# MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE 

National Aerospace University "Kharkiv Aviation Institute"

Andriy Okhrimovskyy, Oksana Podshyvalova, Serhii Oliinyk

# ELECTRICITY AND MAGNETISM 

Guidance Manual for Recitation

УДК 537 (075.8)
О-39
Подано варіанти задач для одинадцяти практичних занять з фізики, що охоплюють такі теми: електричне поле, теорема Гаусса, електричний потенціал, ємність і діелектрики, електричний струм, магнітне поле та його джерела, закон електромагнітої індуції, рівняння Максвелла і електромагнітні хвилі. До кожної теми наведено таблицю з формулами.

Для англомовних студентів.

## Reviewers: Dr. of Science in Physics \& Mathematics, Prof. I. O. Girka, Dr. of Science in Physics \& Mathematics, Prof. O. I. Spolnik

## Okhrimovskyy, A.M.

O-39 Electricity and Magnetism: guidance manual for recitation / A. M. Okhrimovskyy, O. V. Podshyvalova, S. V. Oliinyk. Kharkiv: National Aerospace University "KhAI", 2013. - 116 p.

Manual contains a set of problems to eleven recitation classes offered by the Physics department of the National Aerospace University "Kharkiv Aviation Institute". Guidance embraces the following topics: Electric Field, Gasus's Law, Electric Potential, Capacitance and Dielectrics, Electric Current (AC \& DC), Magnetic Field and its sources, Faraday's Law, Maxwell's equations, and Electromagnetic waves. Each chapter is supplied by a table with basic equations.

For english-speaking students.
Figs. 54. Tables 11. Bibl.: 5 items

## UDK 537 (075.8)

(C) Okhrimovskyy A.M., Podshyvalova O.V., Oliinyk S.V., 2013
(c) National Aerospace University "Kharkiv Aviation Institute", 2013

## Contents

Introduction ..... 4
Chapter 1. Electric Fields ..... 6
Chapter 2. Gauss's Law ..... 16
Chapter 3. Electric Potential ..... 25
Chapter 4. Electric Field in a Medium. Capacitance ..... 34
Chapter 5. Current and Resistance. DC Circuits ..... 44
Chapter 6. Effects of Magnetic Fields ..... 55
Chapter 7. Sources of Magnetic Fields ..... 61
Chapter 8. Faraday's Law ..... 70
Chapter 9. Magnetic Field in a Medium. Inductance ..... 79
Chapter 10. Electrical Oscillations and AC Circuits ..... 86
Chapter 11. Maxwell's Equations. Electromagnetic Waves ..... 94
Answers ..... 102
Appendix ..... 111
Bibliography ..... 115

## INTRODUCTION

Problem solving is an important part of studying physics. It is evident that there are better chances of a student's understanding a topic if he or she can apply his or her knowledge in more than one way. The course is arranged so that a student meets a given topic in a variety of ways: in reading assignments in the text [1-3], in demonstration lectures, in supplementary notes issued to students, in recitation and problem drill in problem sessions [4], in both the study and the performance of laboratory experiments [5], in homework problem sets, and in quizzes and examinations. These various types of presentation are synchronized so that, it is hoped, their impact on the student will have a maximum effectiveness.

Recitation can be an exciting part of the course, or it can be drudgery, depending upon your attitude toward it. If you regard it merely as an impediment to your getting through the course, probably, you will not enjoy it and, furthermore, derive very little benefit from it. On the other hand, if you approach the class with the thought that it is an opportunity to learn, and with a desire to make the most out of it, then it is almost certain you will find the time you spend on it both profitable and interesting.

This manual is a logical continuation of the guidance manual for the recitations "Mechanics and Thermodynamics" [4]. It offers a wide range of problems covering the fields of electromagnetism as a part of the course "Experimental and Theoretical Physics". We chose to use once proved structure, with a table containing main definitions and physical laws at the beginning of every chapter and references to corresponding chapters in the textbooks $[1-3]$. Most of the problems were taken from the textbooks $[1-3]$, and students can find examples of how to solve typical problems there. Those who want to be sure that their solutions
are correct can compare obtained results with numerical answers given at the end of the manual.

We would like to thank our reviewers and colleagues at Physics department, especially Professor Anatoliy Taran, Iryna Skresanova, and Mykola Aleksandrov for stimulating discussions about physics pedagogy and friendly criticism of our work. We also want to express our gratitude to our spouses for their love, support, and emotional sustenance during the writing of this manual.

We welcome suggestions and comments from our readers and wish our students great success in studying physics.

## Chapter 1 <br> ELECTRIC FIELDS

|  | Equation | Equation title | Comments |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |
| 1.1 | $\overrightarrow{\boldsymbol{F}}=k_{C} \frac{q_{1} q_{2}}{r^{2}} \frac{\overrightarrow{\boldsymbol{r}}}{r}$ | Interaction force between two point charges at rest (Coulomb's law) | $q_{1}, q_{2}$ are charges of particles; $\overrightarrow{\boldsymbol{r}}$ is a position vector of one particle with respect to another; $k_{C}$ is a Coulomb's constant |
| 1.2 | $\overrightarrow{\boldsymbol{E}}=\frac{\overrightarrow{\boldsymbol{F}}}{q}$ | Electric field | $\overrightarrow{\boldsymbol{F}}$ is a force exerted on a point charge $q$ |
| 1.3 | $\overrightarrow{\boldsymbol{E}}=k_{C} \frac{q}{r^{2}} \frac{\overrightarrow{\boldsymbol{r}}}{r}$ | Electric field created by a stationary point charge $q$ | $\overrightarrow{\boldsymbol{r}}$ is a position vector with respect to charge |
| 1.4 | $\overrightarrow{\boldsymbol{E}}=\sum \overrightarrow{\boldsymbol{E}}_{i}$ | Electric field due to the system of charges (superposition principle) | $\overrightarrow{\boldsymbol{E}}_{i}$ are electric fields due to individual charges |


| 1 | 2 | 3 | 4 |
| :---: | :--- | :--- | :--- |
| 1.5 | $\overrightarrow{\boldsymbol{p}}_{e}=q \overrightarrow{\boldsymbol{\ell}}_{-+}$ | Electric dipole mo- <br> ment | $\overrightarrow{\boldsymbol{\ell}}_{-+}$is a position <br> vector of a positive <br> charge $+q$ with re- <br> spect to the negative <br> $-q$ |
| 1.6 | $\overrightarrow{\boldsymbol{E}}=k_{C}\left(\frac{3\left(\overrightarrow{\boldsymbol{p}}_{e} \cdot \overrightarrow{\boldsymbol{r}}\right) \overrightarrow{\boldsymbol{r}}}{r^{5}}-\right.$ <br> $\left.-\frac{\overrightarrow{\boldsymbol{p}}_{e}}{r^{3}}\right)$ <br> $\boldsymbol{\tau}=\overrightarrow{\boldsymbol{p}}_{e} \times \overrightarrow{\boldsymbol{E}}$ | Electric field of a <br> point dipole | $\overrightarrow{\boldsymbol{r}}$ is a position vector <br> with respect to the <br> dipole; $\ell_{-+} \ll r$ |
| Torque on an elec- <br> tric dipole | $\overrightarrow{\boldsymbol{E}}$ is a uniform exter- <br> nal electric field |  |  |

Pre-Class Reading: [1], chap. 21; [2], chap. 22; [3], chap. 23.

## Case 1.1

1.1.1. An average human weighs about 640 N . If two such humans each carry 0.1 C of excess charge, one positive and one negative, how far apart would they have to be for the electric attraction between them to equal their $640-\mathrm{N}$ weight?
1.1.2. An electric dipole with a dipole moment $\overrightarrow{\boldsymbol{p}}$ is in a uniform electric field $\overrightarrow{\boldsymbol{E}}$. (a) Find the orientations of the dipole for which the torque on the dipole is zero. (b) Which of the orientations in part (a) is stable, and which is unstable? (Hint: Consider a small displacement away from the equilibrium position and see what happens.) (c) Show that for the stable orientation in part (b), the dipole's own electric field tends to oppose the external field.
1.1.3. A proton is traveling horizontally to the right at $4.00 \times 10^{6} \mathrm{~m} / \mathrm{s}$. (a) Find the magnitude and direction of the weakest electric field that can bring the proton uniformly to rest over a distance of 1.67 cm . (b) How
much time does it take the proton to stop after entering the field?
1.1.4. A $-1.00-\mathrm{nC}$ point charge is at the origin, and a $+4.00-\mathrm{nC}$ point charge is on the $y$-axis at $y=2.00 \mathrm{~m}$. (a) Find the electric field (magnitude and direction) at each of the following points on the $y$-axis: (i) $y=3.00 \mathrm{~m}$; (ii) $y=1.00 \mathrm{~m}$; (iii) $y=-2.00 \mathrm{~m}$. (b) Find the net electric force that the two charges would exert on an electron placed at each point in part (a).
1.1.5. Positive charge $+q$ is distributed uniformly around a


Figure 1.1. Problem 1.1.5
semi-circle of radius $a$ that lies in the $3^{\text {rd }}$ and $4^{\text {th }}$ quadrants, with the center of curvature at the origin as shown in Fig. 1.1. Find the $x$ - and $y$-components of the net electric field at the origin.

## Case 1.2

1.2.1. A positive point charge $Q$ is placed on the $-x$-axis at $x=-a$, and a negative point charge $-Q$ is placed on the $+x$-axis at $x=a$. A positive point charge $q$ is located at some point on the $+y$-axis. (a) In a free-body diagram, show the forces that act on the charge $q$. (b) Find the $x$ - and $y$-components of the net force that the two charges $Q$ and $-Q$ exert on $q$. (Your answer should involve only $k, q, Q, a$ and the coordinate $y$ of the third charge.) (c) What is the net force on the charge $q$ when it is at the origin $(y=0)$ ? (d) Graph the $x$-component of the net force on the charge $q$ as a function of $y$ for values of $y$ between $-4 a$
and $+4 a$.
1.2.2. Negative charge $q$ is distributed uniformly along the $y$-axis


Figure 1.2. Problem 1.2.2
from $y=0$ to $y=a$. A positive point charge $Q$ is located on the positive $y$-axis at $y=a+r$ (Fig. 1.2). (a) Calculate the electric field (magnitude and direction) produced by the charge distribution $q$ at points on the positive $y$-axis where $y>a$. (b) Calculate the force (magnitude and direction) that the charge distribution $q$ exerts on $Q$. (c) Show that if $r=y \gg a$, the magnitude of the force in part (b) is approximately $k_{C} Q|q| / r^{2}$. Explain why this result is obtained.
1.2.3. An electron is projected with an initial speed $v_{0}=1.6 \cdot 10^{5} \mathrm{~m} / \mathrm{s}$


Figure 1.3. Problem 1.2.3
into the uniform electric field between the parallel plates in Fig. 1.3. Assume that the field between the plates is uniform and directed vertically
upward, and that the field outside the plates is zero. The electron enters the field at a point midway between the plates. (a) If the electron just misses the upper plate as it emerges from the field, find the magnitude of the electric field. (b) Suppose that in Fig. 1.3 the electron is replaced by a proton with the same initial speed $v_{0}$. Would the proton hit one of the plates? If the proton would not hit one of the plates, what would be the magnitude and direction of its vertical displacement as it exits the region between the plates? (c) Compare the paths traveled by the electron and the proton and explain the differences. (d) Discuss whether it is reasonable to ignore the effects of gravity for each particle.
1.2.4. Three charges are situated at the corners of an isosceles trian-


Figure 1.4. Problem 1.2.4
gle as shown in Fig. 1.4. The $\pm 2.00-\mathrm{nC}$ charges form a dipole. (a) Find the force (magnitude and direction) the $+5.00-\mu \mathrm{C}$ charge exerts on the dipole. (b) For an axis perpendicular to the line connecting the $\pm 2.00-\mathrm{nC}$ charges at the midpoint of this line, find the torque (magnitude and direction) exerted on the dipole by the $+5.00-\mu \mathrm{C}$ charge.
1.2.5. Sodium chloride ( NaCl , ordinary table salt) is made up of positive sodium ions $\left(\mathrm{Na}^{+}\right)$and negative chloride ions $\left(\mathrm{Cl}^{-}\right)$. (a) If a point charge with the same charge and mass as all the $\mathrm{Na}^{+}$ions in 0.010 mol of NaCl is 9.6 mm from a point charge with the same charge
and mass as all the $\mathrm{Cl}^{-}$ions, what is the magnitude of the attractive force between these two point charges? (b) If the negative point charge in part (a) is held in place and the positive point charge is released from rest, what is its initial acceleration? (Atomic mass of Na-atom is $23 \mathrm{~g} / \mathrm{mol}$.) (c) Does it seem reasonable that the ions in NaCl could be separated in this way? Why or why not? (In fact, when sodium chloride dissolves in water, it breaks up into $\mathrm{Na}^{+}$and $\mathrm{Cl}^{-}$ions. However, in this situation, there are additional electric forces exerted by the water molecules on the ions.)

## Case 1.3

1.3.1. Three point charges are arranged on a line. Charge $Q=-15.00 \mathrm{nC}$ and is at the origin. Charge $q_{2}=+25.00 \mathrm{nC}$ is at $z=-5.0 \mathrm{~cm}$. Charge $q_{1}$ is at $z=-1.00 \mathrm{~cm}$. What is $q_{1}$ (magnitude and sign) if the net force on $Q$, is zero?
1.3.2. A ring-shaped conductor with a radius $a=3.00 \mathrm{~cm}$ has a


Figure 1.5. Problem 1.3.2
total positive charge $Q=+1.25 \mathrm{nC}$ uniformly distributed around it, as shown in Fig. 1.5. The center of the ring is at the origin of coordinates $O$. (a) What is the electric field (magnitude and direction) at a point $P$ which is on the $Y$-axis at $y=4.00 \mathrm{~cm}$ ? (b) A point charge $q=-5.00 \mu \mathrm{C}$
is placed at the point $P$ described in part (a). What are the magnitude and direction of the force exerted by the charge $q$ on the ring?
1.3.3. A point charge is at the origin. With this point charge as the source point, what is the unit vector $\overrightarrow{\boldsymbol{r}} / r$ in the direction of (a) the field point at $x=-12.00 \mathrm{~cm}, y=0 \mathrm{~cm}$; (b) the field point at $x=2.0 \mathrm{~mm}$, $y=-2.0 \mathrm{~mm}$; (c) the field point at $x=8.00 \mathrm{~m}, y=6.00 \mathrm{~m}$ ? Express your results in terms of unit vectors $\overrightarrow{\boldsymbol{i}}$ and $\overrightarrow{\boldsymbol{j}}$.
1.3.4. Point charges $q_{1}=+5.0 \mathrm{nC}$ and $q_{2}=-5.0 \mathrm{nC}$ are separated by 4.0 cm forming an electric dipole. (a) Find the electric dipole moment (magnitude and direction). (b) The charges are in a uniform electric field whose direction makes an angle of $30.0^{\circ}$ with the line connecting the charges. What is the magnitude of this field if the torque exerted on the dipole has a magnitude of $8.0 \cdot 10^{-9} \mathrm{~N} \cdot \mathrm{~m}$ ?
1.3.5. Two identical beads each have a mass $m$ and a charge $q$.


Figure 1.6. Problem 1.3.5
When placed in a hemispherical bowl of a radius $R$ with frictionless nonconducting walls, the beads move. At equilibrium, they are a distance $d<2 R$ apart (Fig. 1.6). (a) Determine the charge $q$ on each bead. (b) Determine the charge required for $d$ to become equal to $R$ if $m=17.3 \mathrm{~g}$ and $R=30 \mathrm{~cm}$.

## Case 1.4

1.4.1. A molecule of DNA (deoxyribonucleic acid) is $2.17 \mu \mathrm{~m}$ long. The ends of the molecule become singly ionized: negative on one end, positive on the other. The helical molecule acts like a spring and com-
presses $1.00 \%$ upon becoming charged. Determine the effective spring constant of the molecule.
1.4.2. Two small spheres with a mass $m=10.0 \mathrm{~g}$ are hung by silk


Figure 1.7. Problem 1.4.2
threads of a length $L=1.50 \mathrm{~m}$ from a common point (Fig. 1.7). When the spheres are given equal negative charge, so that $q_{1}=q_{2}=q$, each thread hangs at $\theta=45.0^{\circ}$ from the vertical. (a) Draw a diagram showing the forces acting on each sphere. Treat the spheres as point charges. (b) Find the magnitude of $q$. (c) Both threads are now shortened to half of their original length $\left(L_{1}=0.75 \mathrm{~m}\right)$ while the charges $q_{1}$ and $q_{2}$ remain unchanged. What angle will each thread make with the vertical? (Hint: This part of the problem can be solved numerically by using trial values for $\theta$ and adjusting the values of $\theta$ until a self-consistent answer is obtained.)
1.4.3. A uniformly charged ring of a radius 3.0 cm has a total charge of 30.0 nC . Find the electric field on the axis of the ring at (a) 4.00 cm , (b) 1.0 mm , and (c) 1.00 m from the center of the ring. (d) Find the strongest electric field on the axis of the ring.
1.4.4. A water molecule has a permanent electric dipole moment
of magnitude $6 \cdot 10^{-30} \mathrm{C} \cdot \mathrm{m}$. Estimate the value of the electric field it produces at the position of a neighboring water molecule which is $3 \cdot 10^{-9} \mathrm{~m}$ away.
1.4.5. Positive electric charge is distributed along the $x$-axis with a charge per unit length $\lambda$. (a) Consider the case where charge is distributed only between the points $x=a$ and $x=-a$. For points on the $+y$-axis, graph the $y$-component of the electric field as a function of $y$ for values of $y$ between $y=a / 4$ and $y=16 a$. (b) Consider instead the case where the charge is distributed along the entire $x$-axis with the same charge per unit length $\lambda$. Using the same graph as in part (a), plot the $y$-component of the electric field as a function of $y$ for values of $y$ between $y=a / 4$ and $y=16 a$. Label which graph refers to which situation.

## Case 1.5

1.5.1. A charge $q$ is split into two parts, $q=q_{1}+q_{2}$. In order to maximize the repulsive Coulomb force between $q_{1}$ and $q_{2}$, what fraction of the original charge $q$ should $q_{1}$ and $q_{2}$ have?
1.5.2. An electric dipole consists of two opposite charges of a mag-


Figure 1.8. Problem 1.5.2
nitude $6.0 \mu \mathrm{C}$ placed 2.0 cm apart (Fig. 1.8). The dipole is placed in a uniform electric field of $5.0 \mathrm{~N} / \mathrm{C}$ along the $x$-axis, with the direction of $\overrightarrow{\boldsymbol{p}}$ at an angle of $+30^{\circ}$ from the $x$-axis in the $x y$-plane. Determine the
torque on the dipole.
1.5.3. Calculate the electric field due to an infinitely long thin uniformly charged rod with a charge density of $5 \cdot 10^{-7} \mathrm{C} / \mathrm{m}$ at a distance of 9 cm from the rod. Assume that the rod is aligned with the $x$-axis.
1.5.4. A rod of a length $2 l$ is charged uniformly over its length with a total positive charge $q$. Another positive point charge $q$ is placed at a distance $2 l$ from the midpoint of the rod along a line perpendicular to the rod. Calculate the magnitude of the total electric field at a point halfway between the point charge and the center of the rod.
1.5.5. A uniformly charged disk has a radius of 3.00 cm and carries a total charge of $1.0 \cdot 10^{-9} \mathrm{C}$. (a) Find the electric field (magnitude and direction) on the $x$-axis at $x=30.0 \mathrm{~cm}$. (b) Show that for $x \gg R$, $E=k_{C} Q / x^{2}$, where $Q$ is the total charge on the disk. (c) Is the magnitude of the electric field you calculated in part (a) larger or smaller than the electric field 30.0 cm from a point charge that has the same total charge as this disk?

## Chapter 2

## GAUSS'S LAW

|  | Equation | Equation title | Comments |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |
| 2.1 | $\Phi_{E}=E A \cos \theta$ | Electric flux of a uniform electric field $\overrightarrow{\boldsymbol{E}}$ through plain surface | $A$ is a surface area; $\theta$ is an angle between the $\overrightarrow{\boldsymbol{E}}$ and a unit vector $\overrightarrow{\boldsymbol{n}}$ of normal to the surface |
| 2.2 | $d \Phi_{E}=\overrightarrow{\boldsymbol{E}} \cdot d \overrightarrow{\boldsymbol{A}}$ | Infinitesimal electric flux | $d \overrightarrow{\boldsymbol{A}}=\overrightarrow{\boldsymbol{n}} d A$ is a vector of infinitesimal surface area |
| 2.3 | $\Phi_{E}=\iint_{A} \overrightarrow{\boldsymbol{E}} \cdot d \overrightarrow{\boldsymbol{A}}$ | Electric flux through the surface $A$ (general case) | Nonuniform electric field $\overrightarrow{\boldsymbol{E}}$; arbitrary surface $A$ |
| 2.4 | $\oiint_{A} \overrightarrow{\boldsymbol{E}} \cdot d \overrightarrow{\boldsymbol{A}}=\frac{q_{n e t}^{e n c}}{\varepsilon_{0}}$ | Gauss's law for electric field (total electric flux through a closed surface $A$ ) | $q_{n e t}^{e n c}$ is a net electric charge enclosed by the surface $A ; \varepsilon_{0}$ is an electric constant |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 2.5 | $\lambda=\frac{d q}{d \ell}$ | Linear charge density | $d q$ is an infinitesimal charge distributed over an infinitesimal segment of length $d \ell$ |
| 2.6 | $\sigma=\frac{d q}{d A}$ | Surface charge density | $d q$ is an infinitesimal charge distributed over an infinitesimal surface of area $d A$ |
| 2.7 | $\rho=\frac{d q}{d V o l}$ | Volume charge density | $d q$ is an infinitesimal charge distributed within an infinitesimal spatial element of a volume $d V o l$ |
| 2.8 | $\overrightarrow{\boldsymbol{E}}=\frac{\lambda}{2 \pi \varepsilon_{0} R} \frac{\overrightarrow{\boldsymbol{R}}}{R}$ | Electric field of an infinite uniformly charged wire | $\overrightarrow{\boldsymbol{R}}$ is a radial position vector in the plane perpendicular to the wire |
| 2.9 | $\overrightarrow{\boldsymbol{E}}=\frac{\sigma}{2 \varepsilon_{0}} \overrightarrow{\boldsymbol{n}}$ | Electric field of an infinite uniformly charged plane | $\overrightarrow{\boldsymbol{n}}$ is a unit vector perpendicular to the plane |
| 2.10 | $\overrightarrow{\boldsymbol{E}}=\frac{\rho \overrightarrow{\boldsymbol{r}}}{3 \varepsilon_{0}}$ | Electric field inside a uniformly charged solid sphere | $\overrightarrow{\boldsymbol{r}}$ is a position vector with respect to the center of the sphere |

Pre-Class Reading: [1], chap. 22; [2], chap. 23; [3], chap. 24.

## Case 2.1

2.1.1. A flat sheet of paper of area $0.4 \mathrm{~m}^{2}$ is oriented so that the sheet is at an angle of $30^{\circ}$ to a uniform electric field of a magnitude 5.0 N/C. (a) Find the magnitude of the electric flux through the sheet. (b) Does the answer to part (a) depend on the shape of the sheet? Why or why not? (c) For what angle $\phi$ between the normal to the sheet and the electric field is the magnitude of the flux through the sheet (i) the largest and (ii) the smallest? Explain your answer.
2.1.2. A charged paint is spread in a very thin uniform layer over the surface of a plastic sphere of diameter 4.0 cm giving it a charge of +30.0 nC . Find the electric field (a) just inside the paint layer; (b) just outside the paint layer; (c) 1.00 cm outside the surface of the paint layer.
2.1.3. How many excess electrons must be added to an isolated spherical conductor 9.0 cm in diameter to produce an electric field of $1.6 \cdot 10^{3} \mathrm{~N} / \mathrm{C}$ just outside the surface?
2.1.4. The electric field $\overrightarrow{\boldsymbol{E}}$ in Fig. 2.1 is everywhere parallel to the


Figure 2.1. Problem 2.1.4
$x$-axis, so the components $E_{y}$ and $E_{z}$ are zero. The $x$-component of the field $E_{x}$ depends on $x$ but not on $y$ and $z$. At points in the $y z$-plane (where $x=0$ ), $E_{x}=200 \mathrm{~N} / \mathrm{C}$. (a) What is the electric flux through the surface $\mathbf{I}$ in Fig. 2.1? (b) What is the electric flux through the surface

II? (c) The volume shown in the figure is a small section of a very large insulating slab 1.0 m thick. If there is a total charge of -8.85 nC within the volume shown, what are the magnitude and direction of $\overrightarrow{\boldsymbol{E}}$ at the face opposite the surface $\mathbf{I}$ ? (d) Is the electric field produced only by charges within the slab, or is the field also due to charges outside the slab? How can you tell?
2.1.5. Positive charge $+3.00 \mu \mathrm{C}$ is distributed uniformly over the surface of a thin spherical insulating shell with a radius of 2.00 cm . Calculate the force (magnitude and direction) that the shell exerts on a positive point charge -2.00 nC located (a) a distance 3.00 cm from the center of the shell and (b) a distance of 1.00 cm from the center of the shell.

## Case 2.2

2.2.1. You measure an electric field of $9.00 \cdot 10^{5} \mathrm{~N} / \mathrm{C}$ at a distance of 0.10 m from a point charge. (a) What is the electric flux through a sphere at that distance from the charge? (b) What is the magnitude of the charge?
2.2.2. The electric field at a distance of 6.00 cm from the surface of a solid insulating sphere with a radius of 2.00 cm is $3.0 \cdot 10^{3} \mathrm{~N} / \mathrm{C}$. (a) Assuming the sphere's charge is uniformly distributed, what is the charge density inside it? (b) Calculate the electric field inside the sphere at a distance of 1.00 cm from the center.
2.2.3. A $5.0-\mathrm{g}$ piece of Styrofoam carries a net charge of $+17.7 \mu \mathrm{C}$ and is suspended in equilibrium above the center of a large horizontal sheet of plastic that has a uniform charge density on its surface. What is the charge per unit area on the plastic sheet?
2.2.4. A long coaxial cable consists of an inner cylindrical conductor with a radius $a$ and an outer coaxial cylinder with an inner radius $b$ and an outer radius $c$. The outer cylinder is mounted on insulating supports and has no net charge. The inner cylinder has a uniform negative charge per unit length $\lambda$. Calculate the electric field (a) at any point between the cylinders a distance $r$ from the axis and (b) at any point outside
the outer cylinder. (c) Graph the magnitude of the electric field as a function of the distance $r$ from the axis of the cable, from $r=0$ to $r=2 c$. (d) Find the charge per unit length on the inner surface and on the outer surface of the outer cylinder.
2.2.5. A nonuniform but spherically symmetric distribution of charge has a charge density $\rho(r)$ given as follows:

$$
\begin{cases}\rho(r)=\rho_{0}(1-r / R) & \text { for } r \leq R \\ \rho(r)=0 & \text { for } r \geq R,\end{cases}
$$

where $\rho_{0}=3 Q / \pi R^{3}$ is a positive constant. (a) Show that the total charge contained in the charge distribution is $Q$. (b) Show that the electric field in the region $r \geq R$ is identical to that produced by a point charge $Q$ at $r=0$. (c) Obtain an expression for the electric field in the region $r \leq R$. (d) Graph the electric-field magnitude $E$ as a function of $r$. (e) Find the value of $r$ at which the electric field is maximum and find the value of that maximum field.

## Case 2.3

2.3.1. An infinitely long cylindrical conductor has a radius $R$ and uniform surface charge density $\sigma$. (a) In terms of $\sigma$ and $R$, what is the charge per unit length $\lambda$ for the cylinder? (b) In terms of $\sigma$, what is the magnitude of the electric field produced by the charged cylinder at a distance $r>R$ from its axis? (c) Express the result of part (b) in terms of $\lambda$ and show that the electric field outside the cylinder is the same as if all the charge were on the axis. Compare your result to the result for a line of charge.
2.3.2. A solid sphere of radius 30.0 cm has a total positive charge of $12.0 \mu \mathrm{C}$ uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm , (b) 3.0 cm , (c) 30.0 cm , and (d) 60.0 cm from the center of the sphere.
2.3.3. A hemispherical surface with a radius $r$ in a region of a uniform electric field $\overrightarrow{\boldsymbol{E}}$ has its axis aligned parallel to the direction of the field.

Calculate the flux through the surface.
2.3.4. A small sphere with a mass of 0.5 g and carrying a charge


Figure 2.2. Problem 2.3.4
of $8.85 \mu \mathrm{C}$ hangs from a thread near a very large charged conducting sheet, as shown in Fig. 2.2. The charge density on the sheet is $10.0 \mathrm{nC} / \mathrm{m}^{2}$. Find the angle of the thread.
2.3.5. (a) An insulating sphere with a radius $a$ has a uniform charge


Figure 2.3. Problem 2.3.5
density $\rho$. The sphere is not centered at the origin but at $\overrightarrow{\boldsymbol{r}}=\overrightarrow{\boldsymbol{b}}$. Find the electric field inside the sphere. (b) An insulating sphere of a radius $R$ has a spherical hole of a radius $a$ located within its volume and centered a distance $b$ from the center of the sphere, where $a<b<R$ (Fig. 2.3 shows a cross section of the sphere). The solid part of the sphere has a uniform volume charge density $\rho$. Find the magnitude and direction of
the electric field $\overrightarrow{\boldsymbol{E}}$ inside the hole, and show that $\overrightarrow{\boldsymbol{E}}$ is uniform over the entire hole. [Hint: Use the principle of superposition and the result of part (a).]

## Case 2.4

2.4.1. A 17.7 nC point charge is at the center of a cube with sides 0.500 m long. (a) What is the electric flux through one of the six faces of the cube? (b) How would your answer to part (a) change if the sides were 0.250 m long? Explain.
2.4.2. An infinitely long line charge having a uniform charge per


Figure 2.4. Problem 2.4.2
unit length $\lambda$ lies a distance $d$ from a point $O$ as shown in Fig. 2.4. Determine the total electric flux through the surface of a sphere of radius $R$ centered at $O$ resulting from this line charge. Consider both cases, where (a) $R<d$ and (b) $R>d$.
2.4.3. Consider a long cylindrical charge distribution of a radius $R$ with a uniform charge density $\rho$. Find the electric field at a distance $r$ from the axis, where $r<R$.
2.4.4. A solid conducting sphere with a radius $R$ that carries a positive charge $Q$ is concentric with a very thin insulating shell of a radius
$2 R$ that also carries a charge $Q$. The charge $Q$ is distributed uniformly over the insulating shell. (a) Find the electric field (magnitude and direction) in each of the regions $0<r<R, R<r<2 R$, and $r>2 R$. (b) Graph the electric-field magnitude as a function of $r$.
2.4.5. A positive charge $Q$ is distributed uniformly over each of two


Figure 2.5. Problem 2.4.5
spherical volumes with a radius $R$. One sphere of charge is centered at the origin and the other at $x=2 R$ (Fig. 2.5). Find the magnitude and direction of the net electric field due to these two distributions of the charge at the following points on the $x$-axis: (a) $x=0$; (b) $x=R / 2$; (c) $x=R$; (d) $x=3 R$.

## Case 2.5

2.5.1. An electric field of a magnitude $400.0 \mathrm{~N} / \mathrm{C}$ is applied along the $x$ axis. Calculate the electric flux through a rectangular plane 3.0 m wide and 2.0 m long (a) if the plane is parallel to the $y z$ plane, (b) if the plane is parallel to the $x y$ plane, and (c) if the plane contains the $y$ axis and its normal makes an angle of $60.0^{\circ}$ with the $x$ axis.
2.5.2. In the air over a particular region at an altitude of 400 m above the ground, the electric field is $140 \mathrm{~N} / \mathrm{C}$ directed upward. At 500 m above the ground, the electric field is $120 \mathrm{~N} / \mathrm{C}$ upward. What is the average volume charge density in the layer of air between these two elevations? Is it positive or negative?
2.5.3. A long cylindrical shell of an inner radius $r_{1}$ and an outer
radius $r_{2}$ carries a uniform volume charge density $\rho$. Find the electric field due to this distribution of charge everywhere in space.
2.5.4. A small conducting spherical shell with an inner radius $a$


Figure 2.6. Problem 2.5.4
and an outer radius $b$ is concentric with a larger conducting spherical shell with an inner radius $c$ and an outer radius $d$ (Fig. 2.6). The inner shell has a total charge $+2 q$, and the outer shell has a charge $+4 q$. (a) Calculate the electric field (magnitude and direction) in terms of $q$ and the distance $r$ from the common center of the two shells for (i) $r<a$; (ii) $a<r<b$; (iii) $b<r<c$; (iv) $c<r<d$; (v) $r>d$. Show your results in a graph of the radial component of $\overrightarrow{\boldsymbol{E}}$ as a function of $r$. (b) What is the total charge on the (i) inner surface of the small shell; (ii) outer surface of the small shell; (iii) inner surface of the large shell; (iv) outer surface of the large shell?
2.5.5. A given region has an electric field that is a sum of two contributions, a field due to a charge $q=17.7 \cdot 10^{-8} \mathrm{C}$ at the origin and a uniform field of strength $E_{0}=1.0 \cdot 10^{3} \mathrm{~N} / \mathrm{C}$ in the $-x$-direction. Calculate the flux through each side of a cube with sides 1.0 m long that are parallel to the $x$-, $y$-, and $z$-directions. The cube is centered at the origin.

Chapter 3
ELECTRIC POTENTIAL

|  | Equation | Equation title | Comments |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |
| 3.1 | $\oint_{L} \overrightarrow{\boldsymbol{E}} \cdot d \overrightarrow{\boldsymbol{\ell}}=0$ | Condition for potentiality of electrostatic field | $d \overrightarrow{\boldsymbol{\ell}}$ is an infinitesimal element of an arbitrary closed loop $L$ |
| 3.2 | $\begin{aligned} & V_{i}-V_{f}=\frac{W_{i f}}{q}= \\ = & \int_{i}^{f} \overrightarrow{\boldsymbol{E}} \cdot d \overrightarrow{\boldsymbol{\ell}} \end{aligned}$ | Electric potential difference | $W_{i f}$ is a work done by electric field $\overrightarrow{\boldsymbol{E}}$ on a charge $q$ to move it form the $i$ nitial position to the $f$ inal position |
| 3.3 | $V=\frac{1}{4 \pi \varepsilon_{0}} \frac{q}{r}=k_{C} \frac{q}{r}$ | Electric potential due to a point charge $q$ | $r$ is a distance from the charge to the point under consideration |
| 3.4 | $V=\sum V_{j}$ | Electric potential due to the system of charges | $V_{j}$ is an electric potential due to an individual charge |
| 3.5 | $W_{i f}=q\left(V_{i}-V_{f}\right)$ | Work done by an electric field on a charge $q$ | $V_{i}$ and $V_{f}$ are potentials at initial and $f$ inal positions |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 3.6 | $\begin{aligned} & \overrightarrow{\boldsymbol{E}}=-\operatorname{grad} V= \\ & =-\vec{\nabla} V \end{aligned}$ | Relation between an electrostatic field and a potential | grad is a vector differential operator (see Appendix for details) |
| 3.7 | $V_{i}-V_{f}=E d_{i f}$ | Potential difference in a uniform electric field $E$ | $d_{i f}$ is a distance between two points along the field line |
| 3.8 | $\begin{aligned} & \quad V_{i}-\frac{V_{f}}{}= \\ & =\frac{\lambda}{2 \pi \varepsilon_{0}} \ln \frac{r_{f}}{r_{i}}= \\ & =2 k_{C} \lambda \ln \frac{r_{f}}{r_{i}} \end{aligned}$ | Potential difference in the vicinity of the uniformly charged infinite wire | $\lambda$ is a linear charge density; $r_{i}$ and $r_{f}$ are the distances from the wire |

Pre-Class Reading: [1], chap. 23; [2], chap. 24; [3], chap. 25.

## Case 3.1

3.1.1. A point charge $q_{1}=+2.00 \mathrm{nC}$ is held stationary at the origin. A second point charge $q_{2}=-4.50 \mathrm{nC}$ moves from the point $x_{i}=0.300 \mathrm{~m}, y_{i}=0.400$ to the point $x_{f}=0.600 \mathrm{~m}, y_{f}=0.800 \mathrm{~m}$. How much work is done by the electric force on $q_{2}$ ?
3.1.2. A uniform electric field is directed due west. Point $B$ is 2.00 m west of point $A$, point $C$ is 2.00 m east of point $A$, and point $D$ is 2.00 m north of $A$. For each point, $B, C$, and $D$, is the potential at that point larger, smaller, or the same as at point $A$ ? Give the reasoning behind your answers.
3.1.3. A uniformly charged thin ring has a radius of 8.0 cm and a total charge of +18.2 nC . An electron is placed on the ring's axis 6.0 cm from the center of the ring and is constrained to stay on the axis of the ring. The electron is then released from rest. (a) Describe the subsequent motion of the electron. (b) Find the speed of the electron when it reaches
the center of the ring.
3.1.4. In a certain region of space, the electric potential is $V(x, y, z)=A z^{2}+B y-C y z$ where $A, B$, and $C$ are positive constants. (a) Calculate the $x$-, $y$-, and $z$-components of the electric field. (b) At which points is the electric field equal to zero?
3.1.5. The electric potential immediately outside a charged conducting sphere is 270 V .5 .0 cm farther from the center of the sphere, the potential is 220 V . Determine (a) the radius of the sphere and (b) the charge on it. The electric potential immediately outside another charged conducting sphere is 200 V .18 .0 cm farther from the center the magnitude of the electric field is being $100 \mathrm{~V} / \mathrm{m}$. Determine (c) the radius of the sphere and (d) its charge on it. (e) Are the answers to parts (c) and (d) unique?

## Case 3.2

3.2.1. How much work is needed to assemble an atomic nucleus containing three protons (such as Li ) if we model it as an equilateral triangle of a side $2.56 \cdot 10^{-15} \mathrm{~m}$ with a proton at each vertex? Assume the protons started from very far away.
3.2.2. A charge of 12.0 nC is placed in a uniform electric field that is directed vertically upward and has a magnitude of $1.00 \cdot 10^{4} \mathrm{~V} / \mathrm{m}$. What work is done by the electric force when the charge moves (a) 0.50 m to the left; (b) 0.50 m downward; (c) 0.50 m at an angle of $30.0^{\circ}$ upward from the horizontal?
3.2.3. A very long wire carries a uniform linear charge density $\lambda$. Using a voltmeter to measure potential difference, you find that when one probe of the meter is placed 10.0 cm from the wire and the other one is 17.2 cm farther from the wire, the meter reads 630 V . (a) What is $\lambda$ ? (b) If you now place one probe at 27.2 cm from the wire and the other probe 17.2 cm farther away, will the voltmeter read 630 V ? If not, will it read more or less than 630 V ? Why? (c) If you place both probes 10.0 cm from the wire but 17.2 cm from each other, what will
the voltmeter read?
3.2.4. The electric potential inside a charged spherical conductor of radius $R$ is given by $V=k_{C} q / R$, and the potential outside is given by $V=k_{C} q / r$. Using $\overrightarrow{\boldsymbol{E}}=-\vec{\nabla} V$, derive the electric field (a) inside and (b) outside this charge distribution.
3.2.5. A particle with a charge of +8.00 nC is in a uniform electric field directed to the right. Another force, in addition to the electric force, acts on the particle so that when it is released from rest, it moves to the left. After it has moved 5.00 cm , the additional force has done $73.6 \mu \mathrm{~J}$ of work and the particle has $41.6 \mu \mathrm{~J}$ of kinetic energy. (a) What work was done by the electric force? (b) What is the potential of the starting point with respect to the end point? (c) What is the magnitude of the electric field?

## Case 3.3

3.3.1. A small metal sphere, carrying a net charge of $q_{1}=-5.0 \mu \mathrm{C}$


Figure 3.1. Problem 3.3.1
is held in a stationary position by insulating supports. A second small metal sphere with a net charge of $q_{2}=-6.0 \mu \mathrm{C}$ and mass 1.08 g is projected toward $q_{1}$. When the two spheres are 1.0 m apart, $q_{2}$ is moving toward $q_{1}$ with the speed of $30.0 \mathrm{~m} / \mathrm{s}$ (Fig. 3.1). Assume that the two spheres can be treated as point charges. You can ignore the force of gravity. (a) What is the speed of $q_{2}$ when the spheres are 0.5 m apart? (b) How close does $q_{2}$ get to $q_{1}$ ?
3.3.2. Consider a ring of a radius $R$ with the total charge $q$ spread
uniformly over its perimeter. What is the potential difference between the point at the center of the ring and a point on its axis a distance $\frac{3}{4} R$ from the center?
3.3.3. Two point charges, $q_{1}=+3.2 \mathrm{nC}$ and $q_{2}=-4.80 \mathrm{nC}$, are


Figure 3.2. Problem 3.3.3
0.50 m apart. Point $A$ is midway between them; point $B$ is 0.40 m from $q_{1}$ and 0.30 m from $q_{2}$ (Fig. 3.2). Take the electric potential to be zero at infinity. Find (a) the potential at point $A ;(\mathrm{b})$ the potential at point $B$; (c) the work done by the electric field on a charge of $+2.5 \mu \mathrm{C}$ that travels from point $A$ to point $B$.
3.3.4. A metal sphere with a radius $a$ is supported on an insulating stand at the center of a hollow metal spherical shell with radius $b$. There is a charge $+Q$ on the inner sphere and a charge $-Q$ on the outer spherical shell. Calculate the potential $V(r)$ for (i) $r<a$; (ii) $a<r<b$; (iii) $r>b$. (Hint: The net potential is the sum of the potentials due to the individual spheres.) Take $V$ to be zero when $r$ is infinite.
3.3.5. A vacuum tube diode consists of concentric cylindrical electrodes, the negative cathode and the positive anode. Because of the accumulation of charge near the cathode, the electric potential between the electrodes is not a linear function of the position, even with planar geometry, but is given by $V(x)=C x^{4 / 3}$ where $x$ is the distance from
the cathode and $C$ is a constant, characteristic of a particular diode and operating conditions. Assume that the distance between the cathode and anode is 8.0 mm and the potential difference between electrodes is 320 V . (a) Determine the value of $C$. (b) Obtain a formula for the electric field between the electrodes as a function of $x$. (c) Determine the force on an electron when the electron is halfway between the electrodes.

## Case 3.4

3.4.1. An insulating rod having linear charge density $\lambda=10 \mu \mathrm{C} / \mathrm{m}$


Figure 3.3. Problem 3.4.1
and linear mass density $\mu=30.0 \mathrm{~g} / \mathrm{m}$ is released from rest in a uniform electric field $E=500 \mathrm{~V} / \mathrm{m}$ directed perpendicular to the rod (Fig. 3.3). (a) Determine the speed of the rod after it has traveled 3.0 m . (b) How does your answer to part (a) change if the electric field is not perpendicular to the rod? Explain.
3.4.2. A point charge $q_{1}=4.00 \mathrm{nC}$ is placed at the origin, and a second point charge $q_{2}=-3.00 \mathrm{nC}$ is placed on the $x$-axis at $x=+20.0 \mathrm{~cm}$. A third point charge $q_{3}=2.00 \mathrm{nC}$ is to be placed on the $x$-axis between $q_{1}$ and $q_{2}$. (Take the potential energy of the three charges as zero when they are infinitely far apart.) (a) What is the potential energy of the system of the three charges if $q_{3}$ is placed at $x=+10.0 \mathrm{~cm}$ ? (b) Where should $q_{3}$ be placed to make the potential
energy of the system equal to zero?
3.4.3. Figure 3.4 shows eight point charges arranged at the corners of


Figure 3.4. Problem 3.4.3
a cube with sides of length $d$. The values of the charges are $+q$ and $-q$, as shown. This is a model of one cell of a cubic ionic crystal. In sodium chloride $(\mathrm{NaCl})$, for instance, the positive ions are $\mathrm{Na}^{+}$and the negative ions are $\mathrm{Cl}^{-}$. (a) Calculate the potential energy $U$ of this arrangement. (Take the potential energy of the eight charges as zero when they are infinitely far apart.) (b) In part (a), you should have found that $U<0$. Explain the relationship between this result and the observation that such ionic crystals exist in nature.
3.4.4. A very long insulating cylindrical shell of a radius 5.0 cm carries the charge of a linear density $111 \mathrm{nC} / \mathrm{m}$ spread uniformly over its outer surface. What would a voltmeter read if it were connected between (a) the surface of the cylinder and a point 5.0 cm above the surface, and (b) the surface and a point 2.0 cm from the central axis of the cylinder?
3.4.5. A very long cylinder of a radius 1.00 cm carries a uniform charge density of $0.5 \mathrm{nC} / \mathrm{m}$. (a) Describe the shape of the equipotential
surfaces for this cylinder. (b) Taking the reference level for the zero of potential to be the surface of the cylinder, find the radius of equipotential surfaces having potentials of $9.0 \mathrm{~V}, 18.0 \mathrm{~V}$, and 27.0 V . (c) Are the equipotential surfaces equally spaced? If not, do they get closer together or farther apart as $r$ increases?

## Case 3.5

3.5.1. How much work is required to assemble eight identical charged particles, each of a magnitude $Q$, at the corners of a cube of side $s$ ?
3.5.2. A positive charge $+q$ is located at the point $x=0, y=-a$, and another positive charge $+q$ is located at the point $x=0$, $y=+a$. (a) Show the positions of the charges in a diagram. (b) Derive an expression for the potential $V$ at points on the $x$-axis as a function of the coordinate $x$. Take $V$ to be zero at an infinite distance from the charges. (c) Graph $V$ at points on the $x$-axis as a function of $x$ over the range from $x=-4 a$ to $x=+4 a$. (d) What is the answer to part (b) if the one of charges are replaced by $-q$ ?
3.5.3. Two large parallel metal plates carry opposite charges of equal magnitude. They are separated by 45.0 mm , and the potential difference between them is 360 V . (a) What is the magnitude of the electric field (assumed to be uniform) in the region between the plates? (b) What is the magnitude of the force this field exerts on a particle with the charge of +2.40 nC ? (c) Use the results of part (b) to compute the work done by the field on the particle as it moves from the higher-potential plate to the lower one. (d) Compare the result of part (c) to the change of potential energy of the same charge, computed from the electric potential.
3.5.4. A metal sphere with a radius $r_{a}$ is supported on an insulating stand at the center of a hollow metal spherical shell with a radius $r_{b}$. There is charge $-q$ on the inner sphere and charge $+q$ on the outer spherical shell. Find the potential of the inner sphere with respect to the outer one.
3.5.5. A long metal cylinder with a radius $a$ is supported on an insulating stand on the axis of a long hollow metal tube with a radius
$b$. The positive charge per unit length on the inner cylinder is $\lambda$, and there is an equal negative charge per unit length on the outer cylinder. (a) Calculate the potential $V(r)$ for (i) $r<a$; (ii) $a<r<b$; (iii) $r>b$. (Hint: The net potential is the sum of the potentials due to the individual conductors.) Take $V=0$ at $r=b$. (b) Express the the magnitude of electric field at any point between cylinders through their potential difference.

Chapter 4

## ELECTRIC FIELD IN A MEDIUM. CAPACITANCE

|  | Equation | Equation title | Comments |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |
| 4.1 | $C=\frac{q}{V}$ | Capacitance of an isolated conductor | $q$ and $V$ are charge and potential of the conductor, respectively |
| 4.2 | $C=4 \pi \varepsilon_{0} R=\frac{R}{k_{C}}$ | Capacitance of a conducting sphere | $R$ is a radius of the sphere |
| 4.3 | $C=\frac{q}{V_{12}}$ | Capacitance of a capacitor | $q$ is a charge on one of the plates of the capacitor; $V_{12}$ is a potential difference across the capacitor |
| 4.4 | $C=\frac{\varepsilon_{0} A}{d}$ | Capacitance of an air parallel-plate capacitor | $A$ is an area of the capacitor's plates; $d$ is a distance between them |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 4.5 | $C=\frac{2 \pi \varepsilon_{0} l}{\ln \left(R_{\mathrm{o}} / R_{\mathrm{i}}\right)}$ | Capacitance of an air cylindrical capacitor | $R_{\mathrm{o}}$ and $R_{\mathrm{i}}$ are radii of the outer and inner cylinders |
| 4.6 | $C=4 \pi \varepsilon_{0} \frac{R_{\mathrm{i}} R_{\mathrm{o}}}{R_{\mathrm{o}}-R_{\mathrm{i}}}$ | Capacitance of an air spherical capacitor | $R_{\mathrm{o}}$ and $R_{\mathrm{i}}$ are radii of the outer and inner spheres |
| 4.7 | $C_{p a r}=\sum C_{i}$ | Equivalent capacitance of a battery of capacitors connected in parallel | $C_{i}$ is a capacitance of an individual capacitor in the battery |
| 4.8 | $\frac{1}{C_{s e r}}=\sum \frac{1}{C_{i}}$ | Equivalent capacitance of a battery of capacitors connected in series | $C_{i}$ is a capacitance of an individual capacitor in the battery |
| 4.9 | $\overrightarrow{\boldsymbol{P}}=\lim _{V o l \rightarrow 0} \frac{\sum \overrightarrow{\boldsymbol{p}}_{e i}}{V o l}$ | Electric polarization vector | $\overrightarrow{\boldsymbol{p}}_{e i}$ is an electric dipole moment of an individual dipole within the element of volume Vol |
| 4.10 | $\sigma_{i}=P_{n}$ | Induced surfacecharge density | $P_{n}$ is a normal component of the polarization vector |
| 4.11 | $\overrightarrow{\boldsymbol{D}}=\varepsilon_{0} \overrightarrow{\boldsymbol{E}}+\overrightarrow{\boldsymbol{P}}$ | Electric field displacement vector | $\varepsilon_{0}$ is an electric constant |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 4.12 | $\oiiint_{A} \overrightarrow{\boldsymbol{D}} \cdot d \overrightarrow{\boldsymbol{A}}=q_{\text {free }}^{\text {enc }}$ | General form of Gauss's law; see also the equation (2.4) | $q_{\text {free }}^{\text {enc }}$ is a net free charge enclosed by an arbitrary surface A |
| 4.13 | $\overrightarrow{\boldsymbol{P}}=\varepsilon_{0} \varkappa_{e} \overrightarrow{\boldsymbol{E}}$ | Definition of electric susceptibility $\varkappa_{e}$ of a substance | Valid for nonferroelectric substances |
| 4.14 | $\varepsilon_{r}=1+\varkappa_{e}$ | Definition of relative dielectric permittivity of a substance | For vacuum $\varepsilon_{r} \equiv 1$, $\varkappa_{e} \equiv 0$; for air $\varepsilon_{r} \approx 1, \varkappa_{e} \approx 0$ |
| 4.15 | $\overrightarrow{\boldsymbol{D}}=\varepsilon_{0} \varepsilon_{r} \overrightarrow{\boldsymbol{E}}=\varepsilon_{0} \overrightarrow{\boldsymbol{E}}_{0}$ | Displacement vector in a uniform medium | $\overrightarrow{\boldsymbol{E}}_{0}$ is an electric filed in vacuum created by the same system of free chargers |
| 4.16 | $C=\varepsilon_{r} C_{0}$ | Capacitance of a capacitor with a dielectric between plates | $C_{0}$ is a capacitance of an air (vacuum) capacitor |
| 4.17 | $\begin{aligned} & U_{C}=\frac{q V_{12}}{2}=\frac{q^{2}}{2 C}= \\ & =\frac{C V_{12}^{2}}{2} \end{aligned}$ | Energy of a charged capacitor with capacitance C | $V_{12}$ is a potential difference across the capacitor |
| 4.18 | $u_{E}=\frac{\overrightarrow{\boldsymbol{E}} \cdot \overrightarrow{\boldsymbol{D}}}{2}=\frac{\varepsilon_{0} \varepsilon_{r} E^{2}}{2}$ | Electric field energy density | $\overrightarrow{\boldsymbol{E}}$ is a local electric field |

Pre-Class Reading: [1], chap. 24; [2], chap. 25; [3], chap. 26.

## Case 4.1

4.1.1. A parallel-plate air capacitor of capacitance 250 pF has a charge of magnitude $0.125 \mu \mathrm{C}$ on each plate. The plates are 0.328 mm apart. (a) What is the potential difference between the plates? (b) What is the area of each plate? (c) What is the electric field magnitude between the plates? (d) What is the surface charge density on each plate?
4.1.2. In Fig. 4.1, each capacitor has $C=5.0 \mu \mathrm{~F}$ and $V_{a b}=65.0 \mathrm{~V}$.


Figure 4.1. Problem 4.1.2

Calculate (a) the charge on each capacitor; (b) the potential difference across each capacitor; (c) the potential difference between points $a$ and $d$.
4.1.3. A 800 pF capacitor is charged to 300 V . Then a wire is connected between the plates. How many joules of thermal energy are produced as the capacitor discharges if all of the energy that was stored goes into heating the wire?
4.1.4. The dielectric to be used in a parallel-plate capacitor has a dielectric constant of 4.00 and a dielectric strength of $2.00 \cdot 10^{7} \mathrm{~V} / \mathrm{m}$. The capacitor is to have a capacitance of $1.77 \cdot 10^{-8} \mathrm{~F}$ and must be able to withstand a maximum potential difference of 6.0 kV . What is the minimum area the plates of the capacitor may have?
4.1.5. A capacitor of unknown capacitance has been charged to a potential difference of 250 V and then disconnected from the battery. When the charged capacitor is then connected in parallel to an uncharged
$12.0-\mu \mathrm{F}$ capacitor, the potential difference across the combination is 200 V . Calculate the unknown capacitance.

## Case 4.2

4.2.1. A $12.0 \mu \mathrm{~F}$ parallel-plate capacitor with circular plates is connected to a $10.0-\mathrm{V}$ battery. (a) What is the charge on each plate? (b) How much charge would be on the plates if their separation were doubled while the capacitor remained connected to the battery? (c) How much charge would be on the plates if the capacitor were connected to the $10.0-\mathrm{V}$ battery after the radius of each plate was doubled without changing their separation?
4.2.2. In Fig. 4.2, $C_{1}=45.0 \mathrm{nF}$ and $V_{a b}=5.00 \mathrm{~V}$. The charge on


Figure 4.2. Problem 4.2.2
the capacitor $C_{1}$ is 90.0 nC . Calculate the voltage across the other two capacitors.
4.2.3. A parallel-plate vacuum capacitor with a plate area $A$ and a separation $x$ has charges $+q$ and $-q$ on its plates. The capacitor is disconnected from the source of charge, so the charge on each plate remains fixed. (a) What is the total energy stored in the capacitor? (b) The plates are pulled apart an additional distance $d x$. What is the change in the stored energy? (c) If $F$ is the force with which the plates attract each other, then the change in the stored energy must equal the work $d W=F d x$ done in pulling the plates apart. Find an expression for $F$. (d) Explain why $F$ is not equal to $q E$ where $E$ is the electric
field between the plates.
4.2.4. A parallel-plate capacitor in the air has a plate separation of 8.85 cm and a plate area of $15.0 \mathrm{~cm}^{2}$. The plates are charged to a potential difference of 300 V and disconnected from the source. The capacitor is then immersed into distilled water $\left(\varepsilon_{r}=80\right)$. Assume the liquid is an insulator. Determine (a) the charge on the plates before and after immersion, (b) the capacitance and potential difference after immersion, and (c) the change in energy of the capacitor.
4.2.5. Electronic flash units for cameras contain a capacitor for storing the energy used to produce the flash. In one such unit, the flash lasts for $\frac{1}{750} \mathrm{~s}$ with an average light power output of $2.50 \cdot 10^{5} \mathrm{~W}$. (a) If the conversion of electrical energy to light is $98 \%$ efficient (the rest of the energy goes to thermal energy), how much energy must be stored in the capacitor for one flash? (b) The capacitor has a potential difference between its plates of 100 V when the stored energy equals the value calculated in part (a). What is the capacitance?

## Case 4.3

4.3.1. A capacitor is made of two hollow coaxial iron cylinders, one inside the other. The inner cylinder is negatively charged and the outer one is positively charged; the magnitude of the charge on each is 20.0 pC . The inner cylinder has a radius of 3.00 mm , the outer one has a radius of 6.00 mm , and the length of each cylinder is 6.93 cm . (a) What is the capacitance? (b) What applied potential difference is necessary to produce these charges on the cylinders?
4.3.2. Two parallel-plate vacuum capacitors have areas $A_{1}$ and $A_{2}$ and equal plate spacings $d$. Show that when the capacitors are connected in parallel, the equivalent capacitance is the same as for a single capacitor with a plate area $A_{1}+A_{2}$ and a spacing $d$.
4.3.3. A storm cloud and the ground represent the plates of a capacitor. During a storm, the capacitor has a potential difference of $1.00 \cdot 10^{8} \mathrm{~V}$ between its plates and a charge of 50.0 C . A lightning strike delivers $0.50 \%$ of the energy of the capacitor to a tree on the ground.

How much sap in the tree can be boiled away? Model the sap as water initially at $20.0^{\circ} \mathrm{C}$. Water has a specific heat of $4186 \mathrm{~J} / \mathrm{kg} \cdot \mathrm{K}$, a boiling point of $100^{\circ} \mathrm{C}$, and a latent heat of vaporization of $2.26 \cdot 10^{6} \mathrm{~J} / \mathrm{kg}$.
4.3.4. A capacitor has parallel plates of area $12 \mathrm{~cm}^{2}$ separated by 5.0 mm . The space between the plates is filled with polystyrene with the dielectric strength $2 \cdot 10^{7} \mathrm{~V} / \mathrm{m}$. (a) Find the maximum permissible voltage across the capacitor to avoid dielectric breakdown. (b) When the voltage equals the value found in part (a), find the surface charge density on each plate and the induced surface-charge density on the surface of the dielectric.
4.3.5. For the capacitor network shown in Fig. 4.3, the potential


Figure 4.3. Problem 4.3.5
difference across $a b$ is 8.0 V . Taking $C_{1}=3.0 \mathrm{pF}, C_{2}=2.0 \mathrm{pF}$, $C_{3}=6.0 \mathrm{pF}, C_{4}=12.0 \mathrm{pF}$, and $C_{5}=2.0 \mathrm{pF}$ find (a) the total energy stored in this network and (b) the energy stored in the capacitor $C_{2}$.

## Case 4.4

4.4.1. A spherical capacitor contains a charge of 2.50 nC when connected to a potential difference of 125 V . If its plates are separated by vacuum and the inner radius of the outer shell is 9.00 cm , calculate: (a) the capacitance; (b) the radius of the inner sphere; (c) the electric field just outside the surface of the inner sphere.
4.4.2. Consider three capacitors $C_{1}, C_{2}$, and $C_{3}$ and a battery. If only $C_{1}$ is connected to the battery, the charge on $C_{1}$ is $30.0 \mu \mathrm{C}$. Now $C_{1}$ is disconnected, discharged, and connected in series with $C_{2}$. When
the series combination of $C_{2}$ and $C_{1}$ is connected across the battery, the charge on $C_{1}$ is $15.0 \mu \mathrm{C}$. The circuit is disconnected, and both capacitors are discharged. Next, $C_{3}, C_{1}$, and the battery are connected in series, resulting in a charge on $C_{1}$ of $10.0 \mu \mathrm{C}$. If, after being disconnected and discharged, $C_{1}, C_{2}$, and $C_{3}$ are connected in series with one another and with the battery, what is the charge on $C_{1}$ ?
4.4.3. For the capacitor network shown in Fig. 4.4, the potential


Figure 4.4. Problem 4.4.3
difference across $a b$ is 100 V . Taking $C_{1}=2.5 \mu \mathrm{~F}$ and $C_{2}=7.5 \mu \mathrm{~F}$ find (a) the total charge stored in this network; (b) the charge on each capacitor; (c) the total energy stored in the network; (d) the energy stored in each capacitor; (e) the potential difference across each capacitor.
4.4.4. When a $20-\mathrm{nF}$ air capacitor $\left(1 \mathrm{nF}=10^{-9} \mathrm{~F}\right)$ is connected to a power supply, the energy stored in the capacitor is $9.00 \cdot 10^{-6} \mathrm{~J}$. While the capacitor is kept connected to the power supply, a slab of dielectric is inserted that completely fills the space between the plates. This increases the stored energy by $6.00 \cdot 10^{-6} \mathrm{~J}$. (a) What is the potential difference between the capacitor plates? (b) What is the dielectric constant of the slab?
4.4.5. Three capacitors having capacitances of $8.0,8.0$, and $4.0 \mu \mathrm{~F}$ are connected in series across a 30 V potential difference. (a) What is the charge on the $4.0-\mu \mathrm{F}$ capacitor? (b) What is the total energy stored in all three capacitors? (c) The capacitors are disconnected from the potential difference without allowing them to discharge. Then they are reconnected in parallel with each other, with the positively charged plates connected together. What is the voltage across each capacitor in
the parallel combination? (d) What is the total energy now stored in the capacitors?

## Case 4.5

4.5.1. A 90.0 m length of a coaxial cable has an inner conductor that has a diameter of 1.00 mm and carries a charge of $4.0 \mu \mathrm{C}$. The surrounding conductor has an inner diameter of 2.72 mm and a charge of $10.0 \mu \mathrm{C}$. Assume the region between the conductors is air. (a) What is the capacitance of this cable? (b) What is the potential difference between the two conductors?
4.5.2. Let it be $C_{2}=1 \mu \mathrm{~F}, C_{3}=12 \mu \mathrm{~F}, C_{4}=6 \mu \mathrm{~F}$, and $C_{5}=15 \mu \mathrm{~F}$


Figure 4.5. Problem 4.5.2
in a circuit of Fig. 4.5. What would be the capacitance $C_{1}$ to have an equivalent capacitance between points $a$ and $b$ equal to $5 \mu \mathrm{~F}$ ?
4.5.3. A cylindrical air capacitor 16.0 m long stores $4.00 \cdot 10^{-9} \mathrm{~J}$ of energy when the potential difference between the two conductors is 3.00 V . (a) Calculate the magnitude of the charge on each conductor. (b) Calculate the ratio of the radii of the outer and inner conductors.
4.5.4. A $9.0-\mu \mathrm{F}$ capacitor is connected to a power supply that keeps a constant potential difference of 20.0 V across the plates. A piece of material having a dielectric constant of 5.00 is placed between the plates, completely filling the space between them. (a) How much energy is stored in the capacitor before and after the dielectric is inserted? (b) By how much did the energy change during the insertion? Did it increase or

## decrease?

4.5.5. A parallel-plate capacitor has the space between the plates


Figure 4.6. Problem 4.5.5
filled with two slabs of dielectric, one with relative dielectric permittivity constant $\varepsilon_{1}$ and one with constant $\varepsilon_{2}$ (Fig. 4.6). Each slab has a thickness $d / 2$, where $d$ is the plate separation. Find the capacitance of the capacitor.

## Chapter 5

## CURRENT AND RESISTANCE. DC CIRCUITS

|  | Equation | Equation title | Comments |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |
| 5.1 | $\overrightarrow{\boldsymbol{j}}=q_{0} n \overrightarrow{\boldsymbol{v}}_{d}$ | Electric current density | $\overrightarrow{\boldsymbol{v}}_{d}$ is a drift velocity of charge carriers with a charge $q_{0}$ and their number per unit volume $n$ |
| 5.2 | $\overrightarrow{\boldsymbol{j}}=\sigma \overrightarrow{\boldsymbol{E}}=\frac{\overrightarrow{\boldsymbol{E}}}{\rho}$ | Ohm's law in a differential form | $\sigma$ and $\rho$ are electroconductivity and resistivity of the substance, respectively |
| 5.3 | $\rho(T)=\rho_{0}(1+\alpha \Delta T)$ | Temperature dependence of resistivity | $\rho_{0}$ is a resistivity at a reference temperature $T_{0} ; \quad \alpha$ is a temperature coefficient of resistivity; $\Delta T=T-T_{0}$ |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 5.4 | $I=\frac{d q}{d t}=\iint_{A} \overrightarrow{\boldsymbol{j}} \cdot d \overrightarrow{\boldsymbol{A}}$ | Electric current | $d q$ is an infinitesimal charge passed trough the crosssection $A$ of the conductor during time $d t$ |
| 5.5 | $R_{12}=\frac{V_{12}}{I}$ | Resistance of a conductor | $V_{12}$ is potential difference across the conductor |
| 5.6 | $R=\rho \frac{\ell}{A_{\perp}}$ | Resistance of a uniform "cylindrical" conductor of length $\ell$ | $A_{\perp} \quad$ is a crosssectional area of the conductor; $\rho$ is resistivity |
| 5.7 | $\pm V= \pm \mathcal{E}-I r$ | Terminal voltage of a battery with emf $\mathcal{E}$ | $r$ is an internal resistance of the battery; use "-" for recharging the battery |
| 5.8 | $R_{s e r}=\sum R_{i}$ | Equivalent resistance of resistors connected in series | $R_{i}$ is a resistance of an individual resistor |
| 5.9 | $\frac{1}{R_{p a r}}=\sum \frac{1}{R_{i}}$ | Equivalent resistance of resistors connected in parallel | $R_{i}$ is a resistance of an individual resistor |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 5.10 | $\sum_{j u n c t} I_{i}^{\text {in }}=\sum_{j u n c t} I_{j}^{\text {out }}$ | Kirchhoff's junction rule | $I_{i}^{\text {in }}$ and $I_{j}^{\text {out }}$ are individual inward and outward currents in the junction, respectively |
| 5.11 | $\sum_{\text {loop }} I_{i} R_{j}=\sum_{\text {loop }} \mathcal{E}_{k}$ | Kirchhoff's loop rule | $I_{i} R_{j}$ are potential differences across resistors around the loop; $\mathcal{E}_{k}$ are emf withing the loop |
| 5.12 | $P_{12}=V_{12} I=I^{2} R_{12}$ | Power delivered by current $I$ to a conductor (Joule's law) | $R_{12}$ is a resistance of the conductor; $V_{12}$ is a potential difference across it |
| 5.13 | $\tau=R C$ | Relaxation time (time constant) of an $R C$-circuit | $R$ and $C$ are resistance and capacitance |
| 5.14 | $q(t)=q_{0} e^{-t / \tau}$ | Time dependence for charge decay in an $R C$-circuit | $q_{0}$ is an initial charge stored on the capacitor |
| 5.15 | $q(t)=\mathcal{E} C\left(1-e^{-t / \tau}\right)$ | Time dependence for charge growth in an $R C$-circuit with $\operatorname{emf} \mathcal{E}$ | $C$ is the capacitance of the circuit |

Pre-Class Reading: [1], chap. $25 \& 26 ;$ [2], chap. $26 \& 27 ; ~[3]$, chap. 27 \& 28 .

## Case 5.1

5.1.1. A 4.80 -A current runs through a copper wire (diameter 2.0 mm ) and through a light bulb. Copper has $8.5 \cdot 10^{28}$ free electrons per cubic meter. (a) How many electrons pass through the light bulb each second? (b) What is the current density in the wire? (c) At what speed does a typical electron pass by any given point in the wire? (d) If you were to use wire of twice the diameter, which of the above answers would change? Would they increase or decrease?
5.1.2. A cylindrical tungsten filament 10.0 cm long with a diameter of 2.00 mm is to be used in a machine for which the temperature will range from the room temperature $\left(20^{\circ} \mathrm{C}\right)$ up to $120^{\circ} \mathrm{C}$. It will carry a current of 9.00 A at all temperatures. Temperature coefficient of resistivity of silver is $3.8 \cdot 10^{-3} \mathrm{~K}^{-1}$. (a) What will be the maximum electric field in this filament, and (b) what will be its resistance with that field? (c) What will be the maximum potential drop over the full length of the filament?
5.1.3. The ammeter shown in Fig. 5.1 reads 3.0 A . Find (a) $I_{1}$, (b) $I_{3}$,


Figure 5.1. Problem 5.1.3
and (c) $\mathcal{E}_{2}$. Use following values: $R_{1}=2.0 \Omega, R_{2}=6.0 \Omega, R_{3}=4.0 \Omega$ and $\mathcal{E}_{1}=20.0 \mathrm{~V}$.
5.1.4. In Europe the standard voltage in homes is 240 V instead of the 120 V used in the United States. Therefore a "200-W" European bulb would be intended for use with a $240-\mathrm{V}$ potential difference.
(a) If you bring a "200-W" European bulb home to the United States, what should be its U.S. power rating? (b) How much current will the "200-W" European bulb draw in normal use in the United States?
5.1.5. A car owner forgets to turn off the headlights of his car while it is parked in his garage. If the $12.0-\mathrm{V}$ battery in his car is rated at 60.0 A•h and each headlight requires 20.0 W of power, how long will it take the battery to discharge completely?

## Case 5.2

5.2.1. A rectangular solid of pure silicon measures $9 \mathrm{~cm} \times 9 \mathrm{~cm} \times 18 \mathrm{~cm}$. Assuming that each of its faces is an equipotential surface, what is the resistance between opposite faces that are (a) farthest apart and (b) closest together?
5.2.2. When switch $S$ in Fig. 5.2 is open, the voltmeter $\mathbf{V}$ of the


Figure 5.2. Problem 5.2.2
battery reads 9.00 V . When the switch is closed, the voltmeter reading drops to 6.00 V , and the ammeter $\mathbf{A}$ reads 3.00 A . Find the emf, the internal resistance of the battery $r$, and the circuit resistance $R$. Assume that the two meters are ideal, so they don't affect the circuit.
5.2.3. The heating element of an electric coffee maker operates at 220 V and carries a current of 3.0 A . Assuming the water absorbs all the energy delivered to the resistor, calculate the time interval during which the temperature of 1.1 kg of water rises from the room temperature
$\left(20^{\circ} \mathrm{C}\right)$ to the boiling point. The specific heat is $4.2 \mathrm{~J} /(\mathrm{g} \cdot \mathrm{K})$.
5.2.4. A battery with $\mathcal{E}=12.0 \mathrm{~V}$ and no internal resistance supplies


Figure 5.3. Problem 5.2.4
current to the circuit shown in Fig. 5.3. When the double-throw switch $S$ is open as shown in the figure, the current in the battery is 1.00 A . When the switch is closed in position $a$, the current in the battery is 1.2 A. When the switch is closed in position $b$, the current in the battery is 2.0 A . Find the resistances (a) $R_{1}$, (b) $R_{2}$, and (c) $R_{3}$.
5.2.5. Taking $\mathcal{E}_{1}=26.0 \mathrm{~V}, R_{2}=9.0 \Omega, R_{3}=4.0 \Omega, I_{2}=2.0 \mathrm{~A}$, and


Figure 5.4. Problem 5.2.5
$I_{3}=5.0 \mathrm{~A}$, in the circuit shown in Fig. 5.4, find (a) the current in the resistor $R_{1}$; (b) the resistance $R_{1}$; (c) the unknown emf $\mathcal{E}_{2}$. (d) If the circuit is broken at point $x$, what is the current in the resistor $R_{1}$ ?

## Case 5.3

5.3.1. A certain waffle iron is rated at 3.00 kW when connected to a
$220-\mathrm{V}$ source. (a) What current does the waffle iron carry? (b) What is its resistance?
5.3.2. In the circuit of Fig. 5.5, each resistor represents a light bulb.


Figure 5.5. Problem 5.3.2
Let $R_{1}=R_{2}=R_{3}=R_{4}=3.00 \Omega$ and $\mathcal{E}=7.00 \mathrm{~V}$. (a) Find the current in each bulb. (b) Find the power dissipated in each bulb. Which bulb or bulbs glow the brightest? (c) Bulb $R_{4}$ is now removed from the circuit, leaving a break in the wire at its position. Now what is the current in each of the remaining bulbs $R_{1}, R_{2}$, and $R_{3}$ ? (d) With the bulb $R_{4}$ removed, what is the power dissipated in each of the remaining bulbs? (e) Which light bulb(s) glow brighter as a result of removing $R_{4}$ ? Which bulb(s) glow less brightly? Discuss why there are different effects on different bulbs.
5.3.3. Taking $R_{1}=5 \Omega, R_{2}=20 \Omega, \mathcal{E}_{1}=3 \mathrm{~V}$, and $\mathcal{E}_{2}=6 \mathrm{~V}$, in the


Figure 5.6. Problem 5.3.3
circuit shown in Fig. 5.6, (a) find the value of $R_{3}$ that will give a current
$I_{3}$ of 0.1 A with the indicated direction. (b) Is there a value of $R_{3}$ that will give a current $I_{3}$ with the same magnitude but opposite direction? If so, what is it?
5.3.4. The current in a wire varies with time according to the relationship $I=17 \mathrm{~A}-\left(0.10 \mathrm{~A} / \mathrm{s}^{2}\right) t^{2}$. (a) How many coulombs of charge pass a cross section of the wire in the time interval between $t=0$ and $t=10.0 \mathrm{~s}$ ? (b) What constant current would transport the same charge in the same time interval?
5.3.5. A material of resistivity $\rho$ is formed into a solid truncated cone


Figure 5.7. Problem 5.3.5
of height $h$ and radii $r_{1}$, and $r_{2}$ at either end (Fig. 5.7). (a) Calculate the resistance of the cone between the two flat end faces. (Hint: Imagine slicing the cone into very many thin disks, and calculate the resistance of one such disk.) (b) Show that your result agrees with Eq. (5.6) when $r_{1}=r_{2}$.

## Case 5.4

5.4.1. The capacity of a storage battery such as those used in au-
tomobile electrical systems is rated in ampere-hours (A•h). A 50-A•h battery can supply a current of 50 A for 1.0 h , or 25 A for 2.0 h , and so on. (a) What total energy can be supplied by a $12-\mathrm{V}, 90-\mathrm{A} \cdot \mathrm{h}$ battery if its internal resistance is negligible? (b) If a generator with an average electrical power output of 1.00 kW is connected to the battery, how much time will be required for it to charge the battery fully?
5.4.2. In the circuit shown in Fig. 5.8, the $\mathcal{E}_{1}$ battery is removed


Figure 5.8. Problem 5.4.2
and reinserted with the opposite polarity, so that its negative terminal is now next to point $a$. Taking $\mathcal{E}_{1}=30.0 \mathrm{~V}, r_{1}=10.0 \Omega, \mathcal{E}_{2}=15.0 \mathrm{~V}$, $r_{2}=3.0 \Omega, R_{3}=2.0 \Omega$, and $R_{4}=18.0 \Omega$ find (a) the current in the circuit (magnitude and direction); (b) the terminal voltage $V_{a b}$ of the $\mathcal{E}_{1}$ battery; (c) the potential difference $V_{a c}$ of point $a$ with respect to point c. (d) Graph the potential rises and drops in this circuit.
5.4.3. A capacitor is charged to a potential of 9.0 V and is then connected to a voltmeter having an internal resistance of $2.50 \mathrm{M} \Omega$. After a time of 3.00 s , the voltmeter reads 3.0 V . What are (a) the capacitance and (b) the time constant of the circuit?
5.4.4. The potential difference across the terminals of a battery is 5.0 V when there is a current of 0.5 A in the battery from the negative to the positive terminal. When the current is 1.7 A in the reverse direction, the potential difference becomes 6.5 V . (a) What is the internal resistance
of the battery? (b) What is the emf of the battery?
5.4.5. Two batteries with emf $\mathcal{E}_{1}=6.0 \mathrm{~V}$ and $\mathcal{E}_{2}=9.0 \mathrm{~V}$ are


Figure 5.9. Problem 5.4.5
connected to resistors $R_{1}=300 \Omega, R_{2}=400 \Omega$, and $R_{3}=50 \Omega$ in a circuit as shown in a Fig. 5.9. (a) Calculate the power dissipated in the resistor $R_{3}$. (b) Assume that the terminals on $\mathcal{E}_{1}$ battery a reversed, and repeat your calculation.

## Case 5.5

5.5.1. The electron beam emerging from a certain high energy electron accelerator has a circular cross section of a radius 2.00 mm . (a) The beam current is $15.00 \mu \mathrm{~A}$. Find the current density in the beam assuming that it is uniform throughout. (b) The speed of the electrons is so close to the speed of light that their speed can be taken as $1.00 \cdot 10^{6} \mathrm{~m} / \mathrm{s}$ with a negligible error. Find the electron density in the beam. (c) Over what time interval does Avogadro's number of electrons emerge from the accelerator?
5.5.2. A " $300-\mathrm{W}$ " electric heater is designed to operate from 220V lines. (a) What is its resistance? (b) What current does it draw? (c) If the line voltage drops to 200 V , what power does the heater take? (Assume that the resistance is constant. Actually, it will change because of the change in temperature.) (d) The heater coils are metallic, so that the resistance of the heater decreases with decreasing temperature. If
the change of resistance with temperature is taken into account, will the electrical power consumed by the heater be larger or smaller than what you calculated in part (c)? Explain.
5.5.3. In a circuit shown in Fig. 5.10, find (a) the current in each


Figure 5.10. Problem 5.5.3
resistor and (b) the power delivered to each resistor. Take $\mathcal{E}_{1}=10.0 \mathrm{~V}$, $\mathcal{E}_{2}=6.0 \mathrm{~V}, R_{1}=25.0 \Omega, R_{2}=10.0 \Omega$, and $R_{3}=15.0 \Omega$.
5.5.4. An emf source with $\mathcal{E}=60 \mathrm{~V}$, a resistor with $R=100.0 \Omega$, and a capacitor with $C=12.0 \mu \mathrm{~F}$ are connected in series. As the capacitor charges, when the current in the resistor is 0.2 A , what is the magnitude of the charge on each plate of the capacitor?
5.5.5. Unlike the idealized ammeter, any real ammeter has a nonzero resistance. (a) An ammeter with a resistance $R_{A}$ is connected in series with a resistor $R$ and a battery of an $\operatorname{emf} \mathcal{E}$ and internal resistance $r$. The current measured by the ammeter is $I_{A}$. Find the current through the circuit if the ammeter is removed so that the battery and the resistor form a complete circuit. Express your answer in terms of $I_{A} r, R_{A}$, and $R$. The more "ideal" the ammeter, the smaller the difference between this current and the current $I_{A}$. (b) If $R=5.00 \Omega, \mathcal{E}=7.50 \mathrm{~V}$, and $r=0.3 \Omega$, find the maximum value of the ammeter resistance $R_{A}$ so that $I_{A}$ is within $1.0 \%$ of the current in the circuit when the ammeter is absent. (c) Explain why your answer in part (b) represents a maximum value.

Chapter 6 EFFECTS OF MAGNETIC FIELDS

|  | Equation | Equation title | Comments |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |
| 6.1 | $\overrightarrow{\boldsymbol{F}}=q \overrightarrow{\boldsymbol{v}} \times \overrightarrow{\boldsymbol{B}}$ | Magnetic force on a moving charge $q$ | $\overrightarrow{\boldsymbol{B}}$ is a magnetic field; $\overrightarrow{\boldsymbol{v}}$ is velocity of the charged particle |
| 6.2 | $R=\frac{m v}{\|q\| B}$ | Radius of a circular path of the particle | $m$ is a mass of the particle |
| 6.3 | $\overrightarrow{\boldsymbol{\omega}}=-\frac{q}{m} \overrightarrow{\boldsymbol{B}}$ | Cyclotron angular velocity | $q / m$ is a specific charge of the particle |
| 6.4 | $d \overrightarrow{\boldsymbol{F}}=I d \overrightarrow{\boldsymbol{\ell}} \times \overrightarrow{\boldsymbol{B}}$ | Magnetic force on a wire with current I | $d \overrightarrow{\boldsymbol{\ell}}$ is an infinitesimal element of the wire length along the current |
| 6.5 | $\overrightarrow{\boldsymbol{p}}_{m}=\overrightarrow{\boldsymbol{n}} I A^{\text {enc }}$ | Magnetic dipole moment of a current loop | $A^{e n c}$ is an area enclosed by the loop; $\overrightarrow{\boldsymbol{n}}$ is a unit vector of normal to the plain of the loop |
| 6.6 | $\overrightarrow{\boldsymbol{\tau}}=\overrightarrow{\boldsymbol{p}}_{m} \times \overrightarrow{\boldsymbol{B}}$ | Torque on a current loop | $\overrightarrow{\boldsymbol{B}}$ is a uniform magnetic field |


| 1 | $\|c\|$ <br> 6.7 <br> 6.8$\Phi_{m}=\iint_{A} \overrightarrow{\boldsymbol{B}} \cdot d \overrightarrow{\boldsymbol{A}}$ | Magnetic flux; see <br> also the equations <br> $(2.1)$ and (2.3) | $A$ is an arbitrary <br> surface |
| :---: | :---: | :--- | :--- |
| Work done by a <br> magnetic field on a <br> loop with current <br> $I$ | $d \Phi_{m}$ is an infinitesi- <br> mal change of mag- <br> netic flux through <br> the surface attached <br> to the loop |  |  |

Pre-Class Reading: [1], chap. 27; [2], chap. 28; [3], chap. 29.

## Case 6.1

6.1.1. A particle with a charge of $-1.2 \cdot 10^{-8} \mathrm{C}$ is moving with the instantaneous velocity $\overrightarrow{\boldsymbol{v}}=\left(4.2 \cdot 10^{4} \mathrm{~m} / \mathrm{s}\right) \overrightarrow{\boldsymbol{i}}+\left(-4.0 \cdot 10^{4} \mathrm{~m} / \mathrm{s}\right) \overrightarrow{\boldsymbol{j}}$. What is the force exerted on this particle by a magnetic field (a) $\overrightarrow{\boldsymbol{B}}=(1.50 \mathrm{~T}) \overrightarrow{\boldsymbol{i}}$ and (b) $\overrightarrow{\boldsymbol{B}}=(1.50 \mathrm{~T}) \overrightarrow{\boldsymbol{k}}$ ?
6.1.2. A particle with a charge $6.4 \cdot 10^{-19} \mathrm{C}$ travels in a circular orbit with a radius 4.0 mm due to the force exerted on it by a magnetic field with magnitude of 1.6 T and perpendicular to the orbit. (a) What is the magnitude of the linear momentum $\overrightarrow{\boldsymbol{p}}$ of the particle? (b) What is the magnitude of the angular momentum $\overrightarrow{\boldsymbol{L}}$ of the particle?
6.1.3. A particle with the initial velocity $\overrightarrow{\boldsymbol{v}}_{0}=\left(6.0 \cdot 10^{3} \mathrm{~m} / \mathrm{s}\right) \overrightarrow{\boldsymbol{j}}$ enters a region of uniform electric and magnetic fields. The magnetic field in the region is $\overrightarrow{\boldsymbol{B}}=-(1.2 \mathrm{~T}) \overrightarrow{\boldsymbol{k}}$. Calculate the magnitude and direction of the electric field in the region if the particle is to pass through undeflected, for a particle of charge (a) +0.640 nC and (b) -0.320 nC . You can ignore the weight of the particle.
6.1.4. A proton moves with a velocity of $\overrightarrow{\boldsymbol{v}}=(\overrightarrow{\boldsymbol{i}}-2 \overrightarrow{\boldsymbol{j}}+\overrightarrow{\boldsymbol{k}}) \mathrm{m} / \mathrm{s}$ in a region in which the magnetic field is $\overrightarrow{\boldsymbol{B}}=(\overrightarrow{\boldsymbol{i}}+2 \overrightarrow{\boldsymbol{j}}-\overrightarrow{\boldsymbol{k}}) \mathrm{T}$. What is the
magnitude of the magnetic force this particle experiences?
6.1.5. A current of 15.7 mA is maintained in a single circular loop of 2.00 m circumference. A magnetic field of 0.800 T is directed parallel to the plane of the loop. (a) Calculate the magnetic moment of the loop. (b) What is the magnitude of the torque exerted by the magnetic field on the loop?

## Case 6.2

6.2.1. A particle of mass 0.195 g carries a charge of $-2.50 \cdot 10^{-8} \mathrm{C}$. The particle is given an initial horizontal velocity that is due north and has magnitude of $4.00 \cdot 10^{4} \mathrm{~m} / \mathrm{s}$. What are the magnitude and direction of the minimum magnetic field that will keep the particle moving in the Earth's gravitational field in the same horizontal northward direction?
6.2.2. A $150-\mathrm{g}$ ball containing $4.00 \cdot 10^{8}$ excess electrons is dropped into a $125-\mathrm{m}$ vertical shaft. At the bottom of the shaft, the ball suddenly enters a uniform horizontal magnetic field that has magnitude of 0.250 T and is directed from east to west. If the air resistance is negligibly small, find the magnitude and direction of the force that this magnetic field exerts on the ball just as it enters the field.
6.2.3. A straight vertical wire carries a current of 1.20 A downward in a region between the poles of a large superconducting electromagnet, where the magnetic field is horizontal and has a magnitude of $B=0.6 \mathrm{~T}$. What are the magnitude and direction of the magnetic force on a $1.00-\mathrm{cm}$ section of the wire that is in this uniform magnetic field, if the magnetic field direction is (a) east; (b) south; (c) $30.0^{\circ}$ south of west?
6.2.4. An electron moves in a circular path perpendicular to a uniform magnetic field with a magnitude of 1.80 mT . If the speed of the electron is $1.60 \cdot 10^{7} \mathrm{~m} / \mathrm{s}$, determine (a) the radius of the circular path and (b) the time interval required to complete one revolution.
6.2.5. A wire is formed into a circle having a diameter of 10.0 cm and is placed in a uniform magnetic field of 2.00 mT . The wire carries a current of 4.00 A . Find the maximum torque on the wire.

## Case 6.3

6.3.1. In a $1.25-\mathrm{T}$ magnetic field directed vertically upward, a particle having a charge of a magnitude $8.0 \mu \mathrm{C}$ and initially moving northward at $5.0 \mathrm{~km} / \mathrm{s}$ is deflected toward the east. (a) What is the sign of the charge of this particle? Make a sketch to illustrate how you found your answer. (b) Find the magnetic force on the particle.
6.3.2. A deuteron (the nucleus of an isotope of hydrogen) has a mass of $3.34 \cdot 10^{-27} \mathrm{~kg}$ and a charge of $+e$. The deuteron travels in a circular path with a radius of 6.68 mm in a magnetic field with a magnitude of 2.50 T . (a) Find the speed of the deuteron. (b) Find the time required for it to make half a revolution. (c) Through what potential difference would the deuteron have to be accelerated to acquire this speed?
6.3.3. A horizontal rod 0.200 m long is mounted on a balance and carries a current. At the location of the rod, a uniform horizontal magnetic field has the magnitude of 0.065 T and the direction perpendicular to the rod. The magnetic force on the rod is measured by the balance and is found to be 0.13 N . What is the current?
6.3.4. A cyclotron designed to accelerate protons has a magnetic field of a magnitude 0.50 T over a region of radius 1.0 m . What are (a) the cyclotron angular frequency and (b) the maximum speed acquired by the protons?
6.3.5. A wire having a mass per unit length of $0.500 \mathrm{~g} / \mathrm{cm}$ carries a $2.00-\mathrm{A}$ current horizontally to the south. What are (a) the direction and (b) the magnitude of the minimum magnetic field needed to lift this wire vertically upward?

## Case 6.4

6.4.1. At a given instant, a particle with a mass of $1.8 \cdot 10^{-3} \mathrm{~kg}$ and a charge of $1.2 \cdot 10^{-8} \mathrm{C}$ has a velocity $\overrightarrow{\boldsymbol{v}}=\left(3.00 \cdot 10^{4} \mathrm{~m} / \mathrm{s}\right) \overrightarrow{\boldsymbol{j}}$. What are the magnitude and direction of the particle's acceleration produced by a uniform magnetic field $\overrightarrow{\boldsymbol{B}}=(1.65 \mathrm{~T}) \overrightarrow{\boldsymbol{i}}+(0.980 \mathrm{~T}) \overrightarrow{\boldsymbol{j}}$ ?
6.4.2. A singly charged ion of ${ }^{7} \mathrm{Li}$ (an isotope of lithium) has a
mass of $1.16 \cdot 10^{-26} \mathrm{~kg}$. It is accelerated through a potential difference of 320 V and then enters a magnetic field with a magnitude of 0.5 T perpendicular to the path of the ion. What is the radius of the ion's path in the magnetic field?
6.4.3. The plane of a $5.0 \mathrm{~cm} \times 8.0 \mathrm{~cm}$ rectangular loop of wire is parallel to a 0.2-T magnetic field. The loop carries a current of 6.4 A . (a) What torque acts on the loop? (b) What is the magnetic moment of the loop? (c) What is the maximum torque that can be obtained with the same total length of wire carrying the same current in this magnetic field?
6.4.4. A particle with a charge $q$ and kinetic energy $E_{K}$ travels in a uniform magnetic field of a magnitude $B$. If the particle moves in a circular path of a radius $R$, find expressions for (a) its speed and (b) its mass.
6.4.5. A singly charged ion of mass $m$ is accelerated from rest by a potential difference $\Delta V$. It is then deflected by a uniform magnetic field (perpendicular to the ion's velocity) into a semicircle of a radius $R$. Now a doubly charged ion of mass $m^{\prime}$ is accelerated through the same potential difference and deflected by the same magnetic field into a semicircle of a radius $R^{\prime}=2 R$. What is the ratio of the masses of the ions?

## Case 6.5

6.5.1. An electron experiences a magnetic force of a magnitude $4.8 \cdot 10^{-16} \mathrm{~N}$ when moving at an angle of $30^{\circ}$ with respect to a magnetic field of a magnitude $4.0 \cdot 10^{-3} \mathrm{~T}$. Find the speed of the electron.
6.5.2. (a) What is the speed of a beam of electrons when the simultaneous influence of an electric field of $1.6 \cdot 10^{4} \mathrm{~V} / \mathrm{m}$ and a magnetic field of $5.0 \cdot 10^{-3} \mathrm{~T}$ (with both fields normal to the beam and to each other) produces no deflection of the electrons? (b) In a diagram, show the relative orientation of the vectors $\overrightarrow{\boldsymbol{v}}, \overrightarrow{\boldsymbol{E}}$, and $\overrightarrow{\boldsymbol{B}}$. (c) When the electric field is removed, what is the radius of the electron orbit? What is the
period of the orbit?
6.5.3. In the Bohr model of the hydrogen atom (see Section 38.5 [1]) in the lowest energy state, the electron orbits the proton at a speed of $2.188 \cdot 10^{6} \mathrm{~m} / \mathrm{s}$ in a circular orbit of a radius $5.292 \cdot 10^{-11} \mathrm{~m}$. (a) What is the orbital period of the electron? (b) If the orbiting electron is considered to be a current loop, what is the current $I$ ? (c) What is the magnetic moment of the atom due to the motion of the electron?
6.5.4. The picture tube in an old black-and-white TV-set uses magnetic deflection coils rather than electric deflection plates. Suppose an electron beam is accelerated through a $50.0-\mathrm{kV}$ potential difference and then through a region of uniform magnetic field 1.00 cm wide. The screen is located 10.0 cm from the center of the coils and is 50.0 cm wide. When the field is turned off, the electron beam hits the center of the screen. Ignoring relativistic corrections, what field magnitude is necessary to deflect the beam to the side of the screen?
6.5.5. A 100-turn circular coil of a radius 5.00 cm can be oriented in any direction in a uniform magnetic field having a magnitude of 0.20 T . If the coil carries a current of 40.0 mA , find the magnitude of the maximum possible torque exerted on the coil.

Chapter 7

## SOURCES OF MAGNETIC FIELDS

|  | Equation | Equation title | Comments |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |
| 7.1 | $\overrightarrow{\boldsymbol{B}}=\frac{\mu_{0}}{4 \pi} \frac{q \overrightarrow{\boldsymbol{v}} \times \overrightarrow{\boldsymbol{r}}}{r^{3}}$ | Magnetic field of a charge $q$ moving with a velocity $\overrightarrow{\boldsymbol{v}}$; compare with the equation (1.3) | $\mu_{0}$ is a magnetic constant; $\overrightarrow{\boldsymbol{r}}$ is an instantaneous position vector with respect to charge |
| 7.2 | $d \overrightarrow{\boldsymbol{B}}=\frac{\mu_{0}}{4 \pi} \frac{I d \overrightarrow{\boldsymbol{\ell}} \times \overrightarrow{\boldsymbol{r}}}{r^{3}}$ | Magnetic field of a conductor element with current $I$ (Biot-Savart law) | $d \overrightarrow{\boldsymbol{\ell}}$ is an infinitesimal element of the conductor length in the direction of the current |
| 7.3 | $B=\frac{\mu_{0}}{2 \pi} \frac{I}{a}$ | Magnetic field due to a straight thin infinite wire with current I | $a$ is a distance from the wire |
| 7.4 | $\frac{F_{B}}{\ell}=\frac{\mu_{0}}{2 \pi} \frac{I_{1} I_{2}}{d}$ | Magnetic interaction force per unit length between two parallel infinite wires | $I_{1}$ and $I_{2}$ are currents through the wires; $d$ is a separation distance between them |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 7.5 | $B=\frac{\mu_{0} I}{4 \pi a}\left(\cos \theta_{i}-\cos \theta_{f}\right)$ | Magnetic field at a point due to a straight thin wire of a finite length with current $I$ at a distance $a$ from the wire | $\theta_{i, f}$ are angles between the direction of the current and the position vectors of a point with respect to the initial and final points of the conductor |
| 7.6 | $\overrightarrow{\boldsymbol{B}}=\frac{\mu_{0}}{2 \pi} \frac{\overrightarrow{\boldsymbol{p}}_{m}}{r^{3}}$ | Magnetic field on the axis of a current circular loop with a magnetic dipole moment $\overrightarrow{\boldsymbol{p}}_{m}$ at a distance $x$ from the plane of the loop | $p_{m}=\pi I R^{2}$ where $I$ is a current through the loop and $R$ is a radius of the loop; $r=\left(R^{2}+x^{2}\right)^{1 / 2}$ is a distance from the loop to the point |
| 7.7 | $\oint_{L} \overrightarrow{\boldsymbol{B}} \cdot d \overrightarrow{\boldsymbol{\ell}}=\mu_{0} I^{e n c}$ | Ampere's circulation law for magnetic field; compare with the equation (3.1) | $I^{e n c}$ is a net current passing through a surface enclosed by the loop $L$ |
| 7.8 | $B=\frac{\mu_{0} I N}{2 \pi r}$ | Magnetic filed inside a toroidal solenoid | $N$ is a number of turns; $r$ is a distance from the center |
| 7.9 | $B=\frac{\mu_{0} I N}{\ell}=\mu_{0} n I$ | Magnetic filed inside a solenoid of infinite length | $n=N / \ell$ is a number of turns per unit length |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :--- | :--- |
| 7.10 | $\oiint$ |  |  |
| $A$ |  |  |  | \(\left.\overrightarrow{\boldsymbol{B}} \cdot d \overrightarrow{\boldsymbol{A}}=0 \quad \begin{array}{l}Gauss's law for <br>

magnetic field; <br>
compare with the <br>
equation (2.4)\end{array} \quad $$
\begin{array}{l}A \text { is an arbitrary } \\
\text { closed surface }\end{array}
$$\right]\)

Pre-Class Reading: [1], chap. 28; [2], chap. 29; [3], chap. 30.

## Case 7.1

7.1.1. Figure 7.1 shows, in cross section, several conductors that


Figure 7.1. Problem 7.1.1
carry currents through the plane of the figure. The currents have the magnitudes $I_{1}=4.0 \mathrm{~A}, I_{2}=6.0 \mathrm{~A}$, and $I_{3}=2.0 \mathrm{~A}$, and the directions shown. Four paths, labeled $a$ through $d$, are given. What is the line integral $\oint \overrightarrow{\boldsymbol{B}} \cdot d \overrightarrow{\boldsymbol{\ell}}$ for each path? Each integral involves going around the path in the counterclockwise direction. Explain your answers.
7.1.2. $\mathrm{A}+4.00-\mu \mathrm{C}$ point charge is moving at a constant speed of $8.00 \cdot 10^{6} \mathrm{~m} / \mathrm{s}$ in the $+y$-direction, relative to a reference frame. At the instant when the point charge is at the origin of this reference frame, what is the magnetic-field vector $\overrightarrow{\boldsymbol{B}}$ it produces at the following points: (a) $x=0.50 \mathrm{~m}, y=0, z=0$; (b) $x=0, y=-0.50 \mathrm{~m}, z=0$; (c) $x=0$,
$y=0, z=+0.50 \mathrm{~m}$; (d) $x=0, y=-0.50 \mathrm{~m}, z=+0.50 \mathrm{~m}$ ?
7.1.3. A very long, straight horizontal wire carries a current such that $3.50 \cdot 10^{18}$ electrons per second pass any given point going from west to east. What are the magnitude and direction of the magnetic field this wire produces at a point 4.00 cm directly above it?
7.1.4. Two long parallel wires carry currents of $I_{1}=4.0 \mathrm{~A}$ and


Figure 7.2. Problem 7.1.4
$I_{2}=6.0 \mathrm{~A}$ in the directions indicated in Fig. 7.2. (a) Find the magnitude and direction of the magnetic field at a point midway between the wires. (b) Find the magnitude and direction of the magnetic field at point $P$, located $d=20.0 \mathrm{~cm}$ above the wire carrying the $6.0-\mathrm{A}$ current.
7.1.5. As a new electrical technician, you are designing a large solenoid to produce a uniform 0.4 T magnetic field near the center of the solenoid. You have enough wire for 5000 circular turns. This solenoid must be 1.571 m long and 5.0 cm in diameter. What current will you need to produce the necessary field?

## Case 7.2

7.2.1. A $-5.0-\mu \mathrm{C}$ charge is moving at a constant speed of $4.0 \cdot 10^{5} \mathrm{~m} / \mathrm{s}$ in the $+x$-direction relative to a reference frame. At the instant when the point charge is at the origin, what is the magnetic-field vector it produces at the following points: (a) $x=0.500 \mathrm{~m}, y=0$, $z=0$; (b) $x=0, y=0.500 \mathrm{~m}, z=0$; (c) $x=0.500 \mathrm{~m}, y=0.500 \mathrm{~m}$,
$z=0$; (d) $x=0, y=0, z=0.500 \mathrm{~m}$ ?
7.2.2. Two long, parallel transmission lines, 4.0 cm apart, carry $1.0-\mathrm{A}$ and $3.0-\mathrm{A}$ currents. Find all locations where the net magnetic field of the two wires is zero if these currents are in (a) the same direction and (b) the opposite direction.
7.2.3. A solid conductor with a radius $a$ is supported by insulating


Figure 7.3. Problem 7.2.3 \& 7.3.3
disks on the axis of a conducting tube with an inner radius $b$ and an outer radius $c$ (Fig. 7.3). The central conductor and tube carry equal currents $I$ in opposite directions. The currents are distributed uniformly over the cross sections of each conductor. Derive an expression for the magnitude of the magnetic field (a) at points outside the central solid conductor but inside the tube $(a<r<b)$ and (b) at points outside the tube ( $r>c$ ).
7.2.4. Two long, parallel conductors, separated by 2.0 cm , carry currents in the same direction. The first wire carries a current $I_{1}=1.00 \mathrm{~A}$, and the second carries $I_{2}=5.00 \mathrm{~A}$. (a) What is the magnitude of the magnetic field created by $I_{1}$ at the location of $I_{2}$ ? (b) What is the force per unit length exerted by $I_{1}$ on $I_{2}$ ? (c) What is the magnitude of the magnetic field created by $I_{2}$ at the location of $I_{1}$ ?
(d) What is the force per length exerted by $I_{2}$ on $I_{1}$ ?
7.2.5. Calculate the magnitude and direction of the magnetic field


Figure 7.4. Problem 7.2.5
at point $P$ due to the current in the semicircular section of wire shown in Fig. 7.4. (Hint: Does the current in the long straight section of the wire produce any field at $P$ ?)

## Case 7.3

7.3.1.Calculate the magnitude of the magnetic field at a point 4.0 cm from a long, thin conductor carrying a current of 5.0 A .
7.3.2. Four long parallel power lines carry 600 A currents each. A


Figure 7.5. Problem 7.3.2
cross-sectional diagram of these lines is a square, 12.0 cm on each side. For each of the three cases shown in Fig. 7.5, calculate the magnetic field at the center of the square.
7.3.3. A solid conductor with a radius $a$ is supported by insulating disks on the axis of a conducting tube with an inner radius $b$ and an outer radius $c$ (Fig. 7.3). The central conductor and tube carry equal currents $I$ in the same directions. The currents are distributed uniformly over the cross sections of each conductor. Derive an expression for the magnitude of the magnetic field (a) at points outside the central solid conductor but inside the tube ( $a<r<b$ ) and (b) at points outside the
tube $(r>c)$.
7.3.4. The magnetic coils of a tokamak fusion reactor are in the shape of a toroid having an inner radius of 0.900 m and an outer radius of 1.60 m . The toroid has 800 turns of large-diameter wire, each of which carries a current of 18.0 kA . Find the magnitude of the magnetic field inside the toroid along (a) the inner radius and (b) the outer radius.
7.3.5. Two parallel wires separated by 2.00 cm attract to each other with a force per unit length of $3.00 \cdot 10^{-4} \mathrm{~N} / \mathrm{m}$. The current in one wire is 4.00 A . (a) Find the current in the other wire. (b) Are the currents in the same direction or in opposite directions? (c) What would happen if the direction of one current were reversed and doubled?

## Case 7.4

7.4.1. (a) A conducting loop in the shape of a square of an edge length


Figure 7.6. Problem 7.4.1
$\ell=0.20 \mathrm{~m}$ carries a current $I=5.0 \mathrm{~A}$ as shown in Fig. 7.6. Calculate the magnitude and direction of the magnetic field at the center of the square. (b) If this conductor is reshaped to form a circular loop and carries the same current, what is the value of the magnetic field at the center?
7.4.2. Two long parallel wires are separated by a distance of 15 cm . The force per unit length that each wire exerts on the other is $2.0 \cdot 10^{-5} \mathrm{~N} / \mathrm{m}$, and the wires attract to each other. The current in one wire is 3.0 A . (a) What is the current in the second wire? (b) Are
the two currents in the same direction or in opposite directions?
7.4.3. A long straight cylindrical wire of a radius $R=10.0 \mathrm{~cm}$ carries a current uniformly distributed over its cross section. At what location is the magnetic field produced by this current equal to half of its largest value? Consider points inside and outside the wire.
7.4.4. A plastic circular loop of a radius $R$ and a positive charge $q$ is distributed uniformly around the circumference of the loop. The loop is then rotated around its central axis, perpendicular to the plane of the loop with angular speed $\omega$. If the loop is in a region where there is a uniform magnetic field $\overrightarrow{\boldsymbol{B}}$ directed parallel to the plane of the loop, calculate the magnitude of the magnetic torque on the loop.
7.4.5. A long straight wire carries a current $I$. A right-angle bend


Figure 7.7. Problem 7.4.5
is made in the middle of the wire. The bend forms an arc of a circle of radius r as shown in Fig. 7.7. Determine the magnetic field at point $P$, the center of the arc.

## Case 7.5

7.5.1. A wooden ring whose mean diameter is 18.0 cm is wound with a closely spaced toroidal winding of 900 turns. Compute the magnitude of the magnetic field at the center of the cross section of the windings when the current in the windings is 0.50 A .
7.5.2. A solenoid is designed to produce a magnetic field of 0.0314 T at its center. It has a radius of 1.0 cm and a length of 50.0 cm , and the wire can carry a maximum current of 25.0 A . (a) What minimum
number of turns per unit length must the solenoid have? (b) What total length of wire is required?
7.5.3. An infinitely long wire carrying a current $I$ is bent at a right


Figure 7.8. Problem 7.5.3
angle as shown in Fig. 7.8. Determine the magnetic field at point $P$ located a distance $x$ from the corner of the wire.
7.5.4. A current path shaped as shown in Fig. 7.9 produces a mag-


Figure 7.9. Problem 7.5.4
netic field at $P$, the center of the arc. If the arc subtends an angle of $\theta=1.0 \mathrm{rad}$ and the radius of the arc is 0.5 m , what are the magnitude and direction of the field produced at $P$ if the current is 2.0 A ?
7.5.5. Two long parallel wires are attracted to each other by a force per unit length of $120 \mu \mathrm{~N} / \mathrm{m}$. One wire carries a current of 30.0 A to the right and is located along the line $y=1.0 \mathrm{~m}$. The second wire lies along the $x$ axis. Determine the value of $y$ for the line in the plane of the two wires along which the total magnetic field is zero.

Chapter 8
FARADAY'S LAW

|  | Equation | Equation title | Comments |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |
| 8.1 | $\mathcal{E}_{i}=-\frac{d \Phi_{m}}{d t}$ | EMF induced in a closed conducting loop (Faraday's law) | $\Phi_{m}$ is a magnetic flux through the surface attached to the loop; see the equation (6.7) |
| 8.2 | $\mathcal{E}_{i}=B A N \omega \sin (\omega t)$ | EMF induced in the coil with $N$ turns rotating in a uniform magnetic field $B$ | $A$ is an area enclosed by the coil; $\omega$ is an angular speed of rotation |
| 8.3 | $\|\mathcal{E}\|=\|(\overrightarrow{\boldsymbol{v}} \times \overrightarrow{\boldsymbol{B}}) \cdot \overrightarrow{\boldsymbol{\ell}}\|$ | Motional emf induced in a conducting rod of length $\ell$ moving with velocity $\overrightarrow{\boldsymbol{v}}$ in a uniform magnetic field $\overrightarrow{\boldsymbol{B}}$ | $\overrightarrow{\boldsymbol{\ell}}$ is a vector along the rod with the magnitude equal to the rod's length |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :--- | :--- |
| 8.4 | $\oint \underset{L}{\boldsymbol{E}} \cdot d \overrightarrow{\boldsymbol{\ell}}=$ |  |  |
| $=-\frac{d}{d t}\left(\iint_{A_{L}} \overrightarrow{\boldsymbol{B}} \cdot d \overrightarrow{\boldsymbol{A}}\right)$ | General form of <br> Faraday's law; <br> compare with the <br> equation (3.1) | $\overrightarrow{\boldsymbol{E}}$ is a nonconserva- <br> tive electric field; $d \overrightarrow{\boldsymbol{\ell}}$ <br> is an element along <br> the closed loop $L ;$ <br> $A_{L}$ is a surface at- <br> tached to the loop |  |

Pre-Class Reading: [1], chap. 29; [2], chap. 30; [3], chap. 31.

## Case 8.1

8.1.1. A flat rectangular coil consisting of 100 turns measures 25 cm by 20 cm . It is placed in a 0.5 T uniform magnetic field, with the plane of the coil parallel to the field. In 0.25 s, it is rotated so that the plane of the coil is perpendicular to the field. (a) What is the change in the magnetic flux through the coil due to this rotation? (b) Find the magnitude of the average emf induced in the coil during this rotation.
8.1.2. The current in Fig. 8.1 obeys the equation $I(t)=\frac{I_{0} \tau^{2}}{t^{2}+\tau^{2}}$,


Figure 8.1. Problem 8.1.2
where $\tau$ in seconds. Find the direction (clockwise or counterclockwise) of the current induced in the round coil for $t>0$.
8.1.3. A $0.5-\mathrm{m}$ length of wire is held in an east-to-west direction and moves horizontally to the north with a speed of $2.0 \mathrm{~m} / \mathrm{s}$. In this
region, the Earth's magnetic field is of magnitude $60.0 \mu \mathrm{~T}$ and is directed northward and $53.1^{\circ}$ below the horizontal. (a) Calculate the magnitude of the induced emf between the ends of the wire and (b) determine which end is positive.
8.1.4. An electric generator in Fig. 8.2 was designed to produce a


Figure 8.2. Problem 8.1.4
peak voltage of 157 V AC by rotating a 25-turn loop at 3000 rpm in a constant magnetic field of 0.4 T . What is the area of the loop?
8.1.5. A solenoid of a radius $R$ wound with $n$ turns per unit length carries a current given by $I=I_{0} e^{-\beta t}$, where $t$ is the time and $\beta>0$. What are the magnitude and direction of the induced electric field just outside the solenoid?

## Case 8.2

8.2.1. A closely wound search coil has an area of $2.50 \mathrm{~cm}^{2}, 200$ turns, and a resistance of $60.0 \Omega$. It is connected to a charge-measuring instrument whose resistance is $40.0 \Omega$. When the coil is rotated quickly from a position parallel to a uniform magnetic field to a position perpendicular to the field, the instrument indicates a charge of $5.0 \cdot 10^{-5} \mathrm{C}$. What is the magnitude of the field?
8.2.2. A generator produces 18.0 V when turning at 600 rpm . What emf does it produce when turning at 800 rpm ?
8.2.3. How fast (in $\mathrm{m} / \mathrm{s}$ and $\mathrm{km} / \mathrm{h}$ ) would a $5.00-\mathrm{cm}$ copper bar have to move at right angles to a $0.750-\mathrm{T}$ magnetic field to generate 1.50 V (the same as a AA battery) across its ends? Does this seem like
a practical way to generate electricity?
8.2.4. The conducting rod $a b$ shown in Fig. 8.3 makes a contact with


Figure 8.3. Problem 8.2.4
metal rails $a c$ and $b d$ separated by a distance $l$ of 1.00 m . The apparatus is in a uniform magnetic field of 0.750 T , perpendicular to the plane of the figure. (a) Find the magnitude of the emf induced in the rod when it is moving toward the left with a speed $8.00 \mathrm{~m} / \mathrm{s}$. (b) In what direction does the current flow in the rod? (c) If the resistance of the circuit abdc is $3.00 \Omega$ (assumed to be constant), find the force (magnitude and direction) required to keep the rod moving to the left with a constant speed of $8.00 \mathrm{~m} / \mathrm{s}$. You can ignore friction. (d) Compare the rate at which mechanical work is done by the force $(F v)$ with the rate at which thermal energy is developed in the circuit $\left(I^{2} R\right)$.
8.2.5. A long thin solenoid has 1000 turns per meter and a radius of 2.50 cm . The current in the solenoid is increasing at a uniform rate of $50.0 \mathrm{~A} / \mathrm{s}$. What is the magnitude of the induced electric field at a point near the center of the solenoid and (a) 1.00 cm from the axis of the solenoid; (b) 3.14 cm from the axis of the solenoid?

## Case 8.3

8.3.1. A 100 -turn circular coil of wire has a diameter of 20.0 cm . It is placed with its plain perpendicular to the direction of the Earth's magnetic field of $50.0 \mu \mathrm{~T}$ and then in 0.200 s is flipped $180^{\circ}$. An average
emf of what magnitude is generated in the coil?
8.3.2. The current in the long, straight wire $A B$ shown in Fig. 8.4


Figure 8.4. Problem 8.3.2
is downward and increases steadily at a rate $d i / d t$. (a) At an instant when the current is $i$, what are the magnitude and direction of the field $\overrightarrow{\boldsymbol{B}}$ at a distance $r$ to the right of the wire? (b) What is the magnetic flux $d \Phi_{m}$ through the narrow shaded strip? (c) What is the total flux through the loop? (d) What is the induced emf in the loop? (e) Evaluate the numerical value of the induced emf if $a=14.5 \mathrm{~cm}, b=39.5 \mathrm{~cm}$, $L=25.0 \mathrm{~cm}$, and $d i / d t=10.0 \mathrm{~A} / \mathrm{s}$.
8.3.3. The circuit shown in Fig. 8.5 has a resistance of $12.0 \Omega$ and


Figure 8.5. Problem 8.3.3
consumes 3.0 W of power; the rod has a width of 0.25 m between the tracks and moves to the right at $4.0 \mathrm{~m} / \mathrm{s}$. What is the strength of the magnetic field?
8.3.4. A long solenoid has $n=500$ turns per meter and carries a


Figure 8.6. Problem 8.3.4
current given by $I=I_{0} e^{-t / \tau}$, where $I_{0}=27.2 \mathrm{~A}$ and $\tau=2.0 \mathrm{~s}$. Inside the solenoid and coaxial with it is a coil that has a radius of $R=5.00 \mathrm{~cm}$ and consists of a total of $N=400$ turns of fine wire (Fig. 8.6). What emf is induced in the coil at $t=2.0 \mathrm{~s}$ ?
8.3.5. A long thin solenoid has 500 turns per meter and a radius of 1.0 cm . The current in the solenoid is increasing at a uniform rate $d i / d t$. The induced electric field at a point near the center of the solenoid and 3.14 cm from its axis is $5.00 \cdot 10^{-6} \mathrm{~V} / \mathrm{m}$. Calculate $d i / d t$.

## Case 8.4

8.4.1. A circular loop of a flexible iron wire has an initial circumference of 199.0 cm , but its circumference decreases at a constant rate of $6.0 \mathrm{~cm} / \mathrm{s}$ due to a tangential pull on the wire. The loop is in a constant uniform magnetic field oriented perpendicular to the plane of the loop and with the magnitude of 0.500 T . (a) Find the emf induced in the loop at the instant when 7.0 s have passed. (b) Find the direction of the induced current in the loop as viewed looking along the direction of the
magnetic field.
8.4.2. An electromagnet produces a uniform magnetic field of 20.0 mT over a cross-sectional area of $0.1 \mathrm{~m}^{2}$. A coil having 400 turns and a total resistance of $40.0 \Omega$ is placed around the electromagnet. The current in the electromagnet is then smoothly reduced until it reaches zero in 10.0 ms . What is the current induced in the coil?
8.4.3. The coil in an electric generator rotated at 50 Hz has 50 turns with an area of $2 \cdot 10^{-2} \mathrm{~m}^{2}$. The magnetic field is 1.0 T . (a) What is the maximum voltage? (b) What is voltage at $t=0.03$ seconds?
8.4.4. A coil formed by wrapping 80 turns of wire in the shape of a square is positioned in a magnetic field so that the normal to the plane of the coil makes an angle of $60.0^{\circ}$ with the direction of the field. When the magnetic field is increased uniformly from 100 mT to 500 mT in 0.20 s , the emf of magnitude 50.0 mV is induced in the coil. What is the total length of the wire in the coil?
8.4.5. A long solenoid (Fig. 8.7) of a radius $R=5.0 \mathrm{~cm}$ and 200


Figure 8.7. Problem 8.4.5
turns per meter carries an alternating current $I=I_{0} \sin (2 \pi f t)$, where $I_{0}=10.0$ A and $f=50.0 \mathrm{~Hz}$. What are the electric fields induced at initial instant $(t=0)$ (a) within the solenoid at a distance $R / 2$ and (b) outside the solenoid at a distance 2R? [Hint: Apply Faraday's law to the two paths shown, and use symmetry.]

## Case 8.5

8.5.1. A circular loop of wire is in a region of a spatially uniform


Figure 8.8. Problem 8.5.1
magnetic field, as shown in Fig. 8.8. The magnetic field is directed out of the plane of the figure. Determine the direction (clockwise or counterclockwise) of the induced current in the loop when (a) $\overrightarrow{\boldsymbol{B}}$ is increasing; (b) $\overrightarrow{\boldsymbol{B}}$ is decreasing; (c) $\overrightarrow{\boldsymbol{B}}$ is constant with a value $B_{0}$. Explain your reasoning.
8.5.2. A 25 -turn circular coil of radius 2.0 cm and resistance $0.5 \Omega$ is placed in a magnetic field directed perpendicular to the plane of the coil. The magnitude of the magnetic field varies in time according to the expression $B=0.1 t+0.04 t^{2}$, where $B$ is in teslas and $t$ is in seconds. Calculate the current induced in the coil at $t=5.0 \mathrm{~s}$.
8.5.3. The rotating loop in an $A C$ generator is a square 1.0 cm on each side. It is rotated at 50.0 Hz in a uniform field of 1.0 T . Calculate (a) the flux through the loop as a function of time, (b) the emf induced in the loop, (c) the current induced in the loop for a loop resistance of $2.00 \Omega,(\mathrm{~d})$ the power delivered to the loop, and (e) the torque that must be exerted to rotate the loop.
8.5.4. The armature of a small generator consists of a flat square coil
with 100 turns and sides with a length of 2.00 cm . The coil rotates in a magnetic field of 0.025 T . What is the angular speed of the coil if the maximum emf produced is 50.0 mV ?
8.5.5. The uniform magnetic field of the electromagnet, with circular


Figure 8.9. Problem 8.5.5
pole faces of a radius $R_{0}=4.0 \mathrm{~cm}$, increases linearly from 0.9 T to 1.7 T in 6.0 ms . What is the emf induced around the path drawn in Fig. 8.9 (looking down at north pole) that consists of quarter arcs at radial distances $R_{0} / 4$ and $R_{0} / 2$, connected by radial lines? The path is clockwise.

Chapter 9
MAGNETIC FIELD IN A MEDIUM. INDUCTANCE

|  | Equation | Equation title | Comments |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |
| 9.1 | $\overrightarrow{\boldsymbol{M}}=\lim _{V o l \rightarrow 0} \frac{\sum \overrightarrow{\boldsymbol{p}}_{m i}}{V o l}$ | Magnetization vector | $\overrightarrow{\boldsymbol{p}}_{m i}$ is a magnetic dipole moment of an individual atom (molecule) within the spatial element Vol |
| 9.2 | $\overrightarrow{\boldsymbol{H}}=\frac{\overrightarrow{\boldsymbol{B}}}{\mu_{0}}-\overrightarrow{\boldsymbol{M}}$ | Magnetic intensity | $\mu_{0}$ is a magnetic constant |
| 9.3 | $\oint_{L} \overrightarrow{\boldsymbol{H}} \cdot d \overrightarrow{\boldsymbol{\ell}}=I_{c o n d}^{e n c}$ | Ampere's law for magnetic intensity; see also the equation (7.7) | $I_{\text {cond }}^{\text {enc }}$ is a net conduction current enclosed by the loop $L$ |
| 9.4 | $\overrightarrow{\boldsymbol{M}}=\varkappa_{m} \overrightarrow{\boldsymbol{H}}$ | Definition of magnetic susceptibility $\varkappa_{m}$ of a substance | Valid for nonferromagnetic substances |
| 9.5 | $\mu_{r}=1+\varkappa_{m}$ | Definition of relative magnetic permeability of a substance | $\begin{aligned} & \text { For vacuum } \mu_{r} \equiv 1, \\ & \varkappa_{m} \equiv 0 ; \text { for air } \mu_{r} \approx \\ & 1, \varkappa_{m} \approx 0 \end{aligned}$ |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 9.6 | $\overrightarrow{\boldsymbol{B}}=\mu_{r} \mu_{0} \overrightarrow{\boldsymbol{H}}=\mu_{r} \overrightarrow{\boldsymbol{B}}_{0}$ | Magnetic filed in a uniform medium; compare with the equation (4.15) | $\overrightarrow{\boldsymbol{B}}_{0}$ is a magnetic filed in vacuum created by the same system of conductivity currents |
| 9.7 | $\mathcal{E}_{s i}=-L \frac{d I}{d t}$ | Self-induced emf in a closed circuit | $L$ is an inductance of the circuit |
| 9.8 | $L=\frac{N \Phi_{m}}{I}$ | Inductance of a coil with $N$ turns | $\Phi_{m}$ is a magnetic flux through the single turn |
| 9.9 | $L=\mu_{r} \mu_{0} \frac{N^{2} A}{l}$ | Inductance of a long solenoid with $N$ turns | $l$ and $A$ are length end cross-section area, respectively |
| 9.10 | $U_{M}=\frac{1}{2} L I^{2}$ | Magnetic field energy stored in an inductor | $I$ is a current through the inductor with inductance L |
| 9.11 | $\begin{aligned} & u_{M}=\frac{\overrightarrow{\boldsymbol{B}} \cdot \overrightarrow{\boldsymbol{H}}}{2}= \\ = & \frac{\mu_{r} \mu_{0} H^{2}}{2}=\frac{B^{2}}{2 \mu_{r} \mu_{0}} \end{aligned}$ | Magnetic filed energy density; compare with the equation (4.18) | $\overrightarrow{\boldsymbol{B}}$ and $\overrightarrow{\boldsymbol{H}}$ are local values of magnetic field and magnetic intensity, respectively |
| 9.12 | $\tau=\frac{L}{R}$ | Relaxation time (time constant) of an $R L$-circuit | $L$ and $R$ are inductance and resistance of the circuit, respectively |


| 1 | 2 | 3 |  |
| :---: | :---: | :---: | :--- |
| 9.13 | $I(t)=I_{0} e^{-t / \tau}$ | Time dependence <br> for current decay <br> in an $R L$-circuit <br> Time dependence <br> for current growth <br> in an $R L$-circuit <br> with emf $\mathcal{E}$ | $I_{0}$ is an initial <br> current |
| tance of the circuit |  |  |  |

Pre-Class Reading: [1], chap. 28.8 \& 30; [2], chap. 31 \& 32; [3], chap. 30.6 \& 32.

## Case 9.1

9.1.1. A toroidal solenoid with 200 turns of wire and a mean radius of 3.0 cm carries a current of 0.3 A . The relative permeability of the core is 100 . (a) What is the magnetic field in the core? (b) What part of the magnetic field is due to atomic currents?
9.1.2. A permalloy magnet is 1.0 cm in diameter, 30 cm long, and has the magnetization $M=1.5 \cdot 10^{4} \mathrm{~A} / \mathrm{m}$ at its pole. How many turns must an empty solenoid of the same dimensions have to give rise to the same magnetic field if it carries a current of 7.5 A ?
9.1.3. When the current in a toroidal solenoid changes at a rate of $0.025 \mathrm{~A} / \mathrm{s}$, the magnitude of the induced emf is 12.0 mV . When the current equals 1.50 A , the average flux through each turn of the solenoid is 0.003 Wb . How many turns does the solenoid have?
9.1.4. The magnetic field inside a superconducting solenoid is 4.0 T . The solenoid has an inner radius of 3.2 cm and a length of 25.0 cm . Determine (a) the magnetic energy density in the field and (b) the energy stored in the magnetic field within the solenoid.
9.1.5. A 500 -turn solenoid has a radius of 8.00 mm and an overall length of 16.0 cm . (a) What is its inductance? (b) If the solenoid is
connected in series with a $2.0-\Omega$ resistor and a battery, what is the time constant of the circuit?

## Case 9.2

9.2.1. The current in the windings of a toroidal solenoid is 2.5 A . There are 500 turns, and the mean radius is 25.00 cm . The toroidal solenoid is filled with a magnetic material. The magnetic field inside the windings is found to be 2.0 T . Calculate (a) the relative permeability and (b) the magnetic susceptibility of the material that fills the toroid.
9.2.2. The Earth's magnetic moment is about $10^{23} \mathrm{~A} \cdot \mathrm{~m}^{2}$. If the core, which is responsible for the magnetic moment, is about $20 \%$ of the Earth's volume, what would be the core's magnetization? Assume the Earth is a sphere with the radius of $6.4 \cdot 10^{3} \mathrm{~km}$.
9.2.3. An air-filled toroidal solenoid has a mean radius of 15.0 cm and a cross-sectional area of $4.00 \mathrm{~cm}^{2}$. When the current is 30 A , the energy stored is 6.0 J . How many turns does the winding have?
9.2.4. A $24.0-\mathrm{V}$ battery with a negligible internal resistance, a $45.0-\Omega$ resistor, and a $0.9-\mathrm{mH}$ inductor with negligible resistance are all connected in series with an open switch. The switch is suddenly closed. (a) How long after closing the switch will the current through the inductor reach one-half of its maximum value? (b) How long after closing the switch will the energy stored in the inductor reach one-half of its maximum value?
9.2.5. In an $R L$-circuit connected to a $12-\mathrm{V}$ battery, the current is measured to be 0.25 A after $1.386 \cdot 10^{-4} \mathrm{~s}$, and 0.5 A after 20 s . What are the values of $R$ and $L$ ?

## Case 9.3

9.3.1. A cylindrical rod of a radius 0.5 cm and 10.0 cm long, of palladium (magnetic susceptibility $\varkappa_{m}=8 \cdot 10^{-4}$ ), is placed in and aligned with a uniform magnetic field of 2.0 T . What is the magnetic dipole moment of the rod?
9.3.2. An ideal cylindrical solenoid carrying a current of 0.1 A has
a winding density of 10 turns $/ \mathrm{cm}$. If the core is filled with iron $\left(\varkappa_{m}=\right.$ $=4999)$ what is the energy density contained within the magnetic field?
9.3.3. At the instant when the current in an inductor increases at a rate of $0.06 \mathrm{~A} / \mathrm{s}$, the magnitude of the self-induced emf is 0.015 V . (a) What is the inductance of the inductor? (b) If the inductor is a solenoid with 300 turns, what is the average magnetic flux through each turn when the current is 0.48 A ?
9.3.4. A solenoid 12.56 cm long and with a cross-sectional area of $0.5 \mathrm{~cm}^{2}$ contains 300 turns of wire and carries a current of 20.0 A . Calculate: (a) the magnetic field in the solenoid; (b) the energy density in the magnetic field if the solenoid is filled with air; (c) the total energy contained in the coil's magnetic field (assume the field is uniform); (d) the inductance of the solenoid.
9.3.5. A series $R L$ circuit with $L=2.0 \mathrm{mH}$ and a series $R C$ circuit with $C=8.0 \mathrm{nF}$ have equal time constants. If the two circuits contain the same resistance $R$, (a) what is the value of $R$ ? (b) What is the time constant?

## Case 9.4

9.4.1. A thin toroidal coil of a total length 60.0 cm is wound with 1500 turns of wire. A current of 0.5 A flows through the wire. (a) What is the magnitude of $\overrightarrow{\boldsymbol{B}}$ inside the torus if the core consists of a ferromagnetic material of magnetic susceptibility $\varkappa_{m}=2.0 \cdot 10^{3}$ ? (b) What is the magnitude of $\overrightarrow{\boldsymbol{H}}$ ?
9.4.2. A straight wire carries a current $I=10.0$ A. (a) Find the energy density in the surrounding magnetic field as a function of the distance $r$ from the wire. (b) At what distance from the wire does the energy density equal that of a parallel-plate capacitor with a charge of $2.0 \mu \mathrm{C}$ and a capacitance of 4.0 nF if the separation between the plates is 2.0 mm ?
9.4.3. On a clear day at a certain location, a $100-\mathrm{V} / \mathrm{m}$ vertical electric field exists near the Earth's surface. At the same place, the Earth's magnetic field has a magnitude of $60.0 \mu \mathrm{~T}$. Compute the energy densities
of (a) the electric field and (b) the magnetic field.
9.4.4. It is proposed to store $1.0 \mathrm{~kW} \cdot \mathrm{~h}=3.6 \cdot 10^{6} \mathrm{~J}$ of electrical energy in a uniform magnetic field with a magnitude of 1.0 T . (a) What volume (in vacuum) must the magnetic field occupy to store this amount of energy? (b) If instead this amount of energy is to be stored in a volume (in vacuum) equivalent to a cube 1.0 m on a side, what magnetic field is required?
9.4.5. A toroidal solenoid of a rectangular cross section with width $w$, height $h$, and inner radius $R$ is wound with $N$ uniformly spaced turns of wire. The toroid is wound on a nonmagnetic core. (a) Find a magnetic flux through each turn of the toroid if there is a current $I$ in the coil. Do not assume the field is uniform over the cross section. (b) Find the inductance of the toroid. (c) Show that your result for part (b) agrees with Eq. (9.9) for inductance of a long solenoid under assumption $R \gg w$. Use the approximation $\ln (1+z) \approx z$, that is valid for $z \ll 1$. (d) Compute the inductance of a $1000-$ turn toroid for which $R=8.0 \mathrm{~cm}, w=4.0 \mathrm{~cm}$, and $h=4.0 \mathrm{~cm}$.

## Case 9.5

9.5.1. The coil of a solenoid wound with a turn density of 30 turns $/ \mathrm{cm}$ is tilled with a material of unknown magnetic susceptibility $\varkappa_{m}$. When the wire carries 0.5 A , the magnetic field within is 1.885 T . What is $\varkappa_{m}$ ?
9.5.2. (a) What is the magnetic field energy density inside a straight wire of a radius $a$ that carries current $I$ uniformly over its area? (b) What is the total magnetic field energy per unit length inside the wire? Calculate value in part (b) for 2.0-A current.
9.5.3. A $20.0-\Omega$ resistor and a coil are connected in series with a $12.0-$ V battery with negligible internal resistance and a closed switch. (a) At 1.00 ms after the switch is opened, the current has decayed to 0.3 A . Calculate the inductance of the coil. (b) Calculate the time constant of the circuit. (c) How long after the switch is closed will the current reach
$5.0 \%$ of its original value?
9.5.4. Calculate the energy associated with the magnetic field of a 200 -turn solenoid in which a current of 1.6 A produces a magnetic flux of $3.2 \cdot 10^{-4} \mathrm{~Wb}$ in each turn.
9.5.5. A solenoid of a radius 4.0 cm has 250 turns and is 0.4 m long. Find (a) its inductance and (b) the rate at which current must change through it to produce an emf of 50.0 mV .

Chapter 10

## ELECTRICAL OSCILLATIONS AND AC CIRCUITS

|  | Equation | Equation title | Comments |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |
| 10.1 | $-L \frac{d I}{d t}=\frac{q}{C}$ | Differential equation for free undamped oscillations in an $L C$-circuit | $L$ and $C$ are inductance and capacitance of the circuit, respectively |
| 10.2 | $\omega_{0}=\frac{1}{\sqrt{L C}}$ | Angular frequency of undamped oscillations in an $L C$ circuit | $T=2 \pi \sqrt{L C}$ is a period of undamped oscillations in an $L C$-circuit |
| 10.3 | $q(t)=q_{m} \cos \left(\omega_{0} t+\varphi_{0}\right)$ | Time dependence of charge for simple harmonic oscillations | $q_{m}$ is a maximum (amplitude) value of charge; $\varphi_{0}$ is an initial phase of oscillations |
| 10.4 | $L \frac{d I}{d t}+R I+\frac{q}{C}=0$ | Differential equation of free oscillations in an $R L C$ circuit | $R$ is a resistance of a circuit |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 10.5 | $\begin{aligned} & q(t)=q_{\max } e^{-\beta t} \times \\ & \times \cos \left(\omega_{d} t+\varphi_{0}\right) \end{aligned}$ | Time dependence of charge for underdamped oscillations | $\beta=\frac{R}{2 L}$ is a damping coefficient; $\omega_{d}=\sqrt{\omega_{0}^{2}-\beta^{2}}$ is an angular frequency of damped oscillations |
| 10.6 | $R=2 \sqrt{\frac{L}{C}}$ | Condition for critical damping | $\sqrt{\frac{L}{C}}$ is a wave resistance |
| 10.7 | $\begin{aligned} & L \frac{d I}{d t}+R I+\frac{q}{C}= \\ = & \mathcal{E}_{\max } \sin (\omega t) \end{aligned}$ | Differential equation of driven oscillations | $\mathcal{E}_{\text {max }}$ is an amplitude of a driving emf; $\omega$ is a driven frequency |
| 10.8 | $X_{L}=\omega L$ | Inductive reactance | $L$ is an inductance of the circuit |
| 10.9 | $X_{C}=\frac{1}{\omega C}$ | Capacitive reactance | $C$ is a capacitance of the circuit |
| 10.10 | $Z=\sqrt{R^{2}+\left(X_{L}-X_{C}\right)^{2}}$ | Impedance of an $R L C$-circuit | $R$ is a resistance of a circuit |
| 10.11 | $\phi=\arctan \left(\frac{X_{L}-X_{C}}{R}\right)$ | Phase shift between current and voltage | $\begin{aligned} & \cos \phi=\frac{R}{Z} \\ & \sin \phi=\frac{X_{L}-X_{C}}{Z} \end{aligned}$ |
| 10.12 | $I(t)=\frac{\mathcal{E}_{\max }}{Z} \sin (\omega t-\phi)$ | AC current in an $R L C$-circuit | "Ohm's law" for AC-current |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 10.13 | $\begin{aligned} & V_{\mathrm{rms}}=\mathcal{E}_{\text {max }} / \sqrt{2} \\ & I_{\mathrm{rms}}=V_{\mathrm{rms}} / Z \end{aligned}$ | Effective voltage and current in AC circuit | Root mean-square values |
| 10.14 | $P_{\mathrm{av}}=I_{\mathrm{rms}} V_{\mathrm{rms}} \cos \phi$ | Average power in AC circuit | $\phi$ is a currentvoltage phase shift; $\cos \phi$ is a power factor |
| 10.15 | $\omega_{0}=\frac{1}{\sqrt{L C}}$ | Resonant frequency of an $R L C$-circuit | $L$ and $C$ are inductance and capacitance of the circuit, respectively |
| 10.16 | $Q=\frac{\omega_{0}}{\Delta \omega}=\frac{\sqrt{L / C}}{R}$ | $Q$-factor of an $R L C$-circuit | $\Delta \omega$ is the bandwidth (a width of the peak at halfpower) |
| 10.17 | $\frac{V_{2}}{V_{1}}=\frac{I_{1}}{I_{2}}=\frac{N_{2}}{N_{1}}$ | Transformer equation. $\quad V_{1}\left(I_{1}\right)$ and $V_{2}\left(I_{2}\right)$ are the voltages across (currents through) the primary and secondary coil, respectively | $N_{1}$ and $N_{2}$ are the numbers of turns in the primary and secondary coil, respectively |

Pre-Class Reading: [1], chap.31; [2], chap. 33; [3], chap. 33.

## Case 10.1

10.1.1. A $7.50-\mathrm{nF}$ capacitor is charged up to 12.0 V , then disconnected from the power supply and connected in series through a coil. The period of oscillation of the circuit is then measured to be $6.28 \cdot 10^{-5} \mathrm{~s}$. Calculate: (a) the inductance of the coil; (b) the maximum charge on the capacitor; (c) the total energy of the circuit; (d) the maximum current in the circuit.
10.1.2. An $R L C$-circuit is composed of a resistor $R=1.00 \Omega$, an inductor $L=2.00 \mathrm{mH}$, and a capacitor $C=8.00 \mathrm{nF}$, all arranged in series. What is the angular frequency of current oscillations in this circuit?
10.1.3. A series AC circuit contains a resistor, an inductor of 150 mH , a capacitor of $5.00 \mu \mathrm{~F}$, and a source with the amplitude voltage of 240 V operating at 50.0 Hz . The maximum current in the circuit is 100 mA . Calculate (a) the inductive reactance, (b) the capacitive reactance, (c) the impedance, (d) the resistance in the circuit, and (e) the phase angle between the current and the source voltage.
10.1.4. The power of a certain CD player operating at 240 V rms is 40.0 W . Assuming that the CD player behaves like a pure resistance, find (a) the maximum instantaneous power; (b) the rms current; (c) the resistance of this player.
10.1.5. A $16-\mu \mathrm{F}$ capacitor is connected in series with a coil which resistance is $30 \Omega$ and which inductance can be varied. The circuit is connected across a $12-\mathrm{V}, 50-\mathrm{Hz}$ generator. What is the potential difference across the inductor-resistor combination when the frequency is the resonant frequency?

## Case 10.2

10.2.1. A $1.00-\mu \mathrm{F}$ capacitor is charged by a $40.0-\mathrm{V}$ power supply. The fully charged capacitor is then discharged through a $10.0-\mathrm{mH}$ inductor. Find the maximum current in the resulting oscillations.
10.2.2. An $R L C$-circuit has $L=0.8 \mathrm{H}, C=20 \mu \mathrm{~F}$, and resis-
tance $R$. (a) What is the angular frequency of the circuit when $R=0$ ? (b) What value must $R$ have to give a $50.0 \%$ decrease in angular frequency compared to the value calculated in part (a)?
10.2.3. A serious $R L C$-circuit of frequency 50 Hz has maximum current of 94.2 mA . (a) What is the maximum charge on the capacitor? (b) If the impedance is $50 \Omega$, what is the emf?
10.2.4. The primary coil of a transformer has $N_{1}=340$ turns, and the secondary coil has $N_{2}=1414$ turns. If the input voltage across the primary coil is $\mathcal{E}=170 \cos \omega t$, where $\mathcal{E}$ is in volts and $t$ is in seconds, what rms voltage is developed across the secondary coil?
10.2.5. In an $R L C$-series circuit, $R=500 \Omega, L=0.5 \mathrm{mH}$, and $C=20.0 \mathrm{nF}$. When the AC source operates at the resonance frequency of the circuit, the current amplitude is 0.3 A . (a) What is the voltage amplitude of the source? (b) What is the amplitude of the voltage across the resistor, across the inductor, and across the capacitor? (c) What is the average power supplied by the source?

## Case 10.3

10.3.1. Design an $L C$ circuit (give values for $L$ and $C$ ) that has an angular frequency of $2.5 \cdot 10^{4} \mathrm{rad} / \mathrm{s}$ and a stored energy of 0.1 mJ . The maximum voltage drop across the capacitor must be 10.0 V .
10.3.2. Consider an $L C$ circuit in which $L=0.25 \mathrm{H}$ and $C=160 \mathrm{nF}$. (a) What is the resonance angular frequency $\omega_{0}$ ? (b) If a resistance of $500 \Omega$ is introduced into this circuit, what is the frequency of damped oscillations? (c) By what percentage does the frequency of the damped oscillations differ from the resonance frequency?
10.3.3. You have a $80-\Omega$ resistor, a 0.04 -H inductor, a $25.0-\mu \mathrm{F}$ capacitor, and a variable-frequency AC source with an amplitude of 12.0 V . You connect all four elements together to form a series circuit. (a) At what angular frequency will the current in the circuit be the greatest? What will be the current amplitude at this frequency? (b) What will be the current amplitude at an angular frequency of $500 \mathrm{rad} / \mathrm{s}$ ? At this
frequency, will the source voltage lead or lag the current?
10.3.4. When a coil draws 200 W from a $V_{\mathrm{rms}}=110-\mathrm{V}, 60-\mathrm{Hz}$ line, the power factor is 0.600 . (a) If the same coil with a capacitor added in series is to draw the same power from a $V_{\text {rms }}=220-\mathrm{V}, 60-\mathrm{Hz}$ line, what must the capacitance be? (b) If the aim were to maintain the same power factor rather than the same rms power, how would your answer change?
10.3.5. A $10.0-\Omega$ resistor, $10.0-\mathrm{mH}$ inductor, and $100-\mu \mathrm{F}$ capacitor are connected in series to a $50.0-\mathrm{V}(\mathrm{rms})$ source having variable frequency. If the operating frequency is twice the resonance frequency, find the energy delivered to the circuit during one period.

## Case 10.4

10.4.1. An $L C$ circuit containing an $125.00-\mathrm{mH}$ inductor and a $0.80-\mathrm{nF}$ capacitor oscillates with a maximum current of 2.00 A . Calculate: (a) the maximum charge on the capacitor and (b) the angular frequency of the circuit. (c) Assuming the capacitor had its maximum charge at time $t=0$, calculate the energy stored in the inductor after $15.71 \mu \mathrm{~s}$ of oscillation.
10.4.2. Consider an $R L C$-circuit at critical damping, with $L=60 \mathrm{mH}$. What is the value of $R$ if the current decays by 2 percent in 15.0 ms ?
10.4.3. In an $R L C$ series circuit that includes a source of alternating current operating at fixed frequency and voltage, the resistance $R$ is equal to the inductive reactance. If the plate separation of the parallel-plate capacitor is reduced to one-half its original value, the current in the circuit doubles. Find the initial capacitive reactance in terms of $R$.
10.4.4. You plan to take your hair blower to the United States where the electrical outlets put out 120 V instead of the 240 V seen in Europe. The blower puts out 1600 W at 240 V . (a) What could you do to operate your blower via the $120-\mathrm{V}$ line in US? (b) What current will your blower draw from a US outlet? (c) What resistance will your blower appear to
have when operated at 120 V ?
10.4.5. A radar transmitter contains an $L C$ circuit oscillating at $1.00 \cdot 10^{10} \mathrm{~Hz}$. (a) For a one-turn loop having an inductance of 250 pH to resonate at this frequency, what capacitance is required in series with the loop? (b) The capacitor has square parallel plates separated by 1.00 mm of air. What should the edge length of the plates be? (c) What is the common reactance of the loop and capacitor at resonance?

## Case 10.5

10.5.1. An $L C$-circuit oscillates with an angular frequency of $5.0 \cdot 10^{4} \mathrm{rad} / \mathrm{s}$. When a second capacitor is inserted in series with the original one, the angular frequency becomes $6.0 \cdot 10^{4} \mathrm{rad} / \mathrm{s}$. If the capacitors are replaced by a resistor of $0.01 \Omega$, the current drops to $1 / 2$ of its initial value in 6.93 ms . What are the values of the two capacitors and of the inductance $L$ ?
10.5.2. Consider an $R L C$ series circuit with a $0.40-\mathrm{H}$ inductor, a $2.50-\mu \mathrm{F}$ capacitor, and a $200-\Omega$ resistor. The source has terminal rms voltage $V_{\mathrm{rms}}=100.0 \mathrm{~V}$ and variable angular frequency $\omega$. (a) What is the resonance angular frequency $\omega_{0}$ of the circuit? (b) What is the rms current through the circuit at resonance, $I_{\mathrm{rms}-0}$ ? (c) For what two values of the angular frequency, $\omega_{1}$ and $\omega_{2}$, is the rms current half the resonance value? (d) The quantity $\left|\omega_{1}-\omega_{2}\right|$ defines the resonance width. Calculate $I_{\mathrm{rms}-0}$ and the resonance width for (i) $R=20.0 \Omega$, (ii) $2.0 \Omega$, and (ii) $0.2 \Omega$.
10.5.3. (a) At what angular frequency does the voltage amplitude across the resistor in an $R L C$ series circuit reach its maximum value? (b) At what angular frequency does the voltage amplitude across the capacitor reach its maximum value? (c) At what angular frequency does the voltage amplitude across the inductor reach its maximum value?
10.5.4. House current, which has an rms voltage of 220 V and frequency of 50 Hz , drives a resistor of variable resistance set at $R=100 \Omega$, a capacitor of fixed capacitance $C=10 \mu \mathrm{~F}$, and an inductor of variable inductance, connected in series. (a) What is the power absorbed by the
circuit if $L=10 \mathrm{mH}$ ? (b) What power would be drawn if the resistance were halved without changing the setting of the inductance? (c) What is the maximum power drawn in part (a)?
10.5.5. A voltage $\mathcal{E}=50 \sin \omega t$, where $\mathcal{E}$ is in volts and $t$ is in seconds, is applied across a series combination of a $0.10-\mathrm{H}$ inductor, a $10.0-\mu \mathrm{F}$ capacitor, and a $10.0-\Omega$ resistor. (a) Determine the angular frequency $\omega_{0}$ at which the power delivered to the resistor is a maximum. (b) Calculate the average power delivered at that frequency. (c) Determine two angular frequencies $\omega_{1}$ and $\omega_{2}$ at which the power is one-half the maximum value. (d) Determine the $Q$-factor of the circuit. Note: the $Q$-factor of the circuit is $\frac{\omega_{0}}{\left|\omega_{2}-\omega_{1}\right|}$.

## Chapter 11

## MAXWELL'S EQUATIONS. ELECTROMAGNETIC WAVES

|  | Equation | Equation title | Comments |
| :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 |
| 11.1 | $\begin{gathered} \oint_{L} \overrightarrow{\boldsymbol{H}} \cdot d \overrightarrow{\boldsymbol{\ell}}=I_{\text {cond }}^{\text {enc }}+ \\ +\frac{d}{d t}\left(\int_{A_{L}} \overrightarrow{\boldsymbol{D}} \cdot d \overrightarrow{\boldsymbol{A}}\right) \end{gathered}$ | Ampere's law with Maxwell's displacement current; see also the equations (7.7) and (9.3) | $\overrightarrow{\boldsymbol{H}}$ is a magnetic intensity; $I_{\text {cond }}^{\text {enc }}$ is a net conduction current passing through a surface $A_{L}$ enclosed by the loop $L$ |
| 11.2 | $\overrightarrow{\boldsymbol{j}}_{D}=\frac{\partial \overrightarrow{\boldsymbol{D}}}{\partial t}$ | Displacement current density | $\vec{D}$ is an electric filed displacement vector |
| 11.3 | $\begin{aligned} \operatorname{div} \overrightarrow{\boldsymbol{D}} & =\rho_{\text {free }} \\ \operatorname{div} \overrightarrow{\boldsymbol{B}} & =0 \\ \operatorname{rot} \overrightarrow{\boldsymbol{E}} & =-\frac{\partial \overrightarrow{\boldsymbol{B}}}{\partial t} \\ \operatorname{rot} \overrightarrow{\boldsymbol{H}} & =\overrightarrow{\boldsymbol{j}}_{\text {cond }}+\frac{\partial \overrightarrow{\boldsymbol{D}}}{\partial t} \end{aligned}$ | Maxwell's equations in differential form; for integral form, check the equations (4.12), (7.10), (8.4), and (11.1) | $\overrightarrow{\boldsymbol{E}}$ and $\overrightarrow{\boldsymbol{B}}$ are electric and magnetic fields, respectively; $\rho_{\text {free }}$ is a volume density of free charge; $\overrightarrow{\boldsymbol{j}}_{\text {cond }}$ is a conduction current density; see details for div and rot in Appendix |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 11.4 | $\begin{aligned} & \overrightarrow{\boldsymbol{D}}=\varepsilon_{r} \varepsilon_{0} \overrightarrow{\boldsymbol{E}} \\ & \overrightarrow{\boldsymbol{B}}=\mu_{r} \mu_{0} \overrightarrow{\boldsymbol{H}} \\ & \overrightarrow{\boldsymbol{j}}_{\text {cond }}=\sigma \overrightarrow{\boldsymbol{E}} \end{aligned}$ | Constitutive relations; see also the equations (4.11), (9.2), and (5.2) | $\varepsilon_{r}$ and $\mu_{r}$ are relative permittivity and permeability of the substance, respectively; $\sigma$ is a conductivity |
| 11.5 | $\overrightarrow{\boldsymbol{F}}=q(\overrightarrow{\boldsymbol{E}}+\overrightarrow{\boldsymbol{v}} \times \overrightarrow{\boldsymbol{B}})$ | Force exerted by an electromagnetic field on a point charge $q$ (Lorentz force) | $\overrightarrow{\boldsymbol{v}}$ is velocity of the charged particle; see also the equations (1.2) and (6.1) |
| 11.6 | $\begin{aligned} & \frac{\partial^{2} \overrightarrow{\boldsymbol{E}}}{\partial t^{2}}=\frac{1}{\varepsilon_{r} \varepsilon_{0} \mu_{r} \mu_{0}} \Delta \overrightarrow{\boldsymbol{E}} \\ & \frac{\partial^{2} \overrightarrow{\boldsymbol{H}}}{\partial t^{2}}=\frac{1}{\varepsilon_{r} \varepsilon_{0} \mu_{r} \mu_{0}} \Delta \overrightarrow{\boldsymbol{H}} \end{aligned}$ | Wave equations for $\overrightarrow{\boldsymbol{E}}$ and $\overrightarrow{\boldsymbol{H}}$ fields in 3D case | $\Delta=\nabla^{2}=\vec{\nabla} \cdot \vec{\nabla}$ <br> is a Laplace operator (see Appendix for details) |
| 11.7 | $\frac{\partial^{2} \overrightarrow{\boldsymbol{E}}}{\partial t^{2}}=v^{2} \frac{\partial^{2} \overrightarrow{\boldsymbol{E}}}{\partial x^{2}}$ | Wave equation of a plane wave propagating along $x$ axis | $v=\frac{1}{\sqrt{\varepsilon_{r} \varepsilon_{0} \mu_{r} \mu_{0}}}$ is a propagation speed of the electromagnetic wave |
| 11.8 | $c=\frac{1}{\sqrt{\mu_{0} \varepsilon_{0}}}$ | Speed of electromagnetic (EM) wave in vacuum | Speed of light in vacuum |
| 11.9 | $n=\frac{c}{v}=\sqrt{\mu_{r} \varepsilon_{r}}$ | Refractive index of the substance | $v$ is a speed of electromagnetic wave in the substance |


| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 11.10 | $\begin{aligned} & \overrightarrow{\boldsymbol{E}}=\overrightarrow{\boldsymbol{B}} \times \overrightarrow{\boldsymbol{v}} \\ & \overrightarrow{\boldsymbol{H}}=\overrightarrow{\boldsymbol{v}} \times \overrightarrow{\boldsymbol{D}} \end{aligned}$ | Instantaneous electric and magnetic fields in an EM wave | $\overrightarrow{\boldsymbol{v}}$ is a propagation velocity of the electromagnetic wave |
| 11.11 | $u_{E M W}=\overrightarrow{\boldsymbol{E}} \cdot \overrightarrow{\boldsymbol{D}}=\overrightarrow{\boldsymbol{B}} \cdot \overrightarrow{\boldsymbol{H}}$ | Energy density in a propagating electromagnetic wave | Energy stored by the electromagnetic wave in a unit volume; see also the equations (4.18) and (9.11) |
| 11.12 | $\overrightarrow{\boldsymbol{S}}=u_{E M W} \overrightarrow{\boldsymbol{v}}=\overrightarrow{\boldsymbol{E}} \times \overrightarrow{\boldsymbol{H}}$ | Energy flux density of an electromagnetic wave (Poynting vector) | Power delivered by the wave per unit area perpendicular to the direction of propagation |
| 11.13 | $\begin{aligned} & E=E_{0} \cos (k x-\omega t) \\ & B=B_{0} \cos (k x-\omega t) \end{aligned}$ | Electric and magnetic fields in a plane harmonic wave propagating in $+x$ direction | $k$ and $\omega$ are wave number and angular frequency, respectively |
| 11.14 | $v=\frac{c}{n}=\frac{\omega}{k}=\lambda f$ | Propagation speed of a harmonic wave | $\lambda=\frac{2 \pi}{k}$ is a wavelength; $f=\frac{\omega}{2 \pi}$ is a frequency of the wave |
| 11.15 | $I=\langle \| \overrightarrow{\boldsymbol{S}}\| \rangle$ | Intensity of an electromagnetic wave | Average power delivered by the wave per unit area |

Pre-Class Reading: [1], chap.29.7\&32; [2], chap.29.5\&34; [3], chap.34.

## Case 11.1

11.1.1. A dielectric with permittivity constant 3.95 completely fills the volume between two capacitor plates. For $t>0$, the electric flux through the dielectric is $\left(8.0 \cdot 10^{3} \mathrm{~V} \cdot \mathrm{~m} / \mathrm{s}^{3}\right) t^{3}$. The dielectric is ideal and nonmagnetic; the conduction current in the dielectric is zero. At what time does the displacement current in the dielectric equal $21 \mu \mathrm{~A}$ ?
11.1.2. A $0.400-\mathrm{A}$ current is charging a capacitor that has circular plates 20.0 cm in radius. If the plate separation is 1.00 mm , (a) what is the time rate of the electric field increase between the plates? (b) What is the magnetic field between the plates 2.00 cm from the center?
11.1.3. The electric field of a sinusoidal electromagnetic wave obeys the equation
$E=-(375 \mathrm{~V} / \mathrm{m}) \sin \left[\left(6.28 \cdot 10^{15} \mathrm{rad} / \mathrm{s}\right) t+\left(2.094 \cdot 10^{7} \mathrm{rad} / \mathrm{m}\right) x\right]$. (a) What are the amplitudes of the electric and magnetic fields of this wave? (b) What are the frequency, wavelength, and period of the wave? Is this light visible to humans? (c) What is the speed of the wave?
11.1.4. A sinusoidal electromagnetic wave is propagating in a vacuum in the $-y$-direction. If at a particular instant and at a certain point in space the electric field is in the $+z$-direction and has a magnitude of $4.50 \mathrm{~V} / \mathrm{m}$, what are the magnitude and direction of the magnetic field of the wave at this same point in space and instant in time?
11.1.5. In a region of free space, the electric field at an instant of time is $\overrightarrow{\boldsymbol{E}}=(1 \vec{i}+3 \overrightarrow{\boldsymbol{j}}+4 \overrightarrow{\boldsymbol{k}}) \mathrm{V} / \mathrm{m}$ and the magnetic field is $\overrightarrow{\boldsymbol{H}}=(1 \overrightarrow{\boldsymbol{i}}+5 \overrightarrow{\boldsymbol{j}}-4 \overrightarrow{\boldsymbol{k}}) \mathrm{A} / \mathrm{m}$. (a) Show that the two fields are perpendicular to each other. (b) Determine the Poynting vector for these fields.

## Case 11.2

11.2.1. The electric flux through a certain area of a dielectric is $\left(5.0 \cdot 10^{3} \mathrm{~V} \cdot \mathrm{~m} / \mathrm{s}^{4}\right) t^{4}$. The displacement current through this area is 8.85 pA at a time $t=20.0 \mathrm{~ms}$. Calculate the dielectric permittivity
constant for the dielectric.
11.2.2. Consider the situation shown in Fig. 11.1. An electric field


Figure 11.1. Problem 11.2.2
of $500 \mathrm{~V} / \mathrm{m}$ is confined to a circular area $d=10.0 \mathrm{~cm}$ in diameter and directed toward perpendicular to the plane of the figure. If the field is increasing at a rate of $36.0 \mathrm{~V} /(\mathrm{m} \cdot \mathrm{s})$, what are (a) the direction and (b) the magnitude of the magnetic field at the point $P, r=20.0 \mathrm{~cm}$ from the center of the circle?
11.2.3. At a certain location on the Earth, the rms value of the magnetic field caused by solar radiation is $1.20 \mu \mathrm{~T}$. From this value, calculate (a) the rms electric field due to solar radiation, (b) the average energy density of the solar component of electromagnetic radiation at this location, and (c) the average magnitude of the Poynting vector for the Sun radiation.
11.2.4. An electromagnetic wave has an electric field given by $\overrightarrow{\boldsymbol{E}}(x, t)=\left(3.0 \cdot 10^{5} \mathrm{~V} / \mathrm{m}\right) \overrightarrow{\boldsymbol{j}} \sin \left[k x-\left(12 \pi \cdot 10^{14} \mathrm{rad} / \mathrm{s}\right) t\right]$. (a) In which direction is the wave traveling? (b) What is the wavelength of the wave? (c) Write the vector equation for $\overrightarrow{\boldsymbol{B}}(y, t)$.
11.2.5. In SI units, the electric field in an electromagnetic wave is described by $E_{y}=300 \sin \left(3.14 \cdot 10^{7} x-\omega t\right)$. Find (a) the amplitude of the corresponding magnetic field oscillations, (b) the wavelength $\lambda$, and (c) the frequency $f$.

## Case 11.3

11.3.1. A sinusoidal electromagnetic wave having a magnetic field of amplitude $5.0 \mu \mathrm{~T}$ and a wavelength of $0.5 \mu \mathrm{~m}$ is traveling in the $+x$-direction through the empty space. (a) What is the frequency of this wave? (b) What is the amplitude of the associated electric field? (c) Write the equations for the electric and magnetic fields as functions of $x$ and $t$ in the form of Eqs. (11.13).
11.3.2. An electromagnetic wave has a magnetic field given by $\overrightarrow{\boldsymbol{B}}(z, t)=\left(6.0 \cdot 10^{-9} \mathrm{~T}\right) \overrightarrow{\boldsymbol{j}} \sin \left[\left(1.57 \cdot 10^{4} \mathrm{rad} / \mathrm{m}\right) z+\omega t\right]$. (a) In which direction is the wave traveling? (b) What is the frequency $f$ of the wave? (c) Write the vector equation for $\overrightarrow{\boldsymbol{E}}(z, t)$.
11.3.3. A parallel-plate air-filled capacitor is being charged as in


Figure 11.2. Problem 11.3.3 \& 11.4.1

Fig. 11.2. The circular plates have a radius of 4.00 cm , and at a particular instant, the conduction current in the wires is 0.280 A . (a) What is the displacement current density $j_{D}$ in the air space between the plates? (b) What is the rate at which the electric field between the plates is changing? What is the induced magnetic field between the plates at a
distance (c) of 2.00 cm from the axis; (d) of 1.00 cm from the axis?
11.3.4. Why is the following situation impossible? An electromagnetic wave travels through empty space with electric and magnetic fields described by:

$$
\begin{gathered}
E=1.5 \cdot 10^{4} \cos \left[\left(5.0 \cdot 10^{6}\right) x-\left(2.0 \cdot 10^{15}\right) t\right] \\
B=5.0 \cdot 10^{-5} \cos \left[\left(5.0 \cdot 10^{6}\right) x-\left(2.0 \cdot 10^{15}\right) t\right]
\end{gathered}
$$

where all numerical values and variables are in SI units.
11.3.5. Important news are transmitted by radio waves to people sitting next to their radios 150 km from the station and by sound waves to people sitting across the newsroom 3.3 m from the newscaster. Taking the speed of sound in air to be $330 \mathrm{~m} / \mathrm{s}$, who receives the news first? Explain.

## Case 11.4

11.4.1. Suppose that the parallel plates in Fig. 11.2 have an area of $2.00 \mathrm{~cm}^{2}$ and are separated by a $8.85-\mathrm{mm}$-thick sheet of dielectric that completely fills the volume between the plates. The dielectric has dielectric constant 5.0. (You can ignore fringing effects.) At a certain instant, the potential difference between the plates is 50 V and the conduction current $i_{c}$ equals 7.00 mA . At this instant, what are (a) the charge $q$ on each plate; (b) the rate of change of charge on the plates; (c) the displacement current in the dielectric?
11.4.2. An electromagnetic wave of wavelength 400 nm is traveling in vacuum in the $-y$-direction. The electric field has amplitude of $1.50 \cdot$ $10^{-3} \mathrm{~V} / \mathrm{m}$ and is parallel to the $z$-axis. What are (a) the frequency and (b) the magnetic-field amplitude? (c) Write the vector equations for $\overrightarrow{\boldsymbol{E}}(y, t)$ and $\overrightarrow{\boldsymbol{B}}(y, t)$.
11.4.3. An electromagnetic wave with frequency 100.0 Hz travels in an insulating magnetic material that has dielectric constant 10.0 and relative permeability 2.5 at this frequency. The electric field has an amplitude of $3.0 \cdot 10^{-3} \mathrm{~V} / \mathrm{m}$. (a) What is the speed of propagation of the wave? (b) What is the wavelength of the wave? (c) What is the
amplitude of the magnetic field? (d) What is the intensity of the wave?
11.4.4. If the intensity of sunlight at the Earth's surface under a fairly clear sky is $999 \mathrm{~W} / \mathrm{m}^{2}$, how much electromagnetic energy per cubic meter is contained in sunlight?
11.4.5. In 1965, Arno Penzias and Robert Wilson discovered the cosmic microwave radiation left over from the big bang expansion of the Universe. Suppose the energy density of this background radiation is $4.00 \cdot 10^{-14} \mathrm{~J} / \mathrm{m}^{3}$. Determine the corresponding magnetic field amplitude.

## Case 11.5

11.5.1. A $26.55-\mathrm{mA}$ current is charging a capacitor that has square plates 15.0 cm on each side. The plate separation is 2.00 mm . Find (a) the time rate of change of electric flux between the plates and (b) the displacement current between the plates.
11.5.2. At the upper surface of the Earth's atmosphere the time averaged magnitude of the Poynting vector is referred to as the solar constant and is given by $\langle | \overrightarrow{\boldsymbol{S}}\left\rangle=1.35 \mathrm{~kW} / \mathrm{m}^{2}\right.$. If you assume that the Sun's electromagnetic radiation is a plane sinusoidal wave, what are the magnitudes of (a) the electric and (b) magnetic fields?
11.5.3. Consider each of the following electric and magnetic-field orientations. In each case, what is the direction of propagation of the wave? (a) $\overrightarrow{\boldsymbol{E}}=E \overrightarrow{\boldsymbol{i}}, \overrightarrow{\boldsymbol{B}}=-B \overrightarrow{\boldsymbol{j}}$; (b) $\overrightarrow{\boldsymbol{E}}=E \overrightarrow{\boldsymbol{j}}, \overrightarrow{\boldsymbol{B}}=B \overrightarrow{\boldsymbol{i}}$; (c) $\overrightarrow{\boldsymbol{E}}=-E \overrightarrow{\boldsymbol{k}}$, $\overrightarrow{\boldsymbol{B}}=-B \overrightarrow{\boldsymbol{\boldsymbol { i }}}$; (d) $\overrightarrow{\boldsymbol{E}}=E \overrightarrow{\boldsymbol{i}}, \overrightarrow{\boldsymbol{B}}=-B \overrightarrow{\boldsymbol{k}}$.
11.5.4. What is the average magnitude of the Poynting vector 1.0 km from a radio transmitter broadcasting isotropically (equally in all directions) with an average power of 100 kW ?
11.5.5. Assume the intensity of solar radiation incident on the cloud tops of the Earth is $1350 \mathrm{~W} / \mathrm{m}^{2}$. Taking the average Earth-Sun separation to be $150 \cdot 10^{6} \mathrm{~km}$, calculate the total power radiated by the Sun.

## ANSWERS

Chapter 1
1.1.1 375 m
$\mathbf{1 . 1 . 2} \quad$ a) $\overrightarrow{\boldsymbol{p}} \| \overrightarrow{\boldsymbol{E}} \quad$ b) if $\overrightarrow{\boldsymbol{p}} \uparrow \uparrow \overrightarrow{\boldsymbol{E}}$,
then the orientation is stable; if
$\overrightarrow{\boldsymbol{p}} \uparrow \downarrow \overrightarrow{\boldsymbol{E}} \Rightarrow$ unstable
1.1 .3 a) $5.00 \cdot 10^{6} \mathrm{~N} / \mathrm{C}$
b) $8.35 \cdot 10^{-9} \mathrm{~s}$
1.1 .4 a) $\overrightarrow{\boldsymbol{E}}_{(\mathrm{i})}=35 \overrightarrow{\boldsymbol{j}} \mathrm{~N} / \mathrm{C}$,
$\overrightarrow{\boldsymbol{E}}_{(\mathrm{ii)}}=-45 \overrightarrow{\boldsymbol{j}} \mathrm{~N} / \mathrm{C}, \quad \overrightarrow{\boldsymbol{E}}_{(\mathrm{iii})}=0$
b) $\overrightarrow{\boldsymbol{F}}_{(\mathrm{i})}=-5.6 \cdot 10^{-18} \overrightarrow{\boldsymbol{j}} \mathrm{~N}$,
1.3.2 a) $3600 \overrightarrow{\boldsymbol{i}} \mathrm{~N} / \mathrm{C}$
b) $-18.0 \cdot 10^{-3} \vec{i} \mathrm{~N}$
1.3 .3 a) $-\overrightarrow{\boldsymbol{i}}$
b) $\frac{\overrightarrow{\boldsymbol{i}}-\overrightarrow{\boldsymbol{j}}}{\sqrt{2}}$
c) $0.8 \overrightarrow{\boldsymbol{i}}+0.6 \overrightarrow{\boldsymbol{j}}$
1.3 .4 a) $2.0 \cdot 10^{-10} \mathrm{C} \cdot \mathrm{m}$ from $q_{2}$ toward $q_{1}$ b) $80.0 \mathrm{~N} / \mathrm{C}$
1.3.5 a) $d \sqrt{\frac{m g}{k_{C}} \frac{d}{\sqrt{4 R^{2}-d^{2}}}}$
b) $\approx 1 \mu \mathrm{C}$
1.4.1 $2.25 \cdot 10^{-9} \mathrm{~N} / \mathrm{m}$
1.4 .2 b) $7.07 \mu \mathrm{C} \quad$ c) $67^{\circ}$
1.4 .3 a) $8.64 \cdot 10^{4} \mathrm{~N} / \mathrm{C}$
b) $10^{4} \mathrm{~N} / \mathrm{C}$
c) $270 \mathrm{~N} / \mathrm{C}$
d) $1.15 \cdot 10^{5} \mathrm{~N} / \mathrm{C}$
1.4.4 $2 \cdot 10^{6} \mathrm{~N} / \mathrm{C}$
1.2.2 a) $\frac{-k_{C} q \overrightarrow{\boldsymbol{j}}}{y(y+a)} \quad$ b) $\frac{-k_{C} q Q \overrightarrow{\boldsymbol{j}}}{y(y+a)}$
1.2 .3 a) $0.91 \mathrm{~N} / \mathrm{C}$ b) no,
$2.72 \mu \mathrm{~m}$ above midpoint
1.4 .5 a) $E_{y}=\frac{2 k_{C} \lambda a}{y \sqrt{y^{2}+a^{2}}}$
b) $E_{y}=\frac{2 k_{C} \lambda}{y}$
1.2.4 a) 0.90 N , along the dipole from the negative toward the positive charge b) $7.79 \cdot 10^{-3} \mathrm{~N} \cdot \mathrm{~m}$, clockwise
$\mathbf{1 . 2 . 5}$ a) $9 \cdot 10^{19} \mathrm{~N}$
b) $3.91 \cdot 10^{23} \mathrm{~m} / \mathrm{s}^{2} \quad$ c) no
1.3.1 +1.00 nC
1.5.1 $q_{1}=q_{2}=\frac{1}{2} q$
1.5.2 $3.0 \cdot 10^{-7} \mathrm{~N} \cdot \mathrm{~m}$
1.5.3 $10^{5} \mathrm{~N} / \mathrm{C}$
1.5.4 $k_{C} \frac{q}{l^{2}}\left(1-\frac{\sqrt{2}}{2}\right)$
$\mathbf{1 . 5 . 5}$ a) $\overrightarrow{\boldsymbol{E}}=99.50 \overrightarrow{\boldsymbol{i}} \mathrm{~N} / \mathrm{C}$
c) smaller

## Chapter 2

2.1 .1 a) $1.0 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}$
$\mathbf{2 . 1 . 2}$ a) $0 \quad$ b) $6.75 \cdot 10^{5} \mathrm{~N} / \mathrm{C}$
c) $3.00 \cdot 10^{5} \mathrm{~N} / \mathrm{C}$
$2.1 .3 \quad 2.25 \cdot 10^{9}$
2.1 .4 a) $2.00 \cdot 10^{3} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C} \quad$ b) 0
c) $300 \overrightarrow{\boldsymbol{i}} \mathrm{~N} / \mathrm{C}$
c) $300 \boldsymbol{\imath} \mathrm{~N} / \mathrm{C}$
2.1 .5 a) $6 \cdot 10^{-2} \mathrm{~N}$, toward the cen- $E=2 k_{C} \frac{Q}{r^{2}}$ ter of the shell b) 0
2.2 .1 a) $1.31 \cdot 10^{5} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}$
b) $1.00 \cdot 10^{-6} \mathrm{C}$
2.2 .2 a) $6.37 \cdot 10^{-5} \mathrm{C} / \mathrm{m}^{3}$
b) $2.40 \cdot 10^{4} \mathrm{~N} / \mathrm{C}$
2.2.3 $5.0 \cdot 10^{-8} \mathrm{C} / \mathrm{m}^{2}$
2.2 .4 a) $\frac{2 k_{C} \lambda}{r}$, radially inward
b) $\frac{2 k_{C} \lambda}{r}$, radially inward
d) inner: $-\lambda$; outer: $\lambda$
$\mathbf{2 . 2 . 5} \quad$ a) $\frac{1}{3} \pi \rho_{0} R^{3}$
c) $\frac{\rho_{0} r}{3 \varepsilon_{0}}\left(1-\frac{3 r}{4 R}\right)$
e) $\frac{2}{3} R, \frac{\rho_{0} R}{9 \varepsilon_{0}}$
2.3.1 a) $\lambda=2 \pi R \sigma$
b) $\frac{\sigma R}{\varepsilon_{0} r}$
$2.3 .2 \quad$ a) 0
b) $1.2 \cdot 10^{5} \mathrm{~N} / \mathrm{C}$
c) $1.2 \cdot 10^{6} \mathrm{~N} / \mathrm{C} \quad$ d) $3.0 \cdot 10^{5} \mathrm{~N} / \mathrm{C}$
2.3.3 $\pi r^{2} E$
2.3.4 $45^{\circ}$
$\mathbf{2 . 3 . 5}$ a) $\overrightarrow{\boldsymbol{E}}=\frac{\rho}{3 \varepsilon_{0}}(\overrightarrow{\boldsymbol{r}}-\overrightarrow{\boldsymbol{b}})$
b) $\overrightarrow{\boldsymbol{E}}=\frac{\rho \overrightarrow{\boldsymbol{b}}}{3 \varepsilon_{0}}$
$\mathbf{2 . 4 . 5}$ a) $-\frac{1}{4} \frac{k_{C} Q^{2}}{R^{2}} \overrightarrow{\boldsymbol{i}} \quad$ b) $\frac{1}{18} \frac{k_{C} Q}{R^{2}} \overrightarrow{\boldsymbol{i}}$
$\mathbf{2 . 4 . 5}$ a) $-\frac{1}{4} \frac{k_{C} Q_{\overrightarrow{2}}^{R^{2}} \overrightarrow{\boldsymbol{i}}}{} \quad$ b) $\frac{1}{18} \frac{k_{C} Q}{R^{2}} \overrightarrow{\boldsymbol{i}}$
$\mathbf{2 . 4 . 5}$ a) $-\frac{1}{4} \frac{k_{C} Q^{2}}{R^{2}} \overrightarrow{\boldsymbol{i}} \quad$ b) $\frac{1}{18} \frac{k_{C} Q}{R^{2}} \overrightarrow{\boldsymbol{i}}$
c) $0 \quad$ d) $\frac{10}{9} \frac{k_{C} Q}{R^{2}} \overrightarrow{\boldsymbol{i}}$
c) $0 \quad$ d) $\frac{10}{9} \frac{k_{C} Q_{\vec{~}}}{R^{2}} \overrightarrow{\boldsymbol{i}}$
2.5 .1 a) $2.40 \cdot 10^{3} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C} \quad$ b) 0
c) $1.20 \cdot 10^{3} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}$
2.5.2-1.77•10-12 $\mathrm{C} / \mathrm{m}^{3}$
2.5.3 $r<r_{1}: 0$;
$r_{1}<r<r_{2}: \frac{\rho}{2 \varepsilon_{0} r}\left(r^{2}-r_{1}^{2}\right) \frac{\overrightarrow{\boldsymbol{r}}}{r} ;$
b) $\frac{k_{C} Q}{r^{2}} \quad r>r_{2}: \frac{\rho}{2 \varepsilon_{0} r}\left(r_{1}^{2}-r_{1}^{2}\right) \frac{\overrightarrow{\boldsymbol{r}}}{r}$
$\mathbf{2 . 4 . 1}$ a) $333 \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}$
b) no change
$\mathbf{2 . 4 . 2}$ a) $0 \quad$ b) $\frac{2 \lambda}{\varepsilon_{0}} \sqrt{R^{2}-d^{2}}$
2.4.3 $\overrightarrow{\boldsymbol{E}}=\frac{\rho \overrightarrow{\boldsymbol{r}}}{2 \varepsilon_{0}}$
2.4 .4 a) $0<r<R, E=0 ;$
$R<r<2 R, E=k_{C} \frac{Q}{r^{2}} ; r>2 R$,
$\mathbf{2 . 5 . 4}$ a) (i) 0 , (ii) 0 , (iii) $k_{C} \frac{2 q}{r^{2}}$,
(iv) $0,\left(\right.$ v) $k_{C} \frac{6 q}{r^{2}} \quad$ b) i) $0,($ ii $)+2 q$,
(iii) $-2 q$, (iv) $+6 q$
2.5.5 $3.33 \cdot 10^{3} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}$ out of the sides parallel to $x y$ - or $y z$ - planes; $4.33 \cdot 10^{3} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}$ out of the side perpendicular to $-x$-axis;
$2.33 \cdot 10^{3} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}$ out of the side perpendicular to $+x$-axis

Chapter 3
3.1.1 - 81 nJ
3.1.2 $V_{C}>V_{A}, V_{B}<V_{A}, V_{D}=r_{2}=7.39 \mathrm{~cm}, r_{3}=20.09 \mathrm{~cm}$ $V_{A}$
3.1.3 b) $1.2 \cdot 10^{7} \mathrm{~m} / \mathrm{s}$
3.1.4 a) $E_{x}=0, E_{y}=(C z-B)$, $E_{z}=(C y-2 A z)$
b) $\forall x, y=-2 A B / C^{2}, z=B / C$
3.1 .5 a) $22.0 \mathrm{~cm} \quad$ b) 6.6 nC
c) 2.0 cm or $1.62 \mathrm{~m} \quad$ d) 0.444 nC or 36.0 nC
e) No
3.2.1 $2.7 \cdot 10^{-13} \mathrm{~J}=1.69 \mathrm{MeV}$
3.2.2 a) $0 \quad$ b) $-60 \mu \mathrm{~J} \quad$ c) $30 \mu \mathrm{~J}$
3.2 .3 a) $3.50 \cdot 10^{-8} \mathrm{C} / \mathrm{m} \quad$ b) less then 630 V c) 0
3.2 .4 a) $0 \quad$ b) $k_{C} \frac{q}{r^{2}} \frac{\overrightarrow{\boldsymbol{r}}}{r}$
$\mathbf{3 . 4 . 4}$ a) $1.38 \mathrm{kV} \quad$ b) 0
3.4 .5 b) $r_{1}=2.72 \mathrm{~cm}$,
c) No, they get farther apart
3.5.1 $22.8 k_{C} \frac{Q^{2}}{s}$
3.5.2 b) $\frac{2 k_{C} q}{\sqrt{x^{2}+a^{2}}}$ d) 0
3.5.3 a) $8.00 \mathrm{kV} / \mathrm{m} \quad$ b) $19.2 \mu \mathrm{~N}$
c) $0.864 \mu \mathrm{~J}$ d) $-0.864 \mu \mathrm{~J}$
3.5.4 $V=k_{C} \frac{q\left(r_{a}-r_{b}\right)}{r_{a} r_{b}}$
3.5.5 a) (i) $V_{a b}=2 k_{C} \lambda \ln (b / a)$
(ii) $V(r)=2 k_{C} \lambda \ln (b / r)$
(iii) $0 \quad$ b) $E(r)=\frac{V_{a b}}{r \ln (a / b)}$

## Chapter 4

3.2 .5 a) $-32.0 \mu \mathrm{~J} \quad$ b) -4.00 kV
c) $80 \mathrm{kV} / \mathrm{m}$
3.3.1 a) $20 \mathrm{~m} / \mathrm{s}$ b) 0.357 m
4.1.1 a) 500 V
b) $50.0 \mathrm{~cm}^{2}$
c) $2.82 \cdot 10^{6} \mathrm{~V} / \mathrm{m}$
3.3.2 $0.8 \cdot k_{C} \frac{q}{R}$
d) $25.0 \cdot 10^{-6} \mathrm{C} / \mathrm{m}^{2}$
3.3.3 a) -57.6 V
b) -72.0 V
c) $36.0 \mu \mathrm{~J}$
3.3.4 (i) $k_{C} Q\left(\frac{1}{a}-\frac{1}{b}\right)$
4.1.2 a) $q_{4}=195 \mu \mathrm{C}$,
$q_{1}=130 \mu \mathrm{C}, q_{2}=q_{3}=65 \mu \mathrm{C}$
b) $V_{2}=V_{3}=13 \mathrm{~V}, V_{1}=26 \mathrm{~V}$, $V_{4}=39 \mathrm{~V}$
(ii) $k_{C} Q\left(\frac{1}{r}-\frac{1}{b}\right)$
(iii) 0
c) $V_{a d}=26 \mathrm{~V}$
4.1.3 $36.0 \mu \mathrm{~J}$
$\mathbf{3 . 3 . 5}$ a) $2.0 \cdot 10^{5} \mathrm{~V} / \mathrm{m}^{4 / 3}$
4.1.4 $0.15 \mathrm{~m}^{2}$
b) $E=-\frac{4}{3} C x^{1 / 3} \quad$ c) $6.77 \cdot 10^{-15} \mathrm{~N}$
3.4.1 a) $1.00 \mathrm{~m} / \mathrm{s}$ b) No change
3.4.2 a) $0.36 \mu \mathrm{~J}$ b) 7.4 cm
$\mathbf{3 . 4 . 3}$ a) $-5.824 \cdot k_{C} \frac{q^{2}}{d}$
4.1.5 $48 \mu \mathrm{~F}$
4.2.1 a) $120 \mu \mathrm{C}$
b) $60 \mu \mathrm{C}$
c) $480 \mu \mathrm{C}$
4.2.2 $V_{2}=2.00 \mathrm{~V}, V_{3}=3.00 \mathrm{~V}$
4.2.3 a) $\frac{q^{2} x}{2 \varepsilon_{0} A}$
b) $\frac{q^{2}}{2 \varepsilon_{0} A} d x \quad$ c) $\frac{q^{2}}{2 \varepsilon_{0} A}$
4.2 .4 a) $45 \cdot 10^{-12} \mathrm{C}$
b) $12.0 \mathrm{pF}, 3.75 \mathrm{~V}$
c) $6.66 \cdot 10^{-9} \mathrm{~J}$
4.2 .5 a) 340 J
b) 68 mF
4.3.1 a) 5.56 pF
b) 3.6 V
4.3.2 $\frac{\varepsilon_{0}\left(A_{1}+A_{2}\right)}{d}$
4.3 .34 .82 kg
4.3 .4 a) $1.0 \cdot 10^{5} \mathrm{~V}$
b) $2.8 \cdot 10^{-4} \mathrm{C} / \mathrm{m}^{2}$,
$4.6 \cdot 10^{-4} \mathrm{C} / \mathrm{m}^{2}$,
4.3 .5 a) $32.0 \cdot 10^{-12} \mathrm{~J}$
b) $16.0 \cdot 10^{-12} \mathrm{~J}$
4.4 .1 a) 20 pF
b) 6.00 cm
c) $6.25 \mathrm{kV} / \mathrm{m}$
4.4.2 $7.5 \mu \mathrm{C}$
4.4 .3 a) $1.0 \cdot 10^{-3} \mathrm{C}$
b) $2.5 \cdot 10^{-4} \mathrm{C}, 7.5 \cdot 10^{-4} \mathrm{C}$
c) 0.5 J d$) 0.125 \mathrm{~J}, 0.375 \mathrm{~J}$
e) 100 V
4.4 .4 a) 30.0 V
b) 1.667
4.4 .5 a) $6.00 \cdot 10^{-5} \mathrm{C}$
b) $1.80 \cdot 10^{-3} \mathrm{~J}$
c) 9.0 V
d) $8.1 \cdot 10^{-4} \mathrm{~J}$
4.5.1 a) $5.0 \cdot 10^{-9} \mathrm{~F} \quad$ b) 800 V
4.5.2 $2.5 \mu \mathrm{~F}$
4.5.3 a) $2.67 \mathrm{nC} \quad$ b) 2.72
4.5.4 а) $1.80 \mathrm{~mJ}, 9.00 \mathrm{~mJ}$
b) 7.2 mJ , increase
4.5.5 $\frac{\varepsilon_{0} A}{d} \cdot \frac{2 \varepsilon_{1} \varepsilon_{2}}{\varepsilon_{1}+\varepsilon_{2}}$

## Chapter 5

5.1.1 a) $3.0 \cdot 10^{19} \mathrm{~s}^{-1}$
b) $1.53 \cdot 10^{6} \mathrm{~A} / \mathrm{m}^{2}$
c) $1.12 \cdot 10^{-4} \mathrm{~m} / \mathrm{s}$
5.1 .2 a) $5.81 \cdot 10^{-2} \mathrm{~V} / \mathrm{m}$
b) $6.46 \cdot 10^{-4} \Omega \quad$ c) 5.81 mV
5.1 .3 a) 1.0 A , b) 2.0 A c) 26.0 V
5.1.4 a) $50.0 \mathrm{~W} \quad$ b) 0.417 A
5.1.5 $65 \cdot 10^{3} \mathrm{~S}$
5.2 .1 a) $51 \mathrm{k} \Omega \quad$ b) $13 \mathrm{k} \Omega$
5.2.2 $9.00 \mathrm{~V}, 2.00 \Omega, 1.00 \Omega$
5.2.3 560 s
5.2.4 a) $2.00 \Omega$ b) $4.00 \Omega$
c) $6.00 \Omega$
$\mathbf{5 . 2 . 5}$ a) 3.00 A b) $2.00 \Omega$
c) $38.0 \mathrm{~V} \mathrm{d)} 4.33 \mathrm{~A}$
5.3.1 a) 13.6 A b) $16.2 \Omega$
5.3 .2 а) $I_{1}=1.75 \mathrm{~A}$,
$I_{2}=I_{3}=I_{4}=0.583 \mathrm{~A}$
b) $P_{1}=9.19 \mathrm{~W}$,
$P_{2}=P_{3}=P_{4}=1.02 \mathrm{~W}$
c) $I_{1}=1.56 \mathrm{~A}, I_{2}=I_{3}=0.78 \mathrm{~A}$
d) $P_{1}=7.30 \mathrm{~W}, P_{2}=P_{3}=1.82 \mathrm{~W}$
$\mathbf{5 . 3 . 3}$ a) $32 \Omega$ b) No
5.3.4 a) 140 C b) 14 A
5.3.5 $\frac{\rho h}{\pi r_{1} r_{2}}$
5.4 .1 a) $3.90 \mathrm{MJ} \quad$ b) 3900 s
$\mathbf{5 . 4 . 2}$ a) 1.36 A , clockwise
b) 25.9 V c) 1.44 V
$\mathbf{5 . 4 . 3}$ a) $1.09 \mu \mathrm{~F} \quad$ b) 2.73 s
5.4 .4 а) $1.25 \Omega$
b) 4.37 V
$\mathbf{5 . 4 . 5}$ a) 54 mW
b) 3.5 mW
5.5.1 a) $1.19 \mathrm{~A} / \mathrm{m}^{2}$
b) $7.44 \cdot 10^{12} \mathrm{~m}^{-3} \quad$ c) $6.44 \cdot 10^{9} \mathrm{~s}$
5.5.2 a) $161 \Omega$
b) 1.36 A
c) 248 W
5.5 .3 a) $I_{1}=0.207 \mathrm{~A}$,
$I_{2}=0.116 \mathrm{~A}, I_{3}=0.323 \mathrm{~A}$
b) $P_{1}=1.07 \mathrm{~W}, P_{2}=0.135 \mathrm{~W}$,
$P_{3}=1.56 \mathrm{~W}$
5.5.4 $480 \mu \mathrm{C}$
$\mathbf{5 . 5 . 5}$ a) $I=I_{A}\left(1+\frac{R_{A}}{r+R}\right)$
b) $53.5 \mathrm{~m} \Omega$

Chapter 6
6.1.1 a) $-(7.2 \overrightarrow{\boldsymbol{k}}) \cdot 10^{-4} \mathrm{~N}$
b) $(7.2 \overrightarrow{\boldsymbol{i}}+7.56 \overrightarrow{\boldsymbol{j}}) \cdot 10^{-4} \mathrm{~N}$
6.1.2 a) $4.1 \cdot 10^{-21} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}$
b) $1.64 \cdot 10^{-21} \mathrm{~J} \cdot \mathrm{~s}$
$\mathbf{6 . 1 . 3}$ a) $7.2 \overrightarrow{\boldsymbol{i}} \mathrm{kV} / \mathrm{m}$
b) $-7.2 \overrightarrow{\boldsymbol{i}} \mathrm{kV} / \mathrm{m}$
6.1.4 $F=7.16 \cdot 10^{-19} \mathrm{~N}$
6.1 .5 a) $5.0 \cdot 10^{-3} \mathrm{~A} \cdot \mathrm{~m}^{2}$
b) $4.0 \cdot 10^{-3} \mathrm{~N} \cdot \mathrm{~m}$
6.2.1 1.95 T , east
6.2.2 $8.0 \cdot 10^{-10} \mathrm{~N}$, south
6.2.3 a) south b) west c) $30^{\circ}$ west of north $7.2 \cdot 10^{-3} \mathrm{~N}$
$\mathbf{6 . 2 . 4}$ a) 5.06 cm
b) 19.9 ns
6.2.5 $6.28 \cdot 10^{-5} \mathrm{~N} \cdot \mathrm{~m}$
6.3.1 a) positive b) 50 mN
6.3.2 a) $8.0 \cdot 10^{5} \mathrm{~m} / \mathrm{s} \quad$ b) 26.2 ns
c) 6.68 kV
6.3.3 10 A
6.3 .4 a) $4.8 \cdot 10^{7} \mathrm{rad} / \mathrm{s}$
b) $4.8 \cdot 10^{7} \mathrm{~m} / \mathrm{s}$
6.3.5 a) east b) 0.25 T
6.4.1 $-0.33 \overrightarrow{\boldsymbol{k}} \mathrm{~m} / \mathrm{s}^{2}$
6.4.2 1.36 cm
6.4 .3 a) $5.12 \cdot 10^{-3} \mathrm{~N} \cdot \mathrm{~m}$
b) $2.56 \cdot 10^{-2} \mathrm{~A} \cdot \mathrm{~m}^{2}$
c) $6.89 \cdot 10^{-3} \mathrm{~N} \cdot \mathrm{~m}$
6.4.4 a) $v=2 E_{K} /(q B R)$
b) $m=(q B R)^{2} / 2 E_{K}$
6.4.5 $\mathrm{m}^{\prime} / \mathrm{m}=8$
6.5.1 $1.5 \cdot 10^{6} \mathrm{~m} / \mathrm{s}$
$\mathbf{6 . 5 . 2}$ a) $3.2 \cdot 10^{6} \mathrm{~m} / \mathrm{s} \quad$ c) 3.6 mm
6.5.3 a) $1.52 \cdot 10^{-16} \mathrm{~s}$ b) 1.05 mA
c) $9.27 \cdot 10^{-24} \mathrm{~A} \cdot \mathrm{~m}^{2}$
6.5.4 70 mT
6.5.5 $6.28 \cdot 10^{-3} \mathrm{~N} \cdot \mathrm{~m}$

## Chapter 7

7.1.1 a) $8 \pi \cdot 10^{-7} \mathrm{~T} \cdot \mathrm{~m} \quad$ b) 0
c) $16 \pi \cdot 10^{-7} \mathrm{~T} \cdot \mathrm{~m} \quad$ d) 0
7.1.2 a) $-1.28 \cdot 10^{-5} \overrightarrow{\boldsymbol{k}} \mathrm{~T} \quad$ b) 0
c) $1.28 \cdot 10^{-5} \boldsymbol{i} \mathrm{~T} \quad$ d) $6.4 \cdot 10^{-6} \overrightarrow{\boldsymbol{i}} \mathrm{~T}$
7.1.3 $2.8 \cdot 10^{-6} \mathrm{~T}$, north
7.1.4 a) $4.0 \cdot 10^{-6} \mathrm{~T}$
b) $8.25 \cdot 10^{-6} \mathrm{~T}$
7.1.5 100 A
7.2.1 a) 0
b) $-8.00 \cdot 10^{-7} \overrightarrow{\boldsymbol{k}} \mathrm{~T}$
c) $-2.83 \cdot 10^{-7} \boldsymbol{k} \mathrm{~T}$
d) $8.00 \cdot 10^{-7} \boldsymbol{j} \mathbf{~ T}$
7.2.2 a) 1.0 cm from lower current, between wires b) 2.0 cm from lower current, outside of wires; both cases along the line parallel to wires in their plain
7.2 .3 a) $\frac{\mu_{0} I}{2 \pi r}$
b) 0
7.2 .4 a) $1.0 \cdot 10^{-5} \mathrm{~T}$
b) $5.0 \cdot 10^{-5} \mathrm{~N} / \mathrm{m}$
c) $5.0 \cdot 10^{-5} \mathrm{~T}$
d) $5.0 \cdot 10^{-5} \mathrm{~N} / \mathrm{m}$
7.2.5 $\frac{\mu_{0} I}{4 R}$, from the paper
7.3.1 $2.5 \cdot 10^{-5} \mathrm{~T}$
$\mathbf{7 . 3 . 2}$ a) $0 \quad$ b) $0 \quad$ c) $4.0 \cdot 10^{-3} \mathrm{~T}$
7.3 .3 a) $\frac{\mu_{0} I}{2 \pi r} \quad$ b) $\frac{\mu_{0} I}{\pi r}$
$\mathbf{7 . 3 . 4}$ a) $3.2 \mathrm{~T} \quad$ b) 1.8 T
7.3 .5 a) 7.5 A b) the same
c) repel with $6.0 \cdot 10^{-4} \mathrm{~N} / \mathrm{m}$
7.4.1 a) $28.3 \mu \mathrm{~T}$ toward the page
b) $24.7 \mu \mathrm{~T}$
7.4.2 a) 5.0 A b) the same
7.4.3 $r=5.0 \mathrm{~cm} \& r=20.0 \mathrm{~cm}$
7.4.4 $\frac{1}{2} q \omega R^{2} B$
7.4.5 $\frac{\mu_{0} I}{2 r}\left(\frac{1}{4}+\frac{1}{\pi}\right)$
7.5.1 $1.0 \cdot 10^{-3} \mathrm{~T}$
7.5.2 a) $1000 \mathrm{~m}^{-1} \quad$ b) 31.4 m
7.5.3 $\frac{\mu_{0} I}{4 \pi x}$
7.5.4 $4.0 \cdot 10^{-7} \mathrm{~T}$, toward the page
7.5.5 $y=0.4 \mathrm{~m}$

## Chapter 8

8.1.1 a) 2.5 Wb
b) 10.0 V
8.1.2 clockwise
8.1.3 a) $48.0 \mu \mathrm{~V} \quad$ b) west end is positive
8.1.4 $5.0 \cdot 10^{-2} \mathrm{~m}^{2}$
8.1.5 $E=\frac{1}{2} \beta R \mu_{0} n I_{0} e^{-\beta t}$ in the same direction as the current in the solenoid
8.2.1 0.10 T
8.2.2 24.0 V
8.2.3 $40 \mathrm{~m} / \mathrm{s}=144 \mathrm{~km} / \mathrm{h}$
8.2.4 a) 6.0 V
b) counterclockwise c) 1.5 N to the left d) $P_{\text {mech }}=P_{\text {elec }}=12.0 \mathrm{~W}$
8.2.5 $0 \quad$ a) $3.14 \cdot 10^{-4} \mathrm{~V} / \mathrm{m}$
b) $6.25 \cdot 10^{-4} \mathrm{~V} / \mathrm{m}$
8.3.1 $1.57 \cdot 10^{-3} \mathrm{~V}$
8.3 .2 e) $5.0 \cdot 10^{-7} \mathrm{~V}$
8.3.3 6.0 T
8.3.4 9.87 mV
8.3.5 5.0 A/s
8.4 .1 a) $7.5 \cdot 10^{-3} \mathrm{~V}$ b) clockwise
8.4.2 2.0 A
$\mathbf{8 . 4 . 3}$ a) 314 V b) 0 V
8.4 .48 .0 m
$\mathbf{8 . 4 . 5}$ a) $9.87 \cdot 10^{-3} \mathrm{~V} / \mathrm{m}$
$\begin{array}{llll}\text { b) } 9.87 \cdot 10^{-3} \mathrm{~V} / \mathrm{m} & \mathbf{9 . 4 . 2} & \text { a) } \frac{\mu_{0} I^{2}}{8 \pi^{2} r^{2}} & \text { b) } 2.4 \mathrm{~mm}\end{array} l$
8.5.1 a) clockwise b) counter-
$\mathbf{9 . 4 . 3}$ a) $4.42 \cdot 10^{-8} \mathrm{~J} / \mathrm{m}^{3}$
clockwise c) no current induced
b) $1.43 \cdot 10^{-3} \mathrm{~J} / \mathrm{m}^{3}$
$\mathbf{9 . 4 . 4}$ a) $9.05 \mathrm{~m}^{3} \quad$ b) 3.0 T
8.5.2 31.4 mA
8.5.3 a) $10^{-4} \cos (100 \pi t) \mathrm{Wb}$
b) $3.14 \cdot 10^{-2} \sin (100 \pi t) \mathrm{V}$
c) $1.57 \cdot 10^{-2} \sin (100 \pi t) \mathrm{A}$
d) $4.93 \cdot 10^{-4} \sin ^{2}(100 \pi t) \mathrm{W}$
e) $1.57 \cdot 10^{-6} \sin ^{2}(100 \pi t) \mathrm{N} \cdot \mathrm{m}$
9.4 .5 a) $\frac{\mu_{0} N I h}{2 \pi} \ln \left(\frac{R+w}{R}\right)$
b) $\frac{\mu_{0} N^{2} h}{2 \pi} \ln \left(\frac{R+w}{R}\right)$
8.5.4 $50.0 \mathrm{rad} / \mathrm{s}$
8.5.5 31.4 mV

Chapter 9
9.1.1 a) 40.0 mT
b) $99 \%$
b) $10^{-7} \mathrm{~J} / \mathrm{m}$
$\mathbf{9 . 5 . 3}$ a) $28.8 \mathrm{mH} \quad$ b) 1.44 ms
c) $72.1 \mu \mathrm{~s}$
9.1.3 240
$\begin{array}{lll}9.1 .4 & \text { a) } 6.37 \mathrm{MJ} / \mathrm{m}^{3} & \text { b) } 5.12 \mathrm{~kJ}\end{array}$
9.1.5 a) $0.4 \mathrm{mH} \quad$ b) 2.0 ms
9.2 .1 a) $2.0 \cdot 10^{3}$
b) $1.999 \cdot 10^{3}$
9.2.2 $4.55 \cdot 10^{2} \mathrm{~A} / \mathrm{m}$
9.2.3 5000
9.5.4 51.2 mJ
9.5.5 a) $9.87 \mathrm{mH} \quad$ b) $5.0 \mathrm{~A} / \mathrm{s}$
Chapter 10
$\mathbf{1 0 . 1 . 1}$ a) $13.3 \mathrm{mH} \quad$ b) 90.0 nC
c) $540 \mathrm{~nJ} \quad$ d) 9.0 mA
10.1.2 $2.5 \cdot 10^{5} \mathrm{rad} / \mathrm{s}$
9.2.4
a) $13.86 \mu \mathrm{~s}$
b) $24.56 \mu \mathrm{~s}$
10.1.3
a) $47.1 \Omega$
b) $637 \Omega$
9.2.5 $24.0 \Omega, 4.8 \mathrm{mH}$
c) $2.40 \mathrm{k} \Omega$
d) $2.33 \mathrm{k} \Omega$
e) $-14.2^{\circ}$
9.3.1 $10^{-2} \mathrm{~A} \cdot \mathrm{~m}^{2}$
$\mathbf{1 0 . 1 . 4}$ a) $80.0 \mathrm{~W} \quad$ b) 0.167 A
9.3.2 $31.4 \mathrm{~J} / \mathrm{m}^{3}$
c) $1.44 \mathrm{k} \Omega$
$\mathbf{9 . 3 . 3}$ a) 0.25 H
b) $4.0 \cdot 10^{-4} \mathrm{~Wb}$
10.1.5 81 V
9.3.4 a) 60 mT
b) $1.43 \mathrm{~kJ} / \mathrm{m}^{3}$
c) 9.0 mJ d) $45.0 \mu \mathrm{H}$
9.3 .5 a) $0.5 \mathrm{k} \Omega$
b) $4.0 \mu \mathrm{~s}$
10.2.1 0.4 A
$\mathbf{9 . 4 . 1}$ a) 3.14 T
b) $1.25 \cdot 10^{3} \mathrm{~A} / \mathrm{m}$
10.2 .2 a) $250 \mathrm{rad} / \mathrm{s}$ b) $346 \Omega$
$\mathbf{1 0 . 2 . 3}$ a) $3.0 \cdot 10^{-4} \mathrm{C} \quad$ b) 4.71 V
10.2.4 500 V
c) $\frac{\mu_{0} N^{2}}{2 \pi R} h w \quad$ d) 3.2 mH
9.5.1 999
9.5 .2 a) $\frac{\mu_{0} I^{2} r^{2}}{8 \pi^{2} a^{4}}$
10.2.5 a) 150 V
b) 150 V ,
$1500 \mathrm{~V}, 1500 \mathrm{~V}$
c) 22.5 W
10.5.5 a) $10^{3} \mathrm{~s}^{-1}$
b) 125 W
c) $951 \mathrm{~s}^{-1}, 1051 \mathrm{~s}^{-1}$ d) 10
10.3.1 $2.0 \mu \mathrm{~F}, 0.8 \mathrm{mH}$
$\mathbf{1 0 . 3 . 2}$ a) $5000 \mathrm{rad} / \mathrm{s}$
b) $4899 \mathrm{rad} / \mathrm{s} \quad$ c) $-2.0 \%$
10.3 .3 a) $1.0 \cdot 10^{3} \mathrm{rad}, 0.15 \mathrm{~A}$

## Chapter 11

11.1.1 5.0 s
11.1 .2 а) $4.0 \cdot 10^{10} \mathrm{~V} /(\mathrm{m} \cdot \mathrm{s})$
b) $4.0 \cdot 10^{-8} \mathrm{~T}$
11.1 .3 a) $375 \mathrm{~V} / \mathrm{m}, 12.5 \mu \mathrm{~T}$
b) $10^{15} \mathrm{~Hz}, 10^{-15} \mathrm{~s}, 3.0 \cdot 10^{-7} \mathrm{~m}$
c) $3.0 \cdot 10^{8} \mathrm{~m} / \mathrm{s}$
11.1.4 $1.50 \cdot 10^{-8} \mathrm{~T}$, in the $-x$ direction
11.1 .5 b) $(-32 \overrightarrow{\boldsymbol{i}}+8 \overrightarrow{\boldsymbol{j}}+2 \overrightarrow{\boldsymbol{k}}) \mathrm{W} / \mathrm{m}^{2}$
11.2.1 6.25
11.2.2 $2.5 \cdot 10^{-18} \mathrm{~T}$
11.2 .3 a) $360 \mathrm{~V} / \mathrm{m}$
b) $1.14 \cdot 10^{-6} \mathrm{~J} / \mathrm{m}^{3} \quad$ c) $342 \mathrm{~W} / \mathrm{m}^{2}$
11.2.4 a) $+x$-direction b) $0.5 \mu \mathrm{~m}$
c) $\left(10^{-3} \mathrm{~T}\right) \overrightarrow{\boldsymbol{k}} \sin \left[\left(4 \pi \cdot 10^{6} \mathrm{rad} / \mathrm{m}\right) x-\right.$ $\left.-\left(12 \pi \cdot 10^{14} \mathrm{rad} / \mathrm{s}\right) t\right]$
$\mathbf{1 0 . 5 . 2}$ a) $10^{3} \mathrm{rad} / \mathrm{s} \quad$ b) 0.5 A
c) $1756 \mathrm{rad} / \mathrm{s} \& 890 \mathrm{rad} / \mathrm{s}$
d) $866 \mathrm{rad} / \mathrm{s} \mathrm{d}$ ) (i) $5.0 \mathrm{~A}, 87 \mathrm{rad} / \mathrm{s}$;
(ii) $50 \mathrm{~A}, 8.7 \mathrm{rad} / \mathrm{s}$; (ii) 500 A ,
$0.9 \mathrm{rad} / \mathrm{s}$
10.5 .3 a) $\frac{1}{\sqrt{L C}}$
b) $\sqrt{\frac{1}{L C}-\frac{R^{2}}{2 L^{2}}}$
c) $\sqrt{\left(L C-\frac{R^{2} C^{2}}{2}\right)^{-1}}$
10.5.4 a) $43 \mathrm{~W} \quad$ b) 23 W
c) 484 W
$11.2 .5 \quad$ a) $1.0 \mu \mathrm{~T} \quad$ b) $2.0 \cdot 10^{-7} \mathrm{~m}$
c) $1.5 \cdot 10^{15} \mathrm{~Hz}$
11.3 .1 a) $6.0 \cdot 10^{14} \mathrm{~Hz}$
b) $1.5 \mathrm{kV} / \mathrm{m}$
c) $1.5 \mathrm{kV} / \mathrm{m} \sin \left[4 \pi \cdot 10^{6}\left(x-3 \cdot 10^{8} t\right)\right]$,
$5.0 \mu \mathrm{~T} \sin \left[4 \pi \cdot 10^{6}\left(x-3 \cdot 10^{8} t\right)\right]$
11.3.2 a) - $z$-direction
b) $7.5 \cdot 10^{11} \mathrm{~Hz} \quad$ c) $(-1.8 \mathrm{~V} / \mathrm{m}) \overrightarrow{\boldsymbol{i}} \times$
$\times \sin \left[1.57 \cdot 10^{4} x+4.71 \cdot 10^{12} t\right]$,
11.3 .3 a) $55.7 \mathrm{~A} / \mathrm{m}^{2}$
b) $6.29 \cdot 10^{12} \mathrm{~V} /(\mathrm{m} \cdot \mathrm{s})$
c) $0.70 \mu \mathrm{~T}$
d) $0.35 \mu \mathrm{~T}$
11.3.4 $4 \cdot 10^{8} \mathrm{~m} / \mathrm{s}>c$
11.3.5 $5.0 \cdot 10^{-4} \mathrm{~s}<10^{-2} \mathrm{~s}$, radio $\mathbf{1 1 . 4 . 4} 3.33 \cdot 10^{-6} \mathrm{~J} / \mathrm{m}^{3}$
audience first

## 11.4 .1 a) 5.00 nC

b) $7.00 \cdot 10^{-3} \mathrm{C} / \mathrm{s} \quad$ c) 7.00 mA
11.4 .2 a) $7.50 \cdot 10^{14} \mathrm{~Hz}$
b) $5.0 \mathrm{pT} \quad$ c) $\left(1.5 \cdot 10^{-3} \mathrm{~V} / \mathrm{m}\right) \overrightarrow{\boldsymbol{k}} \times$ $\times \sin \left[5 \pi \cdot 10^{6} y+15 \pi \cdot 10^{14} t\right]$,
$(5.0 \mathrm{pT}) \overrightarrow{\boldsymbol{i}} \sin \left[5 \pi \cdot 10^{6} y+15 \pi \cdot 10^{14} t\right]$
11.4 .3 a) $6.0 \cdot 10^{7} \mathrm{~m} / \mathrm{s}$
b) $6.0 \cdot 10^{5} \mathrm{~m}$
c) $5.0 \cdot 10^{-11} \mathrm{~T}$
d) $23.9 \cdot 10^{-9} \mathrm{~W} / \mathrm{m}^{2}$
11.4.5 $2.24 \cdot 10^{-10} \mathrm{~T}$
11.5.1 a) $3.00 \cdot 10^{9} \mathrm{~V} \cdot \mathrm{~m} / \mathrm{s}$
b) $2.655 \cdot 10^{-2} \mathrm{~A}$
11.5 .2 a) $1.01 \mathrm{kV} / \mathrm{m} \quad$ b) $3.4 \mu \mathrm{~T}$
11.5.3
a) $-z$
b) $-z$
c) $+y$
d) $+y$
11.5.4 $7.96 \cdot 10^{-3} \mathrm{~W} / \mathrm{m}^{2}$
11.5.5 $3.82 \cdot 10^{26} \mathrm{~W}$

## APPENDIX

## Universal physical constants

$e=1.602176565 \cdot 10^{-19} \mathrm{C} \quad$ Elementary charge $\quad e \approx 1.60 \cdot 10^{-19} \mathrm{C}$
$c=299792458 \mathrm{~m} / \mathrm{s} \quad$ Speed of light $\quad c \approx 3.00 \cdot 10^{8} \mathrm{~m} / \mathrm{s}$ in vacuum
$h=6.62606957 \cdot 10^{-34} \mathrm{~J} \cdot \mathrm{~s} \quad$ Plank's constant $\quad h \approx 6.63 \cdot 10^{-34} \mathrm{~J} \cdot \mathrm{~s}$
$\mu_{0}=4 \pi \cdot 10^{-7} \mathrm{H} / \mathrm{m} \quad$ Magnetic constant $\quad \mu_{0} \approx 1.26 \cdot 10^{-6} \mathrm{H} / \mathrm{m}$
$\varepsilon_{0}=\frac{1}{c^{2} \mu_{0}}$
Electric constant $\quad \varepsilon_{0} \approx 8.85 \cdot 10^{-12} \mathrm{~F} / \mathrm{m}$
$k_{C}=\frac{1}{4 \pi \varepsilon_{0}}$
Coulomb's constant $k_{C} \approx 8.99 \cdot 10^{9} \mathrm{Nm}^{2} / \mathrm{C}^{2}$
$\hbar=\frac{h}{2 \pi}$
Reduced Plank's $\quad \hbar \approx 1.05 \cdot 10^{-34} \mathrm{~J} \cdot \mathrm{~s}$ constant
$m_{e}=9.10938291 \cdot 10^{-31} \mathrm{~kg} \quad$ Rest mass of $\quad m_{e} \approx 9.11 \cdot 10^{-31} \mathrm{~kg}$ electron
$m_{p}=1.672621777 \cdot 10^{-27} \mathrm{~kg}$ Rest mass of proton $m_{p} \approx 1.67 \cdot 10^{-27} \mathrm{~kg}$
$\mu_{B}=\frac{e \hbar}{2 m_{e}}$
Bohr magneton $\quad \mu_{B} \approx 9.27 \cdot 10^{-24} \mathrm{~A} \cdot \mathrm{~m}^{2}$

## Some Electric \& Magnetic Quantities and their Units

Electric Charge
Electric Field
Electric Potential
Electric Capacitance
Electric Current
Electric Resistance
Power
Magnetic Flux
Magnetic Field Inductance
$q \quad \mathrm{C}=\mathrm{A} \cdot \mathrm{s} \quad$ coulomb
E V/m $\mathrm{m} / \mathrm{C}$
$V \quad \mathrm{~V}=\mathrm{J} / \mathrm{C} \quad$ volt
$C \quad \mathrm{~F}=\mathrm{V} / \mathrm{C}$ farad
$I \quad \mathrm{~A}=\mathrm{C} / \mathrm{s} \quad$ ampere
$R \quad \Omega=\mathrm{V} / \mathrm{A} \quad$ ohm
$P \quad \mathrm{~W}=\mathrm{V} \cdot \mathrm{A} \quad$ watt
$\Phi_{m} \quad \mathrm{~Wb}=\mathrm{V} \cdot \mathrm{s} \quad$ weber
$B \quad \mathrm{~T}=\mathrm{Wb} / \mathrm{m}^{2}$ tesla
$L \quad \mathrm{H}=\mathrm{Wb} / \mathrm{A}$ henry

Relative Dielectric Permittivity of the Substances

| Material | Relative di- <br> alectic per- <br> mittivity $\varepsilon_{r}$ | Material | Relative di- <br> alectic per- <br> mittivity $\varepsilon_{r}$ |
| :--- | :--- | :--- | :--- |
| Bakelite | 4.9 | Polyvinyl chloride | 3.4 |
| Mylar | 3.2 | Porcelain | 6 |
| Neoprene rubber | 6.7 | Pyrex glass | 5.6 |
| Nylon | 3.4 | Silicone oil | 2.5 |
| Paper | 3.7 | Strontium titanate | 233 |
| Polystyrene | 2.6 | Water | 80 |
| Teflon | 2.1 | Air (dry) | 1.0006 |

## Electrical Resistivity of the Substances

| Material | Electrical resis- <br> tivity $\rho, \Omega \cdot \mathrm{m}$ | Material | Electrical resis- <br> tivity $\rho, \Omega \cdot \mathrm{m}$ |
| :--- | :--- | :--- | :--- |
| Silver | $1.47 \cdot 10^{-8}$ | Copper | $1.7 \cdot 10^{-8}$ |
| Gold | $2.44 \cdot 10^{-8}$ | Aluminum | $2.82 \cdot 10^{-8}$ |
| Tungsten | $5.6 \cdot 10^{-8}$ | Iron | $10 \cdot 10^{-8}$ |
| Platinum | $11 \cdot 10^{-8}$ | Lead | $22 \cdot 10^{-8}$ |
| Nichrome | $1.00 \cdot 10^{-6}$ | Carbon | $3.5 \cdot 10^{-5}$ |
| Germanium | 0.46 | Silicon | $2.3 \cdot 10^{3}$ |
| Glass | $10^{10}$ to $10^{14}$ | Quartz (fused) | $7.5 \cdot 10^{16}$ |

## Magnetic Susceptibility of the Substances

| Material | Magnetic <br> susceptibility | Material | Magnetic <br> susceptibility |
| :--- | :--- | :--- | :--- |
| $\varkappa_{m}$ |  | $\varkappa_{m}$ |  |
| Copper | $-1.0 \cdot 10^{-5}$ | Mercury | $-2.9 \cdot 10^{-5}$ |
| Silver | $-2.6 \cdot 10^{-5}$ | Bismuth | $-1.7 \cdot 10^{-4}$ |
| Lead | $-1.8 \cdot 10^{-5}$ | Sodium chloride | $-1.4 \cdot 10^{-5}$ |
| Oxygen gas | $0.19 \cdot 10^{-5}$ | Sodium | $0.72 \cdot 10^{-5}$ |
| Aluminum | $2.2 \cdot 10^{-5}$ | Platinum | $2.6 \cdot 10^{-4}$ |
| Tungsten | $6.8 \cdot 10^{-5}$ | Uranium | $4 \cdot 10^{-4}$ |
| Iron | $4.5 \cdot 10^{3}$ | Nickel | $6 \cdot 10^{2}$ |
| Cobalt | $2.5 \cdot 10^{2}$ | Permalloy | $8 \cdot 10^{3}$ |
| $\mu-$ metal | $1 \cdot 10^{5}$ | Metglas | $1 \cdot 10^{6}$ |

## Vector Algebra

$$
\begin{array}{r}
\overrightarrow{\boldsymbol{a}}=a_{x} \overrightarrow{\boldsymbol{i}}+a_{y} \overrightarrow{\boldsymbol{j}}+a_{z} \overrightarrow{\boldsymbol{k}} \\
\overrightarrow{\boldsymbol{b}}=b_{x} \overrightarrow{\boldsymbol{i}}+b_{y} \overrightarrow{\boldsymbol{j}}+b_{z} \overrightarrow{\boldsymbol{k}}
\end{array}
$$

Dot (scalar) product of vectors

$$
\begin{aligned}
& \overrightarrow{\boldsymbol{a}} \cdot \overrightarrow{\boldsymbol{b}} \stackrel{\text { def }}{=}|\overrightarrow{\boldsymbol{a}}||\overrightarrow{\boldsymbol{b}}| \cos (\angle \overrightarrow{\boldsymbol{a}} \overrightarrow{\boldsymbol{b}})=\overrightarrow{\boldsymbol{b}} \cdot \overrightarrow{\boldsymbol{a}} \\
& \vec{i} \cdot \vec{i}=\vec{j} \cdot \vec{j}=\vec{k} \cdot \vec{k}=1 \\
& \vec{i} \cdot \vec{j}=\vec{j} \cdot \vec{k}=\vec{k} \cdot \vec{i}=0 \\
& \overrightarrow{\boldsymbol{a}} \cdot \overrightarrow{\boldsymbol{b}}=a_{x} b_{x}+a_{y} b_{y}+a_{z} b_{z}
\end{aligned}
$$

Cross (vector) product of vectors

$$
\begin{gathered}
|\overrightarrow{\boldsymbol{a}} \times \overrightarrow{\boldsymbol{b}}| \stackrel{\text { def }}{=}|\overrightarrow{\boldsymbol{a}}||\overrightarrow{\boldsymbol{b}}| \sin (\angle \overrightarrow{\boldsymbol{a}} \overrightarrow{\boldsymbol{b}}) \\
\overrightarrow{\boldsymbol{i}} \times \overrightarrow{\boldsymbol{i}}=\overrightarrow{\boldsymbol{j}} \times \overrightarrow{\boldsymbol{j}}=\overrightarrow{\boldsymbol{k}} \times \overrightarrow{\boldsymbol{k}}=0 \\
\overrightarrow{\boldsymbol{a}} \perp(\overrightarrow{\boldsymbol{a}} \times \overrightarrow{\boldsymbol{b}}) \perp \overrightarrow{\boldsymbol{b}} \\
\overrightarrow{\boldsymbol{a}} \times \overrightarrow{\boldsymbol{b}}=-\overrightarrow{\boldsymbol{b}} \times \overrightarrow{\boldsymbol{a}} \\
\overrightarrow{\boldsymbol{a}} \times \overrightarrow{\boldsymbol{b}}=\overrightarrow{\boldsymbol{i}}\left(a_{y} b_{z}-a_{z} b_{y}\right)+\overrightarrow{\boldsymbol{j}}\left(a_{z} b_{x}-a_{x} b_{z}\right)+\overrightarrow{\boldsymbol{k}}\left(a_{x} b_{y}-a_{y} b_{x}\right) \\
\overrightarrow{\boldsymbol{a}} \times \overrightarrow{\boldsymbol{j}}=\left|\begin{array}{ccc}
\overrightarrow{\boldsymbol{i}} & \overrightarrow{\boldsymbol{j}} & \overrightarrow{\boldsymbol{k}} \\
a_{x} & a_{y} & a_{z} \\
b_{x} & b_{y} & b_{z}
\end{array}\right|
\end{gathered}
$$

Scalar triple product

$$
\vec{a} \cdot(\vec{b} \times \vec{c})=\vec{b} \cdot(\vec{c} \times \vec{a})=\vec{c} \cdot(\vec{a} \times \vec{b})=-\vec{c} \cdot(\vec{b} \times \vec{a})
$$

Vector triple product

$$
\vec{a} \times(\vec{b} \times \vec{c})=\vec{b}(\vec{a} \cdot \vec{c})-\vec{c}(\vec{a} \cdot \vec{b})
$$

## Vector Calculus

$$
\begin{gathered}
\overrightarrow{\boldsymbol{\nabla}} \stackrel{\text { def }}{=} \frac{\partial}{\partial x} \overrightarrow{\boldsymbol{i}}+\frac{\partial}{\partial y} \overrightarrow{\boldsymbol{j}}+\frac{\partial}{\partial z} \overrightarrow{\boldsymbol{k}} \\
\Delta \stackrel{\text { def }}{=} \nabla^{2}=\vec{\nabla} \cdot \overrightarrow{\boldsymbol{\nabla}}=\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+\frac{\partial^{2}}{\partial z^{2}}
\end{gathered}
$$

Gradient of the Scalar Field and Nabla Operator

$$
\begin{gathered}
\overrightarrow{\boldsymbol{E}}=-\overrightarrow{\boldsymbol{\nabla}} V=-\operatorname{grad} V=-\left(\frac{\partial V}{\partial x} \overrightarrow{\boldsymbol{i}}+\frac{\partial V}{\partial y} \overrightarrow{\boldsymbol{j}}+\frac{\partial V}{\partial z} \overrightarrow{\boldsymbol{k}}\right) \\
\overrightarrow{\boldsymbol{E}}=E_{x} \overrightarrow{\boldsymbol{i}}+E_{y} \overrightarrow{\boldsymbol{j}}+E_{z} \overrightarrow{\boldsymbol{k}}
\end{gathered}
$$

Divergence of the Vector Field and Laplace Operator

$$
\operatorname{div} \overrightarrow{\boldsymbol{E}} \stackrel{\text { def }}{=} \vec{\nabla} \cdot \overrightarrow{\boldsymbol{E}}=\frac{\partial E_{x}}{\partial x}+\frac{\partial E_{y}}{\partial y}+\frac{\partial E_{z}}{\partial z}
$$

$$
\Delta V \stackrel{\text { def }}{=} \operatorname{div}(\operatorname{grad} V)=(\vec{\nabla} \cdot \vec{\nabla}) V=\nabla^{2} V=\frac{\partial^{2} V}{\partial x^{2}}+\frac{\partial^{2} V}{\partial y^{2}}+\frac{\partial^{2} V}{\partial z^{2}}
$$

## Curl of the Vector Field

$$
\begin{gathered}
\operatorname{rot} \overrightarrow{\boldsymbol{E}} \stackrel{\text { def }}{=} \overrightarrow{\boldsymbol{\nabla}} \times \overrightarrow{\boldsymbol{E}}=\left|\begin{array}{ccc}
\overrightarrow{\boldsymbol{i}} & \overrightarrow{\boldsymbol{j}} & \overrightarrow{\boldsymbol{k}} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
E_{x} & E_{y} & E_{z}
\end{array}\right| \\
\operatorname{rot} \overrightarrow{\boldsymbol{E}}=\left(\frac{\partial E_{z}}{\partial y}-\frac{\partial E_{y}}{\partial z}\right) \overrightarrow{\boldsymbol{i}}+\left(\frac{\partial E_{x}}{\partial z}-\frac{\partial E_{z}}{\partial x}\right) \overrightarrow{\boldsymbol{j}}+\left(\frac{\partial E_{y}}{\partial x}-\frac{\partial E_{x}}{\partial y}\right) \overrightarrow{\boldsymbol{k}} \\
\operatorname{rot}(\operatorname{rot} \overrightarrow{\boldsymbol{H}})=\overrightarrow{\boldsymbol{\nabla}} \times(\overrightarrow{\boldsymbol{\nabla}} \times \overrightarrow{\boldsymbol{H}})=\operatorname{grad}(\operatorname{div} \overrightarrow{\boldsymbol{H}})-\Delta \overrightarrow{\boldsymbol{H}}
\end{gathered}
$$

Gauss-Ostrogradsky Divergence Theorem

$$
\oiint_{\partial V o l} \overrightarrow{\boldsymbol{E}} \cdot d \overrightarrow{\boldsymbol{A}}=\iiint_{V o l} \operatorname{div} \overrightarrow{\boldsymbol{E}} d V o l
$$

Stokes Theorem

$$
\oint_{\partial A} \overrightarrow{\boldsymbol{H}} \cdot d \overrightarrow{\boldsymbol{\ell}}=\iint_{A} \operatorname{rot} \overrightarrow{\boldsymbol{H}} \cdot d \overrightarrow{\boldsymbol{A}}
$$

## Bibliography

1. Sears and Zemansky's University Physics: with Modern Physics. 12th ed./Hugh D. Young, Roger A. Freedman; contributing author, A. Lewis Ford. - Pearson Education, Inc., publishing as Pearson Addison-Wesley. - 2008. - 1632 p.
2. Physics for Scientists and Engineers with Modern Physics. - Third ed. / Paul M. Fishbane, Stephen G. Gasiorowicz, Stephen T. Thornton. - Pearson Prentice Hall Pearson Education. Inc. - 2005. 1379 p.
3. Physics for Scientists and Engineers with Modern Physics. - 8th ed. / Raymond A. Serway, John W. Jewett - Brooks/Cole Cengage Leaning. - 2010. - 1558 p.
4. Okhrimovskyy, A. M., Mechanics and Thermodynamics: guidance manual for recitation / A. M. Okhrimovskyy, O. V. Podshyvalova. - Kharkiv: National Aerospace University "KhAI". - 2012. 72 p.
5. Okhrimovskyy, A. M., Physics: guidance manual for laboratory experiments / A. M. Okhrimovskyy, O. V. Podshyvalova. - Kharkiv: National Aerospace University "KhAI". - 2011. - 144 p.

# Охрімовський Андрій Михайлович Подшивалова Оксана Володимирівна Олійник Сергій Володимирович 

ЕЛЕКТРИКА I МАГНЕТИЗМ<br>(Англійською мовою)<br>Редактор Є.В. Пизіна<br>Технічний редактор Л. О. Кузьменко

Зв. план, 2013
Підписано до друку 20.05.2013
Формат 60x84 1/16. Папір офс. № 2. Офс. друк
Ум. друк. арк. 6,4. Обл.-вид. арк. 7,25. Наклад 300 пр.
Замовлення 183. Ціна вільна
Національний аерокосмічний університет ім. М. Є. Жуковського "Харківський авіаційний інститут" 61070, Харків-70, вул. Чкалова, 17
http://www.khai.edu
Видавничий центр "XAI" 61070, Харків-70, вул. Чкалова, 17 izdat@khai.edu

Свідоцтво про внесення суб'єкта видавничої справи до Державного реестру видавців, виготовлювачів і розповсюджувачів видавничої продукції сер. ДК № 391 від 30.03.2001

