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DESIGNING OF AIRPLANE’S UNITS MADE OF COMPOSITES

Textbook

Kharkiv “KhAI” 2015

UDC 629.735.33.023.01+629.735.33.002.3:620.22(075.8)
BLC 39.53я73
T19

Подано матеріали курсу лекцій “Проектування агрегатів літаків із композитів” (англійською мовою), які допомагають зрозуміти узагальнені шляхи щодо проектування і конструювання основних силових елементів літаків з композитів – балок, оболонок обертання, стрижнів, з’єднань, панелей. Розглянуто методики вибору відповідних до експлуатаційних умов конструктивних рішень силових елементів, оцінювання фізичних і механічних властивостей деталей із композитів з довільною структурою, методи підвищення місцевої й загальної стійкості тонкостінних композитних деталей, методи розрахунку напружено-деформованого стану металокомпозитних конструкцій і особливості розроблення конструкторської документації на креслення з композитів.

Для студентів, що вивчають курси " Проектування агрегатів літаків із композитів", “Designing of airplane’s units made of composites”.

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Taranenko, I. M.
T16 Designing of airplane’s units of composites [Text]: textbook / I. M. Taranenko.– Kh.: Nat. Aerosp. Univer “Khark. Aviat. Inst.”, 2015.– 140 p.

The textbook contains materials for studying course “Designing of airplane’s units made of composites” and is developed for helping students to understand generalized approaches of designing main structural elements of aircrafts made of composites – beams, shells of revolution, rods, joints, panels etc. Methods of selection of exact structural solutions of composite load-carrying elements (in accordance with operational conditions), estimation of physical and mechanical properties of composite articles with arbitrary stacking sequence, increasing local and global stability of thin-walled composite articles, stress state analysis of metal-composite structures and distinctive features of drawings preparation on composite articles and assemblies are considered.

For the students studying course “ Designing of airplane’s units made of composites”.

Fig. 87. Tabl. 3. Bibliogr.: 11 sources
UDC 629.735.33.023.01+629.735.33.002.3:620.22(075.8)
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CONTENTS

Introduction

Nowadays composites have reached such global embedding in all branches of national economy that nobody can imagine our current life and future without them. Being a very important, developing and wide-spread structural material at first in military and space area for the last 50-60 years they now overcome to civil aviation, auto- and railway-building, chemical, shipbuilding and other branches. Taking into account high cost of some composites and distinctive features of their manufacturing, maintenance and handling in further operation the most efficiency of composites application can be shown in their implementation in heavy- and medium-loaded load-carrying elements of structural skeletons.

Both young and mature designer know that to create reliable structure with required level of life-cycle quality and satisfying necessary operational conditions they have to conduct such several stages of preliminary analysis as material components selection, developing of force diagrams, critical load-cases selection etc. For design and stress analysis of structures made of conventional (mainly isotropic) structural materials (metals and alloys, homogenious plastics, glasses, ceramics, elastomers) some reference values of physical and mechanical parameters can be found in literature sources. But composite structures (mainly anisotropic) having arbitrary reinforcement arrangement engineers need special procedures for mentioned properties estimation. Moreover taking into account such specific composites features as relatively low interlaminar shear strength, bearing strength brittle behavior of matrix one has to use special structural solutions of articles and units in which composites are used.

The main objective of this book is to help reader to understand approaches how to estimate composite article properties if properties of original semi-finished components are known, what main groups of general structural mechanics equations can be used for composite structure strength analysis and what exact structural solutions can be implemented in definite load case.

For deeper analysis of definite question of beams, shells, rods, panels and other aircraft structural elements design the list of recommended literature is suggested.

Theme 1. FORMULATION OF THE DESIGN PROBLEM FOR COMPOSITE STRUCTURES. DESIGN PRINCIPLES OF ARTICLES AND UNITS MADE OF COMPOSITES

1.1. Formulation of the design problem for composite structures

Technical progress, from the one hand, requires working out new structural materials, from another hand, technical progress is stipulated by working out of materials abilities. One way of existing structures perfection is application of up-to-date materials which allow realization of new structural solutions and manufacturing processes. For the last 40 years technical progress in all branches of engineering, national economy is closely related with composites application. To improve physical-mechanical properties of composites it is necessary to study properly their mechanical behavior. Successful realization of high composites potential abilities depends on designer's knowledge level about these abilities, principles of composites design and analysis methods. Moreover the majority of literature sources about composites is oriented on pure science but not on applied engineering methods. That is why quite actual problem is to work out quite simple, brief and reliable methods of composites properties analysis, design approaches and structures manufacturing techniques. Generally mechanics of materials as pure and applied science is quite alive one so its branches are under development now.

The main idea of this course is to attempt to compose design approaches according to the rule "**from MATERIAL properties to STRUCTURE properties**".

Combination of various substances is one of the basic way for new materials creation. The majority of modern structural materials are compositions which allow technical products to possess certain operational properties (for example, concrete reinforced with metal rods, glass plastic pressure vessels, automobile tires etc). In all these cases it means creation of the system of different materials; moreover each of these components fulfill definite role in considered finished article. Teamwork of different materials gives the effect equivalent to creation a new material, which property both quantitatively and qualitatively differ from properties of each its components.

Any composite material as structural one carries operational loads (mechanical, thermal, environmental influence etc). Therefore these factors define structural and operational requirements to composite materials. That is why knowledge of laws defining material physical, mechanical, thermal, manufacturing and other properties allows to use efficiently existing materials and to create new ones.

Composite materials are artificial heterogeneous systems obtained from at least of two components with individual properties. Following distinctive features are typical for composite materials:

– composition and shape of components are previously defined;

– quantity of each component guarantees required properties of composite;

–components possess different (from each other) chemical composition and well-defined boundary can be seen between different components;

– final composite possesses new properties not inherent to separated components;

– final composite is uniform at macro-level and non-uniform at micro-level.

In majority of composites its components differ from each other by geometrical feature. **Matrix** (binder) is continuous through composite volume substance. **Reinforcing material** is discontinuous through composite volume substance.

To study mechanics of composites engineer should remember main directions of this scientific reciprocal development: structural mechanics, building mechanics, fracture mechanics and technological mechanics [1]. **Structural mechanics** studies dependence of composite properties on its components properties, composite arrangement and type. **Building mechanics** (or mechanics of solids) studies composites behavior under external loading, stress analysis of structural elements. **Fracture mechanics** studies ultimate states and fracture criteria of composites. **Technological mechanics** deals with composites properties dependence on its manufacturing parameters.

From the course of Aviation material we know about main composites advantages and drawbacks. Analysis of these properties shows that high efficiency of composites depends on the following factors:

– high strength bonding between reinforcing materials and matrix through composite volume (solidity of composite);

– application of matrices with possessing values of maximum allowable deformations as close as it possible to reinforcing material deformation or more then this value (selection of such kind of matrices permits to realize full strength properties of reinforcing material);

– to escape possible negative thermal-elastic phenomena of structural behavior and reducing influence of composite drawbacks (low bearing strength, shear strength, peeling strength etc) due to using special rational structural and manufacturing solutions;

– maximum possible realization of reinforcing material strength, rigidity and special properties.

Practical realization permits to compose the following **main principles of composites design**:

1. It is necessary to transfer loads directly by reinforcing materials or by geometrically shortest way.

2. Only thermally-balanced reinforcing schemes have to be selected as load-carrying schemes of structural units.

3. Components of a composite have to be chemically and mechanically compatible with each other.

4. Selected manufacturing process for making composite should ensure required level of design properties.

Demonstration of these principles is shown at the Fig. 1.1.

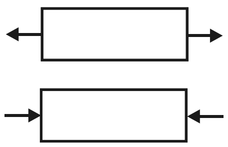
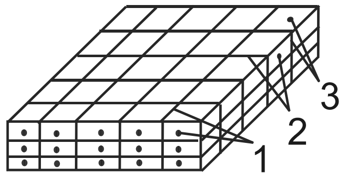
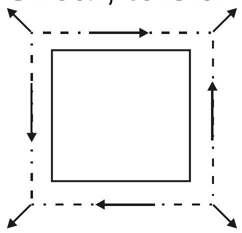
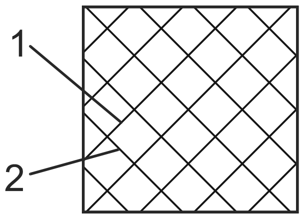
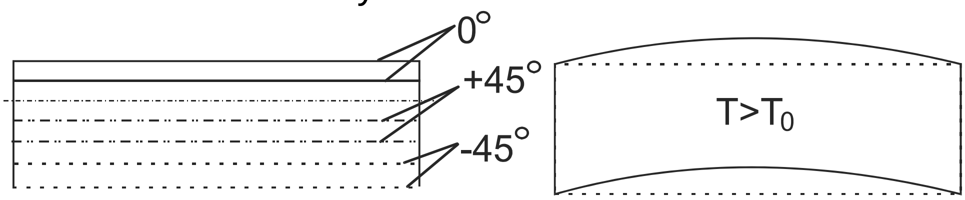
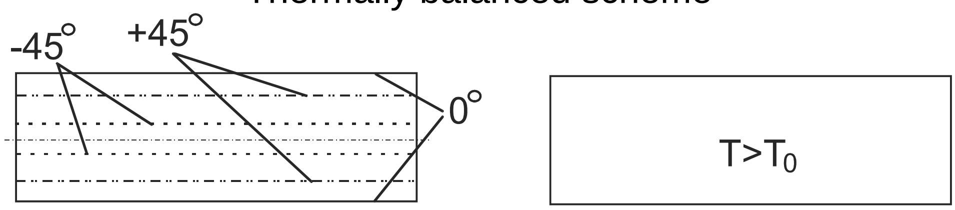
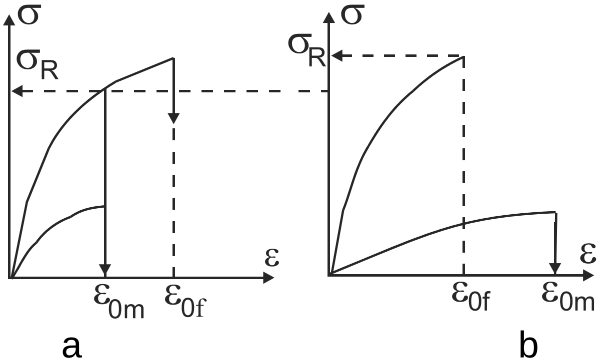
<table>
<tr><th rowspan="3">Principle 1</th><th>Loading type</th><th>Reinforcing scheme</th><th>Reinforcing agent purpose</th></tr>
<tr><td>
Axial</td><td></td><td>1 – to transfer longitudinal loads;
2 – to transfer lateral loads ;
3 – to transfer transversal loads;
2, 3 – to prevent lateral and transversal peeling and interlaminar shear</td></tr>
<tr><td>Shear, torsion
</td><td></td><td>1,2 – to carry shear forces</td></tr>
<tr><th rowspan="2">Principle 2</th><td colspan="3">Thermally non-balanced scheme
</td></tr>
<tr><td colspan="3">Thermally balanced scheme
</td></tr>
<tr><th rowspan="2">Principle 3</th><td colspan="2" rowspan="2"></td><td>a – fiber stress-strain diagram;
b – matrix stress-strain diagram;
σ_R – realized stress level in fibers</td></tr>
<tr><td>Absence of chemical interaction between fibers and matrix has to be guaranteed</td></tr>
<tr><th>Principle 4</th><td colspan="3">Selection of proper manufacturing process type, parameters of this process (time, temperature, pressure etc) to achieve required properties (quality parameters) of final composite</td></tr>
</table>

Fig. 1.1. Realization of composites design principles

1.2. Main concepts of composites articles design

To choose rational structural and manufacture solution for an article made of composite we should accept the concept of article mainframe arrangement.

There are two basic concepts of article mainframe forming in aviation according to design principles of composite articles (Fig. 1.2).

The first concept contains synthesis of frame members. Reinforcing fibers in these members are directed in the way due to ensure the best resistance of a structure to all regulated types of loads (see Fig. 1.2, a).

This concept is called **synthesizing** or **integrating** one.

The second concept includes structurally underlined members. Every of these elements can withstand definite type of regulated load and almost can't carry other types of loads (see Fig. 1.2, b). This principle is called **differential** design concept.

Masses of designed structures according to synthesizing and differential concepts are significantly different.

In pure realization the synthesizing concept isn't used practically. The cause of this rear use is the following. Majority of panel and shell structures is thin-walled and consists of small quantity of layers. That is why their general or local stability but not their strength defines the main load carrying ability of these structures.

To increase stability of mainframe members of panel and shell type one can use so called **sandwich structure** (Fig. 1.3). This structure has increased integral stiffness. It consists of thin load carrying layers and light (usually honeycomb or foam) filler. Layers can take all internal loads and filler ensure combined deformation of layers at their loading. Usually filler are loaded with shear.

Let consider two panels: smooth two-layered and sandwich. One can see that stiffness of sandwich panel is 10…1000 times more than smooth panel. Instability of sandwich panel is unlikely.

It's difficult to use reinforcement in different directions in thin layers (as synthesizing concept needs) because of low manufacturability of this process – labor-content of individual layer (monolayers) cutting is high, presents of defects (porosities, folders) in joining zone is guaranteed, therefore mass of such structure is too high. That is why synthesizing design principle can be used for average loading level through entire article area. So reinforcing scheme of entire article is the same (is defined by the most loaded article zone). As a conclusion one can see that synthesizing concept doesn't permit to decrease structure mass significantly due to mentioned restrictions.

Nowadays for synthesizing design concept the following reinforcing scheme are widely used $\left[0^{\circ}_{n}, \pm\varphi^{\circ}_{m}, 90^{\circ}_{k}\right]$. Indexes n, m, k means quantity of monolayers in correspondent direction $\left(0^{\circ}, 90^{\circ} \text{ or } \pm\varphi^{\circ}\right)$. Generally direction with 0° means longitudinal axis of a unit.

Differential concept is widely used in aircraft structures for designing wing spars, control surfaces, wings and fuselage.

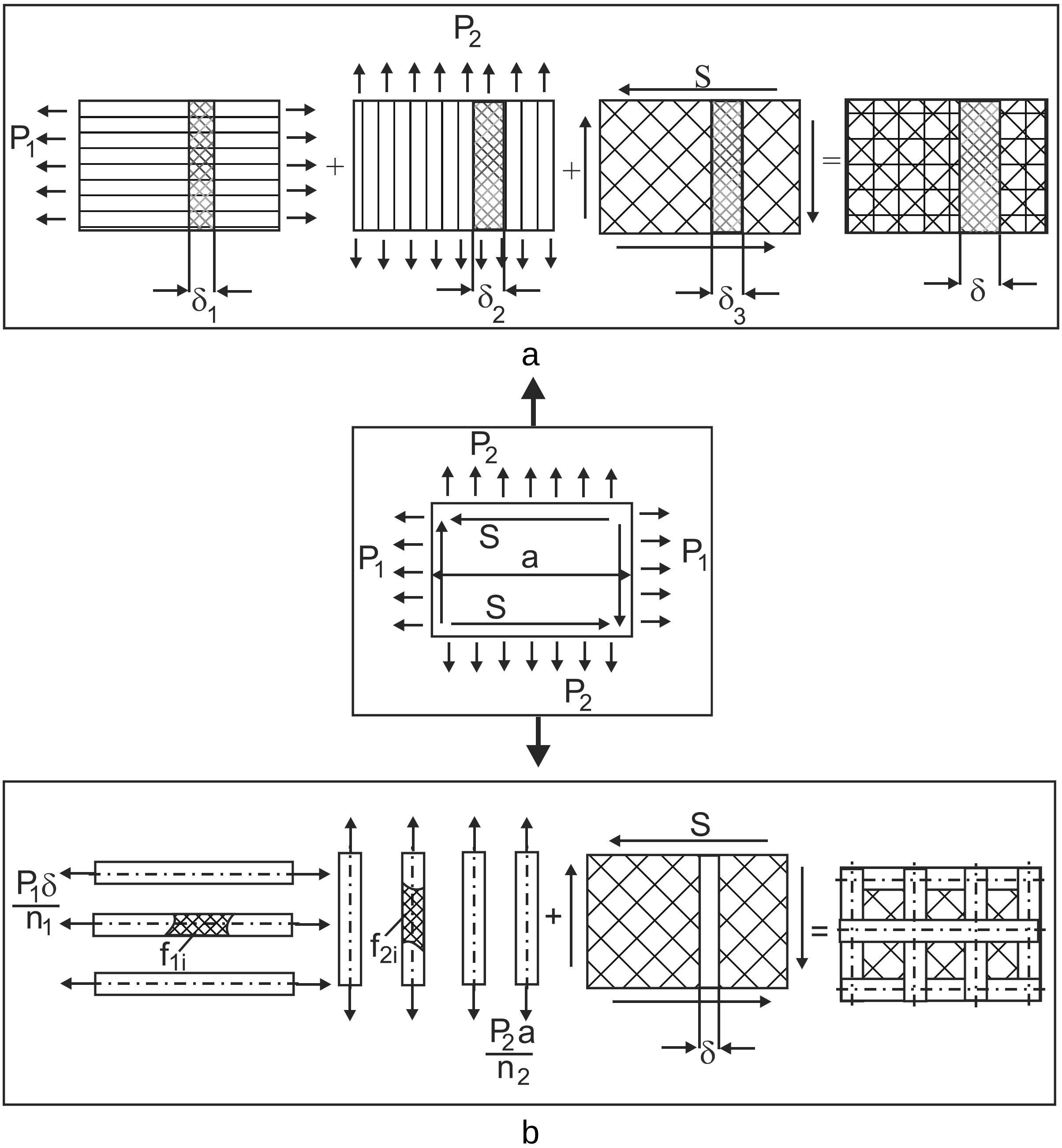


Fig. 1.2. Scheme of composite articles and units design main concepts: a– synthesizing (integrating) concept; b– differential concept

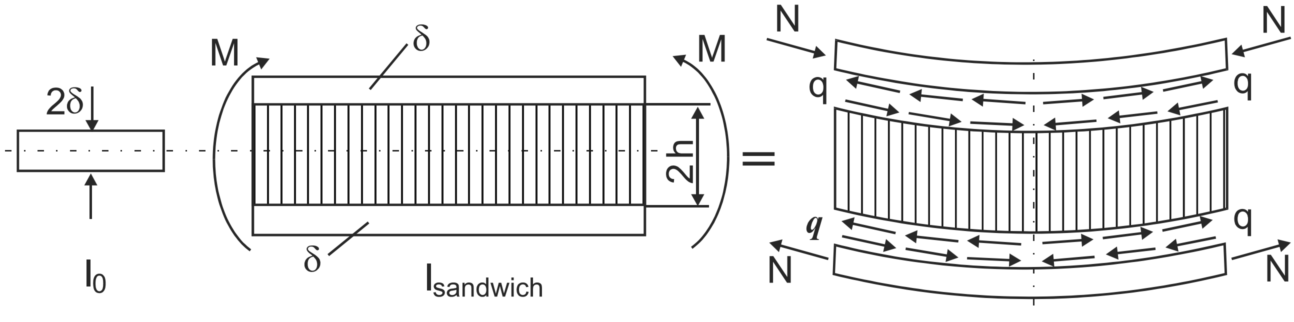


Fig. 1.3. Rigidity efficiency of sandwich panel

1.3. Typical reinforcing schemes of aircraft articles and units

All typical structural elements of aircraft can be divided into four groups: rods (pivots), panels, shells and solids. All these structural elements can be analyzed with the following analytical schemes.

Rods are elements which length is approximately more than 10 times comparing with width and height (thickness).

Panels are elements having width and length of the same order but thickness is approximately 10 times less comparing with width and length. Panels can be flat and curved (with single and double curvature). Moreover less radius of curvature should be at least 10 times more than less plane dimensions (width or length).

Shells are closed elements with radius not less than length or open elements with radius of curvature of the same order with less plane dimension (width or length).

Solids are elements having all three dimensions of the same order.

Classification of typical structural elements of aircraft and recommended for their design reinforcing schemes are shown at the Fig. 1.4.

All aircraft articles and units are designed on the basis of above-mentioned structural elements (analysis schemes). Moreover possibilities of technological equipment permit to manufacture all articles of an assembly at the same manufacturing stage or separately. So complex structures made of the single manufacturing cycle are known as **integral structures** (in this case it doesn't matter that some of articles were previously produced during another manufacturing cycle). We should draw attention that this term refers to manufacturing method but not to load-carrying scheme of a unit.

Checking-up questions

1. Formulate main milestones of the problem of composite articles design.
2. What are main design principles one has to satisfy at composite structures developing?
3. What is the main idea of sandwich structure application?
4. What does synthesizing (integrating) and differential concepts of design mean?
5. What kinds of analytical schemes can be used for analysis of aircrafts typical structural elements?
6. Analyze recommended reinforcing schemes for typical composite articles of aircrafts.

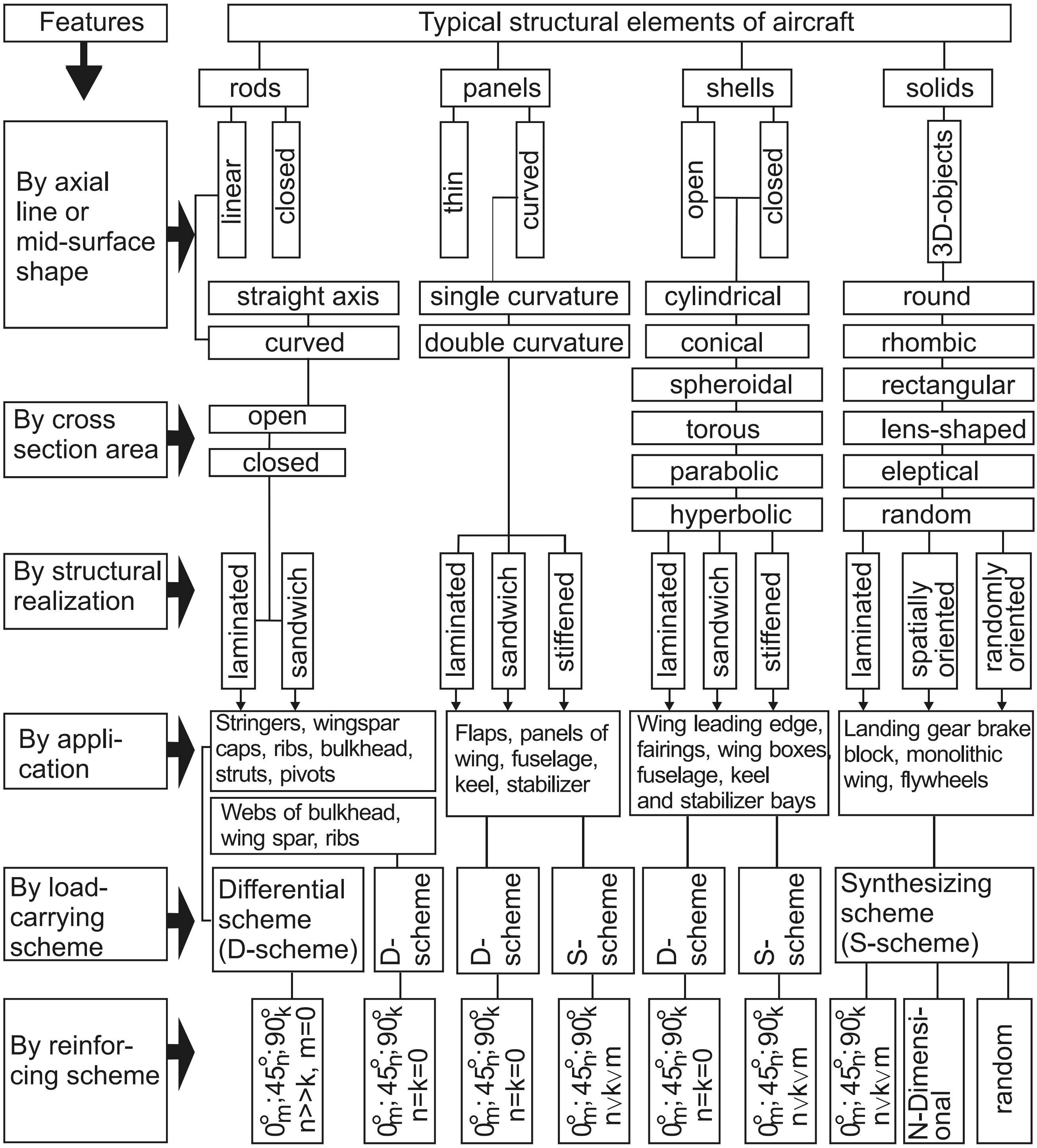


Fig. 1.4. Classification of typical structural elements

Theme 2. METHOD OF DETERMINATION OF ELASTIC CONSTANTS OF LAMINATED COMPOSITE

2.1. Elastic properties of laminated composite

Laminated composite (or composite material of laminated structure) is composite material consisting of package of consequently laid-up individual layers (monolayers), each of them is characterized by individual thickness and stacking angle related to adopted coordinate system. It is considered that the ideal adhesion exists between layers, as a result they deform together at any pack loading, i.e. layers do not slip related to each other.

To research physical and mechanical properties of laminated composite material V.V. Vasiliev’s model is used (Fig. 2.1) [2, 3, 6]. An orthotropic strip is the representative element in this theory. This strip has definite stiffness at tension, compression and shear. Elastic constants of monolayers are defined theoretically by above-mentioned formulas or by experimental way.

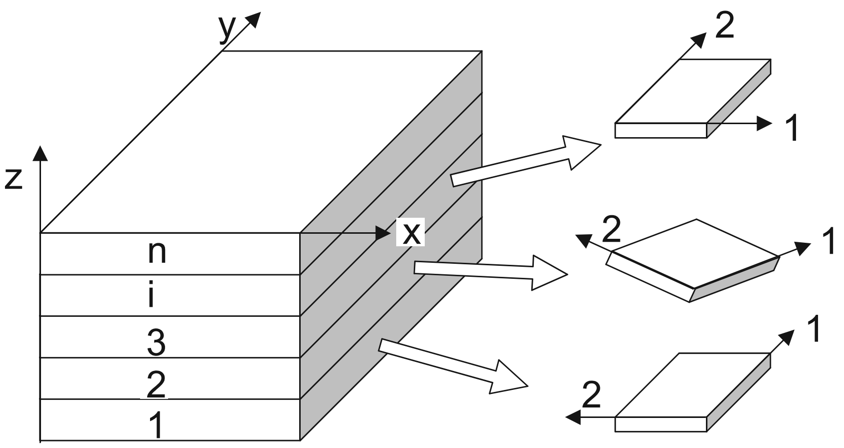


Fig. 2.1. Laminated model of composite material (Vasiliev’s model)

Vasiliev’s model of laminated composite material. Materials consist of any orthotropic layers (for examples, unidirectional ones) can be analyzed by means of this model. An orthotropic strip, possessing by definite stiffness at tension (compression) along the axes 1, 2 and at shear in the layer plane, is the main seriated representative element of composite structure. Strips are assumed to be uniform material, there is the ideal adhesion between layers (so they are joined together).

Physical and mechanical characteristics of laminated composite can be expressed by means of properties of layers (which in their turn can be determined by means of previous model or by experimental way), reinforcing angle of each layer and layers quantity. Now this model is widely used either for prediction of laminated composites or structure design, strength analysis etc.

Let consider composite material consisting of any layers with thickness

δ_i; orthotropy axes of these layers apply angles φ_i with axis *x* of basic (global) coordinate system (Fig. 2.2). In general case equations of physical law for anisotropic material have the following form:

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \mu_{yx}\frac{\sigma_y}{E_y} + \eta_{xy,x}\frac{\tau_{xy}}{G_{xy}};$$

$$\varepsilon_y = -\mu_{xy}\frac{\sigma_x}{E_x} + \frac{\sigma_y}{E_y} + \eta_{xy,y}\frac{\tau_{xy}}{G_{xy}}; \qquad (2.1)$$

$$\gamma_{xy} = \eta_{x,xy}\frac{\sigma_x}{E_x} + \eta_{y,xy}\frac{\sigma_y}{E_y} + \frac{\tau_{xy}}{G_{xy}},$$

where E_x, E_y, G_{xy}, μ_{xy}, μ_{yx}, $\eta_{x,xy}$, $\eta_{y,xy}$, $\eta_{xy,x}$, $\eta_{xy,y}$ – elastic constants, which should be expressed by means of anisotropic layer characteristics; $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ - pack strain; $\sigma_x, \sigma_y, \tau_{xy}$ - average stress along the pack thickness.

Let deform pack of layers up to strains $\varepsilon_x, \varepsilon_y, \gamma_{xy}$, then define stresses $\sigma_x, \sigma_y, \tau_{xy}$, which cause these strains.

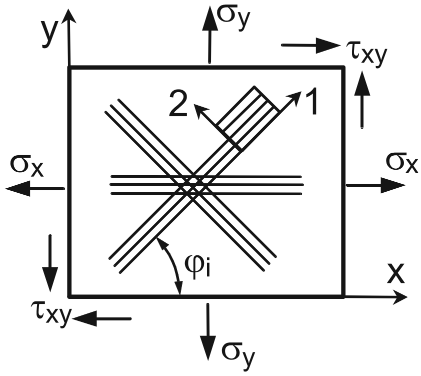


Fig. 2.2. Model of laminated composite material

The strains of each individual layer in local coordinate system can be determined by well-known formulas because of compatible deformation of entire package:

$$\varepsilon_{1i} = \varepsilon_x \cos^2\varphi_i + \varepsilon_y \sin^2\varphi_i + \gamma_{xy}\sin\varphi_i\cos\varphi_i;$$

$$\varepsilon_{2i} = \varepsilon_x \sin^2\varphi_i + \varepsilon_y \cos^2\varphi_i - \gamma_{xy}\sin\varphi_i\cos\varphi_i; \qquad (2.2)$$

$$\gamma_{12i} = (\varepsilon_y - \varepsilon_x)\sin 2\varphi_i + \gamma_{xy}\cos 2\varphi_i.$$

Generalized Hook's law for each orthotropic layer has the form:

$$\varepsilon_{1i} = \frac{\sigma_{1i}}{E_{1i}} - \mu_{21i}\frac{\sigma_{2i}}{E_{2i}};$$
$$\varepsilon_{2i} = \frac{\sigma_{2i}}{E_{2i}} - \mu_{12i}\frac{\sigma_{1i}}{E_{1i}}; \quad (2.3)$$
$$\gamma_{12i} = \frac{\tau_{12i}}{G_{12i}}.$$

If we solve these equations related to stress, one can obtain:

$$\sigma_{1i} = \bar{E}_{1i}(\varepsilon_{1i} + \mu_{21i}\varepsilon_{2i});$$
$$\sigma_{2i} = \bar{E}_{2i}(\varepsilon_{2i} + \mu_{12i}\varepsilon_{1i}); \quad (2.4)$$
$$\tau_{12i} = G_{12i}\gamma_{12i,}$$

where

$$\bar{E}_{1i} = \frac{E_{1i}}{1-\mu_{12i}\mu_{21i}}; \qquad \bar{E}_{2i} = \frac{E_{2i}}{1-\mu_{12i}\mu_{21i}}. \quad (2.5)$$

Let substitute dependences (2.2) to (2.4) ones to express stresses $\sigma_{1i}, \sigma_{2i}, \tau_{12i}$ by means of pack strains ε_x, ε_y, γ_{xy}. Then

$$\sigma_{1i} = \bar{E}_{1i}\Big[\varepsilon_x \cos^2\varphi_i + \varepsilon_y \cos^2\varphi_i + \gamma_{xy}\sin\ \varphi_i \cos\ \varphi_i +$$
$$+\mu_{21i}\left(\varepsilon_x \sin^2\varphi_i + \varepsilon_y \sin^2\varphi_i - \gamma_{xy}\sin\varphi_i\cos\ \varphi_i\right)\Big];$$
$$\sigma_{2i} = \bar{E}_{2i}\Big[\varepsilon_x \sin^2\varphi_i + \varepsilon_y \sin^2\varphi_i - \gamma_{xy}\sin\ \varphi_i\cos\varphi_i + \quad (2.6)$$
$$+\mu_{12i}\left(\varepsilon_x \cos^2\varphi_i + \varepsilon_y \cos^2\varphi_i + \gamma_{xy}\sin\ \varphi_i\cos\varphi_i\right)\Big];$$
$$\tau_{12i} = G_{12i}\Big[\left(\varepsilon_y - \varepsilon_x\right)\sin 2\varphi_i + \gamma_{xy}\cos 2\ \varphi_i\Big].$$

Let find projections of these stresses on x, y axes by known formulas of elasticity theory:

$$\sigma_{xi} = \sigma_{1i}\cos^2\varphi_i + \sigma_{2i}\sin^2\varphi_i - \tau_{12i}\sin 2\varphi_i;$$
$$\sigma_{yi} = \sigma_{1i}\sin^2\varphi_i + \sigma_{2i}\cos^2\varphi_i + \tau_{12i}\sin 2\varphi_i; \quad (2.7)$$
$$\tau_{xyi} = (\sigma_{1i} - \sigma_{2i})\sin\varphi_i\cos\varphi_i + \tau_{12i}\cos 2\varphi_i,$$

or, taking into consideration expressions (2.6),

$$\sigma_{xi} = \varepsilon_x B_{11i} + \varepsilon_y B_{12i} + \gamma_{xy} B_{13i};$$
$$\sigma_{yi} = \varepsilon_x B_{21i} + \varepsilon_y B_{22i} + \gamma_{xy} B_{23i}; \quad (2.8)$$
$$\tau_{xyi} = \varepsilon_x B_{31i} + \varepsilon_y B_{32i} + \gamma_{xy} B_{33i}.$$

Here

$$\begin{aligned}
&B_{11i} = \bar{E}_{1i}\cos^4\varphi_i + 2\bar{E}_{1i}\mu_{21i}\sin^2\varphi_i\cos^2\varphi_i + \bar{E}_{2i}\sin^4\varphi_i + G_{12i}\sin^2 2\varphi_i;\\
&B_{12i} = B_{21i} = (\bar{E}_{1i} + \bar{E}_{2i})\sin^2\varphi_i\cos^2\varphi_i + \bar{E}_{1i}\mu_{21i}(\sin^4\varphi_i + \cos^4\varphi_i) - G_{12i}\sin^2 2\varphi_i;\\
&B_{22i} = \bar{E}_{1i}\sin^4\varphi_i + 2\bar{E}_{1i}\mu_{21i}\sin^2\varphi_i\cos^2\varphi_i + \bar{E}_{2i}\cos^4\varphi_i + G_{12i}\sin^2 2\varphi_i;\\
&B_{13i} = B_{31i} = \sin\varphi_i\cos\varphi_i\cdot\left[\bar{E}_{1i}(1-\mu_{21i})\cos^2\varphi_i - \bar{E}_{2i}(1-\mu_{12i})\sin^2\varphi_i - 2G_{12i}\cos 2\varphi_i\right];\\
&B_{33i} = (\bar{E}_{1i} + \bar{E}_{2i} - 2\bar{E}_{1i}\mu_{21i})\sin^2\varphi_i\cos^2\varphi_i + G_{12i}\cos^2 2\varphi_i;\\
&B_{23i} = B_{32i} = \sin\varphi_i\cos\varphi_i\cdot\left[\bar{E}_{1i}(1-\mu_{21i})\sin^2\varphi_i - \bar{E}_{2i}(1-\mu_{12i})\cos^2\varphi_i + 2G_{12i}\cos 2\varphi_i\right].
\end{aligned} \tag{2.9}$$

Let compose equilibrium equations on x and y axes:

$$\sum_{i=1}^{n}\sigma_{xi}\delta_i = \sigma_x\delta_\Sigma; \qquad \sum_{i=1}^{n}\sigma_{yi}\delta_i = \sigma_y\delta_\Sigma; \qquad \sum_{i=1}^{n}\tau_{xyi}\delta_i = \tau_{xy}\delta_\Sigma, \tag{2.10}$$

where n – total number of layers, $\delta_\Sigma = \sum_{i=1}^{n}\delta_i$ - total package thickness.

After substitution of (2.8) dependences to (2.10) ones we can obtain formulas for stresses $\sigma_x, \sigma_y, \tau_{xy}$ expressed by $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ strains:

$$\begin{aligned}
\sigma_x &= \frac{1}{\delta_\Sigma}\left(B_{11}\varepsilon_x + B_{12}\varepsilon_y + B_{13}\gamma_{xy}\right);\\
\sigma_y &= \frac{1}{\delta_\Sigma}\left(B_{21}\varepsilon_x + B_{22}\varepsilon_y + B_{23}\gamma_{xy}\right);\\
\tau_{xy} &= \frac{1}{\delta_\Sigma}\left(B_{31}\varepsilon_x + B_{32}\varepsilon_y + B_{33}\gamma_{xy}\right).
\end{aligned} \tag{2.11}$$

Here

$$B_{kl} = \sum_{i=1}^{n}\delta_i B_{kli}, \tag{2.12}$$

where к, l take values 1, 2 and 3.

Equations (2.11) are generalized Hook's law, which for design stage can be written as the following:

$$\begin{aligned}
N_x &= \sigma_x\delta_\Sigma = B_{11}\varepsilon_x + B_{12}\varepsilon_y + B_{13}\gamma_{xy};\\
N_y &= \sigma_y\delta_\Sigma = B_{21}\varepsilon_x + B_{22}\varepsilon_y + B_{23}\gamma_{xy};\\
q_{xy} &= \tau_{xy}\delta_\Sigma = B_{31}\varepsilon_x + B_{32}\varepsilon_y + B_{33}\gamma_{xy},
\end{aligned} \tag{2.13}$$

where N_x, N_y, q_{xy}- forces per unit width (force, acting on place with width of one linear unit).

Let solve (2.11) equations system related to strains $\varepsilon_x, \varepsilon_y, \gamma_{xy}$:

$$\varepsilon_x=\frac{\delta_\Sigma}{B}\left[\sigma_x\left(B_{22}B_{33}-B_{23}^2\right)-\sigma_y\left(B_{12}B_{33}-B_{13}B_{23}\right)+\tau_{xy}\left(B_{12}B_{23}-B_{22}B_{13}\right)\right];$$

$$\varepsilon_y=\frac{\delta_\Sigma}{B}\left[-\sigma_x\left(B_{12}B_{33}-B_{13}B_{23}\right)+\sigma_y\left(B_{11}B_{33}-B_{13}^2\right)+\tau_{xy}\left(B_{12}B_{13}-B_{11}B_{23}\right)\right];\quad(2.14)$$

$$\gamma_{xy}=\frac{\delta_\Sigma}{B}\left[\sigma_x\left(B_{12}B_{23}-B_{13}B_{22}\right)+\sigma_y\left(B_{12}B_{13}-B_{11}B_{23}\right)+\tau_{xy}\left(B_{11}B_{22}-B_{12}^2\right)\right],$$

where

$$B=B_{33}(B_{11}B_{22}-B_{12}^2)+2B_{12}B_{13}B_{23}-B_{22}B_{13}^2-B_{11}B_{23}^2. \quad(2.15)$$

If we compare coefficients at stresses in the equation systems (2.1) and (2.14) one can obtain:

$$\frac{1}{E_x}=\frac{\delta_\Sigma}{B}\left(B_{22}B_{33}-B_{23}^2\right);\qquad \frac{\mu_{yx}}{E_y}=\frac{\delta_\Sigma}{B}\left(B_{12}B_{33}-B_{13}B_{23}\right);$$

$$\frac{\eta_{xy,x}}{G_{xy}}=\frac{\delta_\Sigma}{B}\left(B_{12}B_{23}-B_{22}B_{13}\right);\qquad \frac{\mu_{xy}}{E_x}=\frac{\delta_\Sigma}{B}\left(B_{12}B_{33}-B_{13}B_{23}\right);$$

$$\frac{1}{E_y}=\frac{\delta_\Sigma}{B}\left(B_{11}B_{33}-B_{13}^2\right);\qquad \frac{\eta_{xy,y}}{G_{xy}}=\frac{\delta_\Sigma}{B}\left(B_{12}B_{13}-B_{11}B_{23}\right);\quad(2.16)$$

$$\frac{\eta_{x,xy}}{E_x}=\frac{\delta_\Sigma}{B}\left(B_{12}B_{23}-B_{22}B_{13}\right);\qquad \frac{\eta_{y,xy}}{E_y}=\frac{\delta_\Sigma}{B}\left(B_{12}B_{13}-B_{11}B_{23}\right);$$

$$\frac{1}{G_{xy}}=\frac{\delta_\Sigma}{B}\left(B_{11}B_{22}-B_{12}^2\right).$$

Formulas for determination of elastic properties of laminated composite materials follow from these equations:

$$E_x=\frac{B}{\delta_\Sigma(B_{22}B_{33}-B_{23}^2)};\; E_y=\frac{B}{\delta_\Sigma(B_{11}B_{33}-B_{13}^2)};\; G_{xy}=\frac{B}{\delta_\Sigma(B_{11}B_{22}-B_{12}^2)};$$

$$\mu_{xy}=\frac{B_{12}B_{33}-B_{13}B_{23}}{B_{22}B_{33}-B_{23}^2};\qquad \mu_{yx}=\frac{B_{12}B_{33}-B_{13}B_{23}}{B_{11}B_{33}-B_{13}^2};$$

$$\eta_{xy,x}=\frac{B_{12}B_{23}-B_{22}B_{13}}{B_{11}B_{22}-B_{12}^2};\qquad \eta_{xy,y}=\frac{B_{12}B_{13}-B_{11}B_{23}}{B_{11}B_{22}-B_{12}^2};\quad(2.17)$$

$$\eta_{x,xy}=\frac{B_{12}B_{23}-B_{22}B_{13}}{B_{22}B_{33}-B_{23}^2};\qquad \eta_{y,xy}=\frac{B_{12}B_{13}-B_{11}B_{23}}{B_{11}B_{33}-B_{13}^2}.$$

Following equations sequence from (2.16) expressions:

$$\frac{\eta_{x,xy}}{G_{xy}}=\frac{\eta_{xy,x}}{E_x};\qquad \frac{\eta_{y,xy}}{G_{xy}}=\frac{\eta_{xy,y}}{E_y};\qquad \frac{\mu_{xy}}{E_x}=\frac{\mu_{yx}}{E_y}. \quad(2.18)$$

Analysis of (2.11) equations shows that material is orthotropic in ***x***, ***y*** axes in that case when the following conditions are fulfilled simultaneously:

$$B_{13}=B_{23}=B_{31}=B_{32}=0\,. \tag{2.19}$$

In that case formulas (2.15) and (2.17) simplify to form:

$$B=B_{33}(B_{11}B_{22}-B_{12}^2); \tag{2.20}$$

$$E_x=\frac{B_{11}B_{22}-B_{12}^2}{\delta_\Sigma B_{22}};\qquad E_y=\frac{B_{11}B_{22}-B_{12}^2}{\delta_\Sigma B_{11}};\qquad G_{xy}=\frac{B_{33}}{\delta_\Sigma};$$
$$\mu_{xy}=\frac{B_{12}}{B_{22}};\qquad \mu_{yx}=\frac{B_{12}}{B_{11}};\qquad \eta_{x,xy}=\eta_{y,xy}=\eta_{xy,x}=\eta_{xy,y}=0. \tag{2.21}$$

Let consider in detail some particular structures, which are widely used in practice.

Example 2.1. The pack consists of one layer (***n***=1) with reinforcing angle φ and thickness $\delta=\delta_\Sigma$*.* Then

$$B_{11}=\delta\left(\bar{E}_1\cos^4\varphi+2\bar{E}_1\mu_{21}\sin^2\varphi\cos^2\varphi+\bar{E}_2\sin^4\varphi+G_{12}\sin^2 2\varphi\right);$$
$$B_{12}=\delta\left[\left(\bar{E}_1+\bar{E}_2\right)\sin^2\varphi\cos^2\varphi+\bar{E}_1\mu_{21}(\sin^4\varphi+\cos^4\varphi)-G_{12}\sin^2 2\varphi\right];$$
$$B_{22}=\delta(E_1\sin^4\varphi+2\bar{E}_1\mu_{21}\sin^2\varphi\cos^2\varphi+\bar{E}_2\cos^4\varphi+G_{12}\sin^2 2\varphi);$$
$$B_{33}=\delta\left[\left(\bar{E}_1+\bar{E}_2-2E_1\mu_{21}\right)\sin^2\varphi\cos^2\varphi+G_{12}\cos^2 2\varphi\right];$$
$$B_{13}=B_{31}=\delta\sin\varphi\cos\varphi\left[\bar{E}_1(1-\mu_{21})\cos^2\varphi-\bar{E}_2(1-\mu_{12})\sin^2\varphi-2G_{12}\cos 2\varphi\right];$$
$$B_{23}=B_{32}=\delta\sin\varphi\cos\varphi\left[\bar{E}_1(1-\mu_{21})\sin^2\varphi-\bar{E}_2(1-\mu_{12})\cos^2\varphi-2G_{12}\cos 2\varphi\right]. \tag{2.22}$$

It is obvious, that application of these equations for determination of elastic properties of composite material by means of (2.17) formulas leads to huge dependences, which are not useful for qualitative analysis of the results. Let derive formulas for elastic properties by another way, taking into consideration that composite material is statically definable system.

Let stresses $\sigma_x,\sigma_y,\tau_{xy}$ act in composite material element (Fig. 2.3). Then in 1, 2 axes we obtain:

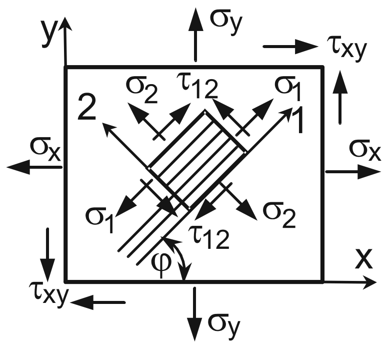


Fig. 2.3. Determination of elastic properties of composite material with arbitrary reinforcing angle

$$\sigma_1 = \sigma_x \cos^2\varphi + \sigma_y \sin^2\varphi + \tau_{xy} \sin 2\varphi;$$
$$\sigma_2 = \sigma_x \sin^2\varphi + \sigma_y \cos^2\varphi - \tau_{xy} \sin 2\varphi; \qquad (2.23)$$
$$\tau_{12} = \left(\sigma_y - \sigma_x\right)\sin\varphi\cos\varphi + \tau_{xy} \cos 2\varphi.$$

These stresses stipulates strains $\varepsilon_1, \varepsilon_2, \gamma_{12}$:

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - \mu_{21}\frac{\sigma_2}{E_2} = \sigma_x\left(\frac{\cos^2\varphi}{E_1} - \mu_{21}\frac{\sin^2\varphi}{E_2}\right) +$$
$$+\sigma_y\left(\frac{\sin^2\varphi}{E_1} - \mu_{21}\frac{\cos^2\varphi}{E_2}\right) + \tau_{xy}\sin 2\varphi\left(\frac{1}{E_1} + \frac{\mu_{21}}{E_2}\right); \qquad (2.24)$$
$$\varepsilon_2 = \sigma_x\left(\frac{\sin^2\varphi}{E_2} - \mu_{21}\frac{\cos^2\varphi}{E_1}\right) + \sigma_y\left(\frac{\cos^2\varphi}{E_2} - \mu_{21}\frac{\sin^2\varphi}{E_1}\right) - \tau_{xy}\sin 2\varphi\left(\frac{1}{E_2} + \frac{\mu_{21}}{E_1}\right);$$
$$\gamma_{12} = -\sigma_x\frac{\sin\varphi\cos\varphi}{G_{12}} + \sigma_y\frac{\sin\varphi\cos\varphi}{G_{12}} + \tau_{xy}\frac{\cos 2\varphi}{G_{12}}.$$

Strains in axes *x, y* can be calculated by the formulas:

$$\varepsilon_x = \varepsilon_1\cos^2\varphi + \varepsilon_2\sin^2\varphi - \gamma_{12}\sin\varphi\cos\varphi;$$
$$\varepsilon_y = \varepsilon_1\sin^2\varphi + \varepsilon_2\cos^2\varphi + \gamma_{12}\sin\varphi\cos\varphi; \qquad (2.25)$$
$$\gamma_{xy} = \left(\varepsilon_1 - \varepsilon_2\right)\sin 2\varphi + \gamma_{12}\cos 2\varphi,$$

which after substitution with expressions (2.24) and some transformations will obtain the form:

$$\varepsilon_x = \sigma_x\left[\frac{\cos^4\varphi}{E_1} + \frac{\sin^4\varphi}{E_2} + \sin^2\varphi\cos^2\varphi\left(\frac{1}{G_{12}} - 2\frac{\mu_{12}}{E_1}\right)\right] +$$
$$+\sigma_y\left[\sin^2\varphi\cos^2\varphi\left(\frac{1}{E_1} + \frac{1}{E_2} + 2\frac{\mu_{12}}{E_1} - \frac{1}{G_{12}}\right) - \frac{\mu_{12}}{E_1}\right] +$$
$$+\tau_{xy}\sin 2\varphi\left(\frac{1+\mu_{12}}{E_1}\cos^2\varphi - \frac{1+\mu_{21}}{E_2}\sin^2\varphi - \frac{\cos 2\varphi}{2G_{12}}\right);$$

$$\varepsilon_y = \sigma_x\left[\sin^2\varphi\cos^2\varphi\left(\frac{1}{E_1} + \frac{1}{E_2} + 2\frac{\mu_{12}}{E_1} - \frac{1}{G_{12}}\right) - \frac{\mu_{12}}{E_1}\right] +$$
$$+\sigma_y\left[\frac{\sin^4\varphi}{E_1} + \frac{\cos^4\varphi}{E_2} + \sin^2\varphi\cos^2\varphi\left(\frac{1}{G_{12}} - 2\frac{\mu_{12}}{E_1}\right)\right] +$$

$$+\tau_{xy}\sin 2\varphi\left(\frac{1+\mu_{12}}{E_1}\sin^2\varphi-\frac{1+\mu_{21}}{E_2}\cos^2\varphi-\frac{\cos 2\varphi}{2G_{12}}\right);$$

(2.26)

$$\gamma_{xy}=\sigma_x\sin 2\varphi\left(\frac{1+\mu_{12}}{E_1}\cos^2\varphi-\frac{1+\mu_{21}}{E_2}\sin^2\varphi-\frac{\cos 2\varphi}{2G_{12}}\right)+$$
$$+\ \sigma_y\sin 2\varphi\left(\frac{1+\mu_{12}}{E_1}\sin^2\varphi-\frac{1+\mu_{21}}{E_2}\cos^2\varphi+\frac{\cos 2\varphi}{2G_{12}}\right)+$$
$$+\tau_{xy}\left[\sin^2 2\varphi\left(\frac{1}{E_1}+\frac{1}{E_2}+2\frac{\mu_{12}}{E_1}\right)+\frac{\cos^2 2\varphi}{G_{12}}\right].$$

Following relationships can be obtained after comparison coefficients at stresses in this equations system with general notation of physical law (2.1):

$$\frac{1}{E_x}=\frac{\cos^4\varphi}{E_1}+\frac{\sin^4\varphi}{E_2}+\sin^2\varphi\cos^2\varphi\left(\frac{1}{G_{12}}-\frac{2\mu_{12}}{E_1}\right);$$
$$\frac{1}{E_y}=\frac{\sin^4\varphi}{E_1}+\frac{\cos^4\varphi}{E_2}+\sin^2\varphi\cos^2\varphi\left(\frac{1}{G_{12}}-\frac{2\mu_{12}}{E_1}\right);\qquad (2.27)$$
$$\frac{1}{G_{xy}}=\sin^2 2\varphi\left(\frac{1+\mu_{12}}{E_1}+\frac{1+\mu_{21}}{E_2}-\frac{1}{G_{12}}\right)+\frac{1}{G_{12}}.$$

$$\frac{\mu_{xy}}{E_x}=\frac{\mu_{yx}}{E_y}=\frac{\mu_{12}}{E_1}-\sin^2\varphi\cos^2\varphi\left(\frac{1+\mu_{12}}{E_1}+\frac{1+\mu_{21}}{E_2}-\frac{1}{G_{12}}\right);$$
$$\frac{\eta_{x,xy}}{G_{xy}}=\frac{\eta_{xy,x}}{E_x}=\sin 2\varphi\left(\frac{1+\mu_{12}}{E_1}\cos^2\varphi-\frac{1+\mu_{21}}{E_2}\sin^2\varphi-\frac{\cos 2\varphi}{2G_{12}}\right);\qquad (2.28)$$
$$\frac{\eta_{y,xy}}{G_{xy}}=\frac{\eta_{xy,y}}{E_y}=\sin 2\varphi\left(\frac{1+\mu_{12}}{E_1}\sin^2\varphi-\frac{1+\mu_{21}}{E_2}\cos^2\varphi+\frac{\cos 2\varphi}{2G_{12}}\right).$$

Graphical dependences of $E_x(\varphi)$, $E_y(\varphi)$, $G_{xy}(\varphi)$, $\mu_{xy}(\varphi)$, $\eta_{x,xy}(\varphi)$, $\eta_{y,xy}(\varphi)$ (Fig. 2.4) shows that reinforcing angle change influences material elastic properties significantly. Formulas (2.27) and (2.28) are proved experimentally and show enough validity for majority of composites.

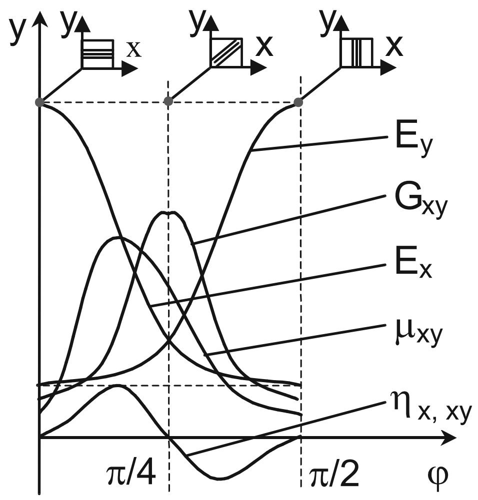


Fig. 2.4. Dependence of unidirectional composite material elastic properties on reinforcing direction

Example 2.2. Pack consists of two layers of the same material with reinforcing along the axes ***x*** and ***y****.* This composite material are usually called orthogonal reinforced composite.

Rigidity characteristics of package can be obtained from formulas (2.12), taking into consideration (2.9) ones (n=2, $\delta_1=\delta_1$, $\delta_2=\delta_2$, $\varphi_1=0$, $\varphi_2=0$, $\delta_\Sigma=\delta_1+\delta_2$):

$$B_{11}=\delta_1\bar{E}_1+\delta_2\bar{E}_2;\ \ B_{12}=(\delta_1+\delta_2)\bar{E}_1\mu_{21};\ \ B_{22}=\delta_1\bar{E}_2+\delta_2\bar{E}_1;$$
$$B_{33}=(\delta_1+\delta_2)G_{12};\ \ B_{13}=B_{31}=B_{23}=B_{32}=0. \qquad (2.29)$$

Elastic constants can be obtained by formulas (2.21) and (2.29) because of material orthotropy:

$$E_x=\frac{1}{\delta_1+\delta_2}\left(B_{11}-\frac{B_{12}^2}{B_{22}}\right)=\frac{\delta_1\bar{E}_1+\delta_2\bar{E}_2}{\delta_1+\delta_2}-\frac{(\delta_1+\delta_2)\bar{E}_1^2\mu_{21}^2}{\delta_1\bar{E}_2+\delta_2\bar{E}_1};$$
$$E_y=\frac{1}{\delta_1+\delta_2}\left(B_{22}-\frac{B_{12}^2}{B_{11}}\right)=\frac{\delta_1\bar{E}_2+\delta_2\bar{E}_1}{\delta_1+\delta_2}-\frac{(\delta_1+\delta_2)\bar{E}_1^2\mu_{21}^2}{\delta_1\bar{E}_1+\delta_2\bar{E}_2}; \qquad (2.30)$$
$$G_{xy}=G_{12};\qquad \mu_{xy}=\frac{B_{12}}{B_{22}}=\frac{(\delta_1+\delta_2)\bar{E}_1\mu_{21}}{\delta_1\bar{E}_2+\delta_2\bar{E}_1}.$$

Let introduce the following notations:

$\Psi_1=\dfrac{\delta_1}{\delta_1+\delta_2}$ - volume fraction of longitudinal layers,

$\Psi_2=\dfrac{\delta_2}{\delta_1+\delta_2}=1-\Psi_1$ - volume fraction of lateral layers.

Taking into consideration these notations previous expressions will take the form:

$$E_x = \Psi_1 \bar{E}_1 + (1-\Psi_1)\bar{E}_2 - \frac{\bar{E}_1^2 \mu_{21}^2}{\Psi_1 \bar{E}_2 + (1-\Psi_1)\bar{E}_1};$$

$$E_y = \Psi_1 \bar{E}_2 + (1-\Psi_1)\bar{E}_1 - \frac{\bar{E}_1^2 \mu_{21}^2}{\Psi_1 \bar{E}_1 + (1-\Psi_1)\bar{E}_2}; \qquad (2.31)$$

$$G_{xy} = G_{12}; \qquad \mu_{xy} = \frac{\bar{E}_1 \mu_{21}}{\Psi_1 \bar{E}_2 + (1-\Psi_1) E_1}.$$

These dependences evident about invariance of elastic constants of perpendicular reinforced composites to absolute pack thickness, but elastic constants depend on relative ratio of these layers thickness (Fig. 2.5). In some cases to determine elasticity modulus of structure [0°,90°] in practical calculations the well-known rule of mixture can be used, i.e.

$$E_x^* = \Psi_1 E_1 + \Psi_2 E_2 = \Psi_1 E_1 + (1-\Psi_1) E_2. \qquad (2.32)$$

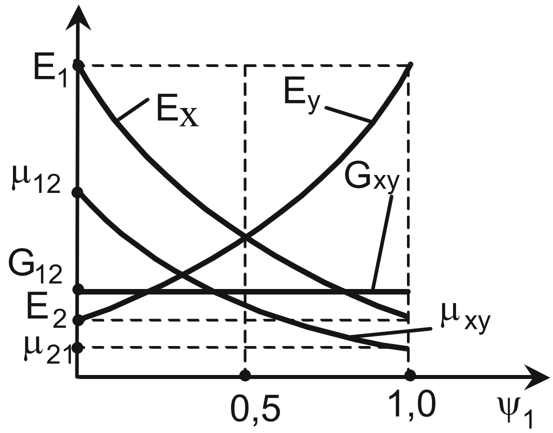


Fig. 2.5. Elastic constants of orthogonal reinforced composite

Error of such kind calculation lays in the range 5…10%, that is permissible for design stage.

Example 2.3. Pack consists of two layers of the same material and same thickness $\delta_1 = \delta_2 = \delta/2$ and reinforcing angles $\varphi_1 = -\varphi_2 = \varphi$. Such materials are called cross-plied composites.

Let find rigidity characteristics of the pack by formulas (2.12):

$$B_{11} = \delta\left(\bar{E}_1 \cos^4\varphi + 2\bar{E}_1\mu_{21}\sin^2\varphi\cos^2\varphi + E_2\sin^4\varphi + G_{12}\sin^2 2\varphi\right);$$

$$B_{12} = \delta\left[\left(\bar{E}_1 + \bar{E}_2\right)\sin^2\varphi\cos^2\varphi + \bar{E}_1\mu_{21}\left(\sin^4\varphi + \cos^4\varphi\right) - G_{12}\sin^2 2\varphi\right];$$

$$B_{22} = \delta\left(\bar{E}_1 \sin^4\varphi + 2\bar{E}_1\mu_{21}\sin^2\varphi\cos^2\varphi + E_2\cos^4\varphi + G_{12}\sin^2 2\varphi\right); \qquad (2.33)$$

$$B_{33} = \delta\left[\left(\bar{E}_1 + \bar{E}_2 - 2\bar{E}_1\mu_{21}\right)\sin^2\varphi\cos^2\varphi + G_{12}\cos^2 2\varphi\right];$$

$$B_{13} = B_{31} = B_{23} = B_{32} = 0.$$

This material is orthotropic in axes ***x*** and ***y***, for it coefficients B_{13}, B_{31}, B_{23},

B_{32} are equal to zero.

Elastic constants are calculated by formulas (2.21). Comparison of dependences (2.22) and (2.33) shows that coefficients B_{11}, B_{22}, B_{12}, B_{33} are equal to zero for composites with structures $[+\varphi]$ and $[\pm\varphi]$. In connection with abovementioned fact, it is interesting to know, which of these materials has the larger elasticity modulus. Let consider, for example, E_x. The first formula of the system (2.17) one can write after definite transformations:

$$E_x = \frac{1}{\delta}\left[B_{11} - \frac{B_{12}^2}{B_{22}} - \frac{\left(B_{22}B_{13} - B_{12}B_{23}\right)^2}{B_{22}\left(B_{22}B_{33} - B_{23}^2\right)}\right]. \qquad (2.34)$$

Comparison of this expressions with the following ones from (2.18) for composite with reinforcing $[\pm\varphi]$

$$E_x = \frac{1}{\delta}\left(B_{11} - \frac{B_{12}^2}{B_{22}}\right) \qquad (2.35)$$

shows, that cross-plied composite material has large rigidity. Thus it is more efficient to use structure $[\pm\varphi]$ instead of skew reinforcing $[+\varphi]$ or $[-\varphi]$ at the same structure thickness (i.e. structure mass).

Example 2.4. Pack consists of four layers of the same material: $n = 4$, $\delta_1 = \delta_1$, $\delta_2 = \delta_2$, $\delta_3 = \delta_4 = \delta$, $\varphi_1 = 0°$, $\varphi_2 = 90°$, $\varphi_3 = -\varphi_4 = \varphi$.

Such kind of structure is frequently used in composite constructions.

By formulas (2.12) we can obtain:

$$\begin{aligned}
&B_{11} = \delta_1\bar{E}_1 + \delta_2\bar{E}_2 + 2\delta\left(\bar{E}_1\cos^4\varphi + \bar{E}_2\sin^4\varphi + 2\bar{E}_1\mu_{21}\sin^2\varphi\cos^2\varphi + G_{12}\sin^2 2\varphi\right);\\
&B_{22} = \delta_1\bar{E}_2 + \delta_2\bar{E}_1 + 2\delta\left(\bar{E}_1\sin^4\varphi + \bar{E}_2\cos^4\varphi + 2\bar{E}_1\mu_{21}\sin^2\varphi\cos^2\varphi + G_{12}\sin^2 2\phi\right)\varphi;\\
&B_{12} = \left(\delta_1 + \delta_2\right)\bar{E}_1\mu_{21} + 2\delta\left[\left(\bar{E}_1 + \bar{E}_2\right)\sin^2\varphi\cos^2\varphi + \bar{E}_1\mu_{21}\left(\sin^4\varphi + \cos^4\varphi\right) - G_{12}\sin^2 2\varphi\right]; \qquad (2.36)\\
&B_{33} = \left(\delta_1 + \delta_2\right)G_{12} + 2\delta\left[\left(\bar{E}_1 + \bar{E}_2 - 2\bar{E}_1\mu_{21}\right)\sin^2\varphi\cos^2\varphi + G_{12}\cos^2 2\varphi\right];\\
&B_{13} = B_{23} = B_{31} = B_{32} = 0.
\end{aligned}$$

Find elastic constants from (2.20) equations. If we introduce notations

$$\psi_1 = \delta_1 / \delta_\Sigma; \qquad \psi_2 = \delta_2 / \delta_\Sigma; \qquad (2.37)$$

where

$$\delta_\Sigma = \delta_1 + \delta_2 + 2\delta, \qquad (2.38)$$

that

$$2\delta = \delta_\Sigma\left(1 - \psi_1 - \psi_2\right). \qquad (2.39)$$

This permits to rewrite (2.36) and (2.20) in the form:

$$B_{11}=\delta_\Sigma\Big[\psi_1\bar{E}_1+\psi_2\bar{E}_2+(1-\psi_1-\psi_2)\Big(\bar{E}_1\cos^4\varphi+\bar{E}_2\sin^4\varphi+$$
$$+2\bar{E}_1\mu_{21}\sin^2\varphi\cos^2\varphi+G_{12}\sin^2 2\varphi\Big)\Big]=\delta_\Sigma\bar{B}_{11};$$
$$B_{22}=\delta_\Sigma\Big[\psi_1\bar{E}_2+\psi_2\bar{E}_1+(1-\psi_1-\psi_2)\Big(\bar{E}_1\sin^4\varphi+\bar{E}_2\cos^4\varphi+$$
$$+2\bar{E}_1\mu_{21}\sin^2\varphi\cos^2\varphi+G_{12}\sin^2 2\varphi\Big)\Big]=\delta_\Sigma\bar{B}_{22};\qquad(2.40)$$
$$B_{12}=\delta_\Sigma\Big[(\psi_1+\psi_2)\bar{E}_1\mu_{21}+(1-\psi_1-\psi_2)\Big[\Big(\bar{E}_1+\bar{E}_2\Big)\times\sin^2\varphi\times$$
$$\times\cos^2\varphi+\bar{E}_1\mu_{21}\Big(\sin^4\varphi+\cos^4\varphi\Big)-G_{12}\sin^2 2\varphi\Big]\Big]=\delta_\Sigma\bar{B}_{12};$$
$$B_{33}=\delta_\Sigma\Big[(\psi_1+\psi_2)G_{12}+(1-\psi_1-\psi_2)\Big[\Big(\bar{E}_1+\bar{E}_2-2\bar{E}_1\mu_{21}\Big)\times$$
$$\times\sin^2\varphi\cos^2\varphi+G_{12}\cos^2 2\varphi\Big]\Big]=\delta_\Sigma\bar{B}_{33};$$
$$E_x=\bar{B}_{11}-\frac{\bar{B}_{12}^2}{\bar{B}_{22}};\qquad E_y=\bar{B}_{22}-\frac{\bar{B}_{12}^2}{\bar{B}_{11}};\qquad G_{xy}=\bar{B}_{33};$$
$$\mu_{xy}=\frac{\bar{B}_{12}}{\bar{B}_{22}};\qquad \mu_{yx}=\frac{\bar{B}_{12}}{\bar{B}_{11}}\ .$$

Thus, elasticity moduli, Poisson's ratios of such kind of composite material do not depend on absolute pack thickness, but depend on layers thickness ratio.

Obtained above formulas for determination of set of elastic constants of laminated composite material with any structure are classical now and all analysis and design of composite structure can be provided by means of these formulas. This conclusion is based on following fact: in local coordinate system each layer is orthotropic, i.e. this layer must not be unidirectional. Examples of this structures are layers based on woven reinforcement; groups of layers for which axes 1, 2 are orthotropy axes and properties of these groups of layers are known in these axes; braided fabrics which frequently have reinforcement $[\pm\varphi]$, and isotropic materials, for example, metal sheets.

2.2. Thermomechanical characteristics of laminated composites

2.2.1. Linear temperature expansion coefficients of laminated composites

Composite pack of layers obtains temperature deformations $\alpha_x\Delta T$, $\alpha_y\Delta T$, $\alpha_{xy}\Delta T$ (Fig. 2.6), which are sequence of temperature deformations of layers at temperature change.

If all layers deform all together, that it is obvious, at pack of layers arbitrary reinforcing layers restrict each other to deform free because of presence of individual LCTE α_{1i} and α_{2i}. Because of this fact stresses appear in layers, for entire pack this system of stresses is self-balanced.

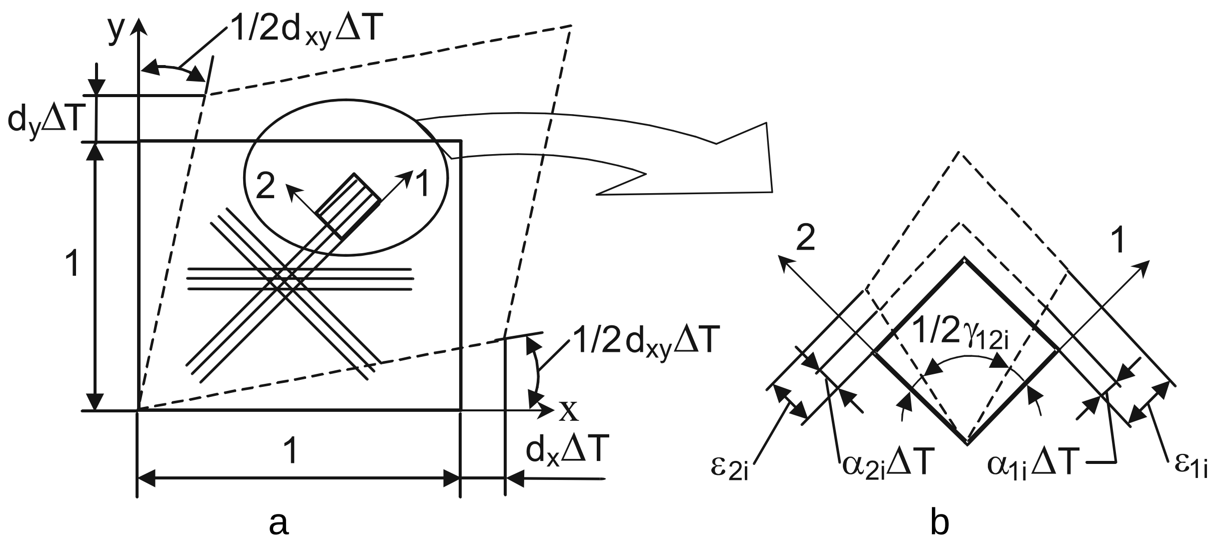


Fig. 2.6. Temperature deformation of composite material

The following layers strains corresponds to pack deformations $\alpha_x\Delta T$, $\alpha_y\Delta T$, $\alpha_{xy}\Delta T$:

$$\varepsilon_{1i} = \Delta T\left(\alpha_x \cos^2\varphi_i + \alpha_y \sin^2\varphi_i + \alpha_{xy}\sin\varphi_i\cos\varphi_i\right);$$
$$\varepsilon_{2i} = \Delta T\left(\alpha_x \sin^2\varphi_i + \alpha_y \cos^2\varphi_i - \alpha_{xy}\sin\varphi_i\cos\varphi_i\right); \qquad (2.41)$$
$$\gamma_{12i} = \Delta T\left[\left(\alpha_y - \alpha_x\right)\sin 2\varphi_i + \alpha_{xy}\cos 2\varphi_i\right].$$

Equations of generalized Hook's law for individual layer according to Duamel-Neumann hypothesis can be written:

$$\varepsilon_{1i} = \frac{\sigma_{1i}}{E_{1i}} - \mu_{21i}\frac{\sigma_{2i}}{E_{2i}} + \alpha_{1i}\Delta T;$$
$$\varepsilon_{2i} = \frac{\sigma_{2i}}{E_{2i}} - \mu_{12i}\frac{\sigma_{1i}}{E_{1i}} + \alpha_{2i}\Delta T; \qquad (2.42)$$
$$\gamma_{12i} = \frac{\tau_{12i}}{G_{12i}}.$$

One can obtain, solving this system related to stresses:

$$\sigma_{1i} = \bar{E}_{1i}\left[\left(\varepsilon_{1i} - \alpha_{1i}\Delta T\right) + \mu_{21i}\left(\varepsilon_{2i} - \alpha_{2i}\Delta T\right)\right];$$
$$\sigma_{2i} = \bar{E}_{2i}\left[\left(\varepsilon_{2i} - \alpha_{2i}\Delta T\right) + \mu_{21i}\left(\varepsilon_{1i} - \alpha_{1i}\Delta T\right)\right]; \qquad (2.43)$$
$$\tau_{12i} = G_{12i}\gamma_{12i}.$$

Differences $\left(\varepsilon_{1i} - \alpha_{1i}\Delta T\right)$ and $\left(\varepsilon_{2i} - \alpha_{2i}\Delta T\right)$ are deformations, which correspond to stresses σ_{1i} and σ_{2i}.

Let compose equilibrium equations of pack, taking into consideration formulas of stresses rotation (2.7) and absence of internal loads:

$$N_x = \sum_{i=1}^{n} \sigma_{xi}\delta_i = \sum_{i=1}^{n} \delta_i \left(\sigma_{1i} \cos^2 \varphi_i + \sigma_{2i} \sin^2 \varphi_i - \tau_{12i} \sin 2\varphi_i \right) = 0;$$

$$N_y = \sum_{i=1}^{n} \sigma_{yi}\delta_i = \sum_{i=1}^{n} \delta_i \left(\sigma_{1i} \sin^2 \varphi_i + \sigma_{2i} \cos^2 \varphi_i + \tau_{12i} \sin 2\varphi_i \right) = 0; \qquad (2.44)$$

$$q_{xy} = \sum_{i=1}^{n} \tau_{xyi}\delta_i = \sum_{i=1}^{n} \delta_i \left[\left(\sigma_{1i} - \sigma_{2i} \right) \sin\varphi_i \cos\varphi_i + \tau_{12i} \cos 2\varphi_i \right] = 0 \ .$$

Substitute dependences (2.41) to (2.43), and obtained result to (2.44). Equilibrium equations (2.44) obtain the following form after series of transformations:

$$\alpha_x B_{11} + \alpha_y B_{12} + \alpha_{xy} B_{13} = A_{T1};$$

$$\alpha_x B_{21} + \alpha_y B_{22} + \alpha_{xy} B_{23} = A_{T2}; \qquad (2.45)$$

$$\alpha_x B_{31} + \alpha_y B_{32} + \alpha_{xy} B_{33} = A_{T3},$$

where B_{ij} coefficients are defined by (2.12), but coefficients A_{T1}, A_{T2}, A_{T3} - according to formulas:

$$A_{T1} = \sum_{i=1}^{n} \delta_i a_{T1i} = \sum_{i=1}^{n} \delta_i \left[\alpha_{1i}\bar{E}_{1i} \left(\cos^2 \varphi_i + \mu_{21i} \sin^2 \varphi_i \right) + \alpha_{2i}\bar{E}_{2i} \left(\sin^2 \varphi_i + \mu_{12i} \cos^2 \varphi_i \right) \right];$$

$$A_{T2} = \sum_{i=1}^{n} \delta_i a_{T2i} = \sum_{i=1}^{n} \delta_i \left[\alpha_{1i}\bar{E}_{1i} \left(\sin^2 \varphi_i + \mu_{21i} \cos^2 \varphi_i \right) + \alpha_{2i}\bar{E}_{2i} \left(\cos^2 \varphi_i + \mu_{12i} \sin^2 \varphi_i \right) \right]; \quad (2.46)$$

$$A_{T3} = \sum_{i=1}^{n} \delta_i a_{T3i} = \sum_{i=1}^{n} \delta_i \sin\varphi_i \cos\varphi_i \left[\alpha_{1i}\bar{E}_{1i} \left(1 - \mu_{21i} \right) - \alpha_{2i}\bar{E}_{2i} \left(1 - \mu_{12i} \right) \right].$$

From system of equation (2.45) one can derive formulas for determination of linear expansion coefficient:

$$\alpha_x = \frac{1}{B}\left[A_{T1}\left(B_{22}B_{33} - B_{23}^2 \right) + A_{T2}\left(B_{13}B_{23} - B_{33}B_{12} \right) + A_{T3}\left(B_{12}B_{23} - B_{22}B_{13} \right) \right];$$

$$\alpha_y = \frac{1}{B}\left[A_{T1}\left(B_{12}B_{33} - B_{13}B_{23} \right) + A_{T2}\left(B_{13}^2 - B_{11}B_{33} \right) + A_{T3}\left(B_{11}B_{23} - B_{12}B_{13} \right) \right]; \quad (2.47)$$

$$\alpha_{xy} = \frac{1}{B}\left[A_{T1}\left(B_{12}B_{23} - B_{22}B_{13} \right) + A_{T2}\left(B_{12}B_{13} - B_{11}B_{23} \right) + A_{T3}\left(B_{11}B_{22} - B_{12}^2 \right) \right].$$

For orthotropic in axes ***x***, ***y*** laminated composite:

$$B_{13} = B_{31} = B_{23} = B_{32} = A_{T3} = 0.$$

Thus for orthotropic composites (2.47) formulas transform to form:

$$\alpha_x = \frac{A_{T1}B_{22} - A_{T2}B_{12}}{B_{11}B_{22} - B_{12}^2}; \quad \alpha_y = \frac{A_{T2}B_{11} - A_{T1}B_{12}}{B_{11}B_{22} - B_{12}^2}; \quad \alpha_{xy} = 0. \quad (2.48)$$

Equality $\alpha_{xy} = 0$ means, that orthotropic composite does not warp at heating, i.e. shear deformation does not appear. However, it does not mean absence of shear stresses in separate layers. These stresses can be determined from (2.43).

2.2.2. Shrinkage coefficients of laminated composites

Scheme of composite material deformation at shrinkage of its components in polymerization process is analogous to deformation scheme at temperature change as was shown above. That is why we write the final results, skipping intermediate transformations. Equation system for determination of shrinkage coefficients ξ_x, ξ_y, ξ_{xy} has the form:

$$\begin{aligned} &\xi_x B_{11} + \xi_y B_{12} + \xi_{xy} B_{13} = A_{y1}; \\ &\xi_x B_{21} + \xi_y B_{22} + \xi_{xy} B_{23} = A_{y2}; \\ &\xi_x B_{31} + \xi_y B_{32} + \xi_{xy} B_{33} = A_{y3}, \end{aligned} \tag{2.49}$$

where

$$\begin{aligned} A_{y1} &= \sum_{i=1}^{n} \delta_i a_{y1i} = \sum_{i=1}^{n} \delta_i \Big[\xi_{1i} \bar{E}_{1i} \left(\cos^2 \varphi_i + \mu_{21i} \sin^2 \varphi_i \right) + \\ &\qquad + \xi_{21} \bar{E}_{2i} \left(\sin^2 \varphi_i + \mu_{12i} \cos^2 \varphi_i \right) \Big]; \\ A_{y2} &= \sum_{i=1}^{n} \delta_i a_{y2i} = \sum_{i=1}^{n} \delta_i \Big[\xi_{1i} \bar{E}_{1i} \left(\sin^2 \varphi_i + \mu_{21i} \cos^2 \varphi_i \right) + \\ &\qquad + \xi_{21} \bar{E}_{2i} \left(\cos^2 \varphi_i + \mu_{12i} \sin^2 \varphi_i \right) \Big]; \end{aligned} \tag{2.50}$$

$$A_{y3} = \sum_{i=1}^{n} \delta_i a_{y3i} = \sum_{i=1}^{n} \delta_i \Big[\sin\varphi_i \cos\varphi_i \big[\xi_{1i} \bar{E}_{1i} (1 - \mu_{21i}) - \xi_{2i} \bar{E}_{2i} (1 - \mu_{12i}) \big] \Big].$$

From the equations system (49) we find ξ_x, ξ_y, ξ_{xy}:

$$\begin{aligned} \xi_x &= \frac{1}{B} \Big[A_{y1} \left(B_{22} B_{33} - B_{23}^2 \right) + A_{y2} \left(B_{13} B_{23} - B_{33} B_{12} \right) + A_{y3} \left(B_{11} B_{23} - B_{22} B_{13} \right) \Big]; \\ \xi_y &= \frac{1}{B} \Big[A_{y1} \left(B_{12} B_{33} - B_{13} B_{23} \right) + A_{y2} \left(B^2_{13} - B_{11} B_{33} \right) + A_{y3} \left(B_{11} B_{23} - B_{12} B_{13} \right) \Big]; \\ \xi_{xy} &= \frac{1}{B} \Big[A_{y1} \left(B_{12} B_{23} - B_{22} B_{13} \right) + A_{y2} \left(B_{12} B_{13} - B_{11} B_{23} \right) + A_{y3} \left(B_{11} B_{22} - B_{12}^2 \right) \Big]. \end{aligned} \tag{2.51}$$

Shrinkage coefficients of orthotropic composite are defined by formulas:

$$\xi_x = \frac{A_{y1} B_{22} - A_{y2} B_{12}}{B_{11} B_{22} - B_{12}^2}; \qquad \xi_x = \frac{A_{y2} B_{11} - A_{y1} B_{12}}{B_{11} B_{22} - B_{12}^2}; \qquad \xi_{xy} = 0. \tag{2.52}$$

In conclusion we write notation of physical law, taking into consideration temperature and shrinkage deformations in accordance with Duamel-Neumann hypothesis:

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \mu_{yx}\frac{\sigma_y}{E_y} + \eta_{xy,x}\frac{\tau_{xy}}{G_{xy}} + \alpha_x \Delta T + \xi_x;$$

$$\varepsilon_y = -\mu_{xy}\frac{\sigma_x}{E_x} + \frac{\sigma_y}{E_y} + \eta_{xy,y}\frac{\tau_{xy}}{G_{xy}} + \alpha_y \Delta T + \xi_y; \qquad (2.53)$$

$$\gamma_{xy} = \eta_{x,xy}\frac{\sigma_x}{E_x} + \eta_{y,xy}\frac{\sigma_y}{E_y} + \frac{\tau_{xy}}{G_{xy}} + \alpha_{xy} \Delta T + \xi_{xy}.$$

For composite material orthotropic in axes x, y formulas (2.53) transform to the following appearance:

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \mu_{yx}\frac{\sigma_y}{E_y} + \alpha_x \Delta T + \xi_x;$$

$$\varepsilon_y = -\mu_{xy}\frac{\sigma_x}{E_x} + \frac{\sigma_y}{E_y} + \alpha_y \Delta T + \xi_y; \qquad (2.54)$$

$$\xi_{xy} = \frac{\tau_{xy}}{G_{xy}}.$$

Reverse notations of these systems (after solution related to stresses) have the form:

$$\sigma_x = \frac{1}{\delta_\Sigma}\left(B_{11}\varepsilon_x^* + B_{12}\varepsilon_y^* + B_{13}\gamma_{xy}^*\right);$$

$$\sigma_y = \frac{1}{\delta_\Sigma}\left(B_{21}\varepsilon_x^* + B_{22}\varepsilon_y^* + B_{23}\gamma_{xy}^*\right); \qquad (2.55)$$

$$\tau_{xy} = \frac{1}{\delta_\Sigma}\left(B_{31}\varepsilon_x^* + B_{32}\varepsilon_y^* + B_{33}\gamma_{xy}^*\right),$$

where

$$\varepsilon_x^* = \varepsilon_x - \alpha_x \Delta T - \xi_x;$$

$$\varepsilon_y^* = \varepsilon_y - \alpha_y \Delta T - \xi_y; \qquad (2.56)$$

$$\gamma_{xy}^* = \gamma_{xy} - \alpha_{xy} \Delta T - \xi_{xy}.$$

For orthotropic composite these formulas transform to:

$$\sigma_x = \frac{1}{\delta_\Sigma}\left(B_{11}\varepsilon_x^* + B_{12}\varepsilon_y^*\right);$$

$$\sigma_y = \frac{1}{\delta_\Sigma}\left(B_{12}\varepsilon_x^* + B_{22}\varepsilon_y^*\right); \qquad (2.57)$$

$$\tau_{xy} = \frac{1}{\delta_\Sigma} B_{33}\gamma_{xy}^*,$$

where

$$\gamma_{xy}^{*} = \gamma_{xy}. \qquad (2.58)$$

Thus, obtained dependences for determination of elastic constants of unidirectional and laminated composite materials permit to express the single meaning of stress by means of strain and vice versa. If metal elastic characteristics can be found in guidebooks that for composite materials it is necessary to define these properties by means of known physical and mechanical characteristics of composite material components (for unidirectional materials) or by means of monolayers characteristics obtained theoretically or by experimental way (for laminated composites).

Checking-up questions

1. What does orthotropic composite material mean?
2. Main assumptions used for laminated composite stress analysis in accordance with Vasiliev's model.
3. Write generalized 2-dimensional physical law (Hook's law) in terms of global stress and strains.
4. Write relationships for determination of elastic and thermal constants of laminated composites.
5. What is the physical meaning of coefficients of reciprocal influence?
6. What is the physical meaning of shrinkage coefficients?
7. Derive relationships between global and local stress and strains.
8. What is the typical view of UD-composite elastic properties dependence on reinforcing angle? Draw graphs.

Theme 3. PREDICTION OF STRENGTH PROPERTIES OF LAMINATED COMPOSITES

3.1 Fundamentals of composites strength estimation

To design any structure engineer must estimate its workability either functional point of view or determine strength of construction (or safety factor) at defined level of operational loads. The most valid estimation method of construction strength is experiment, in which real operational conditions can be realized. But it is not always possible to make testing of the structure because of large dimensions (for examples, large ship, broadcasting tower, bridge etc.), non-determined load types or load conditions. Acting stresses are determined by analytical methods based on analysis schemes (models) – rod, beam, plate etc, which quite precisely describe stressed-strained state of structure. The conclusion about article workability is made after comparison of acting stresses with mechanical properties of material.

Strength of construction made of uniform (isotropic) material at simple loading schemes (tension, compression, torsion) can be estimated by comparison of calculated stress with yield stress or with experimentally determined strength. Theories of strength, based on large amount of theoretical and experimental researches at complex types of loading (for example, tension with compression or shear), are used. In general case theories of strength permit to predict structure and its element workability at compatible action of some load types and known strength properties of material at simple loading (ultimate strength at tension, compression, shear) [2, 3, 6].

Notion of strength directly relates to notion of breakage, because strength is ability of construction to withstand definite level of mechanical (thermal-mechanical) loading without breakage. A construction is strong up to appearance of first breakage feature and breakage is the upper margin of structure carrying ability. This margin includes a large amount of factors related to material cracking breakage, loosing stability, fatigue etc.

Description of composite material breaking process becomes more complicated because of large amount of such interrelated forms of breakage as fiber bending, delamination, discontinuities in adhesion between fiber and matrix, binder cracking as a result or temperature stress, low-quality impregnation and other. Above-mentioned phenomena, accompanying breakage, make the problem at microlevel (on the level of interaction of fiber and matrix) consideration more complicated. That is why engineering criterion of breakage or strength criterion, which is analogous to well-known theories of strength, cannot be formulated based on analysis of mechanisms of these phenomena and these phenomena interaction.

Engineer strength criteria are based on date of material macrovolume behavior and strength, i.e. engineer strength criteria have phenomenological character.

Strength (breakage) criteria are worked out to estimate structure carrying ability at complicated stressed state. The most important demands to strength criteria are quite precise description of experimental results and simplicity of application. All engineer criteria are phenomenological (phenomena, which take place on microlevel, influence properties of material macrovolume) now.

There is no unique approach to formulation of strength criteria of composite material. It is stipulated by the following facts:

– complexity of breakage mechanisms;

– dependence of composite properties on technology of composite components preparation and technology of article manufacture;

– not enough data of statistical experiments.

Two following approaches for research of laminated composite material strength are spread widely now.

According to the first approach material consists of uniform and orthotropic connected to each other layers and strength criterion is written for each individual layer. Ultimate carrying ability is defined as beginning of any layer breakage. For mathematical description of strength criterion of monolayer it is necessary to know the following values:

– four elastic constants of individual layer (elasticity moduli at tension-compression in two directions and at shear and one of Poisson's ratios;)

– five strength properties (ultimate strength at tension and compression in two directions and shear strength);

– definite functional dependence between above-mentioned values.

All these data can be determined analytically by means of formulas obtained earlier or take them from guidebooks. The drawback of this approach is impossibility to find the final result – strength properties of entire pack, which are dominant at the stage of selection of structural material class.

According to the second approach (pack of layers is considered to be uniform and isotropic) strength criterion is written for entire pack. In this case it is necessary to know:

– six elastic constants of pack (elasticity moduli at tension-compression in two directions and at shear, Poisson's ratios and coefficients of reciprocal influence);

– five strength characteristics (ultimate strength at tension and compression in two directions and in shear);

– definite functional dependence between above-mentioned strength and elastic properties.

It is obvious from considered above that application of this approach at the stage of composite structure design is possible in the case of presence of prediction method of pack strength properties.

Designer analyses a number of pack structures and material components at the stage of design. That is why prediction methods of composite properties at different design levels are necessary for designer.

The notion of carrying ability of structure includes many aspects – strength, stability, stiffness, long serviceability, durability, survivability and other. But ensuring of carrying ability begins from satisfaction of strength conditions. The essence of strength condition is the following: material must not break in any point of material. That is why strength analysis of composite structure includes determination of presence or absence of breakage of material at definite stressed state (Fig. 3.1, a). In this case designer knows package structure, physical and mechanical properties of layers and forces, applied to entire package.

Generally, each layer is subjected to complex stressed state (Fig. 3.1, b) and for estimation of its properties many worked out criteria are used. But the most wide spread criteria are the following:

a) Criterion of maximum stress consists of absence of breakage in any arbitrary directions, i.e. following conditions should be fulfilled:

$$\text{abs}\ \sigma_{1i} \le F_{1i}; \quad \text{abs}\,\sigma_{2i} \le F_{2i}; \quad \text{abs}\,\tau_{12i} \le F_{12i}, \tag{3.1}$$

where F_1, F_2, F_{12} - ultimate strength values along the orthotropy axes 1, 2 and at shear, at that

$$F_{1i} = \begin{cases} F_{1it} & \text{at } \sigma_{1i} > 0; \\ F_{1ic} & \text{at } \sigma_{1i} < 0; \end{cases}$$

$$F_{2i} = \begin{cases} F_{2it} & \text{at } \sigma_{2i} > 0; \\ F_{2ic} & \text{at } \sigma_{2i} < 0. \end{cases} \tag{3.2}$$

Here indexes «t» and «c» mean tension and compression.

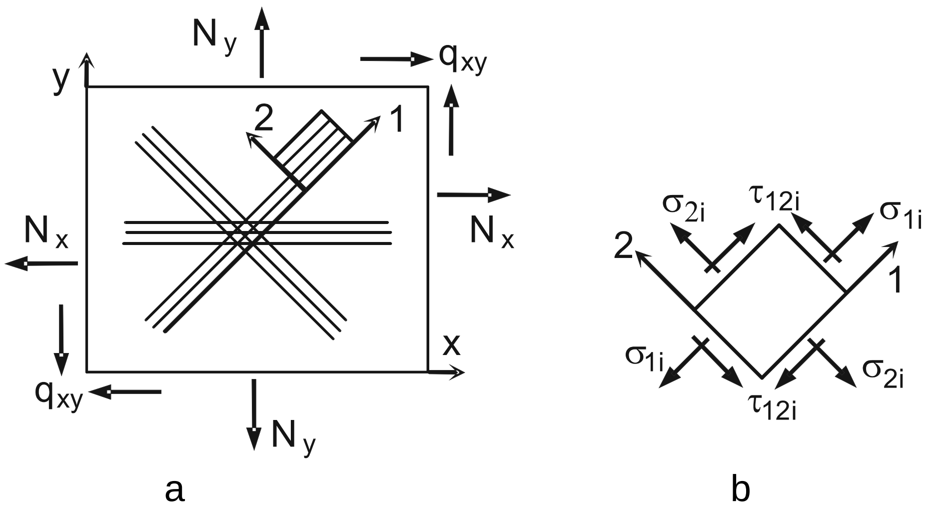


Fig. 3.1. To strength analysis of laminated composites

b) Criterion of maximum strains consists of the following assumption: deformation of material does not exceed ultimate value, i.e.

$$\varepsilon_{1i} \le \frac{F_{1i}}{E_{1i}}; \quad \varepsilon_{2i} \le \frac{F_{2i}}{E_{2i}}; \quad \gamma_{12i} \le \frac{F_{12i}}{G_{12i}}, \tag{3.3}$$

or taking into consideration physical law (3.3):

$$\begin{aligned} &abs\left(\sigma_{1i}-\mu_{12i}\sigma_{2i}\right)\le F_{1i};\\ &abs\left(\sigma_{2i}-\mu_{21i}\sigma_{1i}\right)\le F_{2i};\\ &abs\,\tau_{12i}\le F_{12i}, \end{aligned} \tag{3.4}$$

where F_{1i}, F_{2i} are determined by (3.2).

c) Mises-Hill energy criterion based on assumption: deformation energy does not exceed its ultimate value. Mathematical notation of this criterion has the form

$$\frac{\sigma_{1i}^2}{F_{1i}^2}-\frac{\sigma_{1i}\sigma_{2i}}{F_{1i}F_{2i}}+\frac{\sigma_{2i}^2}{F_{2i}^2}+\frac{\tau_{12i}^2}{F_{12i}^2}\le 1. \tag{3.5}$$

For some composite materials other strength criteria are used, but their application does not differ sharply of above-mentioned ones.

3.2. Strength property of composite monolayer in arbitrary direction

Let consider the method of application of criteria (3.1), (3.3) and (3.5) for prediction of composite monolayer strength in arbitrary direction (Fig. 3.2).

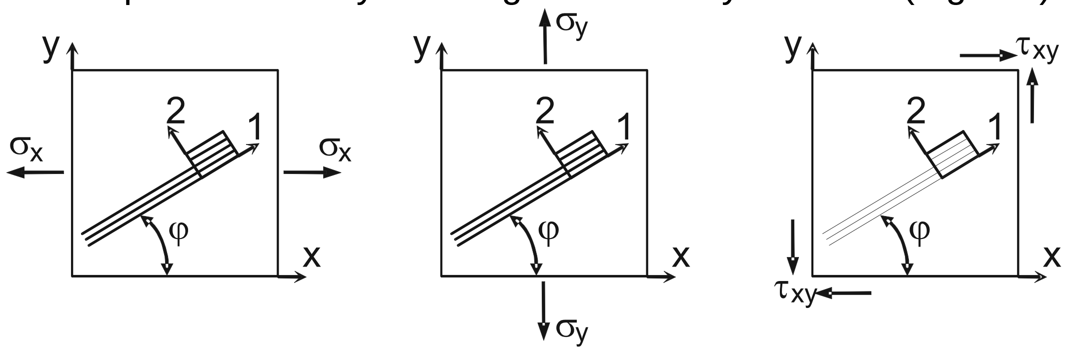


Fig. 3.2. To determination of composite monolayer strength in arbitrary direction

Let find stresses in axes 1, 2 to determine strength along axis ***x*** (Fig. 3.2, a):

$$\sigma_1=\sigma_x\cos^2\varphi;\qquad \sigma_2=\sigma_x\sin^2\varphi;\qquad \tau_{12}=-\sigma_x\sin\varphi\cos\varphi. \tag{3.6}$$

Substitute these expressions to criterion of maximum stress (3.1) and obtain

$$\sigma_x\cos^2\varphi\le F_{1t};\quad \sigma_x\sin^2\varphi\le F_{2t};\quad \sigma_x\sin\varphi\cos\varphi\le F_{12}. \tag{3.7}$$

We obtain system of non-equalities for determination of σ_x ultimate values. In the last non-equality minus sign is lost, because invariance of shear forces to direction for orthotropic composite:

$$\sigma_x\le\frac{F_{1t}}{\cos^2\varphi};\quad \sigma_x\le\frac{F_{2t}}{\sin^2\varphi};\quad \sigma_x\le\frac{F_{12}}{\sin\varphi\cos\varphi}\ . \tag{3.8}$$

Then ultimate tensile strength along ***x*** axis can be written

$$F_{xt} = \min\left(\frac{F_{1t}}{\cos^2\varphi}; \quad \frac{F_{2t}}{\sin^2\varphi}; \quad \frac{F_{12}}{\sin\varphi\cos\varphi} \right). \tag{3.9}$$

By analogous way

$$F_{xc} = \min\left(\frac{F_{1c}}{\cos^2\varphi}; \quad \frac{F_{2c}}{\sin^2\varphi}; \quad \frac{F_{12}}{\sin\varphi\cos\varphi} \right). \tag{3.10}$$

Graphical dependences (3.8) for two materials – unidirectional and woven reinforcements are shown on the Fig. 3.3.

Criterion of maximum stresses permits to predict either value of ultimate strength or breakage type. At $0 \le \varphi \le \varphi_1$ (see Fig. 3.3) tearing of fibers (warp threads of fabric) takes place, at $\varphi_1 \le \varphi \le \varphi_2$ binder breakage from shear in planes parallel to fibers (warp threads) takes place, at $\varphi_1 \le \varphi \le \pi/2$ binder breakage (weft thread of fabric) in lateral direction takes place.

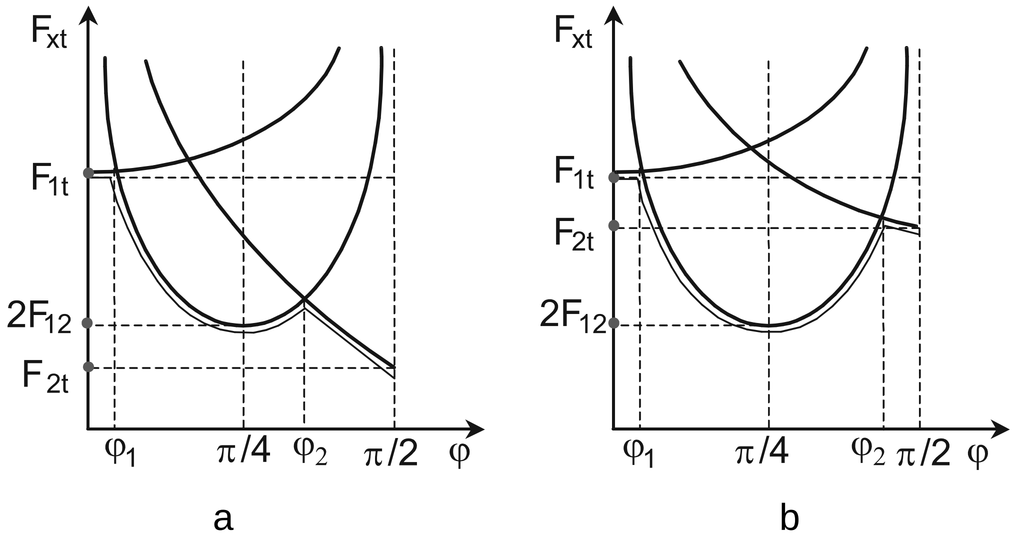


Fig. 3.3. Dependence of composite ultimate strength at tension on reinforcing direction (criterion of maximum stress): a – unidirectional composite; b – composite based on fabric

We obtain the following strength equations after substitution of formulas (3.6) to criterion of maximum strain:

$$\sigma_x \le \frac{F_1}{\mathrm{abs}\left(\cos^2\varphi - \mu_{12}\sin^2\varphi\right)};$$

$$\sigma_x \le \frac{F_2}{\mathrm{abs}\left(\sin^2\varphi - \mu_{21}\cos^2\phi\right)}; \tag{3.11}$$

$$\sigma_x \le \frac{F_{12}}{\sin\varphi\cos\varphi}.$$

Here

$$F_1 = F_{1t} \quad \text{at} \quad \cos^2\varphi - \mu_{12}\sin^2\varphi > 0,$$
$$F_1 = F_{1c} \quad \text{at} \quad \cos^2\varphi - \mu_{12}\sin^2\varphi < 0, \tag{3.12}$$
$$F_2 = F_{2t} \quad \text{at} \quad \sin^2\varphi - \mu_{12}\cos^2\varphi > 0,$$
$$F_2 = F_{2c} \quad \text{at} \quad \sin^2\varphi - \mu_{12}\cos^2\varphi < 0.$$

Ultimate strength at tension along ***x*** axis is calculated by formula

$$F_{xt} = \min\left(\frac{F_1}{\text{abs}\left(\cos^2\varphi - \mu_{12}\sin^2\varphi\right)};\ \frac{F_2}{\text{abs}\left(\sin^2\varphi - \mu_{12}\cos^2\varphi\right)};\ \frac{F_{12}}{\sin\varphi\cos\varphi} \right). \tag{3.13}$$

Graphical dependence of this expression is shown on the Fig. 3.4 and is analogous to previous dependence and permits to predict breakage character.

For ultimate strength at compression we can obtain dependence (3.13), in which:

$$F_1 = F_{1c} \quad \text{at} \quad \cos^2\varphi - \mu_{12}\sin^2\varphi > 0,$$
$$F_1 = F_{1t} \quad \text{at} \quad \cos^2\varphi - \mu_{12}\sin^2\varphi < 0, \tag{3.14}$$
$$F_2 = F_{2c} \quad \text{at} \quad \sin^2\varphi - \mu_{12}\cos^2\varphi > 0,$$
$$F_2 = F_{2t} \quad \text{at} \quad \sin^2\varphi - \mu_{12}\cos^2\varphi < 0.$$

We can obtain the following results after substitution expressions (3.6) to Mises-Hill criterion:

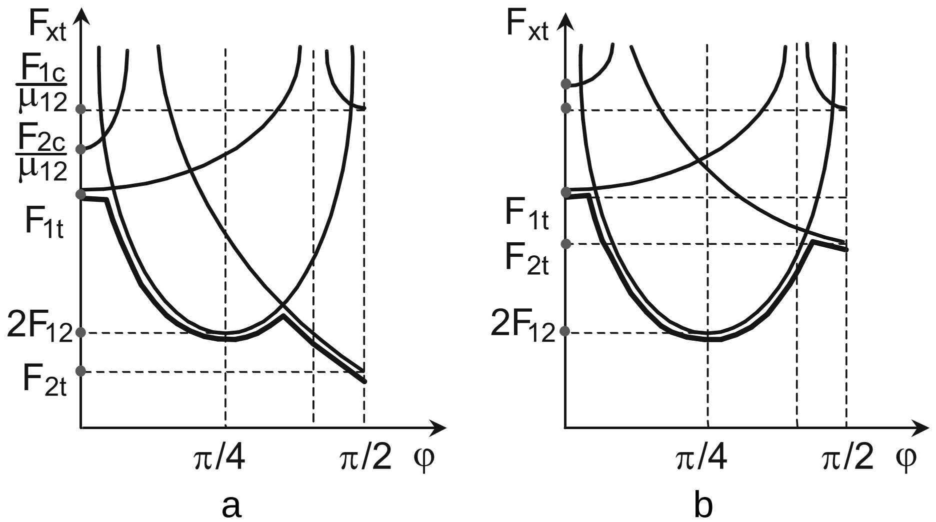


Fig. 3.4. Dependence of composite ultimate strength at tension on reinforcing direction (criterion of maximum strain): a – unidirectional composite; b – composite material, reinforced with fabric

$$\sigma_x \le \left(\frac{\cos^4\varphi}{F_{1t}^2} - \frac{\sin^2\varphi\cos^2\varphi}{F_{1t}F_{2t}} + \frac{\sin^4\varphi}{F_{2t}^2} + \frac{\sin^2\varphi\cos^2\varphi}{F_{12}^2} \right)^{-0.5} \tag{3.15}$$

or

$$F_{xt}=\left(\frac{\cos^4\varphi}{F_{1t}^2}-\frac{\sin^2\varphi\cos^2\varphi}{F_{1t}F_{2t}}+\frac{\sin^4\varphi}{F_{2t}^2}+\frac{\sin^2\varphi\cos\varphi^2}{F_{12}^2}\right)^{-0.5}. \qquad (3.16)$$

Formula (3.16) is continuous function of reinforcing angle φ, but it does not permit to predict character of breaking (Fig. 3.5). At $\phi=\pi/4$

$$F_{xt}=F_{45}=2\left(\frac{1}{F_{1t}^2}-\frac{1}{F_{1t}F_{2t}}+\frac{1}{F_{2t}^2}+\frac{1}{F_{12}^2}\right)^{-0.5}. \qquad (3.17)$$

Ultimate strength at compression can be calculated by the formula

$$F_{xc}=\left(\frac{\cos^4\varphi}{F_{1c}^2}-\frac{\sin^2\varphi\cos^2\varphi}{F_{1c}F_{2c}}+\frac{\sin^4\varphi}{F_{2c}^2}+\frac{\sin^2\varphi\cos^2\varphi}{F_{12}^2}\right)^{-0.5}. \qquad (3.18)$$

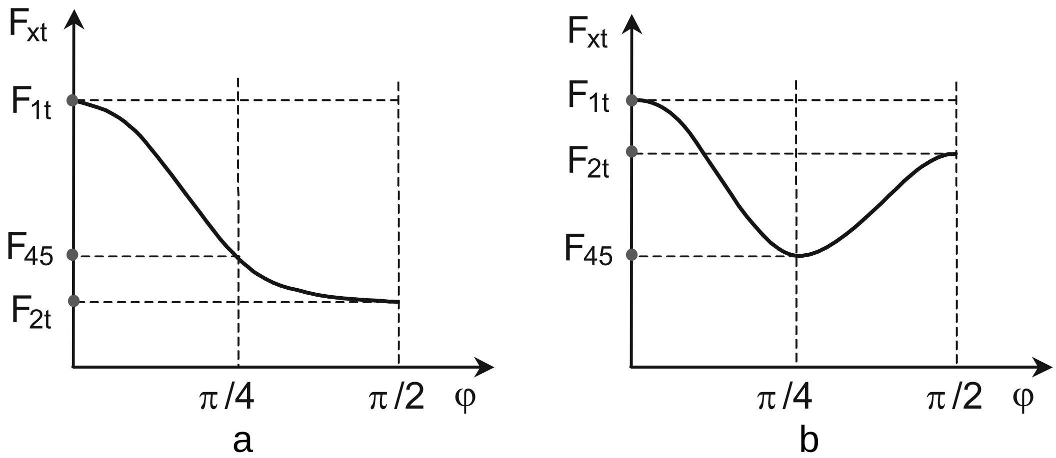


Fig. 3.5. Dependence of composite ultimate strength at tension on reinforcing direction (Mises-Hill criterion): a - unidirectional composite; b– composite material, reinforced with fabric

Replacing argument φ on $\pi/2-\phi$ in previous formulas, we can determine ultimate strength at tension/compression along the ***y*** axis (see Fig. 3.2, b).

Shear strength (see Fig. 3.2, c) can be derived by substitution expressions for stresses σ_1, σ_2, τ_{12} at $\sigma_x=\sigma_y=0$ to correspondent strength criteria:

$$\sigma_1=\tau_{xy}\sin 2\varphi; \quad \sigma_2=-\tau_{xy}\sin 2\varphi; \quad \tau_{12}=\tau_{xy}\cos 2\varphi. \qquad (3.19)$$

From (3.1) criteria one can obtain:

– at $\tau_{xy}>0$

$$\tau_{xy}\le\frac{F_{1t}}{\sin 2\varphi}; \quad \tau_{xy}\le\frac{F_{2c}}{\sin 2\varphi}; \quad \tau_{xy}\le\frac{F_{12}}{\mathrm{abs}(\cos 2\varphi)}; \qquad (3.20)$$

$$F_{xy}^{(+)}=\min\left(\frac{F_{1t}}{\sin 2\varphi}; \quad \frac{F_{2c}}{\sin 2\varphi}; \quad \frac{F_{12}}{\mathrm{abs}(\cos 2\varphi)}\right); \qquad (3.21)$$

– at $\tau_{xy}<0$

$$\tau_{xy} \le \frac{F_{1c}}{\sin 2\varphi}; \quad \tau_{xy} \le \frac{F_{2t}}{\sin 2\varphi}; \quad \tau_{xy} \le \frac{F_{12}}{\text{abs }(\cos 2\varphi)}; \qquad (3.22)$$

$$F_{xy}^{(-)} = \min\left(\frac{F_{1c}}{\sin 2\varphi}; \quad \frac{F_{2t}}{\sin 2\varphi}; \quad \frac{F_{12}}{\text{abs}(\cos 2\varphi)}\right). \qquad (23)$$

Dependences (3.20) – (3.23) are shown on the Fig. 3.6.

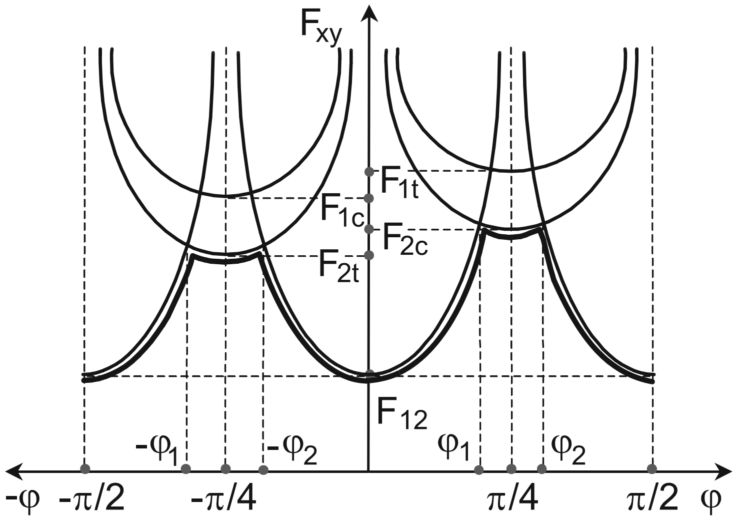


Fig. 3.6. Dependence of composite shear strength on reinforcing direction (maximal stress criterion)

One can obtain from criterion of maximum strain (3.4) after some transformation:

– at $\tau_{xy} > 0$ $\tau_{xy} \le \frac{F_{1t}}{1+\mu_{12}}$; $\tau_{xy} \le \frac{F_{1c}}{1+\mu_{21}}$; $\tau_{xy} \le \frac{F_{12}}{\text{abs}(\cos 2\varphi)}$; (3.24)

$$F_{xy}^{(+)} = \min\left(\frac{F_{1t}}{1+\mu_{12}}; \quad \frac{F_{2c}}{1+\mu_{21}}; \quad \frac{F_{12}}{\text{abs}(\cos 2\varphi)}\right); \qquad (3.25)$$

– at $\tau_{xy} < 0$

$$\tau_{xy} \le \frac{F_{1c}}{1+\mu_{12}}; \quad \tau_{xy} \le \frac{F_{2t}}{1+\mu_{21}}; \quad \tau_{xy} \le \frac{F_{12}}{\text{abs}(\cos 2\varphi)}; \qquad (3.26)$$

$$F_{xy}^{(-)} = \min\left(\frac{F_{1c}}{1+\mu_{12}}; \quad \frac{F_{2t}}{1+\mu_{21}}; \quad \frac{F_{12}}{\text{abs}(\cos 2\varphi)}\right). \qquad (3.27)$$

Criteria of maximum stress and maximum strain (Fig. 3.7) permit to predict character of material breakage. At $0 \le \varphi \le \varphi_1$ and $\varphi_2 \le \varphi \le \pi/2$ binder breakage at shear in planes parallel to fibers takes place; at $\varphi_1 \le \varphi \le \varphi_2$ - tensile breakage along fibers or compression breakage across fiber takes place (see Fig. 3.6, 3.7).

We can obtain the following expression for shear strength from Mises-Hill criterion (3.5):

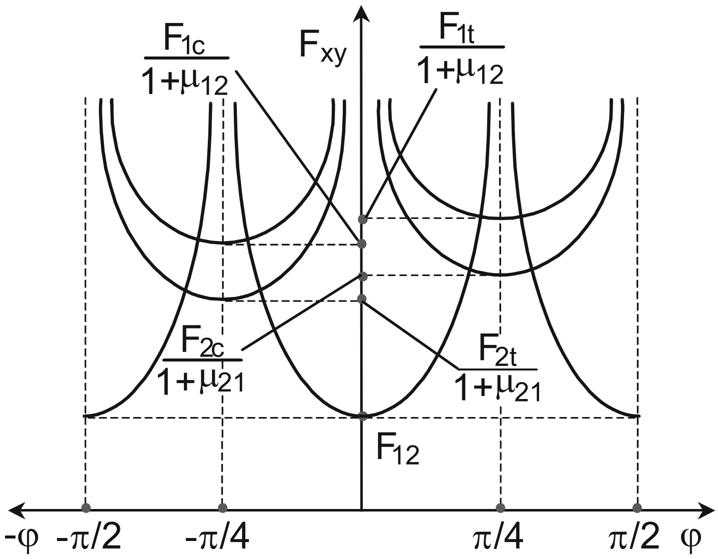


Fig. 3.7. Dependence of composite shear strength on reinforcing direction(maximum strain criterion)

- at $\tau_{xy} > 0$

$$\tau_{xy} \le \left[\sin^2 2\varphi \left(\frac{1}{F_{1t}^2} + \frac{1}{F_{1t}F_{2c}} + \frac{1}{F_{2c}^2} + \frac{1}{F_{12}^2} \right) - \frac{1}{F_{12}^2} \right]^{-0.5}; \tag{3.28}$$

$$F_{xy}^{(+)} = \left[\sin^2 2\varphi \left(\frac{1}{F_{1t}^2} + \frac{1}{F_{1t}F_{2c}} + \frac{1}{F_{2c}^2} + \frac{1}{F_{12}^2} \right) - \frac{1}{F_{12}^2} \right]^{-0.5}; \tag{3.29}$$

- at $\tau_{xy} < 0$

$$\tau_{xy} \le \left[\sin^2 2\varphi \left(\frac{1}{F_{1c}^2} + \frac{1}{F_{1c}F_{2t}} + \frac{1}{F_{2t}^2} + \frac{1}{F_{12}^2} \right) - \frac{1}{F_{12}^2} \right]^{-0.5}; \tag{3.30}$$

$$F_{xy}^{(-)} = \left[\sin^2 2\varphi \left(\frac{1}{F_{1c}^2} + \frac{1}{F_{1c}F_{2t}} + \frac{1}{F_{2t}^2} + \frac{1}{F_{12}^2} \right) - \frac{1}{F_{12}^2} \right]^{-0.5}. \tag{3.31}$$

It is necessary to note that acting stresses σ_1 and σ_2 must be substituted to criterion (3.5) with their signs that is why all members in parenthesis are positive.

Shear strength according to Mises-Hill criterion (either tensile or compression) is described by continuous function, but this criterion does not predict breakage character (Fig. 3.8). At $\varphi = 45^\circ$ composite material withstands the following stresses:

$$F_{45}^{(+)}=\left(\frac{1}{F_{1t}^2}+\frac{1}{F_{1t}F_{2c}}+\frac{1}{F_{2c}^2}\right)^{-0.5};\qquad F_{45}^{(-)}=\left(\frac{1}{F_{1c}^2}+\frac{1}{F_{1c}F_{2t}}+\frac{1}{F_{2t}^2}\right)^{-0.5}. \qquad (3.32)$$

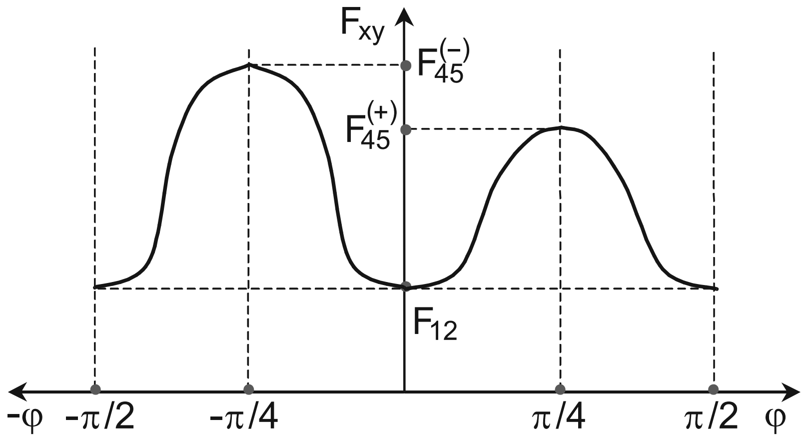


Fig. 3.8. Dependence of composite shear strength on reinforcing direction (Mises-Hill criterion)

3.3. Strength properties of laminated composite

To apply dependences (3.1), (3.4) and (3.5) for laminated composite material of arbitrary structure it is necessary to express stresses σ_{1i}, σ_{2i} and τ_{12i} by means of σ_x, σ_y and τ_{xy}. For this purpose it is necessary to use condition of compatible deformation of layers because of statically uncertainly of laminated composite (layer quantity is more than two).

Since pack structure and physical-mechanical characteristics of all individual layers are to be known elastic constants of composite material E_x, E_y, G_{xy}; μ_{xy}, μ_{yx}, $\eta_{x,xy}$; $\eta_{y,xy}$, $\eta_{xy,x}$, $\eta_{xy,y}$ are calculated at first and then - package global strains ε_x, ε_y, γ_{xy}:

$$\varepsilon_x=\frac{\sigma_x}{E_x}-\mu_{yx}\frac{\sigma_y}{E_y}+\eta_{xy,x}\frac{\tau_{xy}}{G_{xy}};$$

$$\varepsilon_x=-\mu_{xy}\frac{\sigma_x}{E_x}+\frac{\sigma_y}{E_y}+\eta_{xy,y}\frac{\tau_{xy}}{G_{xy}}; \qquad (3.33)$$

$$\gamma_{xy}=\eta_{x,xy}\frac{\sigma_x}{E_x}+\eta_{y,xy}\frac{\sigma_y}{E_y}+\frac{\tau_{xy}}{G_{xy}}.$$

Let define each layer deformations in local coordinate system by formulas (3.2):

$$\varepsilon_{1i}=\frac{\sigma_x}{E_x}\left(\cos^2\varphi_i-\mu_{xy}\sin^2\varphi_i+\eta_{xy,x}\sin\varphi_i\cos\varphi_i\right)+\frac{\sigma_y}{E_y}\left(\sin^2\varphi_i-\mu_{yx}\cos^2\varphi_i+\eta_{y,xy}\sin_i\cos\varphi_i\right)+$$

$$+\frac{\tau_{xy}}{G_{xy}}\left(\eta_{xy,x}\cos^2\varphi_i+\eta_{xy,y}\sin^2\varphi_i+\sin\varphi_i\cos\varphi_i\right)\ ;$$

$$\varepsilon_{2i}=\frac{\sigma_x}{E_x}\left(\sin^2\varphi_i-\mu_{xy}\cos^2\varphi_i-\eta_{x,xy}\sin\varphi_i\cos\varphi_i\right)+$$

$$+\frac{\sigma_y}{E_y}\left(\cos^2\varphi_i-\mu_{yx}\sin^2\varphi_i-\eta_{y,xy}\sin\varphi_i\cos\varphi_i\right)+ \qquad (3.34)$$

$$+\frac{\tau_{xy}}{G_{xy}}\left(\eta_{xy,x}\sin^2\varphi_i+\eta_{xy,y}\cos^2\varphi_i-\sin\varphi_i\cos\varphi_i\right)\ ;$$

$$\gamma_{12i}=\frac{\sigma_x}{E_x}\left[-\sin2\varphi_i\left(1+\mu_{xy}\right)+\eta_{x,xy}\cos2\varphi_i\right]+\frac{\sigma_y}{E_y}\left[\left(1+\mu_{yx}\right)\sin2\varphi_i+\eta_{y,xy}\cos2\varphi_i\right]+$$

$$+\frac{\tau_{xy}}{G_{xy}}\left[\left(\eta_{xy,y}-\eta_{xy,x}\right)\sin2\varphi_i+\cos2\varphi_i\right]\ .$$

Let write these expressions in the form:

$$\varepsilon_{1i}=\sigma_x a_{11i}+\sigma_y a_{12i}+\tau_{xy}a_{13i};$$
$$\varepsilon_{2i}=\sigma_x a_{21i}+\sigma_y a_{22i}+\tau_{xy}a_{23i}; \qquad (3.35)$$
$$\gamma_{12i}=\sigma_x a_{31i}+\sigma_y a_{32i}+\tau_{xy}a_{33i},$$

where

$$a_{11i}=\frac{1}{E_x}\left(\cos^2\varphi_i-\mu_{xy}\sin^2\varphi_i+\eta_{xy,x}\sin\varphi_i\cos\varphi_i\right);$$

$$a_{12i}=\frac{1}{E_y}\left(\sin^2\varphi_i-\mu_{yx}\cos^2\varphi_i+\eta_{xy,y}\sin\varphi_i\cos\varphi_i\right);$$

$$a_{13i}=\frac{1}{G_{xy}}\left(\eta_{x,xy}\cos^2\varphi_i+\eta_{y,xy}\sin^2\varphi_i+\sin\varphi_i\cos\varphi_i\right);$$

$$a_{21i}=\frac{1}{E_x}\left(\sin^2\varphi_i-\mu_{xy}\cos^2\varphi_i-\eta_{xy,x}\sin\varphi_i\cos\varphi_i\right);$$

$$a_{22i}=\frac{1}{E_y}\left(\cos^2\varphi_i-\mu_{yx}\sin^2\varphi_i-\eta_{xy,y}\sin\varphi_i\cos\varphi_i\right);$$

$$a_{23i}=\frac{1}{G_{xy}}\left(\eta_{x,xy}\sin^2\varphi_i+\eta_{y,xy}\cos^2\varphi_i-\sin\varphi_i\cos\varphi_i\right);$$

$$a_{31i} = \frac{1}{E_x}\left[-\left(1+\mu_{xy}\right)\sin 2\varphi_i + \eta_{xy,x}\cos 2\varphi_i\right]; \qquad (3.36)$$

$$a_{32i} = \frac{1}{E_y}\left[\left(1+\mu_{yx}\right)\sin 2\varphi_i + \eta_{xy,y}\cos 2\varphi_i\right];$$

$$a_{33i} = \frac{1}{G_{xy}}\left[\left(\eta_{y,xy} - \eta_{x,xy}\right)\sin 2\varphi_i + \cos 2\varphi_i\right].$$

Necessary expressions for stresses σ_{1i}, σ_{2i}, τ_{12i} can be found from equations of physical law:

$$\sigma_{1i} = \bar{\bar{E}}_{1i}\left[\sigma_x\left(a_{11i}+\mu_{21i}a_{21i}\right) + \sigma_y\left(a_{12i}+\mu_{21i}a_{22i}\right) + \tau_{xy}\left(a_{13i}+\mu_{21i}a_{23i}\right)\right];$$

$$\sigma_{2i} = \bar{\bar{E}}_{2i}\left[\sigma_x\left(\mu_{12i}a_{11i}+a_{21i}\right) + \sigma_y\left(\mu_{12i}a_{12i}+a_{22i}\right) + \tau_{xy}\left(\mu_{12i}a_{13i}+a_{23i}\right)\right]; \qquad (3.37)$$

$$\tau_{12i} = G_{12i}\left(\sigma_x a_{31i} + \sigma_y a_{32i} + \tau_{xy}a_{33i}\right).$$

Generalized procedure for definition of any ultimate strength consists of the following steps: components of internal stresses σ_x, σ_y, τ_{xy} are assumed to be equal to zero, then expressions (3.35) or (3.37) are substitute to criteria (3.1), (3.3), (3.5), from which formulas for ultimate strength definitions can be obtained.

Let consider procedure of definition ultimate tensile strength F_{xt} along the axis ***x***. Let assume, for this purpose, stresses σ_y, τ_{xy} to be equal to zero in (3.37) formulas. According to criterion of maximum stress (3.1) we can obtain:

$$\sigma_x \bar{\bar{E}}_{1i}\,\mathrm{abs}\left(a_{11i}+\mu_{21i}a_{21i}\right) \le F_{1i};$$

$$\sigma_x \bar{\bar{E}}_{2i}\,\mathrm{abs}\left(\mu_{12i}a_{11i}+a_{21i}\right) \le F_{2i}; \qquad (3.38)$$

$$\sigma_x G_{12i}\,\mathrm{abs}\,a_{31i} \le F_{12i}.$$

From here

$$\sigma_x \le \frac{F_{1i}}{\bar{\bar{E}}_{1i}\,\mathrm{abs}\left(a_{11i}+\mu_{21i}a_{21i}\right)};$$

$$\sigma_x \le \frac{F_{2i}}{\bar{\bar{E}}_{2i}\,\mathrm{abs}\left(\mu_{12i}a_{11i}+a_{21i}\right)}; \qquad (3.39)$$

$$\sigma_x \le \frac{F_{12i}}{G_{12i}\,\mathrm{abs}\,a_{31i}}.$$

The following equation for ultimate strength determination can be found from non-equalities (3.39):

$$F_x = \min_{(i)}\left[\frac{F_{1i}}{\bar{\bar{E}}_{1i}\mathrm{abs}\left(a_{11i}+\mu_{21i}a_{21i}\right)};\ \frac{F_{2i}}{\bar{\bar{E}}_{2i}\mathrm{abs}\left(\mu_{12i}a_{11i}+a_{21i}\right)};\ \frac{F_{12i}}{G_{12i}\mathrm{abs}(a_{31i})}\right]. \qquad (3.40)$$

Individual layers characteristics F_{1i}, F_{2i} are equal to the following expressions at determination of ultimate tensile stress F_{xt}:

$$F_{1j}=\begin{cases}F_{1it} & \text{at } (a_{11j}+\mu_{21j}a_{21j})>0;\\ F_{1ic} & \text{at } (a_{11j}+\mu_{21j}a_{21j})<0;\end{cases}$$

$$F_{2i}=\begin{cases}F_{2it} & \text{at } (\mu_{12i}a_{11j}+a_{21j})>0;\\ F_{2ic} & \text{at } (\mu_{12i}a_{11j}+a_{21j})<0\ ;\end{cases} \tag{3.41}$$

and for determination of ultimate compressive strength F_{xc}:

$$F_{1j}=\begin{cases}F_{1ic} & \text{at } (a_{11j}+\mu_{21j}a_{21j})>0;\\ F_{1it} & \text{at } (a_{11j}+\mu_{21j}a_{21j})<0;\end{cases}$$

$$F_{2i}=\begin{cases}F_{2ic} & \text{at } (\mu_{12i}a_{11}+a_{21j})>0;\\ F_{2it} & \text{at } (\mu_{12i}a_{11}+a_{21j})<0.\end{cases} \tag{3.42}$$

Prediction of ultimate tensile strength along ***x*** axis based on maximum deformation criterion is carried by substitution of deformations

$$\varepsilon_{1j}=\sigma_x a_{11j};\quad \varepsilon_{2i}=\sigma_x a_{21j};\quad \gamma_{12i}=\sigma_x a_{31j} \tag{3.43}$$

to non-equalities (3.3). As result we obtain:

$$\sigma_x\le\frac{F_{1j}}{\bar{\bar{E}}_{1j}a_{11j}};\quad \sigma_x\le\frac{F_{2i}}{\bar{\bar{E}}_{2i}a_{21j}};\quad \sigma_x\le\frac{F_{12i}}{G_{12i}a_{31j}}; \tag{3.44}$$

$$F_x=\min_{(i)}\left(\frac{F_{1j}}{\bar{\bar{E}}_{1j}\mathrm{abs}(a_{11j})};\quad \frac{F_{2i}}{\bar{\bar{E}}_{2i}\mathrm{abs}(a_{21j})};\ \frac{F_{12i}}{G_{12i}\mathrm{abs}(a_{31j})}\right), \tag{3.45}$$

where at $F_x=F_{xt}$

$$F_{1j}=\begin{cases}F_{1it} & \text{at } a_{11i}>0;\\ F_{1ic} & \text{at } a_{11i}<0;\end{cases}\quad F_{2i}=\begin{cases}F_{2it} & \text{at } a_{21i}>0;\\ F_{2ic} & \text{at } a_{21i}<0,\end{cases} \tag{3.46}$$

and at $F_x=F_{xc}$

$$F_{1j}=\begin{cases}F_{1ic} & \text{at } a_{11i}>0;\\ F_{1it} & \text{at } a_{11i}<0;\end{cases}\quad F_{2i}=\begin{cases}F_{2ic} & \text{at } a_{21i}>0;\\ F_{2it} & \text{at } a_{21i}<0.\end{cases} \tag{3.47}$$

We can see from expressions (3.40) and (3.45), those criteria of maximum stress and maximum strain permit to predict character of composite material breakage, i.e. to define what layer and from what stresses is broken the first.

We can obtain the following expressions after analyzing Mises-Hill criterion:

$$\sigma_x \le \left[\frac{\bar{E}_{1i}^2 (a_{11i} + \mu_{21i} a_{21i})}{F_{1i}^2} - \right.$$

$$\left. - \frac{\bar{E}_{1i}\bar{E}_{2i}(a_{11i} + \mu_{21i}a_{21i})(\mu_{12i}a_{11i} + a_{21i})}{F_{1i}F_{2i}} + \frac{E_{2i}^2(\mu_{12i}a_{11i} + a_{21i})^2}{F_{2i}^2} + \frac{G_{12i}^2 a_{31i}^2}{F_{12}^2} \right]^{-0.5};$$

(3.48)

$$F_x = \min_{(i)} \left[\frac{\bar{E}_{1i}^2 (a_{11i} + \mu_{21i} a_{21i})^2}{F_{1i}^2} + \frac{\bar{E}_{2i}^2 (\mu_{12i} a_{11i} + a_{21i})^2}{F_{2i}^2} - \right.$$

$$\left. - \frac{\bar{E}_{1i}\bar{E}_{2i}(a_{11i} + \mu_{21i}a_{21i})(\mu_{12i}a_{11i} + a_{21i})}{F_{1i}F_{2i}} + \frac{G_{12i}^2 a_{31i}^2}{F_{12i}^2} \right]^{-0.5}, \qquad (3.49)$$

where F_{1i} and F_{2i} are defined from conditions (3.41) or (3.42).

Dependences for determination of ultimate tensile strength along *y* axis, ultimate compression strength along *y* axis and shear strength can be obtained by analogous way. Here is the final result.

Example 3.1. Maximum stress criterion

$$F_y = \min_i \left[\frac{F_{1i}}{\bar{\bar{E}}_{1i} \operatorname{abs}(a_{12i} + \mu_{21i} a_{22i})}; \frac{F_{2i}}{\bar{\bar{E}}_{2i} \operatorname{abs}(\mu_{12i} a_{12i} + a_{22i})}; \frac{F_{12i}}{G_{12i} \operatorname{abs} a_{32i}} \right], \qquad (3.50)$$

where at $F_y = F_{yt}$

$$F_{1i} = \begin{cases} F_{1ti} & \text{at } a_{12i} + \mu_{21i} a_{22i} > 0; \\ F_{1ci} & \text{at } a_{12i} + \mu_{21i} a_{22i} < 0; \end{cases}$$

$$F_{2i} = \begin{cases} F_{2ti} & \text{at } \mu_{12i} a_{12i} + a_{22i} > 0; \\ F_{2ci} & \text{at } \mu_{12i} a_{12i} + a_{22i} < 0, \end{cases} \qquad (3.51)$$

and at $F_y = F_{yc}$

$$F_{1i} = \begin{cases} F_{1ci} & \text{at } a_{12i} + \mu_{21i} a_{22i} > 0; \\ F_{1ti} & \text{at } a_{12i} + \mu_{21i} a_{22i} < 0; \end{cases}$$

$$F_{2i} = \begin{cases} F_{2ci} & \text{at } \mu_{12i} a_{12i} + a_{22i} > 0; \\ F_{2ti} & \text{at } \mu_{12i} a_{12i} + a_{22i} < 0\ ; \end{cases} \qquad (3.52)$$

$$F_{xy} = \min_{(i)} \left[\frac{F_{1i}}{\bar{\bar{E}}_{1i} \operatorname{abs}(a_{13i} + \mu_{21i} a_{23i})}; \right.$$

$$\left. \frac{F_{2i}}{\bar{\bar{E}}_{2i} \operatorname{abs}(\mu_{12i} a_{13i} + a_{23i})}; \quad \frac{F_{12i}}{G_{12i} \operatorname{abs} a_{33i}} \right], \qquad (3.53)$$

where at $F_{xy} = F_{xy}^{(+)}$

$$F_{1i} = \begin{cases} F_{1ti} & \text{at } a_{13i} + \mu_{21i} a_{23i} > 0; \\ F_{1ci} & \text{at } a_{13i} + \mu_{21i} a_{23i} < 0; \end{cases}$$
$$F_{2i} = \begin{cases} F_{2ti} & \text{at } \mu_{12i} a_{13i} + a_{23i} > 0; \\ F_{2ci} & \text{at } \mu_{12i} a_{13i} + a_{23i} < 0 \end{cases}, \tag{3.54}$$

and at $F_{xy} = F_{xy}^{(-)}$

$$F_{1i} = \begin{cases} F_{1ci} & \text{at } a_{13i} + \mu_{21i} a_{23i} > 0; \\ F_{1ti} & \text{at } a_{13i} + \mu_{21i} a_{23i} < 0; \end{cases}$$
$$F_{2i} = \begin{cases} F_{2ci} & \text{at } \mu_{12i} a_{13i} + a_{23i} > 0; \\ F_{2ti} & \text{at } \mu_{12i} a_{13i} + a_{23i} < 0 \end{cases}. \tag{3.55}$$

Example 3.2. Maximum strain criterion

$$F_y = \min_{(i)} \left[\frac{F_{1i}}{\overline{\overline{E}}_{1i} \text{abs}\, a_{12i}}; \quad \frac{F_{2i}}{\overline{\overline{E}}_{2i} \text{abs}\, a_{22i}} \quad ; \quad \frac{F_{12i}}{G_{12i} \text{abs}\, a_{32i}} \right], \tag{3.56}$$

where $F_y = F_{yt}$

$$F_{1i} = \begin{cases} F_{1ti} & \text{at } a_{12i} > 0; \\ F_{1ci} & \text{at } a_{12i} < 0; \end{cases} \qquad F_{2i} = \begin{cases} F_{2ti} & \text{at } a_{22i} > 0; \\ F_{2ci} & \text{at } a_{22i} < 0, \end{cases} \tag{3.57}$$

and at $F_y = F_{yc}$

$$F_{1i} = \begin{cases} F_{1ci} & \text{at } a_{12i} > 0; \\ F_{1ti} & \text{at } a_{12i} < 0; \end{cases} \qquad F_{2i} = \begin{cases} F_{2ci} & \text{at } a_{22i} > 0; \\ F_{2ti} & \text{at } a_{22i} < 0, \end{cases} \tag{3.58}$$

$$F_{xy} = \min_{(i)} \left[\frac{F_{1i}}{\overline{\overline{E}}_{1i}\, \text{abs}\, a_{13i}}; \; \frac{F_{2i}}{\overline{\overline{E}}_{2i}\, \text{abs}\, a_{23i}} \; ; \; \frac{F_{12i}}{G_{12i}\, \text{abs}\, a_{33i}} \right], \tag{3.59}$$

where at $F_{xy} = F_{xy}^{(+)}$

$$F_{1i} = \begin{cases} F_{1ti} & \text{at } a_{13i} > 0; \\ F_{1ci} & \text{at } a_{13i} < 0; \end{cases} \qquad F_{2i} = \begin{cases} F_{2ti} & \text{at } a_{23i} > 0; \\ F_{2ci} & \text{at } a_{23i} < 0, \end{cases} \tag{3.60}$$

and at $F_{xy} = F_{xy}^{(-)}$

$$F_{1i} = \begin{cases} F_{1ci} & \text{at } a_{13i} > 0; \\ F_{1ti} & \text{at } a_{13i} < 0; \end{cases} \qquad F_{2i} = \begin{cases} F_{2ci} & \text{at } a_{23i} > 0; \\ F_{2ti} & \text{at } a_{23i} < 0. \end{cases} \tag{3.61}$$

Example 3.3. Mises-Hill criterion

$$F_y = \min_{(i)} \left[\frac{\overline{E}_{1i}^2 \left(a_{12i} + \mu_{21i} a_{22i}\right)^2}{F_{1i}^2} + \frac{\overline{E}_{2i}^2 \left(\mu_{12i} a_{12i} + a_{22i}\right)^2}{F_{2i}^2} - \right.$$
$$\left. - \frac{\overline{E}_{1i} \overline{E}_{2i} \left(a_{12i} + \mu_{21i} a_{22i}\right)\left(\mu_{12i} a_{12i} + a_{22i}\right)}{F_{1i} F_{2i}} + \frac{G_{12i}^2 a_{32i}^2}{F_{12i}^2} \right], \tag{3.62}$$

where F_{1i} and F_{2i} are defined according to conditions (3.51) and (3.52);

$$F_{xy} = \min_{(i)} \left[\frac{\bar{E}_{1i}^2 (a_{13i} + \mu_{21i} a_{23i})^2}{F_{1i}^2} + \frac{\bar{E}_{2i}^2 (\mu_{12i} a_{13i} + a_{23i})^2}{F_{2i}^2} - \right.$$

$$\left. - \frac{\bar{E}_{1i} \bar{E}_{2i} (a_{13i} + \mu_{21i} a_{22i})(\mu_{12i} a_{13i} + a_{23i})}{F_{1i} F_{2i}} + \frac{G_{12i}^2 a_{33i}^2}{F_{12i}^2} \right], \qquad (3.63)$$

where F_{1i} and F_{2i} are defined according to conditions (3.54) and (3.55).

If composite material is orthotropic medium in axes ***x***, ***y*** ultimate strength values are defined by written above dependences, in which the following notations are accepted:

$$a_{11i} = \frac{\cos^2 \varphi_i - \mu_{xy} \sin^2 \varphi_i}{E_x}; \quad a_{12i} = \frac{\sin^2 \varphi_i - \mu_{yx} \cos^2 \varphi_i}{E_y}; \quad a_{13i} = \frac{\sin \varphi_i \cos \varphi_i}{G_{xy}};$$

$$a_{21i} = \frac{\sin^2 \varphi_i - \mu_{xy} \cos^2 \varphi_i}{E_x}; \quad a_{22i} = \frac{\cos^2 \varphi_i - \mu_{yx} \sin^2 \varphi_i}{E_y}; \quad a_{23i} = -\frac{\sin \varphi_i \cos \varphi_i}{G_{xy}}; \qquad (3.64)$$

$$a_{31i} = -\frac{(1 + \mu_{xy}) \sin 2\varphi_i}{E_x}; \quad a_{32i} = \frac{(1 + \mu_{yx}) \sin 2\varphi_i}{E_y}; \quad a_{33i} = \frac{\cos 2\varphi_i}{G_{xy}}.$$

Let consider as an example the method of application of cross-plied composite material with orthogonal reinforcement (Fig. 3.9), i. e.

$$n = 2, \quad \delta_1 = \delta_1, \quad \delta_2 = \delta_2, \quad \varphi_1 = 0, \quad \varphi_2 = 90^\circ.$$

Let materials of all layers are the same. Then

$$E_{11} = E_{12} = E_1, \quad E_{21} = E_{22} = E_2, G_{121} = G_{122} = G_{12}, \quad \mu_{121} = \mu_{122} = \mu_{12},$$

$$F_{11t} = F_{12t} = F_{1t}; \quad F_{21t} = F_{22t} = F_{2t}.$$

We can obtain from (3.64) expressions:

$$a_{111} = \frac{1}{E_x}; \quad a_{121} = -\frac{\mu_{yx}}{E_y}; \quad a_{131} = 0; \; a_{211} = -\frac{\mu_{xy}}{E_x}; \quad a_{221} = \frac{1}{E_y}; \quad a_{231} = 0;$$

$$a_{311} = a_{321} = 0; \quad a_{331} = \frac{1}{G_{xy}};$$

$$(3.65)$$

$$a_{112} = -\frac{\mu_{xy}}{E_x}; \quad a_{122} = \frac{1}{E_y}; \quad a_{132} = 0; \; a_{212} = \frac{1}{E_x}; \quad a_{222} = -\frac{\mu_{yx}}{E_y}; \quad a_{232} = 0;$$

$$a_{312} = a_{322} = 0; \quad a_{332} = \frac{1}{G_{xy}};$$

$$a_{111} + \mu_{21} a_{211} = \frac{1}{E_x} - \mu_{21} \frac{\mu_{xy}}{E_x} = \frac{1}{E_x}(1 - \mu_{xy}\mu_{21}); \quad \mu_{12} a_{111} + a_{121} = \frac{1}{E_x}(\mu_{12} - \mu_{xy});$$

$$a_{112}+\mu_{21}a_{212}=\frac{\mu_{21}-\mu_{xy}}{E_x}; \qquad \mu_{11}a_{112}+a_{212}=\frac{1-\mu_{xy}\mu_{11}}{E_x}.$$

Values F_{1i} and F_{2i} are defined by conditions (3.41) and (3.42).

Equations (3.65) for unidirectional monolayer can be transformed to:

$$\frac{1}{E_x}\left(1-\mu_{xy}\mu_{21}\right)>0; \qquad \frac{1}{E_x}\left(\mu_{12}-\mu_{xy}\right)>0;$$
$$\frac{1}{E_x}\left(\mu_{21}-\mu_{xy}\right)<0; \qquad \frac{1}{E_x}\left(1-\mu_{12}\mu_{xy}\right)>0. \qquad (3.66)$$

These expressions mean that stresses in layers have the directions shown on the Fig. 3.9.

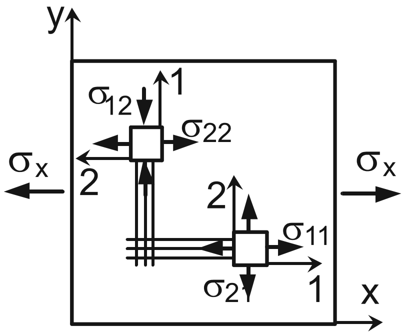


Fig. 3.9. Stressed state of composite material layers
with orthogonal reinforcement

In accordance with this fact it is necessary to assume the following notations for determination F_{xt}:

$$F_{11}=F_{1t}; \quad F_{21}=F_{2t}; \quad F_{12}=F_{1c}; \quad F_{22}=F_{2t}.$$

We can obtain from expression (3.40), taking into consideration found values of monolayers strength

$$F_{xt}=\min\left[\frac{F_{1t}E_x}{\bar{\bar{E}}_1(1-\mu_{xy}\mu_{21})};\frac{F_{2t}E_x}{\bar{\bar{E}}_2(\mu_{12}-\mu_{xy})};\frac{F_{12}}{0}; \ldots \frac{F_{1c}E_x}{\bar{\bar{E}}_1(\mu_{xy}-\mu_{21})};\frac{F_{2t}E_x}{\bar{\bar{E}}_2(1-\mu_{12}\mu_{xy})};\frac{F_{12}}{0}\right]. \quad (3.67)$$

The following is sequent from this fact: it is impossible to break orthogonal composite material by layers shear:

$$F_{xc}=E_x\min\left[\frac{F_{1c}}{\bar{\bar{E}}_1\left(1-\mu_{xy}\mu_{21}\right)}; \quad \frac{F_{2c}}{\bar{\bar{E}}_2\left(\mu_{12}-\mu_{xy}\right)}; \quad \frac{F_{12}}{0}; \ldots\right.$$
$$\left.\ldots\frac{F_{1t}}{\bar{\bar{E}}_1\left(\mu_{xy}-\mu_{21}\right)}; \quad \frac{F_{2c}}{\bar{\bar{E}}_2\left(1-\mu_{12}\mu_{xy}\right)}; \quad \frac{F_{12}}{0}\right]. \qquad (3.68)$$

The following dependences can be found for ultimate tensile and compression strength along the x axis from maximum strains criterion (3.45):

$$F_{xt} = E_x \min\left[\frac{F_{1t}}{E_1};\ \frac{F_{2c}}{E_2\mu_{xy}};\ \frac{F_{12}}{0};\ \frac{F_{1c}}{E_1\mu_{xy}};\ \frac{F_{2t}}{E_2};\ \frac{F_{12}}{0}\right];$$

$$F_{xc} = E_x \min\left[\frac{F_{1c}}{E_1};\ \frac{F_{2t}}{E_2\mu_{xy}};\ \frac{F_{12}}{0};\ \frac{F_{1t}}{E_1\mu_{xy}};\ \frac{F_{2c}}{E_2};\ \frac{F_{12}}{0}\right]. \qquad (3.69)$$

The following expression for F_{xt} can be found by means of Mises-Hill criterion:

$$F_{xt} = E_x \min\left\{\left[\frac{\bar{E}_1^2\left(1-\mu_{xy}\mu_{21}\right)^2}{F_{1t}^2} - \frac{\bar{E}_1\bar{E}_2\left(1-\mu_{xy}\mu_{21}\right)\left(\mu_{12}-\mu_{xy}\right)}{F_{1t}F_{2t}} + \frac{\bar{E}_2^2\left(\mu_{12}-\mu_{xy}\right)^2}{F_{2t}^2}\right];\ \ldots\right.$$

$$\left.\ldots\ \left[\frac{\bar{E}_1^2\left(\mu_{21}-\mu_{xy}\right)^2}{F_{1c}^2} - \frac{\bar{E}_1\bar{E}_2\left(\mu_{21}-\mu_{xy}\right)\left(1-\mu_{12}\mu_{xy}\right)}{F_{1c}F_{2t}} + \frac{\bar{E}_2^2\left(1-\mu_{12}\mu_{xy}\right)^2}{F_{2t}^2}\right]^{-0.5}\right\}. \qquad (3.70)$$

Formulas for other ultimate strength values can be found by analogous way.

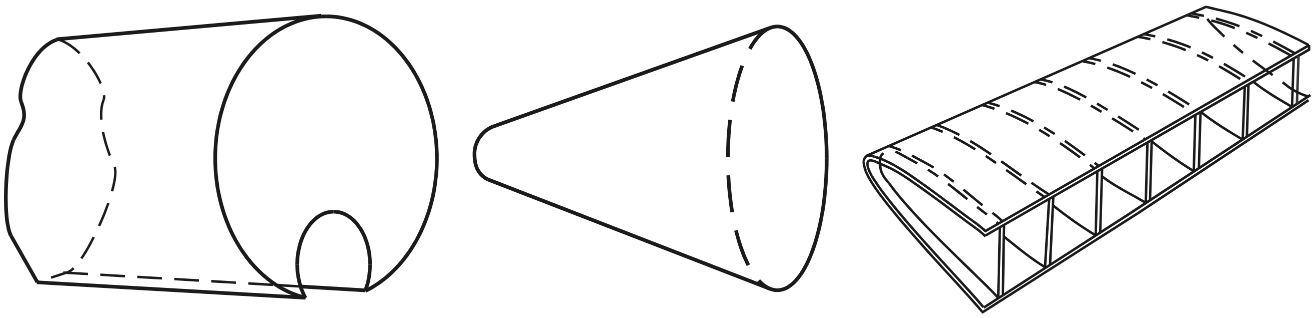
Checking-up questions

1. What are typical failure modes of laminated composites at macro- and micro-levels?
2. Describe two main approaches used for estimation of laminated composite strength.
3. What initial data engineer has to know for estimation of strength of laminated composite (per each of above-mentioned two approaches)?
4. What three main strength criteria can be applied for composite strength analysis?
5. Write expression(s) of maximum stress criterion.
6. Write expression(s) of maximum strain criterion.
7. Write expression(s) of Mises-Hill criterion.
8. Draw and analyze typical dependence of UD-composite strength properties as function of reinforcing angle using different strength criteria.

Theme 4. DESIGN OF SHELLS OF REVOLUTION OBTAINED BY WINDING AT SYMMETRICAL LOADING

4.1. Fundamentals of shells design

Shell structures are widely used in different branches of national economy. In aerospace industry shell structures are represented by rocket cases, fuselage sections of small aircraft (closed cylindrical, conical and parabolic shape), radome fairings, leading edges of wing, control surfaces, cowlings (open shells) etc (Fig. 4.1). In machine-building we can see capacities, pressure vessels, floats, cisterns. High-efficiency and automatize winding process is generally used for manufacturing above-mentioned articles. Fibers, tows, rovings, tapes (unidirectional and woven) and fabrics (in combination with different polymeric binders) are used as reinforcing material for these articles manufacturing. Overall dimensions of such sells exceed to 20..30 m in length and 4…5 m in diameter.



a b c

Fig. 4.1. Aircraft shell structures: a– fuselage section; b– conical rocket radome (closed shell); c– typical wing leading section (open shell)

Analysis of operational conditions of aircraft articles modeled by shell structure shows that these articles can be loaded with (Fig. 4.2) [4, 6]:

– internal and external pressure, uniform tension or compression (by contour length). In this case we consider shells as **symmetrically loaded** (refer to central axis);

– randomly non-symmetrically loaded (with local longitudinal or lateral forces, bending moments, torque etc. In this case one should consider shell as **non-symmetrically loaded**.

Shells manufactured by winding are geometrically symmetrical ones, therefore its thickness and physical and mechanical properties don’t depend on hoop (angle) coordinate but can vary along shell length.

Due to low interlaminar strength of composite structure it is more desirable to orient fibers at winding in such directions to ensure shear stress in layers close to zero. That is why **criterion of shear stress absence** in composite package is commonly used design criterion for **symmetrically loaded shells**. Mathematically this design criterion can be written as

$$\tau_{12i} = 0 = G_{12}\gamma_{12i} = \left(\varepsilon_y - \varepsilon_x\right)\sin 2\varphi_i = 0. \tag{4.1}$$

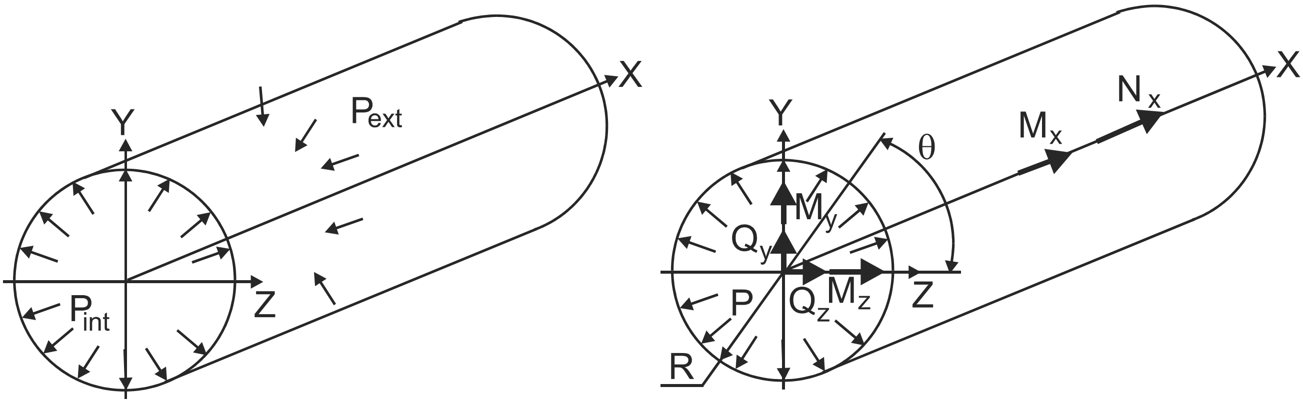


Fig. 4.2. Typical analysis scheme for shell structure: a– symmetrically loaded; b– non-symmetrically loaded

Composite obtained by winding is always orthotropic one in axes x and y. Axis x is directed along shell length, axis y is perpendicular to shell contour (see Fig. 4.2).

Criterion (4.1) can be satisfied by two possible independent solutions:

$$\varepsilon_y - \varepsilon_x = 0; \tag{4.2}$$

$$\sin 2\varphi_i = 0. \tag{4.3}$$

Physical law for composite package considering above-mentioned design criterion is the following

$$\begin{aligned} N_x &= B_{11}\varepsilon_x + B_{12}\varepsilon_y; \\ N_y &= B_{12}\varepsilon_x + B_{22}\varepsilon_y, \end{aligned} \tag{4.4}$$

where N_x, N_y – longitudinal and hoop forces per unit length applied to shell, N/m; B_{11}, B_{12}, B_{22} – rigidity coefficients of composite package defined by formulas:

$$\begin{aligned} B_{11} &= \sum_{i=1}^{n}\delta_i\left(\bar{E}_{1i}\cos^4\varphi_i + 2\bar{E}_{1i}\mu_{21i}\sin^2\varphi_i\cos^2\varphi_i + \bar{E}_{2i}\sin^4\varphi_i + G_{12i}\sin^2 2\varphi_i\right); \\ B_{12i} &= \sum_{i=1}^{n}\delta_i\left((\bar{E}_{1i} + \bar{E}_{2i})\sin^2\varphi_i\cos^2\varphi_i + \bar{E}_{1i}\mu_{21i}(\sin^4\varphi_i + \cos^4\varphi_i) - G_{12i}\sin^2 2\varphi_i\right); \\ B_{22i} &= \sum_{i=1}^{n}\delta_i\left(\bar{E}_{1i}\sin^4\varphi_i + 2\bar{E}_{1i}\mu_{21i}\sin^2\varphi_i\cos^2\varphi_i + \bar{E}_{2i}\cos^4\varphi_i + G_{12i}\sin^2 2\varphi_i\right). \end{aligned} \tag{4.5}$$

First of all the most possible solution is (4.2) relationship.

From physical law (4.4) we can find strains ε_x, ε_y:

$$\varepsilon_x = \frac{N_x B_{22} - N_y B_{12}}{B_{11}B_{22} - B_{12}^2}; \quad \varepsilon_y = \frac{N_y B_{11} - N_x B_{12}}{B_{11}B_{22} - B_{12}^2}. \tag{4.6}$$

Qualitative analysis of this expressions as mathematical functions permits to make the following conclusions:

– solution (4.2) gives that deformations ε_x, ε_y should have the same sign (both positive or both negative), i.e.

$$\begin{cases} N_x B_{22} - N_y B_{12} > 0; \\ N_y B_{11} - N_x B_{12} > 0; \end{cases} \quad \begin{cases} N_x B_{22} - N_y B_{12} < 0; \\ N_y B_{11} - N_x B_{12} < 0; \end{cases} \tag{4.7}$$

– coefficients B_{ij} are positive ones, therefore forces N_x, N_y should have the same sign too

$$N_x > 0; N_y > 0 \text{ or } N_x < 0; N_y < 0 \quad (N_x \neq 0; N_y \neq 0). \tag{4.8}$$

It means that condition (4.2) can be satisfied at **biaxial loading** only.

Condition (4.3) can be satisfied at realization of composite reinforcing scheme like [0°], [90°] or their combination.

4.2 Design procedure for shell manufactured by winding with woven fabric

Design criterion (4.2) together and physical law (4.6) give condition of shell optimality:

$$N_x (B_{22} + B_{12}) - N_y (B_{11} + B_{12}) = 0. \tag{4.9}$$

Using formulas for rigidity coefficients and transformations we can obtain

$$\sum_{i=1}^{n} \delta_i \left[\bar{E}_{1i} (1+\mu_{21i}) \left(N_x \sin^2 \varphi_i - N_y \cos^2 \varphi_i \right) + \bar{E}_{2i} (1+\mu_{12i}) \left(N_x \cos^2 \varphi_i - N_y \sin^2 \varphi_i \right) \right] = 0. \tag{4.10}$$

Requirements of design principle demand symmetrical composite package realization. That is why each layer with reinforcing angle $+\varphi_i$ and thickness δ_i corresponds to the layer with the same thickness but reinforcing angle $-\varphi_i$. Such design objective contains $n/2$ variable of thickness and $n/2$ variable of reinforcing angle.

Condition (4.10) considers dependence between composite rigidity parameters and external loading, therefore, to define numerical values of variable we should add any strength criterion. The most simple and visual is criterion of maximum stress

$$\sigma_{1i} \le F_{1i}; \ \sigma_{2i} \le F_{2i}. \tag{4.11}$$

Condition (4.2) together with physical law (4) permits to obtain dependence

$$\varepsilon_x = \varepsilon_y = \frac{N_x + N_y}{\sum_{i=1}^{n} \delta_i \left[\bar{E}_{1i} (1+\mu_{21i}) + \bar{E}_{2i} (1+\mu_{12i}) \right]}. \tag{4.12}$$

Formulas for strains calculation at axes rotation are known

$$\varepsilon_{1i} = \varepsilon_x \cos^2\varphi_i + \varepsilon_y \sin^2\varphi_i + \gamma_{xy}\sin\varphi_i\cos\varphi_i;$$
$$\varepsilon_{2i} = \varepsilon_x \sin^2\varphi_i + \varepsilon_y \cos^2\varphi_i - \gamma_{xy}\sin\varphi_i\cos\varphi_i; \quad (4.13)$$
$$\gamma_{21i} = (\varepsilon_y - \varepsilon_x)\sin 2\varphi_i + \gamma_{xy}\cos 2\varphi_i.$$

From equations (4.12), (4.13) and second possible solution of design criterion (4.3) one can obtain

$$\varepsilon_{1i} = \varepsilon_{2i} = \varepsilon_x = \varepsilon_y. \quad (4.14)$$

Therefore stresses in local coordinate system 1, 2 can be estimated by the following formulas

$$\sigma_{1i} = \frac{\left(N_x + N_y\right)\bar{E}_{1i}\left(1+\mu_{21i}\right)}{\sum_{i=1}^{n}\delta_i\left[\bar{E}_{1i}\left(1+\mu_{21i}\right)+\bar{E}_{2i}\left(1+\mu_{12i}\right)\right]}, \quad \sigma_{2i} = \frac{\left(N_x + N_y\right)\bar{E}_{2i}\left(1+\mu_{12i}\right)}{\sum_{i=1}^{n}\delta_i\left[\bar{E}_{1i}\left(1+\mu_{21i}\right)+\bar{E}_{2i}\left(1+\mu_{12i}\right)\right]}. \quad (4.15)$$

These dependencies show that stress in axes 1,2 doesn't depend on reinforcing angles, but on elastic constants only. Moreover stresses σ_{1i} and σ_{1i} have the same sign.

To obtain the **highest efficiency** of shell structure designer has to exceed simultaneous strength criterion (4.11) satisfaction, therefore $\sigma_{1i} = F_{1i}$, $\sigma_{2i} = F_{2i}$. If we solve together strength criterion (4.11) and equations (4.15) we obtain two possible variants of full-strength conditions

$$\text{– at } N_x > 0; N_y > 0 \quad \frac{F_{1it}}{F_{2it}} = \frac{\bar{E}_{1i}\left(1+\mu_{21i}\right)}{E_{2i}\left(1+\mu_{12i}\right)} = K_i; \quad (4.16)$$

$$\text{– at } N_x < 0; N_y < 0 \quad \frac{F_{1ic}}{F_{2ic}} = \frac{\bar{E}_{1i}\left(1+\mu_{21i}\right)}{E_{2i}\left(1+\mu_{12i}\right)} = K_i. \quad (4.17)$$

It means that simultaneous breakage of warp fiber and weft fiber is possible at definite combination between strength and elastic properties and doesn't depend on applied loads. In this case any of condition (4.11) can be used for design.

If shell is considered to be made of the same material it is necessary to use for design one condition of shell optimality (4.10) and one strength criterion, quantity of variables is equal to n. It means that any structure satisfied to (4.10) and (4.11) is optimal one.

If shell is considered to be made of different materials one should use one condition (4.10) and n/2 conditions of strength.

If conditions (4.16) and (4.17) aren't satisfied one should clarify in what direction breakage is most probably fastest (in warp direction or in weft direction) and then use this strength condition.

For example, let's find condition at which the first strength criterion $\sigma_{1i} = F_{1i}$ is fulfilled as equality and the second strength condition is simply satisfied $\sigma_{2i} \le F_{2i}$.

$$\frac{\left(N_x+N_y\right)\bar{E}_{1i}\left(1+\mu_{21i}\right)}{\sum_{i=1}^{n}\delta_i\left[\bar{E}_{1i}\left(1+\mu_{21i}\right)+\bar{E}_{2i}\left(1+\mu_{12i}\right)\right]}=F_{1i},\ \frac{\left(N_x+N_y\right)\bar{E}_{2i}\left(1+\mu_{12i}\right)}{\sum_{i=1}^{n}\delta_i\left[\bar{E}_{1i}\left(1+\mu_{21i}\right)+\bar{E}_{2i}\left(1+\mu_{12i}\right)\right]}\le F_{2i} \quad (4.18)$$

It is obvious that:

– if $\frac{F_{1i}}{F_{2i}}\le K_i$ that warp breaks the fist;

– if $\frac{F_{1i}}{F_{2i}}\ge K_i$ that weft breaks the fist.

Example 4.1. Let's consider design of a shell is assumed to be manufactured by spiral symmetrical winding from the same material (angle $+\varphi_i$ and $-\varphi_i$ are used) (Fig. 4.3). It is necessary to find angle $+\varphi_1=+\varphi_2=\varphi$ and layer thickness $+\delta_1=+\delta_2=\delta$. Quantity of equation n is equal to 2.

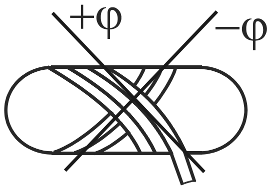


Fig. 4.3. Spiral symmetrical winding

$E_{1i}=E_1;\ E_{2i}=E_2;\ K_i=K;\ \mu_{12i}=\mu_{12};\ \mu_{21i}=\mu_{21};\ F_{1i}=F_1;\ F_{2i}=F_2.$

Then optimality condition (4.10) is the following (for ***n***=1)

$$E_1\left(1+\mu_{21}\right)\left(N_x\sin^2\varphi_i-N_y\cos^2\varphi_i\right)+E_2\left(1+\mu_{12}\right)\left(N_x\cos^2\varphi_i-N_y\sin^2\varphi_i\right)=0. \quad (4.19)$$

$$tg^2\varphi=\frac{N_yK-N_x}{N_xK-N_y}. \quad (4.20)$$

In this case we obtain unique solution for angle φ. Analysis of this solution gives two possible variants:

$$\begin{cases}N_yK-N_x>0;\\ N_xK-N_y>0;\end{cases} \quad (4.21)$$

$$\begin{cases}N_yK-N_x>0;\\ N_xK-N_y>0.\end{cases} \quad (4.22)$$

Thus

– at $K>1$ therefore $\frac{1}{K}\le\frac{N_x}{N_y}\le K;$ (4.23)

– at $K<1$ therefore $K \le \frac{N_x}{N_y} \le \frac{1}{K}$. (4.24)

At $E_1(1+\mu_{21}) = E_2(1+\mu_{12})$ optimal structure can be obtained at $N_x = N_y$ only.

If composite breakage begins from warp fibers then

$$\frac{K(N_x+N_y)}{2\delta(K+1)} = F_1, \tag{4.25}$$

Therefore shell thickness is equal to

$$2\delta = \frac{K(N_x+N_y)}{F_1(K+1)}. \tag{4.26}$$

If composite strength is defined by weft fibers wall thickness is defined by

$$2\delta = \frac{(N_x+N_y)}{F_2(K+1)}. \tag{4.27}$$

Values obtained from conditions (4.26) or (4.27) have to be rounded up to nearest largest technologically realized values.

Example 4.2. Design of shell manufactured by combination of longitudinal and spiral winding from the same material (Fig. 4.4).

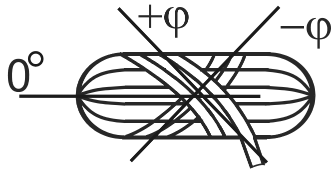


Fig. 4.4. Combination of longitudinal and spiral winding

In this case we have the following initial data:
$n=3$; $\delta_1 = \delta_1$; $\delta_2 = \delta_3 = \delta_2$; $\varphi_1 = 0°$; $\varphi_2 = -\varphi_3 = \varphi$; $E_{1i} = E_1$; $E_{2i} = E_2$; $K_i = K$;
$\mu_{12i} = \mu_{12}$; $F_{1i} = F_1$; $F_{2i} = F_2$.

Optimality condition (4.10) for this shell has the following view;

$$\delta_1(N_x - N_y K) + 2\delta_2\left[K\left(N_x \sin^2\varphi_i - N_y \cos^2\varphi_i\right) + \left(N_x \cos^2\varphi_i - N_y \sin^2\varphi_i\right)\right] = 0. \tag{4.28}$$

After this function analysis we can obtain that

– if $K>1$ and $\frac{1}{K} \le \frac{N_x}{N_y} \le K$ (4.29)

– or if $K<1$ and $K \le \frac{N_x}{N_y} \le \frac{1}{K}$ (4.30)

reinforcing angle φ can take any value satisfying the following condition

$$tg^2\varphi > \frac{N_y K - N_x}{N_x K - N_y}. \qquad (4.31)$$

At other combinations of loading parameters, elastic and strength properties of material and selected reinforcing scheme optimality condition can't be satisfied.

Thickness of longitudinal and spiral layers can be estimated using strength condition of warp fist breakage (4.32) or weft breakage (4.33).

$$\delta_1 + 2\delta_2 = \frac{K(N_x + N_y)}{F_1(K+1)}, \qquad (4.32)$$

$$\delta_1 + 2\delta_2 = \frac{(N_x + N_y)}{F_2(K+1)}. \qquad (4.33)$$

Obtained thicknesses have to be rounded to large side up to even number of monolayers. Since $\delta_1 = n_1\delta_0$, $\delta_2 = n_2\delta_0$ it is more desirable to select such φ value to obtain shell minimum mass. Therefore final parameters of this shell scheme can be obtained from following system of equations:

$$n_1 + 2n_2 = \frac{K(N_x + N_y)}{F_1\delta_0(K+1)} \quad \text{or} \quad n_1 + 2n_2 = \frac{(N_x + N_y)}{F_2\delta_0(K+1)};$$

$$\frac{n_1}{2n_2} = \frac{K\left(N_x \sin^2\varphi_1 - N_y \cos^2\varphi_1\right) + \left(N_x \cos^2\varphi_1 - N_y \sin^2\varphi_1\right)}{N_y K - N_x}. \qquad (4.34)$$

Example 4.3. Design of shell manufactured by combination of lateral and spiral winding from the same material (Fig. 4.5).

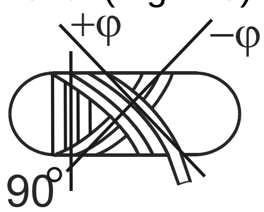


Fig. 4.5. Combination of lateral and spiral winding

The same analysis procedure can be conducted for this case. The following conclusions can be done.

n=3; $\delta_1 = \delta_1$; $\delta_2 = \delta_3 = \delta_2$; $\varphi_1 = 90^\circ$; $\varphi_2 = -\varphi_3 = \varphi$; $E_{1i} = E_1$; $E_{2i} = E_2$; $K_i = K$; $\mu_{12i} = \mu_{12}$; $F_{1i} = F_1$; $F_{2i} = F_2$.

– if $K > 1$ and $\frac{1}{K} \le \frac{N_x}{N_y} \le K$ (4.35)

– or if $K<1$ and $K \le \frac{N_x}{N_y} \le \frac{1}{K}$ (4.36)

$$tg^2\varphi < \frac{N_y K - N_x}{N_x K - N_y}. \quad (4.37)$$

Generally we can make conclusion that:

– structures $[0,\pm\varphi]$ and $[90,\pm\varphi]$ are supplement each other by intervals of allowable reinforcing angle;

– structure $[\pm\varphi]$ occupies intermediate position between them;

– if pure spiral winding is technologically restricted it is recommended to select longitudinally-spiral or laterally-spiral reinforcing scheme;

– considered reinforcing can be used for quite narrow range of loads (required by condition (4.2) $\varepsilon_x = \varepsilon_y$), that is why condition (3) $\sin 2\varphi_i = 0.$ has to be considered.

Example 4.4. Design of shell manufactured by lateral winding from the same material (Fig. 4.6).

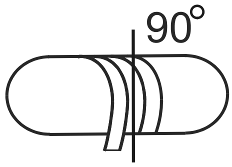


Fig. 4.6. Shell obtained by lateral winding

Initial data for analysis is the following: n=1; $\delta_1 = \delta$; $\varphi = 90^\circ$; $E_{1i} = E_1$; $E_{2i} = E_2$; $K_i = K$; $\mu_{12i} = \mu_{12}$; $F_{1i} = F_1$; $F_{2i} = F_2$.

Rigidity coefficient for this scheme are:

$$B_{11} = \delta \bar{E}_2;\ B_{12} = \delta \bar{E}_1 \mu_{21};\ B_{22} = \delta \bar{E}_1. \quad (4.38)$$

Inserting to expression (4.6) one can see that global strains (4.39), local strains (4.40) and local stress (4.41) are equal to

$$\varepsilon_x = \frac{N_x - N_y \mu_{21}}{\delta E_2}; \quad \varepsilon_y = \frac{N_y - N_x \mu_{12}}{\delta E_1}. \quad (4.39)$$

$$\varepsilon_1 = \varepsilon_x \cos^2\varphi + \varepsilon_y \sin^2\varphi = \varepsilon_y = \frac{N_y - N_x \mu_{12}}{\delta E_1};$$

$$\varepsilon_2 = \varepsilon_x \sin^2\varphi + \varepsilon_y \cos^2\varphi = \varepsilon_x = \frac{N_x - N_y \mu_{21}}{\delta E_2}. \quad (4.40)$$

$$\sigma_1 = \bar{E}_1(\varepsilon_1 + \mu_{21}\varepsilon_2) = \frac{N_y}{\delta};$$
$$\sigma_2 = \bar{E}_2(\varepsilon_1 + \mu_{12}\varepsilon_1) = \frac{N_x}{\delta}. \qquad (4.41)$$

Strength condition (criterion of maximum stress) is the following

$$\sigma_{1i} \le F_{1i};\ \sigma_{2i} \le F_{2i}. \qquad (4.42)$$

Thus shell thickness can be found as

$$\delta = \max\left\{\frac{N_y}{F_1}; \frac{N_x}{F_2}\right\}. \qquad (4.43)$$

Found value has to be rounded to largest even quantity of monolayers.

Example 4.5. Design of shell manufactured by longitudinal winding from the same material (Fig. 4.7).

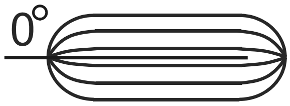


Fig. 4.7. Shell obtained by longitudinal winding

Initial data for analysis is the following: ***n***=1; $\delta_1 = \delta$; $\varphi = 0°$; $E_{1i} = E_1$; $E_{2i} = E_2$; $K_i = K$; $\mu_{12i} = \mu_{12}$; $F_{1i} = F_1$; $F_{2i} = F_2$.

Shell thickness can be found as for this scheme

$$\delta = \max\left\{\frac{N_x}{F_1}; \frac{N_y}{F_2}\right\}. \qquad (4.44)$$

Found value has to be rounded to largest even quantity of monolayers.

One should pay attention that condition (4.8) ($N_x > 0; N_y > 0$ or $N_x < 0$; $N_y < 0$) hasn't been satisfied for two above-mentioned structures.

Example 4.6. Design of shell manufactured by longitudinal-lateral winding from the same material (Fig. 4.8).

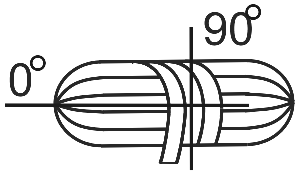


Fig. 4.8. Shell obtained by longitudinal- lateral winding

Initial data for analysis is the following: n=2; $\delta_1 = \delta_1$; $\delta_2 = \delta_2$; $\varphi_1 = 0°$; $\varphi_2 = 90°$; $E_{1i} = E_1$; $E_{2i} = E_2$; $K_i = K$; $\mu_{12i} = \mu_{12}$; $F_{1i} = F_1$; $F_{2i} = F_2$.

Skipping analysis transformations we should recommend following algorithm for shell design:

– to define series of thicknesses δ_1 considering technologically available;
Using conditions

$$\begin{aligned} &\bar{E}_1\left(\varepsilon_x+\mu\varepsilon_{21}\varepsilon_y\right)\le F_1;\ \bar{E}_1\left(\varepsilon_y+\mu\varepsilon_{21}\varepsilon_z\right)\le F_1;\\ &\bar{E}_2\left(\varepsilon_y+\mu\varepsilon_{12}\varepsilon_z\right)\le F_2;\ \bar{E}_2\left(\varepsilon_x+\mu\varepsilon_{12}\varepsilon_y\right)\le F_2; \end{aligned} \tag{4.45}$$

And relationships

$$\begin{aligned} &\varepsilon_x=\frac{N_x\left(\delta_1\bar{E}_2+\delta_2\bar{E}_1\right)-N_y\left(\delta_1+\delta_2\right)\bar{E}_1\mu_{21}}{\left(\delta_1\bar{E}_1+\delta_2\bar{E}_2\right)\left(\delta_1\bar{E}_2+\delta_2\bar{E}_1\right)-\left(\delta_1+\delta_2\right)^2\bar{E}_1^2\mu_{21}^2};\\ &\varepsilon_y=\frac{N_y\left(\delta_1\bar{E}_1+\delta_2\bar{E}_2\right)-N_x\left(\delta_1+\delta_2\right)\bar{E}_1\mu_{21}}{\left(\delta_1\bar{E}_1+\delta_2\bar{E}_2\right)\left(\delta_1\bar{E}_2+\delta_2\bar{E}_1\right)-\left(\delta_1+\delta_2\right)^2\bar{E}_1^2\mu_{21}^2}.\\ &\varepsilon_{11}=\varepsilon_{22}=\varepsilon_x;\varepsilon_{21}=\varepsilon_{21}=\varepsilon_y;\\ &\sigma_{11}=\bar{E}_1\left(\varepsilon_x+\mu_{21}\varepsilon_y\right);\ \sigma_{21}=\bar{E}_2\left(\varepsilon_y+\mu_{12}\varepsilon_x\right);\\ &\sigma_{12}=\bar{E}_1\left(\varepsilon_y+\mu_{21}\varepsilon_x\right);\ \sigma_{22}=\bar{E}_2\left(\varepsilon_x+\mu_{12}\varepsilon_y\right). \end{aligned} \tag{4.46}$$

– to define δ_2 for any δ_1 series considering restrictions using (4.45), (4.46) and round it up to even value;

– to find optimal thickness values considering condition $\delta_1+\delta_2\to\min$;

As the result we can underline that one can apply forces of different signs (shear forces are absent).

4.3 Design procedure for shell manufactured by winding with UD tapes or tows

Main difference of the following design procedure is closing to zero physical and mechanical properties of composite in lateral direction (because of low quantity or even absence of weft fibers and low resin strength comparing with fiber longitudinal properties). Therefore, $E_{2i}=\mu_{12i}=\mu_{21i}=G_{12i}=F_{2i}=0$. Then optimality condition transforms to the following form

$$\sum_{i=1}^{n}\delta_i\bar{E}_{1i}\left(N_x\sin^2\varphi_i-N_y\cos^2\varphi_i\right)=0. \tag{4.47}$$

Local stress in longitudinal direction 1 is equal to

$$\sigma_{1i}=\frac{\left(N_x+N_y\right)\bar{E}_{1i}}{\sum_{i=1}^{n}\delta_i\bar{E}_{1i}};\ \left(\sigma_{2i}=0\right). \tag{4.48}$$

Condition (4.47) allows a large amount of reinforcing schemes satisfying it. Let's consider the most widely used winding schemes.

Table 4.1. Parameters of shell winding by UD-tow

Winding scheme	Reinforcing angle	Shell thickness
Spiral	$tg\varphi = \sqrt{\frac{N_y}{N_x}}$	$2\delta = \frac{N_x + N_y}{F_1}.$
Longitudinal-spiral $\varphi_1 = 0°$	$tg\varphi \geq \sqrt{\frac{N_y}{N_x}}$	$\delta_1 + 2\delta_2 = \frac{N_x + N_y}{F_1}.$
Lateral-spiral $\varphi_1 = 90°$	$tg\varphi \leq \sqrt{\frac{N_y}{N_x}}$	$\delta_1 + 2\delta_2 = \frac{N_x + N_y}{F_1}.$

Checking-up questions

1. Give examples of airplane's units that can be modeled as shells.
2. What typical loading schemes of shells are considered at shell design?
3. What design criteria can be used for shells design?
4. What equipment and manufacturing processes are used for shells manufacturing?
5. What typical reinforcing schemes are used for design of shells produced by fabric and UD-tow (UD-tape) winding?
6. Is it possible to realize maximum strength properties of a fabric at shell winding?

Theme 5. DESIGN OF SHELLS OF REVOLUTION OBTAINED BY WINDING AT GENERAL LOADING

Let's consider geometrically axis-symmetrical cylindrical shell generally loaded with known forces (moments) $N_x, Q_y, Q_z, M_x, M_y, M_z$ and internal and external pressure P (Fig. 5.1) [4, 6].

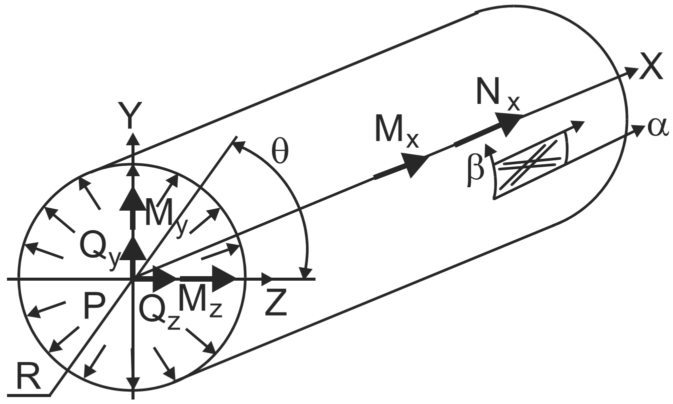


Fig. 5.1. Typical analysis scheme for shell structure with general loading

Using angle coordinate θ internal forces at any point of the shell (in local coordinate system $\alpha\beta$) can be expressed as follows:

$$N_\alpha = \frac{1}{2\pi R^2}\left(N_x R + 2M_z \sin\theta - 2M_y \cos\theta\right);$$
$$N_\beta = PR; \qquad (5.1)$$
$$q_{\alpha\beta} = \frac{1}{2\pi R^2}\left(M_x - 2Q_y R\cos\theta + 2Q_z R\cos\theta\right).$$

Physical law for composite package of shell wall can be written as

$$N_\alpha = \varepsilon_\alpha B_{11} + \varepsilon_\beta B_{12};$$
$$N_\beta = \varepsilon_\alpha B_{12} + \varepsilon_\beta B_{22}; \qquad (5.2)$$
$$q_{\alpha\beta} = \gamma_{\alpha\beta} B_{33}.$$

Generally shell wall includes monolayers oriented in longitudinal direction - $\varphi=0^\circ$; lateral direction - $\varphi=90^\circ$ and spiral direction - $\pm\varphi$. Then total shell thickness is equal to

$$\delta = \delta_1 + \delta_2 + 2\delta_3. \qquad (5.3)$$

Optimality criterion for shell design is criterion of **minimal mass** of unit shell area:

$$\bar{M} = \sum_{i=1}^{4} \delta_i \rho_i = \delta_1\rho_1 + \delta_2\rho_2 + 2\delta_3\rho_3 \to \min, \qquad (5.4)$$

where ρ_1, ρ_2, ρ_3 – density of monolayers with orientation $0^\circ, 90^\circ$ and $\pm\varphi$.

Introducing designation $\delta_1 = \psi_1\delta;\ \delta_2 = \psi_2\delta;\ 2\delta_3 = \delta\left(1-\psi_1-\psi_2\right)$ objective function (5.4) can be written as

$$\bar{M} = \left[\psi_1\rho_1 + \psi_2\rho_2 + \rho_3\left(1 - \psi_1 - \psi_2\right)\right] \to \min. \qquad (5.5)$$

Therefore we have four variables - $\psi_1, \psi_2, \delta, \varphi$.

To simplify shell strength analysis let's select strength criterion "at the point" using criterion of maximum stress and Mises-Hill criterion:

$$\sigma_\alpha = \frac{N_\alpha}{\delta} \le F_\alpha;\ \sigma_\beta = \frac{N_\beta}{\delta} \le F_\beta;\ \tau_{\alpha\beta} = \frac{q_{\alpha\beta}}{\delta} \le F_{\alpha\beta}; \qquad (5.6)$$

$$\frac{\sigma_\alpha^2}{F_\alpha^2} - \frac{\sigma_\alpha\sigma_\beta}{F_\alpha F_\beta} + \frac{\sigma_\beta^2}{F_\beta^2} + \frac{\tau_{\alpha\beta}^2}{F_{\alpha\beta}^2} \le 1, \qquad (5.7)$$

where $F_\alpha, F_\beta, F_{\alpha\beta}$ – ultimate strength for composite package in axes $\alpha\beta$.

$$F_\alpha = \begin{cases} F_{\alpha t} \text{ at } \sigma_\alpha \ge 0; \\ F_{\alpha c} \text{ at } \sigma_\alpha < 0; \end{cases} \quad F_\beta = \begin{cases} F_{\beta t} \text{ at } \sigma_\beta \ge 0; \\ F_{\beta c} \text{ at } \sigma_\beta < 0. \end{cases} \qquad (5.8)$$

Indexes "t" and "c" mean tension and compression correspondingly.

Using conditions (5.6), (5.7) one can find function $\delta = \left(\psi_1, \psi_2, \varphi\right)$:

$$\delta = \left(\psi_1, \psi_2, \varphi\right) = \max\left\{\frac{N_\alpha}{F_\alpha}; \frac{N_\beta}{F_\beta}; \frac{q_{\alpha\beta}}{F_{\alpha\beta}}\right\}. \qquad (5.9)$$

$$\delta = \left(\psi_1, \psi_2, \varphi\right) = \sqrt{\frac{N_\alpha^2}{F_\alpha^2} - \frac{N_\alpha}{F_\alpha}\frac{N_\beta}{F_\beta} + \frac{N_\beta^2}{F_\beta^2} + \frac{q_{\alpha\beta}^2}{F_{\alpha\beta}^2}}. \qquad (5.10)$$

Varying angle coordinate θ and linear coordinate x one can find series of thicknesses; then satisfying objective function (5.5) we can obtain optimal thickness distribution through shell surface.

Above-mentioned design algorithm suggests implementation of **integral** physical and mechanical properties of composite package for its strength estimation (i.e. theoretical forecasting). Practically such approach is not always valid because absence of data comparing experimental and theoretical results. That is why design algorithm can be founded on composite **monolayer** properties (which can be easier obtained experimentally).

According to this second approach function $\delta = \left(\psi_1, \psi_2, \varphi\right)$ can be written

$$\delta = \left(\psi_1, \psi_2, \varphi\right) = \max_{(i)}\left\{\frac{\bar{E}_{1i} A_{1i}}{F_{1i}}; \frac{\bar{E}_{2i} A_{2i}}{F_{2i}}; \frac{G_{12i} A_{12i}}{F_{12i}}\right\}; \qquad (5.11)$$

$$\delta = \left(\psi_1, \psi_2, \varphi\right) = \max_{(i)}\sqrt{\frac{\bar{E}_{1i}^2 A_{1i}^2}{F_{1i}^2} - \frac{\bar{E}_{1i}\bar{E}_{2i} A_{1i} A_{2i}}{F_{1i} F_{2i}} + \frac{\bar{E}_{2i}^2 A_{2i}^2}{F_{2i}^2} + \frac{G_{12i}^2 A_{12i}^2}{F_{12i}^2}}, \qquad (5.12)$$

where

$$F_{1i}=\begin{cases}F_{1it} \text{ at } A_{1i}\geq 0;\\ F_{1ic} \text{ at } A_{1i}<0;\end{cases}\quad F_{2i}=\begin{cases}F_{2it} \text{ at } A_{2i}\geq 0;\\ F_{2ic} \text{ at } A_{2i}<0.\end{cases} \tag{5.13}$$

$$A_{1i}=\frac{N_\alpha}{E_\alpha}Y_{\alpha 1i}+\frac{N_\beta}{E_\beta}Y_{\beta 1i}+\frac{q_{\alpha\beta}}{G_{\alpha\beta}}Y_{\alpha\beta 1i};$$

$$A_{2i}=\frac{N_\alpha}{E_\alpha}Y_{\alpha 2i}+\frac{N_\beta}{E_\beta}Y_{\beta 2i}+\frac{q_{\alpha\beta}}{G_{\alpha\beta}}Y_{\alpha\beta 2i}; \tag{5.14}$$

$$A_{12i}=\frac{N_\alpha}{E_\alpha}Y_{\alpha 12i}+\frac{N_\beta}{E_\beta}Y_{\beta 12i}+\frac{q_{\alpha\beta}}{G_{\alpha\beta}}Y_{\alpha\beta 12i}.$$

$$\begin{gathered}
Y_{\alpha 1i}=\cos^2\varphi_i\left(1-\mu_{21i}\,\mu_{\alpha\beta}\right)+\sin^2\varphi_i\left(\mu_{21i}-\mu_{\alpha\beta}\right);\\
Y_{\alpha 2i}=\cos^2\varphi_i\left(\mu_{12i}-\mu_{\alpha\beta}\right)+\sin^2\varphi_i\left(1-\mu_{12i}\,\mu_{\alpha\beta}\right);\ Y_{\alpha 12i}=-\sin 2\varphi_i\left(1+\mu_{\alpha\beta}\right);\\
Y_{\beta 1i}=\cos^2\varphi_i\left(\mu_{21i}-\mu_{\beta\alpha}\right)+\sin^2\varphi_i\left(1-\mu_{21i}\,\mu_{\beta\alpha}\right);\\
Y_{\beta 2i}=\cos^2\varphi_i\left(1-\mu_{12i}\,\mu_{\beta\alpha}\right)+\sin^2\varphi_i\left(\mu_{12i}-\mu_{\beta\alpha}\right);\\
Y_{\beta 12i}=\sin 2\varphi_i\left(1+\mu_{\beta\alpha}\right);\ Y_{\alpha\beta 1i}=\sin\varphi_i\cos\varphi_i\left(1-\mu_{21i}\right);\\
Y_{\alpha\beta 2i}=\sin\varphi_i\cos\varphi_i\left(\mu_{12i}-1\right);\ Y_{\alpha\beta 12i}=\cos 2\varphi_i.
\end{gathered} \tag{5.15}$$

$$E_\alpha=\bar B_{11}-\frac{\bar B_{12}^2}{\bar B_{22}};\ E_\beta=\bar B_{22}-\frac{\bar B_{12}^2}{\bar B_{11}};\ G_{\alpha\beta}=\bar B_{33};\ \mu_{\alpha\beta}=\frac{\bar B_{12}}{\bar B_{22}};\ \mu_{\beta\alpha}=\frac{\bar B_{12}}{\bar B_{11}}. \tag{5.16}$$

$$\begin{gathered}
\bar B_{11}=\frac{B_{11}}{\delta}=\psi_1\bar E_{11}+\psi_2\bar E_{22}+\left(1-\psi_1-\psi_2\right)\Big(\bar E_{13}\cos^4\varphi+\\
+\bar E_{23}\sin^4\varphi+2\bar E_{13}\,\mu_{213}\sin^2\varphi\cos^2\varphi+G_{123}\sin^2 2\varphi\Big);\\
\bar B_{22}=\frac{B_{22}}{\delta}=\psi_1\bar E_{21}+\psi_2\bar E_{12}+\left(1-\psi_1-\psi_2\right)\Big(\bar E_{13}\sin^4\varphi+\\
+\bar E_{23}\cos^4\varphi+2\bar E_{13}\,\mu_{213}\sin^2\varphi\cos^2\varphi+G_{123}\sin^2 2\varphi\Big);\\
\bar B_{12}=\frac{B_{12}}{\delta}=\psi_1\bar E_{11}\mu_{211}+\psi_2\bar E_{12}\,\mu_{212}+\left(1-\psi_1-\psi_2\right)\Big[\Big(\bar E_{13}+\\
+\bar E_{23}\Big)\sin^2\varphi\cos^2\varphi+\bar E_{13}\,\mu_{213}\left(\sin^4\varphi+\cos^4\varphi\right)-G_{123}\sin^2 2\varphi\Big];\\
\bar B_{33}=\frac{B_{33}}{\delta}=\psi_1 G_{121}+\psi_2 G_{122}+\left(1-\psi_1-\psi_2\right)\Big[\Big(\bar E_{13}+\\
+\bar E_{23}-2\bar E_{13}\,\mu_{213}\Big)\sin^2\varphi\cos^2\varphi+G_{123}\cos^2 2\varphi\Big].
\end{gathered} \tag{5.17}$$

Integral strength properties of composite package by criterion of maximum stress and Mises-Hill criterion can be estimated as:

– strength along local axis α at tension (“t”) and compression (“c”)

$$F_{\alpha t}=\min_{(i)}\left\{\frac{E_\alpha F_{1i}}{\bar{\bar{E}}_{1i}\,\mathrm{abs}Y_{\alpha 1i}};\frac{E_\alpha F_{2i}}{\bar{\bar{E}}_{2i}\,\mathrm{abs}Y_{\alpha 2i}};\frac{E_\alpha F_{12i}}{G_{12i}\,\mathrm{abs}Y_{\alpha 12i}}\right\};$$

$$F_{\alpha t}=\min_{(i)}\left\{E_\alpha\left(\frac{\bar{E}_{1i}^2Y_{\alpha 1i}^2}{F_{1i}^2}-\frac{\bar{E}_{1i}\bar{E}_{2i}\,Y_{\alpha 1i}\,Y_{\alpha 2i}}{F_{1i}F_{2i}}+\frac{\bar{E}_{2i}^2Y_{\alpha 2i}^2}{F_{2i}^2}+\frac{G_{12i}^2Y_{\alpha 12i}^2}{F_{12i}^2}\right)^{-0.5}\right\}, \tag{5.18}$$

where

$$F_{1i}=\begin{cases}F_{1it}\ \text{at}\ Y_{\alpha 1i}\ge 0;\\ F_{1ic}\ \text{at}\ Y_{\alpha 1i}<0;\end{cases}\quad F_{2i}=\begin{cases}F_{2it}\ \text{at}\ Y_{\alpha 2i}\ge 0;\\ F_{2ic}\ \text{at}\ Y_{\alpha 2i}<0.\end{cases} \tag{5.19}$$

$$F_{\alpha c}=\min_{(i)}\left\{\frac{E_\alpha F_{1i}}{\bar{\bar{E}}_{1i}\,\mathrm{abs}Y_{\alpha 1i}};\frac{E_\alpha F_{2i}}{\bar{\bar{E}}_{2i}\,\mathrm{abs}Y_{\alpha 2i}};\frac{E_\alpha F_{12i}}{G_{12i}\,\mathrm{abs}Y_{\alpha 12i}}\right\};$$

$$F_{\alpha c}=\min_{(i)}\left\{E_\alpha\left(\frac{\bar{E}_{1i}^2Y_{\alpha 1i}^2}{F_{1i}^2}-\frac{\bar{E}_{1i}\bar{E}_{2i}\,Y_{\alpha 1i}\,Y_{\alpha 2i}}{F_{1i}F_{2i}}+\frac{\bar{E}_{2i}^2Y_{\alpha 2i}^2}{F_{2i}^2}+\frac{G_{12i}^2Y_{\alpha 12i}^2}{F_{12i}^2}\right)^{-0.5}\right\}, \tag{5.20}$$

where

$$F_{1i}=\begin{cases}F_{1ic}\ \text{at}\ Y_{\alpha 1i}\ge 0;\\ F_{1it}\ \text{at}\ Y_{\alpha 1i}<0;\end{cases}\quad F_{2i}=\begin{cases}F_{2ic}\ \text{at}\ Y_{\alpha 2i}\ge 0;\\ F_{2it}\ \text{at}\ Y_{\alpha 2i}<0.\end{cases} \tag{5.21}$$

– strength along local axis β at tension (“***t***”) and compression (“***c***”)

$$F_{\beta t}=\min_{(i)}\left\{\frac{E_\beta F_{1i}}{\bar{\bar{E}}_{1i}\,\mathrm{abs}Y_{\beta 1i}};\frac{E_\beta F_{2i}}{\bar{\bar{E}}_{2i}\,\mathrm{abs}Y_{\beta 2i}};\frac{E_\beta F_{12i}}{G_{12i}\,\mathrm{abs}Y_{\beta 12i}}\right\};$$

$$F_{\beta t}=\min_{(i)}\left\{E_\beta\left(\frac{\bar{E}_{1i}^2Y_{\beta 1i}^2}{F_{1i}^2}-\frac{\bar{E}_{1i}\bar{E}_{2i}\,Y_{\beta 1i}\,Y_{\beta 2i}}{F_{1i}F_{2i}}+\frac{\bar{E}_{2i}^2Y_{\beta 2i}^2}{F_{2i}^2}+\frac{G_{12i}^2Y_{\beta 12i}^2}{F_{12i}^2}\right)^{-0.5}\right\}, \tag{5.22}$$

where

$$F_{1i}=\begin{cases}F_{1it}\ \text{at}\ Y_{\beta 1i}\ge 0;\\ F_{1ic}\ \text{at}\ Y_{\beta 1i}<0;\end{cases}\quad F_{2i}=\begin{cases}F_{2it}\ \text{at}\ Y_{\beta 2i}\ge 0;\\ F_{2ic}\ \text{at}\ Y_{\beta 2i}<0.\end{cases} \tag{5.23}$$

$$F_{\beta c}=\min_{(i)}\left\{\frac{E_\beta F_{1i}}{\bar{\bar{E}}_{1i}\,\mathrm{abs}Y_{\beta 1i}};\frac{E_\beta F_{2i}}{\bar{\bar{E}}_{2i}\,\mathrm{abs}Y_{\beta 2i}};\frac{E_\beta F_{12i}}{G_{12i}\,\mathrm{abs}Y_{\beta 12i}}\right\};$$

$$F_{\beta c}=\min_{(i)}\left\{E_\beta\left(\frac{\bar{E}_{1i}^2Y_{\beta 1i}^2}{F_{1i}^2}-\frac{\bar{E}_{1i}\bar{E}_{2i}\,Y_{\beta 1i}\,Y_{\beta 2i}}{F_{1i}F_{2i}}+\frac{\bar{E}_{2i}^2Y_{\beta 2i}^2}{F_{2i}^2}+\frac{G_{12i}^2Y_{\beta 12i}^2}{F_{12i}^2}\right)^{-0.5}\right\}, \tag{5.24}$$

where

$$F_{1i}=\begin{cases}F_{1ic} \text{ at } Y_{\beta 1i}\geq 0;\\ F_{1it} \text{ at } Y_{\beta 1i}<0;\end{cases}\quad F_{2i}=\begin{cases}F_{2ic} \text{ at } Y_{\beta 2i}\geq 0;\\ F_{2it} \text{ at } Y_{\beta 2i}<0.\end{cases} \tag{5.25}$$

– shear strength in axes $\alpha\beta$

$$F_{\beta c}=\min_{(i)}\left\{\frac{G_{\alpha\beta}F_{1i}}{\bar{\bar{E}}_{1i}\,\mathrm{abs}Y_{\alpha\beta 1i}};\frac{G_{\alpha\beta}F_{2i}}{\bar{\bar{E}}_{2i}\,\mathrm{abs}Y_{\alpha\beta 2i}};\frac{G_{\alpha\beta}F_{12i}}{G_{12i}\,\mathrm{abs}Y_{\alpha\beta 12i}}\right\};$$

$$F_{\beta c}=\min_{(i)}\left\{G_{\alpha\beta}\left(\frac{\bar{\bar{E}}_{1i}^{2}Y_{\alpha\beta 1i}^{2}}{F_{1i}^{2}}-\frac{\bar{\bar{E}}_{1i}\bar{\bar{E}}_{2i}\,Y_{\alpha\beta 1i}\,Y_{\alpha\beta 2i}}{F_{1i}F_{2i}}+\frac{\bar{\bar{E}}_{2i}^{2}Y_{\alpha\beta 2i}^{2}}{F_{2i}^{2}}+\frac{G_{12i}^{2}Y_{\alpha\beta 12i}^{2}}{F_{12i}^{2}}\right)^{-0.5}\right\}, \tag{5.26}$$

where

$$F_{1i}=\begin{cases}F_{1it} \text{ at } Y_{\alpha\beta 1i}\geq 0;\\ F_{1ic} \text{ at } Y_{\alpha\beta 1i}<0;\end{cases}\quad F_{2i}=\begin{cases}F_{2it} \text{ at } Y_{\alpha\beta 2i}\geq 0;\\ F_{2ic} \text{ at } Y_{\alpha\beta 2i}<0.\end{cases} \tag{5.27}$$

Checking-up questions

1. Draw typical loading scheme for shell at generalized loading.
2. Write dependencies for determiation loads applied to representative element of a shell in local coordinate system.
3. What strength criteria can be used for design of a shell at generalized loading?
4. What does volume fraction of a definite monolayers in total shell thickness mean?
5. How to estimate global (integral) elastic properties of a shell composite package?

Theme 6. DESIGN OF RODS MADE OF COMPOSITE MATERIALS. STRUCTURAL AND MANUFACTURE SOLUTIONS OF RODS AND THEIR CONNECTING TIPS

6.1 Fundamentals of rods design

Efficiency of composite materials application in aircraft structures is defined by load type. That is stipulated by different abilities of composite components (matrix and fibers) to withstand the variety of loads. The maximum of mass decreasing can be reached in those articles and units in which loading is simple, i.e. when high strength ability of composites along the fibers can be fully realized. From this point of view rods and constructions made of them are ideal elements for wide using of composites in aircraft structures [5].

Usually rods are loaded with axis tensile or compressive force only. To withstand this force composite fibers should be oriented in longitudinal direction. The distinctive type of carrying ability loosing is general instability of rods. It can be explained by the following fact: the value of critical force is proportional to longitudinal elastic modulus of rod material and depends on transversal interlayer shear modulus of composite. Shear modulus of majority of composites is quite small that is why this fact should be considered at the stage of rod design.

Another type of rod destroying is their local instability. It can be realized in two forms: symmetrical and non-symmetrical. For metal rod non-symmetrical form of local instability is not necessary to be considered but for composite rods this form of instability is the most important in a majority of analysis cases.

The following problem, which should be taken into consideration, is quite low interlayer strength of polymer composites. So edge effect in the place of joining of composite rod and metal bushing should be analyzed.

The peculiarities of manufacture process of pipe elements made of composites design demand some important restrictions on the connective bushings design. These restrictions should be considered at the stage of composite rod wall design.

6.2. The field of rod system application in aircraft structures

Wide application of rods in aircraft structures is stipulated by their high mass efficiency, especially at low and middle loading intensity and at the case of existing of enough volume for rod system placing.

Rods are the frameworks of rigid connections of aircraft steering surfaces control system and wing mechanization. Truss constructions are widely used for joining of rocket stages, motor frame, light aircraft fuselage skeleton, some aircraft units (wing spars, ribs), installation of hanging brackets for external useful loads etc (Fig. 6.1).

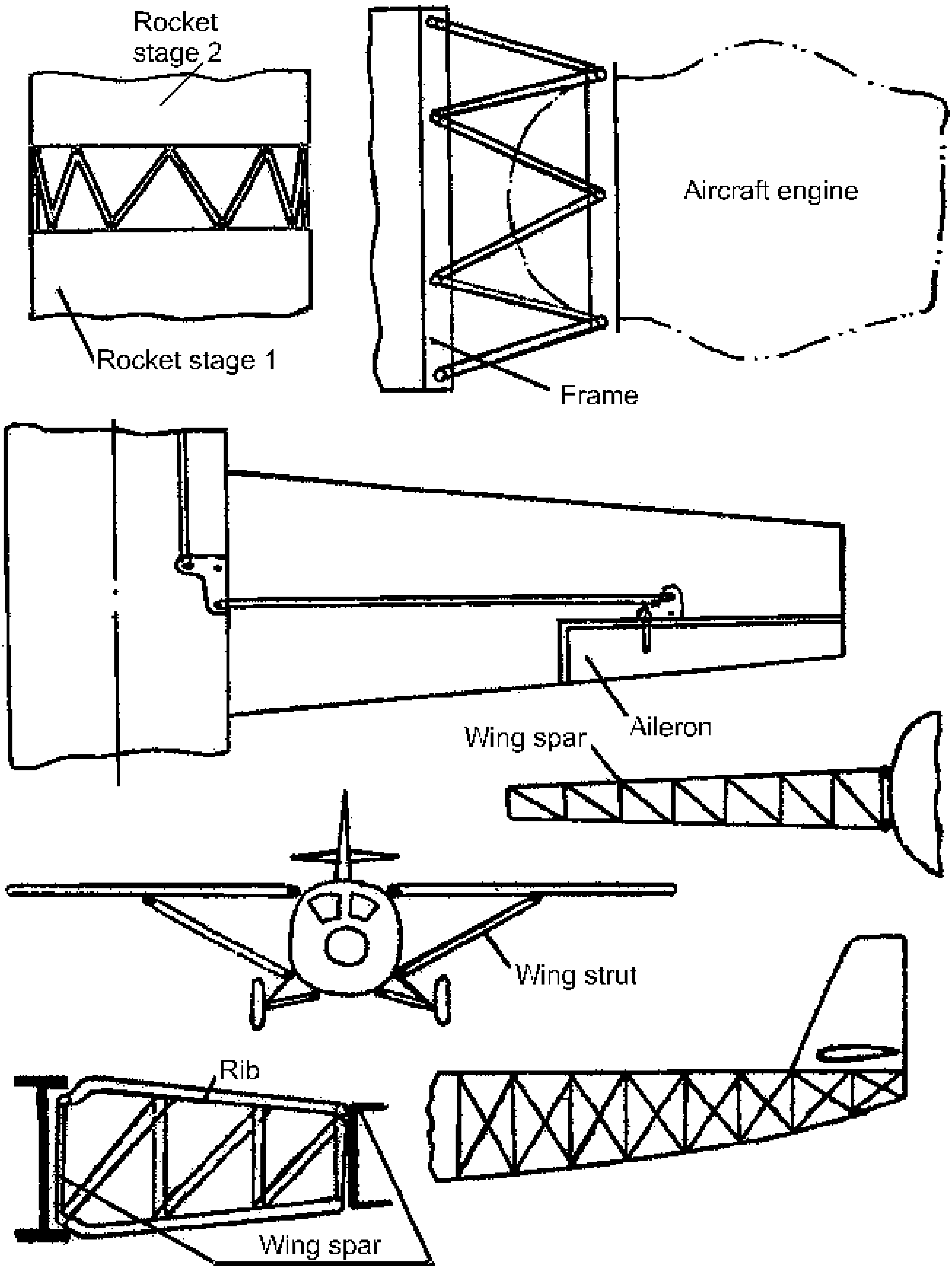


Fig. 6.1. Application of rods in aircraft structures

In addition rod structures are widely used in civil engineering, machine building, lifting gears and vehicles, bridge building etc.

Distinctive rod design features are the following: direct form, constant cross section along the rod length, hinge fixing to other construction parts. These features permit to manufacture rod by means of up-to-date high effective methods of pulltrusion, winding, rolltrusion, extrusion etc.

Predominated loading types for rods are tensile and compression forces according to load case. Design of definite rod (for instance, control rod of aircraft control system with definite length and loads) can be made by means of minimum mass criterion. In rod grounded systems (trusses) system geometry should be determined. That is why rods length, their quantity and their individual loads are functions of general truss scheme. Therefore determination of truss structural parameters is quite complicated problem, especially for statically undetermined systems. Today design of multi-rod constructions is usually made by iteration methods, so the problem of individual rod design is very important.

6.3. The method of rod design

The criteria of minimal rod mass is used for design [5, 6]

$$M=\rho\cdot\ell\cdot f\rightarrow \min. \qquad (6.1)$$

where ρ– is density of rod material; ℓ - rod length; f - rod cross section.

In the first design step parts with $\Delta\ell$ length (Fig. 4.2) of rod don’t take into consideration. The aim of design is to determine rod mean radius R, wall thickness δ and composite structure (stacking sequence).

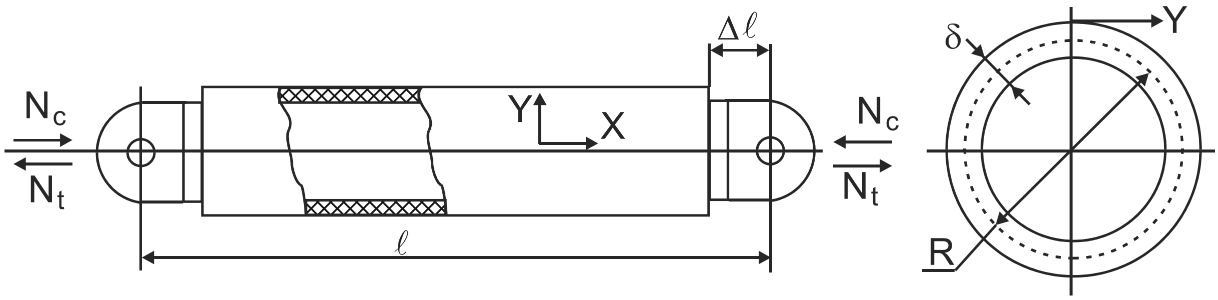


Fig. 4.2. Arrangement of typical rod

The restrictions for design are the following:

a) Strength condition at tension and compression of rod must be provided:

$$2\pi\cdot R\cdot\delta\cdot F_{xt}\geq N_t\,;$$
$$2\pi\cdot R\cdot\delta\cdot F_{xc}\geq N_c\,, \qquad (6.2)$$

where Fxt, Fxc – strength limit of rod material at tension (***t***) and compression (***c***).

b) Condition of rod general stability must be provided:

$$\frac{kD\pi^2}{l^2\left(1+\frac{kD\pi^2}{l^2K_x}\right)} \geq N_C, \qquad (6.3)$$

where D- flexural stiffness of rod wall; K_X – shear stiffness of wall; k- coefficient which characterizes a type of rod tips supporting; k=1 for hinging; k=0.25 - for cantilevered rod; k=4- for double end rigid supporting.

At the first design step the stacking sequence of wall made of composite is undetermined. That is why we assume that wall material is isotropic. By the way such manufacture processes as winding and pulltrusion permit to obtain approximately isotropic rod structure. Therefore we can write

$$D=\frac{\pi R^3\delta B}{B_{22}}, \quad K_x=\pi R\cdot B_{33}, \qquad (6.4)$$

where $B=B_{11}\cdot B_{22}-B_{12}^2$ - for hollow rod;

$B=B_{11}\cdot B_{22}$ - for rod with undeformed contour of cross section (for instance one can fill in internal volume with rigid filler).

B_{ij} - stiffness coefficients.

If wall material is assumed to be isotropic the above-mentioned formulas can be transformed to

$$D=\pi\cdot\delta R^3E_x, \quad K_x=\pi R\cdot\delta G_{xy}, \qquad (6.5)$$

where E_X – elastic modulus of composite at tension or compression;

G_{XY}- shear modulus of composite.

c) Condition of rod local stability (buckling) as thin-walled structure.

Usually one can consider two forms of shell (rod) instability:

- symmetrical instability refer to longitudinal shell axis (Fig. 6.3);
- non-symmetrical instability(Fig. 6.3, b).

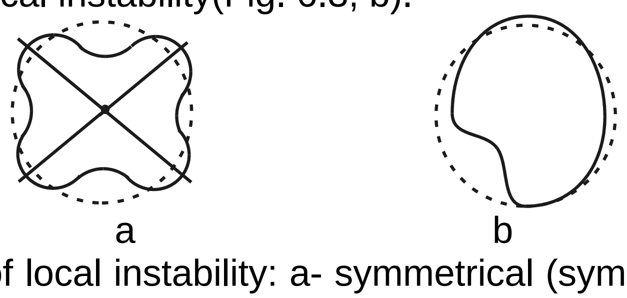


Fig 6.3. Two forms of local instability: a- symmetrical (symmetry of semi-waves can be observed); b- non-symmetrical (no symmetry of deformed contour)

The critical forces N_{cr}^{ax} and N_{cr}^{nax} which can make these instability forms can be determined by the following formulas

$$N_{cr}^{ax}=\frac{2\pi\cdot\delta^2}{\sqrt{3}}\sqrt{\frac{E_x\cdot E_y}{1-\mu_{xy}\cdot\mu_{yx}}}, \qquad (6.6)$$

$$N_{cr}^{nax}=\min_{(m,n)}\left\{\frac{\pi\cdot R\cdot\delta^3}{6\cdot\lambda_m^2}\cdot\left[\bar{E}_x\cdot\lambda_m^4+2\cdot\left(\bar{E}_x\cdot\mu_{yx}+2\cdot G_{xy}\right)\cdot\lambda_m^2\cdot\lambda_n^2+\bar{E}_y\cdot\lambda_n^4\right]+\right.$$

$$\left.+\frac{2\cdot\pi\cdot\delta\cdot\lambda_m^2}{R\cdot\left[\frac{\lambda_m^4}{E_y}+\left(\frac{1}{G_{xy}}-\frac{2\cdot\mu_{xy}}{E_x}\right)\cdot\lambda_m^2\lambda_n^2+\frac{\lambda_n^4}{E_x}\right]}\right\}, \qquad (6.7)$$

$$\bar{E}_x=\frac{E_x}{1-\mu_{xy}\cdot\mu_{yx}};\ \bar{E}_y=\frac{E_y}{1-\mu_{xy}\cdot\mu_{yx}};\ \lambda_m=\frac{\pi\cdot m}{\ell};\ \lambda_n=\frac{n}{R},$$

where N_{cr}^{ax} -critical force of local symmetrical instability;

N_{cr}^{nax} -critical force of local non-symmetrical instability;

E_X, E_Y- longitudinal and lateral elasticity modules of composite;

μ_{xy}, μ_{yx} - Poisson ratios of composite;

m, n- quantity of semi-waves along longitudinal axis of rod and in tangential direction (Fig. 4.4).

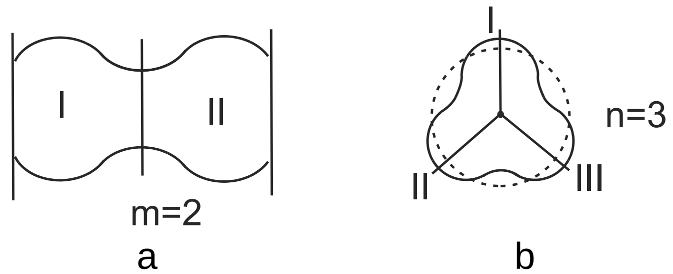


Fig. 4.4. Semi-waves in longitudinal (a) and tangential (b) directions

Condition of local stability can be written:

$$N_{cr}^{ax}\geq N_c,$$

$$N_{cr}^{nax}\geq N_c, \qquad (6.8)$$

It’s known that critical stresses of general and local instability can be more than ultimate stresses of material. So the problem can have solution, which has no physical meaning. This fact can be taken into account by means of two models (Fig. 4.5).

According to the first model (Fig. 4.5, a) the curve section of Euler critical stresses is limited by ultimate strength.

According to the second model (Fig. 4.5, b) critical stresses should be calculated by the formula

$$N_{cr}=2\pi\cdot R\cdot\delta\ F_{xc}\frac{1+\Psi}{1+\Psi+\Psi^2},\ \Psi=\frac{2\pi\cdot R\cdot\delta\ F_{xc}}{N_{cr}^{Eu}}, \qquad (6.9)$$

where N_{cr}^{Eu}- critical force calculated by Euler formula.

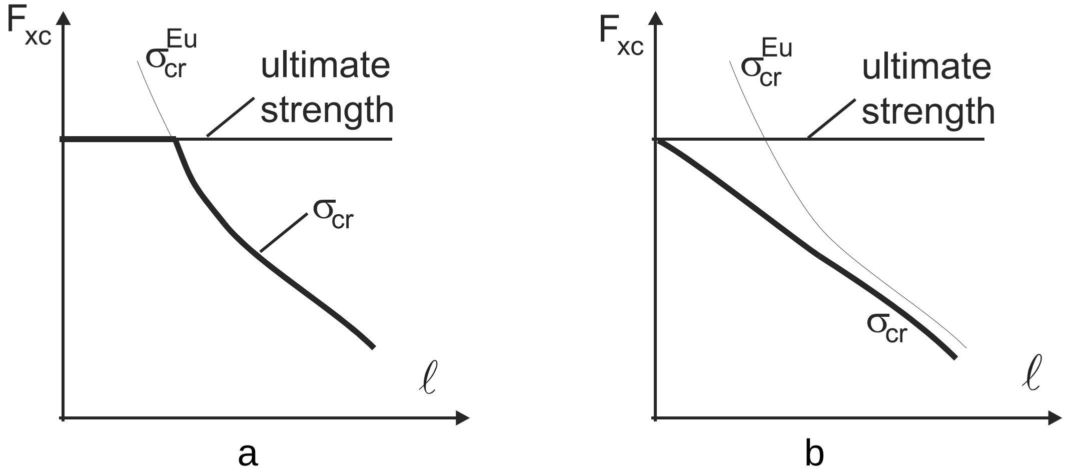


Fig. 4.5. Models of Euler force calculation

Besides limitations which determine load-carrying ability of rod there are design $\left(R_{min}^{D} \text{ or } R_{max}^{D}\right)$ and manufacture limitations $\left(R_{min}^{M} \text{ or } R_{max}^{M}\right)$:

$$R_{min}^{D} \le R \le R_{max}^{M}, \quad \delta = n \cdot \delta_0, \tag{6.10}$$

where R_{min}^{D} - minimal available rod radius. It's determined by the tip dimension, mandrel rigidity etc; R_{max}^{M} - maximum rod radius. The main unit dimensions, for instance airfoil thickness, determine this radius; n, δ_0- quantity of composite layers and prepreg (fabric, tape etc) thickness.

Hence, the problem of rod wall design is to minimize rod mass function M:

$$M = 2\pi \cdot R \cdot \delta \cdot \, l \cdot \rho \to \min. \tag{6.11}$$

The limitations are the following;

$$2\pi R\delta \;\; F_{xt} \ge N_t, \qquad 2\pi R\delta \;\; F_{xc} \ge N_c, \tag{6.12}$$

$$\frac{k\pi^3 R^3 \delta \;\; E_x}{l^2\left(1 + \dfrac{kR^2 E_x \pi^2}{l^2 G_{xy}}\right)} \ge N_c, \tag{6.13}$$

$$2\pi\delta^2 \sqrt{\frac{E_x E_y}{3\left(1 + \mu_{xy}\mu_{yx}\right)}} \ge N_c, \tag{6.14}$$

$$\min_{(m,n)} \left\{ \frac{\pi \cdot R \cdot \delta^3}{6\lambda_m^{\;2}} \left[\bar{E}_x \lambda_m^{\;4} + 2 \cdot \left(\bar{E}_x \mu_{yx} + 2G_{xy} \right) \lambda_m^{\;2} \lambda_m^{\;2} + \bar{E}_y \lambda_n^{\;4} \right] + \right.$$

$$\left. + \frac{2 \cdot \pi \cdot \delta \;\; \lambda_m^{\;2}}{R \cdot \left[\dfrac{\lambda_m^{\;4}}{E_y} + \left(\dfrac{1}{G_{xy}} - \dfrac{2\mu_{xy}}{E_x} \right) \lambda_m^{\;2} \lambda_n^{\;2} + \dfrac{\lambda_n^{\;4}}{E_x} \right]} \right\} \ge N_c. \tag{6.15}$$

Let's consider engineer recommendations for rod design. Let rod is manufactured by pulltrusion. Hence physical and mechanical properties of composite can be described by the following expressions:

$$E_X=E_1;\ E_Y=E_2;\ \mu_{YX}=\mu_{21};\ G_{XY}=G_{12};$$

$$\bar{E}_1=\frac{E_1}{1-\mu_{12}\mu_{21}};\quad \bar{E}_2=\frac{E_2}{1-\mu_{12}\mu_{21}};\ F_x=F_{1c};\ F_{xt}=F_{1t}. \quad (6.16)$$

From the expression (6.14) we can minimal wall thickness

$$\delta_{min}=\sqrt{\frac{N_c}{2\pi}\sqrt{\frac{3\cdot(1-\mu_{xy}\cdot\mu_{yx})}{E_1\cdot E_2}}} \quad (6.17)$$

and from the expressions (6.12) – minimal mean radius of a rod

$$R_{min}=\begin{cases}\dfrac{N_t}{2\pi\cdot\delta_{min}\cdot F_{xt}},\\ \dfrac{N_c}{2\pi\cdot\delta_{min}\cdot F_{xc}}.\end{cases} \quad (6.18)$$

Then designer should check conditions (6.13) and (6.15). If one of these expressions is not satisfied designer should increase R_{min} value until satisfaction of these conditions (6.13) and (6.15).

Let there are design and manufacture limitations on rod radius (R^D or R^M) and rod radius calculated by the (6.13) formula is less then R_{min} we should check conditions (6.13) and (4.15) with value R^D or R^M.

If necessary rod radius is more than R^D_{max} or R^M_{max} designer should increase wall thickness until (6.13) and (6.15) will be satisfied.

Now let's consider a numerical example.

Example 6.1. It's necessary to design rod with hinged tips and length 1000 mm which is manufactured by pulltrusion and made of carbon plastic. Properties of this carbon plastic are concerned to material №1 from the Table 6.1.

Design tension force is 25 kN, compression force is 20 kN.

From the formula (6.17) we obtain

$$\delta_{min}=\sqrt{\frac{20000}{2\cdot 3.14}\sqrt{\frac{3(1-0.35\cdot 0.035)}{10^5\cdot 10^4}}}=0.42\ mm. \quad (6.19)$$

From the equation (6.18) one can obtain R_{min} value

$$R_{min}=\max\begin{Bmatrix}\dfrac{25000}{2\cdot 3.14\cdot 0.42\cdot 900}=10{,}5\ mm\\ \dfrac{20000}{2\cdot 3.14\cdot 0.42\cdot 700}=10{,}8\ mm\end{Bmatrix}=10.8\ \ mm. \quad (6.20)$$

If rod has hinged tips then k=1 and we substitute δ_{min} and R_{min} by their numerical values in the formula (6.13) and obtain

$$\frac{1\cdot 3.14^3\cdot 10.8^3\cdot 0.42\cdot 10^5}{10^3\cdot 10^3\left(1+\dfrac{3.14^2\cdot 10.8^2\cdot 10.5}{10^3\cdot 10^3\cdot 600}\right)}=\frac{1638}{1-0.02}=1671\ \mathrm{N}<20000\ \mathrm{N}. \qquad (6.21)$$

One can see that condition of local stability isn't satisfied so radius should be increased. If we considered that shear influence on critical stresses is negligible then from expression (6.13) one can obtain

$$R=\frac{1}{\pi}\sqrt[3]{\frac{N_c^p\, l^2}{k\delta_{min}E_x}}=\frac{1}{3.14}\sqrt[3]{\frac{20000\cdot 10^6}{1\cdot 0.42\cdot 10^5}}\approx 25\ \mathrm{mm}. \qquad (6.22)$$

To satisfy condition (6.15) designer should use computer to determine the minimal value of this expressions at determined values of m and n.

At chosen initial date the minimal value of expression exceeds at m=48 and n=6:

$$\lambda_m=\frac{3.14\cdot 48}{1000}=0{,}15\ \frac{1}{mm};\qquad \lambda_n=\frac{6}{25}=0.15\ \frac{1}{mm};$$

So,

$$\bar{E}_1=\bar{E}_2=\frac{100000}{1-0.35\cdot 0.035}=101200\ \mathrm{MPa},$$

$$\frac{3.14\cdot 25\cdot 0.42^3\cdot 0.15^4}{6\cdot 0.15^4}\left[101200+2(101200\cdot 0.035+2\cdot 6000)+10120\right]+$$

$$+\frac{2\cdot 3.14\cdot 0.42\cdot 0.15^2}{25\cdot 0.15^4\cdot\left[\dfrac{1}{10000}+\left(\dfrac{1}{6000}-\dfrac{2\cdot 0.35}{100000}\right)+\dfrac{1}{100000}\right]}=20500>20000\ \mathrm{N}. \qquad (6.23)$$

This result means that non-symmetrical form of local instability is possible at loading more than design loading.

Consider case when condition (6.10) is the following

$$10\ \mathrm{mm}\le R\le 15\ \mathrm{mm}.$$

Hence we should put R=15 mm (above calculated value is 25 mm). From the condition (6.13) we will find wall thickness

$$\delta\ge\frac{N_c l^2}{k\pi^3R^3E_x}\left(1+\frac{k\pi^2R^2E_x}{l^2G_{xy}}\right);$$

$$\delta\ge\frac{20000\cdot 10^6}{1\cdot 3.14^3\cdot 15^3\cdot 10^5}\left(1+\frac{1\cdot 3.14^2\cdot 15^2\cdot 10^5}{10^6\cdot 6000}\right). \qquad (6.24)$$

$$\delta\ge 1.91\ \mathrm{mm}.$$

One can check that conditions (6.12) and (6.15) are satisfied. Now we can check the ultimate force:

$$N=2\pi\cdot R\cdot\delta\cdot F_{xc}=2\cdot 3.14\cdot 15\cdot 1.91\cdot 700=125945\ \mathrm{N}.$$

We can see that this force is less than critical force. This example shows that the strictest condition is the condition of general instability. So rational rod parameters are: R=15 mm, δ=1.91 mm.

Table 6.1. Physical and mechanical properties of composites

Composite	1	2	3	4	5	6	7	8	9	10	11
Composite type and structure	Carbon plastic unidi-rectiona l	Carbon plastic unidi-rectiona l	Glass plastic unidi-rectiona l	Organic plastic unidi-rectiona l	Carbon plastic woven	Glass plastic woven	Organic plastic woven	Carbon-organic plastic woven	Carbon-organic plastic woven	Glass-organic plastic woven	Boron-aluminum unidirec-tional
ρ, kg/m^3	1450	1400	2000	1320	1500	1900	1360	1360	1300	1600	3200
E_1, GPa	100	150	45	80	60	24	36	100	50	25	220
E_2, GPa	10	8	10	5,5	60	16	30	50	70	35	70
G_{12}, GPa	6	4	5	2	6	4	3	5	6	5,5	25
μ_{12}	0.35	0.3	0.3	0.31	0.28	0.26	0.22	0.26	0.28	0.25	0.32
F_{1T}, MPa	900	1300	800	1600	400	350	600	1000	800	400	2200
F_{1C}, MPa	700	1200	1000	300	400	280	150	800	500	450	2000
F_{2T}, MPa	50	40	50	16	400	300	550	1200	500	600	400
F_{2C}, MPa	120	100	40	300	400	250	140	200	600	150	400
F_{12}, MPa	75	50	60	30	50	45	40	60	55	50	240
G_{Int}, GPa	7.0	3.6	4.0	2,2	4.0	1,8	2,5	2,2	2,4	2,6	20
τ_{Int}, MPa	60	50	45	50	40	40	50	55	50	60	230
α_1, 10^{-6} 1/K	0	-2.0	8.0	-4.0	1.0	12.0	-3.0	-2.0	-1.0	6.0	5.0
α_2, 10^{-6} 1/K	30	40.0	25.0	60	1.0	16.0	-2,6	-3.0	0	-1.0	10.0
d_0, mm	0.08	0.12	0.15	0.12	0.35	0.25	0.25	0.16	0.28	0.3	0.14
Width ,mm	250	20	50	50	600	900	900	500	1000	1200	100

If considered rod is manufactured by winding with fabric or tape wall thickness of rod should be rounded to larger size up to nearest whole number of layers by multiplying the composite layer thickness to necessary their quantity

δ_0=0.08 mm, h=24 mm, 0.08·24=1.92 mm.

Let's consider method of rod design, wall of which consists of two families of threads or tapes. For instance, longitudinal-lateral winding or the following combinations: winding-pulltrusion, winding-laying up, pulltrusion-laying up.

In this case wall thickness is equal to

$$\delta=\delta_1+\delta_2=\delta_1(1+\Psi), \text{ where } \Psi=\delta_2/\delta_1. \qquad (6.25)$$

Elastic constants of orthotropic material of wall can be calculated by known method:

$$B_{11}=\delta_1\left(A_{11}^{(1)}+\Psi\cdot A_{11}^{(2)}\right), \quad B_{12}=\delta_1\left(A_{12}^{(1)}+\Psi\cdot A_{12}^{(2)}\right),$$
$$B_{22}=\delta_1\left(A_{22}^{(1)}+\Psi\cdot A_{22}^{(2)}\right), \quad B_{33}=\delta_1\left(A_{33}^{(1)}+\Psi\cdot A_{33}^{(2)}\right), \qquad (6.26)$$

where

$$A_{11}^{(i)}=\bar{E}_{1i}\cos^4\varphi_i+\bar{E}_{2i}\sin^4\varphi_i+2\bar{E}_{1i}\mu_{21i}\cos^2\varphi_i\sin^2\varphi_i+G_{12i}\sin^2 2\varphi_i;$$
$$A_{12}^{(i)}=(\bar{E}_{1i}+\bar{E}_{2i})\cos^2\varphi_i\sin^2\varphi_i+\bar{E}_{1i}\mu_{21i}(\sin^4\varphi_i+\cos^4\varphi_i)-G_{12i}\sin^2 2\varphi_i;$$
$$A_{22}^{(i)}=\bar{E}_{1i}\sin^4\varphi_i+\bar{E}_{2i}\cos^4\varphi_i+2\bar{E}_{1i}\mu_{21i}\cos^2\varphi_i\sin^2\varphi_i+G_{12i}\sin^2 2\varphi_i; \qquad (6.27)$$
$$A_{33}^{(i)}=(\bar{E}_{1i}+\bar{E}_{2i}-\bar{E}_{1i}\mu_{21i})\cos^2\varphi_i \,\text{in}^2\varphi_i+G_{12i}\cos^2 2\varphi_i.$$

Angles in formulas (6.26) are equal to 0° and 90°:

$$E_x=\frac{1}{\delta_1(1+\Psi)}\left(B_{11}-\frac{B_{12}^2}{B_{22}}\right), \quad \mu_{xy}=\frac{B_{12}}{B_{22}},$$
$$E_y=\frac{1}{\delta_1(1+\Psi)}\left(B_{22}-\frac{B_{12}^2}{B_{11}}\right), \quad \mu_{yx}=\frac{B_{12}}{B_{11}}, \quad G_{xy}=\frac{B_{33}}{\delta_1(1+\Psi)}. \qquad (6.28)$$

It's necessary to know that elastic constants depend on Ψ coefficient only.

If one assumes φ_1=0°, φ_1=0° the following expressions can be obtained:

$$A_{11}^{(1)}=\bar{E}_{11}, \quad A_{11}^{(2)}=\bar{E}_{22}, \quad A_{12}^{(1)}=\bar{E}_{11}\mu_{211}, \quad A_{12}^{(2)}=\bar{E}_{12}\mu_{212},$$
$$A_{22}^{(1)}=\bar{E}_{21}, \quad A_{22}^{(2)}=\bar{E}_{12}, \quad A_{33}^{(1)}=G_{121}, \quad A_{33}^{(2)}=G_{122}. \qquad (6.29)$$

Elastic constants of this composite according to (6.28) will be equal to:

$$E_x = \frac{1}{1+\Psi}\left[\bar{E}_{11} + \Psi\bar{E}_{22} - \frac{(\bar{E}_{11}\mu_{211} + \Psi\bar{E}_{12}\mu_{212})^2}{\bar{E}_{21} + \Psi\bar{E}_{12}}\right],$$

$$E_y = \frac{1}{1+\Psi}\left[\bar{E}_{21} + \Psi\ \bar{E}_{12} - \frac{(\bar{E}_{11}\mu_{211} + \Psi\bar{E}_{12}\mu_{212})^2}{\bar{E}_{11} + \Psi\bar{E}_{22}}\right], \tag{6.30}$$

$$\mu_{xy} = \frac{\bar{E}_{11}\mu_{211} + \Psi\bar{E}_{12}\mu_{212}}{\bar{E}_{21} + \Psi\bar{E}_{12}},\ \mu_{yx} = \frac{\bar{E}_{11}\mu_{211} + \Psi\bar{E}_{12}\mu_{212}}{\bar{E}_{11} + \Psi\bar{E}_{22}},$$

$$G_{xy} = \frac{1}{1+\Psi}(G_{121} + \Psi G_{122}).$$

Now one can define ultimate strength of this composite. Since rod wall is subjected to uniform stressed state we can use maximum stresses criteria to estimate composite strength (6.31). Designer should know that separate layers are subjected non-uniform stressed state:

$$F_{xt/c} = \min\begin{cases} F_{11t/c}\dfrac{E_x}{\bar{\bar{E}}_{11}(1-\mu_{211}\mu_{xy})}, \\ F_{21t/c}\dfrac{E_x}{\bar{\bar{E}}_{21}(\mu_{121}-\mu_{xy})}, \\ F_{12c/t}\dfrac{E_x}{\bar{\bar{E}}_{12}(\mu_{xy}-\mu_{212})}, \\ F_{22c/t}\dfrac{E_x}{\bar{\bar{E}}_{22}(1-\mu_{122}\mu_{xy})}. \end{cases} \tag{6.31}$$

For class of structural elements to be considered it is advisable to use those pares of composites for which material brakeage happens due to first layer fibers rupture. Then

$$F_{xt} = F_{11t}\frac{E_x}{\bar{\bar{E}}_{11}(1-\mu_{211}\mu_{xy})}, \quad F_{xc} = F_{11c}\frac{E_x}{\bar{\bar{E}}_{11}(1-\mu_{211}\mu_{xy})}. \tag{6.32}$$

Physical and mechanical properties of rod wall will be different even at the same composite layers components if different rod wall layers are made by means of different manufacture process, for instance, internal layer [0°] – by pulltrusion, external layer [90°] – by winding

Determination of rational structural parameters of rod can be made according to the following sequence:

- the row of coefficient Ψ values is defined (these values should consider manufacturer available thickness and statistics data);
- from (6.19) δ_{1min} value is defined for each Ψ;
- from (6.12) minimal radius R_{min} is defined for each pare of Ψ и δ_{1min};
- general stability of rod should be checked according to (6.10) formula and if it is necessary new value of the radius is defined;

- non-symmetrical form of rod wall local stability should be checked. If local stability of rod wall is not provided rod radius should be increased (if (6.10) restriction permits it) or rod wall thickness should be increased (if (6.10) restriction does not permit it);

- the graph of rod mass dependence on Ψ coefficient is drawn (if it is necessary condition (6.10) is taken into consideration).

Described algorithm can be used for design of two layered wall, consisted of two composites with the same reinforcing angles, for instance [0°,0°] and [90°, 90°]. For this axis 1 and 2 of local coordinate system should be oriented properly.

If one analyze (6.31) dependence it can be seen, that the most likely type of orthogonally reinforced composites is brakeage of matrix of ply with angle [90°], i.e. the strength of this composites is defined by the last expression. From the other hand for long rods the instability is the predominated form of carrying ability losing. Due to this fact after determination of structural parameters of the rod it is necessary to make clear the type of composite rupture:

If

$$\frac{F_{22t}}{\bar{\bar{E}}_{22}(1-\mu_{122}\mu_{xy})} < \frac{F_{11t}}{\bar{\bar{E}}_{11}(1-\mu_{211}\mu_{xy})},$$

that the layer [90°] destroys along X axis earlier the layer [0°]. No doubt that if lateral layer supports longitudinal one that such type of rupture is allowable, because of E_y value (in the formulas (6.14) and (6.15)) does not change practically. In this case all calculations should be repeated one more time with suggestion that

$$E_{22} = \mu_{122} = \mu_{212} = G_{122} = 0.$$

For woven semi-articles the strength of orthogonally reinforced composites is defined by the first and last expressions of (6.31) formula.

Winding is widely used for rod production, especially for rod of medium and large diameters. In this case [$\pm\varphi$] structure is realized. Design algorithm for these rods is the same. In this case graph M(φ) is drawn instead of M(Ψ) graph and interval of winding angles φ is defined with taking into consideration of abilities of manufacturer equipment in the form

$$\varphi_1 \le \varphi \le 90^\circ, \qquad (6.33)$$

where φ_1– is minimal available winding angle.

6.4. Structural and manufacture solutions of rods and their connecting tips

Structural and manufacture solutions (SMS) of rods depend on their functional application, manufacture and assembling processes, level and type of loading etc. The form of their cross section defines mass efficiency of the rod because for compressed rods the optimal is that form which has maximal dependence of cross-area to perimeter. That is why hollow round cylindrical rods are optimal (if special restrictions are absent).

Large quantity of rods, used in aircraft structures in control rods, motor frames etc (see Fig. 6.1), should be adjustable length (for compensation of manufacture errors and elimination of assembling stresses) and should have opportunity to disassemble (for maintenance, repair and replacing). On the Fig. 6.6, c typical SMS of control rod connection is shown; this connection consists of tip with screw tail and metal bushing, which is connected with composite rod. The transition zone from composite to metal bushing is the most complicated problem in control rod, struts etc design. So this aspect is considered below.

Connection of tip with rod is defined by manufacture process of tubular elements. Pulltrusion permits to produce tubes of different form and high quality. (Fig. 6.6, a) practically automatically, but in this cross section of this tubes does not change along the length, this fact should be taken into consideration in design process.

By the way pulltruded rods are unidirectional, although there is special tools which permit to co-cure in spinneret one layer of woven tape inside of tube, outside of tube or at the both sides (Fig. 6.6, b).

On the Fig. 6.6, c-f some SMS are shown, these SMS based on tubes turning on universal lathes to create conical surface. It permits to realize reliable adhesive joint of metal bushing and composite tube (it is simple to tight joining surfaces and adhesive thickness is approximately constant), from the other hand maximums of shear stresses in glue at the ends of joint can be decreased. SMS, which is shown at the Fig. 6.6, c, d, e, are usually reinforced by external sub-winding with yarn or thread. For this thermo-shrinking tubes are used (to replace sub-winding).

Some variants of tip joint design are shown at the Fig. 6.6, g, h. The main aim of these SMS is the following: partly-cured (up to keep stable form) pulltruded tubes are specially (see Fig. 6.6, j, k) cut, then bushing is installed and obtained “petals” are tighten by means of sub-winding. Geometry of cut wedge (Fig. 6.6, k) can be calculated form the condition of perimeters equality. In SMS with increasing cone (Fig. 6.6, h) obtained gaps can either be filled with composite or special slots in metal bushing are milled (the perimeter of these slots increases joining area). Typical metal joint with the help of hollow or exploding rivets (shown at the Fig. 6.6) demands additional tube reinforcing by means of sub-winding with definite quantity of fabric layers or woven tapes. This measure is made to compensate removed part of basic unidirectional composite and to decrease influence of local composite destroying under countersunk and locked heads of the rivet. Form the internal part of tube specialists recommend to install washer of special form or use bushing made of ductile metal (alloy). Reinforcing in this type of SMS can be realized both after full cure of tube and after partly-cure. Joint based on tightness of uncured tube (Fig. 6.6, j) and the following gluing of metal inserts can be used in constructions, which do not need length adjusting. Additional cure is usually provided in special furnaces.

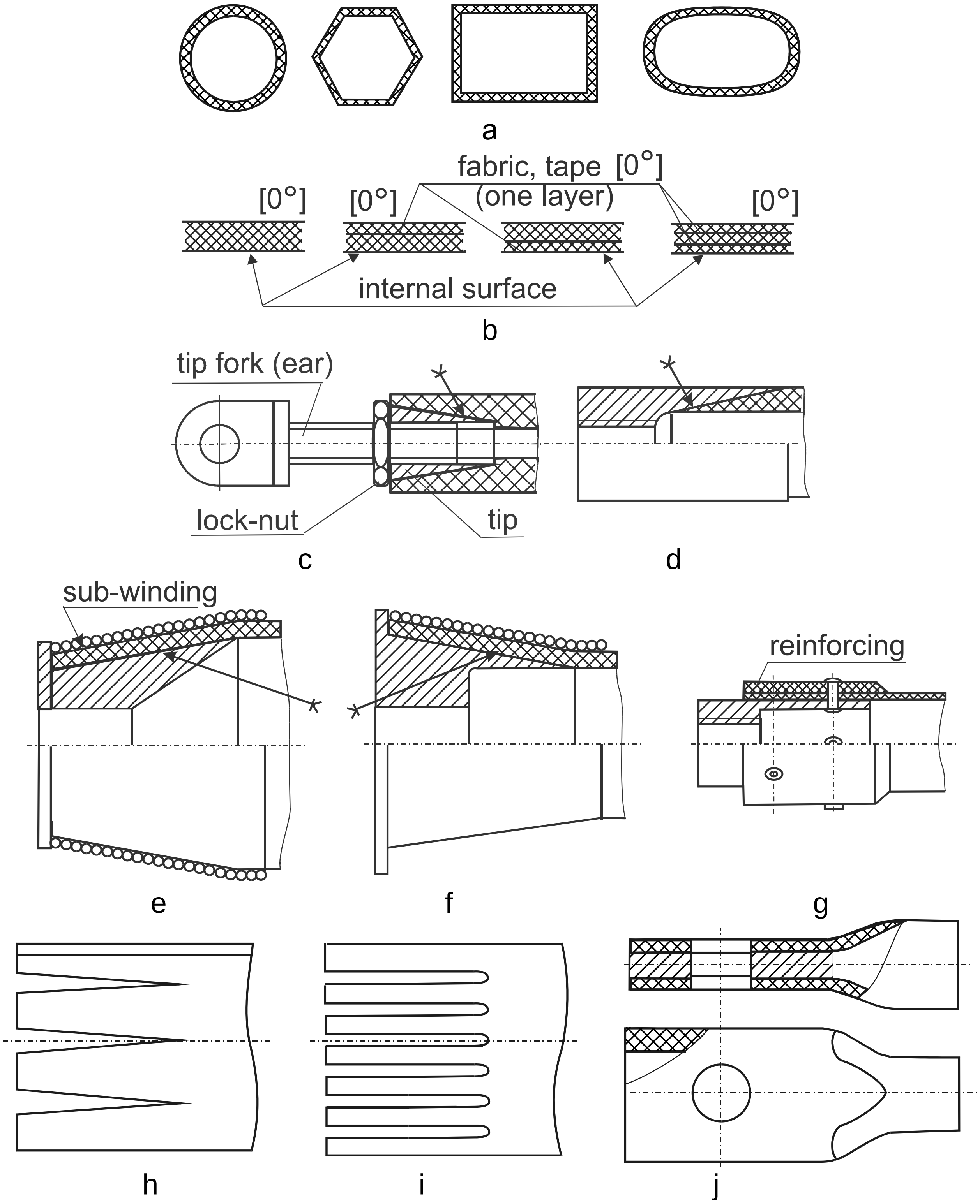


Fig. 6.6. SMS of rods made by means of pulltrusion

Winding can be related to perspective manufacture process of rods because of ability of mechanization a majority of operations. Some rational wall structures are shown at the Fig. 6.7, a.

It is necessary to know that for structure $[\pm\varphi]$ specialists do not recommend to use woven tapes of large width (more than rod diameter) and do not use fabric at all. For longitudinal winding one cannot use threads and yarns. On the Fig. 6.7, b–p some SMS for creating connective tips are shown. The main features of these SMS are the following:

- Fig. 6.7, b - winding is fulfilled on metal bushing directly, this fact should take jig choosing into account;
- Fig. 6.7, h – SMS is analogous SMS shown on the Fig. 6.6, i;
- Fig. 6.7, i, l, p – these joints have lager carrying ability due to wedging on opposite cone surfaces. External tighten bushings made of ductile metal or alloy with “memory”, additional layers (Fig. 6.7, p) or other SMS can be used;
- Fig. 6.7, j – joint tightness can be provide by means of heating or cooling of joint parts;
- Fig. 6.7, k – winding can be carried out in two stages which include embedding bushing between composite layers that method permits to increase carrying ability of joint due to doubled gluing surface;
- Fig. 6.7, m – stepped adhesive joint can be realized in several stages of winding. Bushing can be inserted (with the help of glue) after curing, in this case one can use surfaces with small coneness;
- Fig. 6.7, n – tube winding can be carried out on conical or profiled mandrel, after that external bushing can be installed;
- Fig. 6.7, o – this SMS is characterized by special screw joint, which is fulfilled after rod assembling. Such type of joint is usually used for thick walls or after additional reinforcing (Fig. 6.7, h) and gluing is often used.

In conclusion it is necessary to know that quite effective constructions can be made by means of winding and pulltrusion combination.

For instance, internal layer of the rod, shown on the Fig. 6.7, k, is made by pulltrusion but internal – by winding. In addition these combinations solve the problem of longitudinal reinforcing for winding and spiral or tangent for pulltrusion.

Many rod systems do not need adjustable length and ability to disassemble, for instance, truss wing spar, fuselage etc. There are some difficulties in using pulltruded or wound rods for structures of this type. This problem can be explained by necessity to use special very complicated joining elements. To solve this problem the following SMS can be effective (see Fig. 6.8). These SMS are based on manufacture process of lay-up and autoclave cure.

Rods can have different form of cross section (opened section – Fig. 6.8, b, d and closed section – Fig. 6.8, c) and joining areas of different geometry, that permits to assemble and glue truss structures of necessary form (Fig. 6.8, a) at continuous (non-cut) caps (belts).

Some SMS of wing struts are shown on the Fig. 6.8,e, f, g. These struts are high loaded and responsible rods and usually have mainframe contour (Fig. 6.8, e, f) or mainframe tube with fairing (Fig. 6.8, g).

When anyone researches the peculiarities of stressed-strained state of rod wall in the zone of joining with tip or sub-wound reinforcing it is necessary to take edge effect appearing into consideration. This effect means the appearing of bending and shear forces in glue (adhesive) layer of the tip. It happens as the result of different deformability in radial direction of rod wall up to tip and in the zone of tip.

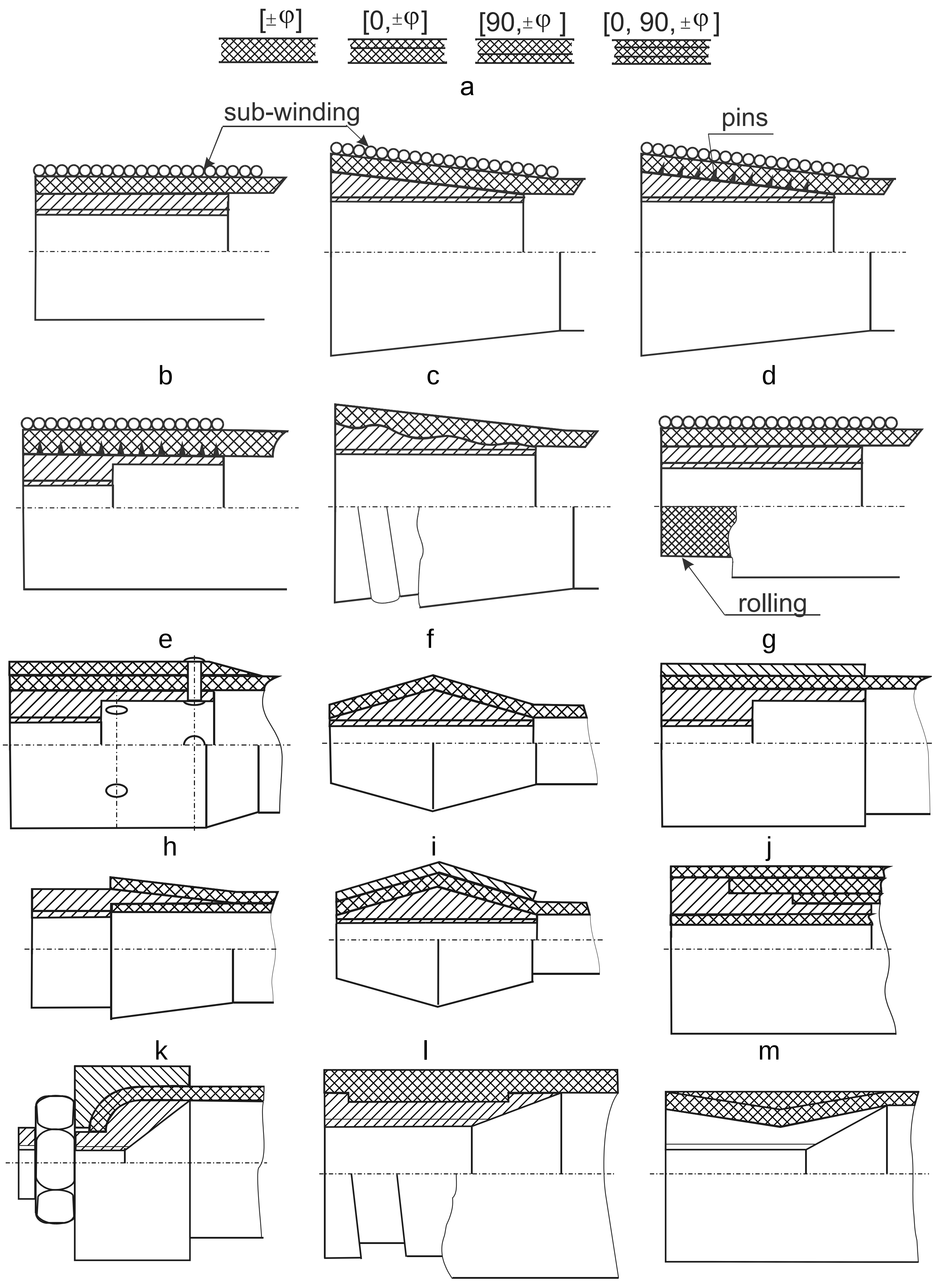


Fig. 6.7. SMS of rods produced by winding and laying-up

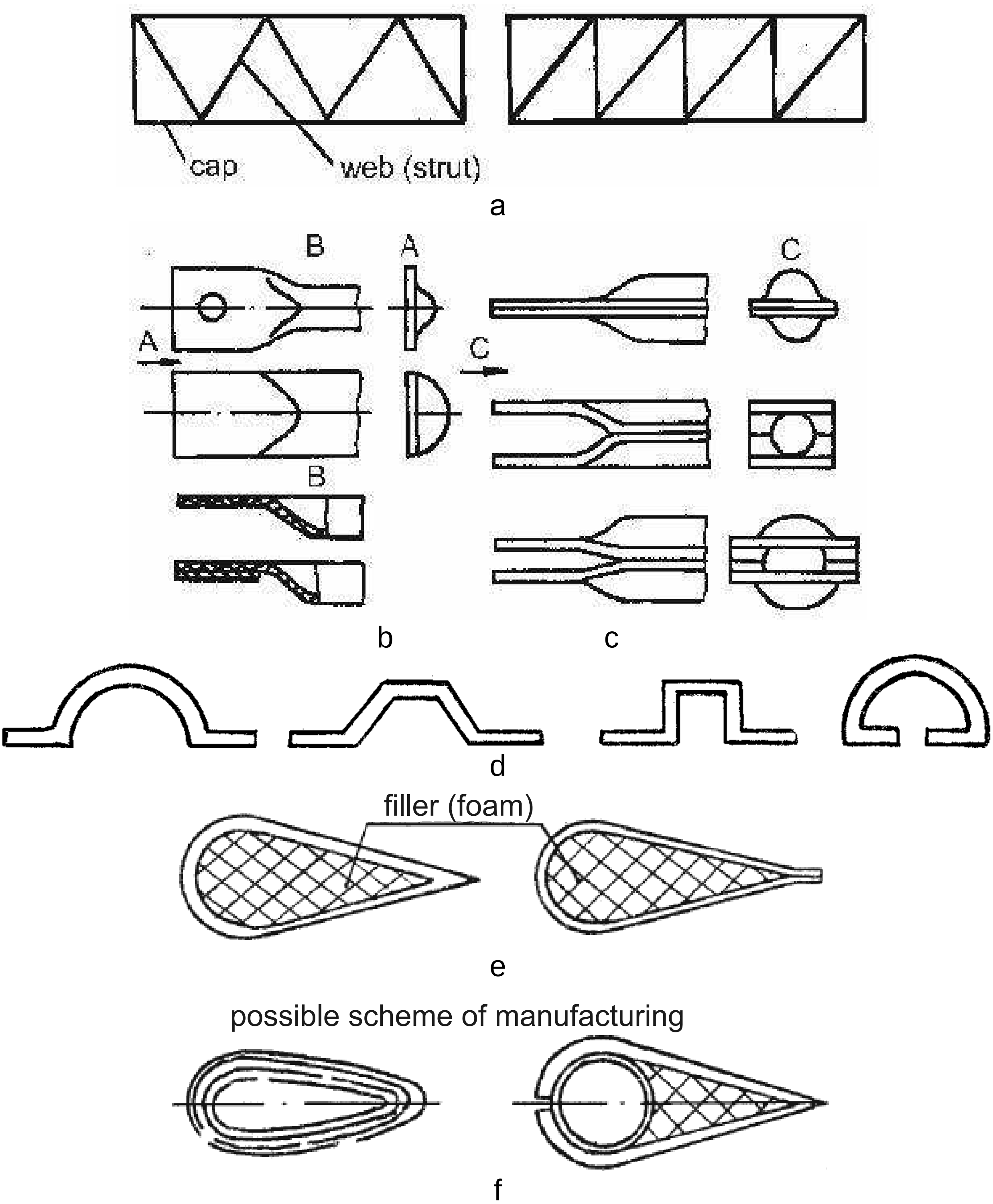


Fig. 6.8. Rods SMS for truss structures

Shear force makes essential influence on adhesive layer loading. The most effective SMS, which provide decreasing of shear forces in adhesive layer, is tangent sub-winding and application of additional cases (see, for instance, Fig. 6.6, e, f, Fig. 6.7, c–e, j, l). Additional sub-winding in the zone of tip can also decrease edge effect.

To decrease stresses concentration in adhesive layer it is necessary to realize smooth change of stiffness of joining articles. The same SMS can essentially “smooth” edge effect. The main role in this question plays special profiled winding (Fig. 6.6 and Fig. 6.7, h) and tapered lap joint of tip (see. Fig. 6.6, c, d, e, h, и Fig. 6.7, f, k, o), due to which joining problems and edge effect “smoothing” can be solved.

Checking-up questions

1. Give examples of rod-contaning structures application in airospace engineering.
2. How does application of composites in rod-contaning structures permit to reach the maximum efficiency of properties realization?
3. Main assumption used at design of circular composite rods.
4. Give definition and illustrate by sketch the phenomena of rods global and local stability.
5. What typical failure modes an engineer has to take into consideration at rods design and stress analysis?
6. Analysing analytical dependencies for rods global and local buckling suggest measures for their prevention.
7. Give examples of manufacturing and structural restrictions used in rods design process.
8. What technological processes are used for composite rods manufacturing?
9. What typical stacking sequences of composite are used in composite rods? What is the function of each sub-group of angles?
10. How to realize practically joining or regular zone of composite rod with another articles and units? Draw sketches of structural solutions of connecting tips.
11. What materials are used for manufacturing of connecting tips?

Theme 7. DESIGN OF BEAMS AND WING SPARS MADE OF COMPOSITES. STRUCTURAL AND MANUFACTURING SOLUTIONS OF BEAMS MADE OF COMPOSITES. SUPPORTS AND FITTINGS USED FOR BEAMS JOINTS

Beams are the most widespread elements of different industrial structures.

Therefore manufacturing high load-carrying structures from composites allows to reduce considerably weight of these structures, to decrease power- and fuel consumption, to increase durability etc. Elements of beams (caps and web) are subjected to simple loading types - tension, compression, shear. This fact defines reinforcing scheme of beam elements and ensures the most efficient realization of composites advantages. At the same time composites reveal series of distinctive phenomena of its own behavior. These phenomena are caused by physical-mechanical properties and operation conditions which have to be taken into consideration during beams design procedure. These distinctive features of beams operation are:

– edge effects appearing due to Poisson's ratio and linear expansion coefficients difference;

– special requirements to composite reinforcing scheme at zones of loading field non-regularities (non-uniformities);

– existing problems of beams elements joints and fittings;

– loosing load-carrying ability due to beam elements instability at enough strength level;

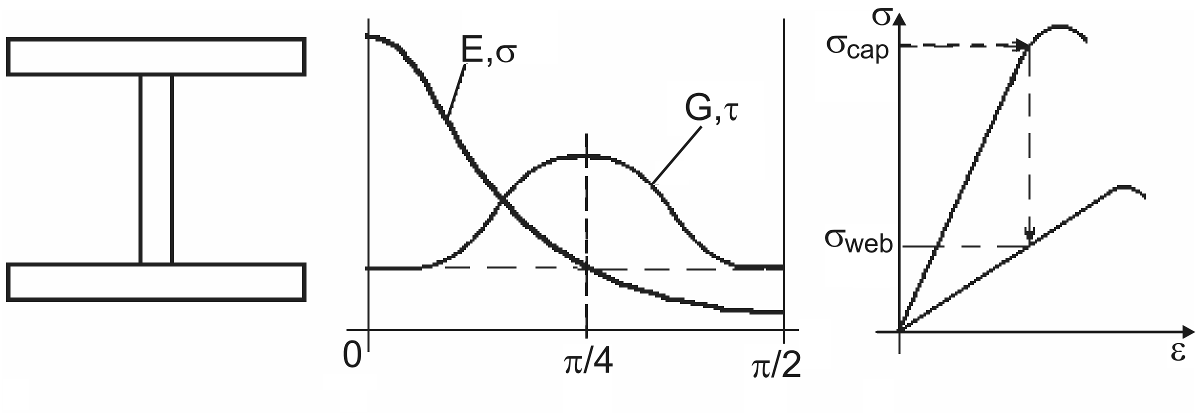
In conventional metal structures mentioned problems were negligible or could be easy eliminated. Decreasing mass of aircraft structure is the most important one. This aim can be realized by manufacturing from composites such high loaded aircraft members as wing spars, empennage, ribs, bulkheads trusses etc.

Thus it is necessary to solve a number of additional problems which concern to of dimensional accuracy, loading distribution between other structurally attaches elements (fairings, skin, membranes, ribs etc.), severe requirements to attachment fittings design. Thus one has to solve the following accompanying questions as required dimensions precision, forces interactions between neighboring elements (skin, rib etc), strong requirements to fittings. Technological questions are very important because of large variety of composites manufacturing techniques, wide range of reinforcing materials types and definite difficulties of non-destructive methods of composites quality control influence significantly on final composite properties.

7.1. Fundamentals of composite beams operation

Beams are the most wide spread such load-carrying elements of aircrafts as wing spars, spars of control surfaces (ailerons, rudders, flaps), floor skeleton etc [6, 7].

Generally beams are loaded with longitudinal and lateral forces. That is why bending moment, lateral force and tension (compression) force appear at any beam cross section (torsion is absent). Typically one can use differential principle considering beam cross section – two caps (upper and lower) and web joining them. Therefore we can suggest that caps withstand tension (compression) forces only (Fig. 7.1, a) and web transfers shear force only. Thus it is recommended to reinforce caps along beam length (with angle 0°) and web with angle ±45°. In this case the highest efficiency of composites application can be achieved (Fig. 7.1, b). Let's analyze realization of above-mentioned differential principle. Normal stress at points of cap-web contact is proportional to cap-web elasticity moduli ratio (Fig. 7.1, c).



a b c

Fig. 7.1. Beam elements properties

Elasticity modulus of composite structure (made of woven fabrics and UD-tapes) with reinforcing [0°] 5...20 times more comparing with packages of the same material but with reinforcing [±45°]. Therefore differential design principle is valid for design stage:

$$\sigma_{web} = \sigma_{cap} \frac{E_{web}}{E_{cap}}, \tag{7.1}$$

where σ_{web}, σ_{cap}, E_{web}, E_{cap} – stress and elasticity moduli of composite web and cap.

Workability of a beam is defined by strength of caps and web joining (Fig. 7.2, a). Considering recommended reinforcing schemes one may suggest adhesive (Fig. 7.2, b) or mechanical joint (Fig. 7.2, c). But structural solution shown at Fig. 7.2, a-c can't be realized due to low adhesive strength, low adhesive area and low composite inerlaminar and bearing strength, thus shear force flow in web hasn't be less adhesive strength, i.e.

$$q_{web} \le \delta_{web} \cdot \tau_{adh}, \tag{7.2}$$

where δ_{web} – web thickness, τ_{adh} – adhesive shear strength.

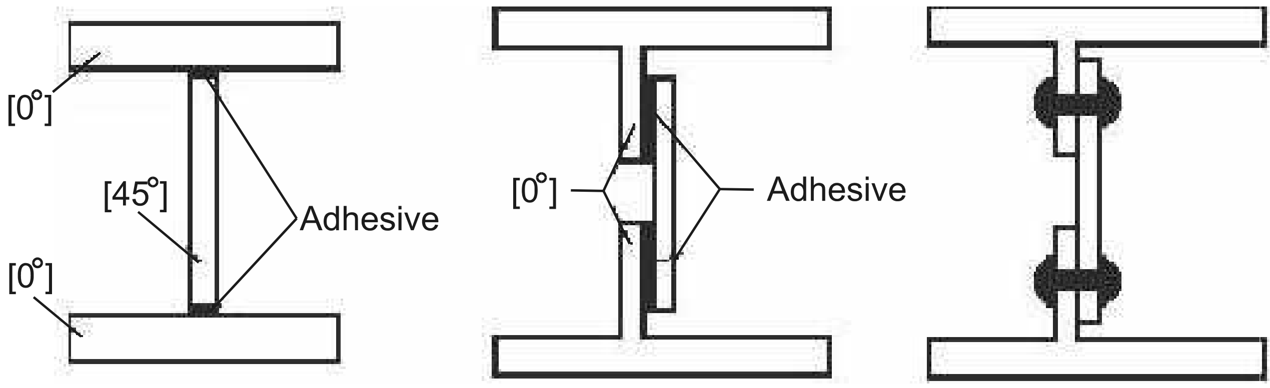


Fig. 7.2. Model for beam analysis

Thus to realize with highest efficiency composite advantages it is necessary to increase area of contact between elements with reinforcing [±45°] (i.e. web) and caps area. Practically such solution can be created by means of forming so-called "shoulders" (see Fig. 7.3). These shoulders ensure uniform shear force distribution between cap and web. Shoulders have the same reinforcing scheme as web has, therefore one has to extent web dimensions and "envelope" caps. Application of shoulders is an example of compromise solution for achieving efficient composite application: from one hand we increase mass of the beam, from another hand we ensure full-scale composite advantages application.

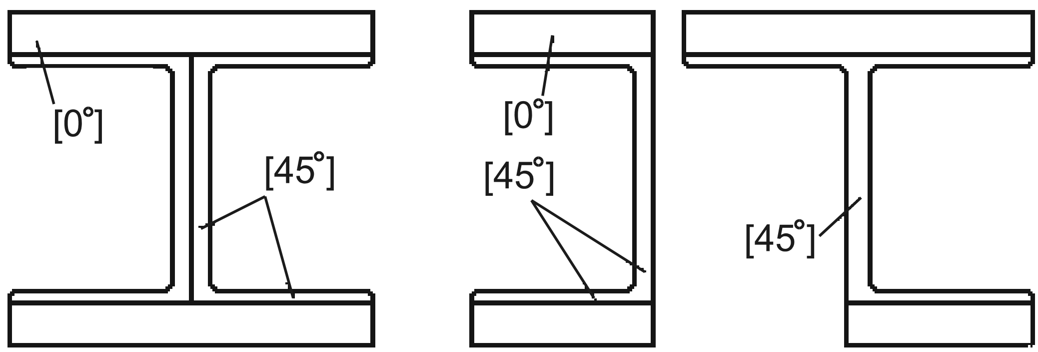


Fig. 7.3. Beam cross-section structural solutions using transition shoulders

7.2. Beam cross section design approach

For analysis of above-mentioned distinctive features of beams operation and suggested reinforcing schemes of web and caps the following generalized analytical scheme of beam design can be suggested (Fig. 7.4).

Design procedure is based on the following assumptions:

– normal stress distribution through caps thickness is uniform due to negligible cap thickness comparing with total beam height;

– web and shoulders transfer shear stress only;

– external loads (bending moment M_z, lateral force Q, and longitudinal force N) and coordinate y_N of longitudinal force N application through cross section are known at any arbitrary cross section.

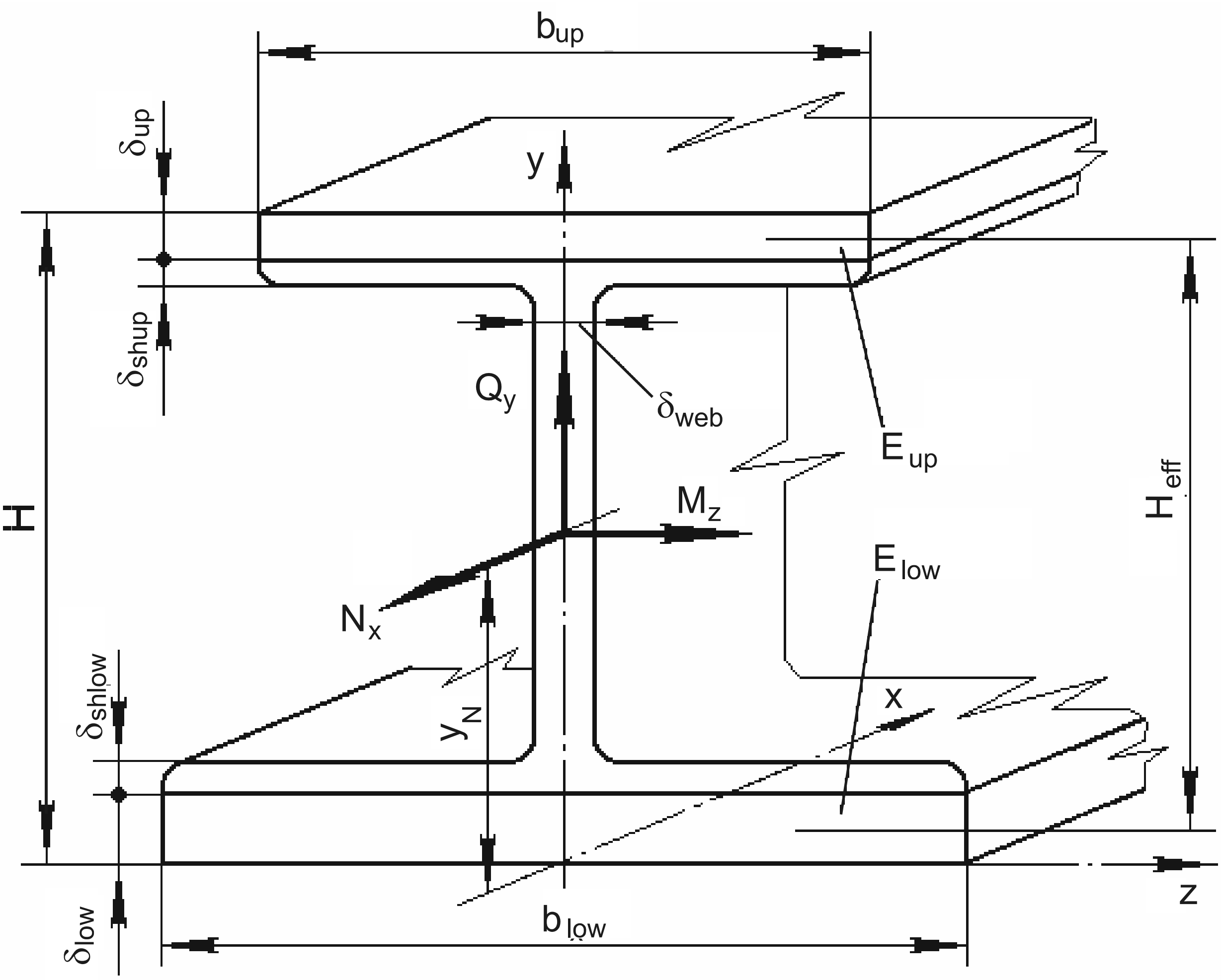


Fig. 7.4. Analytical scheme of beam cross section parameters design

Generally different materials can be selected for upper and lower caps because of different composites strength at tension and compression.

The following dependencies can be written after assumed assumptions analysis:

$$H_{eff} = H - \tfrac{1}{2}\left(\delta_{up} + \delta_{low}\right);$$

$$\delta_{sh} = \tfrac{1}{2}\delta_{web} \text{ – for I-section;} \qquad (7.3)$$

$$\delta_{sh} = \delta_{web} \text{ – for channel section.}$$

Longitudinal force N causes beam bending if applied out of rigidity center. Coordinate y_{rc} of rigidity center is defined from the condition of the same deformation of upper and lower caps from the action of longitudinal force N:

$$y_{rc} = \frac{H_{eff}\delta_{up}b_{up}E_{up}}{\delta_{up}b_{up}E_{up} + \delta_{low}b_{low}E_{low}} + \frac{\delta_{low}}{2}, \qquad (7.4)$$

where E_{up}, E_{low} – elasticity moduli of materials of upper and lower caps correspondingly.

Then total bending moment is equal to

$$M = M_z + \Delta M = M_z - N_x(y_N - y_{rc}). \quad (7.5)$$

Condition of beam optimality is beam minimum mass per unit length

$$\overline{G} = \rho_{up}\delta_{up}b_{up} + \rho_{low}\delta_{low}b_{low} + \rho_{web}\Big[\delta_{web}\left(H - \delta_{up} - \delta_{low}\right) + \delta_{shup}\left(b_{up} - \delta_{web}\right) + \delta_{shlow}\left(b_{low} - \delta_{web}\right)\Big] \to \min. \quad (7.6)$$

Restrictions for beam geometrical parameters are both strength conditions of caps, web and their joint strength:

$$\frac{N_x E_{up}}{\delta_{up}b_{up}E_{up} + \delta_{low}b_{low}E_{low}} + \frac{M}{H_{eff}\delta_{up}b_{up}} \le F_{up}; \quad (7.7)$$

$$\frac{N_x E_{low}}{\delta_{up}b_{up}E_{up} + \delta_{low}b_{low}E_{low}} - \frac{M}{H_{eff}\delta_{low}b_{low}} \le F_{low}; \quad (7.8)$$

$$\frac{Q_y}{H_{eff}\delta_{web}} \le F_{web}; \quad (7.9)$$

$$\frac{Q_y}{H_{eff}b_{up}} \le F_{jup}; \qquad \frac{Q_y}{H_{eff}b_{lw}} \le F_{jlow}, \quad (7.10)$$

where $F_{up}, F_{low}, F_{web}, F_{jup}, F_{jlow}$ – margin of strength of upper cap package, lower cap package, web package, and strength of upper and lower joint between caps and web.

Moreover besides tension, compression or shear beam load-carrying ability can be lost due to loosing local stability (local buckling) of web or compressed cap. At the first design step these phenomena can be excluded from analysis by the following reasons:

– required load-carrying ability of web can be reached by means of application sandwich structure with light filler;

– critical loading of local cap buckling depends significantly on its interaction with other structural elements (for example, wing skin, ribs etc) or one can escape of buckling by means of application special structural and manufacturing solutions preventing buckling.

That is why the following beam design algorithm satisfying restrictions (7.7) – (7.10) and ensuring minimum of objective function (7.6) can be recommended.

a) To define H_{eff} as the first approximation (H is known value)

$$H_{eff} = (0.8...0.95)H. \quad (7.11)$$

b) Minimal caps width is defined according to condition (7.10):

$$b_{up\,min} = \frac{Q_y}{H_{eff}F_{jup}}; \qquad b_{low\,min} = \frac{Q_y}{H_{eff}F_{jlow}}; \quad (7.12)$$

c) Minimal web thickness is defined from (7.9) equation. Obtained value has to be rounded to integer and even monolayers quantity:

$$\delta_{web} = 2\delta_0 \left[int\left(\frac{Q_y}{2H_{eff}F_{web}} \right) + 1 \right], \qquad (7.14)$$

symbol “int” means even part of the number.

d) Caps thickness can be found from conditions (7.7), (7.8) by means of two non-linear equations solution. Obtained value has to be rounded to even monolayers quantity.

e) Estimate new beam effective height H_{eff} (form (7.3)) and continue iteration process from sections a)-e) up to necessary convergence degree.

Obtained bam structure is close to full-strength but not optimal one. One should remember that caps mass can be reduced due to their width increasing. This phenomenon is explained by increasing effective height H_{eff} (Fig. 7.5).

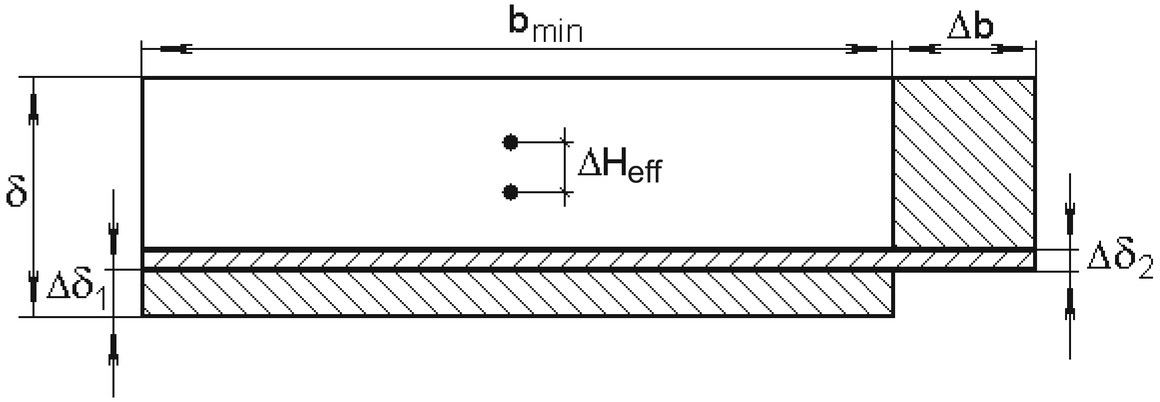


Fig. 7.5. To beam cap cross section optimization

If the following inequality is fulfilled

$$\frac{1}{2} \cdot \frac{\rho_{cap}}{\rho_{web}} \cdot \frac{M^2}{Q_y^3} \cdot \frac{F_j^2 F_{web}}{F_{comp}^2} > 1. \qquad (7.15)$$

If condition (7.15) is valid series of width $b > b_{min}$ has to be used. And optimization procedure $\overline{G} \to min$ has to be achieved.

Generally variant with single cap is possible (if longitudinal force compensates par of bending moment). In this case T-section can be used.

Next step of design procedure should take into consideration the following beam joining with wing elements. Therefore special spacing is needed for installation of wing articles. Separate design questions are devoted to fasteners installation requirements (articles thickness, fasteners installation spacing etc). As the result beam caps thickness can be quite higher comparing with minimal one.

Further considerations will be devoted to questions of beam elements instability.

7.3. Structural and manufacturing solutions of composite beams

Beams cross sections can be divided into the following types (Fig. 7.6): with open section; with close section (box beams); combined (open-close section).

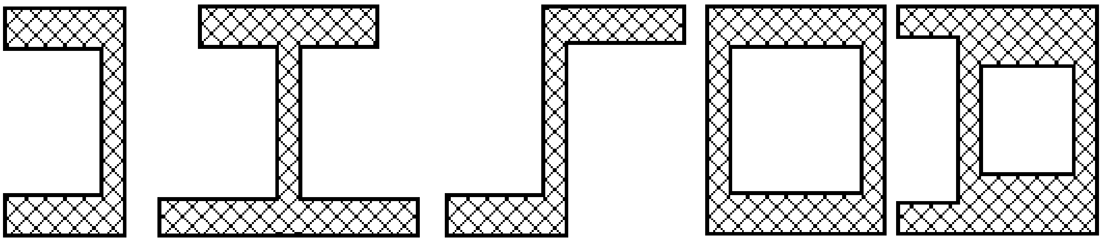


Fig. 7.6. Beams structural and manufacturing solutions

Beams cross sections can be symmetrical (refer to central axis) or non-symmetrical ones (Fig. 7.7). Bevels of cap and web intersection can be symmetrical (refer to central axis) (Fig. 7.8). All mentioned sections can be constant along beam span or variable (Fig. 7.9).

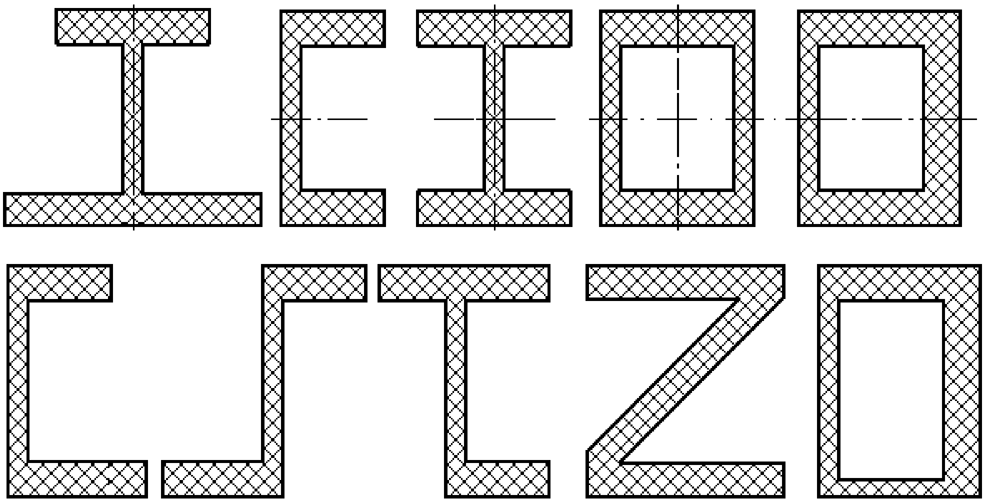


Fig. 7.7. Beam closed and open sections

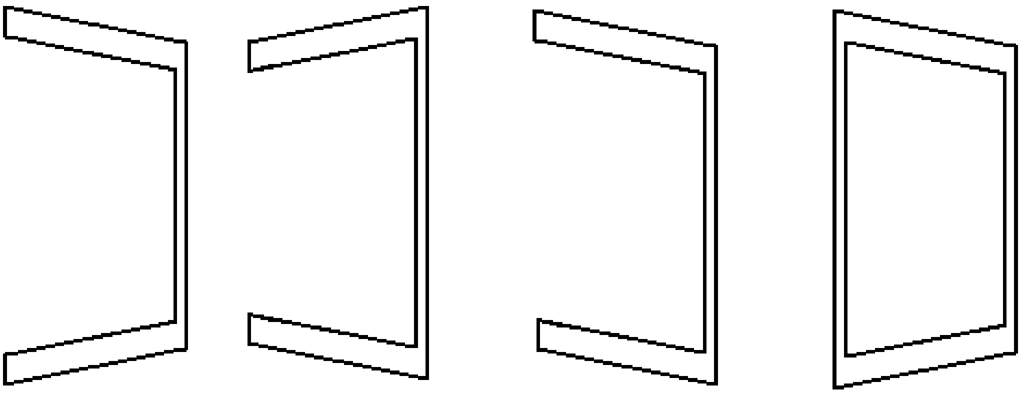


Fig. 7.8. Beam bevels orientation closed and open sections

In spite of used differential principle of loads sharing between caps and web practically all beam elements work together (because bending moment is result of lateral force action). Caps and web possess significantly different composite structure and properties. That is why real structural and manufacturing solutions of beams depend on selected manufacturing process, abilities of used equipment, semi-finished articles properties, allowable dimensions etc.

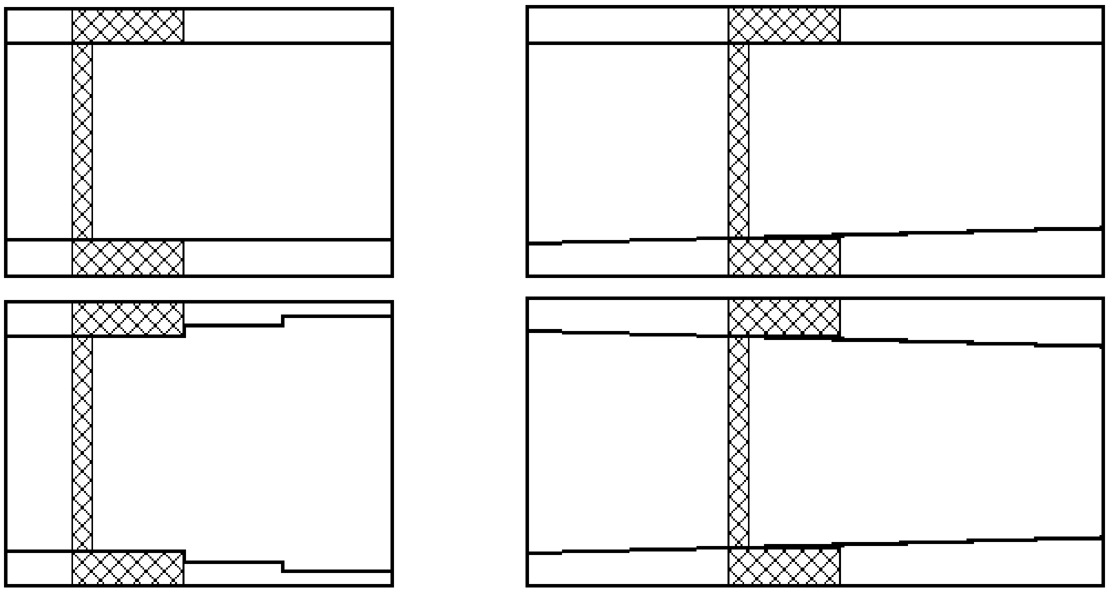


Fig. 7.9. Beam cap laying-up schemes and manufacturing solutions

Generally the following manufacturing process are used for producing beams: vacuum forming, vacuum-autoclave forming, winding, pultrusion and their combinations.

Manufacturing methods suggest the following beam structures – **integral structure** (caps, web and all presented structural elements are joined together at single manufacturing operation simultaneously); **built-up structure** (assembly) – all structural elements are produced separately with consequent assembling; **combination** of two previous methods (for example, caps can be previously polymerized, web is not fully polymerized, then caps and web are joined together through single operation simultaneously).

Recommendations for beam manufacturing process selection are shown in the Table 7.1.

Table 7.1. Recommendations for beams manufacturing process selection

Manufacturing process	Beam cross section structural solution
Vacuum forming, vacuum-autoclave forming	Caps with constant width and thickness; any composite reinforcing scheme can be realized. Webs of any configuration and structure. Beams of integral structure
Winding	Box sections; open shape web, open shape web; Composite structures [±φ], [90, ±φ]; integral structures with caps obtained by pultruzion
Pultruzion	Caps made of unidirectional composite with constant cross section; Stepped variable caps

Typical structural solutions of beams are shown at Fig. 7.10, 7.11. Assembling of beams from previously manufactured elements is conducted by means of adhesive joints, mechanical fasteners or their combination (Fig. 7.12).

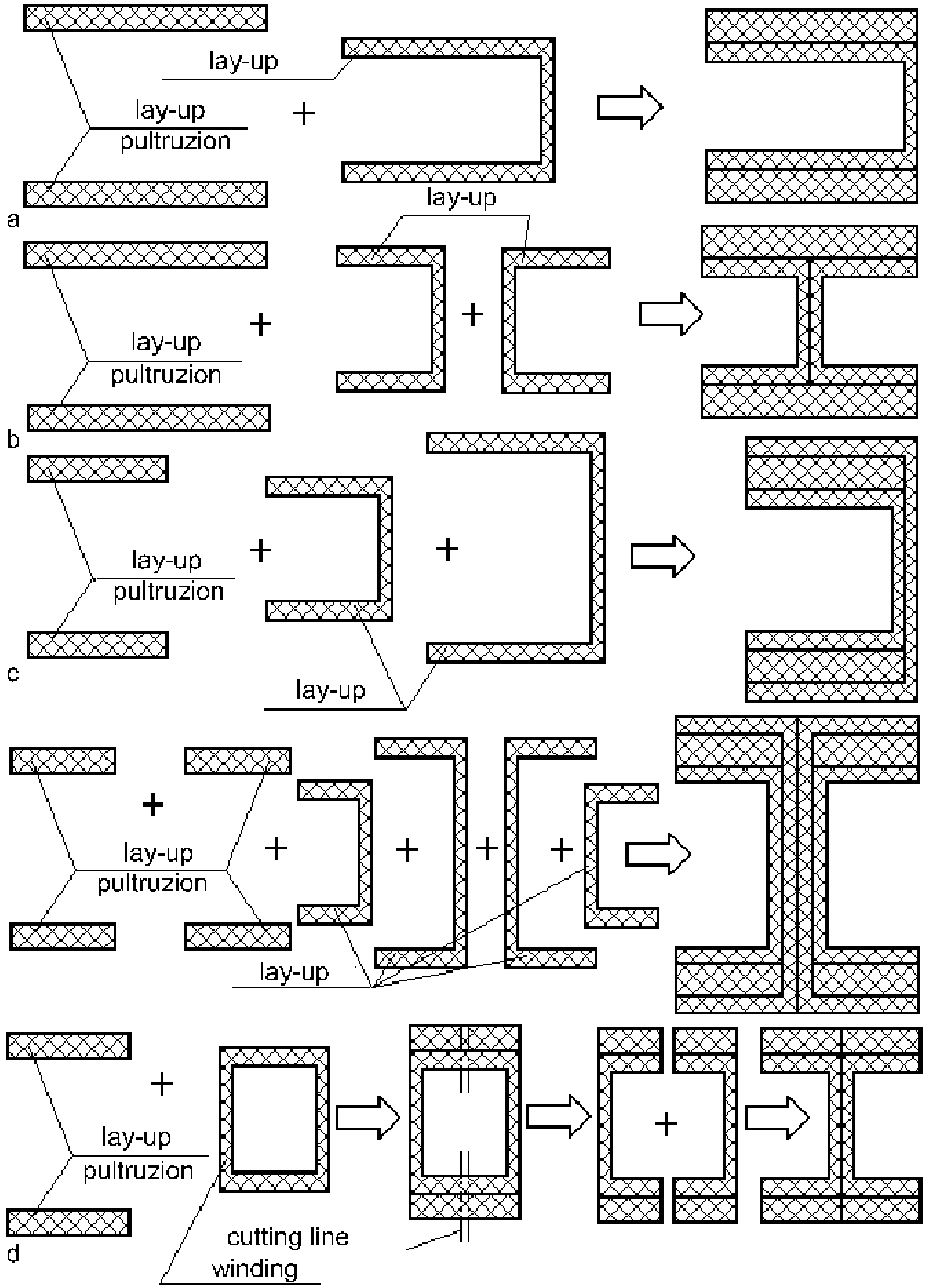


Fig. 7.10. Recommended shapes of beam cross section

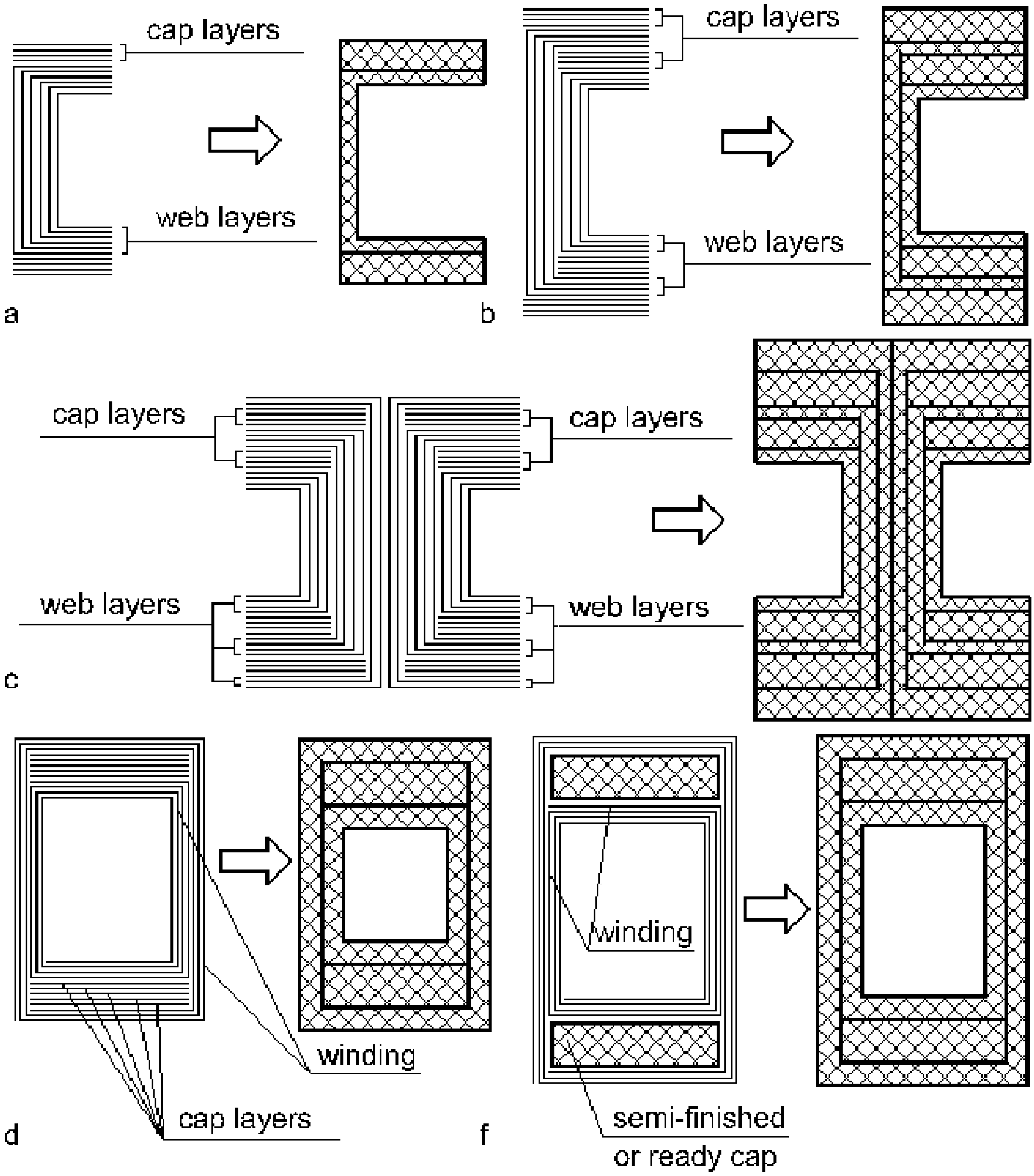


Fig. 7.11. Beam section structural solutions

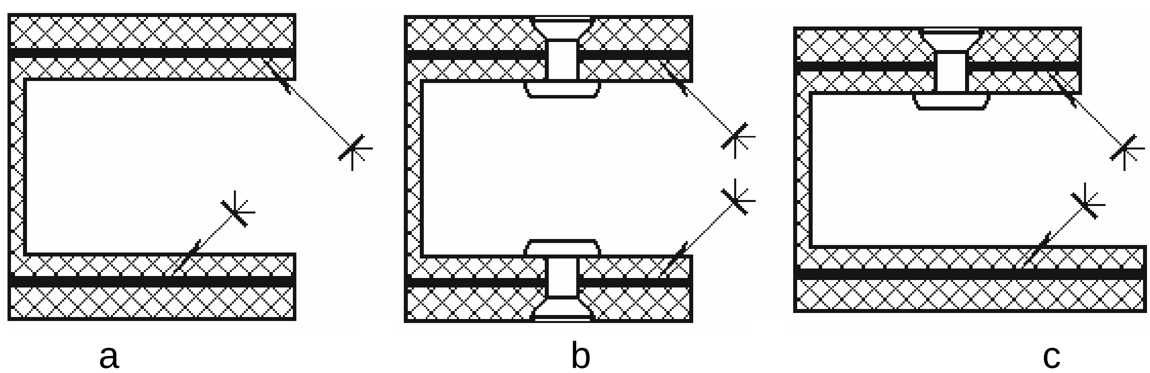


Fig. 7.12. Beams elements joining methods

Selection of beam structural and manufacturing solution has to be based on the following recommendations:

– assembling beam from previously manufactured caps and web requires auxiliary jigs and tools, ensuring their reciprocal pressure and high-precision adjusting of contacting surfaces;

– application of mechanical fasteners isn't desirable solution due reducing "net" section of load-carrying elements and increasing stress concentration factor (which is very important for unidirectional materials);

– structural solution with separated web shoulders is used for reducing intarlaminar stress at boundary cap-web (see Fig. 7.10, c, d; Fig. 7.11, b, c, d, e). Quantity of shoulders sections depends on single section shear rigidity ($G\delta$) (i.e. section thickness and quantity of joining surfaces);

– caps obtained by pultruzion are more desirable due to high degree of composite properties realizing;

– web obtained by winding are overloaded in section corners therefore composite shear strength is lower at those zones;

– box-like beam sections are recommended to be used in the cases of local torque presence or in the cases of general loosing stability.

7.4. Beams loosing stability analysis

It is obvious that structural elements under compression, shear or their combination can loose load-carrying ability because of different modes of loosing stability (global or local buckling). Beams generally can loose stability in the following modes (Fig. 7.13): local buckling of cap under compression (Fig. 7.13, a); web loosing stability from shear (Fig. 7.13, b); general beam loosing stability (beam overturning) (Fig. 7.13, c).

Critical stress σ_{cr} of cap local loosing stability is defined by model of plate under compression (depending on exact beam structural and manufacturing solution) by means of formula

$$\sigma_{cr} = \frac{K\pi^2\sqrt{E_x E_z}}{12\left(1-\mu_{xz}\mu_{zx}\right)} \cdot \left(\frac{b}{\delta}\right)^2, \qquad (7.16)$$

where E_x, E_z – cap elasticity moduli along axes ***x*** and ***z*** (see Fig. 7.4);

b – width of structural element which looses stability (Fig. 7.14);

K – supporting coefficient characterizing support conditions and plate rigidity properties. To define K the following dependencies are used:

– for variant shown at Fig. 7.14, a, b

$$K = 0.2 + 0.3\frac{E_x\mu_{zx}}{\sqrt{E_x E_z}} + \frac{G_{xz}\left(1-\mu_{xz}\mu_{zx}\right)}{\sqrt{E_x E_z}}; \qquad (7.17)$$

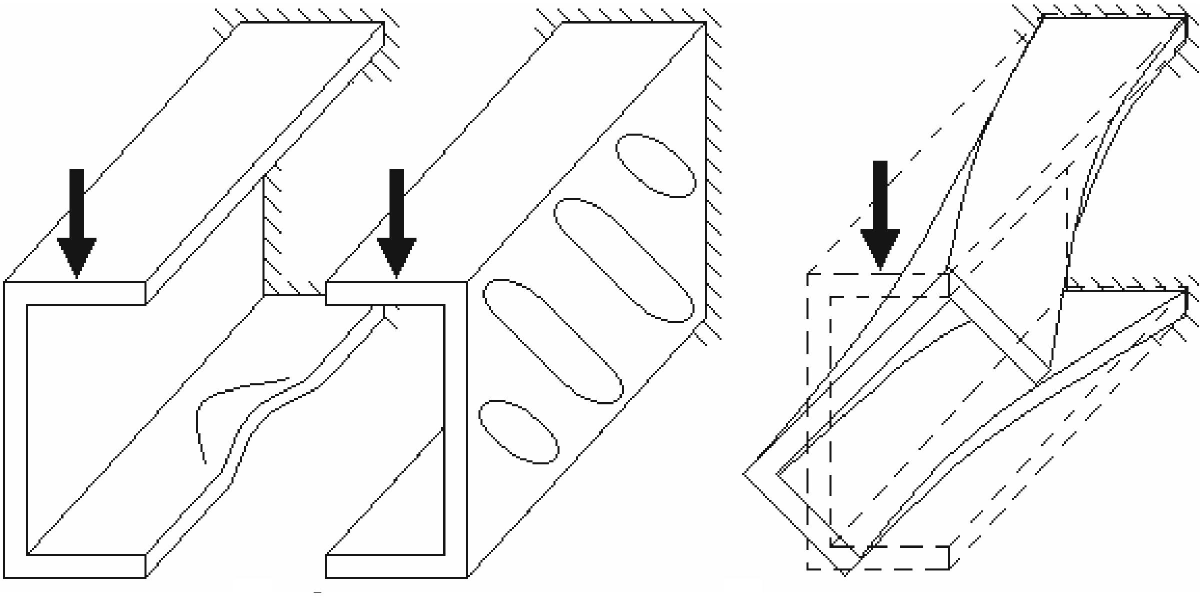


Fig. 7.13. Modes of beam loosing stability

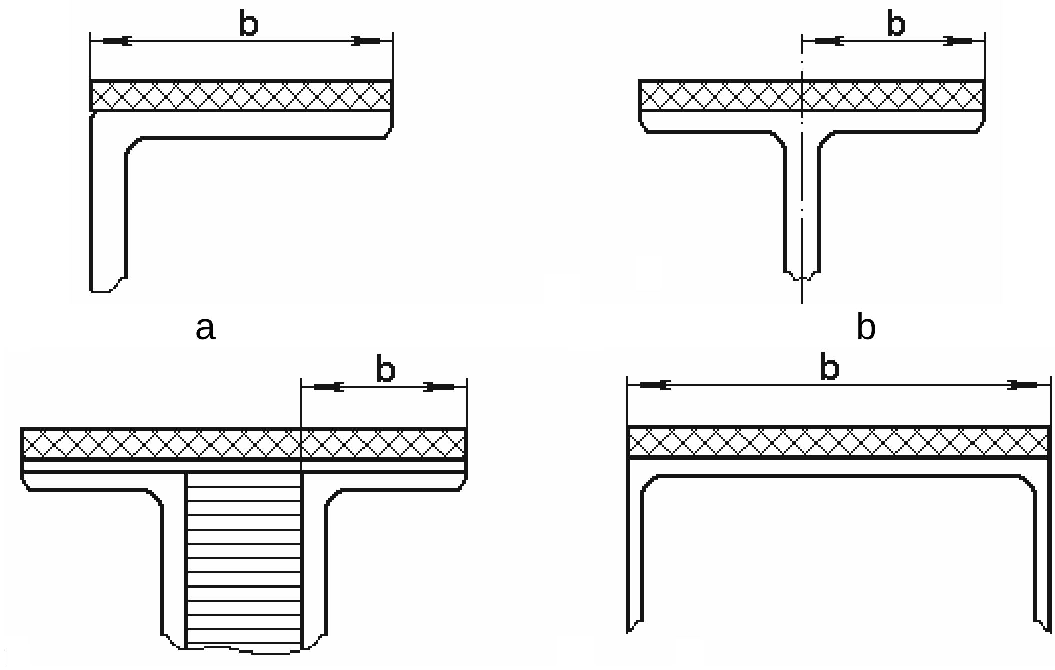


Fig. 7.14. Geometry of loosing stability cap

– for variant shown at Fig. 7.14, c

$$K = 1.15 + \frac{2}{3} \cdot \frac{E_x \mu_{zx}}{\sqrt{E_x E_z}} + \frac{G_{xz}\left(1 - \mu_{xz}\mu_{zx}\right)}{\sqrt{E_x E_z}}; \qquad (7.18)$$

– for variant shown at Fig. 7.14, d

$$K = 2\left(1 + \frac{E_x \mu_{zx} + 2G_{xz}\left(1 - \mu_{xz}\mu_{zx}\right)}{\sqrt{E_x E_z}}\right), \qquad (7.19)$$

where G_{xz} and μ_{xz} – cap shear modulus and Poisson's ratio.

Dependencies (7.17)–(7.19) are valid for composite plates with symmetrical reinforcing scheme under pure compression. For preliminary design stage it is possible to neglect shoulders influence and shear loading influence. Therefore the following non-equalities have to be fulfilled for (for absence of caps loosing stability):

$$\sigma_{cr} \geq F_{cap}; \quad \frac{K\pi^2\sqrt{E_x E_z}}{12(1-\mu_{xz}\mu_{zx})} \cdot \left(\frac{b}{\delta}\right)^2 \geq F_{cap}, \tag{7.20}$$

where F_{cap} – cap compression strength.

Thus we obtain one more condition for caps parameters determination (b, δ).

Web loosing stability (see Fig. 7.13, b) occurs as oblique waves and diagonal tensioned field.

Fig. 7.1, b shows that maximum web **strength** at minimum mass cab be achieved at reinforcing scheme ±45°, but critical shear strength $[\tau]$ always lower than F_{45} value. Thus to escape this mode of loosing stability (at minimum mass) sandwich structures can be recommended (Fig. 7.15).

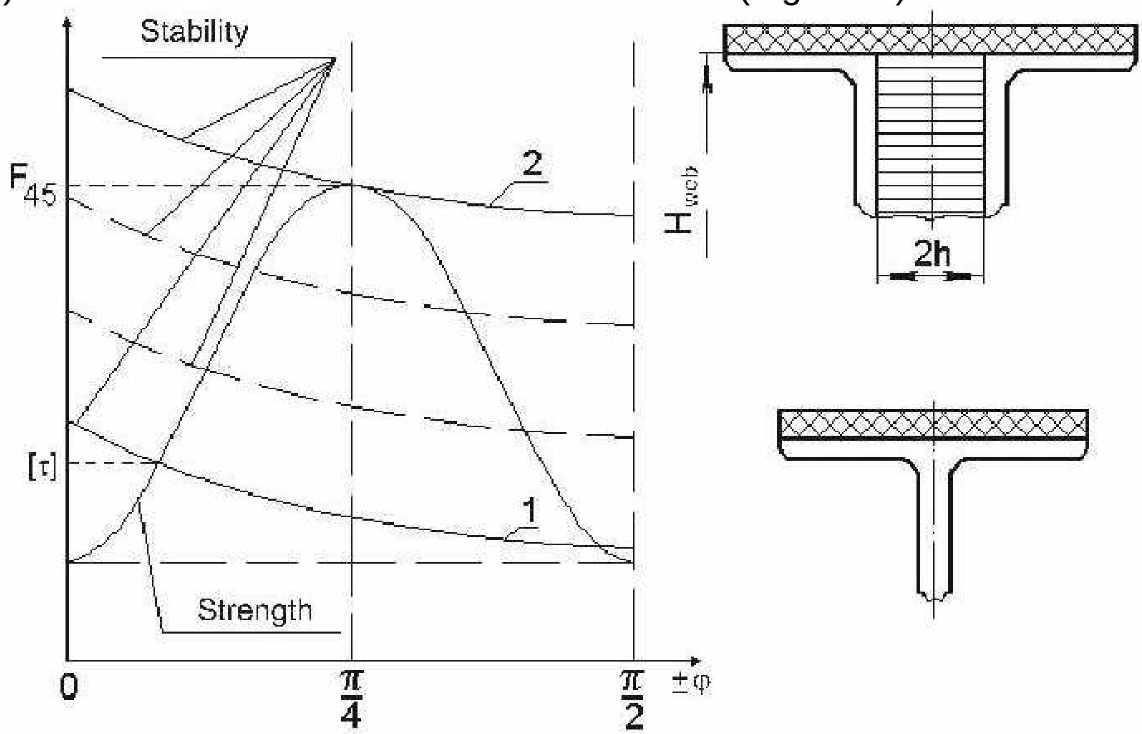


Fig. 7.15. Transition from smooth web to sandwich web

To compare efficiency of sandwich web application the following criterion can be recommended:

$$\frac{1}{[\tau]} > \frac{1}{F_{45}} + \frac{2H_{eff}H_{web}(\rho_f h + \rho_{adh})}{Q_y \rho_{web}(H_{web} + b_{cap})}, \tag{7.21}$$

where $\rho_{web}, \rho_f, \rho_{adh}$ – densities of web material, sandwich structure filler material and arial density of adhesive material (adhesive is necessary for joining web layers and filler); h– semi-thickness of filler (see Fig. 7.15).

If condition (7.21) is fulfilled sandwich structure is more efficient comparing with smooth web.

To analyze the third possible mode of loosing stability (see Fig. 7.13, c) the following beam model can be used (Fig. 7.16). Web is assumed to be loaded with lateral force Q_y only.

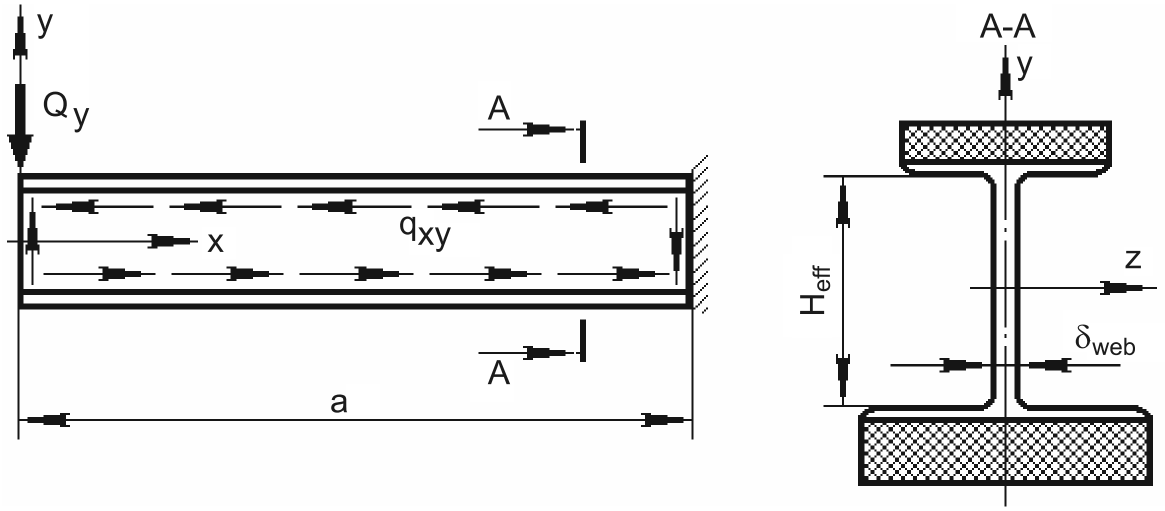


Fig. 7.16. Model of web loosing stability

Lateral force Q_y causes appearing shear force flow q_{xy} (per unit length, N/m). To define critical shear force the following dependence can be used:

$$q_{cr} = \frac{\pi^2 \sqrt{D_x D_y}}{a^2} K, \qquad (7.22)$$

where D_x, D_y – cylindrical rigidities of web package along axes x and y; a– web length; K– supporting coefficient:

$$D_x = \frac{E_x \delta_{web}^3}{12(1-\mu_{xy}\mu_{yx})}; D_y = \frac{E_y \delta_{web}^3}{12(1-\mu_{xy}\mu_{yx})}. \qquad (7.23)$$

Supporting coefficient K depends of web package elastic properties, ratio H_{web}/a and varies with wide range. One can find exact K value in the [1].

7.5. Structural and manufacturing solutions of composite wing spars

Majority of wing spars, control surfaces spars and other load-carrying units of aircraft are beams included to general load-carrying scheme and possess some very important distinctive features:

– beams interact with skin, ribs and other articles, therefore zones for joint realizing have to be provided;

– wing spar height should have ability to be regulated with required precision therefore special structural deformable elements has to be designed;

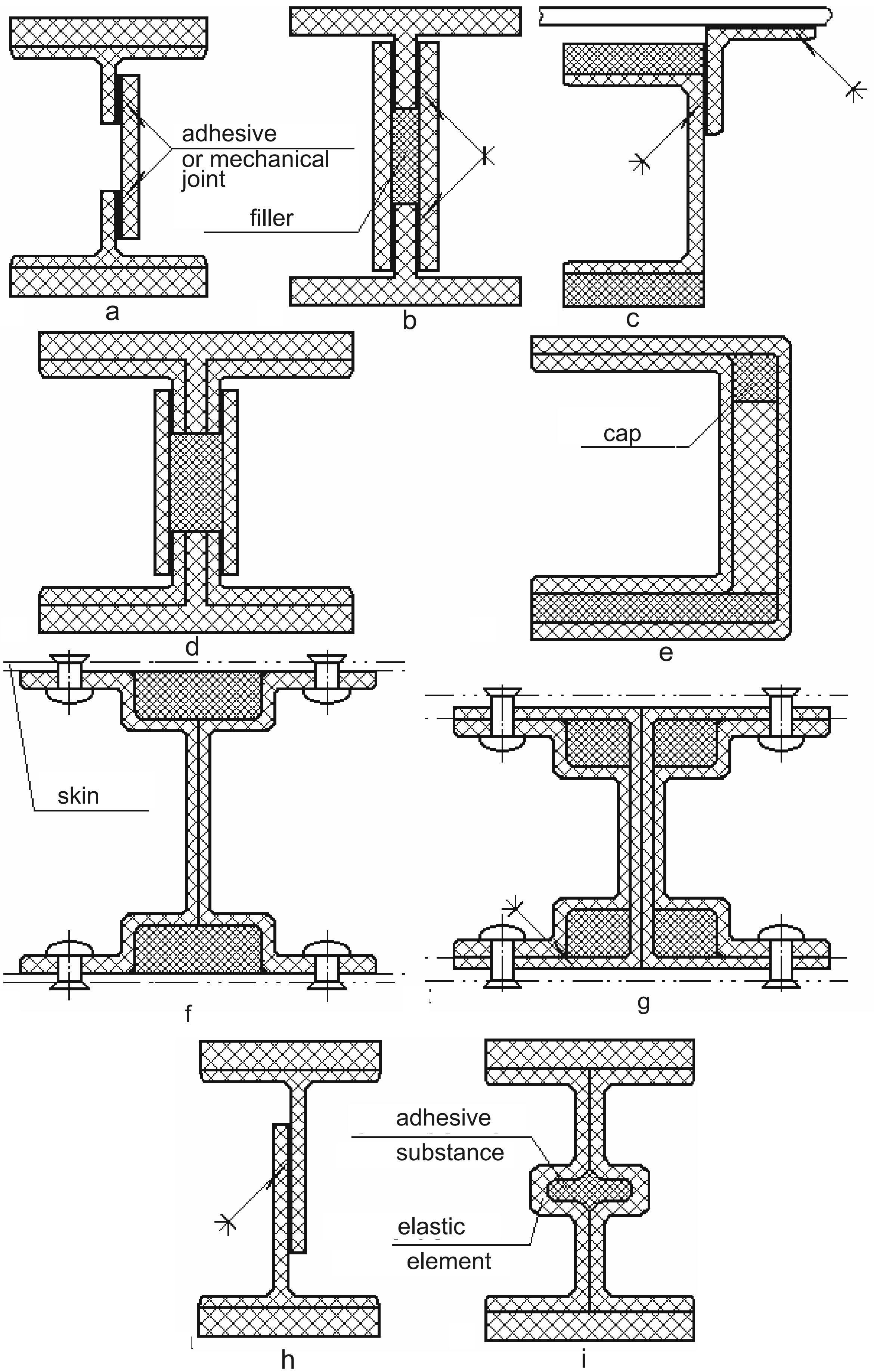


Fig. 7.17. Possible variants of composite wing spars structural solutions

– manufacturing quality of cap and web joining has to be very high that requires special requirements to selected manufacturing and assembling processes.

Practically structural solutions satisfying above-mentioned spars peculiarities can be fulfilled as follows (see Fig. 7.17, 7.18). Suggested solutions permit to regulate spar height (Fig. 7.17, c, h, i); prevent cap fibers cutting (Fig. 7.17, a–d) even at application of mechanical fasteners (Fig. 7.17, f, g).

By means of combinations of these solutions one can solve majority of practical problems.

7.6. Supports and fittings used for beams joints

General approaches of any load-carrying structure design are based on idealized analysis schemes (loading, support conditions), that is why to obtain reliable and workable structure one has to take into consideration real character of forces application, support condition and vary these parameters during design procedure.

Exact solutions of stress problem near zones of loading application and near supports and fitting are very difficult, therefore usage of this approach at the stage of preliminary design is impossible. Thus one should take into consideration the following distinctive features of beams design in non-regular zones. Suggested recommendations are based on Saint-Venant principle (proved experimentally), i.e. length of edge effect zone doesn't exceed beam height. Properties of composite package have to correspond to stress distribution in these non-regular zones.

Considering that beam is loaded with bending moment, lateral force and longitudinal force, one can analyze real stress distribution in beam elements (Fig. 7.18). Since reaction R (lateral force on Fig. 7.18, a is applied to lower beam cap) causes complex stress state: **normal stress σ_y appear** in web besides shear stress τ_{xy}; normal stress is non-uniformly distributed through beam length and height (Fig. 7.18, b, c). Therefore it is necessary to strengthen web with doublers (Fig. 7.18, d) or with ribs to escape of loosing stability in compressed zone (Fig. 7.18, e). exact dimensions these auxiliary structural elements is defined after local strength analysis considering real loads transition between elements.

Any local (point) force application to composite structure is not desirable because of low bearing strength. Thus support reactions have to be distributed per definite area (Fig. 7.18, f) (moreover this solution reduces normal stress σ_y maximum value). Thorough attention should be paid to step bearing (Fig. 7.18, g) – its own deformation can cause point contact therefore it is recommended to fulfill them in the form of micro-beam with variable rigidity (Fig. 7.18, g).

External elements of fixtures and supports have to take beam deformations (Fig. 7.18, i, j) into consideration. In those cases distributed forces can be transformed to concentrated ones. Thus practical solution of this problem is show at (Fig. 7.18, k, l).

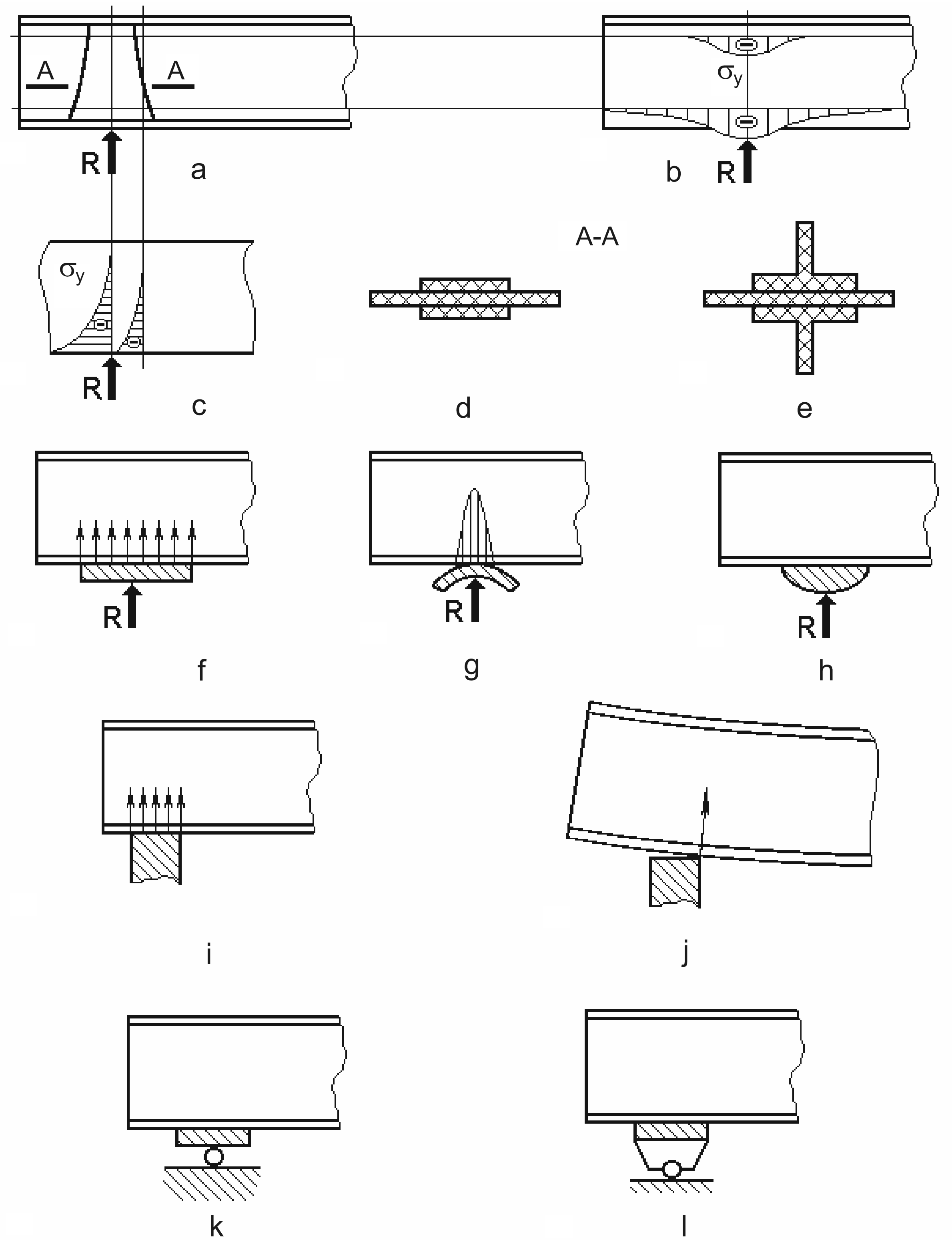


Fig. 7.18. Beam fittings and supports design recommendations

If support is planed to be installed on upper beam cap normal stress σ_y (appearing in web) causes tension (Fig. 7.19, a). In this case buckling is absent but cracks appearing is possible at tensioned zone. (Fig. 7.19, b). To reduce maximum value of σ_y metal doublers with adhesive-mechanical joints can be recommended. (Fig. 7.19, f). Application of this solution permits to simplify wing spar assembling process and to obtain fitting with minimum mass.

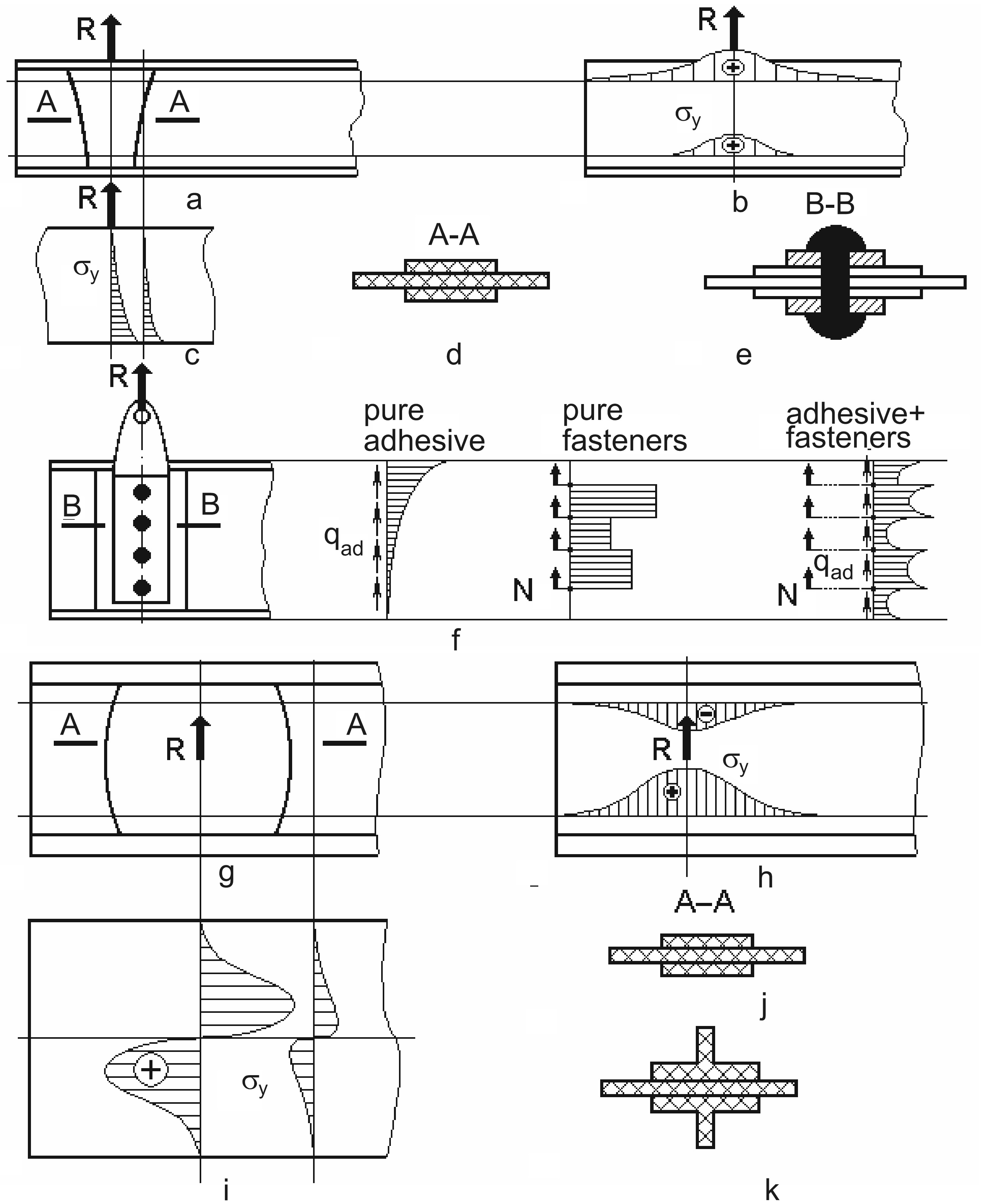


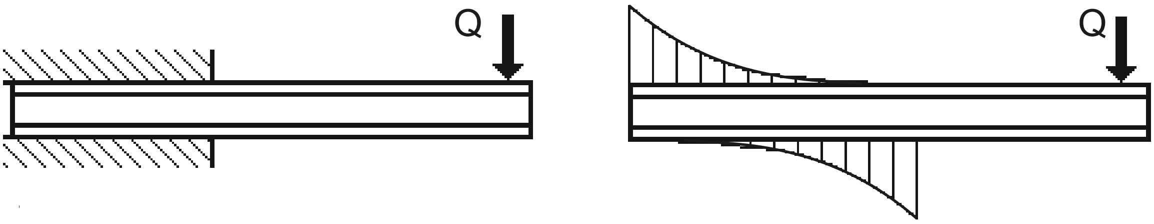
Fig. 7.19. Beam fittings and supports design recommendations

Variant of fittings installation exactly on the web (Fig. 7.19, g) permits to reduce normal stress σ_y but is more complicated in practical realization.

Cantilever beam (Fig. 7.20, a) can be loaded with two reactions distributed through quite small area (Fig. 7.20, b). Quality and precision of touching surfaces and beam deformable properties are very important parameters for this structural solution. Reduction of normal stress σ_y is achieved by increasing of clamping depth and profiling beam internal end close to conical shape.

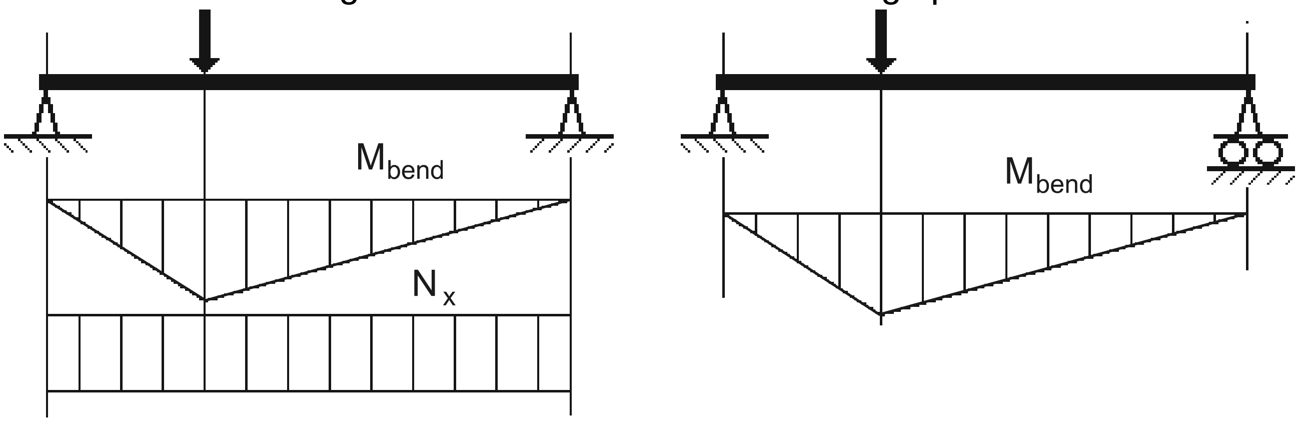
It is necessary to mention that in all above considered variants of beam supporting doublers and ribs installed on web has to be connected to caps to transfer loads.

If beam support are **restrained** auxiliary longitudinal stress (along ***x*** axis) occurs, so-called “chained stress” (Fig. 7.21, a). Therefore one of two supports should be movable (sliding) (Fig. 7.21, b).



a b

Fig. 7.20. Variant of cantilever wing spar



a b

Fig. 7.21. Chain stress appearing

Application of root spar fittings is used for majority of wing spar structures to transmit lateral force and tension-compression forces from caps to fuselage fittings (Fig. 7.22).

Forks or ears of root fitting plate are loaded with bending moment, shear force and tension-compression. Geometrical parameters of this fitting are defined according to approaches considered in the course of Elements of Machines.

Practical implementation of mentioned root fitting is shown at Fig. 7.23. This solution ensures minimum fitting mass due to optimal force distribution between fitting elements.

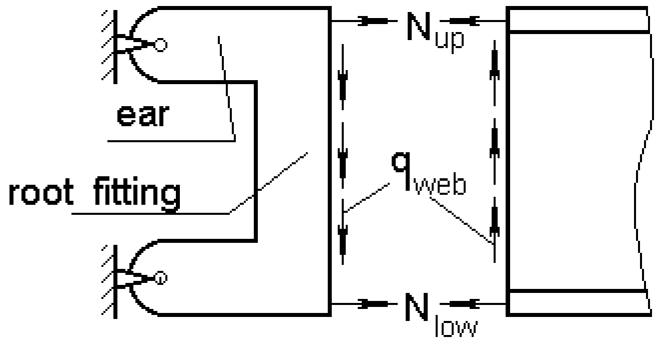


Fig. 7.22. Root fitting functions

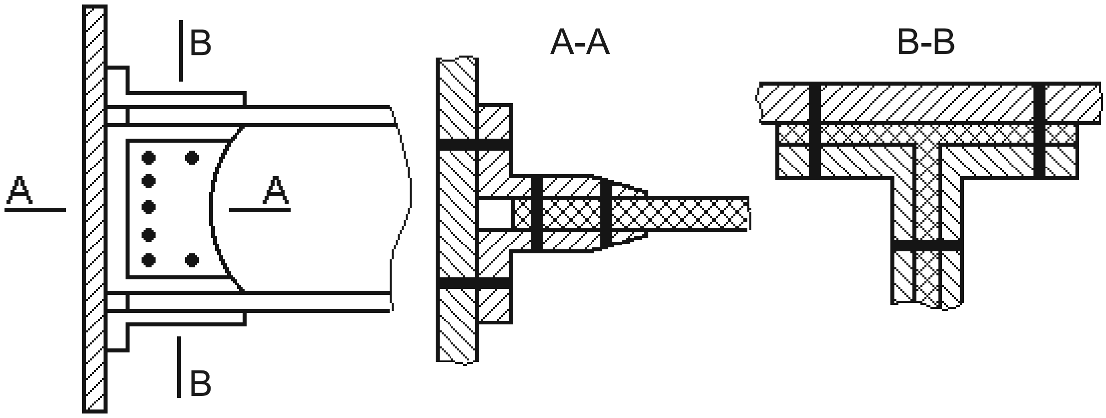


Fig. 7.23. Structural solution of root fitting

Frequently we have to join beam with another one or with strut (this structural scheme is typical for light aircrafts) as shown at Fig. 7.24.

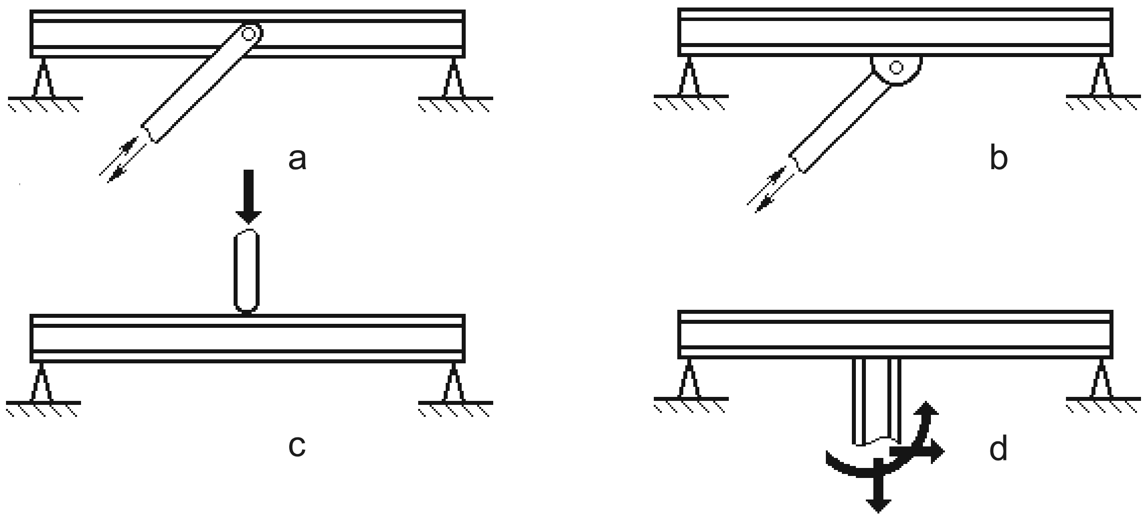


Fig. 7.24. Schemes of beam joining with another beam or strut

Practical realization of above-mentioned joints can be realized as the following structural solutions (Fig. 7.25). Longitudinal force in strut can be reduced on two reactions (R_x and R_y). Reaction R_y can be transmitted by doublers and web thickening (Fig. 7.25, f, g, h). Reaction R_x is desirable to be adopted by caps therefore special web plate fitting is the rational solution (Fig. 7.25, c, e).

Case of two beams joining is quite complicated problem. Practical realization of this joint can be obtained by means of two special fittings combination (Fig. 7.26).

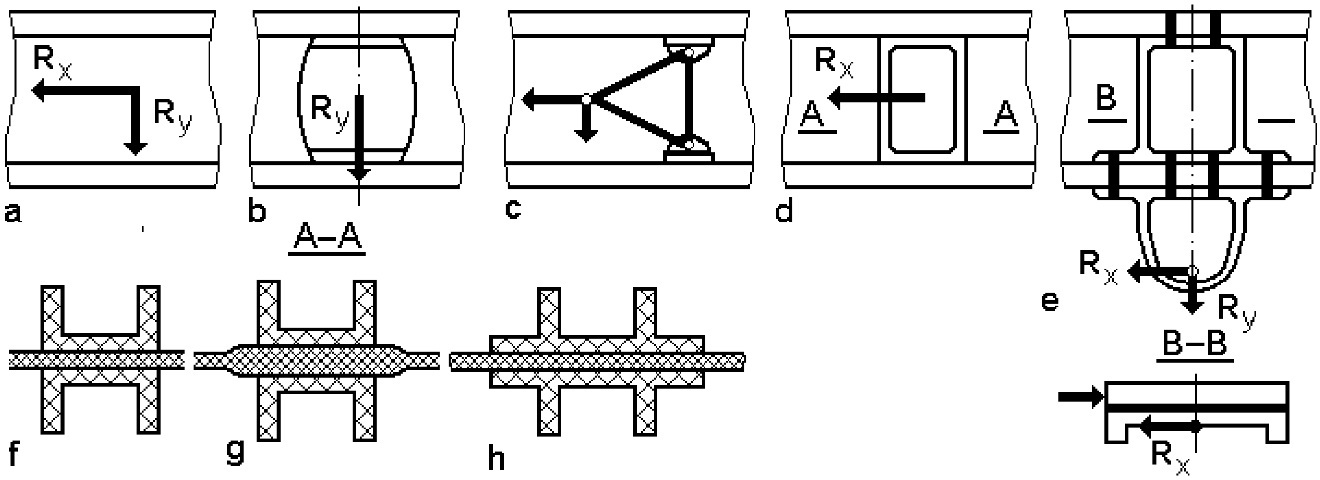


Fig. 7.25. Structural solutions for beam and strut joint

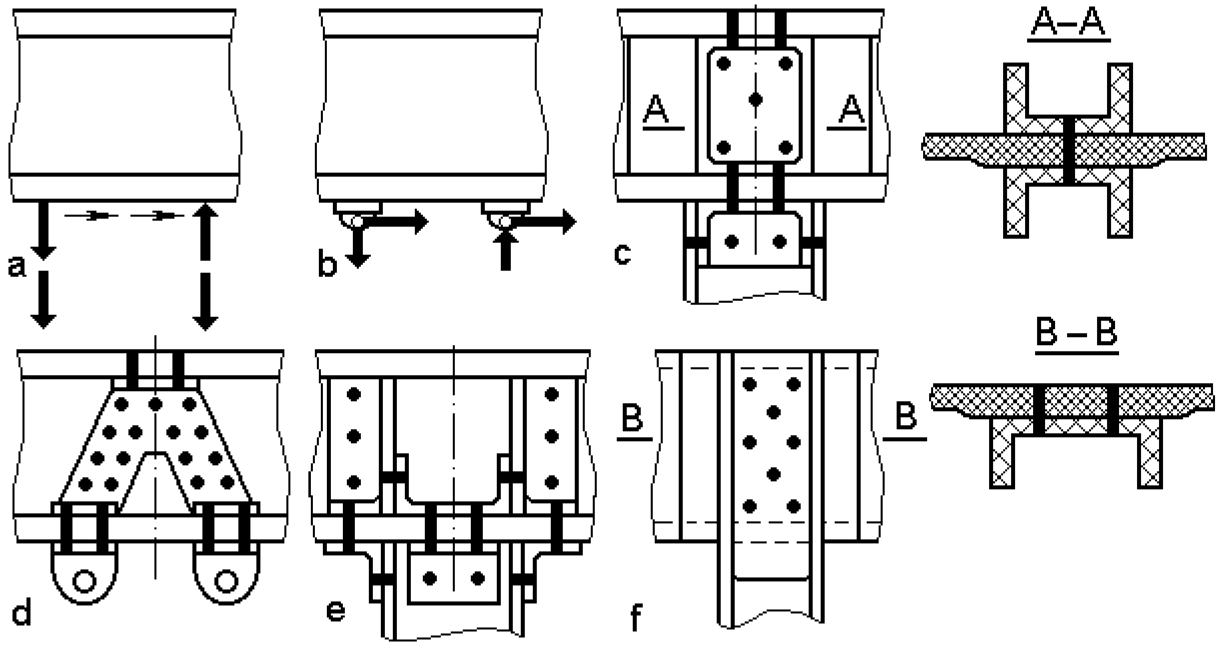


Fig. 7.26. Variant of two beams joining

Generally we can formulate the following main principles of beams fitting and supports design:

1. External beam loading has to be restricted in such way to force each fitting element to withstand definite loading component, moreover to reduce to minimum any eccentricity of force excluding auxiliary torsion or bending out of element plate.

2. Exact proper deformations of fittings and supports and total beam deformation have to be taken into consideration at wing spar fitting and supports design to eliminate appearing of concentrated forces and undesirable stress.

3. For composite beams (especially made of unidirectional composites) one should exclude cracks and fiber breakage appearing; therefore mechanical fasteners can be used in special cases only and together with strengthening measures.

4. Bending rigidity of web in its own plane is higher comparing with caps rigidity. Thus external lateral forces have to be applied exactly to web including caps to loading transferring.

5. Cracks developing has to have self-stopping character but not their growth.

6. Loading application and transferring has to be organized by shortest way and in accordance with natural way for define structural element; moreover symmetrical structural solutions are more desirable.

7. Load transferring for any load-carrying element has to be undoubtedly predicted to exclude uncertainty in element loading.

8. Including in operation any doublers, reinforcing elements have to be as smooth as it possible; therefore sharp thickening or thinning or sharp elasticity modulus changing are impossible (especially for adhesive joints or co-moulded composite layers).

Checking-up questions

1. Give examples of beams application in exact aircraft articles and units.

2. Why maximum efficiency of composites application can be realized in beam-like structures?

3. What are distinctive features of beams operation?

4. What main assumptions are used in the process of composite beams design?

5. What is the difference between beam and wing spar in aircraft wing arrangement?

6. Give the definition of sub-shoulder and draw sketches of their practical realization.

7. What is the typical reinforcing scheme in beam caps and web?

8. What cross-section of beams are used in aircraft structures? Advantages and disadvantages of I-like sections and channel-like sections.

9. What does the notion of "effective height" of a beam mean?

10. What main technological processes are used for composite beams manufacturing?

11. What is the main reason of beams cross-sections application with open and closed bevels?

12. What is the main reason of application of beam caps with step-variable thickness?

13. What three main approaches of beam structures assembling are used in aircraft structures? What are the main criteria of their selection?

14. Stability of what beam elements has to be checked at the stage of beam design?

15. What are the main structural methods of beam elements buckling prevention?

16. Draw the main structural solutions of beams joints with wing skin/

17. What are the main distinctive features of beam joining fittings design?

18. What types of fittings are used for joining beams with other structural elements?

19. What are the main structural solutions of several beams joining between each other?

20. What are the main principles of beams fitting and supports design?

Theme 8. DESIGN OF PANELS MADE OF COMPOSITES

8.1. Fundamentals of panels design

Generally aircraft fuselage, wing skin or other units are composed of panels. The main function of these panels is the same that entire skin fulfils – to keep aerodynamic profile, to withstand aerodynamic loads and transfer this load to other structural elements of aircraft load-carrying scheme [6, 8].

Frequently panels are analytically modeled as thin plate because of relatively small panel thickness refer to its length and width (Fig. 8.1, a). Panels boundaries (dimensions) are restricted by such structural elements as ribs, webs, membranes, bulkheads etc (Fig. 8.1, b).

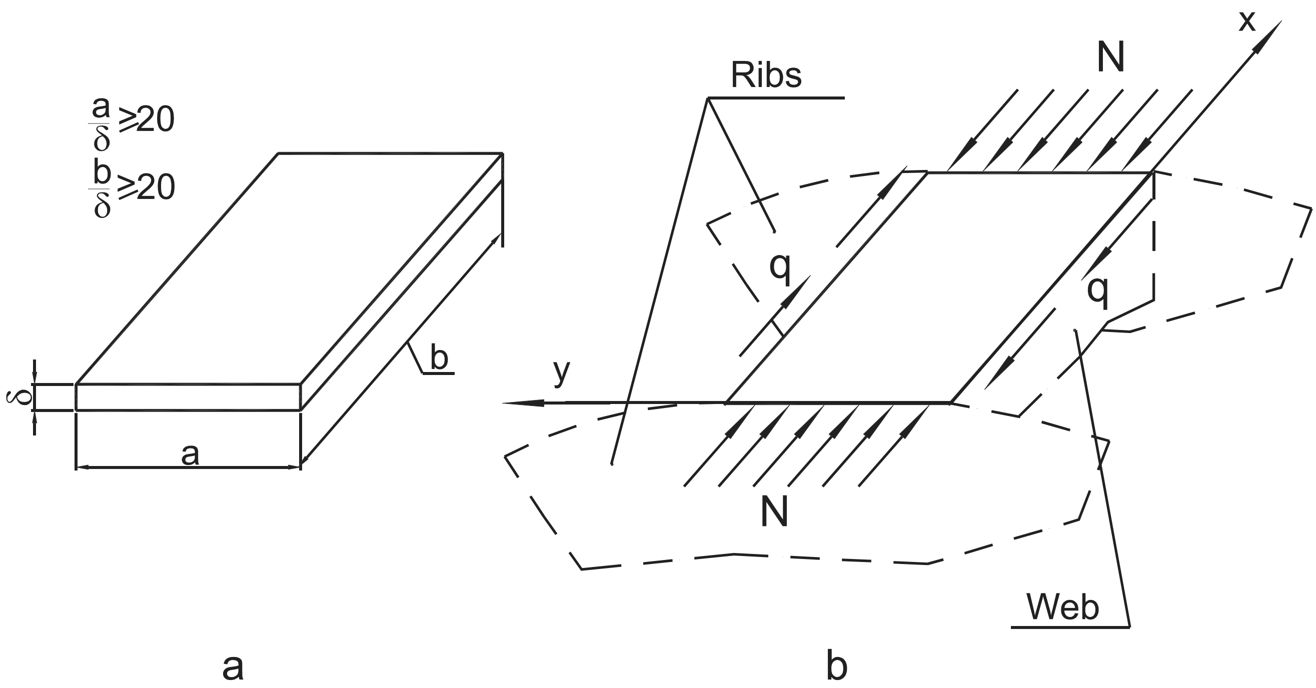


Fig. 8.1. Plate analytical model

Initial data for panel design is loads applied to panel, its geometry (dimensions a and b) and properties of composite monolayers. Generally design procedure is divided into two stages:

a) to define composite package structure and its thickness considering composite strength condition;

b) to correct panel structure and parameters considering auxiliary conditions (absence of loosing stability, deflection restrictions etc).

Panel curvature and loading variation through panel dimensions are neglected (maximum values of loading are used for analysis). Panels of any shape are replaced by rectangular ones in such way to ensure definite safety factor by strength and stability.

Differential principle is used at the first stage of design. According to this principle loads have to transfer between element exactly by fibers in shortest way. Optimal structure of orthotropic package is $\left[0°_n;\ \pm\varphi°_m;\ 90°_k\right]$, where monolayers quantity n, m, k and stacking angles $\pm\varphi°$ are defined according to values of applied load in correspondent directions. Moreover packages with reinforcement $\left[0°_n;\ \pm45°_m;\ 90°_k\right]$ can be used too; in this case panel mass is higher comparing with optimal variant but not more than 10% that is quite allowable. Therefore based on above-mentioned considerations the following assumptions for panels design is adopted:

– longitudinal load N is transferred by layers with stacking angle 0° only;

– shear loads q is transferred by layers with stacking angle ±45° only;

– if lateral forces (along axis y) is presented it is necessary to add layer with stacking angle 90°.

– reciprocal influence of layers with different stacking is neglected.

Then thicknesses of layers groups with different reinforcing are defined as:

$$\delta_0 = N/F_x; \qquad \delta_{45} = q/F_{xy}, \tag{8.1}$$

where F_x, F_{xy} – strength of composite package components stacked with angles 0° and ±45°.

After package structure definition one can conduct strength checking calculation and correct entire package thickness.

Necessity of the second design stage is stipulated by possibility of loosing stability of panel at compression (practically it happens in 60% of cases).

Let's consider structural solutions permitting to increase panel stability. Generally real panel is loaded by complicated system of loads. But we consider the mostly spread case of loading with normal forces ***N*** and flow of tangent forces ***q*** (Fig. 8.1, b). Panel is stable if the following condition is fulfilled:

$$\frac{N}{N_{cr}^0} + \left(\frac{q}{q_{cr}^0}\right)^k \le 1, \tag{8.2}$$

where N, q – applied loads; N_{cr}^0, q_{cr}^0 – flows of critical forces at separate application of normal and shear forces; k – coefficient (has to be proved experimentally) but for analysis can be assumed to be equal $k = 2$.

If panel works at tension

$$N_{cr}^0 = F_{xt}\delta, \tag{8.3}$$

where F_{xt} – composite strength at tension; δ – total package thickness.

Dependence (8.2) can be used at the following conditions:

– if N is compression force then $N > 0$;

– if N is tension force then $N < 0$.

All other parameters have to inserted by modulus.

There are the following structural solutions possessing elevated stability:

– pure sandwich panels;

– sandwich panels stiffened with ordinary rib (this solution is recommended to use for increasing sandwich panel stability by means of panel thickening more than 5 % of panel height);

– smooth skin stiffened with ribs;

– smooth skin stiffened with stringers;

– smooth skin stiffened with ribs and stringers simultaneously.

Panels stability analysis has to take into consideration the following recommendations:

a) composite physical and mechanical properties can be calculated by any technique, for example, Vasiliev's method;

b) if in a composite package the quantity of monolayers is more than **ten** bending rigidity D of a panel can be estimated based on average panel characteristics, i. e.

$$D = \frac{E\delta^3}{12\left(1-\mu_{xy}\mu_{yx}\right)}. \tag{8.4}$$

If quantity of layer is less than **ten** it is necessary to consider real layers coordinate in entire panel thickness.

Let's consider above-mentioned panels structural solutions.

8.2. Smooth panel with stiffeners

At first it is necessary to be sure that smooth panel requires stiffener installation. For this critical loads N^0_{cr}, q^0_{cr} and skin deflection from aerodynamic load (expressed in terms of pressure difference on external and internal skin surfaces) have to be estimated

$$q^0_{cr} = \frac{k_q \pi^2 \sqrt{D_x D_y}}{b^2}; \qquad N^0_{cr} = \frac{k_N \pi^2 \sqrt{D_x D_y}}{a^2}, \tag{8.5}$$

where $D_{x(y)} = \frac{E_{x(y)}\delta^3}{12\left(1-\mu_{xy}\mu_{yx}\right)}$ – cylindrical rigidity; b– panel larger dimension (b>a); a– panel to which compression loading is applied; k_q – supporting coefficient of panel loaded with shear; k_N – supporting coefficient of panel loaded with compression (depends on boundary conditions):

- both panel side edges are free supported,

$$k_N = 2 + \frac{2D_{xy}}{\sqrt{D_x D_y}}; \tag{8.6}$$

- both panel side edges are clamped,

$$k_N = 4.62 + \frac{8D_{xy}}{3\sqrt{D_x D_y}}; \tag{8.7}$$

- one panel side is clamped, the second edge is free supported,

$$k_N = 3.2 + 2.5\frac{D_{xy}}{\sqrt{D_x D_y}}; \qquad (8.8)$$

- one panel side edge is free supported, the second edge is free (not supported) at all,

$$k_N = \frac{12D_к}{\pi^2\sqrt{D_x D_y}}\ ; \qquad (8.9)$$

$D_k = \dfrac{G_{xy}\delta^3}{12}$***;*** $D_{xy} = 2D_k + \mu_{yx} D_x$**.**

Conditions (5) can be transformed to the following

$$q^0_{cr} = \frac{k_q \pi^2 \sqrt{\overline{D}_x \overline{D}_y}}{b^2}\delta^3 = A\frac{\delta^3}{b^2}; \quad N^0_{cr} = \frac{k_N \pi^2 \sqrt{\overline{D}_x \overline{D}_y}}{a^2}\delta^3 = B\frac{\delta^3}{a^2}\ . \qquad (8.10)$$

Considering (10) formula (1) can be rewritten to the following (at ***k***=2)

$$\frac{N}{B}\frac{a^2}{\delta^3} + \left(\frac{q}{A}\right)^2 \frac{b^4}{\delta^6} \le 1. \qquad (8.11)$$

In this formula parameters of material physical and mechanical properties, supporting conditions and geometrical parameters of a panel are separated from each other (considering that k_q depends on unit panel cell dimensions ratio). Therefore variation of panel thickness, its unit cell length (by means of ribs spacing) and cell width (by means of stringer spacing) one can satisfy panel stability conditions.

Required spacing between stringers is defined from condition of panel local stability at compression and from condition of skin allowable deflection.

Required spacing between ribs (ordinary bulkheads) is defined from conditions of ensuring local panel stability and stringers general stability.

The simplest condition restricting skin deflection is the following

$$\frac{\delta}{a} \ge \left[\frac{q(1-\mu_{xy}\mu_{yx})}{32E_y\left[\,\bar{f}\,\right]}\right]^{1/3}, \qquad (8.12)$$

where q– excess pressure (atmospheric) on panel surface; $\left[\,\bar{f}\,\right]$– relative allowable skin deflection (depends on aircraft type and varies in the range 0.001…0.005).

Condition of stringer general stability

$$b = \frac{\pi}{\mu}\sqrt{\frac{(EI)_{str+skn}}{f_{str+skn}\sigma_{cr\,str}}}\ , \qquad (13)$$

where $(EI)_{str+skn}$– bending stiffness of stringer and part of skin joined to this stringer (so-called associated skin).

$$(EI)_{str+skn} = \sum_{i=1}^{n}\left[(EI)_{stri} + f_{stri} * E_{stri}\left(y_{gc.\,stri} - y_{na}\right)^2\right] + $$
$$+E_x\left[\frac{\delta^3 2c}{12} + \delta 2c\left(y_{na} - \delta/2\right)^2\right] \quad ; \tag{8.14}$$

$$f_{str+skn} = \sum_{i=1}^{n} f_{stri} + \delta 2c \quad ; \tag{8.15}$$

2c – width of associated skin;

$$2c = 1.28\delta\left[\frac{E_{xstr}}{\sigma_{cr.str}\left(1-\mu_{xy\,str}\mu_{yx\,str}\right)}\left(\sqrt{\frac{E_x}{E_y}} + \mu_{yx} + 2\frac{G_{xy}}{E_x}\left(1-\mu_{xy}\mu_{yx}\right)\right)\right]^{0.5} \quad ; \tag{8.16}$$

$\sigma_{cr.\,str}$ – minimal value of critical stress corresponding to stringer local loosing stability (stringer section is considered to be series of flat plates);
y_{na} – coordinate of skin-stringer neutral axis

$$y_{na} = \frac{\sum_{i=1}^{n} f_{stri}E_{xstri}y_{stri} + 2cE_x\delta^2/2}{\sum_{i=1}^{n} f_{stri}E_{xstri} + \delta 2cE_x} \quad . \tag{8.17}$$

Mass of unit length panel

$$\overline{M} = \delta a + \frac{M_{rib}}{b} \to \min, \tag{8.18}$$

where M_{rib} – rib or bulkhead mass.
Therefore panel design algorithm is the following:

a) Minimum skin thickness is defined from composite package strength condition $\delta_{strength}$ (equations (8.1)).
b) Minimal panel thickness obtained from condition of panel stability under compressive loading

$$\delta_N > \sqrt[3]{\frac{N}{B}a^2}; \tag{8.19}$$

and from condition of allowable skin deflection by formula (8.12) – δ_{def}.

c) If $\delta_N < \delta_{strength}$ and $\delta_{def} < \delta_{strength}$ is fulfilled one can estimate minimal spacing between ribs from conditions of panel stability at simultaneous action of compression and shear

$$b \le \sqrt[4]{\left(1-\frac{Na^2}{Bb^3}\right)\left(\frac{A}{q}\right)^2\delta^6}. \tag{8.20}$$

If at least one of two conditions $\delta_N < \delta_{strength}$ and $\delta_{def} < \delta_{strength}$ is not valid one has to define at first spacing between stringers from stability condition at compression or from allowable deflection condition and then ribs (bulkheads) installation spacing by formula (8.15).

d) If required rib spacing is more than initial panel dimension it means no auxiliary ordinary ribs are required. If b<a then stringers installation spacing a can be estimated by formula (8.15) (at constant b value).

e) Specific panel mass $\overline{M}$ is calculated for obtained structure and panel thickness is varied according to optimization function (8.18).

At $b < 150$ mm it is recommended to used combination of beam ribs and frame ribs; moreover to ensure that frame rib fulfils role of support one has to keep the following non-equality comparing their rigidity

$$(EI)_{frame\,rib} \geq 5(EI)_{skn}, \tag{8.21}$$

where $(EI)_{skn} = \dfrac{E_x \delta^3 b}{12\left(1 - \mu_{xy}\mu_{yx}\right)}$.

Checking-up questions

1. Give examples of panels application in exact aircraft articles and units.
2. Main structural solutions of panels used in aircraft structures.
3. Main assumptions used in the procedure of smooth and stiffened panels design.
4. In what case curved panel can be analyzed as flat one?
5. What is the typical stacking sequence for smooth and stiffened composite panels?
6. What is the condition of panel general stability at combined loading (tension/compression and shear)?
7. Main methods of composite panels increasing stability.
8. How to calculate smooth composite panel cylindrical rigidities at bending and shear?
9. What is the typical restriction of panel skin from aerodynamic loads? From what factors does it depend?
10. What is the typical shapes, manufacturing processes and stacking sequence of composite stringers?
11. What does the notion of "associated skin" mean?

Theme 9. DESIGN OF JOINTS OF COMPOSITE ARTICLES. JOINTS CLASSIFICATION AND STRUCTURAL SOLUTIONS. METHODS OF INCREASING JOINTS LOAD-CARRYING ABILITY

9.1. Classification of structural joints

Joints of aircraft structures elements are the most important zones defining aircraft load-carrying ability, life-time and reliability. Generally joints increase structure mass by 20..30% and cause about 80% of breakage. From other hand designer can't exclude structure division by bays, sections, parts etc. That is why problem of design joints possessing rational parameters is very actual.

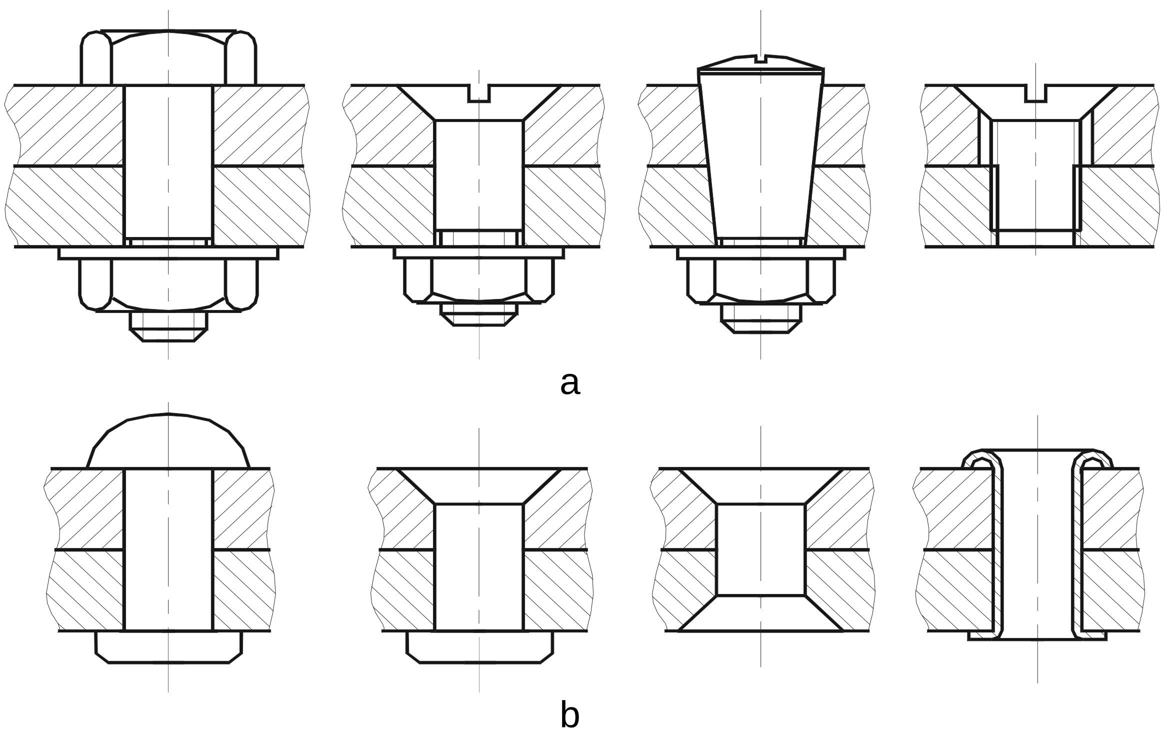
All possible joint types can be divided into **movable** (ensure definite displacement of one article refer another) and **unmovable** (hold reciprocal articles position transfer external and internal loads between structural elements), **splitable** (detachable) and unsplitable (permanent). Moreover by different maintenance principles joints can be classified as follows [9, 10, 6]:

– by physical principles of joining: mechanical, welded, adhesive, soldered;

– by joining elements type: continuous (adhesive, welded etc) and discrete (bolted, riveted, pin etc);

– by geometry of force flows: pointed (bolted, riveted), linear (roller welded), surface (overlapped adhesive, welded, wedged), volumetric (butt welded, butt adhesive).

Nowadays the most wide spread splitable load-carrying joint types for aircraft articles are bolted, screw and riveted joints (Fig. 9.1).



a

b

Fig. 9.1. Discrete mechanical joints

9.2. Design of multi-row mechanical joint with discrete fasteners

General approach to point joist design considers multi-row mechanical joint with fasteners of variable diameter and arrangement through articles area. Thickness of joining articles can vary continuously or stepwise (Fig. 9.2) [10].

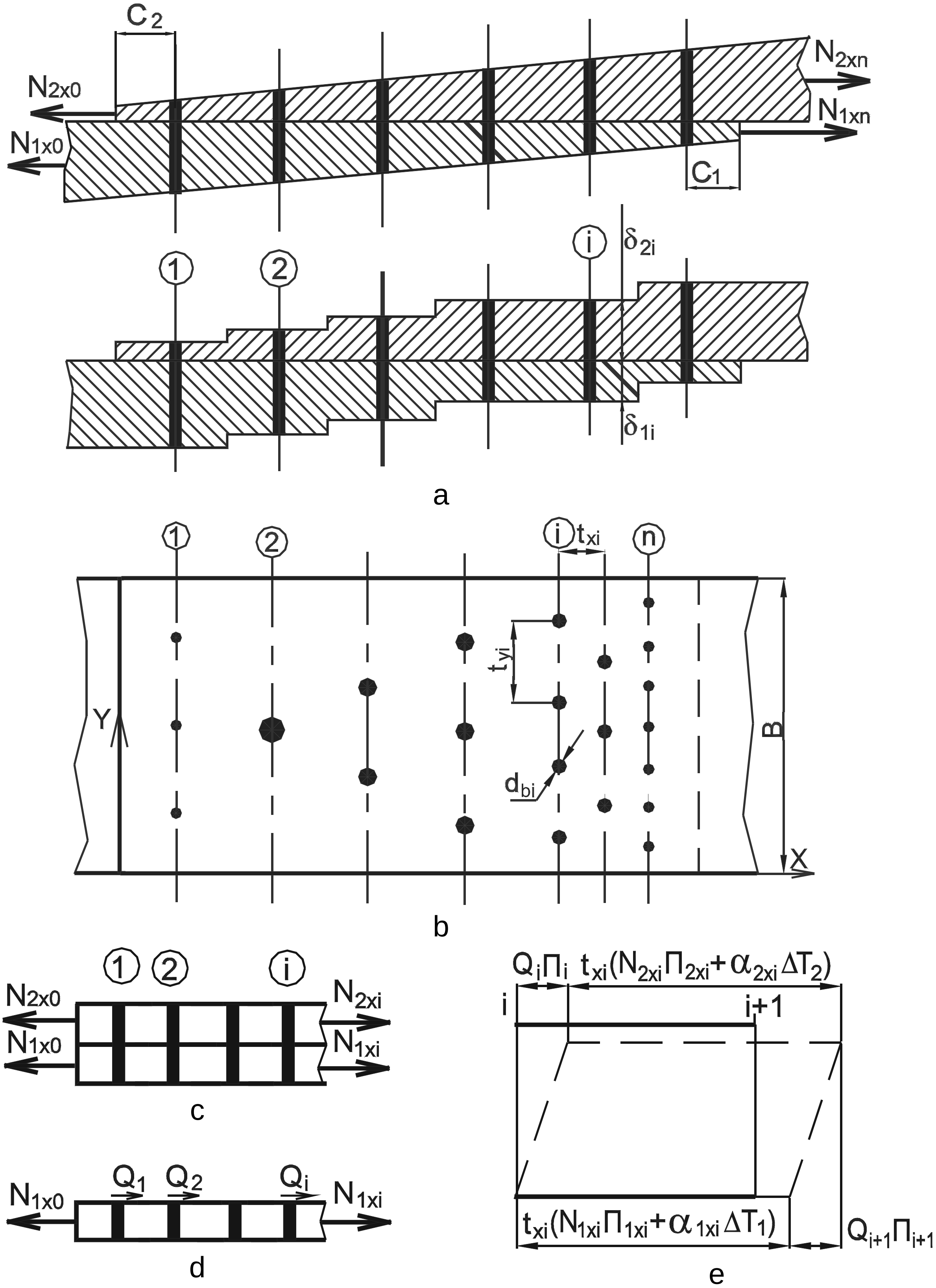


Fig. 9.2. General geometrical model of joint with discrete fasteners

Let the following forces are applied to joining articles edges: N_{1x0}, N_{2x0}, N_{1xn}, N_{2xn} and operating temperature of joint differs from manufacturing temperature.

Assume that each row contains m_i fasteners installed with equal spacing, i. e.

$$t_{yi} = B / m_i \tag{9.1}$$

where B– joint width.

Stress distribution through articles thickness is considered to be uniform (at least for design stage), thus

$$\sigma_{1xi} = N_{1xi} / \delta_{1i}, \qquad \sigma_{2xi} = N_{2xi} / \delta_{2i}. \tag{9.2}$$

Initial data for new desirable joint design are: joining articles physical and mechanical properties (elasticity moduli) – E_{1xi}, E_{2xi} (generally different at each step); joint width B; applied forces N (per unit width); thickness of each joining article a their beginning – δ_{11} and δ_{2n}.

Design variables are: functions of rigidity distribution $(\delta_{1i}E_{1xi})$, $(\delta_{2i}E_{2xi})$, quantity of fastener rows n and fastener quantity in row m; fastener diameter d_b.

Moreover **special requirements** have to be taken into consideration during design: all fasteners should have standard diameter (discrete but not arbitrary), fasteners arrangement requirements (minimal distance between fasteners and from joint edges) etc.

Thus the following design algorithm can be used for joint with point fasteners:

Step 1. Define several standard values of fastener diameter d_b (for example, 2.5 mm; 3 mm; 4 mm; 5 mm, 6 mm etc) and fasteners quantity in row m.

Using conditions ensuring absence of joining article bearing and fastener shear check selected parameters d_b and m :

$$d_b \le \frac{4\delta_{11}\sigma_{bear1x1}}{\pi\tau_b}; \quad d_b \le \frac{4\delta_{2n}\sigma_{bear2xn}}{\pi\tau_b}; \tag{9.3}$$

$$md_b \le B\left(1 - \frac{Nk_{1x1}}{\delta_{11}F_{1x1}}\right); \quad md_b \le B\left(1 - \frac{Nk_{2xn}}{\delta_{2n}F_{2xn}}\right), \tag{9.4}$$

where – F_{1x1}, F_{2xn} – composite articles tension (compression) strength along ***x*** axis; $\sigma_{bear1x1}$; $\sigma_{bear2x1}$ – composite bearing strength along ***x*** axis; τ_b – fastener shear strength; k_{1xi}, k_{2xi} – stress concentration factor near fastener hole;

$$k_{1xi} = \frac{2}{d_b}\sqrt[4]{\frac{1}{2(1+E_b/E_{1xi})}}, \quad k_{2xi} = \frac{2}{d_b}\sqrt[4]{\frac{1}{2(1+E_b/E_{2xi})}}.$$

In formulas (4) we assume that $N_{1x0} = N_{2xn} = N$.

Step 2. Find fastener rows quantity (should be integer) from the condition of fastener shear strength fulfilling

$$n \ge \frac{4NB}{\pi m d_b^2 \tau_b}. \tag{9.5}$$

Step 3. Calculate minimal required thicknesses of joining articles $\delta_{1i}(i=2...n)$ and $\delta_{2i}(i=1...n-1)$ from the conditions of articles bearing and tearing strength in weakest section:

$$\delta_{1i} \geq \frac{NB}{nmd_b\sigma_{bear1xi}}; \quad \delta_{1i} \geq \frac{BN(n+1-i)k_{1xi}}{nF_{1xi}(B-md_b)};$$
$$\delta_{2i} \geq \frac{NB}{nmd_b\sigma_{bear2xi}}; \quad \delta_{2i} \geq \frac{iBNk_{2xi}}{nF_{2xi}(B-md_b)}. \qquad (9.6)$$

Step 4. Assuming that applied load is **uniformly** distributed between fastener rows one has to check correspondence between thicknesses of first and second articles:

$$i\cdot\left(\frac{1}{\delta_{2i}E_{2xi}}+\frac{1}{\delta_{2,i+1}E_{2x,i+1}}\right)-(n-i)\left(\frac{1}{\delta_{1i}E_{1xi}}+\frac{1}{\delta_{1,i+1}E_{1x,i+1}}\right)=$$
$$=\frac{2B}{t_{xi}}\left(\frac{\Pi_{3x2}}{m}-\frac{\Pi_{3x1}}{m}\right); \qquad i=1\ldots(n-1). \qquad (9.7)$$

Defining, for example δ_{1i}, from conditions (9.6) one check correspondence δ_{2i}.

Values of compliance coefficients Π_{3x1}, Π_{3x2} generally has to be obtained experimentally. But for design purpose we can use the following technique. Further formulas are valid for single and double shear joints at considering one joint half (Fig. 9.2, d).

$$\Pi_{3xi}=\frac{d_{bi}}{(EF)_0}\left\{N\alpha_{1i}+(\alpha_{2i}-\alpha_{1i})\sum_{i=1}^{n-1}Q_i-Q_i\alpha_{1i}+\right.$$
$$\left.+\frac{Q_iB}{\pi m_i d_{bi}}\left[\alpha_{1i}k_{q1i}\left(\frac{1-\mu_{1i}^2}{2}+\frac{2}{\beta_1^*}\right)+\alpha_{2i}k_{q2i}\left(\frac{1-\mu_{2i}^2}{2}+\frac{2}{\beta_2^*}\right)\right]\right\}, \qquad (9.8)$$

where $(EF)_0=(EF)_{10}$ – rigidity of article (for example first) to which other rigidities have to be reduced; N – load transferred by entire joint; Q_i –, load transferred by i-th fasteners row; μ_{1i}, μ_{2i} – articles Poisson's ratio:

$$(EF)_0=(EF)_{10}; \qquad \alpha_{1i}=(EF)_0/(EF)_{1i}; \qquad \alpha_{2i}=(EF)_0/(EF)_{2i};$$

$$k_{q1i}=\begin{cases}\left(\dfrac{2}{k_{1i}d_{bi}}\right)^3\dfrac{1}{1+\beta_{1i}}\dfrac{\delta_{1i}}{d_{bi}} & \text{at } \dfrac{\delta_{1i}}{d_{bi}}\geq 1;\\[2ex] \left[\left(\dfrac{2}{k_{1i}d_{bi}}\right)^3\dfrac{1}{1+\beta_{1i}}-1\right]\dfrac{\delta_{1i}}{d_{bi}}+1 & \text{at } \dfrac{\delta_{1i}}{d_{bi}}<1;\end{cases}$$

$$k_{q2i}=\begin{cases}\left(\frac{2}{k_{2i}d_{bi}}\right)^3\frac{1}{1+\beta_{2i}}\frac{\delta_{2i}}{d_{bi}} & \text{at } \frac{\delta_{2i}}{d_{bi}}\geq 1;\\ \left[\left(\frac{2}{k_{2i}d_{bi}}\right)^3\frac{1}{1+\beta_{2i}}-1\right]\frac{\delta_{2i}}{d_{bi}}+1 & \text{at } \frac{\delta_{2i}}{d_{bi}}<1;\end{cases} \quad (9.9)$$

$$k_{1i}=\frac{2}{d_{bi}}\sqrt[4]{\frac{\beta_{1i}^*}{2\beta_{1i}}};\quad k_{2i}=\frac{2}{d_{bi}}\sqrt[4]{\frac{\beta_{2i}^*}{2\beta_{2i}}};$$

$$\beta_{1i}^*=\frac{\beta_{1i}}{1+\beta_{1i}};\quad \beta_{2i}^*=\frac{\beta_{2i}}{1+\beta_{2i}};\quad \beta_{1i}=\frac{E_{bi}}{E_{1i}};\quad \beta_{2i}=\frac{E_{bi}}{E_{2i}};$$

$$(EF)_{1i}=E_{1i}B\delta_{1i};\quad (EF)_{2i}=E_{2i}B\delta_{2i}.$$

If calculated thickness of second (first) article is less than one recommended by (9.6) condition then thickness from condition (9.6) has to be selected and condition (9.7) has to be recalculated by means of the first (second) article thickness increasing.

Step 5. Spacing between rows t_{xi} (in partial case t_x) is defined from the restrictions (articles shear strength conditions)

$$\frac{Q_{x1}}{2mc_2\delta_{21}}\leq\tau_{2xz1};\quad \frac{Q_{x,i+1}}{2mt_{xi}\delta_{2i}}\leq\tau_{2xzi};$$
$$\frac{Q_{xi}}{2mt_{xi}\delta_{1i}}\leq\tau_{1xzi};\quad \frac{Q_{xn}}{2mc_1\delta_{1n}}\leq\tau_{1xzn}. \quad (9.10)$$

Step 6. Mass of obtained joints is estimated by the following dependence

$$G=\sum_{i=1}^{n}\left[\rho_b m_i\frac{\pi d_{bi}^2}{4}(\delta_{1i}+\delta_{2i})+(\delta_{1i}\rho_1+\delta_{2i}\rho_2)\left(Bt_{xi}^*-m_i\frac{\pi d_{bi}^2}{4}\right)\right]+$$
$$+\sum_{i=1}^{n}m_i(m_{1i}+m_{2i}+m_{wi}+m_{ni})\rightarrow\min, \quad (9.11)$$

where – ρ_b, ρ_1, ρ_2– fastener density, joining articles density;

m_{1i}– mass of fastener projected section;

m_{2i}– mass of fastener projected section for nut installation,

m_{wi}, m_{ni}– mass of washer and nut.

Step 7. Graph of joint mass as function on parameters m and d_b is built. Between several variants of joints those with minimal mass has to be selected.

Exact value of composite bearing strength has to be determined experimentally because it depends on a large amount of parameters (in fist turn on fastener diameter, composite package structure, joining articles thickness etc). For preliminary design the following strength values can be recommended (Table 9.1).

Table 9.1. Composites bearing strength

Material	Bearing strength, MPa
Glass fabric + polyester resin	250…300
Glass mat + polyester resin	140…210
Boron fibers [0°;90°] +epoxy resin	1030…1380
Boron fibers [0°;±45°;90°] +epoxy resin	830…1040
Glass-Organic plastic + epoxy resin	310…380
Carbon plastic [0°;90°] +epoxy resin	380…450
Carbon plastic [0°;±45°;90°] +epoxy resin	310…345

Note: Less strength value in mentioned range corresponds to beginning or fastener hole ovalization (ovalization degree ~4 %).

9.3 Adhesive joints design

All composite material exist due to adhesive bonding between fibers and matrix. That is why adhesive joints are quite natural for joining composite articles. Majority of aircraft articles made of composites are plates, shells and other thin-walled structures loaded in their own plane therefore adhesive bonding as method of loads transfer is natural. Composites articles manufacturing operations permit:

– to exclude operations devoted to surface preparation for future adhesive joint;

– to conduct simultaneous co-bonding (co-curing) together with main composite articles manufacturing;

– to shorten duration of manufacturing cycle.

Main drawbacks of adhesive joints are:

– low adhesive strength comparing with composite strength (especially at tearing);

– appearing undesirable interlaminar stress in joining zone;

– low repairability of adhesive joints (especially requiring high temperature and special equipment);

– low efficiency of non-destructive tests for bonding control;

– difficulties at comparison of theoretical models and experimental results.

Lap adhesive joints are the most wide spread one (Fig. 9.3, a, b, c, d). This adhesive joint type transfers axial loads by means of touching neighboring surfaces (Fig. 9.3, a, b, c) and forces out of joint plane (Fig. 9.3, d). Consideration of elementary representative element of lap joint (Fig.3, g) shows that resultant of normal stresses N is in equilibrium with adhesive reaction q. Due to eccentricity of this forces bending moment appears. This moment varies through joint length because of lateral force Q. Moreover this force Q is in equilibrium with vertical reaction R. Thus main conclusion of these consideration that both articles and adhesive are subjected to complex stress state.

The highest stress in adhesive layer is shear one (this fact is proved experimentally). That is why the question about reliable technique for determination of adhesive layer stress state is very important. Nowadays one-dimensional analytical model (along x axis) is practically used. There are two analysis schemes of adhesive joint joining layer: classical one (Fig. 9.3, e) and Volkersen scheme (Fig. 9.3, f).

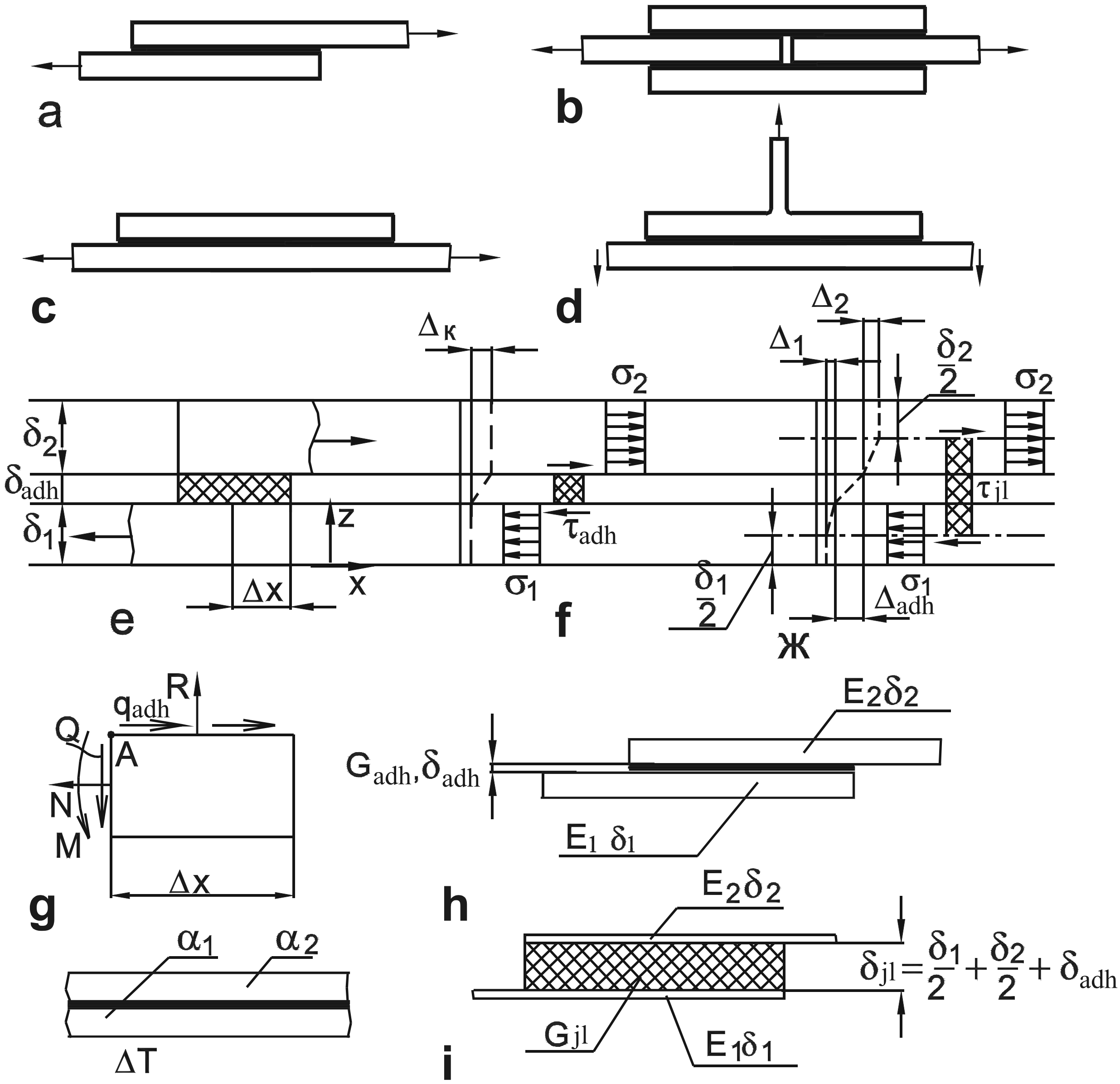


Fig. 9.3. Adhesive joint analytical model

Classical scheme means that shear deformation is adopted by adhesive layer only – joining layer thickness (δ_{jl}) is equal to adhesive thickness (δ_{adh}). According to Volkersen scheme thickness of joining layer δ_{jl} is equal to thickness of pure adhesive film δ_{adh} and semi-thicknesses of two joining articles $(\delta_1+\delta_1)/2$, i.e. $\delta_{jl}=\delta_{adh}+(\delta_1+\delta_2)/2$ (reduced joining layer is used).

Generally one-dimensional analytical schemes of adhesive joints are based on the following assumptions:

– adhesive film withstands shear force only;

– adhesive film and joining articles geometrical and rigidity parameters are the same along joint length;

– normal stress is uniformly distributed through joining articles thickness;

– material of articles is orthotropic in their plane (xy plane).

Stress state analysis of joints with variable parameters permits to make the following conclusions (Fig. 9.4):

– maximum shear strength appears at the end of more rigid article (Fig. 9.4, a);

– classical joining layer scheme (1) gives higher value of shear stress comparing with Volkersen model (9.2) (Fig. 9.4, b);

– maximum stress calculated according to Volkersen model corresponds well to more precise two-dimensional model (Fig. 9.4, b);

– less adhesive compliance less maximum shear stress;

– after achieving definite joint length l_{lim} shear stress asymptotically exceeds definite value (Fig. 9.4, c).

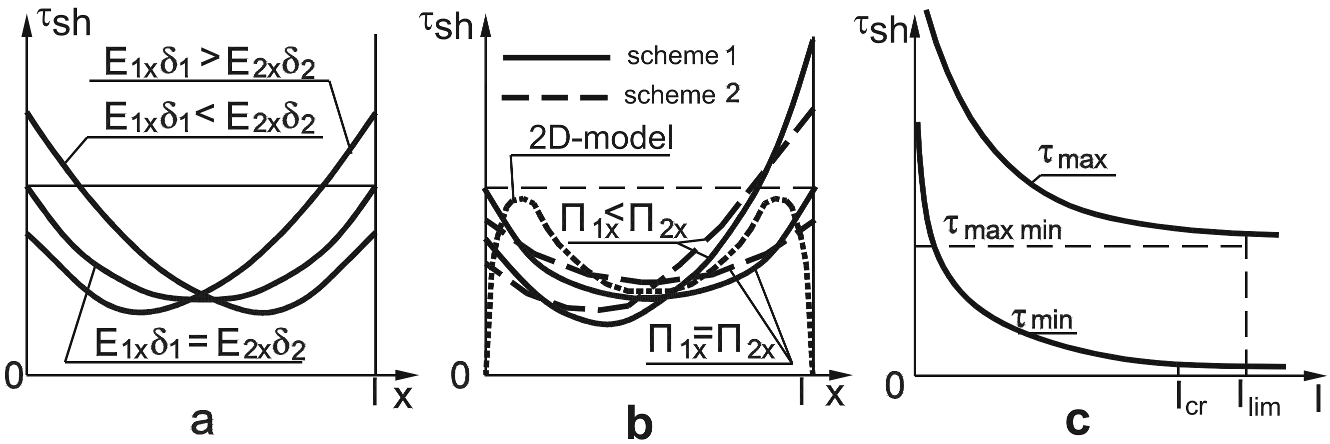


Fig. 9.4. Adhesive joint shears stress as function on joint parameters

Let's consider design procedure of adhesive joint of articles with constant thickness and elasticity moduli (for analysis simplification).

In this case design procedure includes determination joint length and selection of adhesive and adhesive layer thickness. Strength condition for shear stress has the following view:

$$\tau_{max} \le \tau_{adh} \vee \tau_{interlam}, \tag{9.12}$$

where $\tau_{adh}, \tau_{interlam}$ – adhesive shear strength and composite package interlaminar strength.

Maximum shear stress due to mechanical and thermal field can be estimated as

– at $\delta_1 E_{1x} < \delta_2 E_{2x}$

$$\tau_{max} = \frac{N}{k\Pi_{jl}} \frac{\Pi_{1x} \text{ch } kl + \Pi_{2x}}{\text{sh } kl} + \frac{\Delta T(\alpha_{2x} - \alpha_{1x})}{k\Pi_{jl}} \frac{1 - \text{ch } kl}{\text{sh } kl}; \tag{9.13}$$

– at $\delta_1 E_{1x} > \delta_2 E_{2x}$

$$\tau_{max} = \frac{N}{k\Pi_{jl}} \frac{\Pi_{1x} + \Pi_{2x} ch\ kl}{sh\ kl} + \frac{\Delta T(\alpha_{2x} - \alpha_{1x})}{k\Pi_{jl}} \frac{ch\ kl - 1}{sh\ kl}, \qquad (9.14)$$

where joining layer compliance Π_{jl} can be estimated according to the following formulas (depending on selected joint scheme – classical or Volkersen):

$$\Pi_{jl} = \frac{\delta_{adh}}{G_{adh}} \text{ or } \Pi_{jl} = \frac{\delta_1}{2G_{1xz}} + \frac{\delta_2}{2G_{2xz}} + \frac{\delta_{adh}}{G_{adh}}; \qquad (9.15)$$

Π_{1x} – compliances of joining articles,

$$\Pi_{1x} = \frac{1}{\delta_1 E_{1x}},\ \Pi_{2x} = \frac{1}{\delta_2 E_{2x}}; \qquad (9.16)$$

N – load transferred by joint;

$$k^2 = \frac{\Pi_{1x} + \Pi_{2x}}{\Pi_{jl}}; \qquad (9.17)$$

α_{1x}, α_{2x} – articles linear expansion coefficients along ***x*** axis.

To make equality between formulas (9.13), (9.14) and adhesive shear strength or composite interlaminar strength it is possible to solve transcendent equation and find required joint length.

More complicated problem is design of optimal joint with variable articles parameters along joint length. Let’s consider this procedure briefly at the following assumptions (to simplify analysis). Criterion of minimal joint mass is used:

– articles physical and mechanical parameters don’t depend on their thickness;

– thermal loading is absent ($\Delta T = 0$ or $\alpha_{1x} = \alpha_{2x}$);

– initial thickness of each article is known (δ_{10} and δ_{2n});

– classical joint scheme is used ($\delta_{jl} = \delta_{adh}$);

– adhesive parameters are constant through joint length (δ_{adh}, G_{adh});

Step 1. At first it is necessary to define function of articles thickness variation along joint length. Generally linear or parabolic dependencies ca be recommended.

$$\delta_1 = \delta_{10} \frac{N_{1x}}{N} \text{ or } \delta_1 = \frac{N_{1x}}{F_{1x}} \left[\frac{F_{1x}\delta_{10}}{N} - \bar{x} \left(\frac{F_{1x}\delta_{10}}{N} - 1 \right) \right], \qquad (9.18)$$

Using equation of compatible articles deformations

$$\frac{\Pi_{jl}}{l} k_1 = \Pi_{2x} N_{2x} - \Pi_{1x} N_{1x}, \qquad (9.19)$$

where $k_1 = \dfrac{-\Pi_{jl}\ \tau_{adh} \pm \sqrt{\Pi_{jl}^2 \tau_{adh}^2 - 2N^2 \Pi_{10} \Pi_{jl}}}{\Pi_{jl}}$, $\quad l = \dfrac{\Pi_{jl}\ \tau_{adh} \pm \sqrt{\Pi_{jl}^2 \tau_{adh}^2 - 2N^2 \Pi_{10} \Pi_{jl}}}{N \Pi_{10}}$

one can estimate the second article thickness variation function.

Step 2. As alternative variant one can define articles parameters from the condition of **uniform** shear stress distribution inside adhesive layer, i.e.

$$\frac{N_{1x}}{N_{2x}} = \frac{E_{1x}\delta_1}{E_{2x}\delta_2}. \qquad (9.20)$$

Adhesive joint thickness is estimated as $l = N/\tau_{adh}$.

Step 3. Joint variants (from step 1 and 2) mass is calculated. The lightest is more rational one.

Practically the following structural solutions of adhesive joints can be realized in aircraft structures (Fig. 9.5, 9.6). To increase adhesive layer strength at the edges (where maximum shear stress appears) of doublers or joining articles transversal stitching is used.

To reduce shear stress cutting edges of joining articles or doubles are used too (see Fig. 9.6, a, b ,c). Scarf joints (see Fig. 9.6, c) ensures smooth loads transfer. Stepped joints (Fig. 9.6, f, g, h) permit to cut pikes of shear stress.

High quality of adhesive joints can be achieved by application of separating layer between articles and doublers. This method ensures ideal adjusting of joining surfaces and joining materials with different time-temperature manufacturing parameters.

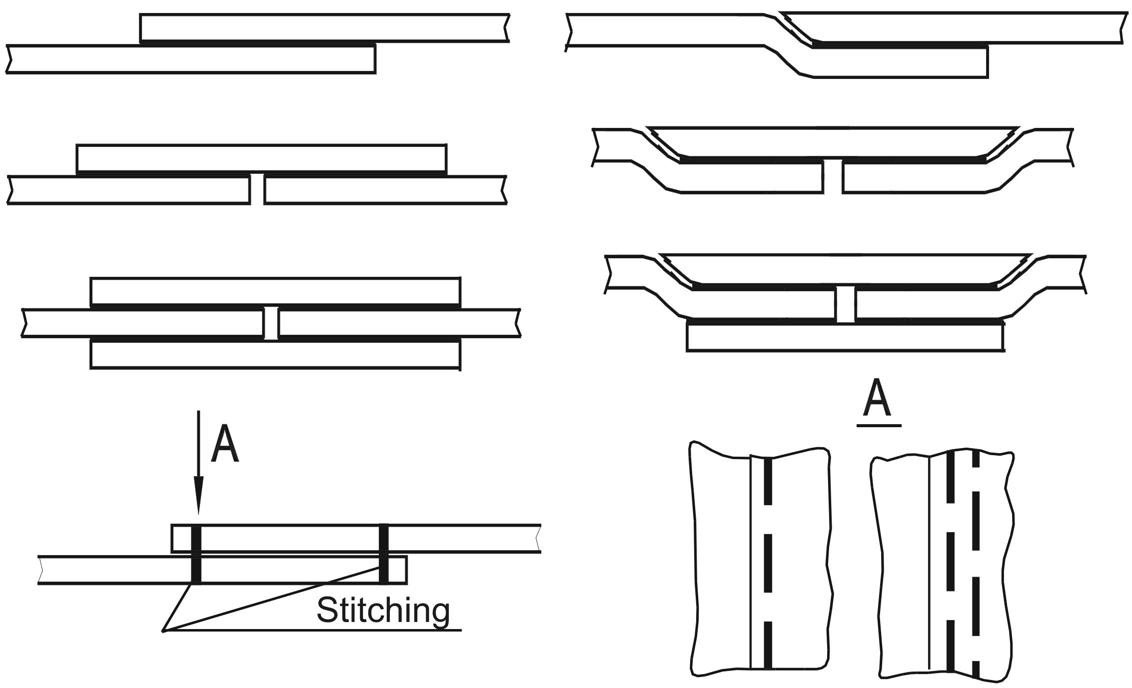


Fig. 9.5. Types of lap adhesive joints

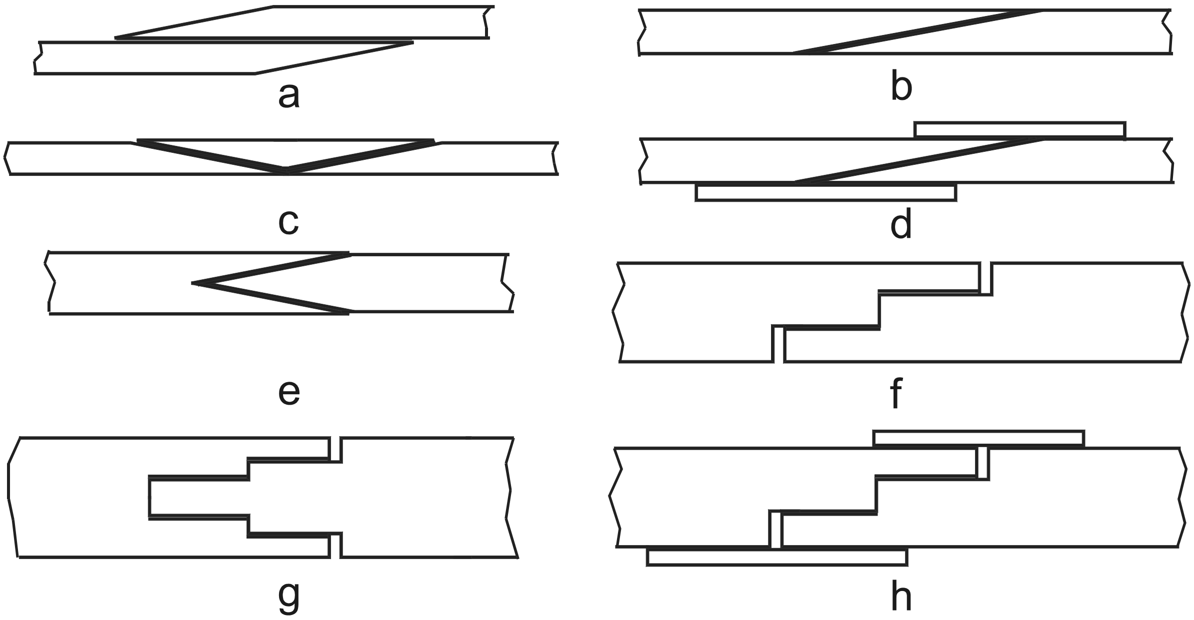


Fig. 9.6. Structural solutions of adhesive joints

Checking-up questions

1. What types of structural joints are used in aircraft articles and units arrangement?

2. Possible structural joints classification and recommendation on application?

3. Main distinctive features of discrete mechanical and adhesive joints.

4. Draw analytical model for design of single-lap multi-row mechanical joint? What are main structural parameters of this model?

5. What are the main strength conditions designer has to satisfy at each section of a joint considered?

6. What does the notion of composite article compliance and mechanical fastener compliance mean?

7. What structural parameters of composite in a joint the bearing strength of composite depends on?

8. What is the analytical model of pure adhesive joint design?

9. What does "classical" and "Volkersen" model of joining layer in adhesive joint mean? How to calculate them?

10. What main assumptions are used for adhesive joints design and checking analysis?

11. How maximum shear stress in adhesive layer depend on joint length and geometrical and rigidity parameters of joining articles?

12. What is the main difference in shear stress calculation between one-dimensional and two-dimensional models of adhesive layer? Draw graphical dependence.

13. What does "critical length" and "limited length" of adhesive joint mean?

14. How to take into account influence of thermal field on adhesive joint stress state?

15. What structural solutions of adhesive joints are used in aircraft structures?

16. What structural methods of shear stress reduction at the edges of adhesive joints are used?

Theme 10. REQUIREMENT TO DESIGN DOCUMENTATION PREPARATION FOR ARTICLES AND ASSEMBLIES MADE OF COMPOSITES

Unified system of design documentation joints state or branch standards, guarantees completeness of technical documentation, rules of approval, registration, storage etc. Unified rules for execution drawings and other design documentation stipulate unified technical language for all enterprises and organizations of any branch of industry.

Application of composites in aircraft structures demands designation of special requirements besides classical designations of material, geometry and type of processing. For composites we have to mention article structure at high level of visualization and definiteness. Nowadays each huge enterprise involved in composites manufacturing creates its own local standard that can cause difficulties at technical information interchange.

10.1. General notions

For further consideration let's mention main notions used for composite article drawing preparation. These notions are based on Unified System of Design Documentations and experience of "Antonov" Research and Scientific Corporation [11]:

Composite article (package) – means structural element consisting of number of monolayers, grouped by functional features and correspondent stacking sequence (reinforcing angles).

Assembly – structural element consisting of two and more composite (or metal) components manufactured by means of assembling operations (bonding, co-curing etc).

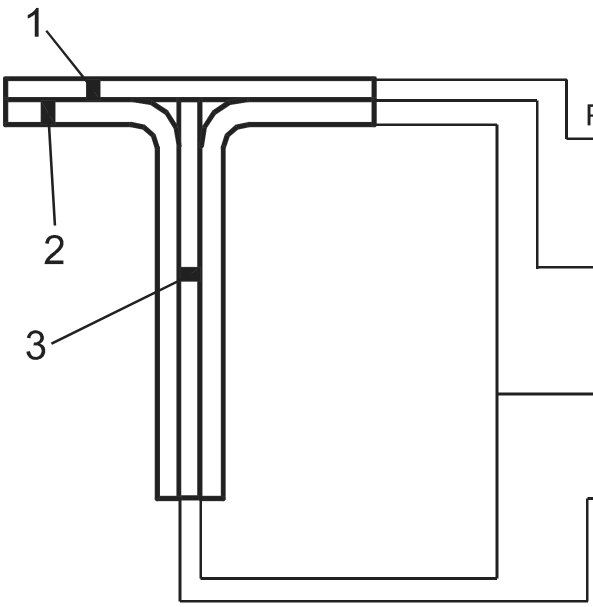
Such approach permits to designate clearly composite package, article and assembly keeping their separated numbering.

10.2. Preparation of drawing for articles made of composites by laying-up method

Generally article made of composite consists of prepreg layers (it doesn't matter what laying-up scheme was used – "dry" or "wet"). These packages are designated by means of two lines with distance between them at least of 2 mm (Fig. 10.1). **Continuous layers numbering** is used for exact package and means sequence of layers laying-up. Number of position is drawn from shaded rectangle with side length 2…3 mm.

It is possible to use base system (reference surface) for designation of origin of layers numbering (especially for automatized design process) (Fig. 10.2).

If article is geometrically symmetrical and layers stacking sequence is not important it is possible not to show reference (base) surface.

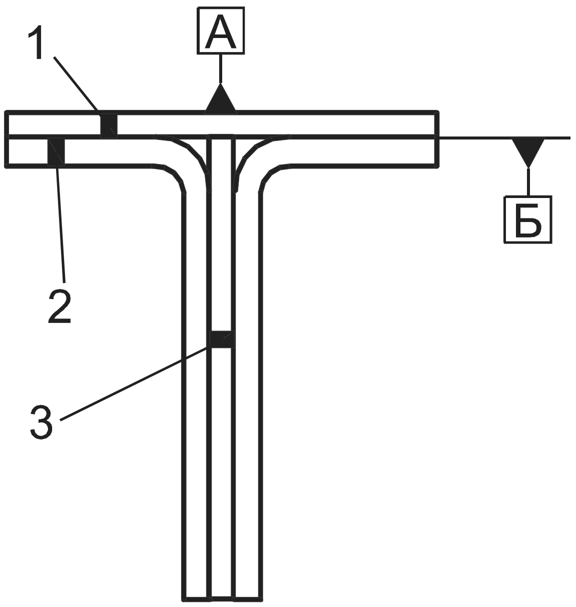


Position	Layer number	Stacking angle	Material
1	1	0°/90°	Hexel T-190-12”-F522
	2 3	+45° -45°	UD CS 50
2	1 2 3 4	0° +45° -45° 90°	UD CS 50
3	1 2 3	+45° 0° -45°	UD CS 50

Fig. 10.1. Drawing of article manufactured by lay-up method

After column “Material” column “Notes” can be presented. In this column auxiliary correction for proper laying-up is mentioned.

One of the article view has to contain designation of coordinate system (especially in laying-up plane). Axes arrows of this coordinate system shows directions of layers laying-up (Fig. 10.3). Plane of layers laying-up means the plate of projection of forming surface on which package is laid. If packages are laid on intersected planes (for example, on perpendicular planes) we should show coordinate systems on every surface.



Position	Layer number	Stacking angle	Material
1	1 - А	0°/90°	СВМ
	2 3	+45° -45°	ЭЛУР П - 0,1
2	1 - Б 2 3 4	0° +45° -45° 90°	ЭЛУР П - 0,1
3	1 2 3	+45° 0° -45°	ЭЛУР П - 0,1

Fig. 10.2. Drawing of article manufactured by lay-up method (method of reference surfaces)

Arrows direction has to coincide with articles main axes of symmetry or assembly axes. Main axes are theoretical axis, article contour, plane or lines of their intersection. This information has to be shown on manufacturing jig.

It is possible to show directions of laying-up for separate articles on article section, article cut, local view (Fig. 10.4). This method of designation is recommended to use when object has quite simple structure or when structural element lays out of base laying-up plane.

Only one laying-up angle has to be shown for unidirectional materials (see Fig. 10.1, Fig. 10.2). For woven materials one has to define two angles, corresponding to fill fibers direction (FFD) and weft (abb) fibers direction (Fig. 10.5).

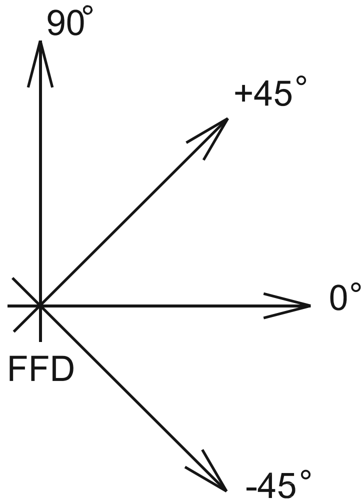


Fig. 10.3. Coordinate system for laying-up (FDD – fill fiber direction)

It is allowable to use scaling (even non-standard) showing package laying-up scheme. In this case reference dimension and exact layers quantity in the package have to be written on a drawing (Fig. 10.6).

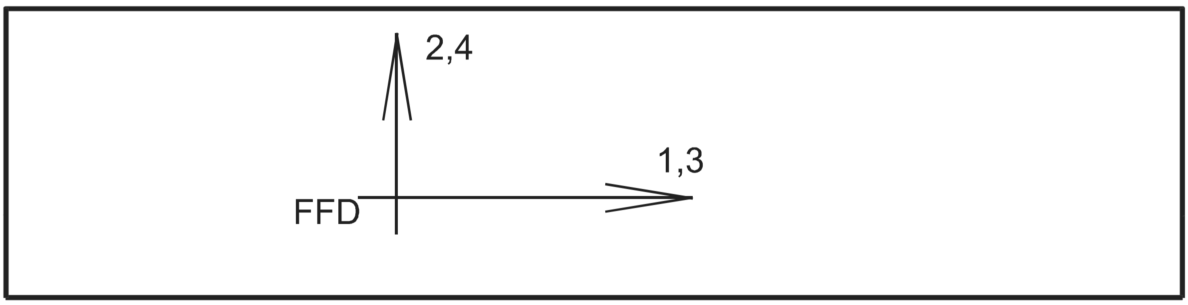


Fig. 10.4. Designation of laying-up direction on local view or cut

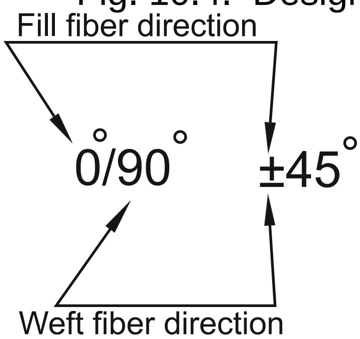
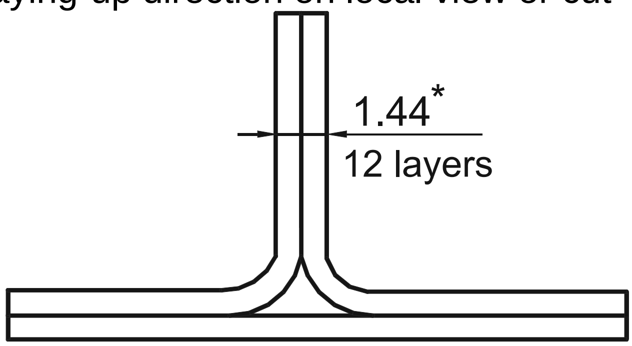


Fig. 10.5. Designation of laying-up direction



* - Reference dimension

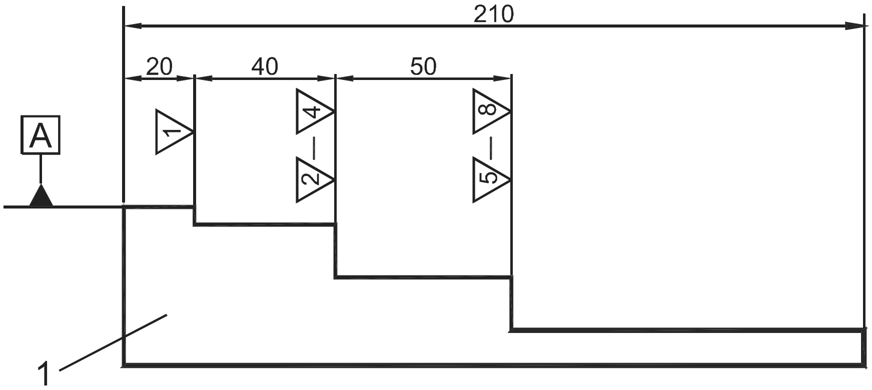
Fig. 10.6. Designation of reference dimension

When one layer or layers of package consist of several semi-finished articles laid-up with different angles it is allowable to mention in a table several laying-up angles, separating them with semicolon; moreover auxiliary views have to be shown at this drawing with designation “Scheme of laying-up for layer #...package position #...” (see Appendix). Each view has to contain designation of material fill fiber direction to escape indefinites of data shown in the table. It is recommended to use two thin lines disposed inside contour of each element not touching element contour (see Appendix).

If designer pans to use wedge-like transition of article contour (for example, drawing of wing spar cap with variable height) it is allowable not to draw separated packages but show boundaries of definite layers cut with sign ▽ disposed on extension line (Fig. 10.7).

Drawing of articles with complicated structure has to contain schemes of individual layers cutting (Fig. 10.8).

Separate drawing has to be prepared for separately manufactured article.



Position	Layer number	Stacking angle	Material
	1 - A	0°/90°	CBM
1	2 3 4 5 6 7 8 9 10	0°/90° ±45° ±45° ±45° ±45° ±45° ±45° ±45° 0°/90°	УТ - 900

Fig. 10.7. Scheme of layer cutting

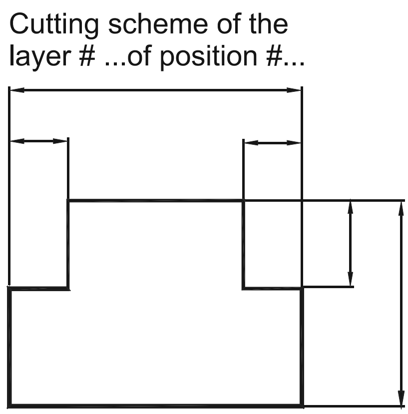


Fig. 10.8. Scheme of layers cutting

Drawings devoted to experimental articles consisting of several simple articles can contain laying-up schemes of different articles (separate drawing can be escaped). In this case assembly drawing of entire unit should contain mentioned laying-up schemes and technical requirements concerning to individual articles manufacturing. For example: "Panel skin position…, consisting of packages 2, 3, 4 and angle section position… consisting of packages 8, 10, 12 are manufactured separately by means of simultaneous pressing of correspondent packages".

For design documentation of complicated assemblies it is allowable to prepare separate drawing for several articles only but mention in technical specifications auxiliary note: «After laying-up operation article has to be transported to further assembly to be partly cured (not fully cured) ».

Drawings have to contain technical specifications written in special field. These specifications (requirements) have to contain necessary information for proper manufacturing, for example, one is shown in the Table 10.1.

Table 10.1. Example of technical specification

<table>
<tr><th>#</th><th>Note</th><th>Condition of application</th></tr>
<tr><td>1</td><td>Theoretical drawing</td><td rowspan="3">Noted if necessary</td></tr>
<tr><td>2</td><td>Unspecified limited deviations of dimensions by quality class H16, h16, IT16/2</td></tr>
<tr><td>3</td><td>Manufacture beginning from external (internal) contour by instruction # …</td></tr>
<tr><td>4</td><td>FFD – Fill fibers direction</td><td>Laying-up scheme (0°, 90°) has to be shown at main view</td></tr>
<tr><td>5</td><td>Fabric overlapping in seams of article position #... 20 – 25 mm
Seam displacement between neighboring layers not less than 100 mm
Keep gaps at laying-up article position #... - 0 – 1 mm
Neighboring layers overlapping is not allowed</td><td>Noted if article dimensions or reinforcing scheme not allow to manufacture it from entire piece of reinforcing material
Noted if necessary or if this information is absent on correspondent drawing field</td></tr>
<tr><td>6</td><td>Checking and acceptance according to …</td><td>Technical instruction is referred</td></tr>
<tr><td>7</td><td>Conduct testing of strength σ_B, σ_{-B} etc in direction 0° (90°) by reference specimens with reinforcing scheme for article position # …</td><td rowspan="5">Noted if necessary</td></tr>
<tr><td>8</td><td>Non-destructive control by instruction #...</td></tr>
<tr><td>9</td><td>Apply coating on…
external surface…
internal surface…</td></tr>
<tr><td>10</td><td>Apply label and stamp with paint by technical instruction #...</td></tr>
<tr><td>11</td><td>Reference dimensions</td></tr>
</table>

One should note that aircraft structures possess a large amount of structural elements differing by some dimensions only (for example, rib caps, rods, plates, bulkhead webs etc). In these cases preparation of group drawings can be recommended (Fig. 10.9).

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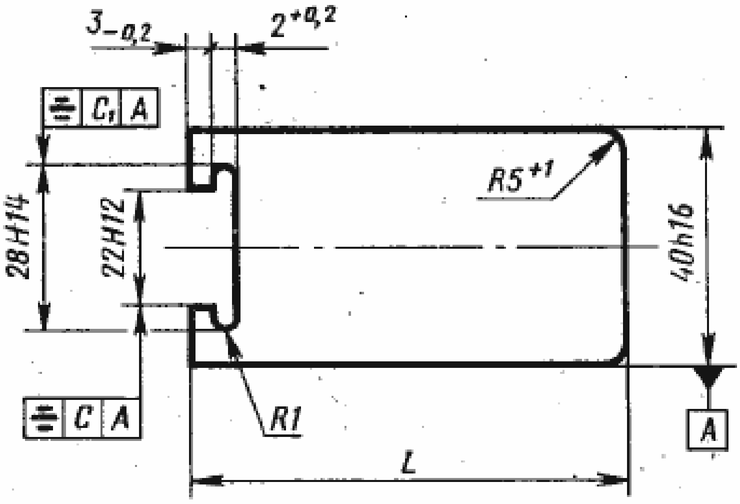


Размеры в мм

Designation	L		C	C_1	Mass, kg
	Nominal	Max. deviation			
АГБВ.838644.012	70	h15	0,1	0,2	0,006
-01	25,5	h16			0,002
-02	30		0,2	0,4	0,003
-03	36	-0,3			0,003
-04	50	-1	0,3	0,6	0,005

Изм.	Лист	№ докум.	Подп.	Дата
1	Зам.1	АГБВ.36		20.07.81
Разраб.		Новикова		01.07.81
Пров.		Телик		06.05.81
Т.контр.		Пахомов		08.07.81
Н.контр.		Назаров		11.07.81
Утв.		Кулыгин		18.07.81

АГБВ.838644.012

Gasket

Лит.	Масса	Масштаб
О	See table	—

Лист | Листов 1

Packing board Б-2,0 ГОСТ 4194—83

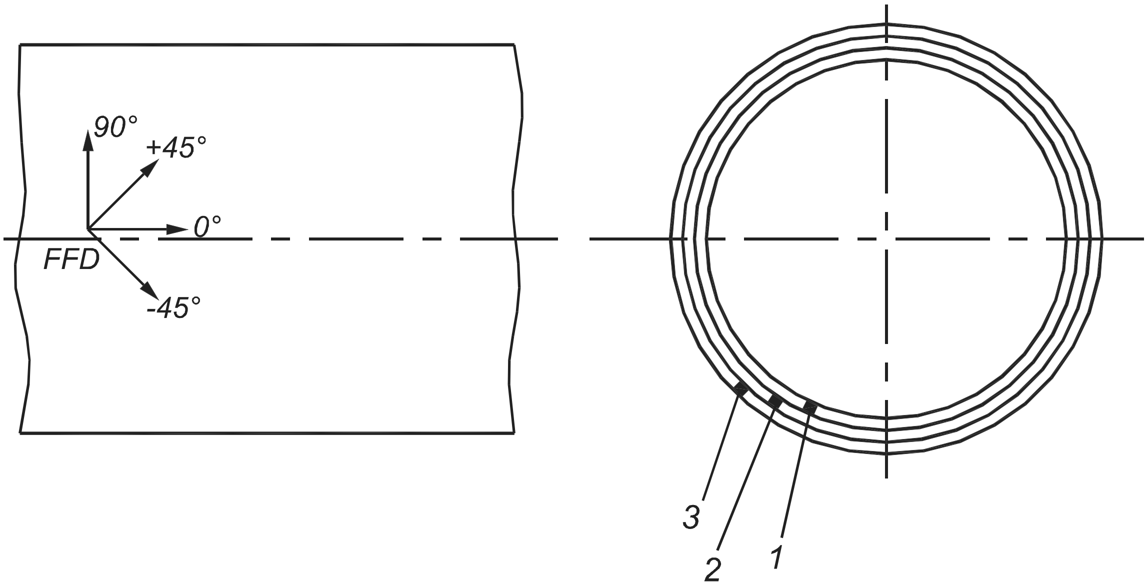
Перв. примен. | Справ. № | Подп. и дата | Инв. № дубл. | Взам. инв. № 1023 | Подп. и дата 20.07.81 | Инв. № подл. 1423

Копировал | Формат А4

Fig. 10.9. Example of group drawing

10.3. Preparation of drawing for articles made of composites by winding method

Distinctive feature of winding process is obtaining symmetrical articles, therefore at any cross section perpendicular to central axis all layers are oriented at the same angle. Moreover guiding device makes at least several the same strokes orienting tapes (fibers) etc. That is why no sense to distinguish separate layers in composite package and write layer number in the table (in other words layer number coincides with package number) (Fig. 10.10). Coordinate system is referred to shell external surface. Packages number correspond to winging (laying) sequence.



Pos.	Layer number	Stacking angle	Material
1	1	+45°; -45°	ЛУ-2 ТУ6-06-И34-83
2	2	90°	Т10-80(115) ГОСТ19170-73
3	3	+45°; -45°	ЛУ-2 ТУ6-06-И34-83

or

Position	Stacking angle	Material
1	+45°; -45°	ЛУ-2 ТУ6-06-И34-83
2	90°	Т10-80(115) ГОСТ19170-73
3	+45°; -45°	ЛУ-2 ТУ6-06-И34-83

Fig. 10.10. Designation of article made by winding process

One should note that for layers of fabric wounded laterally (with 90°). It is necessary to mention the information about reciprocal position of beginning and finishing of winding (Fig. 10.11). For layer of fabric or tape wounded (or laid-up) with angle ±φ° it is necessary to show winding sequence (at correspondent drawing field or in technical specifications) (Fig. 10.12).

Majority of pressure vessels or tubes are generally wounded on non-extracting mandrels (internal shells, liners, carriers, connection-pipes etc), therefore drawings for such articles require preparation of assembly drawings.

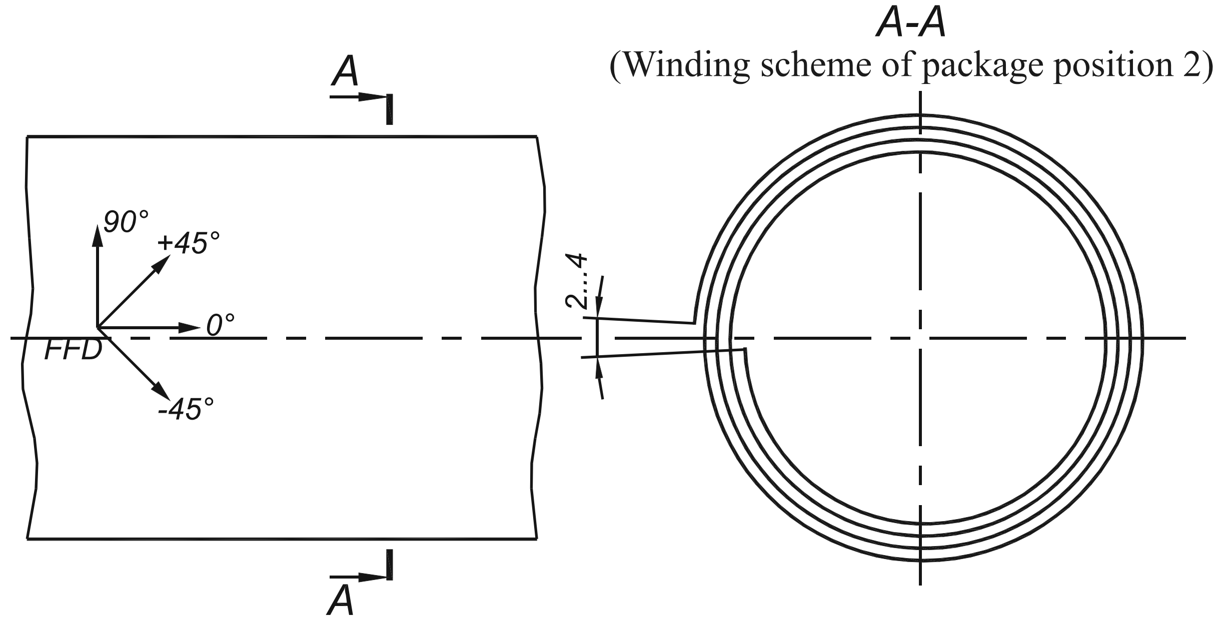


Fig. 10.11. Lateral winding scheme

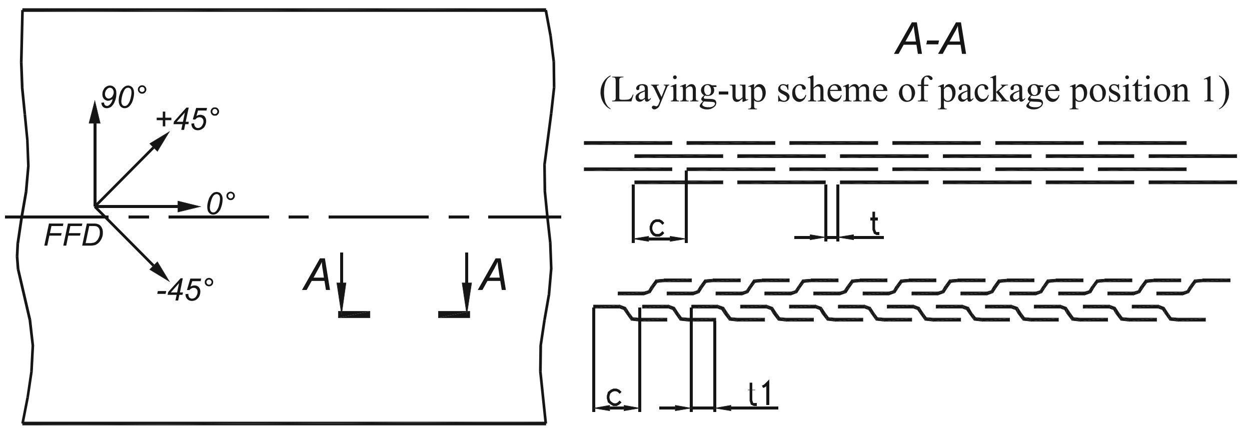


Fig. 10.12. Symmetrical winding scheme

10.4. Assembly drawings of composite articles

Main distinctive feature of assembling drawings of articles made of composites is strong requirements (has to be mentioned in technical specification) what scheme of separate articles joining should be selected – “wet+wet” (forming), “wet+dry” (co-forming or co-curing), “dry+dry” (or bonding).

If assembly providing by “wet+wet” scheme contains inserts (embedded objects like foam plastic bars, wooden bars, tows etc) which can’t be classified as “material” it is recommended to mention all necessary information (i.e. dimensions, package structure, laying-up scheme, cutting schemes etc) for forming articles in drawing specification field. For such kind of drawings separated drawing aren’t prepared. Working drawings are necessary to prepare for articles which were manufactured previously.

If assembly contains several geometrically similar inserts (honeycomb or stiffeners) and one of them is large than other it is possible to mention all necessary dimensions at assembly drawing specifications (excluding separate working drawing for each individual insert). If each insert consist of separate components it is necessary to show places of their joining and joining methods. For honeycomb (as quite anisotropic structure) direction of honeycomb extension has to be shown.

In some cases composite article can be mentioned in specification as "drawingless". Resin can be written to the field "Materials".

10.5 Examples of composite panel drawings

Object 1 – panel with stiffeners obtained by laying-up with consequent forming (Fig. 10.13–10.16). Metal or previously manufactured non-metal inserts (embedded elements) are absent. Therefore one can consider this article as composite one and requires working drawing preparation.

Object 2 – sandwich panel with honeycomb obtained by forming (Fig. 10.17, 10.18). This element has to be considered as assembly because honeycomb element is prepared previously (separately), then bonding operation is conducted. Thus assembly drawing is required.

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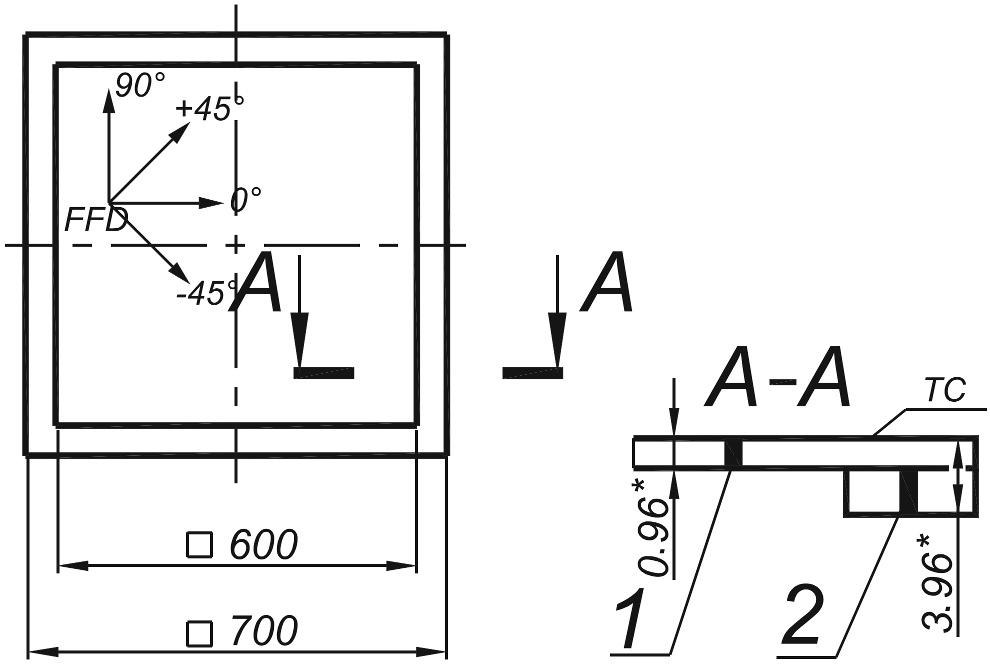

Pos.	Layer #	Stacking angle	Material
1	1...4	±45°;∓45°	Т-10-80(70) ГОСТ19170-73
2	1...30	0°; 90°	ЛСК-ВМ-0,1х100-76 ГОСТ19170-73

1. TC - theoretical contour.
2. FFD - fill fiber direction.
3. * - reference dimension.
4. Shown for layer # 1, for other odd layers dimension has to be increased consequently by 410 mm.
5. Shown for layer # 2, for other evenlayers dimension has to be increased consequently by 410 mm.
6. Place neighbouring layer line-on-line, maximum gap up to 1mm is allowable.
7. Conduct non-destructve test of article quality by instruction #...
8. Label with red paint … on internal panel surface.

Изм.	Лист	№ докум.	Подп.	Дата
Разраб.		Иванов		
Пров.		Петров		
Н. контр.				
Утв.		Сидоров		

A1.407.004.06.31.000

PANEL

Литера	Масса	Масштаб
У		1:10

Лист 1 Листов 3

ХАИ

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Формат А4

Fig. 10.13. Composite panel drawing

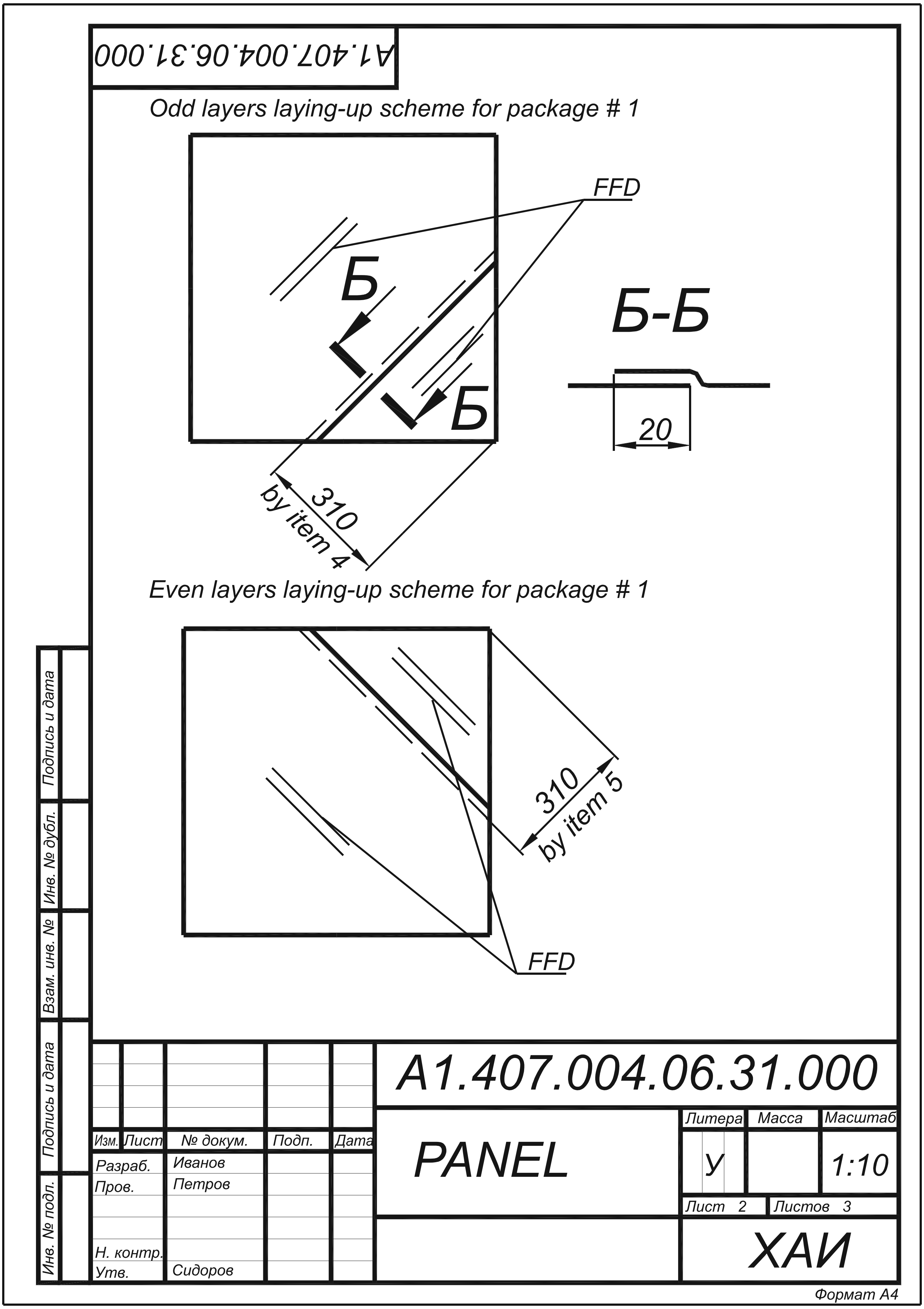

Fig. 10.14. Skin layers laying-up scheme

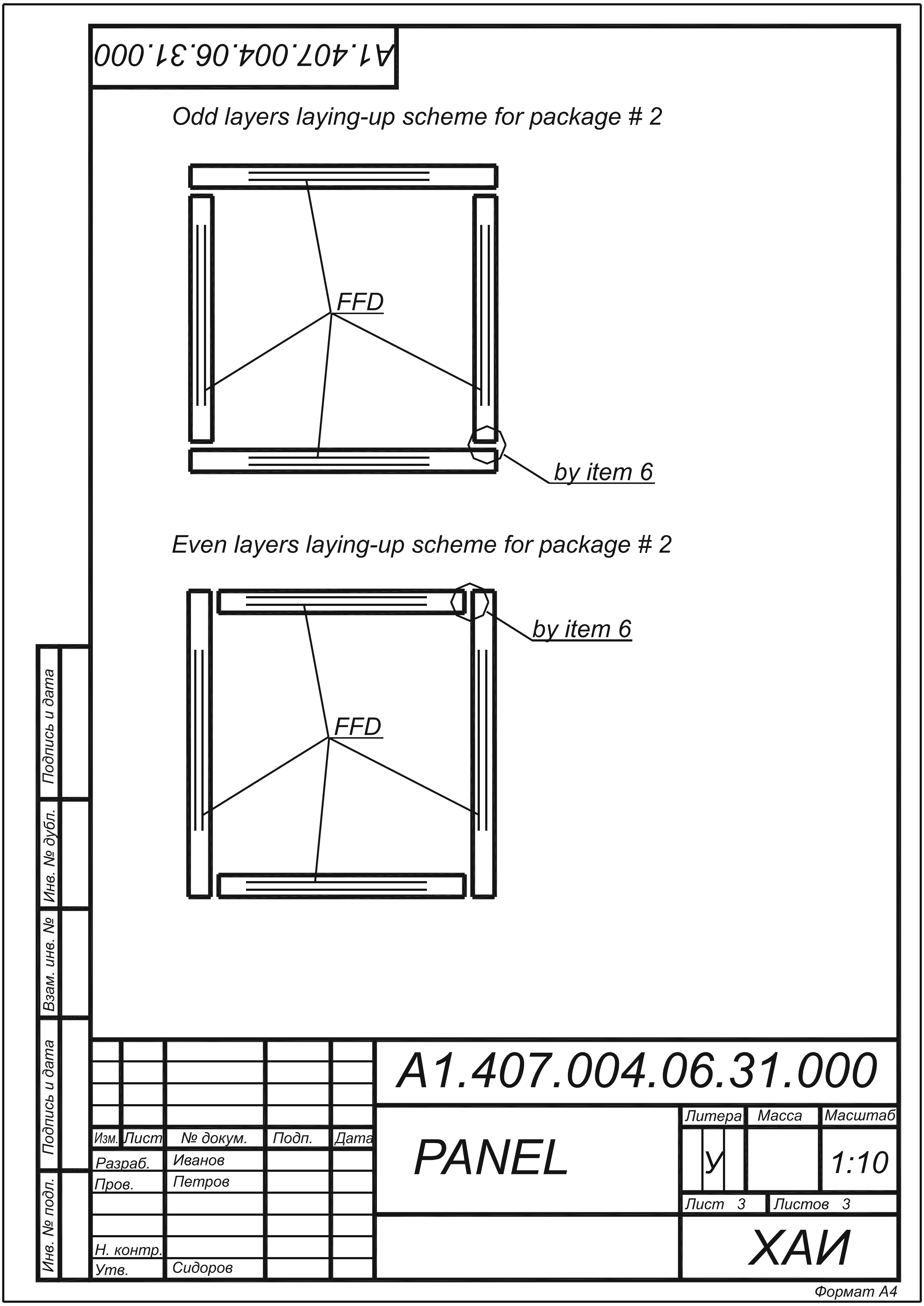

Fig. 10.15. Stiffeners layers laying-up scheme

Инв. № подл.	Подпись и дата	Взам. инв. №	Инв. № дубл.	Подпись и дата

Format	Zone	Pos.	Designation	Title	Quantity	Mass	Material	Примечание
				<u>Documentation</u>				
*			A1.407.004.06.31.000	Working drawing	1	0.8		3 sheets A4
				<u>Articles</u>				
								Laying-up scheme
DL		1	A1.407.004.06.31.001	Skin	1	0.6	Fabric T-10-80(70) ГОСТ19170-73	according to
							* 4 layers 700X700	package pos. 1
DL		2	A1.407.004.06.31.002	Stiffener	1	0.2	Tape ЛСК-ВМ-0,1X100-76	
							ГОСТ19170-73	
							30 layers 700X50	
							30 layers 600X50	

Изм.	Лист	№ докум.	Подп.	Дата	A1.407.004.06.31.000		
Разраб.		Иванов			PANEL	Литера: У	Лист: 1 / Листов: 1
Пров.		Петров				ХАИ	
Н. контр.							
Утв.		Сидоров					

Формат А4

Fig. 10.16. Specification for panel manufacturing

DL – DrawingLess (separate drawing is absent)

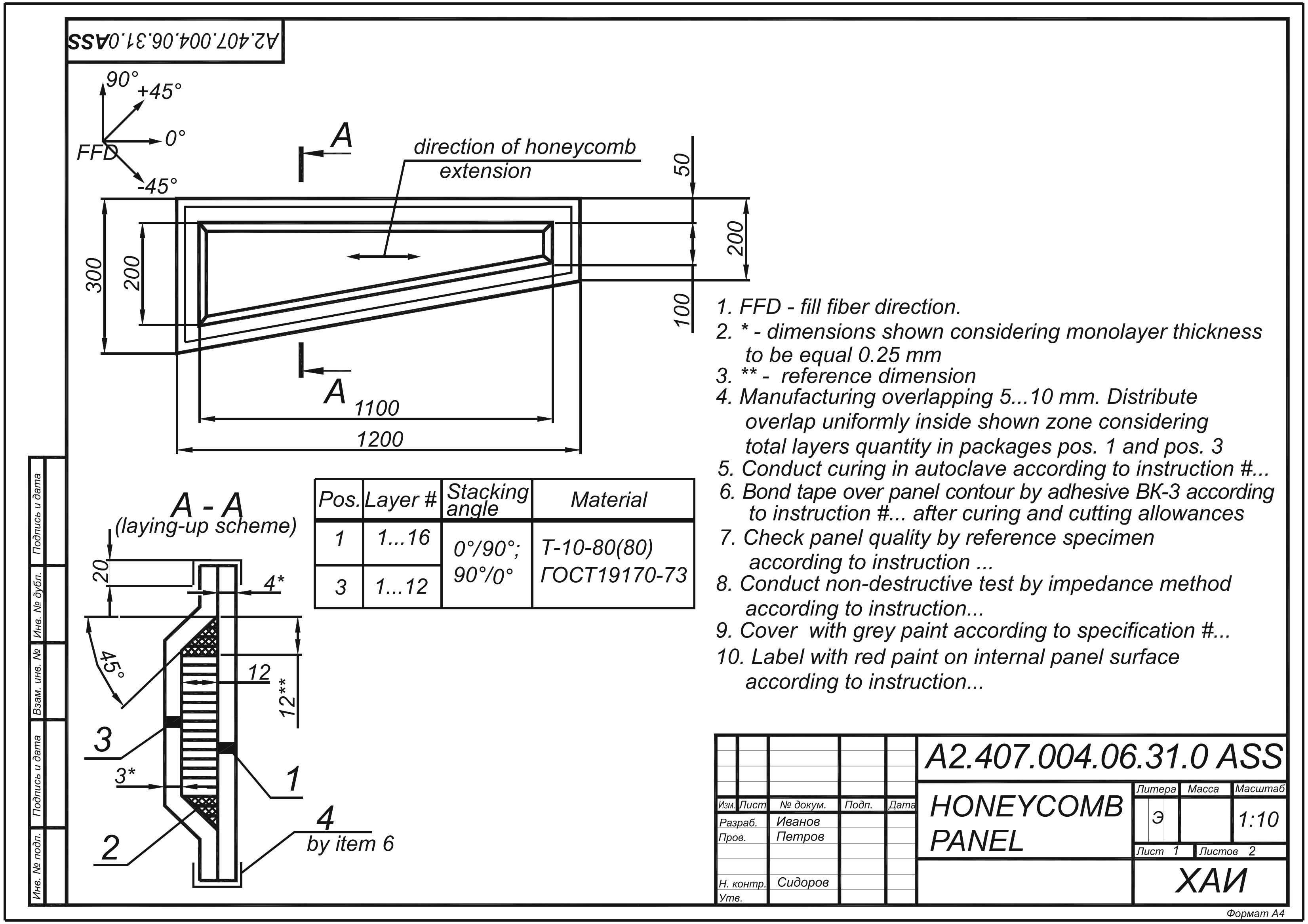

Fig. 10.17. Honeycomb panel assembly drawing

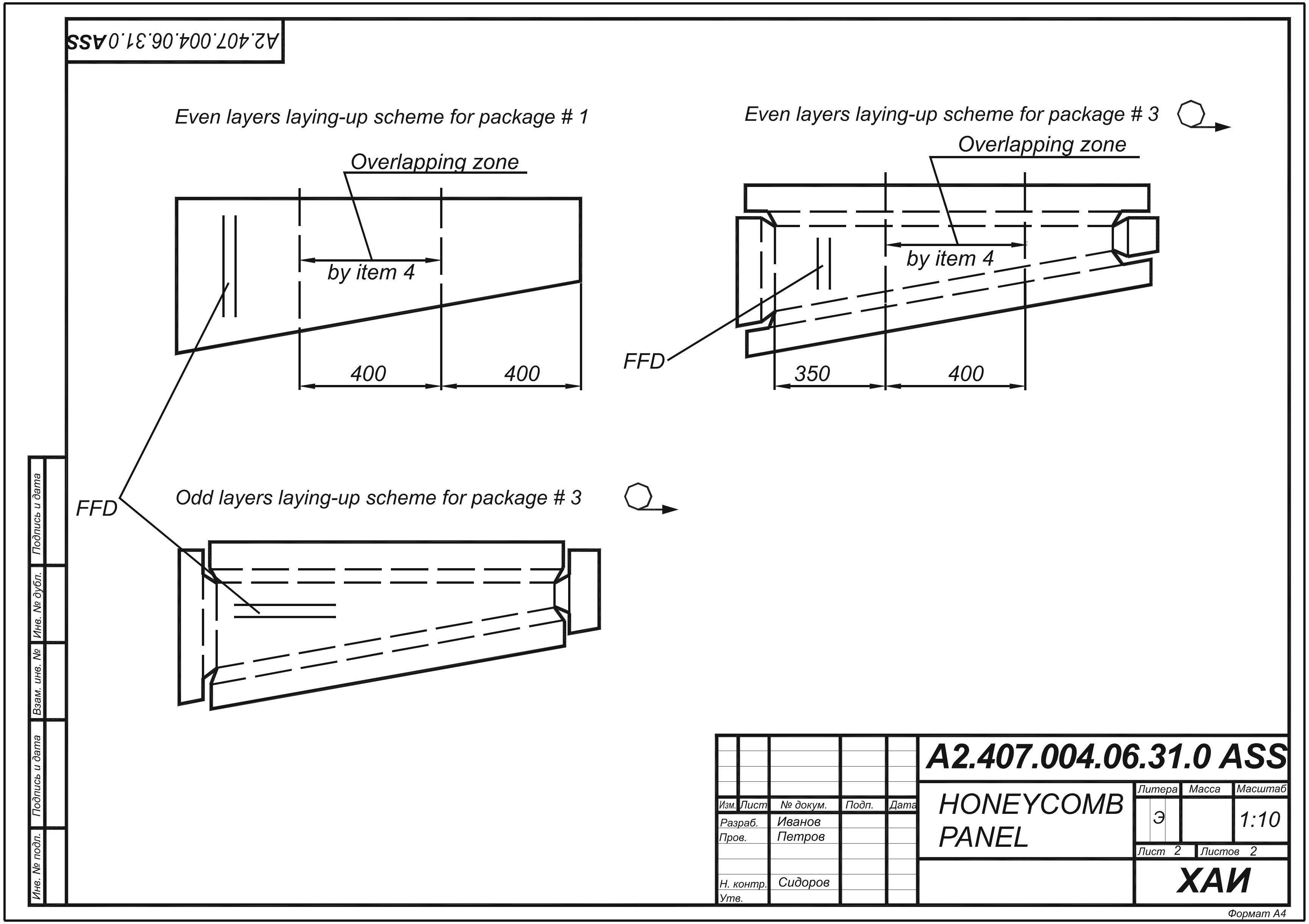

Fig. 10.18. Scheme of panel skin laying-up

Checking-up questions

1. Give the definition of composite article (package).
2. Give the definition of assembly.
3. What main concepts of metal-composite articles assembling can be shown on the design drawings?
4. What does "reference plane" used for composite monolayers enumeration mean? How to select it for exact article?
5. How to show direction of article elements reinforcing?
6. What types of technical requirements are generally shown on composite drawing?
7. What does "asterisk" (*) sign near dimensions designation mean?
8. For what purpose is the "triangle" sign used on drawings of composite articles?
9. What are distinctive features of drawing composite articles produced by winding process?
10. How to show honeycomb and foam filler of sandwich panel on design drawing of composite panel?

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Навчальне видання

Тараненко Ігор Михайлович

ПРОЕКТУВАННЯ АГРЕГАТІВ ЛІТАКІВ З КОМПОЗИТІВ

(Англійською мовою)

Редактор О. В. Галкін
Технічний редактор Л. О. Кузьменко

Зв. план, 2015
Підписано до друку 04.02.2015
Формат 60Х84 $^{1}/_{16}$. Папір. офс. № 2. Офс. друк
Ум. друк. арк. 7,7. Обл.-вид. арк. 8,75. Наклад 200 пр.
Замовлення 67. Ціна вільна

Видавець і виготовлювач
Національний аерокосмічний університет ім. М. Є. Жуковського
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Свідоцтво про внесення суб’єкта видавничої справи
до Державного реєстру видавців, виготовлювачів і розповсюджувачів
видавничої продукції сер. ДК № 391 від 30.03.2001