 – 668	B	450 – 495	668 – 606
G	495 – 570	606 – 526	Y	570 – 590	526 – 508
O	590 – 620	508 – 484	R	620 – 750	484 – 400

Refractive index at $\lambda = 589$ nm of the substances

Substance	Index	Substance	Index	Substance	Index
Vacuume	1.0	Water	1.333	Ice	1.309
Air	1.00029	Ethanol	1.36	Glass	1.52
Carbon dioxide	1.00045	Benzene	1.501	Diamond	2.42

Electron work function ϕ and threshold wavelength λ_{th} for some elements

Element	ϕ, eV	λ_{th}, nm	Element	ϕ, eV	λ_{th}, nm
Ag	4.3	288	Au	5.1	243
Cu	4.7	264	Al	4.1	302
C	5.0	248	Se	5.9	210
Cs	2.14	579	K	2.29	541
Li	2.3	539	Ca	2.87	432

Neutral Atomic Masses for Some Light Nuclides

Isotope	M(u)	Isotope	M (u)	Isotope	M (u)
${}^1_1H$	1.007825	${}^1_0n$	1.008665	${}^2_1H$	2.014102
${}^3_1H$	3.016049	${}^3_2He$	3.016029	${}^4_2He$	4.002603
${}^6_3Li$	6.015122	${}^7_3Li$	7.016004	${}^9_4Be$	9.012182
${}^{10}_5B$	10.012937	${}^{11}_5B$	11.009305	${}^{12}_6C$	12.000000
${}^{13}_6C$	13.003355	${}^{14}_7N$	14.003074	${}^{15}_7N$	15.000109
${}^{16}_8O$	15.994915	${}^{17}_8O$	16.999132	${}^{18}_8O$	17.999160

Derivatives. Basic rules

$$\frac{d(f+g)}{dx} = \frac{df}{dx} + \frac{dg}{dx} \qquad \frac{d(Cf)}{dx} = C\frac{df}{dx} \quad (C = \text{const})$$

$$\frac{d(f\,g)}{dx} = \frac{df}{dx}\,g + f\,\frac{dg}{dx} \qquad \frac{d\left(\frac{f}{g}\right)}{dx} = \frac{\frac{df}{dx}\,g - f\,\frac{dg}{dx}}{g^2}$$

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg}\,\frac{dg}{dx} \qquad \frac{dx}{dy} = \left(\frac{dy}{dx}\right)^{-1} \quad \left(\frac{dy}{dx} \neq 0\right)$$

Derivatives of some functions

$$\frac{dC}{dx} = 0 \qquad \frac{dx}{dx} = 1$$

$$\frac{d}{dx}\left(x^\alpha\right) = \alpha x^{\alpha-1} \qquad \frac{d}{dx}\left(\exp^x\right) = \exp^x$$

$$\frac{d}{dx}\left(\sin x\right) = \cos x \qquad \frac{d}{dx}\left(\cos x\right) = -\sin x$$

$$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x} \qquad \frac{d}{dx}\left(\tan x\right) = \frac{1}{\cos^2 x}$$

Fundamental Theorem of Calculus

$$\int\limits_a^b f(x)\,dx = F(b) - F(a), \text{ where } F(x) - \text{ an antiderivative of } f(x)$$

$$F(x) - \text{ is an antiderivative of } f(x) \Leftrightarrow \frac{dF(x)}{dx} = f(x)$$

Antiderivatives of some functions[1]

$$\int dx = x \qquad \int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1} \quad (\alpha \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| \qquad \int \exp^x\,dx = \exp^x$$

$$\int \sin x\,dx = -\cos x \qquad \int \cos x\,dx = \sin x$$

$$\int \frac{dx}{1+x^2} = \arctan x \qquad \int \frac{dx}{1-x^2} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|$$

[1] An arbitrary constant can be added to the right part of every equations.

Vector Algebra

$$\vec{\boldsymbol{a}} = a_x\vec{\boldsymbol{i}} + a_y\vec{\boldsymbol{j}} + a_z\vec{\boldsymbol{k}},$$

$$\vec{\boldsymbol{b}} = b_x\vec{\boldsymbol{i}} + b_y\vec{\boldsymbol{j}} + b_z\vec{\boldsymbol{k}}$$

Dot (scalar) product of vectors

$$\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}} \stackrel{def}{=} |\vec{\boldsymbol{a}}||\vec{\boldsymbol{b}}| \cos\left(\angle \vec{\boldsymbol{a}}\vec{\boldsymbol{b}}\right) = \vec{\boldsymbol{b}} \cdot \vec{\boldsymbol{a}}$$

$$\vec{\boldsymbol{i}} \cdot \vec{\boldsymbol{i}} = \vec{\boldsymbol{j}} \cdot \vec{\boldsymbol{j}} = \vec{\boldsymbol{k}} \cdot \vec{\boldsymbol{k}} = 1$$

$$\vec{\boldsymbol{i}} \cdot \vec{\boldsymbol{j}} = \vec{\boldsymbol{j}} \cdot \vec{\boldsymbol{k}} = \vec{\boldsymbol{k}} \cdot \vec{\boldsymbol{i}} = 0$$

$$\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}} = a_x b_x + a_y b_y + a_z b_z$$

Cross (vector) product of vectors

$$|\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}}| \stackrel{def}{=} |\vec{\boldsymbol{a}}||\vec{\boldsymbol{b}}| \sin\left(\angle \vec{\boldsymbol{a}}\vec{\boldsymbol{b}}\right)$$

$$\vec{\boldsymbol{i}} \times \vec{\boldsymbol{i}} = \vec{\boldsymbol{j}} \times \vec{\boldsymbol{j}} = \vec{\boldsymbol{k}} \times \vec{\boldsymbol{k}} = 0$$

$$\vec{\boldsymbol{a}} \perp \left(\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}}\right) \perp \vec{\boldsymbol{b}}$$

$$\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}} = -\vec{\boldsymbol{b}} \times \vec{\boldsymbol{a}}$$

$$\vec{\boldsymbol{i}} \times \vec{\boldsymbol{j}} = \vec{\boldsymbol{k}}, \quad \vec{\boldsymbol{j}} \times \vec{\boldsymbol{k}} = \vec{\boldsymbol{i}}, \quad \vec{\boldsymbol{k}} \times \vec{\boldsymbol{i}} = \vec{\boldsymbol{j}}$$

$$\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}} = \vec{\boldsymbol{i}}(a_y b_z - a_z b_y) + \vec{\boldsymbol{j}}(a_z b_x - a_x b_z) + \vec{\boldsymbol{k}}(a_x b_y - a_y b_x)$$

$$\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}} = \begin{vmatrix} \vec{\boldsymbol{i}} & \vec{\boldsymbol{j}} & \vec{\boldsymbol{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Scalar triple product

$$\vec{\boldsymbol{a}} \cdot \left(\vec{\boldsymbol{b}} \times \vec{\boldsymbol{c}}\right) = \vec{\boldsymbol{b}} \cdot (\vec{\boldsymbol{c}} \times \vec{\boldsymbol{a}}) = \vec{\boldsymbol{c}} \cdot \left(\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}}\right) = -\vec{\boldsymbol{c}} \cdot \left(\vec{\boldsymbol{b}} \times \vec{\boldsymbol{a}}\right)$$

Vector triple product

$$\vec{\boldsymbol{a}} \times \left(\vec{\boldsymbol{b}} \times \vec{\boldsymbol{c}}\right) = \vec{\boldsymbol{b}}(\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{c}}) - \vec{\boldsymbol{c}}\left(\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}}\right)$$

Vector Calculus

$$\vec{\boldsymbol{\nabla}} \stackrel{def}{=} \frac{\partial}{\partial x}\vec{\boldsymbol{i}} + \frac{\partial}{\partial y}\vec{\boldsymbol{j}} + \frac{\partial}{\partial z}\vec{\boldsymbol{k}}$$

$$\Delta \stackrel{def}{=} \nabla^2 = \vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{\nabla}} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Gradient of the Scalar Field and Nabla Operator

$$\vec{\boldsymbol{E}} = -\vec{\boldsymbol{\nabla}} V = -\mathbf{grad}\, V = -\left(\frac{\partial V}{\partial x}\vec{\boldsymbol{i}} + \frac{\partial V}{\partial y}\vec{\boldsymbol{j}} + \frac{\partial V}{\partial z}\vec{\boldsymbol{k}}\right)$$

$$\vec{\boldsymbol{E}} = E_x\vec{\boldsymbol{i}} + E_y\vec{\boldsymbol{j}} + E_z\vec{\boldsymbol{k}}$$

Divergence of the Vector Field and Laplace Operator

$$\mathrm{div}\vec{\boldsymbol{E}} \stackrel{def}{=} \vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{E}} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\Delta V \stackrel{def}{=} \mathrm{div}(\mathbf{grad}\, V) = (\vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{\nabla}})V = \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Curl of the Vector Field

$$\mathrm{rot}\vec{\boldsymbol{E}} \stackrel{def}{=} \vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{E}} = \begin{vmatrix} \vec{\boldsymbol{i}} & \vec{\boldsymbol{j}} & \vec{\boldsymbol{k}} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\mathrm{rot}\vec{\boldsymbol{E}} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\vec{\boldsymbol{i}} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\vec{\boldsymbol{j}} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\vec{\boldsymbol{k}}$$

$$\mathrm{rot}(\mathrm{rot}\vec{\boldsymbol{H}}) = \vec{\boldsymbol{\nabla}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{H}}) = \mathbf{grad}(\mathrm{div}\vec{\boldsymbol{H}}) - \Delta\vec{\boldsymbol{H}}$$

Gauss–Ostrogradsky Divergence Theorem

$$\oiint\limits_{\partial Vol} \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \iiint\limits_{Vol} \mathrm{div}\vec{\boldsymbol{E}}\, dVol$$

Stokes Theorem

$$\oint\limits_{\partial A} \vec{\boldsymbol{H}} \cdot d\vec{\boldsymbol{\ell}} = \iint\limits_{A} \mathrm{rot}\vec{\boldsymbol{H}} \cdot d\vec{\boldsymbol{A}}$$

Complex Variables

$$i^2 \overset{def}{=} -1$$

Complex number

$$z = x + iy, \quad \mathrm{Re}(z) = x, \quad \mathrm{Im}(z) = y, \quad |z| = \sqrt{x^2 + y^2}$$

Elementary operations

$$\text{Conjugation: } z^* = (x + iy)^* \overset{def}{=} x - iy$$

$$\text{Addition: } (a + bi) + (c + di) = (a + c) + (b + d)i$$

$$\text{Multiplication: } (a + bi) \cdot (c + di) = (ac - bd) + (bc + ad)i$$

$$|z|^2 = z \cdot z^* = (x + iy) \cdot (x - iy) = x^2 + y^2$$

Polar form

$$z = x + iy = r(\cos\phi + i \sin\phi)$$

$$\text{Magnitude: } r = |z| = \sqrt{x^2 + y^2}$$

$$\text{Phase: } \cos\phi = \frac{x}{r} \;\&\; \sin\phi = \frac{y}{r}$$

Conjugation

$$z^* = (r(\cos\phi + i \sin\phi))^* = r(\cos\phi - i \sin\phi)$$

Multiplication

$$z_1 \cdot z_2 = r_1(\cos\phi_1 + i \sin\phi_1) \cdot r_2(\cos\phi_2 + i \sin\phi_2) =$$

$$= r_1 r_2(\cos(\phi_1 + \phi_2) + i \sin(\phi_1 + \phi_2))$$

Division

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\phi_1 - \phi_2) + i \sin(\phi_1 - \phi_2)]$$

Euler's formula

$$z = r(\cos\phi + i \sin\phi) = re^{i\phi}$$

Conjugation

$$z^* = (re^{i\phi})^* = re^{-i\phi}$$

Multiplication

$$z_1 \cdot z_2 = r_1 e^{i\phi_1} \cdot r_2 e^{i\phi_2} = r_1 r_2 e^{i(\phi_1 + \phi_2)}$$

$$\cos\phi = \frac{e^{i\phi} + e^{-i\phi}}{2}, \quad \sin\phi = \frac{e^{i\phi} - e^{-i\phi}}{2i}$$

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Навчальне видання

Охрімовський Андрій Михайлович
Подшивалова Оксана Володимирівна
Воронович Даніїл Олександрович

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“Харківський авіаційний інститут”
61070, Харків-70, вул. Чкалова, 17
http://www.khai.edu
Видавничий центр “ХАІ”
61070, Харків-70, вул. Чкалова, 17
izdat@khai.edu

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