DEVELOPMENT OF METHODS FOR SOLVING RELAXATION PROBLEMS OF OPTIMIZATION OF POLYEDRON-SPHERIAL CONFIGURATIONS

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The report is devoted to the study of nonlinear optimization methods on a permutation polyhedron, taking into account the specifics of the Euclidean set of permutations and the properties of the functions defined on them.

If $E \subset R^n$ a finite set of points of an arithmetic Euclidean space.

Consider the following discrete optimization problem:

$$f(x) \to \min, \quad (1)$$

$$g_i(x) \le 0,\ i \in J_k, \quad (2)$$

$$g_i(x) = 0,\ i \in J_m \setminus J_k, \quad (3)$$

$$x \in E, \quad (4)$$

where functions $f(x), g_i(x),\ i \in J_m$ are defined and continuously differentiable on E. Hereinafter we denote $J_l = \{1, \ldots, l\}$.

Consider as the set E the following combinatorial set of arithmetic Euclidean space. Let be $A = \{a_1, \ldots, a_n\}$ the set of n real numbers among which $\scriptstyle k$ are different. Without loss of generality, we will assume that $a_i \le a_{i+1},\ i \in J_{n-1}$.

As a result, a set is generated E_{nk} whose elements are ordered sets $x = (x_1, \ldots, x_n)$, where $x_i = a_{\pi_i},\ i \in J_n$, and $\pi = (\pi_1, \ldots, \pi_n)$ is the permutation n of the first natural numbers. If all elements of a set A are different, then such a set is called a Euclidean set of permutations. If the set A contains the same elements, then we have a Euclidean set of permutations with repetitions.

Euclidean sets of permutations and permutations with repetitions are well studied. We note the important fact that they are vertically located, i.e. coincide with the set of vertices of their convex hull. The convex hull of a set E_{nk} is the so-called permutation polyhedron Π_{nk}. It is known that it is described by the following system of linear equations and inequalities

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} a_i;$$

$$\sum_{i \in \omega} x_i \ge \sum_{i=1}^{|\omega|} a_i,\ \forall \omega \subseteq J_n,\ |\omega| < n. \quad (5)$$

where $|\omega|$ is the power of the set ω.

Consider an optimization problem of the form (1) - (4), provided that the set E is described by system (5). It is easy to see that this problem is a nonlinear optimization problem on a permutation polyhedron with additional restrictions.

The report proposes an approach to solving the problem, using a modification of the conditional gradient method. It is known that the conditional gradient method assumes linearization of the objective function in a certain neighborhood of a point with subsequent optimization of a linear function on the set of feasible solutions. The peculiarity of the linear optimization problem functions on a permutation polyhedron is the fact that its solution is written out directly by ordering the coefficients of the objective function. The minimum of a linear function $\varphi(x) = \sum_{i=1}^{n} c_i x_i$ on a set Π_n is reached at the point $x^* = (x_1^*, \dots, x_n^*)$ where $x_{\pi_i}^* = a_i, i \in J_n$, and $\pi = (\pi_1, \dots, \pi_n)$ is the permutation of the first n natural numbers, such that $c_{\pi_i} \geq c_{\pi_{i+1}}, i \in J_{n-1}$.

Thus, it is possible to construct an iterative process of solving a sequence of auxiliary problems of optimizing functions on a permutation polyhedron. At each step in determining the direction of the decrease of a function, the problem of optimizing a linear function is solved, the coefficients of which are the components of its gradient at the corresponding point.

Note that if the functions $f(x), g_i(x), i \in J_1$ are convex on Π_n and the functions $g_i(x), i \in J_m \setminus J_1$ are linear, then the method converges to the exact solution of the problem.

The obtained results are of independent interest from the point of view of solving conditional optimization problems on a permutation polyhedron with allowance for functional constraints. On the other hand, the described tasks can be considered as relaxation in various combinatorial optimization schemes, in particular, on a set of permutations.

References

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