APPLICATION OF ESTIMATES OF COEFFICIENTS OF GENERALIZED ATOMIC WAVELETS EXPANSIONS TO IMAGE PROCESSING

Viktor Makarichev, Associate Professor of Department 405
Vladimir Lukin, Head of Department 504
Iryna Brysina, Associate Professor of Department 405
National Aerospace University "Kharkiv Aviation Institute"

In the current research, we consider generalized atomic wavelets, which were introduced in [1]. This system of functions is applied in discrete atomic compression (DAC) of digital images [2]. We note that DAC is a lossy image compression algorithm. Different metrics of quality loss can be used to evaluate distortions, which are produced by lossy compression. One of them is maximum absolute deviation (MAD):

$$\mathrm{MAD} = \max_{i=1,2,\ldots,n} |x_i - y_i|,$$

where $(x_1, x_2, \ldots, x_n)$ and $(y_1, y_2, \ldots, y_n)$ are the original data and the reconstructed data, respectively. One of the key features of MAD-metric is high sensitivity to even minor distortions. Therefore, it is especially relevant if near lossless image compression is desired. The aim of this research is to obtain technique for controlling quality loss, which is provided by the algorithm DAC, measured by MAD.

Consider some data presented by the function d(x). Application of generalized atomic wavelets provides the following expansion:

$$d(x) = \sum_{k=1}^{n} \ell_k(x),$$

where n is the depth of expansion and $\ell_k(x)$ is a linear combination of shifts of the wavelet $w_k(x)$: $\ell_k(x) = \sum_j \omega_{k,j} w_k\left(x - 2^{k+1} j / N\right)$, $N \neq 0$. Here, the system of functions $\left\{w_k\left(x - 2^{k+1} j / N\right)\right\}$ constitutes a basis of the linear space of shifts of the generalized Fup-functions [1].

If we quantize wavelet coefficients $\omega_{k,j}$, we obtain the function $\tilde{d}(x)$, which is an approximation of the original data $d(x)$.

Consider the set of positive numbers $\{\delta_1, \ldots, \delta_n\}$. It follows from the properties of the generalized Fup-functions and generalized atomic wavelets that if we apply the following quantization and dequantization procedures:

$$\xi_{k,j} = \mathrm{Round}\left(\omega_{k,j} \cdot \frac{N^2}{\delta_k 2^{2k+1}}\right) \text{ and } \tilde{\omega}_{k,j} = \xi_{k,j} \cdot \frac{\delta_k 2^{2k+1}}{N^2},$$

then $\max_{x \in R} \left| d(x) - \tilde{d}(x) \right| \le \delta_1 + ... + \delta_n$. This inequality provides estimate of quality loss measured by MAD-metric. Indeed, if the function $d(x)$ describes the data $(x_1, x_2, ..., x_n)$ and the function $\tilde{d}(x)$, which is obtained from $d(x)$ by the applying the process given above, describes the data $(y_1, y_2, ..., y_n)$, then

$$MAD \le \delta_1 + ... + \delta_n . \quad (1)$$

We see that loss of quality measured by MAD can be controlled by selecting appropriate values of $\{\delta_k\}$.

Now, consider experimental results. In Fig. 1, four test digital images are presented. Each was them was compressed by DAC with different settings. In Table 1, the obtained results are given (we note that $UBMAD = \delta_1 + ... + \delta_n$ is an upper bound of MAD).

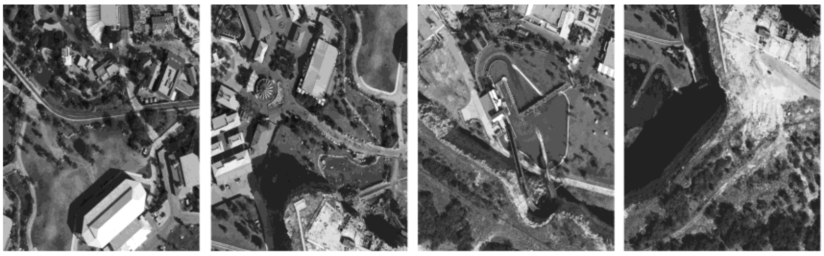

Fig.1. Test remote sensing images

Table 1. Results of compression of test images

UBMAD	16	25	32	50	86	110	158	170	206
MAD (average)	3,25	5	6,25	10,5	18	21,75	32,5	36,25	44,75
MAD (maximum)	4	5	7	12	20	26	34	38	49

The inequality (1) provides an upper estimate of quality loss measured by MAD. We see that the difference between the right and the left parts is significant. Therefore, the correction coefficient c should be used to obtain more precise results: $MAD \le c \cdot (\delta_1 + ... + \delta_n)$. From Table 1, it follows that $c = 0,25$.

So, the estimate (1) in combination with application of correction coefficient provides quality loss control mechanism that can be used to obtain a desired quality in terms of MAD-metric.

References

1. Brysina I.V., Makarichev V.O. Generalized atomic wavelets [Text] // Radioelectronic and Computer Systems. – 2018. – No. 1 (85). – P. 23-31.

2. Lukin V., Brysina I., Makarichev V. Discrete Atomic Compression of Digital Images: A Way to Reduce Memory Expenses [Text] // Advances in Intelligent Systems and Computing, Integrated Computer Technologies in Mechanical Engineering. – 2020. – Vol. 1113. – P. 492-502.