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MATHEMATICAL MODEL OF CYLINDRICAL MANIPULATOR

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Developing of methods of manipulator driving forces and moments determination is important tasks of robotics. It is necessary for the reasonable choice of drives ensuring the mobility of the robot's mechanical system [1].

The work object of analysis is driving forces and moments (hereinafter, generalized forces) in kinematic pairs of a manipulator operating in a cylindrical coordinate system (Fig. 1). To simplify calculations, it is assumed that the center of gravity of the load is located on the axis of link 3, point A in fig. 1. The trajectory and law of movement $v_A(t)$ of the working body of the manipulator are considered given. The inverse problem of kinematics for determining the movements of individual links of the mechanism is solved analytically. Generalized forces are determined using Lagrange equations of the second sort [2]:

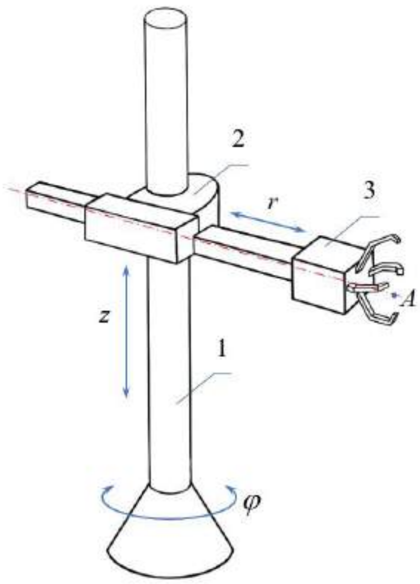


Fig. 1.

$$\begin{cases} I_{Oz}\ddot{\varphi} + 2m_3\left(r - l_{AC_3}\right)\dot{r}\dot{\varphi} + 2m_l r\dot{r}\dot{\varphi} = Q_1, \\ (m_2 + m_3 + m_l)\ddot{z} + (m_2 + m_3 + m_l)g = Q_2, \\ \left(m_3 + m_l\right)\ddot{r} - m_3\left(r - l_{AC_3}\right)\dot{\varphi}^2 - m_l r\dot{\varphi}^2 = Q_3. \end{cases} \quad (1)$$

де

$$q_1 = \varphi;\ q_2 = z;\ q_3 = r.$$

Mass moment of inertia I_{Oz} is function of horizontal coordinate *r*

$$I_{Oz}(r) = I_{1z} + I_{2z} + I_{3Cz} + m_3\left(r - l_{C3}\right)^2 + m_l r^2.$$

Fig. 2 presents the function $I_{Oz}\left(r\right)$. It can be noticed that there is a position where the function has a minimum. The position corresponds situation in which common mass center of link with the load is on the axis of link 1 rotation.

In the first stage of the solution, the friction forces were neglected. The reactions in kinematic pairs were obtained by the kinetostatics method [3]. Later they were used for the second stage of the solution when sliding friction forces were taken into account.

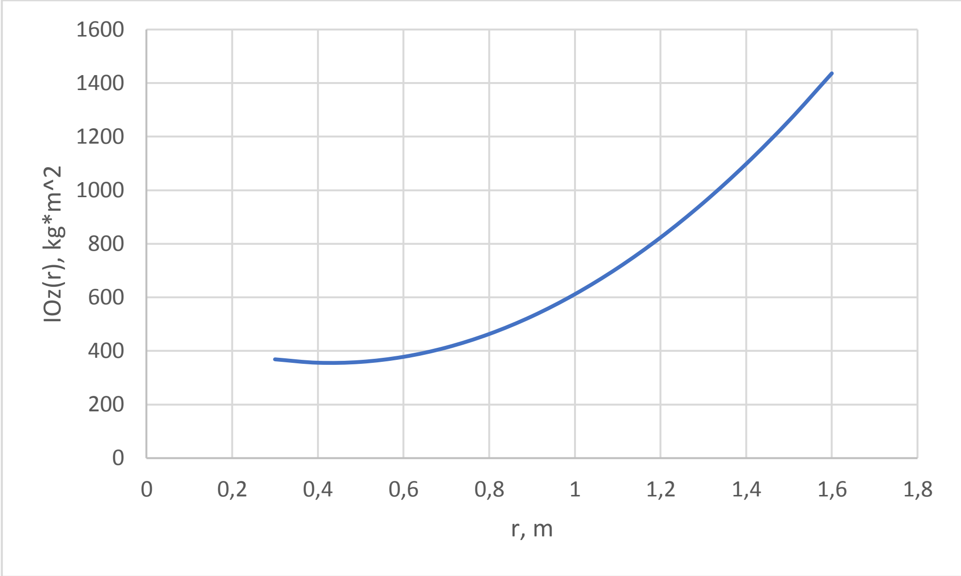
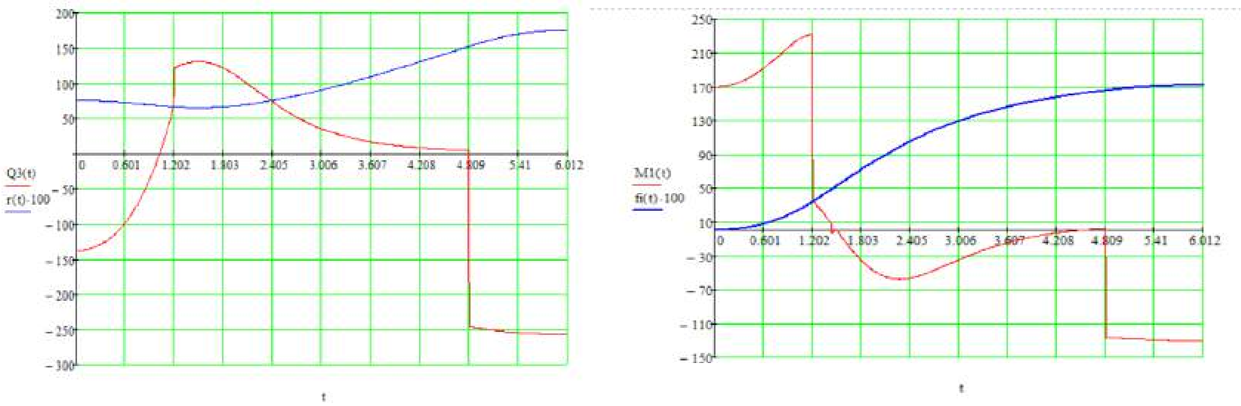


Fig. 2

For some problems, the dependences of generalized forces on time were obtained. Fig. 3 shows the generalized forces and the coordinates to which they correspond, provided that the mass of the load is $m_l = 450$ kg, the mass of the third link is $m_3 = 340$ kg: a) the horizontal coordinate of the working body and the driving force Q_3, N acting on link 3, b) the angle of rotation φ and the driving moment $Q_1 = M_1$, Nm, which ensures the rotation of links 2 and 3.



a) b)

Fig. 3.

The influence of friction forces on the driving force Q_3 acting on the third link is shown in fig. 4, the coefficient of friction in the kinematic pair 2-3 is $\mu = 0.1$: thin solid line - calculation with friction, thick line - without friction. Additionally, in Fig. 4, the dashed line shows the scaled speed of link 3 relative motion. As we can see, it is the forces of friction that largely determine this driving force.

The results make it possible to reasonably choose link drive motors. The mathematical model can also be used to choose the trajectory of the load, which is optimal from the point of view of the load on the mechanism.

The fact that the inverse problem of kinematics for this manipulator is solved analytically and unambiguously allows us to use the developed mathematical model for testing a general software complex that analyzes the kinematics and dynamics of any manipulator with three degrees of freedom.

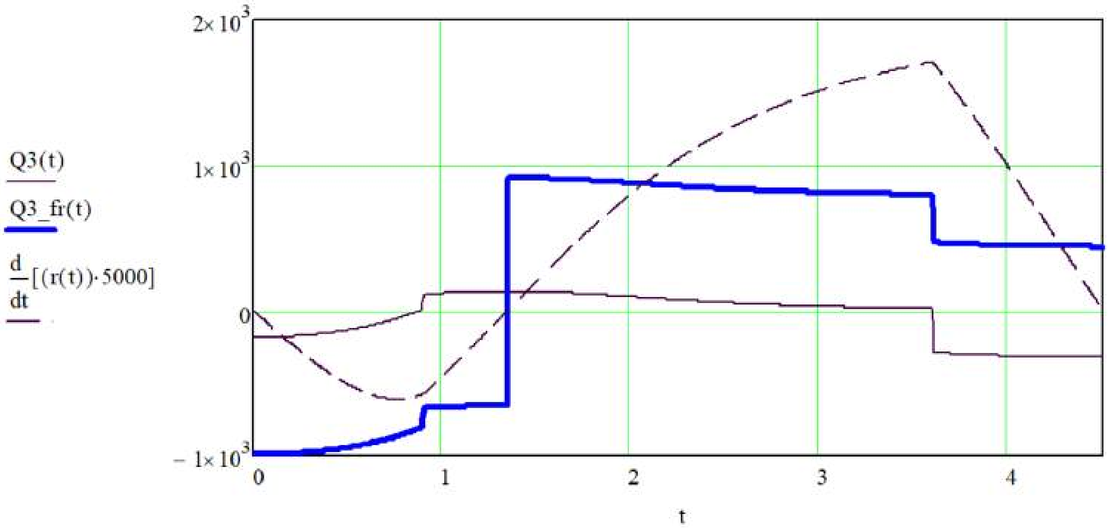


Fig. 4

The analysis of the obtained results allows us to set the problems of further development of the model:

1. Add the possibility to consider the load as a solid body, and not as a partical.
2. Include in the model damping elements that provide damping of vibrations that may occur during movement.

References

1. John J. Craig, Introduction to Robotics Mechanics and Control, Third Edition, 2005
2. Grand R. Fowles, George L. Cassiday. Analytical mechanics, Seventh Edition, 2005
3. Shigley, Josef Edward, Theory of machines and mechanisms, 1981.