# SOME PROBLEMS OF OPTIMIZATION IN THE CONFIGURATION SPACE OF SPHERICAL OBJECTS

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Consider the problem of synthesizing the optional configuration of objects of a spherical shape. Let a system of geometric objects $S_i, i \in J_n$, having the shape of a sphere be given. We associate with each of the balls $S_i$ our own coordinate system by selecting its origin (called the pole) at the center of symmetry of the sphere. Then the geometrical information $g^i = (\{s^i\},\{\mu^i\},\{p^i\})$ about the object $S_i$ will include the form $\{S_i\}$, metric parameter $\{\mu^i\} = r_i$ and placement parameters $\{p^i\} = (x_i, y_i, z_i)$. We use Stoyan-Yakovlev theory of configuration space and form the configuration space of spherical objects as the manifold of the space of geometric information [1-3].

The spatial from of a ball in a fixed coordinate system $O_{xyz}$ is given by parametrically given family of functions, escribed by boundary equation

$$f(\zeta, r) = r^2 - x^2 - y^2 - z^2 = 0. \qquad (1)$$

The equation of the general position of the sphere $S_i$ of radius $r_i$ with the placement parameters $p^i = (x_i, y_i, z_i)$ in the fixed coordinate system $O_{xyz}$ takes the form

$$F(\zeta, x_i, y_i, z_i, r_i) = r_i^2 - (x - x_i)^2 - (y - y_i)^2 - (z - z_i)^2 = 0.$$

Geometric information $g^i = (\{s^i\},\{\mu^i\},\{p^i\})$ induces configuration spaces $\Xi^4(S_i), i \in J_n$ with generalized coordinates $x_i, y_i, z_i, r_i$.

Consider the configuration space of the set of balls $S_i, i \in J_n$ and uniquely which we represent as

$$\Xi^{4n}(S_1,...,S_n) = \Xi^4(S_1) \times ... \times \Xi^4(S_n).$$

The point $g = (r_1, x_1, y_1, z_1, ..., r_n, x_n, y_n, z_n) \in \Xi^{4n}(S_1,...,S_n)$ determines the configuration of the balls $S_i, i \in J_n$ and uniquely determines their location in the fixed coordinate system $O_{xyz}$.

On the set of balls $S_i, i \in J_n$ we introduce the binary relation $\{*\}$, assuming $S_i * S_j$, if $\operatorname{int} S_i \cap \operatorname{int} S_j = \varnothing$. In the configuration space $\Xi^{4n}(S_1,...,S_n)$ the relation $S_i * S_j$ is formalized by means of inequality

$$r_i + r_j - \sqrt{(x - x_i)^2 - (y - y_i)^2 - (z - z_i)^2} \geq 0. \qquad (2)$$

Define a matrix $B = [b_{ij}]_{n \times n}$ with elements

$$b_{ij} = \begin{Bmatrix} 1, \text{if } S_i * S_j \\ 0, \text{otherwisw} \end{Bmatrix}.$$

The configuration of balls $S_i, i \in J_n$ is said to be admissible if condition (2) for any $i, j \in J_n$ such that $b_{ij} = 1$.

Suppose that a certain function to be optimized is given on the set of admissible configurations of balls $S_i, i \in J_n$. We obtain the optimization problem of determining generalized variables $r_1, x_1, y_1, z_1, ..., r_n, x_n, y_n, z_n$ of a configuration space $\Xi^{4n}(S_1,...,S_n)$, which has a lot of geometrical interpretations and practical applications. The report proposes a classification of such problems depending on the type of function being optimized and the choice of generalized variables.

The material described above represents a correct mathematical model for the spherical configuration space problems, which can be used in various programs for finding local optimal solution.

**Literature**

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