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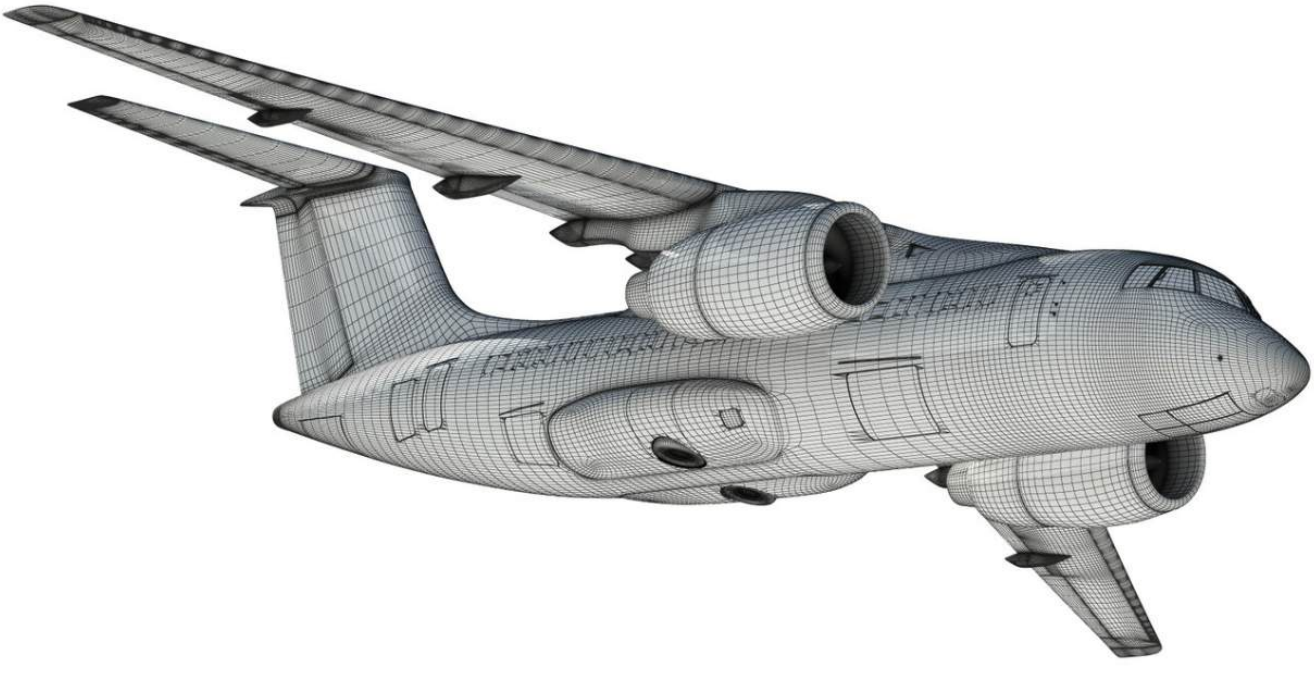
GEOMETRIC CHARACTERISTICS OF PLANE SECTIONS



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MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

National Aerospace University
«Kharkiv Aviation Institute»

GEOMETRIC CHARACTERISTICS OF PLANE SECTIONS

Textbook

Kharkiv «KhAI» 2021

UDC 539.3/.6 (07)
G37

Викладено методику визначення центра ваги, моментів інерції складених плоских фігур. Наведено таблиці довідкових даних, приклади розв'язання задач і рекомендації щодо виконання домашнього завдання.

Для студентів, які вивчають курси «Опір матеріалів» і «Механіка матеріалів і конструкцій», при самостійній роботі.

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G37 **Geometric** characteristics of plane sections [Electronic resource]: textbook / M. M. Grebennikov, V. Y. Miroshnikov, V. B. Myntiuk, D. A. Tkachenko. – Kharkiv: National Aerospace University «Kharkiv Aviation Institute», 2021. – 108 p.

The technique to define the gravity center and the inertia moments of plane sections is explicated. The tables of reference data as well as examples of solving the problems and recommendations on performing the homeworks on this topic.

For students taking the courses «Material resistance» and «Mechanics of materials and structures», for unsupervised work.

Figs 52. Tables 12. Bibliogr.: 12 items

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Introduction

When carrying out calculations of strength, stiffness and stability of structural elements such as a bar (rod), it becomes necessary to use certain geometric characteristics (properties) of bar plane cross-sections (plane figures), that is, the area of the cross-section, the position of its gravity center, static moments, axial and polar and centrifugal inertia moments, radii of gyration, axial and polar moduli of resistance need to be known.

Due to limited applied relevance, these characteristics are not covered in the general Geometry course, but they are closely associated with material and structural mechanics. That is the reason why these characteristics are always included into Material and Structural Mechanics course as an essential part.

Hereinafter, we will equate the concepts «plane figure» and «section», implying «plane cross-section of a bar».

This handbook does not cover all geometric characteristics of plane sections used in Material and Structural Mechanics course. Some of those are applied only in calculations of torsional and bending strength of a bar as well as in stability calculations.

Basic notation keys

d – diameter of circular cross-section;
b – width of a cross-section;
h – height of a cross-section;
A – area of a cross-section;
y_c, z_c – section gravity center coordinates;
S_y, S_z – static moments of a cross-section;
I_y, I_z – axial inertia moments of a cross-section;
I_{yz} – centrifugal inertia moment of a cross-section;
I_ρ – polar inertia moment of a cross-section;
W_y, W_z – modulus of resistance of a cross-section;
W_ρ – polar modulus of resistance of a section;
i_y, i_z – radius of gyration of a cross-section;
α – deflection angle of axes.

1. Static moments of the section

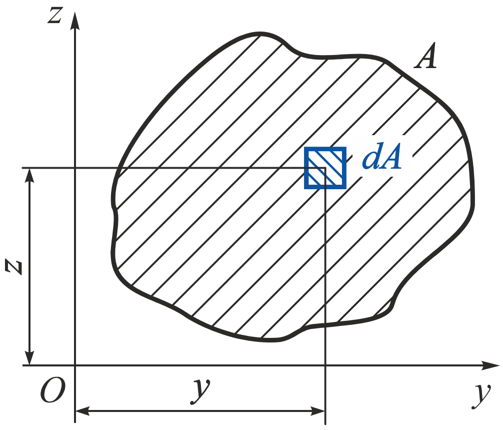


Fig. 1

Consider the cross-section of a bar A, associated with the coordinate system yOz (Fig. 1). Select the unit of area dA with the coordinates y, z.

The product of unit of area dA and distance z from axis Oy

$$dS_y = zdA \tag{1}$$

is called elementary ***static moment*** of the area about the axis Oy.

Similarly, $dS_z = ydA$ – elementary ***static moment*** of the area about the axis Oz.

Adding up values dS_y and dS_z over the cross-section area A, we obtain

$$S_y = \int_A zdA, \qquad S_z = \int_A ydA, \tag{2}$$

where S_y, S_z – are static moments of the section A about the axes y and z, respectively.

It follows from the expressions (2) that static moments can be positive, negative or equal zero and that they are measured in cubed length units (for example, m^3).

Note Static moment of compound area equals the sum of static moments of its components relatively to the same axis:

$$S_y = \sum_{i=1}^{n} S_y^{(i)}; \qquad S_z = \sum_{i=1}^{n} S_z^{(i)}.$$

2. Central axes and center of gravity of the section

Since the resultants of internal forces pass through the «center of gravity» of the section, defining the center of gravity location is an important task.

Consider the change in static moment of the section due to parallel translation of coordinate axes.

Given: A, S_y, S_z, a, b (Fig. 2).

Define S_{y_1}, S_{z_1}, i.e., the way how the static moments of the section change with parallel translation of axes needs to be determined.

By definition (2)

$$S_{y_1} = \int_A z_1 dA,$$
$$S_{z_1} = \int_A y_1 dA. \qquad (3)$$

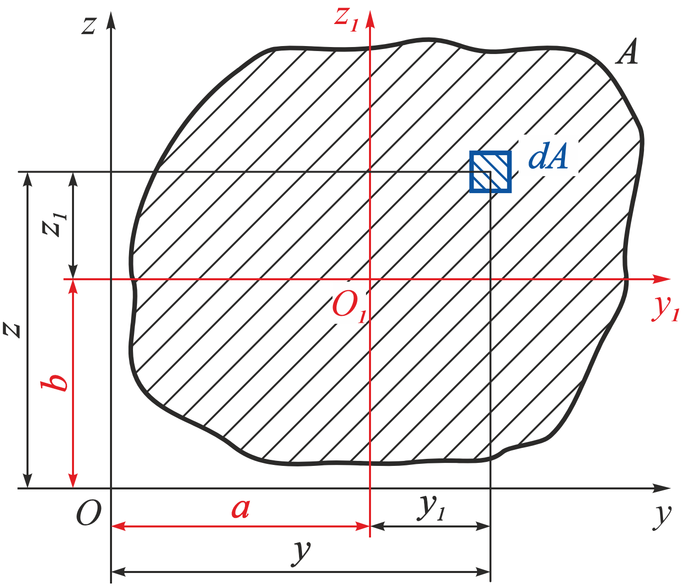


Fig. 2

The Fig. 2 shows that

$$z_1 = z - b,$$
$$y_1 = y - a. \qquad (4)$$

Substituting the values of z_1 and y_1 from the correlations (4) to the expressions (3), we obtain

$$S_{y_1} = \int_A (z-b)dA = \int_A zdA - b\int_A dA,$$
$$S_{z_1} = \int_A (y-a)dA = \int_A ydA - a\int_A dA. \qquad (5)$$

In the expressions (5)

$$\int_A zdA = S_y, \qquad \int_A ydA = S_z, \qquad \int_A dA = A.$$

All these values are given, so finally

$$S_{y_1} = S_y - bA,$$
$$S_{z_1} = S_z - aA. \qquad (6)$$

Since a and b are real numbers, there is ***the only value*** of b that gives $S_{y_1} = 0$, and there is ***the only value*** of a that gives $S_{z_1} = 0$. These values of a and b are the section gravity center coordinates, they are denoted by y_c and z_c, respectively. That is, if $a = y_c$ and $b = z_c$, then

$$0 = S_y - z_c A,$$
$$0 = S_z - y_c A. \qquad (7)$$

- The axes relative to which the static moment of the section equals zero are called the ***central axes*** of this section.
- The cross point of central axes of the section is called its ***gravity center***.

The system of correlations (7) allows solving two types of important problems:

1. Defining the static moments of the section:

a) primitive, if the values of A, z_c, and y_c are known, using the correlations

$$S_y = z_c A,\qquad S_z = y_c A; \tag{8}$$

б) compound, if the values of A_i, z_{c_i}, and $y_{c_i}$are known, using the expressions

$$S_y = \sum_{i=1}^{n} z_{c_i} A_i, \qquad S_z = \sum_{i=1}^{n} y_{c_i} A_i, \tag{9}$$

where n – is the number of elemental parts in the compound section.

The correlations (8) and (9) are the simpler form of the system (2) realization, as far as they avoid the integrating operation when the area of the section and the distance from its gravity center to the axis relative to which the static moment of section is calculated are known.

2. Defining the coordinates of the section center of gravity:

a) primitive, if the values of A, S_y, and S_z are given, using the correlations

$$y_c = \frac{S_z}{A}, \qquad z_c = \frac{S_y}{A}; \tag{10}$$

б) compound, if the values of F_i, $S_y^{(i)}$, and $S_z^{(i)}$are given, using the expressions

$$y_c = \frac{\sum_{i=1}^{n} S_z^{(i)}}{\sum_{i=1}^{n} A_i}; \qquad z_c = \frac{\sum_{i=1}^{n} S_y^{(i)}}{\sum_{i=1}^{n} A_i}, \tag{11}$$

where n – is the number of elemental parts in the compound section.

The symmetry rule. If the ***section has a symmetry axis***, then the static moment of the section about this axis ***is identically equal to zero***. Therefore:

a) a symmetry axis is always a section’s central axis;

b) a section gravity center is always located on its symmetry axis (if present);

c) if a section has two symmetry axes then its center of gravity (geometric center) is located at the cross point of the two symmetry axes.

The Fig. 3 clearly shows the correctness of the symmetry rule, as far as every unit of area dA located above the axis y with $zdA > 0$ has responsive unit of area located below the axis y with $zdA < 0$. Then

$$S_y = \int_A zdA = 0.$$

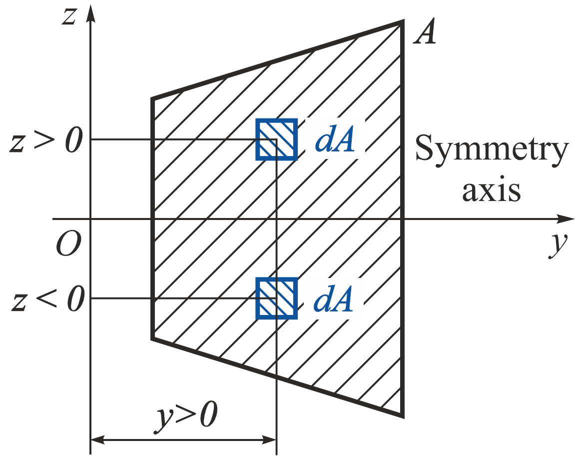


Fig. 3

3. The examples of defining the gravity center coordinates of simple sections

Example 1

Define the gravity center coordinates of the rectangle with the base b and the height h in the system of axes yOz (Fig. 4).

Given: b, h.
Define z_c, y_c.

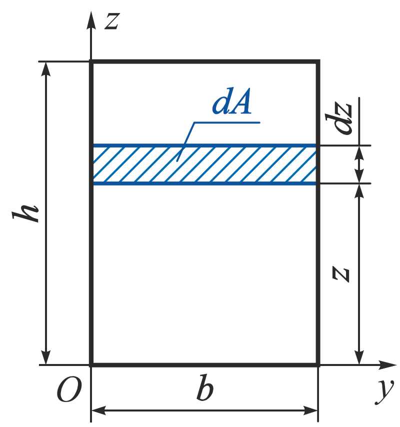


Fig. 4

Solution

Using the expression (10), write

$$z_c = \frac{S_y}{A}, \quad \text{where} \quad A = bh, \quad S_y = \int_A zdA.$$

Taking into account that $dA = bdz,$ move to the integral about the coordinate z:

$$S_y = b\int_0^h zdz = \frac{bh^2}{2}.$$

Then $\quad z_c = \frac{bh^2}{2bh} = +\frac{h}{2}; \qquad$ similarly $\quad y_c = +\frac{b}{2}.$

Example 2

Define the distance from the base b at which the gravity center of the triangle is located (Fig. 5).

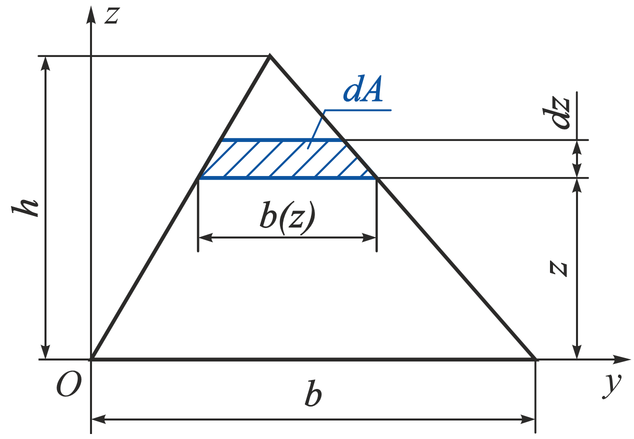


Fig. 5

Given: b, h.
Define z_c.

Solution

To solve the problem, we use the correlation (10):

$$z_c = \frac{S_y}{A}, \text{ where } A = \frac{bh}{2};\ S_y = \int_A z dA.$$

In the last expression $dA = b(z)dz$.

The similarity of triangles gives

$$\frac{b(z)}{b} = \frac{h - z}{h} \quad \Rightarrow \quad b(z) = b\left(1 - \frac{z}{h}\right), \quad \text{then} \quad dA = b\left(1 - \frac{z}{h}\right) dz.$$

Apply dA to the expression

$$S_y = \int_0^h b\left(1 - \frac{z}{h}\right) dz = b\left(\frac{z^2}{2} - \frac{z^3}{3h}\right)\Bigg|_0^h = \frac{bh^2}{6}.$$

Finally $$z_c = \frac{bh^2}{6}\frac{2}{bh} = \frac{h}{3}.$$

Example 3

Define the gravity center coordinates of the triangle with the base b and the height h in the system of axes yOz (Fig. 6).

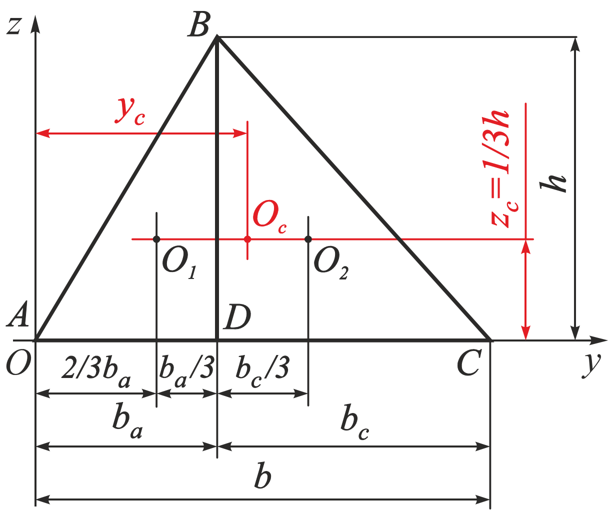


Fig. 6

Given: b, h.
Define z_c, y_c.

Solution

Decompose the triangle ABC into two right angle triangles by drawing a perpendicular line BD from the vertex B to the base AC.

Denote at Fig. 6 the gravity centers of triangles ABD и BCD, which are located at the distance $1/3$ from legs (O_1 and O_2).

Make a sum of static moments of these triangles about the axis Oz:

$$S_z = \sum_{i=1}^{2} S_z^{(i)} = \frac{b_a h}{2}\frac{2}{3}b_a + \frac{b_c h}{2}\left(b_a + \frac{b_c}{3}\right) = \frac{2{b_a}^2 h + b_c h(3b_a + b_c)}{6}.$$

Dividing this value by the area of triangle ABC we obtain the sought abscissa y_c of the gravity center O of the triangle:

$$y_c = \frac{S_z}{A} = \frac{2{b_a}^2 h + b_c h(3b_a + b)}{6\frac{bh}{2}} = \frac{2{b_a}^2 + b_c(3b_a + b_c)}{3b}.$$

Simplify this expression by substituting the value $b_a + b_c = b$:

$$y_c = \frac{2{b_a}^2 + b_c(2b_a + b)}{3b} = \frac{2b_a(b_a + b_c) + b_c b}{3b} = \frac{2b_a + b_c}{3} = \frac{b + b_a}{3}.$$

The gravity centers O_1 and O_2 of the right-angle triangles ABD and BCD lie on the line O_1O_2, which is parallel to the triangle base ABC and is located at the distance of 1/3 height. The gravity center O_c of the triangle ABC lies on the same line.

Therefore, the ordinate of the gravity center of the triangle ABC is

$$z_c = \frac{h}{3}.$$

Example 4

Define the location of gravity center of a semicircle with the radius r about the axis y (Fig. 7).

Given: r.
Define z_c.

Solution

As far as the section is symmetric about the axis z, then the center of gravity C of the semicircle lies on this axis. Therefore, only ordinate z_c of the gravity center needs to be defined.

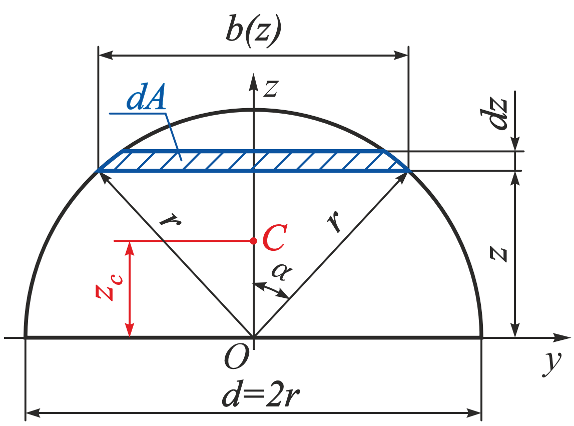


Fig. 7

Separate out the unit of area with width $b(z)$ and height dz located at the distance z from the axis y. The area of this unit is

$$dA = b(z)dz.$$

As it could be seen from Fig. 7, $b(z) = 2r \sin\alpha$ and $z = r\cos\alpha$, therefore,

$$dz = (-r \sin\alpha) d\alpha.$$

Then the static moment about the axis y is

$$S_y = \int_A z dA = \int_0^r z \cdot 2r \sin\alpha \cdot dz = \int_{\frac{\pi}{2}}^0 r\cos\alpha \cdot 2r \sin\alpha\,(-r\sin\alpha) d\alpha =$$

$$= -2r^3 \int_{\frac{\pi}{2}}^0 \sin^2\alpha \cos\alpha\, d\alpha = -2r^3 \left(\frac{\sin^3\alpha}{3}\right)\Bigg|_{\frac{\pi}{2}}^0 = \frac{2}{3} r^3.$$

To find the location of gravity center use the correlation (10)

$$z_C = \frac{S_y}{A} = \frac{\frac{2}{3}r^3}{\frac{\pi r^2}{2}} = \frac{4r}{3\pi} = 0{,}4244r.$$

<u>Example 5</u>

Define the gravity center coordinates of the compound section and show the system of central axes y and z (Fig. 8).

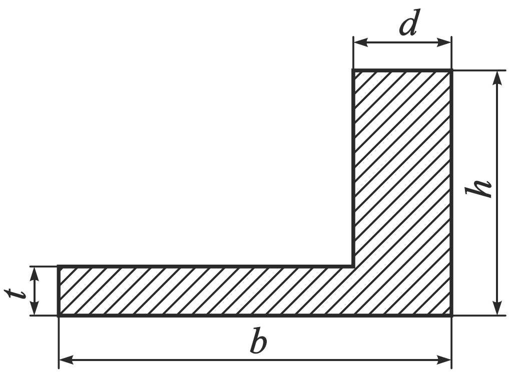


Fig. 8

Given: $h = 5\ cm$, $b = 8\ cm$, $d = 2\ cm$, $t = 1\ cm$.

Define y_C, z_C.

Solution

1. Make a dimensioned drawing of the compound section (Fig. 9).

2. Decompose the section into elemental parts and give them numbers (1 and 2).

3. Locate central coordinate systems $y_i O_i z_i$ in the gravity center of each elemental part of the section (in the present case $y_1 O_1 z_1$ and $y_2 O_2 z_2$).

4. Select an actual (basic) coordinate system to define the coordinates of the whole section gravity center. Here we take the axial system $y_1 O_1 z_1$ as basic one.

<u>Note</u> This problem can be solved in any coordinate system, but in order to minimize and simplify the calculations it is reasonable to use the central axial system of one of the compound section elements as basic system.

5. Write the expressions to define the coordinates of compound section gravity center:

$$y_c = \frac{\sum_{i=1}^{2} S_{z_1}^{(i)}}{\sum_{i=1}^{2} A_i}; \quad z_c = \frac{\sum_{i=1}^{2} S_{y_1}^{(i)}}{\sum_{i=1}^{2} A_i}.$$

6. Calculate the area of compound section

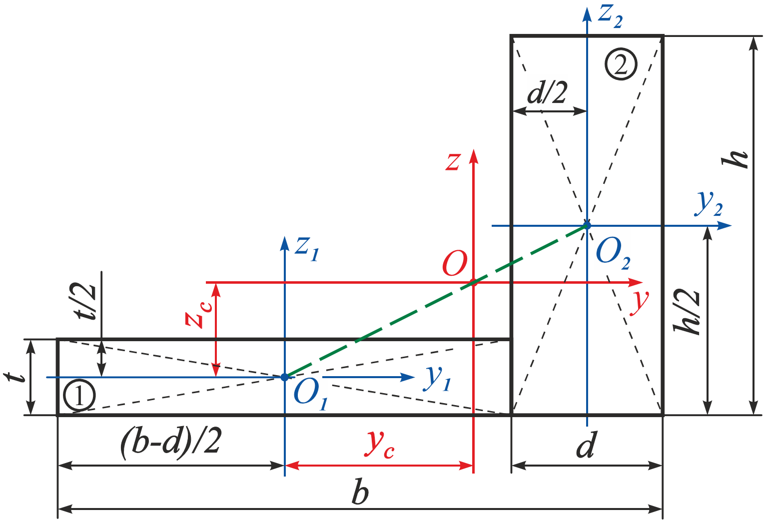


Fig. 9

$$\sum_{i=1}^{2} A_i = A = A_1 + A_2 = (b-d)t + hd = (8-2)\cdot 1 + 5\cdot 2 = 6 + 10 = 16\ cm^2.$$

7. Define the static moments of the compound section about the axes y_1 and z_1:

$$S_{z_1} = \sum_{i=1}^{2} S_{z_1}^{(i)} = S_{z_1}^{(1)} + S_{z_1}^{(2)} = A_1\cdot 0 + A_2\left(\frac{b-d}{2} + \frac{d}{2}\right) = 10\cdot 4 = 40\ cm^3;$$

$$S_{y_1} = \sum_{i=1}^{2} S_{y_1}^{(i)} = S_{y_1}^{(1)} + S_{y_1}^{(2)} = A_1\cdot 0 + A_2\left(\frac{h}{2} - \frac{t}{2}\right) = 10\cdot 2 = 20\ cm^3.$$

8. Substitute the obtained results to the correlations (see art. 5) and calculate the coordinates of compound section gravity center in the axial system $y_1O_1z_1$:

$$y_c = \frac{S_{z_1}}{A} = \frac{40}{16} = 2{,}5\ cm; \qquad z_c = \frac{S_{y_1}}{A} = \frac{20}{16} = 1{,}25\ cm.$$

9. According to the results of the calculations show the system of central axes yOz and gravity center of the compound section (point O) on the Fig. 9.

<u>Solution verification</u>. The gravity center of the compound section composed of two elemental parts always ***lies on the line connecting the gravity centers of elementals***. In addition, the point O divides the interval O_1O_2 into parts which are inversely proportional to the areas of elementals, i.e.

$$\frac{|OO_1|}{|OO_2|} = \frac{A_2}{A_1};$$

$$\frac{|OO_1|}{|OO_2|} = \frac{\sqrt{y_c^2 + z_c^2}}{\sqrt{\left(\frac{b}{2} - y_c\right)^2 + \left(\frac{h}{2} - \frac{t}{2} - z_c\right)^2}} = \frac{2{,}795}{1{,}677} = 1{,}667; \qquad \frac{A_2}{A_1} = \frac{10}{6} = 1{,}667.$$

4. Axial, polar, and centrifugal inertia moments of the section

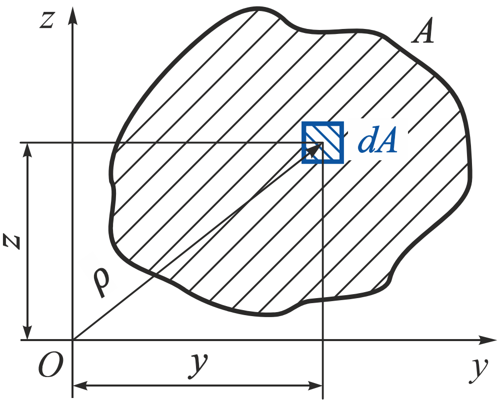


Fig. 10

By analogy with the concept of inertia moment of a body mass about an arbitrary axis, we introduce the concept of inertia moments of the area.

Axial *(equatorial)* ***inertia moment*** of the section about any axis is the sum of products of area units dA by squared distances from their gravity centers to this axis (Fig. 10):

$$I_y = \int_A z^2\, dA; \qquad I_z = \int_A y^2\, dA. \tag{12}$$

Centrifugal inertia moment of the section is the sum of products of area units dA by coordinates of gravity centers of these area units about two orthogonal coordinate axes located in plane of the figure:

$$I_{yz} = \int_A yz\, dA. \tag{13}$$

Polar inertia moment of the section is the sum of products of area units dA by squared distance from their gravity centers to any pole, where the origin of coordinates (the point O in Fig. 10) is usually chosen as a pole:

$$I_\rho = \int_A \rho^2\, dA, \quad \rho = \sqrt{y^2 + z^2}. \tag{14}$$

Inertia moments are measured in length units to the power of four (for example, m^4).

It follows from the correlations (12) – (14) that when $A > 0$:

1) ***axial*** and ***polar*** inertia moments are always ***greater than zero***:

$$I_y > 0; \quad I_z > 0; \quad I_\rho > 0;$$

2) the value of ***centrifugal*** inertia moment I_{yz} can be ***positive, negative*** *or* ***equal zero***.

The symmetry rules

If a section has a symmetry axis, then in the coordinate system that this axis belongs to, the centrifugal inertia moment is ***identically equal to zero***. The correctness of this rule is obvious from the Fig. 3.

- A pair of orthogonal axes that give $I_{yz} = 0$ are called ***principal inertia axes*** of the section.

Therefore, if one of two axes is a symmetry axis then this symmetry axis and any perpendicular axis to it are the principal inertia axes because the centrifugal inertia moment about these axes equals zero.

Note An inertia moment of a compound section consisting of n elementals equals the sum of inertia moments of its elementals about the same axis:

$$I_y = \sum_{i=1}^{n} I_y^{(i)}; \qquad I_z = \sum_{i=1}^{n} I_z^{(i)}; \qquad I_{yz} = \sum_{i=1}^{n} I_{yz}^{(i)},$$

where $I_y^{(i)}, I_z^{(i)}, I_{yz}^{(i)}$ – inertia moments of n-th elementals of the compound section;

n – number of elementals of the whole section.

5. Relation between the values I_ρ, I_y, and I_z

By the definition (14)

$$I_\rho = \int_A \rho^2 \, dA.$$

If the pole O coincides with the origin of coordinates of the system yOz, then

$$\rho^2 = z^2 + y^2$$

and therefore

$$I_\rho = \int_A \rho^2 \, dA = \int_A (z^2 + y^2) \, dA = \int_A z^2 \, dA + \int_A y^2 \, dA = I_y + I_z. \qquad (15)$$

Hence, the sum of axial section inertia moments about ***orthogonal*** axes y and z is equal to the polar section inertia moment about the origin of coordinates and is invariable under coordinate system rotation.

6. Examples of defining inertia moments of simple geometric figures

Example 1

Define inertia moments of rectangular section shown on Fig. 11 about the axes y and z that are the symmetry axes of this section.

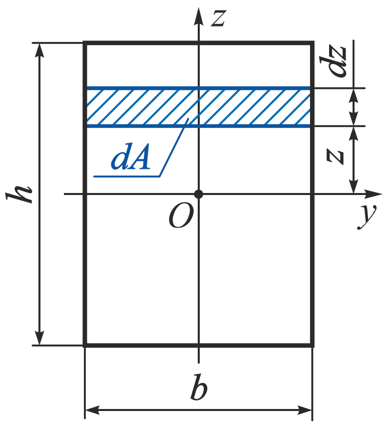


Fig. 11

Given: h, b, O – gravity center of the section.
Define I_y, I_z, I_{yz}, I_ρ.

Solution

1. Define the inertia moment about the axis y.

Separate a unit of area out this rectangle that has the width b and height dz and is located at the distance z from axis y. The area of this unit is

$$dA = bdz.$$

Substitute the value dA into the expression (12) to calculate the axial inertia moment

$$I_y = \int_A z^2 dA = \int_{-\frac{h}{2}}^{\frac{h}{2}} bz^2 dz = b\frac{z^3}{3}\bigg|_{-\frac{h}{2}}^{\frac{h}{2}} = \frac{b}{3}\left(\frac{h^3}{8} + \frac{h^3}{8}\right) = \frac{bh^3}{12}.$$

2. Define the inertia moment about the axis z.

Performing the calculations similar to the previous ones, we obtain

$$I_z = \frac{hb^3}{12}.$$

3. As far as axes y and z are symmetry axes of the section, then they are principal central axes of the section. Therefore, centrifugal inertia moment of the section about the axial system yOz equals zero:

$$I_{yz} = 0.$$

4. Define polar inertia moment

$$I_\rho = I_y + I_z = \frac{bh}{12}(h^2 + b^2) = \frac{A}{12}(h^2 + b^2).$$

Example 2

Define inertia moments of the box-type section assuming that axes y and z are symmetry axes (Fig. 12).

Given: B, H, b, h.
Define I_y, I_z, I_{yz}.

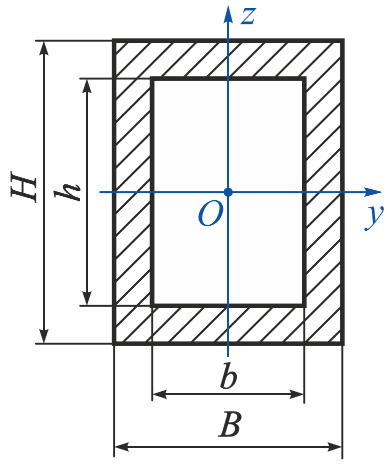


Fig. 12

Solution

1. As far as axes y and z are section symmetry axes, then they are principal central axes of this section, so the location of gravity center of this section is defined: it is the cross point of axes y and z (point O). Then

$$I_{yz} = 0.$$

2. Find I_y. The integral of the sum is equal to the sum of the integrals, therefore

$$I_y = \frac{BH^3}{12} - \frac{bh^3}{12}.$$

3. Define by analogy

$$I_z = \frac{HB^3}{12} - \frac{hb^3}{12}.$$

Example 3

Define inertia moments of a round section (Fig. 13) about central axes y, z.

Given: d.
Define I_y, I_z, I_{yz}, I_ρ.

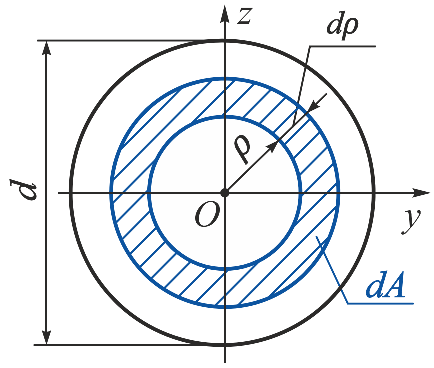


Fig. 13

Solution

1. Define polar inertia moment I_ρ.

Separate circular unit of area with the thickness $d\rho$ and radius ρ. The area of this unit is $dA = 2\pi\rho d\rho$.

Substitute the value dF into the expression (14):

$$I_\rho = \int_A \rho^2 dA = \int_0^{\frac{d}{2}} 2\pi\rho^3 d\rho = 2\pi \frac{\rho^4}{4}\bigg|_0^{\frac{d}{2}} = \frac{2\pi d^4}{4 \cdot 16} = \frac{\pi d^4}{32}.$$

2. Define axial inertia moments I_y and I_z.

Axial inertia moments of the circular section about all axes passing through its gravity center are the same value, i.e. $I_y = I_z$.

Then

$$I_\rho = I_y + I_z = 2I_y = 2I_z.$$

Hence

$$I_y = I_z = \frac{I_\rho}{2} = \frac{\pi d^4}{64}.$$

3. As far as axes y and z are the section symmetry axes then they are principal central axes of this section. Therefore, the centrifugal inertia moment about axial system yOz equals zero:

$$I_{yz} = 0.$$

Example 4

Define inertia moments of a circular section about central axes y and z (Fig. 14).

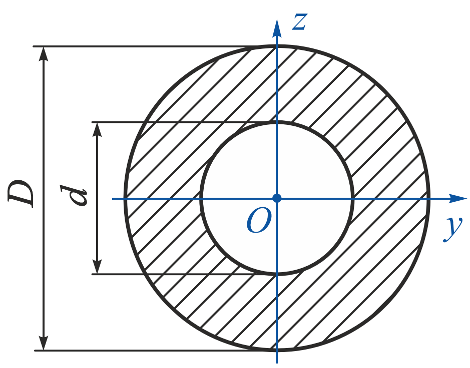


Fig. 14

Given: $D, d, \alpha = {}^{d}/_{D}$.

Define I_y, I_z, I_{yz}, I_ρ.

Solution

1. As far as axes y and z are section symmetry axes, then they are principal central axes of this section, and point O is the gravity center, then

$$I_{yz} = 0.$$

2. Define polar inertia moment

$$I_\rho = \frac{\pi D^4}{32} - \frac{\pi d^4}{32} = \{\text{assuming } d = \alpha D\} = \frac{\pi D^4}{32}(1 - \alpha^4).$$

3. Find axial inertia moments about the axes y and z.

Axial inertia moments of the circular section about all axes passing through its gravity center are the same value, i.e. $I_y = I_z$.

Then

$$I_y = I_z = \frac{\pi D^4}{64} - \frac{\pi d^4}{64} = \{\text{assuming } d = \alpha D\} = \frac{\pi D^4}{64}(1 - \alpha^4).$$

Example 5

Define inertia moments of right-angled triangle about the axes that coincide with legs of this triangle (Fig. 15).

Given: h, b.
Define I_{y_1}, I_{z_1}, $I_{y_1 z_1}$.

Solution

1. Find axial inertia moment about the axis y_1.

Separate a unit of area with width $b(z_1)$ and height dz_1 that is located at the distance z_1 from the axis y_1. The area of this unit is

$$dA = b(z_1)dz_1.$$

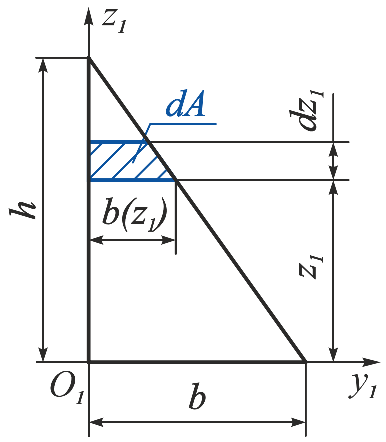


Fig. 15

From the similarity of triangles find $b(z_1)$:

$$\frac{b(z_1)}{b} = \frac{h - z_1}{h} \quad \Rightarrow \quad b(z_1) = b\left(1 - \frac{z_1}{h}\right).$$

Then

$$I_{y_1} = \int_A z_1^2 dA = \int_0^h b z_1^2 \left(1 - \frac{z_1}{h}\right)^2 dz_1 = b\left(\frac{z_1^3}{3} - \frac{z_1^4}{4h}\right)\Bigg|_0^h = \frac{bh^3}{12}.$$

2. By analogy find axial inertia moment about the axis z_1

$$I_{z_1} = \frac{hb^3}{12}.$$

3. Calculate centrifugal inertia moment about the axes y_1 and z_1

$$I_{y_1 z_1} = \int_0^h y_1 z_1 dA = \int_0^h z_1 \left(\int_0^{b(z_1)} y_1 dy_1\right) dz_1 = \int_0^h z_1 \frac{b^2(z_1)}{2} dz_1.$$

By substituting $b(z_1)$ we obtain

$$I_{y_1 z_1} = \int_0^h \frac{b^2}{2} z_1 \left(1 - \frac{z_1}{h}\right)^2 dz_1 = \frac{b^2}{2}\left(\frac{z_1^2}{2} - \frac{2}{3}\frac{z_1^3}{h} + \frac{z_1^4}{4h^2}\right)\Bigg|_0^h =$$

$$= \frac{b^2 h^2}{2}\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) = +\frac{b^2 h^2}{24}.$$

Example 6

Define inertia moments of the right-angled triangle about central y and z (Fig. 16).

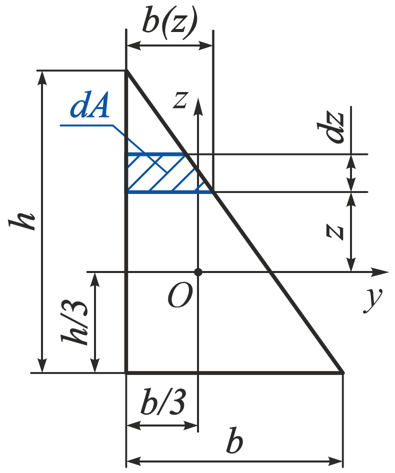


Fig. 16

Given: h, b.

Define I_y, I_z, I_{yz}.

Solution

1. Find axial inertia moment about axis y.

Separate a unit of area with the width $b(z)$ and height dz that is located at the distance z from the axis y. The area of this unit is

$$dA = b(z)dz.$$

Define $b(z)$ from the similarity of triangles:

$$\frac{b(z)}{b} = \frac{\frac{2h}{3} - z}{h} \quad \Rightarrow \quad b(z) = \frac{b}{h}\left(\frac{2h}{3} - z\right).$$

Then

$$I_y = \int_A z^2 dA = \int_{-\frac{h}{3}}^{\frac{2}{3}h} z^2 \frac{b}{h}\left(\frac{2h}{3} - z\right) dz = \frac{b}{3h}\left(\frac{2hz^3}{3} - \frac{3z^4}{4}\right)\Bigg|_{-\frac{h}{3}}^{\frac{2}{3}h} =$$

$$= \frac{b}{3h}\left[\frac{2h\left(\frac{2}{3}h\right)^3}{3} - \frac{3\left(\frac{2}{3}h\right)^4}{4} - \frac{2h\left(-\frac{h}{3}\right)^3}{3} + \frac{3\left(-\frac{h}{3}\right)^4}{4}\right] = \frac{bh^3}{36}.$$

2. By analogy find axial inertia moment relative to the axis z

$$I_z = \frac{hb^3}{36}.$$

3. Calculate centrifugal inertia moment about the axes y and z

$$I_{yz} = \int_0^h yzdF = \int_{-\frac{h}{3}}^{\frac{2}{3}h} z\left(\int_{-\frac{b}{3}}^{b(z)-\frac{b}{3}} ydy\right)dz = \int_{-\frac{h}{3}}^{\frac{2}{3}h} z\frac{\left(\frac{b}{h}\left(\frac{2h}{3} - z\right) - \frac{b}{3}\right)^2 - \left(-\frac{b}{3}\right)^2}{2}dz =$$

$$= \frac{1}{2}\int_{-\frac{h}{3}}^{\frac{2}{3}h}\left(-\frac{2}{3}\frac{b^2z^2}{h} + \frac{b^2z^3}{h^2}\right)dz = \frac{b^2h^2}{2}\left(-\frac{16}{3\cdot 81} + \frac{4}{81} - \frac{2}{3\cdot 81} - \frac{1}{8\cdot 81}\right) = -\frac{b^2h^2}{72}.$$

Notes

1. Every triangle can be presented as a combination of right-angled triangles, which allows using obtained correlations to define inertia moments of arbitrary triangular sections.

2. The centrifugal inertia moment of right-angled triangle about its central axes that are parallel to the legs can be positive as well as negative. The sign of I_{yz} is defined by the collocation of the triangle and its central axes. Consider the options of this collocation (Fig. 17).

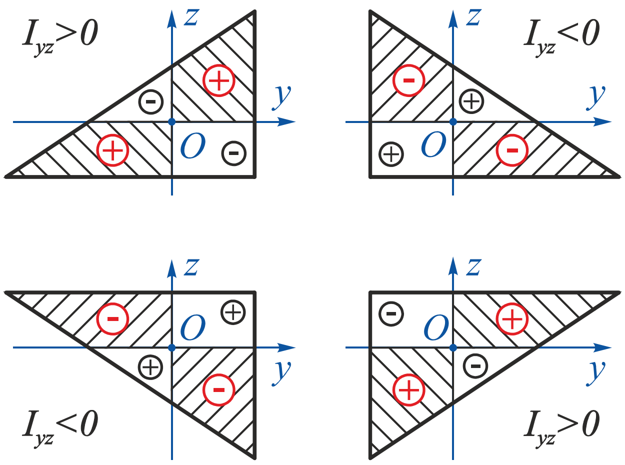


Fig. 17

As far as

$$I_{yz} = \int_A yzdA \quad \text{and} \quad dA > 0,$$

then the sign of I_{yz} is defined by the sign of the product yz, which is positive in first and third quarters and negative in second and forth ones. If the drawing of the section is dimensioned then in most cases the sign of I_{yz} can be defined visually. The hatched areas on the pictures prevail in the process of summating (integrating) the values $yzdA$, which defines the sign of I_{yz}. Therefore finally

$$I_{yz} = \pm\frac{b^2h^2}{72},$$

and the sign is chosen either formally by the results of calculations or by these simple considerations that require understanding the sense of integration operation.

7. Changes in axial and centrifugal inertia moments of the section due to parallel translation of axes

In the strength calculations of engineering structures it is often necessary to define axial and centrifugal inertia moments of compound sections about the ***arbitrarily*** located axes relatively to ***central*** ones. Such axial and centrifugal inertia moments can be found by performing two operations: parallel translation of axes and rotation about the origin of coordinates.

Consider the calculation of axial and centrifugal inertia moments due to switching to axes parallel to central ones (Fig. 18).

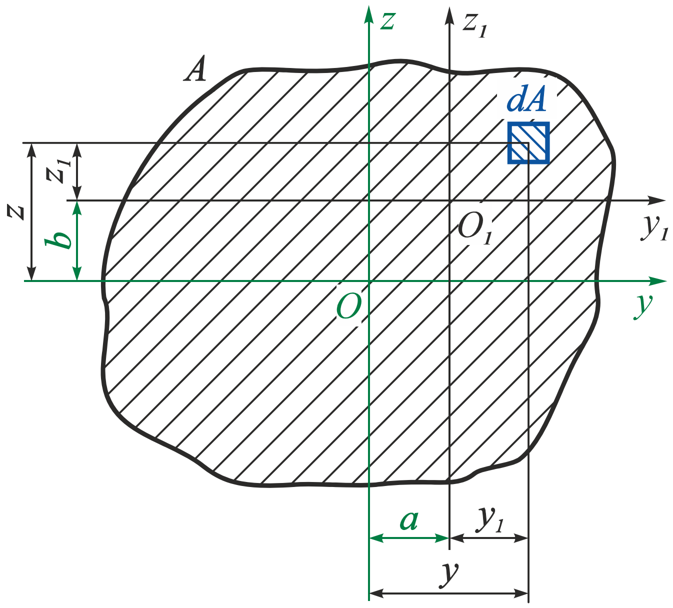


Fig. 18

Given: A, a, b, I_y, I_z, I_{yz}; axes y and z are the central axes of the section A, i.e.

$$S_y = \int_A z dA = 0,$$

$$S_z = \int_A y dA = 0.$$

Define I_{y_1}, I_{z_1}, $I_{y_1 z_1}$.

By the definition (12), (13)

$$I_{y_1} = \int_A z_1^2 \, dA,$$
$$I_{z_1} = \int_A y_1^2 \, dA, \qquad (16)$$
$$I_{y_1 z_1} = \int_A y_1 z_1 \, dA.$$

It can be seen from Fig. 18 that

$$z_1 = z - b,$$
$$y_1 = y - a. \qquad (17)$$

Substituting the values z_1 and y_1 from the equations (17) into correlations (16), we obtain

$$I_{y_1} = \int_A (z-b)^2\,dA = \int_A z^2 dA - 2b\int_A z dA + b^2\int_A dA\,;$$

$$I_{z_1} = \int_A (y-a)^2\,dA = \int_A y^2 dA - 2a\int_A y dA + a^2\int_A dA\,;$$

$$I_{y_1 z_1} = \int_A (z-b)\,(y-a) dA = \int_A yz dA - b\int_A y dA - a\int_A z dA + ab\int_A dA.$$

In the right-hand part of these correlations there are

$$\int_A z^2\,dA = I_y; \quad \int_A y^2\,dA = I_z; \quad \int_A yz\,dA = I_{yz}; \quad \int_A dA = A;$$

$$\int_A z dA = S_y = 0; \quad \int_A y dA = S_z = 0,$$

and all these values are defined, therefore finally we obtain

$$I_{y_1} = I_y + b^2 A;$$
$$I_{z_1} = I_z + a^2 A; \qquad (18)$$
$$I_{yz} = I_{yz} + abA.$$

Hence, an axial inertia moment of a plane section about some axis that lies in plane of the figure's section and is parallel to a central axis, is equal to an axial inertia moment of this figure about central axis plus the product of section area by squared coordinate of gravity center of this section in the new coordinate system.

A centrifugal inertia moment of a section about a pair of orthogonal axes that lie in plane of the section and are parallel to the central axes is equal to centrifugal inertia moment of the section about the pair of orthogonal central axes plus the product of section area by coordinates of section gravity center in the new coordinate system. When determining the ***centrifugal inertia moment***, the ***signs of coordinates*** about the given axes ***must be taken into account***.

Note	It follows from the first and the second correlations of the system (18) that among the whole set of axes parallel to any coordinate direction, the central axis has the minimum value of axial inertia moment about it.

8. Changes in axial and centrifugal inertia moments of the section due to rotation of the axes

Given: A, I_y, I_z, I_{yz}, α. The system y_1Oz_1 is obtained from the original one by means of rotating it by the angle α relative to the origin of coordinates (consider as positive the direction of rotation shown on Fig. 19 – counterclockwise).

Define I_{y_1}, I_{z_1}, I_{yz}.

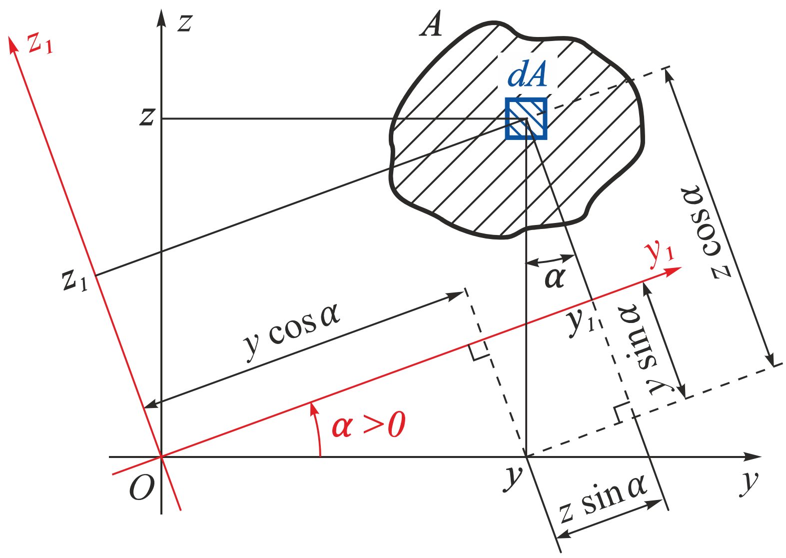


Fig. 19

By the definition (12), (13)

$$
\begin{aligned}
I_{y_1} &= \int_A z_1^2 \, dA; \\
I_{z_1} &= \int_A y_1^2 \, dA; \qquad (19) \\
I_{y_1 z_1} &= \int_A y_1 z_1 \, dA.
\end{aligned}
$$

It can be seen from the Fig. 16 that

$$
\begin{aligned}
z_1 &= z\cos\alpha - y\sin\alpha\,; \\
y_1 &= y\cos\alpha + z\sin\alpha.
\end{aligned} \qquad (20)
$$

Substitute the values z_1 and y_1 to the expressions of the system (19):

$$
\begin{aligned}
I_{y_1} &= \int_A (z\cos\alpha - y\sin\alpha)^2 \, dA = \\
&= \cos^2\alpha \int_A z^2 dA - 2\sin\alpha\cos\alpha \int_A yzdA + \sin^2\alpha \int_A y^2 dA\,;
\end{aligned} \qquad (21)
$$

$$I_{z_1} = \int_A (y\cos\alpha + z\sin\alpha)^2\, dA =$$
$$= \cos^2\alpha \int_A y^2 dA + 2\sin\alpha\cos\alpha \int_A yz dA + \sin^2\alpha \int_A z^2 dA\,; \tag{22}$$

Taking into account that in the expressions (21) and (22)

$$\int_A z^2\, dA = I_y, \quad \int_A y^2\, dA = I_z, \quad \int_A yz\, dA = I_{yz}, \quad 2\sin\alpha\cos\alpha = \sin 2\alpha,$$

and all these values are determined, we finally get the expressions to define axial inertia moments of the section with the rotation of axes:

$$I_{y_1} = I_y\cos^2\alpha + I_z\sin^2\alpha - I_{yz}\sin 2\alpha\,;$$
$$I_{z_1} = I_z\cos^2\alpha + I_y\sin^2\alpha + I_{yz}\sin 2\alpha. \tag{23}$$

Move on to defining centrifugal inertia moment of the section:

$$I_{y_1z_1} = \int_A (z\cos\alpha - y\sin\alpha)\,(y\cos\alpha + z\sin\alpha)dA =$$
$$= \cos^2\alpha \int_A yz dA - \sin\alpha\cos\alpha \int_A y^2 dA + \sin\alpha\cos\alpha \int_A z^2 dA - \sin^2\alpha \int_A yz dA =$$
$$= (\cos^2\alpha - \sin^2\alpha)\int_A yz dA + 2\sin\alpha\cos\alpha\frac{1}{2}\left(\int_A z^2 dA - \int_A y^2 dA\right).$$

Taking into account that in this expression

$$\int_A z^2\, dA = I_y; \quad \int_A y^2\, dA = I_z; \quad \int_A yz\, dA = I_{yz};$$
$$\cos^2\alpha - \sin^2\alpha = \cos 2\alpha\,; \qquad 2\sin\alpha\cos\alpha = \sin 2\alpha,$$

and all these values are determined, we finally obtain

$$I_{y_1z_1} = I_{yz}\cos 2\alpha + \frac{I_y - I_z}{2}\sin 2\alpha. \tag{24}$$

<u>Notes</u>

1. In the expressions (23), (24) initial axes are ***arbitrary*** (not necessarily central).
2. Expressions (23), (24) are periodic functions with minimum period π and are dependent of α.
3. In case of rotation of axes by $\pi/2$ the centrifugal inertia moment changes its sign, and the axial inertia moments always remain positive (with any α).

9. Invariance of the sum of section axial inertia moments relative to rotation of the axes

Find the sum of axial inertia moments from the equations (23):

$$I_{y_1} + I_{z_1} = I_y \cos^2\alpha + I_z \sin^2\alpha - I_{yz}\sin 2\alpha + I_z\cos^2\alpha + I_y \sin^2\alpha + I_{yz}\sin 2\alpha =$$
$$= I_y(\sin^2\alpha + \cos^2\alpha) + I_z(\sin^2\alpha + \cos^2\alpha).$$

Hence, taking into account that $\sin^2\alpha + \cos^2\alpha = 1$, we obtain

$$I_{y_1} + I_{z_1} = I_y + I_z = I_\rho = const. \qquad (25)$$

This proposition, in essence, was proven earlier (see paragraph 5). It was proven (see equation (15)) that

$$I_\rho = I_y + I_z.$$

Thus, with a fixed position of the pole, the value of I_ρ does not change, or, in other words, ***the sum of two axial inertia moments*** about any pair of orthogonal axes emerging from one point is ***a constant value***.

10. Principal axes and principal inertia moments of the section

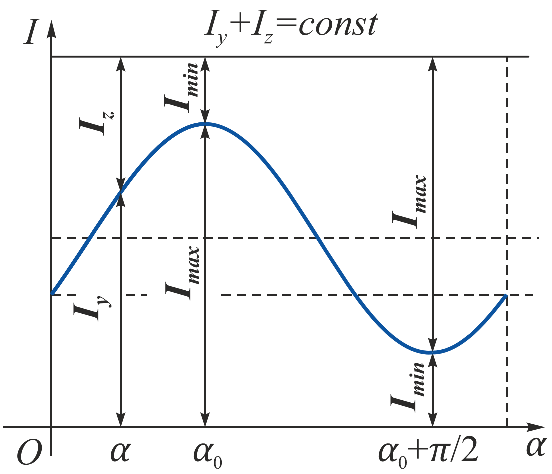


Fig. 20

It can be seen from correlations (23) that functions $I_{y_1} = f_1(\alpha)$ and $I_{z_1} = f_2(\alpha)$ are continuous and periodic with minimum period π. In addition, as it follows from equation (25), the sum of these functions is a constant value, and for the finite dimension section, it is finite value. It is graphically shown on Fig. 20.

This picture shows that within the period there is the value $\alpha = \alpha_0$ (and also $\alpha = \alpha_0 + {}^{\pi}/_{2}$), due to which the section axial inertia moments get extremum values ***simultaneously***.

To define the value of α_0 perform a differentiation one of the correlations (23) (for example, the first one) about α and equate it to zero:

$$\frac{dI_{y_1}}{d\alpha} = I_y 2\cos\alpha_0\,(-\sin\alpha_0) + I_z 2\sin\alpha_0\cos\alpha_0 - 2I_{yz}\cos 2\alpha_0 = 0.$$

Hence

$$\left(I_z - I_y\right)\sin 2\alpha_0 = 2I_{yz}\cos 2\alpha_0 \quad (26)$$

and finally

$$\operatorname{tg} 2\alpha_0 = \frac{2I_{yz}}{\left(I_z - I_y\right)}. \quad (27)$$

Define the value of centrifugal inertia moment of the section with $\alpha = \alpha_0$. Rewrite the expression (26) in a form

$$I_{yz}\cos 2\alpha_0 = \frac{I_z - I_y}{2}\sin 2\alpha_0 \quad (28)$$

and substitute to the equation (24) the value of first summand from the right-hand part by the formula (28)

$$I_{y_1 z_1} = \frac{I_z - I_y}{2}\sin 2\alpha_0 + \frac{I_y - I_z}{2}\sin 2\alpha_0 \equiv 0. \quad (29)$$

Therefore, when $\alpha = \alpha_0$:

1) axial inertia moments of the section have extremum values simultaneously;
2) centrifugal inertia moment of the section becomes zero.

Based on obtained results we can formulate the following definitions:

- The axes relative to which axial inertia moment of the section have extremum values simultaneously, and centrifugal inertia moment of the section becomes zero are referred to as ***principal inertia axes of the section***.

- If principal inertia axes of the section pass through its gravity center then they are referred to as ***principal central inertia axes of the section***.

- Axial inertia moments of the section about its principal axes are referred to as ***principal inertia moments of the section***.

The location of principal inertia axes of the section is found by the formula (27).

To define the values of principal axial inertia moments of the section (with $\alpha = \alpha_0$) rewrite the correlations (23), using trigonometric relations

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}; \qquad \sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

in a form

$$I_{y_1} = \frac{I_y + I_z}{2} + \frac{I_y - I_z}{2}\cos 2\alpha_0 - I_{yz}\sin 2\alpha_0\,;$$

$$I_{z_1} = \frac{I_y + I_z}{2} + \frac{I_z - I_y}{2}\cos 2\alpha_0 + I_{yz}\sin 2\alpha_0\,;$$

or

$$I_{\substack{max\\min}} = \frac{I_y + I_z}{2} \pm \left(\frac{I_z - I_y}{2}\cos 2\alpha_0 + I_{yz}\sin 2\alpha_0\right). \qquad (30)$$

Using the known trigonometric relations

$$\sin 2\alpha_0 = \frac{\mathrm{tg}\,2\alpha_0}{\sqrt{1+\mathrm{tg}^2\,2\alpha_0}}; \qquad \cos 2\alpha_0 = \frac{1}{\sqrt{1+\mathrm{tg}^2\,2\alpha_0}}$$

exclude α_0 by means of expression (27) and obtain

$$\sin 2\alpha_0 = \frac{2I_{yz}}{\sqrt{\left(I_z - I_y\right)^2 + 4I_{yz}^2}}; \qquad \cos 2\alpha_0 = \frac{I_z - I_y}{\sqrt{\left(I_z - I_y\right)^2 + 4I_{yz}^2}}.$$

Then the correlation (30) can be written down in a form

$$I_{\substack{max\\min}} = \frac{I_y + I_z}{2} \pm \left(\frac{I_z - I_y}{2}\frac{I_z - I_y}{\sqrt{\left(I_z - I_y\right)^2 + 4I_{yz}^2}} + I_{yz}\frac{2I_{yz}}{\sqrt{\left(I_z - I_y\right)^2 + 4I_{yz}^2}}\right).$$

After processing we finally get

$$I_{\substack{max\\min}} = I_{\substack{u\\v}} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_z - I_y}{2}\right)^2 + I_{yz}^2} = \frac{I_y + I_z}{2} \pm \frac{1}{2}\sqrt{\left(I_z - I_y\right)^2 + 4I_{yz}^2}. \qquad (31)$$

Notes

1. The angle α_0 obtained using the expression (27) should be laid off ***counterclockwise*** if $\alpha_0 > 0$ and ***clockwise*** if $\alpha_0 < 0$.
2. The axis relative to which the principal inertia moment of the section has maximum value is located at the ***shortest angular distance*** from the central axis (y or z) relative to which the axial inertia moment is greater.
3. It follows from the symmetry rule considered before that if at least one of the axes is a symmetry axis of a section then this system of mutually perpendicular axes is a system of principal inertia axes of a section.

Consider special cases.

1. If $I_y = I_z$ and $I_{yz} = 0$, then it follows from the formula (24)

$$I_{y_1z_1} = I_{yz}\cos 2\alpha + \frac{I_y - I_z}{2}\sin 2\alpha$$

that the value of centrifugal inertia moment about any pair of mutually perpendicular axes $I_{y_1z_1}$ equals zero. Therefore, any axes obtained by means of rotating the coordinate system yOz are principal inertia axes (as well as axes y and z). Hence,

$$I_y = I_z = I_{max} = I_{min} = const.$$

2. If the figure has more than two symmetry axes then its axial inertia moments about all of central axes are equal.

Direct one of the axes (y or z) along one of symmetry axes, and the other one perpendicularly to it. The centrifugal inertia moment about these axes is $I_{yz} = 0$. If the figure has more than two symmetry axes then there is some axis among them that generates an acute angle with the axis z. Denote such axis as z_1, and the one perpendicular to it y_1.

Centrifugal inertia moment $I_{y_1z_1} = 0$, because the axis z_1 is a symmetry axis. In accordance with the formula (24)

$$I_{y_1z_1} = I_{yz}\cos 2\alpha + \frac{I_y - I_z}{2}\sin 2\alpha = 0,$$

but as far as

$$I_{yz} = 0,$$

then

$$I_y = I_z.$$

Then according to Point 1 an inertia moment about any axis has the same value, and any axes obtained by means of rotating the coordinate system yOz are principal inertia axes.

This implies that ***all regular figures*** (square, circle, equilateral triangle etc.) ***have equal inertia moments about all central axes, and all these axes are principal inertia axes***.

3. If $I_y = I_z$ и $I_{yz} \neq 0$, then according to the formula (27)

$$\operatorname{tg} 2\alpha_0 = \frac{2I_{yz}}{(I_z - I_y)};$$

$$\operatorname{tg} 2\alpha_0 = \infty; \qquad 2\alpha_0 = 90°; \qquad \alpha_0 = 45°.$$

In this case principal inertia axes are located at $45°$ angles relative to initial axes y and z.

11. Examples of solving typical problems

Example 1

Define inertia moments of right-angled triangle about central axes parallel to the legs using the formulas of parallel translation (Fig. 21).

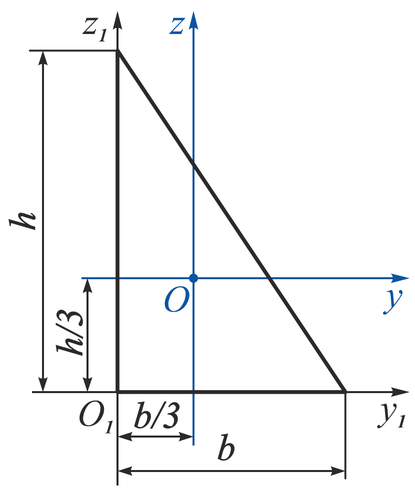


Fig. 21

Given: h, b.
Define I_y, I_z, I_{yz}.

Solution

To solve this problem we use the results of example 5 (see paragraph 6) and correlations (18) of parallel translation of axes. However, we have to take into account that the translation is performed from non-central axes y_1, z_1 to central axes y, z, therefore the formulas of parallel translation are written as

$$I_y = I_{y_1} - b_1^2 A, \qquad I_z = I_{z_1} - a^2 A, \qquad I_{yz} = I_{y_1 z_1} - abA,$$

where

$$b_1 = \frac{h}{3}; \quad a = \frac{b}{3}; \quad A = \frac{bh}{2}; \quad I_{y_1} = \frac{bh^3}{12}; \quad I_{z_1} = \frac{hb^3}{12}; \quad I_{y_1 z_1} = \frac{b^2h^2}{24}.$$

Define axial and centrifugal inertia moments:

$$I_y = \frac{bh^3}{12} - \left(\frac{h}{3}\right)^2 \frac{bh}{2} = \frac{bh^3}{36};$$

$$I_z = \frac{hb^3}{12} - \left(\frac{b}{3}\right)^2 \frac{bh}{2} = \frac{hb^3}{36};$$

$$I_{yz} = \frac{b^2h^2}{24} - \frac{b}{3} \cdot \frac{h}{3} \cdot \frac{bh}{2} = -\frac{b^2h^2}{72}.$$

This option of finding inertia moments in right-angled triangle about central axes parallel to the legs is less labor-consuming compared to the operation of direct integrating using the formulas (12), (13) (see example 6 of paragraph 6).

Example 2

Define inertia moments of isosceles triangle about central axes yOz (Fig. 22).

Given: h, b.
Define I_y, I_z, I_{yz}.

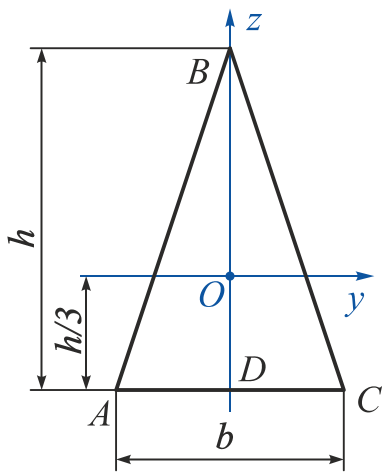


Fig. 22

Solution

1. Decompose the isosceles triangle ABC into two right-angled triangles ABD and BCD.

2. Define I_y.

The inertia moment of isosceles triangle ABC about the axis y is the sum of axial inertia moments of triangles ABD and BCD about the axis y:

$$I_y = I_y^{(ABD)} + I_y^{(BCD)} = 2I_y^{(ABD)} = 2I_y^{(BCD)} = 2 \cdot \frac{\frac{b}{2}h^3}{36} = \frac{bh^3}{36}.$$

Note An inertia moment of any triangle about the central axis y that is parallel to the base is

$$I_y = \frac{bh^3}{36}.$$

3. Define I_z.

The inertia moment of isosceles triangle ABC about the axis z is the sum of axial inertia moments of triangles ABD and BCD about the axis z:

$$I_z = I_z^{(ABD)} + I_z^{(BCD)} = 2I_z^{(ABD)} = 2I_z^{(BCD)} = 2\frac{h\left(\frac{b}{2}\right)^3}{12} = \frac{hb^3}{48}.$$

4. Define I_{yz}.

As far as the axis z is a symmetry axis of this section then it and the axis y perpendicular to it are principal central axes of this section. Therefore, centrifugal inertia moment of this section about the system of axes yOz equals zero:

$$I_{yz} = 0.$$

Example 3

Define the location of principal inertia axes passing through the point *A* (Fig. 23).

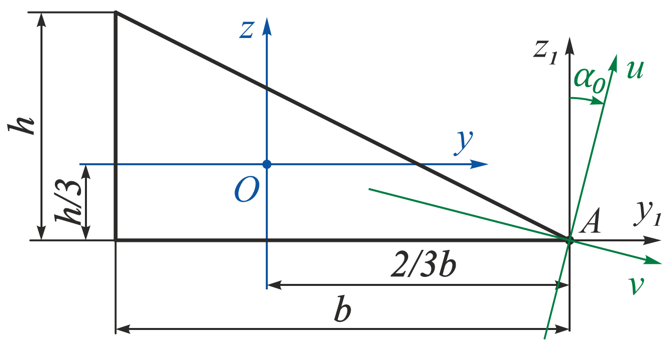


Fig. 23

Given: $h = 3\ cm$, $b = 6\ cm$.
Define α_0.

Solution

1. Define inertia moments relative to the axes y_1, z_1, passing through the point *A* and parallel to the edges of the triangle, using the theorems of parallel translation of axes:

$$I_{y_1} = I_y + \left(\frac{1}{3}h\right)^2 A = \frac{bh^3}{36} + \frac{1}{9}h^2\frac{bh}{2} = \frac{bh^3}{12} = \frac{6\cdot 3^3}{12} = 13{,}5\ cm^4;$$

$$I_{z_1} = I_z + \left(\frac{2}{3}b\right)^2 A = \frac{hb^3}{36} + \frac{4}{9}b^2\frac{bh}{2} = \frac{hb^3}{4} = \frac{3\cdot 6^3}{4} = 162\ cm^4;$$

$$I_{y_1z_1} = I_{yz} + \left(-\frac{2}{3}b\right)\cdot\left(+\frac{1}{3}h\right)A = -\frac{b^2h^2}{72} - \frac{2}{3}b\frac{1}{3}h\frac{bh}{2} = -\frac{b^2h^2}{8} =$$
$$= -\frac{6^2\cdot 3^2}{8} = -40{,}5\ cm^4.$$

2. Define the angle α_0, at which the axes y_1Az_1 should be rotated to become principal:

$$\mathrm{tg}\,2\alpha_0 = \frac{2I_{y_1z_1}}{\left(I_{z_1} - I_{y_1}\right)} = \frac{2(-40{,}5)}{162 - 13{,}5} = -0{,}545;$$

$$2\alpha_0 = -28{,}59°;$$

$$\alpha_0 = -14{,}295° = -14°15'32''.$$

The negative angle α_0 should be laid off clockwise. Here the axis of greater axial moment z_1 becomes the axis of maximum moment u, and the axis of lower axial moments y_1 becomes the axis of minimum moment v. Hence, the condition is fulfilled:

$$I_u > I_{z_1} > I_{y_1} > I_v.$$

Example 4

Define principal central inertia moments of the T-section (z is a symmetry axis) (Fig. 24).

Given: $h = 16\ cm$, $b = 10\ cm$, $t = 4\ cm$, $d = 2\ cm$.

Define I_y, I_z.

Solution

1. As far as axis z is a symmetry axis of this section then the gravity center lies on the axis z, i.e. $y_c = 0$ and $I_{yz} = 0$ (y is any axis perpendicular to z).

Find the second coordinate of the gravity center. Decompose the section into two rectangles and locate the second central axis y_1 and y_2 in the gravity center of each rectangle. Select the coordinate system zO_1y_1 as actual one.

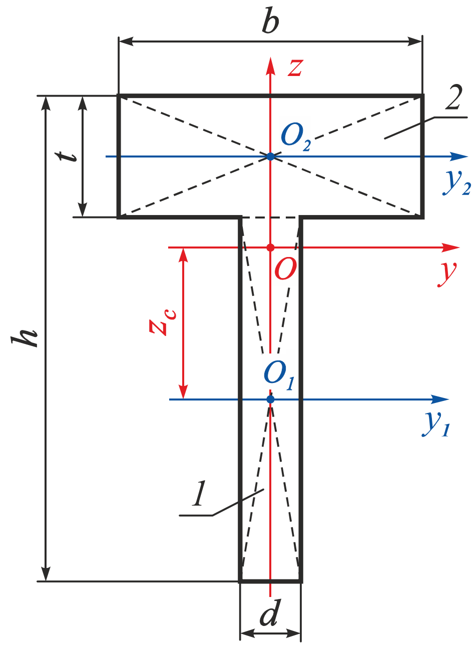


Fig. 24

Then

$$z_c = \frac{\sum_{i=1}^{2} S_{y_1}^{(i)}}{\sum_{i=1}^{2} A_i} = \frac{S_{y_1}^{(1)} + S_{y_1}^{(2)}}{A_1 + A_2} = \frac{0 + bt\left(\frac{h-t}{2} + \frac{t}{2}\right)}{(h-t)d + bt} = \frac{10 \cdot 4 \cdot \frac{16}{2}}{12 \cdot 2 + 10 \cdot 4} = \frac{320}{64} = 5\ cm.$$

2. Calculate the inertia moment about the axis z:

$$I_z = I_z^{(1)} + I_z^{(2)} = \frac{(h-t)d^3}{12} + \frac{tb^3}{12} = \frac{(16-4) \cdot 2^3}{12} + \frac{4 \cdot 10^3}{12} = 341{,}333\ cm^4.$$

3. Find inertia moment about the axis y:

$$I_y = I_y^{(1)} + I_y^{(2)} = 888 + 413{,}333 = 1301{,}333\ cm^4;$$

$$I_y^{(1)} = I_{y_1}^{(1)} + z_c^2 A_1 = \frac{d(h-t)^3}{12} + z_c^2(h-t)d =$$

$$= \frac{2(16-4)^3}{12} + 5^2(16-4) \cdot 2 = 888\ cm^4;$$

$$I_y^{(2)} = I_{y_2}^{(2)} + \left(\frac{h}{2} - z_c\right)^2 A_2 = \frac{bt^3}{12} + \left(\frac{h}{2} - z_c\right)^2 bt =$$

$$= \frac{10 \cdot 4^3}{12} + \left(\frac{16}{2} - 5\right)^2 \cdot 10 \cdot 4 = 413{,}333\ cm^4.$$

Example 5

Define axial and centrifugal inertia moments of the semicircle with radius r about the axes y and z (Fig. 25).

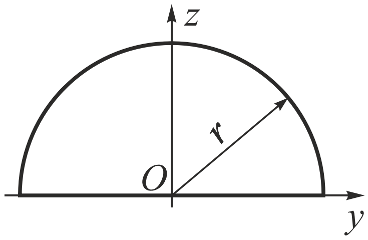


Fig. 25

Given: r.
Define I_y, I_z, I_{yz}.

Solution

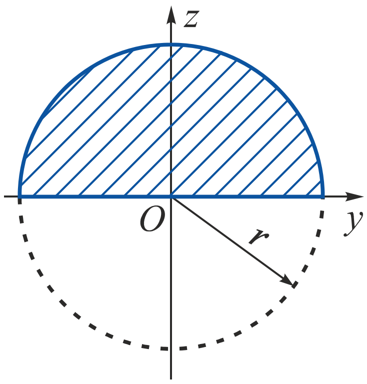


Fig. 26

Add up the semicircle to obtain a circle (Fig. 26). For round cross-sections

$$I_y^{\bigcirc} = I_z^{\bigcirc} = \frac{\pi d^4}{64}; \qquad I_{yz}^{\bigcirc} = 0.$$

Then for the semicircle as a semifigure

$$I_y = I_z = \frac{I_y^{\bigcirc}}{2} = \frac{I_z^{\bigcirc}}{2} = \frac{\pi d^4}{128}; \qquad I_{yz} = \frac{I_{yz}^{\bigcirc}}{2} = 0.$$

Notes

1. It is convenient to use the notion «***semifigure***» for sections that have one or more ***pair*** of mutually perpendicular symmetry axes.

A ***semifigure*** is a figure generated by means of decomposing a plane figure by an arbitrary shaped line that is reversely symmetrical to the symmetry axes. Here, when rotating a semifigure at 180° about the gravity center of the whole figure, the semifigure coincides with the second semifigure (Fig. 27).

The principal central axes of the figure (see Fig. 27) are principal axes of their semifigures. This is obvious if we draw an additional line A_1O (see Fig. 27, a, b, c, d), that is symmetrical to AO, and an additional line A_1C (see Fig. 27, e), that is symmetrical to AC. In the semifigures, the axis y for the part ***I*** and the axis z for the part ***II*** are symmetry axes, and therefore these axes are principal inertia axes for the whole figure.

2. The inertia moments of a semifigure about the principal central axes of the whole figure are half of inertia moments about corresponding principal central axes of the whole figure.

For example, consider the rectangle on the Fig. 27, a:

$$I_y(\text{semifigure } ABCD) = I_y(KK_1CD) + I_y(KAO) - I_y(K_1BO).$$

As far as $I_y(KAO) = I_y(K_1BO)$, we finally obtain

$$I_y = I_y(KK_1CD) = \frac{1}{2}\cdot\frac{bh^3}{12} = \frac{bh^3}{24}; \qquad I_z = \frac{1}{2}\cdot\frac{hb^3}{12} = \frac{hb^3}{24}.$$

For the semifigure on Fig. 27, c $\quad I_y = I_z = \frac{1}{2}\cdot\frac{\pi d^4}{64} = \frac{\pi d^4}{128}.$

For the semifigure on Fig. 27, d $\quad I_y = I_z = \frac{1}{2}\cdot\frac{a^4}{12} = \frac{a^4}{24}.$

For the semifigure on Fig. 27, e $\quad I_y = I_z = \frac{1}{2}\cdot\frac{5\sqrt{3}}{16}a^4 = \frac{5\sqrt{3}}{32}a^4.$

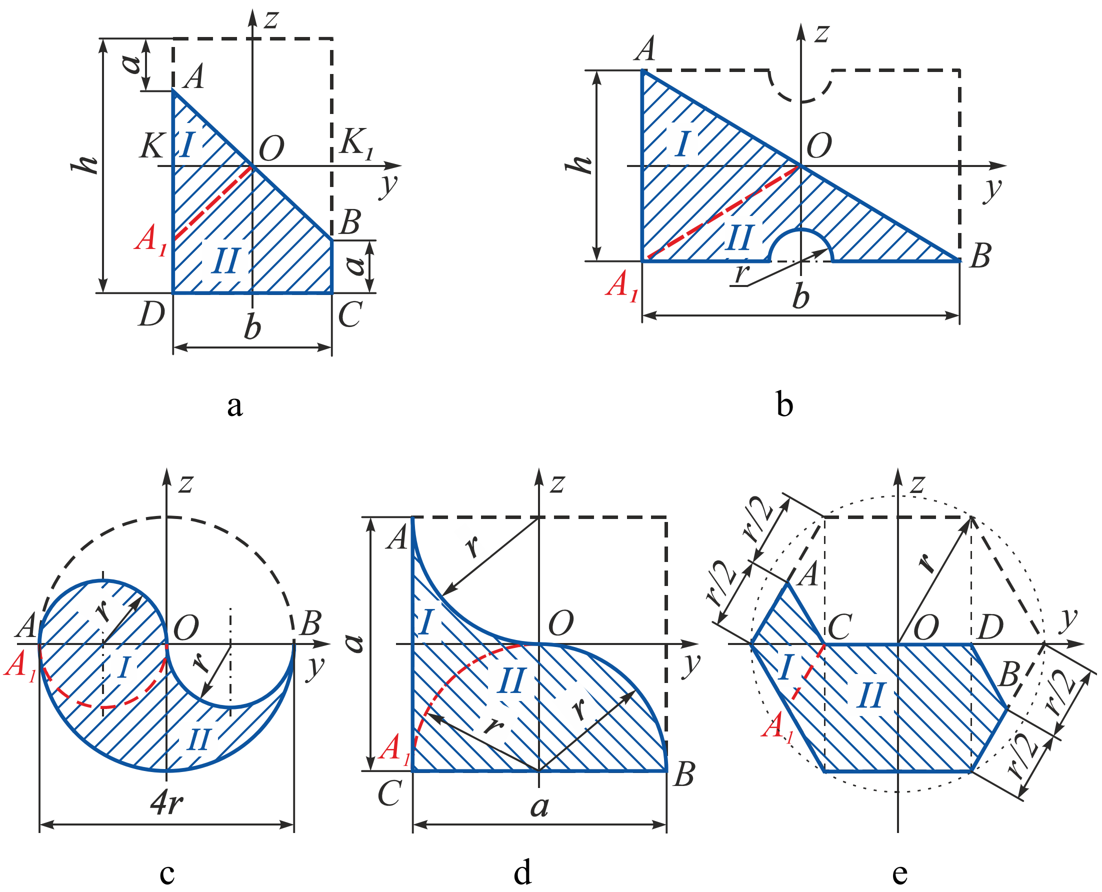


Fig. 27

3. If a figure has two (see Fig. 27, d) and more (see Fig. 27, c, e) ***pairs*** of mutually perpendicular symmetry axes, then any two mutually perpendicular axes passing through the gravity center of the whole figure, are the principal axes of the semifigure.

Example 6

Define the location of principal central inertia axes of given section and the values of principal inertia moments in this axial system (Fig. 28).

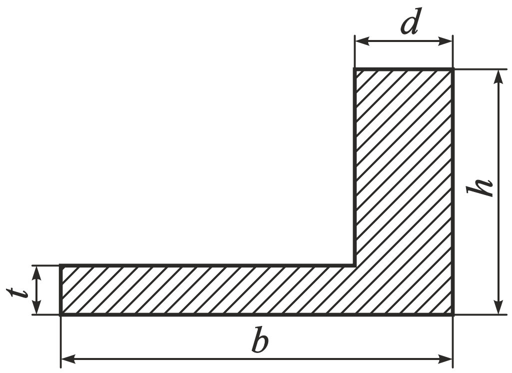


Fig. 28

Given: $h = 5\ cm$, $b = 8\ cm$, $d = 2\ cm$, $t = 1\ cm$.

Define y_c, z_c, α_0, I_u, I_v.

Solution

1. Make a dimensioned drawing of compound section.

2. Decompose the section into elemental parts (rectangle with dimensions $b \times h$ with excision $(b-d) \times (h-t)$) and assign the numbers 1 and 2 to these parts (Fig. 29).

3. Put the central coordinate systems $y_i O_i z_i$ in the gravity center of each elemental part of the section (in our case $y_1 O_1 z_1$ and $y_2 O_2 z_2$).

4. Find the coordinates of gravity center of the compound section.

Select the actual (basic) coordinate system that will be used to define the gravity center coordinates of the whole section. Take the axial system $y_1 O_1 z_1$ as basic one.

Then the formulas for calculating the gravity center coordinates take the following form

$$y_c = \frac{\sum_{i=1}^{2} S_{z_1}^{(i)}}{\sum_{i=1}^{2} A_i}; \qquad z_c = \frac{\sum_{i=1}^{2} S_{y_1}^{(i)}}{\sum_{i=1}^{2} A_i},$$

where the cross-section area is

$$A_1 = bh = 8 \cdot 5 = 40\ cm^2; \qquad A_2 = (b-d)(h-t) = (8-2)(5-1) = 24\ cm^2;$$

$$\sum_{i=1}^{2} A_i = A = A_1 - A_2 = 40 - 24 = 16\ cm^2;$$

static moment about the axis z_1

$$S_{z_1} = \sum_{i=1}^{2} S_{z_1}^{(i)} = S_{z_1}^{(1)} - S_{z_1}^{(2)} = A_1 \cdot 0 - A_2\left(-\frac{d}{2}\right) = 0 - 24 \cdot (-1) = 24\ cm^3;$$

static moment about the axis y_1

$$S_{y_1} = \sum_{i=1}^{2} S_{y_1}^{(i)} = S_{y_1}^{(1)} - S_{y_1}^{(2)} = A_1 \cdot 0 - A_2\left(\frac{t}{2}\right) = 0 - 24 \cdot \frac{1}{2} = -12\ cm^3.$$

Notes

1. When calculating static moments, it is necessary to take into account the signs of gravity center coordinates of elemental parts of the section in actual (basic) coordinate system.
2. If a compound section has an excision (cutout), then the geometric characteristics of this excision (area, static moments, inertia moments) should be taken away.

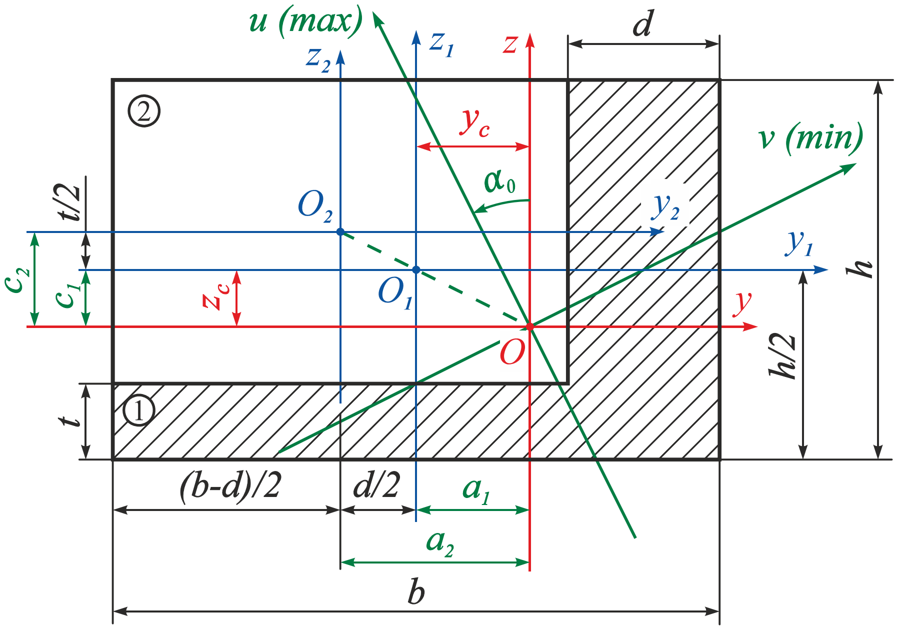


Fig. 29

Calculate the gravity center coordinates (point O) of the compound section in the axial system $y_1 O_1 z_1$:

$$y_c = \frac{S_{z_1}}{A} = \frac{24}{16} = 1{,}5\ cm; \qquad z_c = \frac{S_{y_1}}{A} = -\frac{12}{16} = -0{,}75\ cm.$$

According to the results of the calculations show on Fig. 29 the system of central axes yOz and the gravity center of the compound section (point O).

Define the coordinates of the gravity centers of the elemental parts of this section (points O_1 and O_2) in the system of central axes yOz:

$$a_1 = -y_c = -1{,}5\ cm; \qquad a_2 = -\left(y_c + \frac{d}{2}\right) = -\left(1{,}5 + \frac{2}{2}\right) = -2{,}5\ cm;$$

$$c_1 = |z_c| = 0{,}75\ cm; \qquad c_2 = |z_c| + \frac{t}{2} = |-0{,}75| + \frac{1}{2} = 1{,}25\ cm.$$

Verification of defining a gravity center:

a) graphical check

To validate the location of gravity center connect the points O_1 and O_2 by dot line. The point O should lie on that line.

Note | A gravity center of compound section consisting of two elemental parts ***always lies on a line connecting the gravity centers of elemental parts***, and the ratio of distances from gravity center of the whole figure to gravity centers of elementals is inversely proportional to the ratio of elemental parts' areas.

In this case

$$\frac{|OO_1|}{|OO_2|} = \frac{A_2}{A_1};$$

$$|OO_1| = \sqrt{a_1^2 + c_1^2} = 1{,}677\ cm; \qquad |OO_2| = \sqrt{a_2^2 + c_2^2} = 2{,}795\ cm;$$

$$\frac{|OO_1|}{|OO_2|} = \frac{1{,}677}{2{,}795} = 0{,}6; \qquad \frac{A_2}{A_1} = \frac{40}{24} = 0{,}6; \qquad 0{,}6 = 0{,}6;$$

b) analytic check

Use the property of central axes that the static moment of a compound section about the central axes equals zero:

$$S_y = \sum_{i=1}^{2} S_y^{(i)} = S_y^{(1)} - S_y^{(2)} = bhc_1 - (b-d)(h-t)c_2 = 60 - 60 = 0\ cm^3;$$

$$S_z = \sum_{i=1}^{2} S_z^{(i)} = S_z^{(1)} - S_z^{(2)} = bha_1 - (b-d)(h-t)a_2 = 30 - 30 = 0\ cm^3.$$

Therefore, the location of gravity center of the compound section is defined correctly.

5. Find inertia moments of compound section in the system of central axes yOz.

Define axial and centrifugal inertia moments of elemental parts of the section: for the first rectangle

$$I_{y_1} = \frac{bh^3}{12} = \frac{8 \cdot 5^3}{12} = 83{,}333\ cm^4;$$

$$I_{z_1} = \frac{hb^3}{12} = \frac{5 \cdot 8^3}{12} = 213{,}333\ cm^4; \qquad I_{y_1 z_1} = 0;$$

for the second rectangle

$$I_{y_2} = \frac{(b-d)(h-t)^3}{12} = \frac{6 \cdot 4^3}{12} = 32\ cm^4;$$

$$I_{z_2} = \frac{(h-t)(b-d)^3}{12} = \frac{4 \cdot 6^3}{12} = 72\ cm^4; \qquad I_{y_2 z_2} = 0.$$

Tabulate preliminary results into Table 1.

Table 1

Part of the section	Geometric characteristics					
	A_i, cm^2	I_{y_i}, cm^4	I_{z_i}, cm^4	$I_{y_i z_i}, cm^4$	a_i, cm	c_i, cm
1	40	83,333	213,333	0	-1,5	0,75
2	24	32	72	0	-2,5	1,25

Note All the following calculations need to be performed in the system of ***central axes*** yOz of compound section.

Find axial and centrifugal inertia moments of the compound section in the system of central axes yOz, using the formulas of parallel translation:

$$I_y = I_y^{(1)} - I_y^{(2)} = 105{,}833 - 69{,}5 = 36{,}333\ cm^4;$$
$$I_y^{(1)} = I_{y_1} + c_1^2 A_1 = 83{,}333 + 0{,}75^2 \cdot 40 = 105{,}833\ cm^4;$$
$$I_y^{(2)} = I_{y_2} + c_2^2 A_2 = 32 + 1{,}25^2 \cdot 24 = 69{,}5\ cm^4;$$

$$I_z = I_z^{(1)} - I_z^{(2)} = 303{,}333 - 222{,}0 = 81{,}333\ cm^4;$$
$$I_z^{(1)} = I_{z_1} + a_1^2 A_1 = 213{,}333 + (-1{,}5)^2 \cdot 40 = 303{,}333\ cm^4;$$
$$I_z^{(2)} = I_{z_2} + a_2^2 A_2 = 72 + (-2{,}5)^2 \cdot 24 = 222{,}0\ cm^4;$$

$$I_{yz} = I_{yz}^{(1)} - I_{yz}^{(2)} = -45{,}0 - (-75{,}0) = 30{,}0\ cm^4;$$
$$I_{yz}^{(1)} = I_{y_1 z_1} + a_1 c_1 A_1 = 0 + (-1{,}5) \cdot 0{,}75 \cdot 40 = -45{,}0\ cm^4;$$
$$I_{yz}^{(2)} = I_{y_2 z_2} + a_2 c_2 A_2 = 0 + (-2{,}5) \cdot 1{,}25 \cdot 24 = -75{,}0\ cm^4.$$

Notes

1. The values of axial inertia moments about central axes must be positive, which stems from the definition of an axial inertia moment.
2. If a large area of a compound section belongs to the first and the third quarters then the centrifugal inertia moment is positive, if it belongs to the second and the fourth quarters then the centrifugal inertia moment is negative.

6. Calculate the position of principal inertia axes of the section:

$$\operatorname{tg} 2\alpha_0 = \frac{2I_{yz}}{I_z - I_y} = \frac{2 \cdot 30{,}0}{81{,}333 - 36{,}333} = 1{,}333;$$

$$2\alpha_0 = 53{,}123°; \qquad \alpha_0 = 26{,}5615° = 26°34'.$$

As far as $\alpha_0 > 0$ the rotation of axes y and z by this angle should be counter-clockwise according to the conventional sign rule. The axes obtained as a result of this rotation are principal central inertia axes of the compound section.

7. Define the values of principal inertia moments of the section:

$$I_{\substack{max \\ min}} = I_{\substack{u \\ v}} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_z - I_y}{2}\right)^2 + I_{y_0 z_0}^2} =$$

$$= \frac{36{,}333 + 81{,}333}{2} \pm \sqrt{\left(\frac{81{,}333 - 36{,}333}{2}\right)^2 + 30{,}0^2} = 58{,}833 \pm 37{,}5\ cm^4.$$

Therefore

$$I_{max} = I_u = 96{,}333\ cm^4;$$

$$I_{min} = I_v = 21{,}333\ cm^4.$$

Note	The values of principal central inertia moments must be positive, which stems from the definition of axial inertia moment.

Using the results of calculations show on Fig. 29 the principal central inertia axes of the compound section, namely axes u and v.

As long as $I_z > I_y$, then the axial inertia moment gets its maximum value relative to the axis located at minimum angular distance from the axis z.

8. Validate the solution:

a) check if the correlation is fulfilled

$$I_{max} > I_z > I_y > I_{min}\ (\text{if } I_z > I_y) \text{ or } I_{max} > I_y > I_z > I_{min}\ (\text{if } I_y > I_z).$$

In the considered case

$$96{,}333 > 81{,}333 > 36{,}333 > 21{,}333;$$

b) check the permanence of the sum of axial moment values with the rotation of axes:

$$I_{max} + I_{min} = I_z + I_y;$$

$$96{,}333 + 21{,}333 = 117{,}666; \qquad 81{,}333 + 36{,}333 = 117{,}666;$$

$$117{,}666 = 117{,}666;$$

c) calculate the centrifugal inertia moment about principal central axes which a priori must be equal to zero:

$$I_{uv} = I_{yz} \cos 2\alpha_0 + \frac{I_y - I_z}{2} \sin 2\alpha_0 =$$

$$= 30{,}0 \cdot \cos(2 \cdot 26{,}5615) + \frac{36{,}333 - 81{,}333}{2} \cdot \sin(2 \cdot 26{,}5615) =$$

$$= 30{,}0 \cdot 0{,}6001 + (-22{,}5) \cdot 0{,}7999 = 18{,}003 - 17{,}9978 = 0{,}0052\ cm^4.$$

Computational error

$$\Delta\% = \left|\frac{0{,}0052}{18{,}003}\right| \cdot 100\ \% = 0{,}0289\ \% \leq 1\ \%,$$

hence, the problem is solved correctly.

Notes

1. This task can be solved using different options of decomposing the section into parts (in the example 5 of the section 3 to define the gravity center coordinates another decomposition pattern was used).
2. The decomposition should be performed in a way that helps minimize calculations.
3. The final result should be the same for all options of decomposition.
4. If a compound section consists of two identical parts located at 90° relative to each other, then the gravity center of the whole section lies at the center of a line connecting the gravity centers of elemental parts; one of the principal central axes coincides with this line and the other is perpendicular to it (Fig. 30).

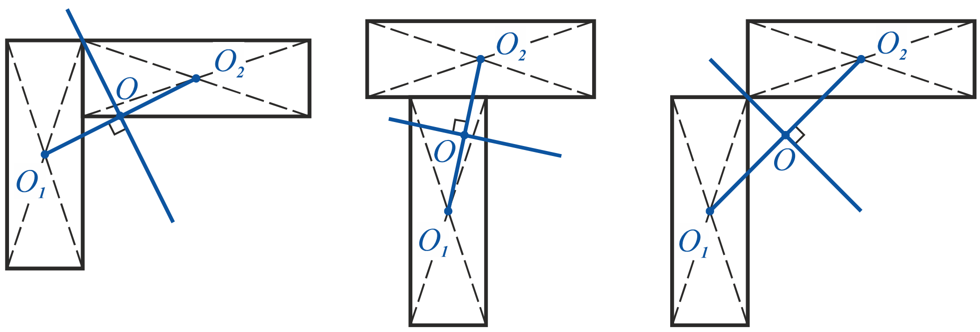


Fig. 30

Example 7

Define the location of principal central inertia axes of a given section and the values of principal inertia moments in the system of these axes (Fig. 31).

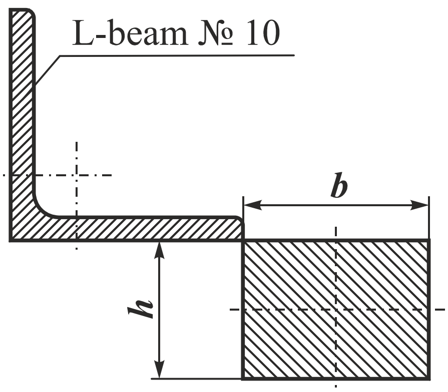


Fig. 31

Given: $h = 6\ cm$, $b = 8\ cm$, L-beam № 10.

Define $y_c, z_c, \alpha_0, I_u, I_v$.

Solution

1. Make a dimensioned drawing of the compound section.

2. Decompose the section into elemental parts (an L-beam and a rectangle) and give these parts the numbers 1 and 2.

3. Put the central coordinate systems $y_i O_i z_i$ (in the considered case $y_1 O_1 z_1$ and $y_2 O_2 z_2$) in the gravity center of each elemental part of the section.

4. Calculate the inertia moments and the area of the rectangle, copy the geometric characteristics of an L-beam № 10 needed for solving this problem from the assortment tables, and tabulate them (Table 2).

Table 2

Part of the section	Geometric characteristics							
	h_i, cm	b_i, cm	A_i, cm^2	I_{y_i}, cm^4	I_{z_i}, cm^4	$I_{y_i z_i}, cm^4$	y_0, cm	z_0, cm
1 (L-beam)	10	10	19,24	178,95	178,95	–110	2,83	2,83
2 (rectangle)	6	8	48	144	256	0	–	–

For the rectangle

$$A_2 = bh = 8 \cdot 6 = 48\ cm^2; \qquad I_{y_2 z_2} = 0;$$

$$I_{y_2} = \frac{bh^3}{12} = \frac{8 \cdot 6^3}{12} = 144\ cm^4; \qquad I_{z_2} = \frac{hb^3}{12} = \frac{6 \cdot 8^3}{12} = 256\ cm^4.$$

Notes

1. If a compound section includes rolled profiles as elemental parts, then their geometric characteristics should be copied from the assortment tables.
2. For the equilateral L-beam $I_y = I_z$.

3. The sign of centrifugal moment of an L-beam is defined by its location relative to own central axes (Fig. 32).

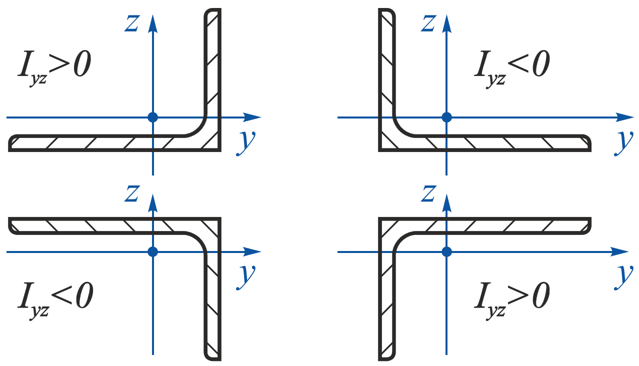


Fig. 32

5. Find the gravity center coordinates of the compound section.

Select an actual (basic) coordinate system to define the coordinates of the whole section gravity center. Here we take the axial system in which the whole section belongs to the first quarter ($y_3O_3z_3$) as basic one (Fig. 33).

Note If the actual (basic) coordinate system is chosen in a way that the whole section lies in the first quarter, then the static moments of this section about the basic axes and its gravity center coordinates are positive, which reduces an error probability in finding the gravity center.

Then the formulas for defining the gravity center coordinates take the form

$$y_c = \frac{\sum_{i=1}^{2} S_{z_3}^{(i)}}{\sum_{i=1}^{2} A_i}; \qquad z_c = \frac{\sum_{i=1}^{2} S_{y_3}^{(i)}}{\sum_{i=1}^{2} A_i},$$

where $$\sum_{i=1}^{2} A_i = A = A_1 + A_2 = 19{,}24 + 48 = 67{,}24\ cm^2;$$

$$S_{z_3} = \sum_{i=1}^{2} S_{z_3}^{(i)} = S_{z_3}^{(1)} + S_{z_3}^{(2)} = A_1 \cdot y_0 + A_2\left(b_1 + \frac{b_2}{2}\right) =$$

$$= 19{,}24 \cdot 2{,}83 + 48 \cdot \left(10 + \frac{8}{2}\right) = 726{,}449\ cm^3;$$

$$S_{y_3} = \sum_{i=1}^{2} S_{y_3}^{(i)} = S_{y_3}^{(1)} + S_{y_3}^{(2)} = A_1 \cdot (h_2 + z_0) + A_2 \left(\frac{h_2}{2}\right) =$$

$$= 19{,}24 \cdot (6 + 2{,}83) + 48 \cdot \frac{6}{2} = 313{,}889\ cm^3.$$

Calculate the gravity center coordinates O of the compound section in the axial system $y_3O_3z_3$:

$$y_c = \frac{S_{z_3}}{A} = \frac{726{,}449}{67{,}24} = 10{,}804\ cm; \qquad z_c = \frac{S_{y_3}}{A} = \frac{313{,}889}{67{,}24} = 4{,}668\ cm.$$

According to the results of calculations shoe on Fig. 33 the system of central axes yOz and the gravity center of the compound section O.

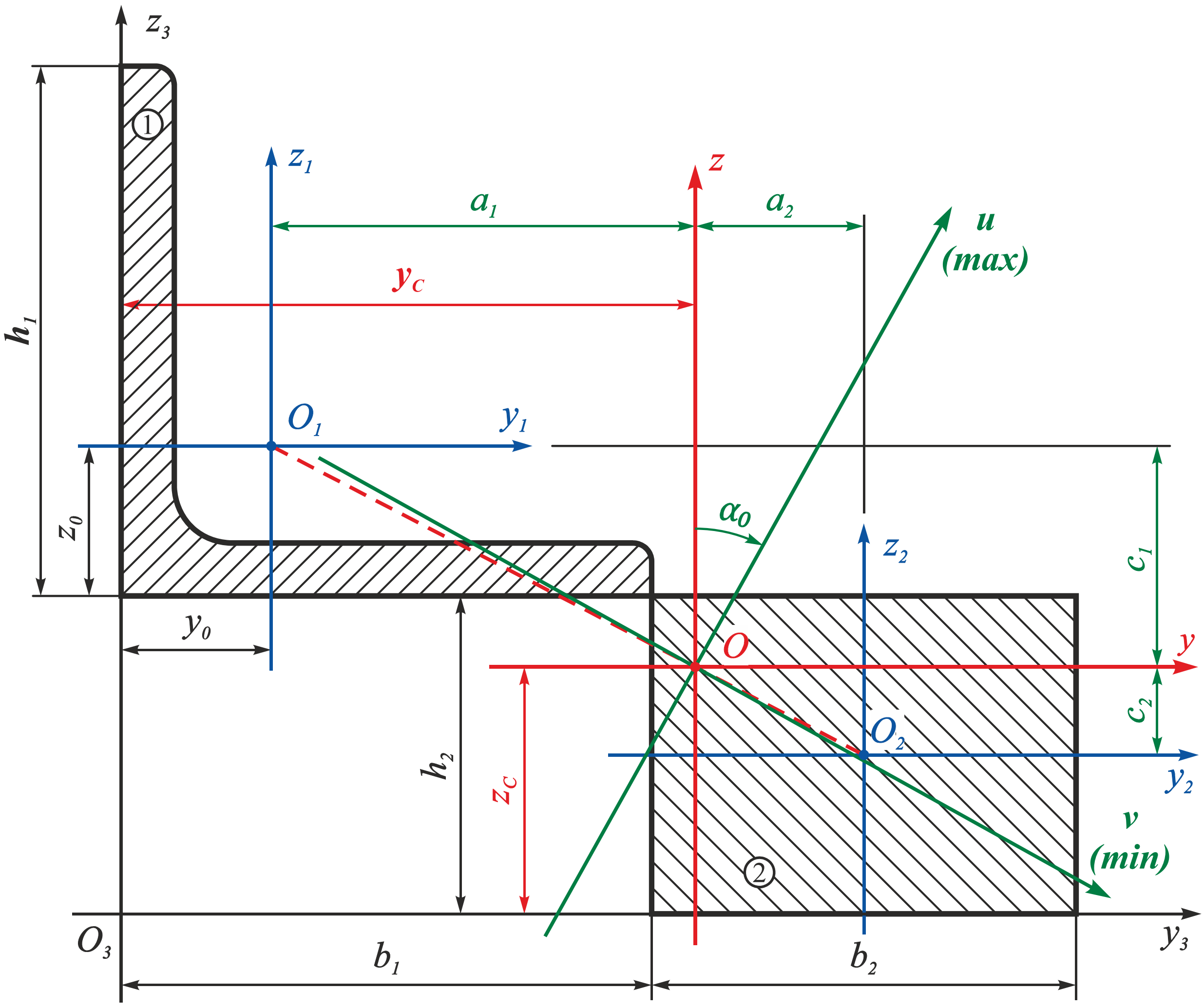


Fig. 33

Define the gravity center coordinates of elemental parts of the section (points O_1 and O_2) in the system of central axes yOz:

$$a_1 = -(y_c - y_0) = -(10{,}804 - 2{,}83) = -7{,}974\ cm;$$

$$a_2 = b_1 + \frac{b_2}{2} - y_c = 10 + \frac{8}{2} - 10{,}804 = 3{,}196\ cm;$$

$$c_1 = h_2 + z_0 - z_c = 6 + 2{,}83 - 4{,}668 = 4{,}162\ cm;$$
$$c_2 = -\left(z_c - \frac{h_2}{2}\right) = -\left(4{,}668 - \frac{6}{2}\right) = -1{,}668\ cm.$$

Verification of defining a gravity center:

a) graphical check

To validate the location of gravity center connect the points O_1 and O_2 by dot line. The point O should lie on that line (see Fig. 33).

Note A gravity center of a compound section consisting of two elemental parts ***always lies on a line connecting the gravity centers of elemental parts.*** As it is, the point O divides the sector O_1O_2 into two parts inversely proportional to the elemental parts' areas:

$$\frac{|OO_1|}{|OO_2|} = \frac{A_2}{A_1};$$

$$|OO_1| = \sqrt{a_1^2 + c_1^2} = 8{,}9976\ cm; \qquad |OO_2| = \sqrt{a_2^2 + c_2^2} = 3{,}6051\ cm;$$

$$\frac{|OO_1|}{|OO_2|} = \frac{8{,}9976}{3{,}6051} = 2{,}496; \qquad \frac{A_2}{A_1} = \frac{48}{19{,}24} = 2{,}495; \qquad 2{,}496 \approx 2{,}495;$$

b) analytic check

Use the property of central axes that the static moment of a compound section about the central axes equals zero:

$$S_y = \sum_{i=1}^{2} S_y^{(i)} = S_y^{(1)} + S_y^{(2)} = A_1c_1 + A_2c_2 = 19{,}24 \cdot 4{,}162 + 48 \cdot (-1{,}668) =$$
$$= 80{,}077 - 80{,}064 = 0{,}013\ cm^3.$$

Relative error

$$\Delta\% = \frac{0{,}013}{80{,}077} \cdot 100\ \% = 0{,}0162\ \% < 1\ \%; \qquad [\Delta\%] \le 1\ \%;$$

$$S_z = \sum_{i=1}^{2} S_z^{(i)} = S_z^{(1)} + S_z^{(2)} = A_1a_1 + A_2a_2 = 19{,}24 \cdot (-7{,}974) + 48 \cdot 3{,}196 =$$
$$= -153{,}412 + 153{,}408 = -0{,}004\ cm^3.$$

Relative error

$$\Delta\% = \left|\frac{-0{,}004}{153{,}408}\right| \cdot 100\ \% = 0{,}0026\ \% < 1\ \%; \qquad [\Delta\%] \le 1\ \%.$$

Therefore, the location of gravity center of the compound section is defined correctly.

6. Find the axial and centrifugal inertia moments of the compound section in the system of central axes yOz, using the formulas of parallel translation:

$$I_y = I_y^{(1)} + I_y^{(2)} = 512{,}230 + 280{,}769 = 792{,}999\ cm^4;$$

$$I_y^{(1)} = I_{y_1} + c_1^2 A_1 = 178{,}95 + 4{,}162^2 \cdot 19{,}24 = 512{,}230\ cm^4;$$

$$I_y^{(2)} = I_{y_2} + c_2^2 A_2 = 144 + (-1{,}668)^2 \cdot 48 = 280{,}769\ cm^4;$$

$$I_z = I_z^{(1)} + I_z^{(2)} = 1402{,}319 + 746{,}292 = 2148{,}611\ cm^4;$$

$$I_z^{(1)} = I_{z_1} + a_1^2 A_1 = 178{,}95 + (-7{,}974)^2 \cdot 19{,}24 = 1402{,}319\ cm^4;$$

$$I_z^{(2)} = I_{z_2} + a_2^2 A_2 = 256 + (3{,}196)^2 \cdot 48 = 746{,}292\ cm^4;$$

$$I_{yz} = I_{yz}^{(1)} + I_{yz}^{(2)} = -748{,}533 + (-255{,}885) = -1004{,}418\ cm^4;$$

$$I_{yz}^{(1)} = I_{y_1 z_1} + a_1 c_1 A_1 = -110 + (-7{,}974) \cdot 4{,}162 \cdot 19{,}24 = -748{,}533\ cm^4;$$

$$I_{yz}^{(2)} = I_{y_2 z_2} + a_2 c_2 A_2 = 0 + 3{,}196 \cdot (-1{,}668) \cdot 48 = -255{,}885\ cm^4.$$

Notes

1. The values of axial inertia moments about central axes must be positive, which stems from the definition of an axial inertia moment.
2. If a large area of a compound section belongs to the first and the third quarters then the centrifugal inertia moment is positive, if it belongs to the second and the fourth quarters then the centrifugal inertia moment is negative.

7. Calculate the location of principal inertia axes of the section:

$$\mathrm{tg}\, 2\alpha_0 = \frac{2I_{yz}}{I_z - I_y} = \frac{2 \cdot (-1004{,}418)}{2148{,}611 - 792{,}999} = -1{,}482;$$

$$2\alpha_0 = -55{,}99°; \qquad \alpha_0 = -27{,}995° = -27°59'42''.$$

As far as $\alpha_0 < 0$ the rotation of axes y and z by this angle should be clockwise according to the conventional sign rule. The axes obtained as a result of this rotation are principal central inertia axes of the compound section.

8. Define the values of principal inertia moments of the section:

$$I_{\substack{max \\ min}} = I_{\substack{u \\ v}} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_z - I_y}{2}\right)^2 + I_{y_0 z_0}^2} = \frac{792{,}999 + 2148{,}611}{2} \pm$$

$$\pm \sqrt{\left(\frac{2148{,}611 - 792{,}999}{2}\right)^2 + (-1004{,}418)^2} = 1470{,}805 \pm 1211{,}725\ cm^4.$$

Therefore

$$I_{max} = I_u = 2682{,}53\ cm^4;$$
$$I_{min} = I_v = 259{,}08\ cm^4.$$

Note	The values of principal central inertia moments should be positive, which stems from the axial inertia moment definition.

Using the results of calculations show on Fig. 33 the principal central inertia axes of the compound section, namely axes u and v.

As long as $I_z > I_y$, then the axial inertia moment gets its maximum value relative to the axis located at minimum angular distance from the axis z.

9. Validate the solution:

a) check if the correlation is fulfilled

$$I_{max} > I_z > I_y > I_{min}\ (\text{if } I_z > I_y) \text{ or } I_{max} > I_y > I_z > I_{min}\ (\text{if } I_y > I_z).$$

In the considered case

$$2682{,}53 > 2148{,}611 > 792{,}999 > 259{,}08.$$

b) check the permanence of the sum of axial moment values with the rotation of axes:

$$I_{max} + I_{min} = I_z + I_y;$$

$$2682{,}53 + 259{,}08 = 2941{,}61; \qquad 2148{,}611 + 792{,}999 = 2941{,}61;$$

$$2941{,}61 = 2941{,}61;$$

c) calculate the centrifugal inertia moment about principal central axes, which a priori equals zero:

$$I_{uv} = I_{yz}\cos 2\alpha_0 + \frac{I_y - I_z}{2}\sin 2\alpha_0 =$$

$$= -1004{,}418 \cdot \cos(2 \cdot (-27{,}995)) + \frac{792{,}999 - 2148{,}611}{2} \cdot \sin(2 \cdot (-27{,}995)) =$$

$$= -561{,}808 + 561{,}860 = 0{,}052\ cm^4.$$

Computational error

$$\Delta\% = \left|\frac{0{,}052}{561{,}860}\right| \cdot 100\ \% = 0{,}0093\ \% \le 1\ \%,$$

therefore, the problem is solved correctly.

Example 8

Define the location of principal central inertia axes of the given section and the values of principal inertia moments in this axial system (Fig. 34).

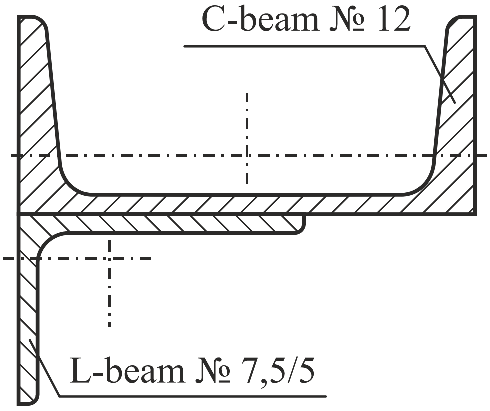


Fig. 34

Given: C-beam № 12, L-beam № 7,5/5.

Define $y_C, z_C, \alpha_0, I_u, I_v$.

Solution

1. Make a dimensioned drawing of the compound section.

2. Decompose the section into elemental parts (a C-beam and an L-beam) and give these parts the numbers 1 and 2, respectively.

3. Put the central coordinate systems $y_i O_i z_i$ in the gravity center of each elemental part of the section (in the considered case $y_1 O_1 z_1$ and $y_2 O_2 z_2$).

4. Copy the geometric characteristics of a C-beam № 12 and an L-beam № 7,5/5 needed for solving this problem from the assortment tables, and tabulate them (Table 3).

Table 3

Part of the section	Geometric characteristics							
	h_i, cm	b_i, cm	A_i, cm^2	I_{y_i}, cm^4	I_{z_i}, cm^4	$I_{y_i z_i}, cm^4$	y_{0_i}, cm	z_{0_i}, cm
1 (C-beam)	12,0	5,2	13,3	31,2	304,0	0	–	1,54
2 (L-beam)	5,0	7,5	6,11	12,47	34,81	12,0	2,39	1,17

Notes

1. If a compound section includes rolled profiles as elemental parts, then their geometric characteristics should be copied from the assortment tables. In this case the position of these parts relative to their own central axes should be taken into account.

2. As far as the axis y_1 is a symmetry axis of a C-beam, then the centrifugal inertia moment of a C-beam is $I_{y_1 z_1} = 0$.

3. The sign of centrifugal moment of an L-beam is defined by its location relative to its own central axes (see Fig. 32).

5. Find the gravity center coordinates of the compound section.

Select an actual (basic) coordinate system to define the coordinates of the whole section gravity center. Here we take the central axial system of a C-beam $y_1 O_1 z_1$ as basic one.

Then the formulas for defining the gravity center coordinates take the form

$$y_c = \frac{\sum_{i=1}^{2} S_{z_1}^{(i)}}{\sum_{i=1}^{2} A_i}; \qquad z_c = \frac{\sum_{i=1}^{2} S_{y_1}^{(i)}}{\sum_{i=1}^{2} A_i},$$

where $\sum_{i=1}^{2} A_i = A = A_1 + A_2 = 13{,}3 + 6{,}11 = 19{,}41\ cm^2$;

$$S_{z_1} = \sum_{i=1}^{2} S_{z_1}^{(i)} = S_{z_1}^{(1)} + S_{z_1}^{(2)} = A_1 \cdot 0 + A_2 \left(-\left(\frac{h_1}{2} - y_{0_2}\right)\right) =$$
$$= 13{,}3 \cdot 0 + 6{,}11 \cdot \left(-\left(\frac{12}{2} - 2{,}39\right)\right) = -22{,}057\ cm^3;$$

$$S_{y_1} = \sum_{i=1}^{2} S_{y_1}^{(i)} = S_{y_1}^{(1)} + S_{y_1}^{(2)} = A_1 \cdot 0 + A_2 \left(-\left(z_{0_1} + z_{0_2}\right)\right) =$$
$$= 13{,}3 \cdot 0 + 6{,}11 \cdot \left(-(1{,}54 + 1{,}17)\right) = -16{,}558\ cm^3.$$

Calculate the gravity center coordinates O of the compound section in the axial system $y_1 O_1 z_1$:

$$y_c = \frac{S_{z_1}}{A} = \frac{-22{,}057}{19{,}41} = -1{,}136\ cm; \qquad z_c = \frac{S_{y_1}}{A} = \frac{-16{,}558}{19{,}41} = -0{,}853\ cm.$$

According to the results of the calculations, show on Fig. 35 the system of central axes yOz and the gravity center of the compound section (point O).

Define the coordinates of the gravity centers of the elemental parts of the compound section (points O_1 and O_2) in the system of central axes yOz:

$$a_1 = |y_c| = 1{,}136\ cm;$$

$$a_2 = -\left(\frac{h_1}{2} - y_{0_2} - |y_c|\right) = -\left(\frac{12}{2} - 2{,}39 - 1{,}136\right) = -2{,}474\ cm;$$

$$c_1 = |z_c| = 0{,}853\ cm;$$

$$c_2 = -\left(z_{0_1} + z_{0_2} - |z_c|\right) = -(1{,}54 + 1{,}17 - 0{,}853) = -1{,}857\ cm.$$

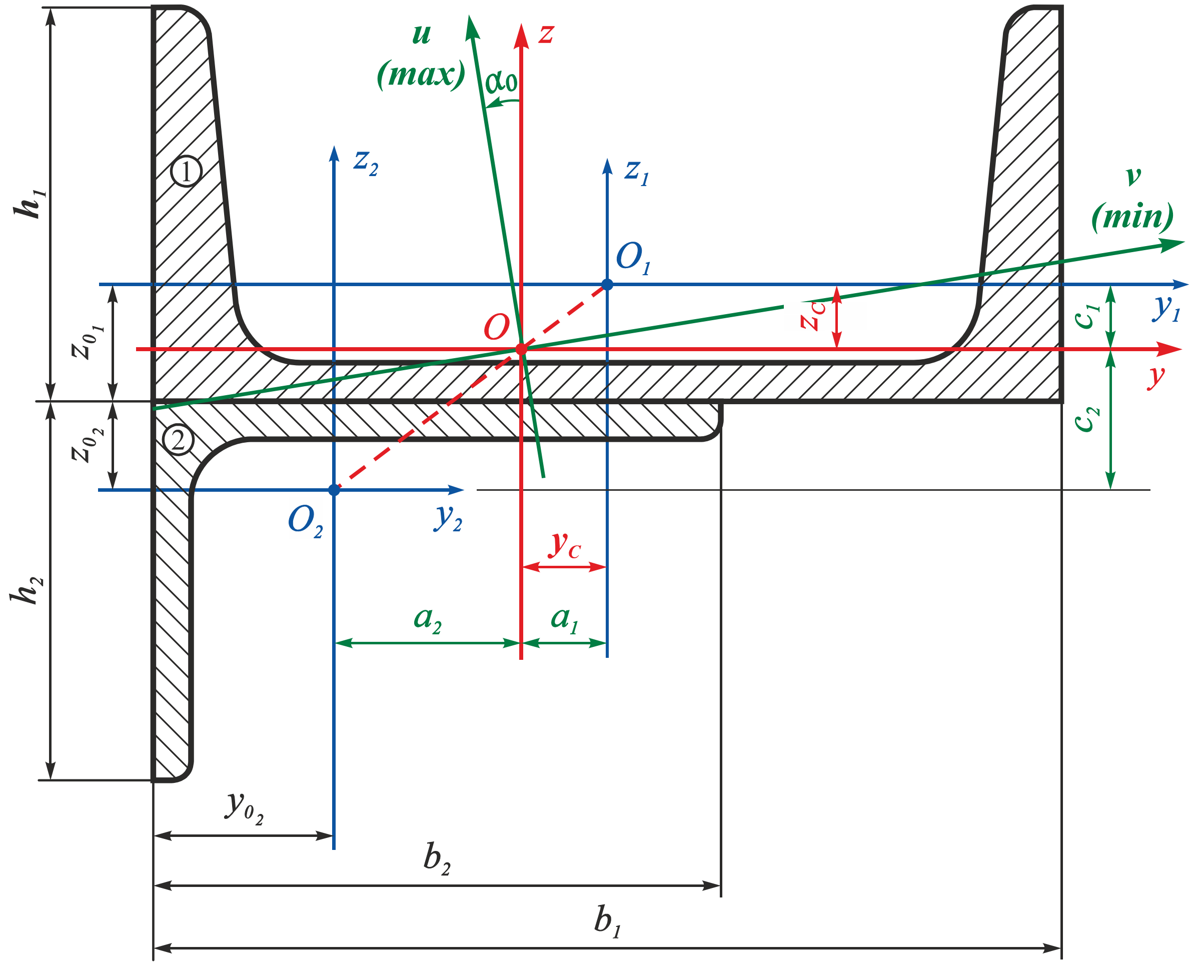


Fig. 35

<u>**Verification of defining a gravity center**</u>:

a) <u>graphical check</u>

To validate the location of gravity center connect the points O_1 and O_2 by dot line. The point O should lie on that line.

<u>Note</u> A gravity center of a compound section consisting of two elemental parts ***always lies on a line connecting the gravity centers of elemental parts.*** As it is, the point O divides the sector O_1O_2 into two parts inversely proportional to the elemental parts' areas:

$$\frac{|OO_1|}{|OO_2|} = \frac{A_2}{A_1};$$

$$|OO_1| = \sqrt{a_1^2 + c_1^2} = 1{,}421\ cm; \qquad |OO_2| = \sqrt{a_2^2 + c_2^2} = 3{,}094\ cm;$$

$$\frac{|OO_1|}{|OO_2|} = \frac{1{,}421}{3{,}094} = 0{,}4593; \qquad \frac{A_2}{A_1} = \frac{6{,}11}{13{,}3} = 0{,}4994; \qquad 0{,}4593 \approx 0{,}4994;$$

b) analytic check

Use the property of central axes that the static moment of a compound section about the central axes equals zero:

$$S_y = \sum_{i=1}^{2} S_y^{(i)} = S_y^{(1)} + S_y^{(2)} = A_1 c_1 + A_2 c_2 = 13{,}3 \cdot 0{,}853 + 6{,}11 \cdot (-1{,}857) =$$

$$= 11{,}3449 - 11{,}3463 = -0{,}0014\ cm^3.$$

Relative error

$$\Delta\% = \left|\frac{-0{,}0014}{11{,}3449}\right| \cdot 100\ \% = 0{,}01234\ \% < 1\ \%; \qquad [\Delta\%] \leq 1\ \%;$$

$$S_z = \sum_{i=1}^{2} S_z^{(i)} = S_z^{(1)} + S_z^{(2)} = A_1 a_1 + A_2 a_2 = 13{,}3 \cdot 1{,}136 + 6{,}11 \cdot (-2{,}474) =$$

$$= 15{,}1088 - 15{,}1161 = -0{,}0073\ cm^3.$$

Relative error

$$\Delta\% = \left|\frac{-0{,}0073}{15{,}1088}\right| \cdot 100\ \% = 0{,}00483\ \% < 1\ \%; \qquad [\Delta\%] \leq 1\ \%.$$

Therefore, the location of gravity center of the compound section is defined correctly.

6. Find the axial and centrifugal inertia moments of the compound section in the system of central axes yOz using the formulas of parallel translation:

$$I_y = I_y^{(1)} + I_y^{(2)} = 40{,}977 + 33{,}540 = 74{,}517\ cm^4;$$

$$I_y^{(1)} = I_{y_1} + c_1^2 A_1 = 31{,}3 + 0{,}853^2 \cdot 13{,}3 = 40{,}977\ cm^4;$$

$$I_y^{(2)} = I_{y_2} + c_2^2 A_2 = 12{,}47 + (-1{,}857)^2 \cdot 6{,}11 = 33{,}540\ cm^4;$$

$$I_z = I_z^{(1)} + I_z^{(2)} = 321{,}164 + 72{,}207 = 393{,}371\ cm^4;$$

$$I_z^{(1)} = I_{z_1} + a_1^2 A_1 = 304{,}0 + 1{,}136^2 \cdot 13{,}3 = 321{,}164\ cm^4;$$

$$I_z^{(2)} = I_{z_2} + a_2^2 A_2 = 34{,}81 + (-2{,}474)^2 \cdot 6{,}11 = 72{,}207\ cm^4;$$

$$I_{yz} = I_{yz}^{(1)} + I_{yz}^{(2)} = 12{,}888 + 28{,}071 = 52{,}959\ cm^4;$$

$$I_{yz}^{(1)} = I_{y_1 z_1} + a_1 c_1 F_1 = 0 + 1{,}136 \cdot 0{,}853 \cdot 13{,}3 = 12{,}888\ cm^4;$$

$$I_{yz}^{(2)} = I_{y_2 z_2} + a_2 c_2 F_2 = 12 + (-2{,}474) \cdot (-1{,}857) \cdot 6{,}11 = 40{,}071\ cm^4.$$

Notes | 1. The values of axial inertia moments about central axes must be positive, which stems from the definition of an axial inertia moment.
2. If a large area of a compound section belongs to the first and the third quarters then the centrifugal inertia moment is positive, if it belongs to the second and the fourth quarters then the centrifugal inertia moment is negative.

7. Calculate the location of principal inertia axes of the section:

$$\operatorname{tg} 2\alpha_0 = \frac{2I_{yz}}{I_z - I_y} = \frac{2 \cdot 52{,}959}{393{,}371 - 74{,}517} = 0{,}332;$$

$$2\alpha_0 = 18{,}366°; \qquad \alpha_0 = 9{,}183° = 9°10'59''.$$

As far as $\alpha_0 > 0$ the rotation of axes y and z by this angle should be counterclockwise according to the conventional sign rule. The axes obtained as a result of this rotation are principal central inertia axes of the compound section.

8. Define the values of principal inertia moments of the section:

$$I_{\substack{max \\ min}} = I_{\substack{u \\ v}} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_z - I_y}{2}\right)^2 + I_{yz}^2} = \frac{74{,}517 + 393{,}371}{2} \pm$$

$$\pm \sqrt{\left(\frac{393{,}371 - 74{,}517}{2}\right)^2 + (52{,}959)^2} = 233{,}944 \pm 167{,}993\ cm^4.$$

Therefore,

$$I_{max} = I_u = 401{,}937\ cm^4;$$
$$I_{min} = I_v = 65{,}951\ cm^4.$$

Note | The values of principal central inertia moments should be positive, which stems from the axial inertia moment definition.

Using the results of calculations show on Fig. 35 the principal central inertia axes of the compound section, namely axes u and v.

As long as $I_z > I_y$, then the axial inertia moment gets its maximum value relative to the axis located at minimum angular distance from the axis z.

9. Validate the solution:

a) check if the correlation is fulfilled

$$I_{max} > I_z > I_y > I_{min} \ (\text{if } I_z > I_y) \ \text{ or } \ I_{max} > I_y > I_z > I_{min} \ (\text{if } I_y > I_z).$$

In the considered case

$$401{,}937 > 393{,}371 > 74{,}517 > 65{,}951;$$

b) check the permanence of the sum of axial moment values with the rotation of axes:

$$I_{max} + I_{min} = I_z + I_y;$$

$$401{,}937 + 65{,}951 = 467{,}888; \qquad 393{,}371 + 74{,}517 = 467{,}888;$$

$$467{,}888 = 467{,}888;$$

c) calculate the centrifugal inertia moment about principal central axes, which a priori equals zero:

$$I_{uv} = I_{yz}\cos 2\alpha_0 + \frac{I_y - I_z}{2}\sin 2\alpha_0 =$$

$$= 52{,}959 \cdot \cos(2 \cdot 9{,}183) + \frac{74{,}517 - 393{,}371}{2} \cdot \sin(2 \cdot 9{,}183) =$$

$$= 52{,}959 \cdot 0{,}949 - 159{,}427 \cdot 0{,}315 = 50{,}2581 - 50{,}2195 = -0{,}0386\ cm^4.$$

Computational error

$$\Delta\% = \left|\frac{-0{,}0386}{50{,}2581}\right| \cdot 100\ \% = 0{,}077\ \% \le 1\ \%,$$

therefore, the problem is solved correctly.

Example 9

Find the centrifugal inertia moment of an L-beam № 7,5/5 about the central axes parallel to the legs (Fig. 36), if the location of principal central inertia axes of the section and the values of principal inertia moments in this axial system are known.

Given: L-beam № 7,5/5.
Define I_{yz}.

Solution

Some reference books do not provide data for the centrifugal moment of inertia of the L-beams (for example, GOST 8510-72). Copy from this document:

$$I_{min} = 7{,}24\ cm^4; \qquad I_y = 34{,}81\ cm^4;$$

$$I_z = 12{,}47\ cm^4; \qquad \mathrm{tg}\,\alpha = 0{,}436.$$

To find the centrifugal inertia moment I_{yz} use the formula of centrifugal inertia moment change due to rotation of axes (transition from principal axes to axes yOz):

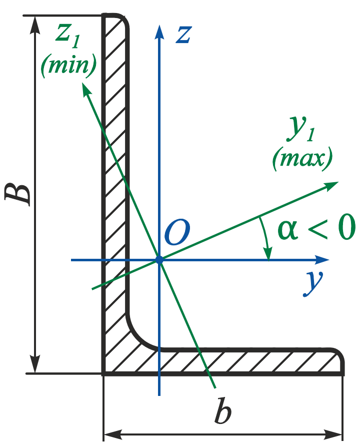


Fig. 36

$$I_{yz} = I_{y_1 z_1} \cos 2\alpha + \frac{I_{max} - I_{min}}{2} \sin 2\alpha.$$

Calculate the value I_{max} from the condition of the sum invariance of axial inertia moments with respect to the rotation of the axes, that is, from the relation

$$I_{max} + I_{min} = I_y + I_z;$$

$$I_{max} + 7{,}24 = 34{,}81 + 12{,}47 \quad \Rightarrow \quad I_{max} = 40{,}04\ cm^4.$$

The centrifugal inertia moment of an L-beam about the principal central inertia axes is identically equal to zero ($I_{y_1 z_1} \equiv 0$).

The deflection angle of the central axes

$$\alpha = -\operatorname{arctg} 0{,}436 = -23{,}557°.$$

In this case, the angle is negative because the shortest way to align the maximum inertia moment axis with the y axis is clockwise.

Therefore, the centrifugal inertia moment of the L-beam is (см. see example 8, Table 3).

$$I_{yz} = 0 + \frac{40{,}04 - 7{,}24}{2} \sin\big(2 \cdot (-23{,}557°)\big) = 16{,}4 \cdot (-0{,}7327) = -12{,}02\ cm^4.$$

Notes

1. The axis of maximum inertia moment of an equilateral L-beam is a symmetry axis, therefore, $\alpha = 45°$.
2. The centrifugal inertia moment of non-equilateral L-beam is easier to calculate by the formula
$$I_{yz} = \frac{\operatorname{tg} 2\alpha\,(I_z - I_y)}{2}.$$
3. In the assortment tables, the centrifugal inertia moment is given in absolute value. Its sign can be defined using the Fig. 32.

Example 10

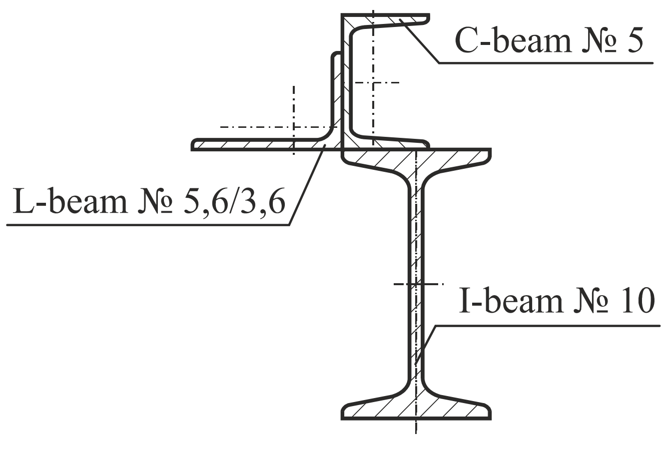


Fig. 37

Define the location of principal central inertia axes of the given section and the values of principal inertia moments in this axial system (Fig. 37).

Given: I-beam № 10, C-beam № 5, L-beam № 5,6/3,6.

Define y_c, z_c, α_0, I_u, I_v.

Solution

1. Make a dimensioned drawing of compound section.

2. Decompose the section into elemental parts and assign them the following numbers: an I-beam – 1, a C-beam – 2, an L-beam – 3 (Fig. 38).

3. Put the central coordinate systems $y_i O_i z_i$ in the gravity center of each elemental part of the section.

4. Copy the geometric characteristics of an I-beam № 10, a C-beam № 5, and an L-beam № 5,6/3,6 from the assortment tables and tabulate them (Table 4).

Table 4

Part of the section	Geometric characteristics							
	h_i, cm	b_i, cm	A_i, cm^2	I_{y_i}, cm^4	I_{z_i}, cm^4	$I_{y_i z_i}, cm^4$	y_{0_i}, cm	z_{0_i}, cm
1 (I-beam)	10,0	5,5	12,0	198,0	17,9	0	–	–
2 (C-beam)	5,0	3,2	6,16	22,8	5,61	0	1,16	–
3 (L-beam)	3,6	5,6	3,58	3,7	11,37	3,74	1,82	0,84

5. Find the coordinates of gravity center of the compound section.

Select the actual (basic) coordinate system that will be used to define the gravity center coordinates of the whole section. Take the central axial system of an I-beam $y_1 O_1 z_1$y as basic one.

Then the formulas for calculating the gravity center coordinates take the form

$$y_c = \frac{\sum_{i=1}^{3} S_{z_1}^{(i)}}{\sum_{i=1}^{3} A_i}; \qquad z_c = \frac{\sum_{i=1}^{3} S_{y_1}^{(i)}}{\sum_{i=1}^{3} A_i},$$

where $\sum_{i=1}^{2} A_i = A = A_1 + A_2 + A_3 = 12{,}0 + 6{,}16 + 3{,}58 = 21{,}74\ cm^2;$

$$S_{z_1} = \sum_{i=1}^{3} S_{z_1}^{(i)} = A_1 \cdot 0 + A_2\left(-\left(\frac{b_1}{2} - y_{0_2}\right)\right) + A_3\left(-\left(\frac{b_1}{2} + y_{0_3}\right)\right) =$$

$$= 12{,}0 \cdot 0 + 6{,}16 \cdot \left(-\left(\frac{5{,}5}{2} - 1{,}16\right)\right) + 3{,}58 \cdot \left(-\left(\frac{5{,}5}{2} + 1{,}82\right)\right) =$$

$$= -26{,}155\ cm^3;$$

$$S_{y_1} = \sum_{i=1}^{3} S_{y_1}^{(i)} = A_1 \cdot 0 + A_2\left(\frac{h_1}{2} + \frac{h_2}{2}\right) + A_3\left(\frac{h_1}{2} + z_{0_3}\right) =$$

$$= 12{,}0 \cdot 0 + 6{,}16 \cdot \left(\frac{10{,}0}{2} + \frac{5{,}0}{2}\right) + 3{,}58 \cdot \left(\frac{10{,}0}{2} + 0{,}84\right) = 67{,}107 \; cm^3.$$

Calculate the gravity center coordinates O of the compound section in the system of axes $y_1 O_1 z_1$:

$$y_c = \frac{S_{z_1}}{A} = \frac{-26{,}155}{21{,}74} = -1{,}203 \; cm; \qquad z_c = \frac{S_{y_1}}{A} = \frac{67{,}107}{21{,}74} = 3{,}087 \; cm.$$

According to the results of calculations show on Fig. 38 the system of central axes yOz and the gravity center of the compound section (point O).

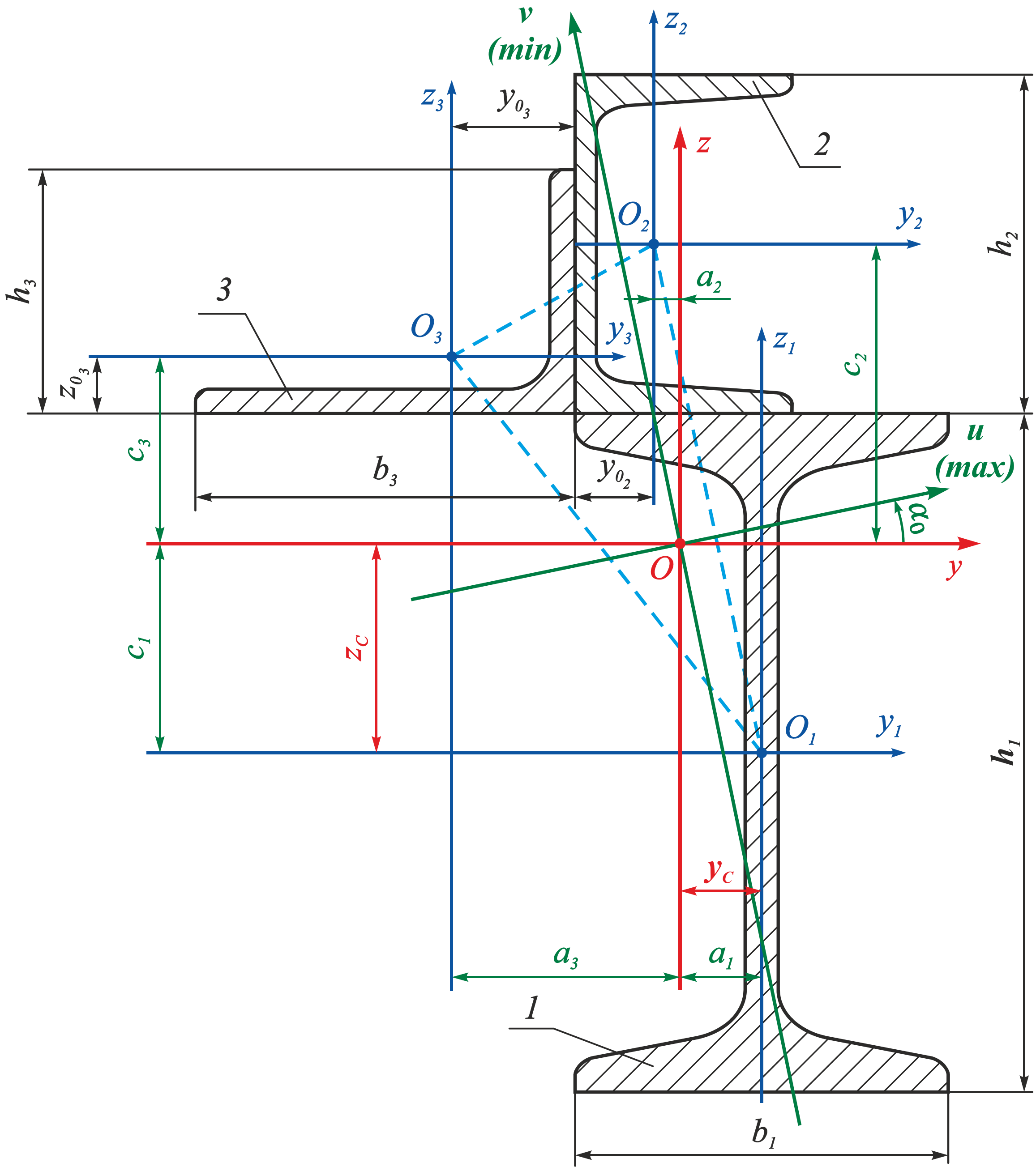


Fig. 38

Note | The gravity center of a section consisting of three and more elemental parts is located within the area bounded by the lines connecting the gravity centers of elemental parts (in the Fig. 38 it is within the triangle $O_1O_2O_3$).

The coordinates of gravity centers of elemental parts of the section (points O_1, O_2, and O_3) in the system of central axes yOz:

$$a_1 = |y_c| = 1{,}203\ cm;$$

$$a_2 = -\left(\frac{b_1}{2} - y_{0_2} - |y_c|\right) = -\left(\frac{5{,}5}{2} - 1{,}16 - 1{,}203\right) = -0{,}387\ cm;$$

$$a_3 = -\left(\frac{b_1}{2} + y_{0_3} - |y_c|\right) = -\left(\frac{5{,}5}{2} + 1{,}82 - 1{,}203\right) = -3{,}367\ cm;$$

$$c_1 = -z_c = -3{,}087\ cm;$$

$$c_2 = \frac{h_1}{2} + \frac{h_2}{2} - z_c = \frac{10{,}0}{2} + \frac{5{,}0}{2} - 3{,}087 = 4{,}413\ cm;$$

$$c_3 = \frac{h_1}{2} + z_{0_3} - z_c = \frac{10{,}0}{2} + 0{,}84 - 3{,}087 = 2{,}753\ cm.$$

Verification of defining a gravity center:

Use the property of central axes that the static moment of a compound section about the central axes equals zero:

$$S_y = \sum_{i=1}^{2} S_y^{(i)} = S_y^{(1)} + S_y^{(2)} + S_y^{(3)} = A_1c_1 + A_2c_2 + A_3c_3 =$$
$$= 12{,}0 \cdot (-3{,}087) + 6{,}16 \cdot 4{,}413 + 3{,}58 \cdot 2{,}753 =$$
$$= -37{,}044 + 27{,}184 + 9{,}856 = -0{,}004\ cm^3.$$

Relative error

$$\Delta\% = \left|\frac{-0{,}004}{37{,}040}\right| \cdot 100\ \% = 0{,}0108\ \% < 1\ \%; \qquad [\Delta\%] \le 1\ \%;$$

$$S_z = \sum_{i=1}^{2} S_z^{(i)} = S_z^{(1)} + S_z^{(2)} + S_z^{(3)} = A_1a_1 + A_2a_2 + A_2a_2 =$$
$$= 12{,}0 \cdot 1{,}203 + 6{,}16 \cdot (-0{,}387) + 3{,}58 \cdot (-3{,}367) =$$
$$= 14{,}436 - 2{,}384 - 12{,}054 = -0{,}002\ cm^3.$$

Relative error

$$\Delta\% = \left|\frac{-0{,}002}{14{,}436}\right| \cdot 100\ \% = 0{,}0139\ \% < 1\ \%; \qquad [\Delta\%] \le 1\ \%.$$

Therefore, the location of gravity center of the compound section is defined correctly.

6. Find the axial and centrifugal inertia moments of the compound section in the system of central axes yOz, using the formulas of parallel translation:

$$I_y = I_y^{(1)} + I_y^{(2)} + I_y^{(3)} = 312{,}355 + 142{,}763 + 30{,}833 = 485{,}951\ cm^4;$$
$$I_y^{(1)} = I_{y_1} + c_1^2 A_1 = 198{,}0 + (-3{,}087)^{\,2} \cdot 12{,}0 = 312{,}355\ cm^4;$$
$$I_y^{(2)} = I_{y_2} + c_2^2 A_2 = 22{,}8 + 4{,}413^2 \cdot 6{,}16 = 142{,}763\ cm^4;$$
$$I_y^{(3)} = I_{y_3} + c_3^2 A_3 = 3{,}7 + 2{,}753^2 \cdot 3{,}58 = 30{,}833\ cm^4;$$

$$I_z = I_z^{(1)} + I_z^{(2)} + I_z^{(3)} = 35{,}267 + 6{,}533 + 51{,}955 = 93{,}755\ cm^4;$$
$$I_z^{(1)} = I_{z_1} + a_1^2 A_1 = 17{,}9 + 1{,}203^2 \cdot 12{,}0 = 35{,}267\ cm^4;$$
$$I_z^{(2)} = I_{z_2} + a_2^2 A_2 = 5{,}61 + (-0{,}387)^2 \cdot 6{,}16 = 6{,}533\ cm^4;$$
$$I_z^{(3)} = I_{z_3} + a_3^2 A_3 = 11{,}37 + (-3{,}367)^2 \cdot 3{,}58 = 51{,}955\ cm^4;$$

$$I_{yz} = I_{yz}^{(1)} + I_{yz}^{(2)} + I_{yz}^{(3)} = -44{,}564 - 10{,}520 - 29{,}444 = -84{,}528\ cm^4;$$
$$I_{yz}^{(1)} = I_{y_1 z_1} + a_1 c_1 A_1 = 0 + 1{,}203 \cdot (-3{,}087) \cdot 12{,}0 = -44{,}564\ cm^4;$$
$$I_{yz}^{(2)} = I_{y_2 z_2} + a_2 c_2 A_2 = 0 + (-0{,}387) \cdot 4{,}413 \cdot 6{,}16 = -10{,}520\ cm^4;$$
$$I_{yz}^{(3)} = I_{y_3 z_3} + a_3 c_3 A_3 = 3{,}74 + (-3{,}367) \cdot 2{,}753 \cdot 3{,}58 = -29{,}444\ cm^4.$$

Note | The large area of the compound section lies in the second and the fourth quarters, therefore the centrifugal inertia moment is negative.

7. Calculate the location of principal inertia axes of the section:

$$\operatorname{tg} 2\alpha_0 = \frac{2I_{yz}}{I_z - I_y} = \frac{2 \cdot (-84{,}528)}{93{,}755 - 485{,}951} = 0{,}431;$$

$$2\alpha_0 = 23{,}316°; \qquad \alpha_0 = 11{,}658° = 11°39'29''.$$

As far as $\alpha_0 > 0$, the rotation of axes y and z by this angle should be counter-clockwise according to the conventional sign rule. The axes obtained as a result of this rotation are principal central inertia axes of the compound section.

8. Define the values of principal inertia moments of the section:

$$I_{\substack{max\\min}} = I_{\substack{u\\v}} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_z - I_y}{2}\right)^2 + I_{y_0 z_0}^2} = \frac{485{,}951 + 93{,}755}{2} \pm$$

$$\pm\sqrt{\left(\frac{93{,}755-485{,}951}{2}\right)^2+(-84{,}528)^2}=289{,}853\pm 213{,}540\ cm^4.$$

Therefore

$$I_{max}=I_u=503{,}393\ cm^4;$$

$$I_{min}=I_v=76{,}313\ cm^4.$$

Using the results of calculations show on Fig. 38 the principal central inertia axes of the compound section, namely axes u and v.

As long as $I_y > I_z$, the axial inertia moment gets its maximum value relative to the axis located at minimum angular distance from the axis y.

9. Validate the solution:

a) check if the correlation is fulfilled

$$I_{max} > I_z > I_y > I_{min}\ (\text{if}\quad I_z > I_y)\ \text{ or }\ I_{max} > I_y > I_z > I_{min}\ \ (\text{if}\quad I_y > I_z).$$

In the considered case

$$503{,}393 > 485{,}951 > 379{,}855 > 76{,}313;$$

b) check the permanence of the sum of axial moment values with the rotation of axes:

$$I_{max}+I_{min}=I_y+I_z;$$

$$503{,}393+76{,}313=485{,}951+93{,}755;$$

$$579{,}706=579{,}706;$$

c) calculate the centrifugal inertia moment about principal central axes, which a priori equals zero ($I_{uv}=0$):

$$I_{uv}=I_{yz}\cos 2\alpha_0+\frac{I_y-I_z}{2}\sin 2\alpha_0=$$

$$=(-84{,}528)\cdot\cos(2\cdot 11{,}658°)+\frac{485{,}951-93{,}755}{2}\cdot\sin(2\cdot 11{,}658°)=$$

$$=-77{,}625+77{,}616=-0{,}009\ cm^4.$$

Computational error

$$\Delta\%=\left|\frac{-0{,}009}{77{,}616}\right|\cdot 100\ \%=0{,}0116\ \%\le 1\ \%,$$

therefore, the problem is solved correctly.

Example 11

Define the location of principal central inertia axes of the given section that represent the simplified section of a wing torsion box, and the values of principal inertia moments in the system of those axes (Fig. 39), if $L = 80\ cm$; $H = 25\ cm$; $h = 15\ cm$; $a = 4\ cm$; $l_1 = 4\ cm$; $l_3 = 3\ cm$; $l_5 = 2\ cm$; $l_7 = 3\ cm$; $\delta_1 = 1\ cm$; $\delta_2 = 0{,}3\ cm$; $\delta_3 = 1{,}5\ cm$; $\delta_4 = 0{,}4\ cm$; $\delta_5 = 1{,}2\ cm$; $\delta_6 = 0{,}2\ cm$; $\delta_7 = 0{,}8\ cm$; $\delta_8 = 0{,}4\ cm$.

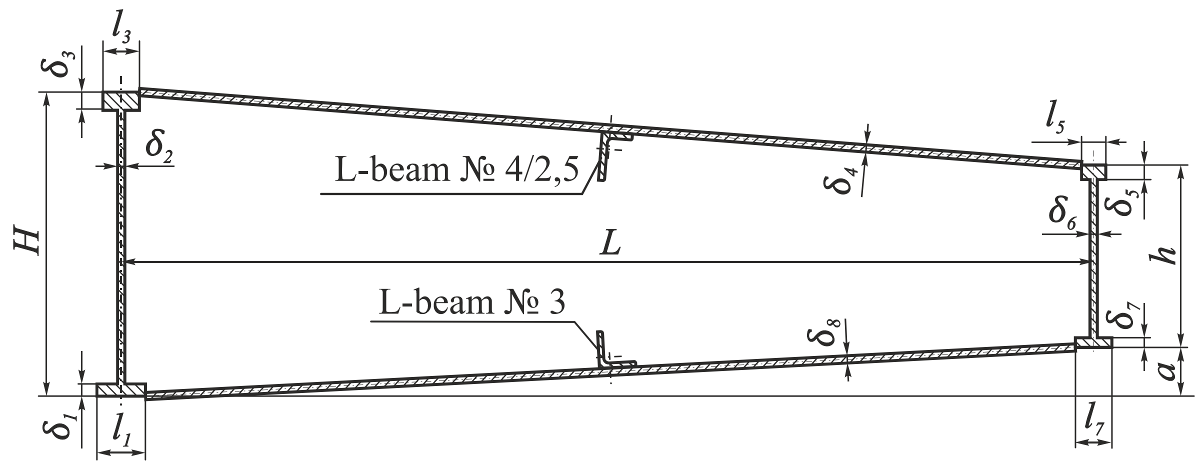


Fig. 39

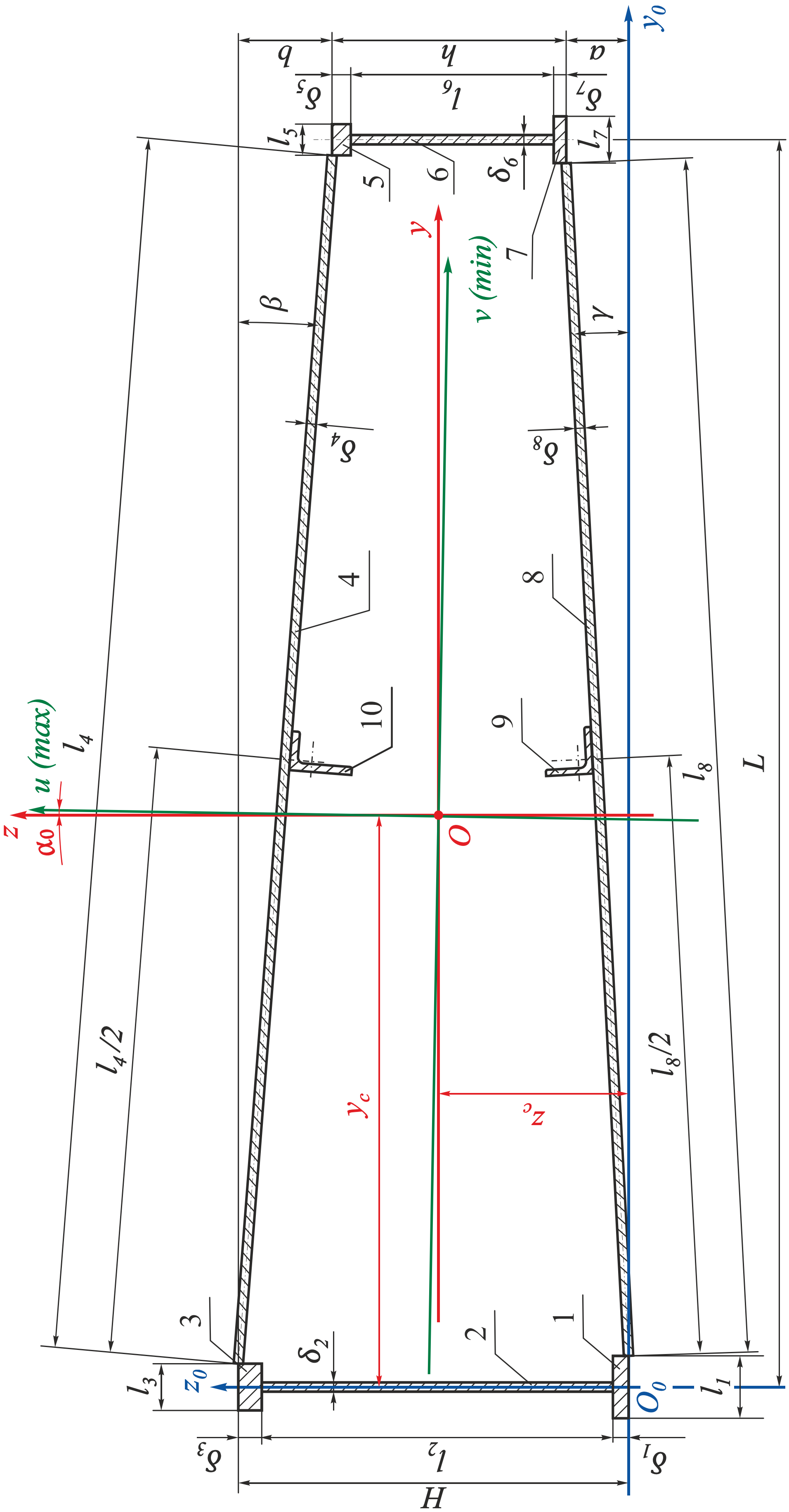
Solution

1. Make a dimensioned drawing of compound section (Fig. 40).

2. Decompose the section into elemental parts and assign them the following numbers (Table 5).

Table 5

Part of the section	Part of the section	Type of the section	Dimensions
1	lower flange of the front spar	rectangle	$l_1 \times \delta_1$
2	wall of the front spar	rectangle	$l_2 \times \delta_2$
3	upper flange of the front spar	rectangle	$l_3 \times \delta_3$
4	upper skin	rectangle	$l_4 \times \delta_4$
5	upper flange of the back spar	rectangle	$l_5 \times \delta_5$
6	wall of the back spar	rectangle	$l_6 \times \delta_6$
7	lower flange of the back spar	rectangle	$l_7 \times \delta_7$
8	lower skin	rectangle	$l_8 \times \delta_8$
9	stringer of the lower panel	L-beam	№ 3
10	stringer of the upper panel	L-beam	№ 4/2,5



y0
a
h
b
δ5
l6
δ7
l5
l7
5
6
δ6
7
y
v (min)
β
γ
δ4
δ8
4
8
10
9
u (max)
l4
z
α0
O
l8
L
l4/2
l8/2
yc
zc
3
2
1
δ2
l3
z0
O0
l1
δ3
l2
δ1
H

Fig. 40

3. Copy the geometric characteristics of L-beams № 3 and № 4/2,5 (Figs 41, 42) from the assortment tables and tabulate them (Table 6).

Table 6

Part of the section	Geometric characteristics							
	h_i, cm	b_i, cm	F_i, cm^2	I_{y_i}, cm^4	I_{z_i}, cm^4	$I_{y_iz_i}, cm^4$	y_{0_i}, cm	z_{0_i}, cm
9 (L-beam)	3,0	3,0	2,27	1,84	1,84	–1,08	0,89	0,89
10 (L-beam)	4,0	2,5	3,03	4,73	1,41	1,44	0,66	1,41

Calculate intermediate values that define the dimensions of the elements as well as their locations (see Fig. 40). The centers of gravity of rectangular elements 1…8 are located at the intersections of diagonals (not shown at Fig. 40).

$$b = H - h - a;$$

$$l_2 = H - \delta_1 - \delta_3; \qquad l_6 = h - \delta_5 - \delta_7;$$

$$l_4 = \sqrt{\left(L - \frac{l_3}{2} - \frac{l_5}{2}\right)^2 + b^2}; \qquad l_8 = \sqrt{\left(L - \frac{l_1}{2} - \frac{l_7}{2}\right)^2 + a^2};$$

$$\Delta_9 = z_{0_9} + \frac{\delta_8}{2}; \qquad \Delta_{10} = z_{0_{10}} + \frac{\delta_4}{2};$$

$$\cos\beta = \frac{L - \frac{l_3}{2} - \frac{l_5}{2}}{l_4}; \qquad \sin\beta = \frac{b}{l_4};$$

$$\cos\gamma = \frac{L - \frac{l_1}{2} - \frac{l_7}{2}}{l_8}; \qquad \sin\gamma = \frac{a}{l_8}.$$

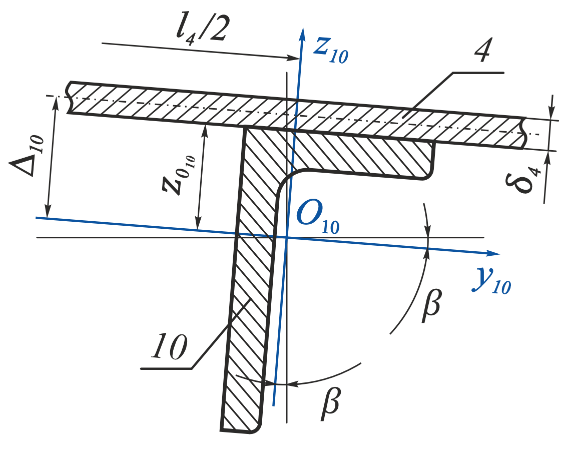


Fig. 41

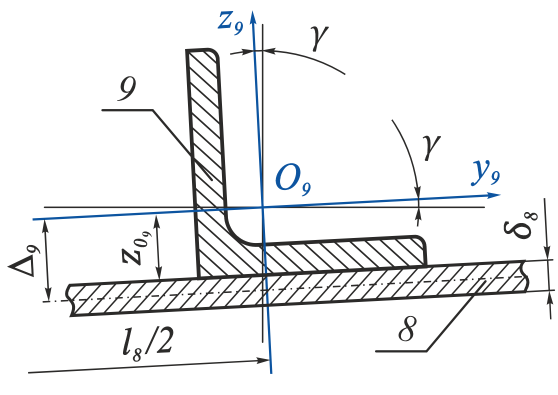


Fig. 42

It is convenient to make all the calculations in an Excel spreadsheet. To do this enter the input data and intermediate values calculated in step 3 into an Excel table (Fig. 43).

F7 =(C4-C11/2-C13/2)/C12

	B	C	D	E	F	G
3						
4	L =	80	cm			
5	H =	25	cm	b=	6	cm
6	h =	15	cm			
7	a=	4	cm	cos(beta)=	0,99701653	1
8				sin(beta)=	0,07718838	
9	l1 =	4	cm	cos(gamma)=	0,9986358	1
10	l2 =	22,5	cm	sin(gamma)=	0,05221625	
11	l3 =	3	cm			
12	l4 =	77,73	cm			
13	l5 =	2	cm			
14	l6 =	13,00	cm			
15	l7 =	3	cm			
16	l8 =	76,60	cm			
17	delta1 =	1	cm			
18	delta2 =	0,3	cm			
19	delta3 =	1,5	cm			
20	delta4 =	0,4	cm			
21	delta5 =	1,2	cm			
22	delta6 =	0,2	cm			
23	delta7 =	0,8	cm			
24	delta8 =	0,4	cm			
25						
26	L-section 9	№3				
27	Delta9 =	1,09	cm	z9 =	0,89	cm
28	L-section 10	№4/2,5				
29	Delta10 =	1,61	cm	z10 =	1,41	cm
30						

Fig. 43

Note | The values given as input data are outlined, the others are intermediate values calculated in step 3.

Fig. 43 contains an example of cell entry (F7) of the calculating formula to define $\cos\beta$.

4. Find the gravity center coordinates of the compound section.

Select $y_0O_0z_0$ as an actual (basic) coordinate system to define the coordinates of the whole section gravity center (see Fig. 40). In this case, the gravity centers of elemental parts of the section will be located in the first quarter.

4.1. Define the areas of the elements of the section:

– for the elements 1…8 the areas A_i are calculated by the formula

$$A_i = l_i \times \delta_i;$$

– for stringers (elements 9, 10) areas A_9 and A_{10} can be taken from the corresponding assortment tables of rolled steel (see Table 6).

4.2. Find the gravity center coordinates of the elemental parts of the section in the basic coordinate system $y_0O_0z_0$ (see Fig. 40):

$$y_1 = 0; \qquad z_1 = \frac{\delta_1}{2};$$

$$y_2 = 0; \qquad z_2 = \delta_1 + \frac{l_2}{2};$$

$$y_3 = 0; \qquad z_3 = H - \frac{\delta_3}{2};$$

$$y_4 = \frac{l_3}{2} + \frac{l_4}{2}\cos\beta; \qquad z_4 = H - \frac{l_4}{2}\sin\beta;$$

$$y_5 = L; \qquad z_5 = a + h - \frac{\delta_5}{2};$$

$$y_6 = L; \qquad z_6 = a + \delta_7 + \frac{l_6}{2};$$

$$y_7 = L; \qquad z_7 = a + \frac{\delta_7}{2};$$

$$y_8 = \frac{\delta_1}{2} + \frac{l_8}{2}\cos\gamma; \qquad z_8 = \frac{l_8}{2}\sin\gamma;$$

$$y_9 = y_8 - \delta_9\sin\gamma; \qquad z_9 = z_8 + \delta_9\cos\gamma;$$

$$y_{10} = y_4 - \delta_{10}\sin\beta; \qquad z_{10} = z_4 - \delta_{10}\cos\beta.$$

4.3. Define static moments of elemental parts of the section about basic axes $y_0O_0z_0$:

$$S_{y_0}^{(i)} = A_i z_i; \qquad S_{z_0}^{(i)} = A_i y_i; \qquad i = 1 \dots 10,$$

where i is a number of the part of the section.

4.4. Find the gravity center coordinates of the compound section y_c and z_c in basic coordinate system $y_0O_0z_0$:

$$y_c = \frac{S_{z_0}}{A} = \frac{\sum_{i=1}^{10} S_{z_0}^{(i)}}{\sum_{i=1}^{10} A_i}; \qquad z_c = \frac{S_{y_0}}{A} = \frac{\sum_{i=1}^{10} S_{y_0}^{(i)}}{\sum_{i=1}^{10} A_i},$$

where A is a cross-section area;

S_{z_0} and S_{y_0} static moments about corresponding axes.

Enter the calculations from steps 4.1 – 4.4 into Excel worksheet (Fig. 44).

M14 =L14*J14

Element number	Ai	yi	zi	Sy0i	Sz0i	yc	zc
1	4,00	0	0,5	2,00	0,00	36,680	12,176
2	6,75	0	12,25	82,69	0,00		
3	4,50	0	24,25	109,13	0,00		
4	31,09	40,25	22,00	684,04	1251,48		
5	2,40	80	18,40	44,16	192,00		
6	2,60	80	11,30	29,38	208,00		
7	2,40	80	4,40	10,56	192,00		
8	30,64	40,25	2,00	61,28	1233,33		
9	2,27	40,19	3,09	7,01	91,24		
10	3,03	40,1257	20,39	61,80	121,58		
Sum	89,68			1092,04	3289,64		
	cm^2	cm	cm	cm^3	cm^3	cm	cm

Fig. 44

According to the results of the calculations, show on Fig. 40 the system of central axes yOz and the gravity center of the compound section (point O).

4.5. Define the coordinates of the gravity centers of the elemental parts of the compound section in the system of central axes yOz:

$$y_{c_i} = y_i - y_c; \qquad z_{c_i} = z_i - z_c;$$

4.6. Validate the defined location of the gravity center.

Use the property of central axes that the static moment of a compound section about the central axes equals zero:

$$S_y = \sum_{i=1}^{10} S_y^{(i)} = \sum_{i=1}^{10} A_i z_{c_i} = 0; \qquad S_z = \sum_{i=1}^{10} S_z^{(i)} = \sum_{i=1}^{10} A_i y_{c_i} = 0.$$

Enter the data from steps 4.5 – 4.6 into Excel worksheet (Fig. 45).

R13 =L13-P11

	Q	R	S	T
9				
10	yci	zci	Syi	Szi
11	-36,68	-11,68	-47	-147
12	-36,68	0,07	0	-248
13	-36,68	12,07	54	-165
14	3,57	9,82	305	111
15	43,32	6,22	15	104
16	43,32	-0,88	-2	113
17	43,32	-7,78	-19	104
18	3,57	-10,18	-312	109
19	3,51	-9,09	-21	8
20	3,45	8,22	25	10
21			0	0
22	cm	cm	cm^3	cm^3

Fig. 45

Therefore, the location of gravity center of the compound section is defined correctly (see zero values in the highlighted row in the table).

5. Find the axial and centrifugal inertia moments of the compound section in the system of central axes yOz.

5.1. Define intrinsic inertia moments of the elements of the compound section about their principal central axes ($y_i O_i z_i$):

– for spar webs (elements 1, 3, 5, 7), upper and lower skin (elements 4 and 8)

$$I_{y_i}^{(i)} = \frac{l_i \delta_i^3}{12}; \qquad I_{z_i}^{(i)} = \frac{\delta_i l_i^3}{12}; \qquad I_{y_i z_i}^{(i)} = 0;$$

– for spar walls (elements 2 and 6)

0

$$I_{y_i}^{(i)} = \frac{\delta_i l_i^3}{12}; \qquad I_{z_i}^{(i)} = \frac{l_i \delta_i^3}{12}; \qquad I_{y_i z_i}^{(i)} = 0;$$

– for stringers we take intrinsic inertia moments about their principal axes parallel to the base from the assortment tables (see Table 6).

5.2. Find the inertia moments of elemental parts of the compound section in the system of central axes $y_i^* O_i z_i^*$ parallel to the central axes of the compound section.

Use the formulas to define axial inertia moments of the section due to rotation of axes.

The rotation angle of elements 1, 2, 3, 5, 6, 7 equals zero, the elements of the upper panel (skin 4 and L-beam 10) are rotated counterclockwise by the angle β, the elements of the lower panel (skin 8 and L-beam 9) are rotated clockwise by the angle γ.

Therefore, we get:

– for spar webs and walls (elements 1, 2, 3, 5, 6, 7):

$$I_{y_i}^{(i)} = I_{y_i^*}^{(i)}; \qquad I_{z_i}^{(i)} = I_{z_i^*}^{(i)}; \qquad I_{y_i z_i}^{(i)} = I_{y_i^* z_i^*}^{(i)};$$

– for the upper panel (skin 4 and L-beam 10)

$$I_{y_i^*}^{(i)} = I_{y_i}^{(i)} \cos^2 \beta + I_{z_i}^{(i)} \sin^2 \beta - I_{y_i z_i}^{(i)} \sin 2\beta\,;$$

$$I_{z_i^*}^{(i)} = I_{z_i}^{(i)} \cos^2 \beta + I_{y_i}^{(i)} \sin^2 \beta + I_{y_i z_i}^{(i)} \sin 2\beta\,;$$

$$I_{y_i^* z_i^*}^{(i)} = I_{y_i z_i}^{(i)} \cos 2\beta + \frac{I_{y_i}^{(i)} - I_{z_i}^{(i)}}{2} \sin 2\beta\,;$$

– for the lower panel (skin 8 and L-beam 9)

$$I_{y_i^*}^{(i)} = I_{y_i}^{(i)} \cos^2 \gamma + I_{z_i}^{(i)} \sin^2 \gamma + I_{y_i z_i}^{(i)} \sin 2\gamma\,;$$

$$I_{z_i^*}^{(i)} = I_{z_i}^{(i)} \cos^2 \gamma + I_{y_i}^{(i)} \sin^2 \gamma - I_{y_i z_i}^{(i)} \sin 2\gamma\,;$$

$$I_{y_i^* z_i^*}^{(i)} = I_{y_i z_i}^{(i)} \cos 2\gamma - \frac{I_{y_i}^{(i)} - I_{z_i}^{(i)}}{2} \sin 2\gamma\,;$$

5.3. Verify the defined intrinsic inertia moments of the elements of the compound section taking into account the rotation of elements 4, 8, 9, 10:

$$\sum_{i=1}^{10} I_{y_i}^{(i)} + \sum_{i=1}^{10} I_{z_i}^{(i)} = \sum_{i=1}^{10} I_{y_i}^{(i)*} + \sum_{i=1}^{10} I_{z_i}^{(i)*}.$$

Enter the data from steps 5.1 and 5.2 into Excel worksheet (Fig. 46).

Y14 =V14*F7^2+U14*F8^2+W14*2*F7*F8

	U	V	W	X	Y	Z	
9							
10	Iyi_native	Izi_native	Iyzi_native	Iyi_native*	Izi_native*	Iyzi_native*	Iy
11	0,33	5,33	0	0,33	5,33	0	
12	0,05	284,77	0	0,05	284,77	0	
13	0,84	3,38	0	0,84	3,38	0	
14	0,41	15655,85	0	93,69	15562,58	-1204,81	
15	0,29	0,80	0	0,29	0,80	0	
16	0,01	36,62	0	0,01	36,62	0	
17	0,13	1,80	0	0,13	1,80	0	
18	0,41	14984,48	0	41,26	14943,62	781,34	
19	1,84	1,84	-1,08	1,73	1,95	-1,07	
20	4,73	1,41	1,44	4,49	1,65	1,68	
21	9,05	30976,27	0,36	142,82	30842,50	-423	
22	cm^4	cm^4	cm^4	cm^4	cm^4	cm^4	cm
23							
24	30985,32		cm^4	30985,32		cm^4	

Fig. 46

The bottom row shows the verification (step 5.3).

5.4. Find the axial and centrifugal inertia moments of the compound section (1…10) in the system of central axes yOz using the formulas of parallel translation:

$$I_y^{(i)} = I_{y_i}^{(i)*} + z_{c_i}^2 A_i; \qquad I_z^{(i)} = I_{z_i}^{(i)*} + y_{c_i}^2 A_i; \qquad I_{yz}^{(i)} = I_{y_i z_i}^{(i)*} + y_{c_i} z_{c_i} A_i;$$

5.5. Define axial and centrifugal inertia moments of the compound section in the system of central axes yOz:

$$I_y = \sum_{i=1}^{10} I_y^{(i)}; \qquad I_z = \sum_{i=1}^{10} I_z^{(i)}; \qquad I_{yz} = \sum_{i=1}^{10} I_{yz}^{(i)}.$$

6. Define the location of principal inertia axes of the section:

$$\operatorname{tg} 2\alpha_0 = \frac{2I_{yz}}{I_z - I_y}; \qquad \alpha_0 = \frac{1}{2}\operatorname{arctg}\left(\frac{2I_{yz}}{I_z - I_y}\right).$$

As far as $\alpha_0 < 0$, the rotation of axes y and z by this angle should be clockwise according to the conventional sign rule. The axes obtained as a result of this rotation are principal central inertia axes of the compound section.

7. Define the values of principal inertia moments of the section:

$$I_{\substack{max \\ min}} = I_{\substack{u \\ v}} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_z - I_y}{2}\right)^2 + I_{yz}^2}.$$

Enter the data from steps 5.4, 5.5, 6 and 7 into Excel worksheet (Fig. 47).

Using the results of calculations show on Fig. 40 the principal central inertia axes of the compound section, namely axes u and v (the angle α_0 is enlarged in the figure).

As long as $I_z > I_y$, then the axial inertia moment gets its maximum value relative to the axis located at minimum angular distance from the axis z.

8. Verification the solution:

a) check the permanence of the sum of axial moment values with the rotation of axes:

$$I_{max} + I_{min} = I_z + I_y;$$

б) calculate the centrifugal inertia moment about principal central axes, which a priori equals zero:

$$I_{uv} = I_{yz}\cos 2\alpha_0 + \frac{I_y - I_z}{2}\sin 2\alpha_0 = 0.$$

AD11 =0,5*ATAN(2*AC21/(AB21-AA21))

	AA	AB	AC	AD	AE	AF	AG	AH
9								
10	Iyi_total	Izi_total	Iyzi_total	alfa0	Iu_max	Iv_min	Verification	
11	545,70	5387,04	1713,18					
12	0,09	9366,40	-18,20	-0,01708			Iy + Iz =(?) Iu + Iv	Iuv =(?) 0
13	656,81	6057,80	-1992,86					
14	3094,18	15958,84	-114,41					
15	93,24	4504,68	647,04	(rad)	66114,91	8133,271		
16	2,01	4915,82	-98,72					
17	145,27	4505,68	-808,51	-0,97887			solution is correct	solution is correct
18	3214,56	15334,14	-331,86					
19	189,21	29,97	-73,55					
20	209,14	37,63	87,48	(degree)				
21	8150,19	66097,99	-990,39					
22	cm^4	cm^4	cm^4		cm^4	cm^4		
23								
24	74248,18		cm^4		74248,18		cm^4	

Fig. 47

Final results of calculation:

$$y_c = 36{,}80\ cm;$$

$$z_c = 12{,}176\ cm;$$

$$\alpha_0 = -0{,}977887°;$$

$$I_{max} = I_u = 66114{,}91\ cm^4;$$

$$I_{min} = I_v = 8133{,}271\ cm^4.$$

Structural mechanics of aircrafts applies a simplified approach of defining geometric characteristics of a wing cross-section. The simplification is that «intrinsic» inertia moments of the elements comprising the wing cross-section are not taken into account. Only «transfer» inertia moments are accounted.

As the result of such simplification, a cross-section turns into a set of points that have areas (discrete model). An inaccuracy of such calculations directly depends on smallness of «intrinsic» inertia moments compared to «transfer» inertia moments. The main advantage, however, is simplicity of calculations.

Let us evaluate the use of discrete model of the wing cross-section considered above. To do this reduce all the elements of the section to point areas. This procedure is shown at the Figs 48 – 49.

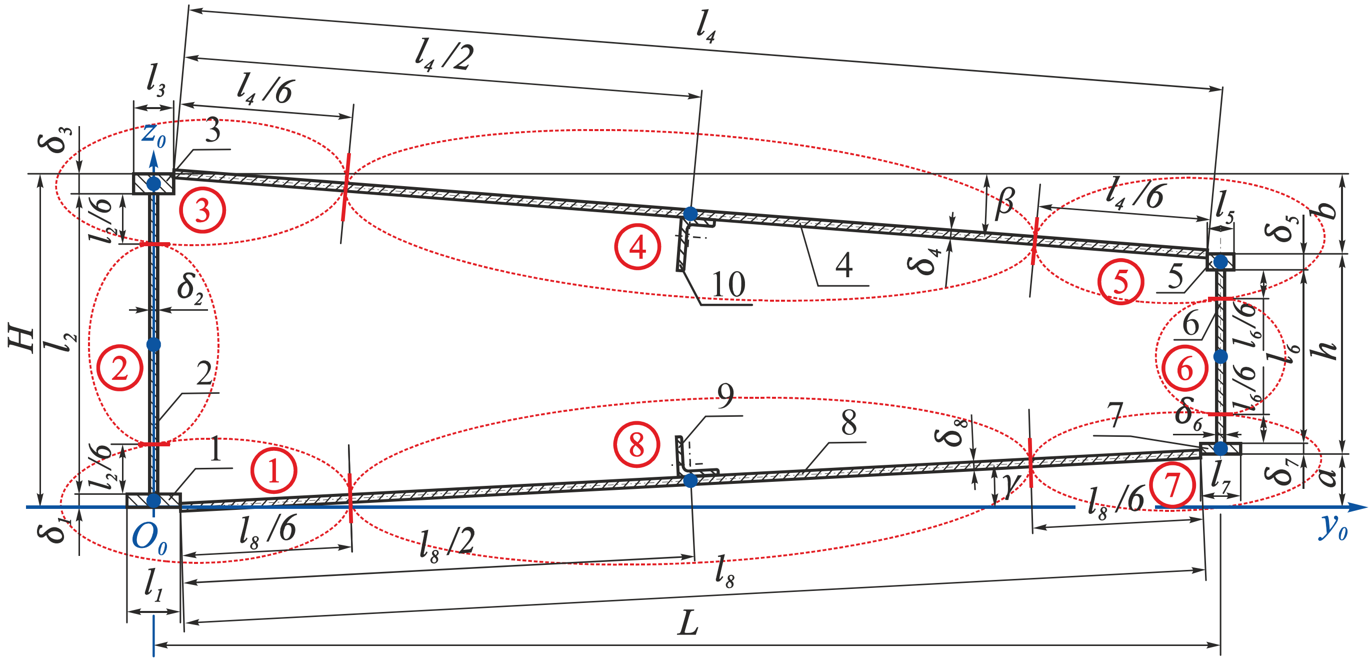


Fig. 48

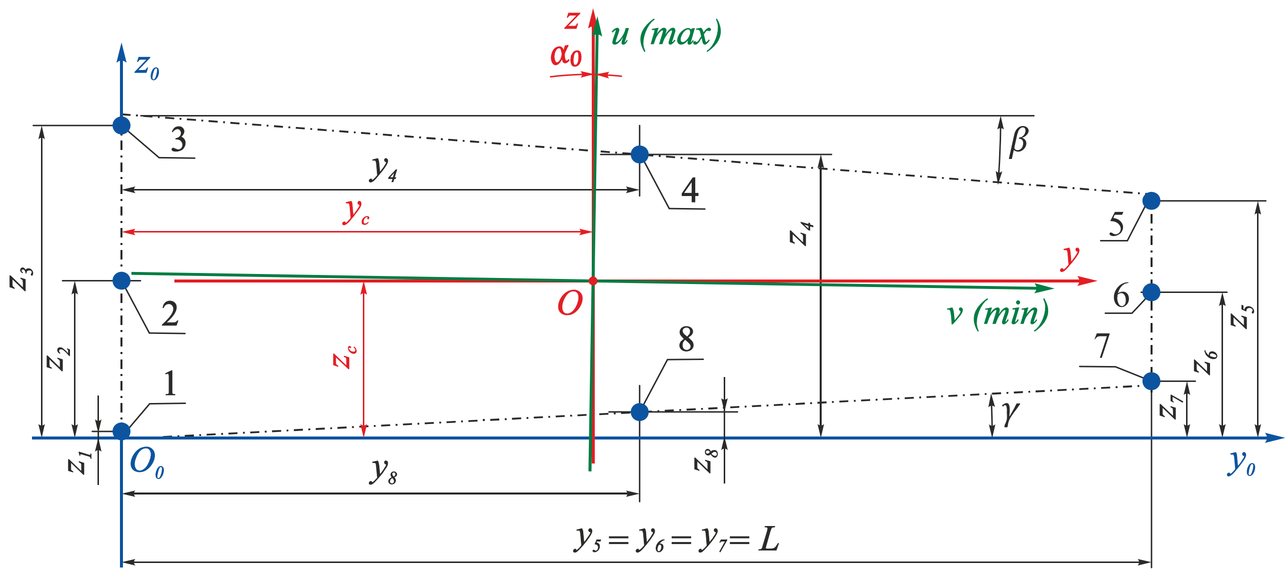


Fig. 49

After that, the procedure of defining principal inertia moments and location of principal axes is down to the following calculations, which are convenient to be made in EXCEL environment (Figs 50 – 52).

The reduced area of the points:

$$A_1^* = A_1 + \frac{A_2}{6} + \frac{A_8}{6}; \qquad A_2^* = \frac{2A_2}{3};$$

$$A_3^* = \frac{A_2}{6} + A_3 + \frac{A_4}{6}; \qquad A_4^* = \frac{2A_4}{3} + A_{10};$$

$$A_5^* = \frac{A_4}{6} + A_5 + \frac{A_6}{6}; \qquad A_6^* = \frac{2A_6}{3};$$

$$A_7^* = \frac{A_6}{6} + A_7 + \frac{A_8}{6}; \qquad A_8^* = \frac{2A_8}{3} + A_9.$$

The gravity center coordinates of the points in the basic coordinate system $y_0 O_0 z_0$ (see Fig. 49):

$$y_1 = 0; \qquad z_1 = \frac{\delta_1}{2};$$

$$y_2 = 0; \qquad z_2 = \delta_1 + \frac{l_2}{2};$$

$$y_3 = 0; \qquad z_3 = H - \frac{\delta_3}{2};$$

$$y_4 = \frac{l_3}{2} + \frac{l_4}{2}\cos\beta; \qquad z_4 = H - \frac{l_4}{2}\sin\beta;$$

$$y_5 = L; \qquad z_5 = a + h - \frac{\delta_5}{2};$$

$$y_6 = L; \qquad z_6 = a + \delta_7 + \frac{l_6}{2};$$

$$y_7 = L; \qquad z_7 = a + \frac{\delta_7}{2};$$

$$y_8 = \frac{l_1}{2} + \frac{l_8}{2}\cos\gamma; \qquad z_8 = \frac{l_8}{2}\sin\gamma;$$

The static moments:

$$S_{y_0}^{(i)} = A_i^* z_i; \qquad S_{z_0}^{(i)} = A_i^* y_i; \qquad i = 1 \dots 8,$$

where i is a number of the points.

The gravity center coordinates of the compound section y_c and z_c in basic coordinate system $y_0 O_0 z_0$:

$$y_c = \frac{S_{z_0}}{A} = \frac{\sum_{i=1}^{8} S_{z_0}^{(i)}}{\sum_{i=1}^{8} A_i^*}; \qquad z_c = \frac{S_{y_0}}{A} = \frac{\sum_{i=1}^{8} S_{y_0}^{(i)}}{\sum_{i=1}^{8} A_i^*}.$$

K13 | =J13+J12/6+J14/6

	H	I	J	K	L	M	N	O	P	Q
9										
10		Element number	Ai	Ai*	yi	zi	Sy0i	Sz0i	yc	zc
11		1	4,00	10,23	0	0,5	5,12	0,00		
12		2	6,75	4,50	0	12,25	55,13	0,00		
13		3	4,50	10,81	0	24,25	262,07	0,00		
14		4	31,09	23,76	40,25	22	522,69	956,28	36,628	12,181
15		5	2,40	8,02	80	18,40	147,48	641,24		
16		6	2,60	1,73	80	11,30	19,59	138,67		
17		7	2,40	7,94	80	4,40	34,94	635,22		
18		8	30,64	22,7	40,25	2,00	45,40	913,59		
19		9	2,27							
20		10	3,03							
21		Sum	89,68	89,68			1092,41	3285,00		
22			cm^2	cm^2	cm	cm	cm^3	cm^3	cm	cm

Fig. 50

The coordinates of the gravity centers of the points in the system of central axes yOz:

$$y_{c_i} = y_i - y_c; \qquad z_{c_i} = z_i - z_c.$$

The verification of the gravity center position determination:

$$S_y = \sum_{i=1}^{8} S_y^{(i)} = \sum_{i=1}^{8} A_i^* z_{c_i} = 0; \qquad S_z = \sum_{i=1}^{8} S_z^{(i)} = \sum_{i=1}^{8} A_i^* y_{c_i} = 0.$$

T13 =S13*K13

	R	S	T	U
9				
10	yci	zci	Syi	Szi
11	-36,63	-11,68	-120	-375
12	-36,63	0,07	0	-165
13	-36,63	12,07	130	-396
14	3,62	9,82	233	86
15	43,37	6,22	50	348
16	43,37	-0,88	-2	75
17	43,37	-7,78	-62	344
18	3,62	-10,18	-231	82
19				
20				
21			0	0
22	cm	cm	cm^3	cm^3

Fig. 51

The axial and centrifugal inertia moments of the compound section in the system of central axes yOz (intrinsic inertia moments of the elements are not taken into account):

$$I_y^{(i)} = z_{c_i}^2 A_i^*; \qquad I_z^{(i)} = y_{c_i}^2 A_i^*; \qquad I_{yz}^{(i)} = y_{c_i} z_{c_i} A_i^*.$$

The axial and centrifugal inertia moments of the compound section in the system of central axes yOz:

$$I_y = \sum_{i=1}^{8} I_y^{(i)}; \qquad I_z = \sum_{i=1}^{8} I_z^{(i)}; \qquad I_{yz} = \sum_{i=1}^{8} I_{yz}^{(i)}.$$

The location of principal inertia axes and principal inertia moments of the section:

$$\operatorname{tg} 2\alpha_0 = \frac{2I_{yz}}{I_z - I_y}; \qquad \alpha_0 = \frac{1}{2}\operatorname{arctg}\left(\frac{2I_{yz}}{I_z - I_y}\right);$$

$$I_{\substack{max \\ min}} = I_{\substack{u \\ v}} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_z - I_y}{2}\right)^2 + I_{yz}^2}.$$

W13 | =K13*R13^2

	V	W	X	Y	Z	AA	AB	AC
9								
10	Iyi_total	Izi_total	Iyzi_total	alfa0	Iu_max	Iv_min	Comparison with exact	
11	1396	13727,6	4377,63					
12	0,02172	6037,36	-11,451	-0,01651				
13	1574,3	14499,2	-4777,7					
14	2290,84	311,627	844,919	(rad)	68164,76	8389,412	3,1004414	3,149294
15	310,053	15077,9	2162,16					
16	1,3439	3260,57	-66,196	-0,9462				
17	480,679	14936,5	-2679,5					
18	2352,48	297,715	-836,88	(degree)				
19								
20								
21	8406	68148	-987					
22	cm^4	cm^4	cm^4		cm^4	cm^4		
23	Iyi_total + Izi_total				Iu_max + Iv_min			
24	76554	cm^4			76554,18	cm^4		

Fig. 52

Final results of calculation:

$$y_c = 36{,}628\ cm;$$

$$z_c = 12{,}181\ cm;$$

$$\alpha_0 = -0{,}9462°;$$

$$I_{max} = I_u = 68164{,}76\ cm^4;$$

$$I_{min} = I_v = 8389{,}41\ cm^4.$$

Therefore, we obtain an errors in defining values that is. The calculation errors are shown in Table 7.

Table 7

y_c	z_c	α_0	$I_{max} = I_u$	$I_{max} = I_u$
0,14 %	0,04 %	3,34 %	3,10 %	3,15 %

These errors can be considerably reduced by using a model with finer discretization.

Self-assessment quiz

1. Name basic geometric characteristics of cross-sections.

2. What are the geometric characteristics of plane sections are needed for?

3. What is the static moment of a plane figure about an axis?

4. What is the dimension of a static moment?

5. What is the static moment about the axis passing through the gravity center of a section?

6. How to determine the gravity center coordinates of simple and compound sections?

7. Which axes are referred to as central axes?

8. What is called axial, polar, and centrifugal inertia moment of a section?

9. What is the dimension of inertia moments of the section?

10. Why axial and polar inertia moments cannot be negative?

11. Relative to which of the parallel axes will the axial inertia moment be the smallest?

12. What form do the transition formulas for calculating the inertia moments with the parallel translation of axes have?

13. What expressions are used to determine the values of the principal inertia moments and the position of the principal axes?

14. Which axes are referred to as principal inertia axes?

15. Which axes are referred to as principal central inertia axes?

16. What properties do principal central inertia moments of the section have?

17. What is the centrifugal inertia moment about the principal central axes?

18. The position of the principal axes of which sections can be specified without calculations?

19. Does the sum of the axial inertia moments about two mutually perpendicular axes change when these axes are rotated?

20. What are the axial inertia moments of a circle and a ring about the axes passing through their gravity centers?

21. What are the axial inertia moments of a rectangle and a right-angled triangle about the axes passing through their gravity centers?

22. How to determine the sign of centrifugal inertia moment of a right-angled triangle and an L-beam about the axes parallel to the base?

The solving procedure and variants of tasks

1. Make a dimensioned drawing of the compound section.
2. Decompose the section into elemental parts.
3. Put the central coordinate systems $y_iO_iz_i$ in the gravity center of each elemental part of the section.
4. Copy the geometric characteristics of rolled profiles from the assortment tables (task 1).

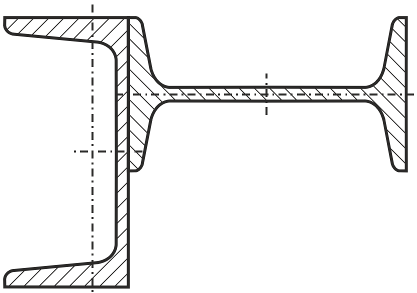
   Calculate the geometric characteristics of simple figures (task 2).

Notes to the task 2	1. Geometric dimensions of simple figures are in centimeters. 2. All triangles are right-angled or equilateral.

5. Find the coordinates of gravity center of the compound section. Verify the calculated location of gravity center.
6. Find the axial and centrifugal inertia moments of the compound section in the system of central axes yOz.
7. Calculate the location of principal central axes of the section, i.e. the angle by which the central axes should be rotated to become principal.
8. Define the values of principal central inertia moments of the section.
9. Validate the solution.

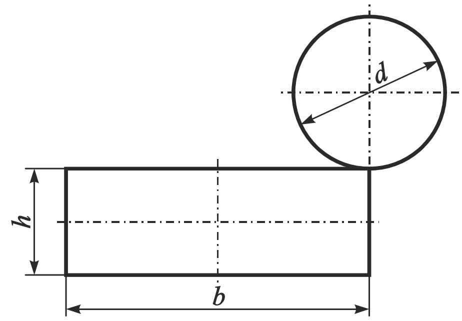
Variant 1

Task 1



№	1	2	3	4	5	6	7	8
№ ⊏	14	14a	16	16a	18	18a	20	20a
№ I	10	12	14	16	18	20	20a	22

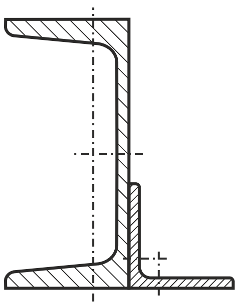
Task 2



№	1	2	3	4	5	6	7	8
b	20	16	12	10	8	14	18	24
h	4	6	4	2	3	4	5	6
d	6	5	6	4	5	8	10	8

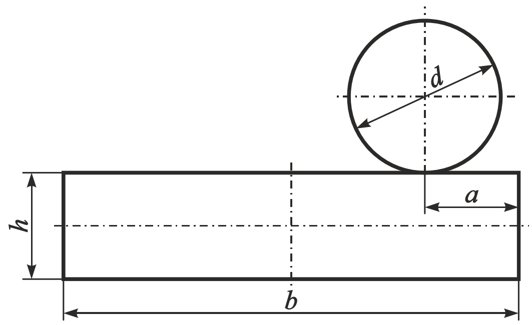
Variant 2

Task 1



№	1	2	3	4	5	6	7	8
№ ⊏	5	6,5	8	10	12	14	14a	16
№ ∟	2	3	3,5	4	4,5	5	6,5	7,5

Task 2



№	1	2	3	4	5	6	7	8
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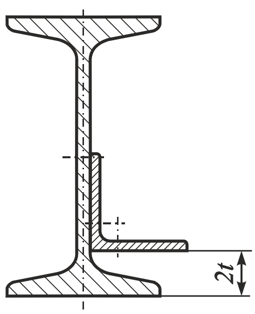
h	6	5	3	4	3	6	4	6
d	8	10	6	5	8	7	8	6
a	3	2	1	2	1	2	2	3

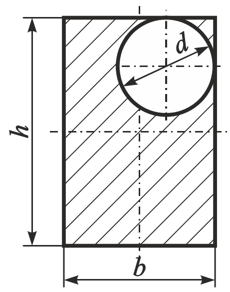
Variant 3

Task 1



№	1	2	3	4	5	6	7	8
№ I	10	12	14	16	18	20	20a	22
№ ∟	3,5	4	4,5	5	6	6,5	7,5	8

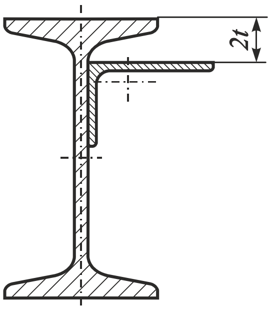
Task 2



№	1	2	3	4	5	6	7	8
b	8	10	14	16	12	18	15	20
h	12	5	16	16	16	24	20	30
d	4	4	6	5	8	10	8	12

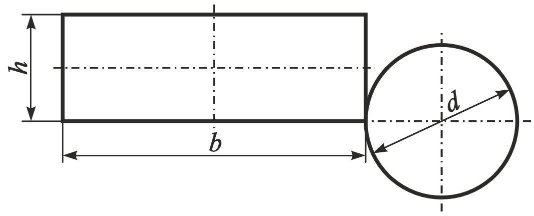
Variant 4

Task 1



№	1	2	3	4	5	6	7	8
№ I	10	12	14	16	18	20	20a	22
№ L	5/3,2	6,3/4	7/4,5	7,5/5	8/5	9/5,6	11/7	14/9

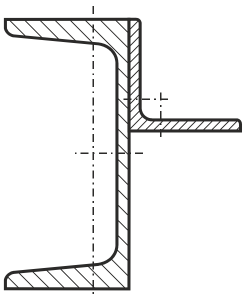
Task 2



№	1	2	3	4	5	6	7	8
b	10	12	9	18	15	8	10	20
h	4	3	5	4	6	2	3	6
d	6	5	7	8	10	5	8	10

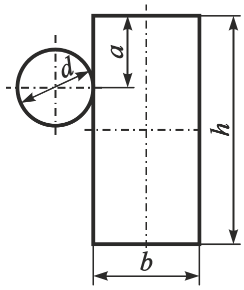
Variant 5

Task 1



№	1	2	3	4	5	6	7	8
№ [	5	6,5	8	10	12	14	14a	18
№ L	2	2,5	3,5	4,5	5	6	6,5	7,5

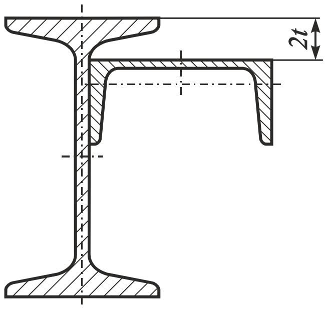
Task 2



№	1	2	3	4	5	6	7	8
b	2	6	4	3	5	4	5	6
h	8	10	9	10	12	14	16	15
d	6	8	7	6	8	6	6	8
a	1	2	3	1	1	2	4	2

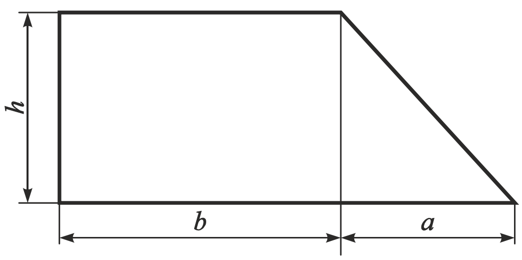
Variant 6

Task 1



№	1	2	3	4	5	6	7	8
№ [	5	6,5	8	10	12	14	14a	16
№ I	10	12	14	16	18	20	20a	22

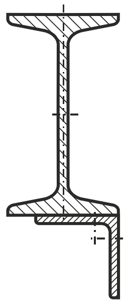
Task 2



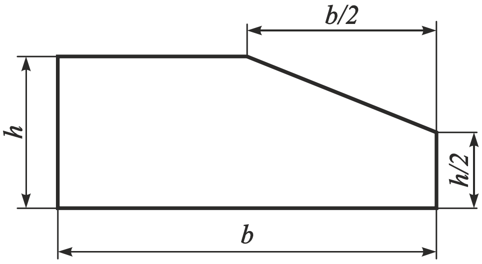
№	1	2	3	4	5	6	7	8
b	8	10	14	7	12	10	11	16
h	9	12	9	6	12	15	15	18
a	6	9	9	7,5	6	9	7,5	15

Variant 7

Task 1



Task 2

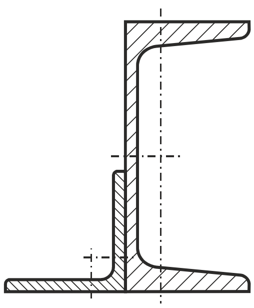


№	1	2	3	4	5	6	7	8
№ I	10	12	14	16	18	20	20a	22
№ ∟	4	4,5	5	6	7	7,5	8	9

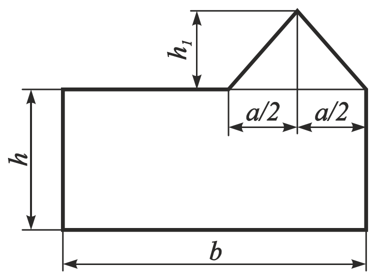
№	1	2	3	4	5	6	7	8
h	6	12	15	24	15	12	18	24
b	9	9	18	18	12	6	12	15

Variant 8

Task 1



Task 2

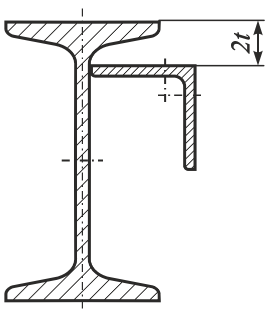


№	1	2	3	4	5	6	7	8
№ [	8	10	12	14	14a	16	16a	18
№ ∟	4	4,5	6	6,3	7,5	8	9	10

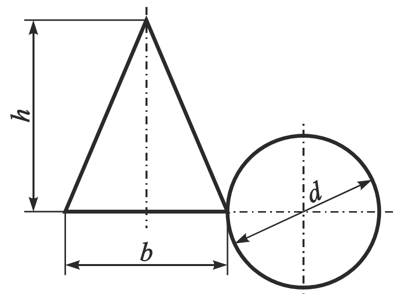
№	1	2	3	4	5	6	7	8
a	8	4	5	6	8	6	10	12
b	10	8	12	14	12	12	19	20
h	6	3	4	4	3	5	6	5
h_1	6	6	9	9	6	12	12	15

Variant 9

Task 1



Task 2

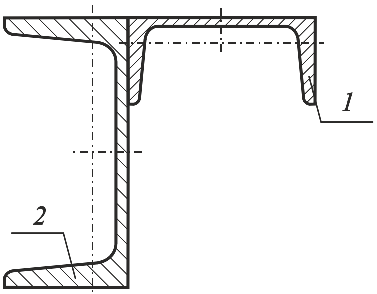


№	1	2	3	4	5	6	7	8
№ I	10	12	14	16	18	20	20a	22
№ ∟	3,5	4,5	5	6	6,3	7,5	8	9

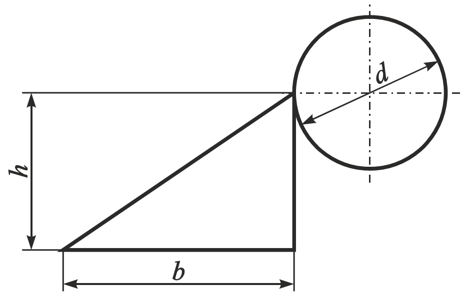
№	1	2	3	4	5	6	7	8
h	10	12	9	9	6	15	18	21
b	8	10	9	8	4	10	12	15
d	8	6	7	8	5	8	10	14

Variant 10

Task 1



Task 2

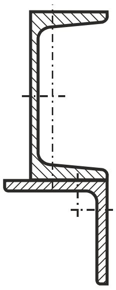


№	1	2	3	4	5	6	7	8
№1 [	5	6,5	8	10	12	14	14a	16
№2 [	6,5	8	10	12	14	14a	16	16a

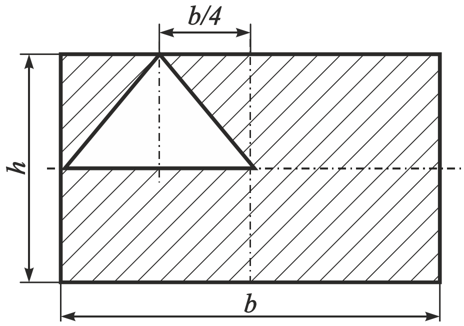
№	1	2	3	4	5	6	7	8
h	6	9	12	15	12	18	6	18
b	9	6	9	9	15	12	6	15
d	8	6	8	10	9	14	5	10

Variant 11

Task 1



Task 2

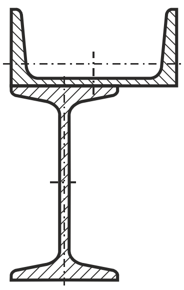


№	1	2	3	4	5	6	7	8
№ [	5	6,5	8	10	12	14	14a	16
№ ∟	3,5	4	4,5	5	6	6,3	7,5	8

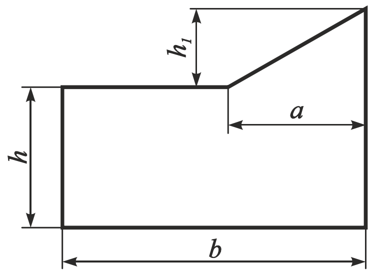
№	1	2	3	4	5	6	7	8
h	9	12	12	15	18	18	6	24
b	8	12	16	10	16	20	8	18

Variant 12

Task 1



Task 2

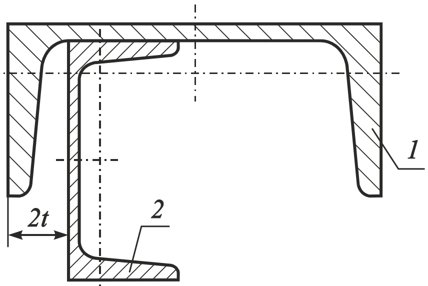


№	1	2	3	4	5	6	7	8
№ [	10	12	14	14a	16	16a	18	18a
№ I	10	12	14	16	18	20	20a	22

№	1	2	3	4	5	6	7	8
h	2	4	3	5	4	3	5	6
b	8	10	8	12	14	10	15	17
a	6	6	9	9	7,5	7,5	9	12
h_1	6	9	6	9	9	7,5	12	9

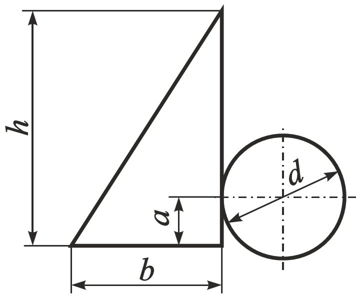
Variant 13

Task 1



№	1	2	3	4	5	6	7	8
№1 [	8	10	12	14	14a	16	16a	18
№2 [	6,5	8	10	12	5	14	14a	16

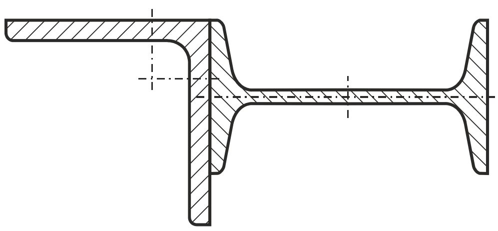
Task 2



№	1	2	3	4	5	6	7	8
h	6	15	12	9	7,5	15	18	21
b	6	9	15	6	6	12	9	15
a	1	3	2	2	1	3	4	3
d	5	7	8	6	4	6	10	10

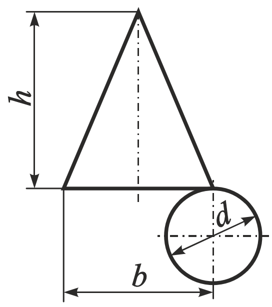
Variant 14

Task 1



№	1	2	3	4	5	6	7	8
№ I	10	12	14	16	18	20	20a	22
№ ∟	7,5	8	9	10	12,5	14	16	18

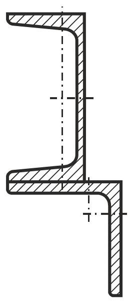
Task 2



№	1	2	3	4	5	6	7	8
h	18	16	15	10	12	14	20	10
b	12	8	10	10	9	8	8	14
d	10	10	12	8	9	6	10	6

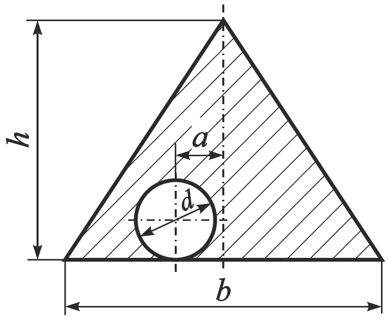
Variant 15

Task 1



№	1	2	3	4	5	6	7	8
№ [	5	6,5	8	10	12	14	14a	16
№ ∟	4,5	5	6	6,3	7,5	8	9	10

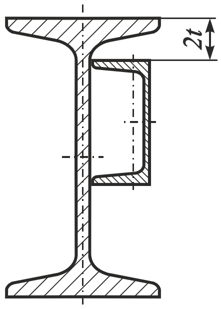
Task 2



№	1	2	3	4	5	6	7	8
h	8	10	12	14	15	16	9	18
b	6	12	8	12	10	12	9	12
a	1	1	1	2	1	1	1,5	2
d	2	4	3	4	4	5	3	5

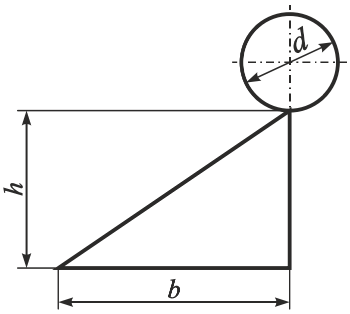
Variant 16

Task 1



№	1	2	3	4	5	6	7	8
№ I	10	12	14	16	18	20	20a	22
№ [	5	6,5	8	10	12	14	14a	16

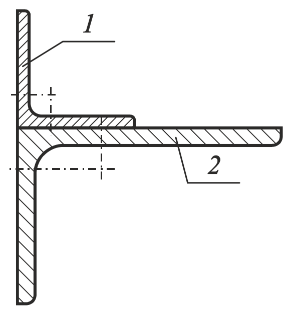
Task 2



№	1	2	3	4	5	6	7	8
b	9	6	9	7,5	9	12	15	18
h	6	6	9	6	7,5	6	9	12
d	6	8	8	7	5	8	10	10

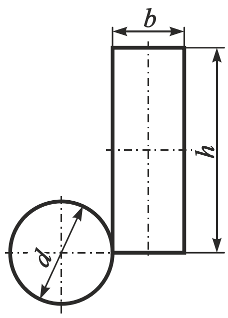
Variant 17

Task 1



№	1	2	3	4	5	6	7	8
№1 ∟	3	3,5	4	4,5	5	6	6,3	7,5
№2 L	3,2/2	4/2,5	5/3,2	6,3/4	7/4,5	8/5	9/5,6	11/7

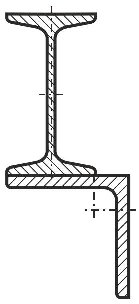
Task 2



№	1	2	3	4	5	6	7	8
b	8	10	8	12	14	14	15	16
h	4	4	2	5	6	4	6	5
d	8	6	6	10	8	9	12	8

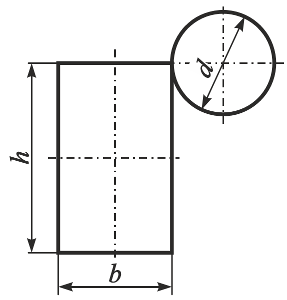
Variant 18

Task 1



№	1	2	3	4	5	6	7	8
№ I	10	12	14	16	18	20	20a	22
№ ∟	7,5	8	9	10	12,5	14	16	18

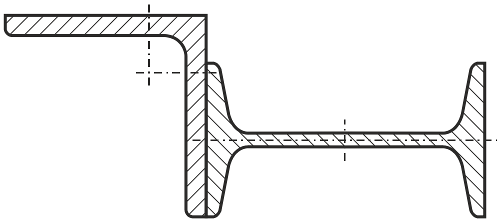
Task 2



№	1	2	3	4	5	6	7	8
h	6	8	10	14	12	16	18	20
b	4	3	4	6	4	5	6	8
d	6	6	8	8	6	10	8	10

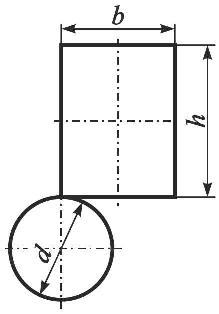
Variant 19

Task 1



№	1	2	3	4	5	6	7	8
№ ⊏	10	12	14	16	18	20	20a	22
№ I	7,5	8	9	10	12,5	14	16	18

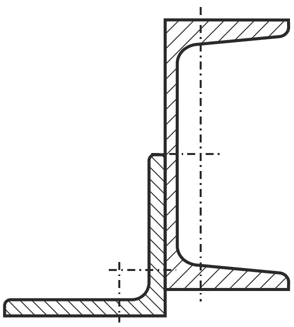
Task 2



№	1	2	3	4	5	6	7	8
h	10	8	12	14	6	6	8	18
b	4	2	4	5	8	4	8	8
d	6	5	6	8	6	8	6	12

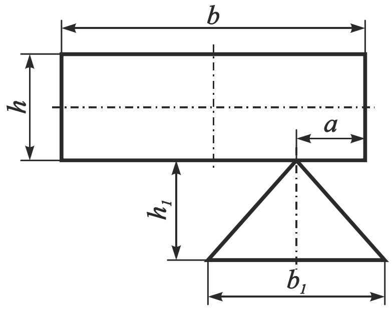
Variant 20

Task 1



№	1	2	3	4	5	6	7	8
№ ⊏	5	6,5	8	10	12	14	14a	18
№ ∟	2,5	3	3,5	4	4,5	5	6	6,3

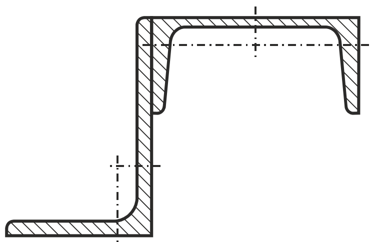
Task 2



№	1	2	3	4	5	6	7	8
h	2	3	4	6	5	4	8	3
b	10	8	14	10	12	12	10	10
h_1	6	3	6	9	7,5	7,5	12	6
b_1	4	6	8	6	8	6	8	6
a	2	1	3	2	3	1	2	1

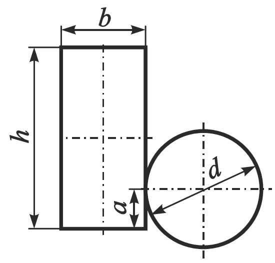
Variant 21

Task 1



№	1	2	3	4	5	6	7	8
№ ⊏	5	6,5	8	10	12	14	14a	16
№ L	4/2,5	5/3,2	6,3/4	7/4,5	7,5/5	8/5	9/5,6	11/7

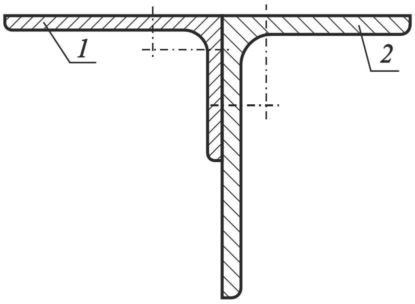
Task 2



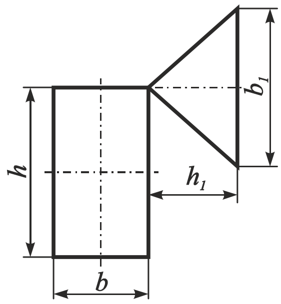
№	1	2	3	4	5	6	7	8
h	10	12	12	8	6	14	15	18
b	6	4	7	5	8	6	4	6
d	4	6	5	6	8	6	8	10
a	1	2	3	2	1	3	2	4

Variant 22

Task 1



Task 2

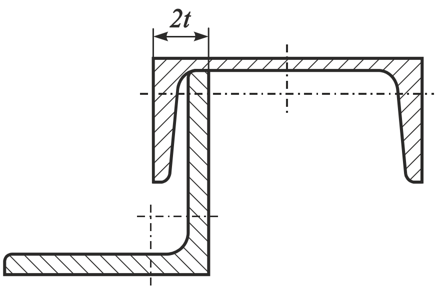


№	1	2	3	4	5	6	7	8
№1 ∟	3,2/2	4/2,5	5/3,2	6,3/4	7/4,5	8/5	9/5,6	11/7
№2 ∟	5/3,2	6,3/4	7/4,5	7,5/5	8/5	9/5,6	11/7	14/9

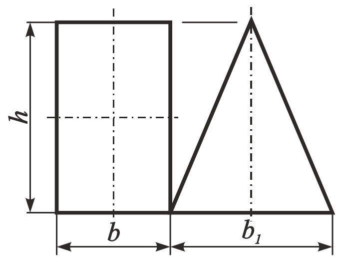
№	1	2	3	4	5	6	7	8
h	6	8	10	9	12	10	7	16
b	6	4	5	4	5	10	4	6
$\boldsymbol{h_1}$	6	6	7,5	9	9	6	6	9
$\boldsymbol{b_1}$	6	8	8	10	6	12	8	12

Variant 23

Task 1



Task 2

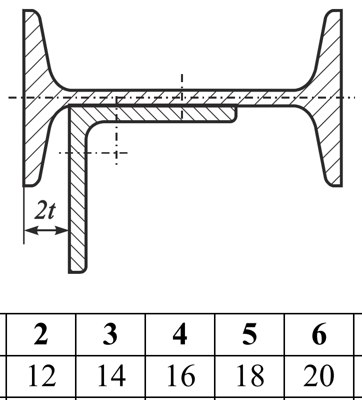


№	1	2	3	4	5	6	7	8
№ [	5	6,5	8	10	12	14	14a	16
№ ∟	5	6	6,3	7,5	8	9	10	12

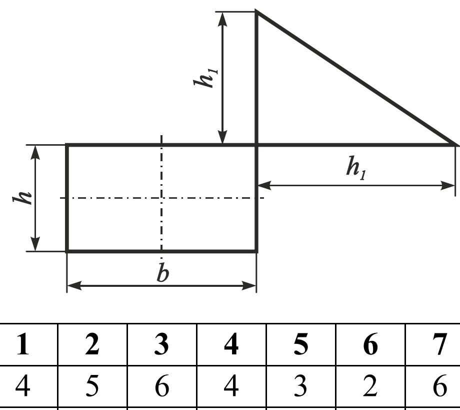
№	1	2	3	4	5	6	7	8
h	6	9	12	6	15	18	21	24
b	4	8	5	8	4	6	10	8
$\boldsymbol{b_1}$	6	6	8	10	10	12	10	12

Variant 24

Task 1



Task 2

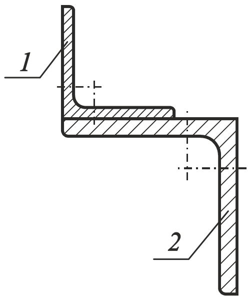


№	1	2	3	4	5	6	7	8
№ I	10	12	14	16	18	20	20a	22
№ ∟	4	4,5	5	6	6,3	7,5	8	9

№	1	2	3	4	5	6	7	8
h	4	5	6	4	3	2	6	8
b	10	10	14	8	10	8	16	20
$\boldsymbol{h_1}$	6	9	9	6	6	3	9	12
$\boldsymbol{b_1}$	6	6	12	3	9	6	9	15

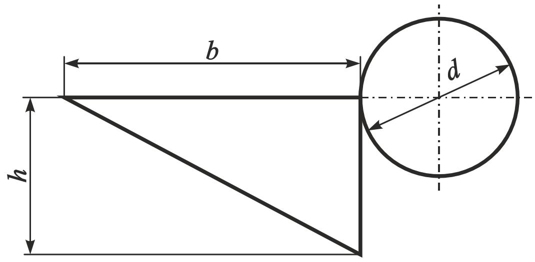
Variant 25

Task 1



№	1	2	3	4	5	6	7	8
№1 ∟	4	4,5	5	6	6,3	7,5	8	9
№2 ∟	6	6,3	7,5	8	9	10	12,5	14

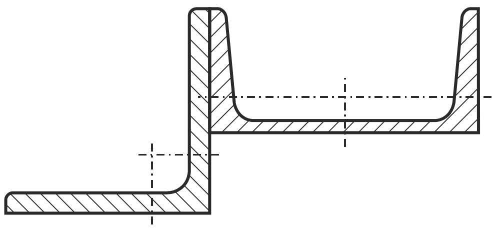
Task 2



№	1	2	3	4	5	6	7	8
h	12	15	9	12	15	18	24	6
b	9	9	6	18	12	15	15	9
d	8	8	6	14	10	12	16	8

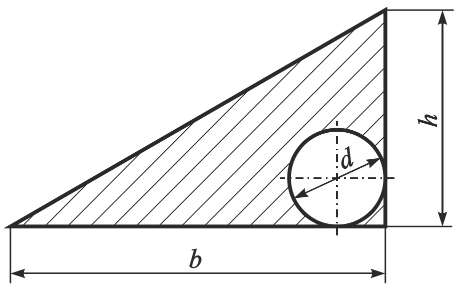
Variant 26

Task 1



№	1	2	3	4	5	6	7	8
№ [	5	6,5	8	10	12	14	14a	16
№ ∟	5	6	6,3	7,5	8	9	10	12,5

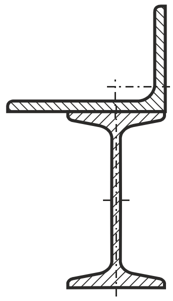
Task 2



№	1	2	3	4	5	6	7	8
h	6	9	9	12	15	18	9	15
b	9	9	12	6	12	12	15	18
d	3	4	5	4	5	7	5	9

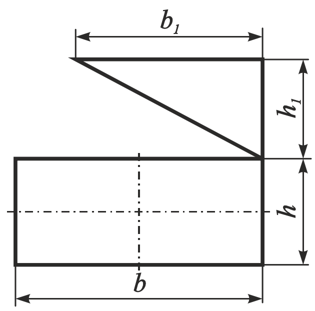
Variant 27

Task 1



№	1	2	3	4	5	6	7	8
№ I	10	12	14	16	18	20	20a	22
№ ∟	6,3/4	7/4,5	7,5/5	8/5	9/5,6	11/7	14/9	19/10

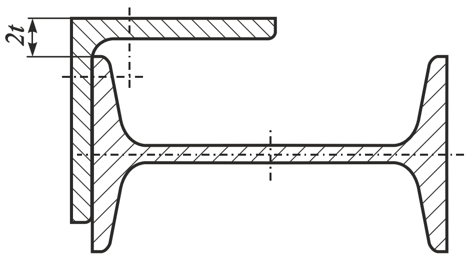
Task 2



№	1	2	3	4	5	6	7	8
h	4	4	5	8	6	5	6	8
b	10	8	9	12	12	10	10	16
h_1	6	5	6	12	9	5	6	9
b_1	6	6	9	12	9	6	12	12

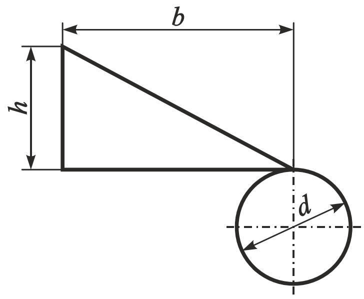
Variant 28

Task 1



№	1	2	3	4	5	6	7	8
№ I	10	12	14	16	18	20	20a	22
№ ∟	8	9	10	12,5	14	16	18	20

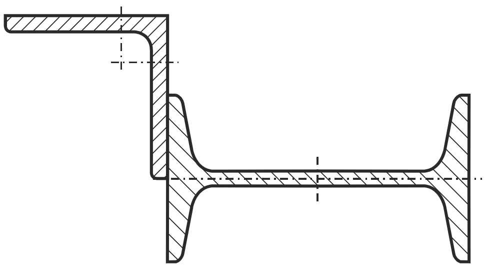
Task 2



№	1	2	3	4	5	6	7	8
h	12	12	9	9	6	12	12	15
b	18	15	12	9	9	6	9	9
d	8	10	8	6	8	8	10	10

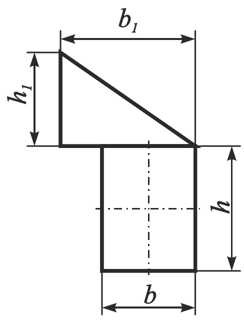
Variant 29

Task 1



№	1	2	3	4	5	6	7	8
№ I	10	12	14	16	18	20	20a	22
№ ∟	7,5	8	9	10	12,5	14	16	18

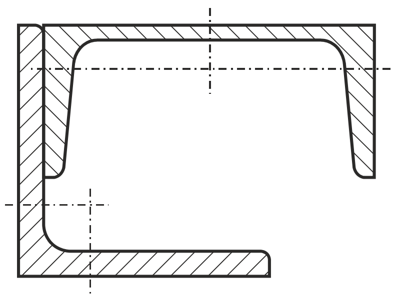
Task 2



№	1	2	3	4	5	6	7	8
h	6	5	7	8	10	9	12	16
b	4	5	5	6	4	8	8	10
h_1	3	6	6	6	9	9	9	15
b_1	6	7	9	12	9	15	12	18

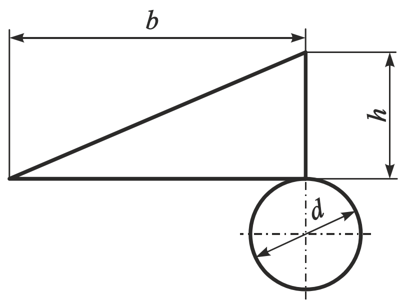
Variant 30

Task 1



№	1	2	3	4	5	6	7	8
№ [	5	6,5	8	10	12	14	14a	16
№ ∟	5	6	6,3	7,5	8	9	10	12,5

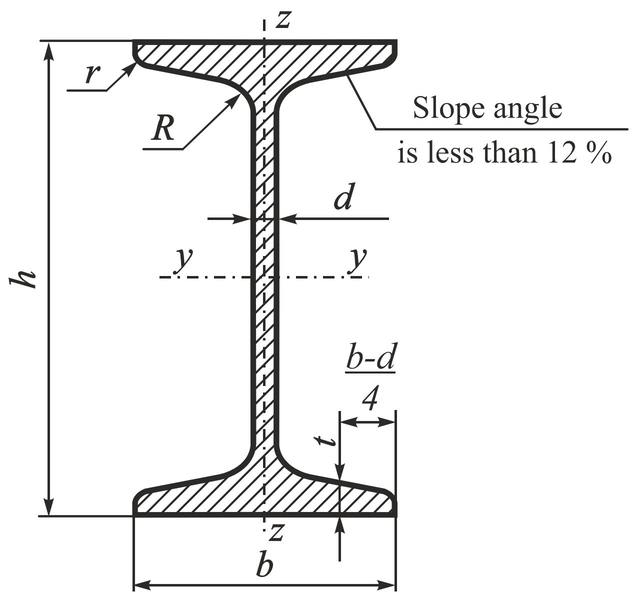
Task 2



№	1	2	3	4	5	6	7	8
h	6	6	3	9	9	6	6	9
b	6	3	6	6	9	9	12	12
d	4	5	4	5	8	7	8	9

Appendix 1

Hot rolled steel assortment.
I-beams. GOST 8239-89



Notes:

1. The cross-section area and the mass of a 1 *m* of I-beam are calculated using nominal dimensions; the density of steel is assumed equal 7,85 g/cm^3.

2. The values of curvature radii, flange inner face pitches, and flange thicknesses indicated in the figure and in the Table Appx. 1, are given for composing gauges and are not controlled on finished production.

3. The following signs are used in the tables:

- ***h*** – a beam height;
- ***b*** – a flange width;
- ***d*** – a wall thickness;
- ***t*** – an average flange thickness;
- ***R*** – an inner curvature radius;
- ***A*** – a cross-section area;
- ***r*** – a flange curvature radius;
- ***I*** – an inertia moment;
- ***W*** – a resistance moment;
- ***S*** – a static moment of a half-section;
- ***i*** – an inertia radius.

4. The I-beams from № 24 to № 60 are not recommended to be used in new designs

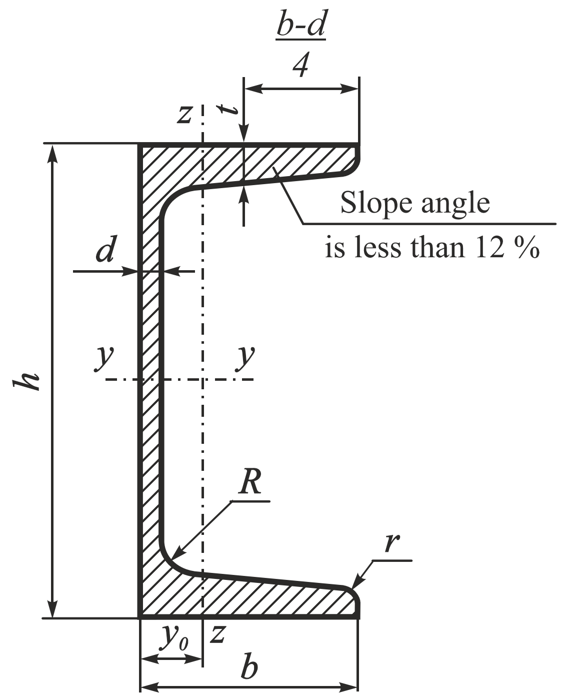
Table Appx. 1.1

Number	**Dimensions**, *mm*						**A**, cm^2	**Mass of 1 m**, *kg*	**Referential data for axes**						
									$y-y$				$z-z$		
	h	b	d	t	R	r			I_y, cm^4	W_y, cm^3	i_y, cm	S_y, cm^3	I_z, cm^4	W_z, cm^3	i_z, cm
10	100	55	4,5	7,2	7,0	2,5	12,0	9,46	198	39,7	4,06	23,0	17,9	6,49	1,22
12	120	64	4,8	7,3	7,5	3,0	14,7	11,5	350	58,4	4,88	33,7	27,9	8,72	1,38
14	140	73	4,9	7,5	8,0	3,0	17,4	13,7	572	81,7	5,73	46,8	41,9	11,5	1,55
16	160	81	5,0	7,8	8,5	3,5	20,2	15,9	873	109,0	6,57	62,3	58,6	14,5	1,70
18	180	90	5,1	8,1	9,0	3,5	23,4	18,4	1290	143,0	7,42	81,4	82,6	18,4	1,88
18a	180	100	5,1	8,3	9,0	3,5	25,4	19,9	1430	159,0	7,51	89,8	114,0	22,8	2,12
20	200	100	5,2	8,4	9,5	4,0	26,8	21,0	1840	184,0	8,28	104,0	115,0	23,1	2,07
20a	200	110	5,2	8,6	9,5	4,0	28,9	22,7	2030	203,0	8,37	114,0	155,0	28,2	2,32
22	220	110	5,4	8,7	10,0	4,0	30,6	24,0	2550	232,0	9,13	131,0	157,0	28,6	2,27
22a	220	120	5,4	8,9	10,0	4,0	32,8	25,8	2790	254,0	9,22	143,0	206,0	34,3	2,50
24	240	115	5,6	9,5	10,5	4,0	34,8	27,3	3460	289,0	9,97	163,0	198,0	34,5	2,37
24a	240	125	5,6	9,8	10,5	4,0	37,5	29,4	3800	317,0	10,10	178,0	260,0	41,6	2,63
27	270	125	6,0	9,8	11,0	4,5	40,2	31,5	5010	371,0	11,20	210,0	260,0	41,5	2,54
27a	270	135	6,0	10,2	11,0	4,5	43,2	33,9	5500	407,0	11,30	229,0	337,0	50,0	2,80
30	300	135	6,5	10,2	12,0	5,0	46,5	36,5	7080	472,0	12,30	268,0	337,0	49,9	2,69
30a	300	145	6,5	10,7	12,0	5,0	49,9	39,2	7780	518,0	12,50	292,0	436,0	60,1	2,95
33	330	140	7,0	11,2	13,0	5,0	53,8	42,2	9840	597,0	13,50	339,0	419,0	59,9	2,79
36	360	145	7,5	12,3	14,0	6,0	61,9	48,6	13380	743,0	14,70	423,0	516,0	71,1	2,89
40	400	155	8,5	13,0	15,0	6,0	72,6	57,0	19062	953,0	16,20	545,0	667,0	86,1	3,03
45	450	160	9,0	14,2	16,0	7,0	84,7	66,5	27696	1231,0	18,10	708,0	808,0	101,0	3,09
50	500	170	10,0	15,2	17,0	7,0	100,0	78,5	39727	1589,0	19,90	919,0	1043,0	123,0	3,23
55	550	180	11,0	16,5	18,0	7,0	118,0	92,6	55962	2035,0	21,80	1181,0	1356,0	151,0	3,39
60	600	190	12,0	17,8	20,0	8,0	138,0	108,0	76806	2560,0	23,60	1491,0	1725,0	182,0	3,54

Appendix 2

Hot rolled steel assortment.

C-beams with flange inner face pitches. GOST 8240-89



Notes:

1. The cross-section area and the mass of a 1 *m* of C-beam are calculated using nominal dimensions; the density of steel is assumed equal 7,85 g/cm^3.

2. The values of curvature radii, flange inner face pitches, and flange thicknesses indicated in the figure and in the Table Appx. 2, are given for composing gauges and are not controlled on finished production.

3. The following signs are used in the tables:

$\boldsymbol{h}$ – a beam height;
$\boldsymbol{b}$ – a flange width;
$\boldsymbol{d}$ – a wall thickness;
$\boldsymbol{t}$ – an average flange thickness;
$\boldsymbol{R}$ – an inner curvature radius;
$\boldsymbol{r}$ – a flange curvature radius;
$\boldsymbol{A}$ – a cross-section area;
$\boldsymbol{I}$ – an inertia moment;
$\boldsymbol{W}$ – a resistance moment;
$\boldsymbol{i}$ – an inertia radius;
$\boldsymbol{S}$ – a static moment of a half-section;
$\boldsymbol{y_0}$ – a distance from axis $\boldsymbol{z-z}$ to outer face of a wall

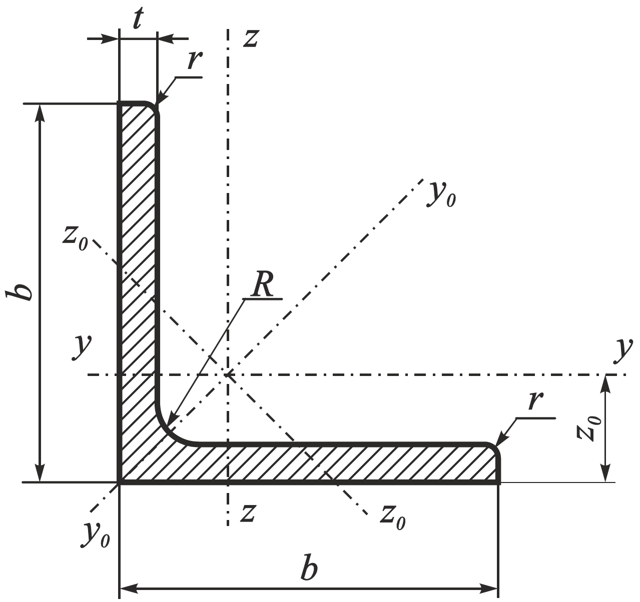
Table Appx. 2.1

Number	Dimensions, *mm*						A, cm^2	Mass of 1 m, *kg*	Referential data for axes							y_0, *cm*
									$y-y$				$z-z$			
	h	b	d	t	R	r			I_y, cm^4	W_y, cm^3	i_y, *cm*	S_y, cm^3	I_z, cm^4	W_z, cm^3	i_z, *cm*	
5	50	32	4,4	7,0	6,0	2,5	6,16	4,84	22,8	9,1	1,92	5,59	5,61	2,75	0,954	1,16
6,5	65	36	4,4	7,2	6,0	2,5	7,51	5,90	48,6	15,0	2,54	9,0	8,7	3,68	1,08	1,24
8	80	40	4,5	7,4	6,5	2,5	8,98	7,05	89,4	22,4	3,16	13,3	12,8	4,75	1,19	1,31
10	100	46	4,5	7,6	7,0	3,0	10,9	8,59	174,0	34,8	3,99	20,4	20,4	6,46	1,37	1,44
12	120	52	4,8	7,8	7,5	3,0	13,3	10,4	304,0	50,6	4,78	29,6	31,2	8,52	1,53	1,54
14	140	58	4,9	8,1	8,0	3,0	15,6	12,3	491,0	70,2	5,6	40,8	45,4	11,0	1,70	1,67
14a	140	62	4,9	8,7	8,0	3,0	17,0	13,3	545,0	77,8	5,66	45,1	57,5	13,3	1,84	1,87
16	160	64	5,0	8,4	8,5	3,5	18,1	14,2	747,0	93,4	6,42	54,1	63,3	13,8	1,87	1,8
16a	160	68	5,0	9,0	8,5	3,5	19,5	15,3	823,0	103,0	6,49	59,4	78,8	16,4	2,01	2,0
18	180	70	5,1	8,7	9,0	3,5	20,7	16,3	1090,0	121,0	7,24	69,8	86,0	17,0	2,04	1,94
18a	180	74	5,1	9,3	9,0	3,5	22,2	17,4	1190,0	132,0	7,32	76,1	105,0	20,0	2,18	2,13
20	200	76	5,2	9,0	9,5	4,0	23,4	18,4	1520,0	152,0	8,07	87,8	113,0	20,5	2,20	2,07
20a	200	80	5,2	9,7	9,5	4,0	25,2	19,8	1670,0	167,0	8,15	95,9	139,0	24,2	2,35	2,28
22	220	82	5,4	9,5	10,0	4,0	26,7	21,0	2110,0	192,0	8,89	110,0	151,0	25,1	2,37	2,21
22a	220	87	5,4	10,2	10,0	4,0	28,8	22,6	2330,0	212,0	8,99	121,0	187,0	30,0	2,55	2,46
24	240	90	5,6	10,0	10,5	4,0	30,6	24,0	2900,0	242,0	9,73	139,0	208,0	31,6	2,6	2,42
24a	240	95	5,6	10,7	10,5	4,0	32,9	25,8	3180,0	265,0	9,84	151,0	254,0	37,2	2,78	2,67
27	270	95	6,0	10,5	11,0	4,5	35,2	27,7	4160,0	308,0	10,9	178,0	262,0	37,3	2,73	2,47
30	300	100	6,5	11,0	12,0	5,0	40,5	31,8	5810,0	387,0	12,0	224,0	327,0	43,6	2,84	2,52
33	330	105	7,0	11,7	13,0	5,0	46,5	36,5	7980,0	484,0	13,1	281,0	410,0	51,8	2,97	2,59
36	360	110	7,5	12,6	14,0	6,0	53,4	41,9	10820,0	601,0	14,2	350,0	513,0	61,7	3,1	2,68
40	400	115	8,0	13,5	15,0	6,0	61,5	48,3	15220,0	761,0	15,7	444,0	642,0	73,4	3,23	2,75

Appendix 3

Hot rolled steel assortment.

Hot rolled equilateral L-section steel (equilateral L-beams). GOST 8509-93



Notes:

1. The cross-section area and the mass of a 1 *m* of L-beam are calculated using nominal dimensions; the density of steel is assumed equal 7,85 g/cm^3.

2. The values of curvature radii, flange inner face pitches, and flange thicknesses indicated in the figure and in the Table Appx. 3, are given for composing gauges and are not controlled on finished production.

3. The following signs are used in the tables:
   - $\boldsymbol{b}$ – a flange width;
   - $\boldsymbol{t}$ – a flange thickness;
   - $\boldsymbol{R}$ – an inner curvature radius;
   - $\boldsymbol{r}$ – a flange curvature radius;
   - $\boldsymbol{A}$ – a cross-section area;
   - $\boldsymbol{I}$ – an inertia moment;
   - $\boldsymbol{W}$ – a resistance moment;
   - $\boldsymbol{i}$ – an inertia radius;
   - $\boldsymbol{z_0}$ – a distance from the center of gravity to the outer face of the flange

Table Appx. 3.1

Number	Dimensions, *mm*				A, cm^2	Mass of 1 m, *kg*	Referential data for axes								I_{yz}, cm^4	z_0, *cm*
							$y-y$			y_0-y_0		z_0-z_0				
	b	t	R	r			I_y, cm^4	W_y, cm^3	i_y, *cm*	I_{y_0max}, cm^4	i_{y_0max}, *cm*	I_{z_0min}, cm^4	W_z, cm^3	i_{z_0min}, *cm*		
2	20	3	3,5	1,2	1,13	0,89	0,40	0,28	0,59	0,63	0,75	0,17	0,20	0,39	0,23	0,60
		4			1,46	1,15	0,50	0,37	0,58	0,78	0,73	0,22	0,24	0,38	0,28	0,64
2,5	25	3	3,5	1,2	1,43	1,12	0,81	0,46	0,75	1,29	0,95	0,34	0,33	0,49	0,47	0,73
		4			1,86	1,46	1,03	0,59	0,74	1,62	0,93	0,44	0,41	0,48	0,59	0,76
2,8	28	3	4,0	1,3	1,62	1,27	1,16	0,58	0,85	1,84	1,07	0,48	0,42	0,55	0,68	0,80
3	30	3	4,0	1,3	1,74	1,36	1,45	0,67	0,91	2,30	1,15	0,60	0,53	0,59	0,85	0,85
		4			2,27	1,78	1,84	0,87	0,80	2,92	1,13	0,77	0,61	0,58	1,08	0,89
3,2	32	3	4,5	1,5	1,86	1,46	1,77	0,77	0,97	2,80	1,23	0,74	0,59	0,63	1,03	0,89
		4			2,43	1,91	2,26	1,00	0,96	3,58	1,21	0,94	0,71	0,62	1,32	0,94
3,5	35	3	4,5	1,5	2,04	1,60	2,35	0,93	1,07	3,72	1,35	0,97	0,71	0,69	1,37	0,97
		4			2,17	2,10	3,01	1,21	1,06	4,76	1,33	1,25	0,88	0,68	1,75	1,01
		5			3,28	2,58	3,61	1,47	1,05	5,71	1,32	1,52	1,02	0,68	2,10	1,05
4	40	3	5,0	1,7	2,35	1,85	3,55	1,22	1,23	5,63	1,55	1,47	0,95	0,79	2,08	1,09
		4			3,08	2,42	4,58	1,60	1,22	7,26	1,53	1,90	1,19	0,78	2,68	1,13
		5			3,79	2,98	5,53	1,95	1,21	8,75	1,52	2,30	1,39	0,78	3,22	1,17
4,5	45	3	5,0	1,7	2,65	2,08	5,13	1,56	1,39	8,13	1,75	2,12	1,24	0,89	3,00	1,21
		4			3,48	2,73	6,63	2,04	1,38	10,52	1,74	2,74	1,54	0,89	3,89	1,26
		5			4,29	3,37	8,03	2,51	1,37	12,74	1,72	3,33	1,81	0,88	4,71	1,30
5	50	3	5,5	1,8	2,96	2,32	7,11	1,94	1,55	11,27	1,95	2,95	1,57	1,00	4,16	1,33
		4			3,89	3,05	9,21	2,54	1,54	14,63	1,94	3,80	1,95	0,99	5,42	1,38
		5			4,80	3,77	11,20	3,13	1,53	17,77	1,92	4,63	2,30	0,98	6,57	1,42
		6			5,69	4,47	13,07	3,69	1,52	20,72	1,91	5,43	2,63	0,98	7,65	1,46
5,6	56	4	6,0	2,0	4,38	3,44	13,10	3,21	1,73	20,79	2,18	5,41	2,52	1,11	7,69	1,52
		5			5,41	4,25	15,97	3,96	1,72	25,36	2,16	6,59	2,97	1,10	9,41	1,57

Table Appx. 3.1 (continued)

Number	Dimensions, *mm*				A, cm^2	Mass of 1 m, kg	Referential data for axes								I_{yz}, cm^4	z_0, cm
							$y-y$			y_0-y_0		z_0-z_0				
	b	t	R	r			I_y, cm^4	W_y, cm^3	i_y, cm	I_{y_0max}, cm^4	i_{y_0max}, cm	I_{z_0min}, cm^4	W_z, cm^3	i_{z_0min}, cm		
6	60	4	7,0	2,3	4,72	3,71	16,21	3,70	1,85	25,69	2,33	6,72	2,93	1,19	9,48	1,62
		5			5,83	4,58	19,79	4,56	1,84	31,40	2,32	8,18	3,49	1,18	11,61	1,66
		6			6,92	5,43	23,21	5,40	1,83	36,81	2,31	9,60	3,99	1,18	13,60	1,70
		8			9,40	7,10	29,55	7,00	1,81	46,77	2,27	12,34	4,90	1,17	17,22	1,78
		10			11,08	8,70	35,32	8,52	1,79	55,64	2,24	15,00	5,70	1,16	20,32	1,85
6,3	63	4	7,0	2,3	4,69	3,90	18,86	4,09	1,95	29,90	2,45	7,81	3,26	1,25	11,00	1,69
		5			6,13	4,81	23,10	5,05	1,94	36,80	2,44	9,52	3,87	1,25	13,70	1,74
		6			7,28	5,72	27,06	5,98	1,93	42,91	2,43	11,18	4,44	1,24	15,90	1,78
7	70	4,5	8,0	2,7	6,20	4,87	29,04	5,67	2,16	46,03	2,72	12,04	4,53	1,39	17,00	1,88
		5			6,86	5,38	31,94	6,27	2,16	50,67	2,72	13,22	4,92	1,39	18,70	1,90
		6			8,15	6,39	37,58	7,43	2,15	59,64	2,71	15,52	5,66	1,38	22,10	1,94
		7			9,42	7,39	42,98	8,57	2,14	68,19	2,69	17,77	6,31	1,37	25,20	1,99
		8			10,67	8,37	48,16	9,68	2,12	76,35	2,68	19,97	6,99	1,37	28,20	2,02
7,5	75	5	9,0	3,0	7,39	5,80	39,53	7,21	2,31	62,65	2,91	16,41	5,74	1,49	23,10	2,02
		6			8,78	6,89	46,57	8,57	2,30	73,87	2,90	19,28	6,62	1,48	27,30	2,06
		7			10,15	7,96	53,34	9,89	2,29	84,61	2,89	22,07	7,43	1,47	31,20	2,10
		8			11,50	9,02	59,84	11,18	2,28	94,89	2,87	24,80	8,16	1,47	35,00	2,15
		9			12,83	10,07	66,10	12,43	2,27	104,72	2,86	27,48	8,91	1,46	38,60	2,188
8	80	5,5	9,0	3,0	8,63	6,78	52,68	9,03	2,47	83,56	3,11	21,80	7,10	1,59	30,90	2,17
		6			9,38	7,36	56,97	9,80	2,47	90,40	3,11	23,54	7,60	1,58	33,40	2,19
		7			10,85	8,51	65,31	11,32	2,45	103,60	3,09	26,97	8,55	1,58	38,30	2,23
		8			12,30	9,65	73,36	12,80	2,44	116,39	3,08	30,32	9,44	1,57	43,00	2,27

Table Appx. 3.1 (continued)

Number	Dimensions, *mm*				A, cm^2	Mass of 1 m, *kg*	Referential data for axes									I_{yz}, cm^4	z_0, *cm*
							$y-y$			y_0-y_0		z_0-z_0				
	b	t	R	r			I_y, cm^4	W_y, cm^3	i_y, *cm*	$I_{y_0 max}$, cm^4	$i_{y_0 max}$, *cm*	$I_{z_0 min}$, cm^4	W_z, cm^3	$i_{z_0 min}$, *cm*		
9	90	6	10,0	3,3	10,61	8,33	82,10	12,49	2,78	130,00	3,50	33,97	9,88	1,79	48,10	2,43
		7			12,28	9,64	94,30	14,45	2,77	149,67	3,49	38,94	11,15	1,78	55,40	2,47
		8			13,93	10,93	106,11	16,36	2,76	168,42	3,48	43,80	12,34	1,77	62,30	2,51
		9			15,60	12,20	118,00	18,29	2,75	186,00	3,46	48,60	13,48	1,77	68,00	2,55
10	100	6,5	12,0	4,0	12,82	10,06	122,10	16,69	3,09	193,46	3,89	50,73	13,38	1,99	71,40	2,68
		7			13,75	10,79	130,59	17,90	3,08	207,01	3,88	54,16	14,13	1,98	76,40	2,71
		8			15,60	12,25	147,19	20,30	3,07	233,46	3,87	60,92	15,66	1,98	86,30	2,75
		10			19,24	15,10	178,95	24,97	3,05	283,83	3,84	74,08	18,51	1,96	110,00	2,83
		12			22,80	17,90	208,90	29,47	3,03	330,95	3,81	86,84	21,10	1,95	122,00	2,91
		14			26,28	20,63	237,15	33,83	3,00	374,98	3,78	99,32	23,49	1,94	138,00	2,99
		16			29,68	23,30	263,82	38,04	2,98	416,04	3,74	111,61	25,79	1,94	152,00	3,06
11	110	7	12,0	4,0	15,15	11,89	175,61	21,83	3,40	278,54	4,29	72,68	17,36	2,19	106,00	2,96
		8			17,20	13,50	198,17	24,77	3,39	314,51	4,28	81,83	19,29	2,18	116,00	3,00
12,5	125	8	14,0	4,6	19,69	15,46	294,36	32,20	3,87	466,76	4,87	121,98	25,67	2,49	172,00	3,36
		9			22,00	17,30	327,48	36,00	3,86	520,00	4,86	135,88	28,26	2,48	192,00	3,40
		10			24,33	19,10	359,82	39,74	3,85	571,04	4,84	148,59	30,45	2,47	211,00	3,45
		12			28,89	22,68	422,23	47,06	3,82	670,02	4,82	174,43	34,94	2,46	248,00	3,53
		14			33,37	26,20	481,76	54,17	3,80	763,90	4,78	199,62	39,10	2,45	282,00	3,61
		16			37,77	29,65	538,56	61,09	3,78	852,84	4,75	224,29	43,10	2,44	315,00	3,68
14	140	9	14,0	4,6	24,72	19,41	465,72	45,55	4,34	739,42	5,47	192,03	35,92	2,79	274,00	3,78
		10			27,33	21,45	512,29	50,32	4,33	813,62	5,46	210,96	39,05	2,78	301,00	3,82
		12			32,49	25,50	602,49	59,66	4,31	956,98	5,43	248,01	44,97	2,76	354,00	3,90

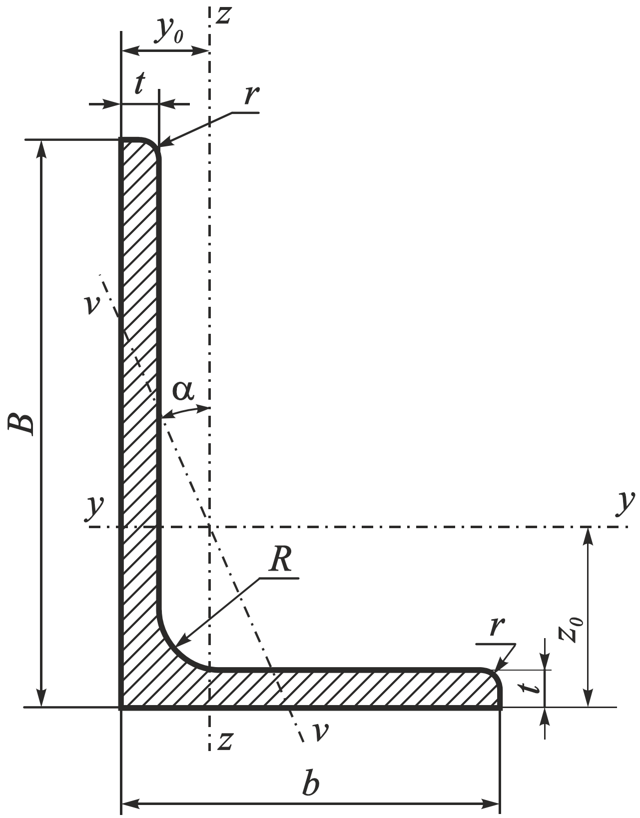
Table Appx. 3.1 (concluded)

Number	Dimensions, *mm*				A, cm^2	Mass of 1 m, *kg*	Referential data for axes								I_{yz}, cm^4	z_0, *cm*
							$y-y$			y_0-y_0		z_0-z_0				
	b	t	R	r			I_y, cm^4	W_y, cm^3	i_y, *cm*	I_{y_0max}, cm^4	i_{y_0max}, *cm*	I_{z_0min}, cm^4	W_z, cm^3	i_{z_0min}, *cm*		
		10			31,43	24,67	774,24	66,19	4,96	1229,10	6,25	319,38	52,52	3,19	455,00	4,30
		11			34,42	27,02	844,21	72,44	4,95	1340,06	6,24	347,77	56,53	3,18	496,00	4,35
		12			37,39	28,35	912,89	78,62	4,94	1450,00	6,23	375,78	60,53	3,17	537,00	4,39
16	160	14	16,0	5,3	43,57	33,97	1046,47	90,77	4,92	1662,13	6,20	430,81	68,15	3,16	615,00	4,47
		16			49,07	38,52	1175,19	102,64	4,89	1865,73	6,17	484,64	75,92	3,14	690,00	4,55
		18			54,79	43,01	1290,24	114,24	4,87	2061,03	6,13	537,46	82,08	3,13	771,00	4,63
		20			60,40	47,44	1418,85	125,60	4,85	2248,26	6,10	589,43	90,02	3,12	830,00	4,70
18	180	11	16,0	5,3	38,80	30,47	1216,44	92,47	5,60	1933,10	7,06	499,78	72,86	3,59	716,00	4,85
		12			42,19	33,12	1316,62	100,41	5,59	2092,78	7,04	540,45	78,15	3,58	776,00	4,89
		12			47,10	36,97	1822,78	124,61	6,22	2896,16	7,84	749,40	98,68	3,99	1073,00	5,37
		13			50,85	39,92	1960,77	134,44	6,21	3116,18	7,83	805,35	105,07	3,98	1156,00	5,42
		14			54,60	42,80	2097,00	144,17	6,20	3333,00	7,81	861,00	111,50	3,97	1236,00	5,46
20	200	16	18,0	6,0	61,98	48,65	2362,57	163,37	6,17	37,55,39	7,78	969,74	123,77	3,96	1393,00	5,54
		20			76,54	60,08	2871,47	200,73	6,12	4560,42	7,72	1181,92	146,62	3,93	1689,00	5,70
		25			94,29	74,02	3466,21	245,59	6,06	5494,04	7,63	1438,38	172,68	3,91	2028,00	5,89
		30			111,54	87,56	4019,60	288,57	6,00	63,51,05	7,55	1698,16	193,06	3,89	2332,00	6,07
22	220	14	21,0	7,0	60,38	47,40	2814,36	175,18	6,83	4470,15	8,60	1158,56	138,62	4,38	1655,00	5,91
		16			68,58	53,83	3175,44	198,71	6,80	5045,37	8,58	1305,52	153,34	4,36	1869,00	53,83
		16			78,40	61,55	4717,10	258,43	7,76	7492,10	9,78	1942,09	203,45	4,98	2775,00	6,75
		18			87,72	68,86	5247,24	288,82	7,73	8336,69	9,75	2157,78	233,39	4,96	3089,00	6,83
		20			96,96	76,11	5764,87	318,76	7,71	9159,73	9,72	2370,01	242,52	4,94	3395,00	6,91
25	250	22	24,0	8,0	106,12	83,31	6270,32	348,26	7,69	9961,60	9,69	2579,04	260,52	4,93	3691,00	7,00
		25			119,71	93,97	7006,39	391,72	7,65	11125,52	9,64	2887,26	287,14	4,91	4119,00	7,11
		28			133,12	104,50	7713,86	434,25	7,61	12243,84	9,59	3189,89	311,98	4,90	4527,00	7,23
		30			141,96	111,44	8176,52	462,11	7,59	12964,66	9,56	3388,98	327,82	4,89	4788,00	7,31

Appendix 4

Hot rolled steel assortment.

Hot rolled non-equilateral L-section steel (non-equilateral L-beams). GOST 8510-86



Notes:

1. The cross-section area and the mass of a 1 *m* of L-beam are calculated using nominal dimensions; the density of steel is assumed equal 7,85 g/cm^3.

2. The values of curvature radii, flange inner face pitches, and flange thicknesses indicated in the figure and in the Table Appx. 4, are given for composing gauges and are not controlled on finished production.

3. The following signs are used in the tables:

$\boldsymbol{B}$, $\boldsymbol{b}$ – the bigger flange and the smaller flange widths;
$\boldsymbol{t}$ – a flange thickness;
$\boldsymbol{R}$ – an inner curvature radius;
$\boldsymbol{r}$ – a flange curvature radius;
$\boldsymbol{A}$ – a cross-section area;
$\boldsymbol{I}$ – an inertia moment;
$\boldsymbol{W}$ – a resistance moment;
$\boldsymbol{i}$ – an inertia radius;
$\boldsymbol{z_0}$, $\boldsymbol{y_0}$ – a distance from the center of gravity to the outer feces of flanges;
tg $\boldsymbol{\alpha}$ **–** an axis inclination angle

Table Appx. 4.1

Number	Dimensions, *mm*					A, cm^2	Mass of 1 m, *kg*	Referential data for axes									y_0, *cm*	z_0, *cm*	I_{yz}, cm^4	tg α
								$y-y$			$z-z$			$v-v$						
	B	b	t	R	r			I_y, cm^4	W_y, cm^3	i_y, *cm*	I_z, cm^4	W_z, cm^3	i_z, *cm*	$I_{v\,min}$, cm^4	W_v, cm^3	$i_{v\,min}$, *cm*				
2,5/1,6	25	16	3	3,5	1,2	1,16	0,91	0,70	0,43	0,78	0,22	0,19	0,44	0,13	0,16	0,34	0,42	0,86	0,22	0,392
3,2/2	32	20	3	3,5	1,2	1,49	1,17	1,52	0,72	1,01	0,46	0,30	0,55	0,28	0,25	0,43	0,49	1,08	0,47	0,382
			4			1,94	1,52	1,93	0,93	1,00	0,57	0,39	0,54	0,35	0,33	0,43	0,53	1,12	0,59	0,374
4/2,5	40	25	3	4,0	1,3	1,89	1,48	3,06	1,14	1,27	0,93	0,49	0,70	0,56	0,41	0,54	0,59	1,32	0,96	0,385
			4			2,47	1,94	3,93	1,49	1,26	1,18	0,63	0,69	0,71	0,52	0,54	0,63	1,37	1,22	0,281
			5			3,03	2,37	4,73	1,82	1,25	1,41	0,77	0,68	0,86	0,64	0,53	0,66	1,41	1,44	0,374
4,5/2,8	45	28	3	5,0	1,7	2,14	1,68	4,41	1,45	1,48	1,32	0,61	0,79	0,79	0,52	0,61	0,64	1,47	1,38	0,382
			4			2,80	2,20	5,68	1,90	1,42	1,69	0,80	0,78	1,02	0,67	0,60	0,68	1,51	1,77	0,379
5/3,2	50	32	3	5,5	1,8	2,42	1,90	6,18	1,82	1,60	1,99	0,81	0,91	1,18	0,68	0,70	0,72	1,60	2,01	0,403
			4			3,17	2,40	7,98	2,38	1,59	2,56	1,05	0,90	1,52	0,88	0,69	0,76	1,65	2,59	0,401
5,6/3,6	56	36	4	6,0	2,0	3,58	2,81	11,37	3,01	1,78	3,70	1,34	1,02	2,19	1,13	0,78	0,84	1,82	3,74	0,406
			5			4,41	3,46	13,82	3,70	1,77	4,48	1,65	1,01	2,65	1,37	0,78	0,88	1,87	4,50	0,404
6,3/4	63	40	4	7,0	2,3	4,04	3,17	16,33	3,83	2,01	5,16	1,67	1,13	3,07	1,41	0,87	0,91	2,03	5,25	0,397
			5			4,98	3,91	19,91	4,72	2,00	6,26	2,05	1,12	3,73	1,72	0,86	0,95	2,08	6,41	0,396
			6			5,90	4,63	23,31	5,58	1,99	7,29	2,42	1,11	4,36	2,02	0,86	0,99	2,12	7,44	0,393
			8			7,68	6,03	29,60	7,22	1,96	9,15	3,12	1,09	5,58	2,60	0,85	1,07	2,20	9,27	0,386
7/4,5	70	45	5	7,5	2,5	5,59	4,39	27,76	5,88	2,23	9,05	2,62	1,27	5,34	2,20	0,98	1,05	2,28	9,12	0,406
7,5/5	75	50	5	8,0	2,7	6,11	4,79	34,81	6,81	2,39	12,47	3,25	1,43	7,24	2,73	1,09	1,17	2,39	12,00	0,436
			6			7,25	5,69	40,92	8,08	2,38	14,60	3,85	1,42	8,48	3,21	1,08	1,21	2,44	14,10	0,435
			8			9,47	7,43	52,38	10,52	2,35	18,52	4,88	1,40	10,87	4,14	1,07	1,29	2,52	17,80	0,430
8/5	80	50	5	8,0	2,7	6,36	4,49	41,64	7,71	2,56	12,68	3,28	1,41	7,57	2,75	1,00	1,13	2,60	13,20	0,387
			6			7,55	5,92	48,98	9,15	2,55	14,85	3,88	1,40	8,88	3,24	1,08	1,17	2,65	15,50	0,386

Table Appx. 4.1 (concluded)

Number	Dimensions, *mm*					A, cm^2	Mass of 1 m, *kg*	Referential data for axes									y_0, *cm*	z_0, *cm*	I_{yz}, cm^4	tg α
								$y-y$			$z-z$			$v-v$						
	B	b	t	R	r			I_y, cm^4	W_y, cm^3	i_y, *cm*	I_z, cm^4	W_z, cm^3	i_z, *cm*	$I_{v\,min}$, cm^4	W_v, cm^3	$i_{v\,min}$, *cm*				
9/5,6	90	56	5,5	9,0	3,0	7,86	6,17	65,28	10,74	2,88	19,67	4,53	1,58	11,77	3,81	1,22	1,26	2,92	20,54	0,384
			6			8,54	6,70	70,58	11,66	2,88	21,22	4,91	1,58	12,70	4,12	1,22	1,28	2,95	22,23	0,384
			8			11,18	8,77	90,87	15,24	2,85	27,08	6,39	1,56	16,29	5,32	1,21	1,36	3,04	28,33	0,380
10/6,3	100	63	6	10,0	3,3	9,58	7,53	98,29	14,52	3,20	30,58	6,27	1,79	18,20	5,27	1,38	1,42	3,23	31,50	0,393
			7			11,09	8,70	112,86	16,78	3,19	34,99	7,23	1,78	20,83	6,06	1,37	1,46	3,28	36,10	0,392
			8			12,57	9,87	126,96	19,01	3,18	39,21	8,17	1,77	23,38	6,82	1,36	1,50	3,32	40,50	0,391
			10			15,47	12,14	153,95	23,32	3,15	47,18	9,99	1,75	28,34	8,31	1,35	1,58	3,40	48,60	0,387
11/7	110	70	6,5	10,0	3,3	11,45	8,98	142,42	19,11	3,53	45,61	8,42	2,00	26,94	7,05	1,53	1,58	3,55	46,80	0,402
			8			13,93	10,93	171,54	23,22	3,51	54,64	10,20	1,98	32,31	8,50	1,52	1,64	3,61	55,90	0,400
12,5/8	125	80	7	11,0	3,7	14,06	11,04	226,53	26,67	4,01	73,73	11,89	2,29	43,40	9,96	1,76	1,80	4,01	74,70	0,407
			8			15,98	12,58	225,98	30,26	4,00	80,95	13,47	2,28	48,82	11,25	1,75	1,84	4,05	84,10	0,406
			10			19,70	15,47	311,61	37,27	3,98	100,47	16,52	2,26	59,33	13,74	1,74	1,92	4,14	102,00	0,404
			12			23,36	18,34	364,79	44,07	3,95	116,84	19,46	2,24	69,47	16,11	1,72	2,00	4,22	118,00	0,400
14/9	140	90	8	12,0	4,0	18,00	14,13	363,68	38,25	4,49	119,79	17,19	2,58	70,27	14,39	1,58	2,03	4,49	121,00	0,411
			10			22,24	17,46	444,45	47,19	4,47	145,54	21,14	2,58	85,51	17,58	1,96	2,12	4,58	147,00	0,409
16/10	160	100	9	13,0	4,3	22,87	17,96	605,97	56,04	5,15	186,03	23,96	2,85	110,40	20,01	2,20	2,24	5,19	194,00	0,391
			10			25,28	19,85	666,59	61,91	5,13	204,09	26,42	2,84	121,16	22,02	2,19	2,28	5,23	213,00	0,390
			12			30,04	23,58	784,22	73,42	5,11	238,75	31,23	2,82	142,14	25,93	2,18	2,36	5,32	249,00	0,388
			14			34,72	27,26	897,19	84,65	5,08	271,60	35,89	2,80	162,49	29,75	2,16	2,43	5,40	282,00	0,385
18/11	180	110	10	14	4,7	28,33	22,20	952,28	78,59	5,80	276,37	32,27	3,12	165,44	26,96	2,42	2,44	5,88	295,00	0,376
			12			33,69	26,40	1122,56	93,33	5,77	324,09	38,20	3,10	194,28	31,83	2,40	2,52	5,97	348,00	0,374
20/12,5	200	125	11	14,0	4,7	34,87	27,37	1449,02	107,31	6,45	446,36	45,98	3,58	263,84	38,27	2,75	2,79	6,50	465,00	0,392
			12			37,89	29,74	1568,19	116,51	6,43	481,93	49,85	3,57	285,04	41,45	2,74	2,83	6,54	503,00	0,392
			14			43,87	34,43	1800,83	134,64	6,41	550,77	57,43	3,54	326,54	47,57	2,73	2,91	6,62	575,00	0,390
			16			49,77	39,07	2026,08	152,41	6,38	616,66	64,83	3,52	366,99	53,56	2,72	2,99	6,71	643,00	0,388

Appendix 5

Geometric characteristics of plane figures

Table Appx. 5.1

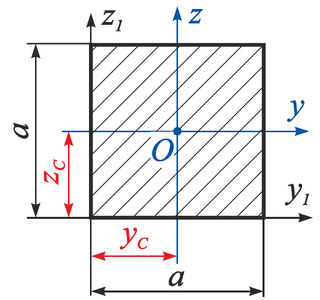
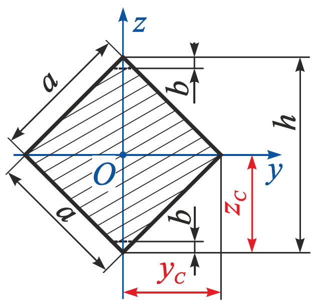
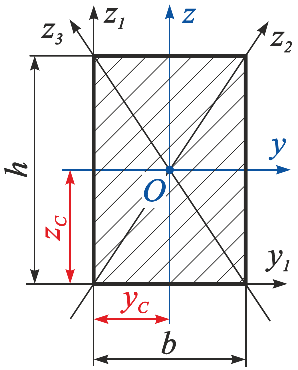
A cross-section shape	A cross-section area	A location of gravity center	Inertia moments	Resistance moments	Inertia radii
Square  y and z are principal central axes	$A = a^2$	$y_c = z_c = \frac{a}{2}$	$I_y = I_z = \frac{a^4}{12}$; $I_{yz} = 0$; $I_{y_1} = I_{z_1} = \frac{a^4}{3}$; $I_{y_1 z_1} = \frac{a^4}{4}$; $I_{\rho o} = \frac{a^4}{6}$; $I_K = 0{,}1406a^4$	$W_y = W_z = \frac{a^3}{6}$; $W_K = 0{,}208a^3$	$i_x = i_y = \frac{a}{\sqrt{12}} =$ $= 0{,}289a$
Edgewise square  y and z are principal central axes	$A = a^2$	$y_c = z_c = \frac{h}{2} =$ $= \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}} =$ $= 0{,}71a$	$I_y = I_z = \frac{a^4}{12} = \frac{h^4}{48}$; $I_{yz} = 0$	$W_y = W_z = \frac{\sqrt{2}}{12} a^3$ When cutting the upper and lower corners by $b = \frac{1}{18} h$ W_y reaches maximum: $W_{y_{cut}} = 0{,}124a^3 =$ $= 0{,}044h^3$	$i_x = i_y = \frac{a}{\sqrt{12}} =$ $= 0{,}289a$

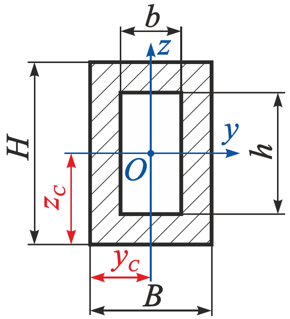
Table Appx. 5.1 (continued)

<table>
<tr><th>A cross-section shape</th><th>A cross-section area</th><th>A location of gravity center</th><th>Inertia moments</th><th>Resistance moments</th><th>Inertia radii</th></tr>
<tr><td>Rectangle


y and z are principal central axes</td><td>$F = bh$</td><td>$y_c = \frac{b}{2}$;

$z_c = \frac{h}{2}$</td><td>$I_y = \frac{bh^3}{12}$; $I_z = \frac{hb^3}{12}$; $I_{yz} = 0$;
$I_{y_1} = \frac{bh^3}{3}$; $I_{z_1} = \frac{hb^3}{3}$;
$I_{y_1z_1} = \frac{b^2h^2}{4}$;
$I_{\rho O} = \frac{bh}{12}(b^2 + h^2)$;
$I_{z_2} = I_{z_3} = \frac{b^3h^3}{6(b^2 + h^2)}$</td><td>$W_y = \frac{bh^3}{6}$;

$W_z = \frac{hb^3}{6}$</td><td>$i_y = 0{,}289h$;

$i_z = 0{,}289b$</td></tr>
<tr><td>Hollow rectangle


y and z are principal central axes</td><td>$F = BH - bh$</td><td>$y_c = \frac{B}{2}$;

$z_c = \frac{H}{2}$</td><td>$I_y = \frac{BH^3}{12} - \frac{bh^3}{12}$;
$I_z = \frac{HB^3}{12} - \frac{hb^3}{12}$;
$I_{yz} = 0$;
$I_{\rho O} = I_y + I_z$</td><td>$W_y = \frac{BH^3 - bh^3}{6H}$;

$W_z = \frac{HB^3 - hb^3}{6B}$</td><td>$i_y = \sqrt{\frac{BH^3 - bh^3}{12(BH - bh)}}$;

$i_z = \sqrt{\frac{HB^3 - hb^3}{12(BH - bh)}}$</td></tr>
</table>

Table Appx. 5.1 (continued)

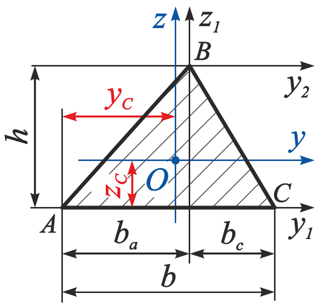
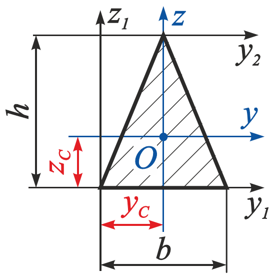
A cross-section shape	A cross-section area	A location of gravity center	Inertia moments	Resistance moments	Inertia radii
Triangle  y and z are central axes	$F = \frac{bh}{2}$	$y_c = \frac{b + b_a}{3}$; $z_c = \frac{h}{3}$	$I_y = \frac{bh^3}{36}$; $I_z = \frac{bh}{36}(b^2 - b_a b_c)$; $I_{yz} = \frac{bh^2}{72}(b - 2b_c)$; $I_{y_1} = \frac{bh^3}{12}$; $I_{y_2} = \frac{bh^3}{4}$; $I_{z_1} = \frac{h}{12}(b_a^3 + b_c^3)$; $I_{y_1 z_1} = \frac{bh^2}{24}(3b - 2b_c)$; $I_{\rho O} = \frac{bh}{36}(h^2 + b_a^2 + b_a b_c + b_c^2)$; $I_{\rho B} = \frac{h}{12}(3bh^2 + b_a^3 + b_c^3)$	For upper fibers $W_y = \frac{bh^2}{24}$; For lower fibers $W_y = \frac{hb^2}{12}$	$i_y = \frac{h}{3\sqrt{2}}$; $i_z = \frac{1}{3\sqrt{2}}\sqrt{b^2 - b_a b_c}$
Isosceles triangle  y and z are principal central axes	$F = \frac{bh}{2}$	$y_c = \frac{b}{2}$; $z_c = \frac{h}{3}$	$I_y = \frac{bh^3}{36}$; $I_z = \frac{hb^3}{48}$; $I_{yz} = 0$; $I_{\rho O} = \frac{bh}{12}\left(\frac{h^2}{3} + \frac{b^2}{4}\right)$; $I_{y_1} = \frac{bh^3}{12}$; $I_{z_1} = \frac{hb^3}{12}$; $I_{y_1 z_1} = \frac{b^2 h^2}{12}$; $I_{y_2} = \frac{bh^3}{4}$ (in equilateral triangle $h = b\sqrt{3}/2$)	For upper fibers $W_y = \frac{bh^2}{24}$; For lower fibers $W_y = \frac{hb^2}{12}$	$i_y = \frac{h}{3\sqrt{2}}$; $i_z = \frac{b}{2\sqrt{6}}$

Table Appx. 5.1 (continued)

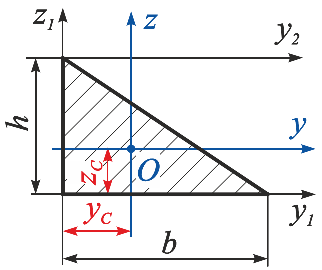
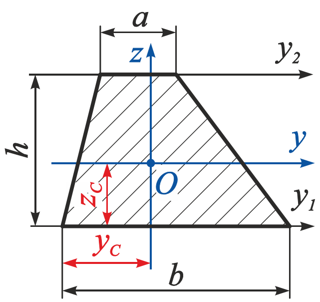
A cross-section shape	A cross-section area	A location of gravity center	Inertia moments	Resistance moments	Inertia radii
Right-angled triangle  y and z are central axes	$F = \frac{bh}{2}$	$y_c = \frac{b}{3}$; $z_c = \frac{h}{3}$	$I_y = \frac{bh^3}{36}$; $I_z = \frac{hb^3}{36}$; $I_{yz} = -\frac{b^2h^2}{72}$; $I_{\rho O} = \frac{bh}{36}(h^2 + b^2)$; $I_{y_1} = \frac{bh^3}{12}$; $I_{z_1} = \frac{hb^3}{12}$; $I_{yz} = \frac{b^2h^2}{24}$; $I_{\rho O} = \frac{bh}{36}(h^2 + b^2)$; $I_{y_2} = \frac{bh^3}{4}$	For upper fibers $W_y = \frac{bh^2}{24}$; For lower fibers $W_y = \frac{hb^2}{12}$	$i_y = \frac{h}{3\sqrt{2}}$; $i_z = \frac{b}{3\sqrt{2}}$
Trapezium  y and z are central axes	$F = \frac{(a+b)}{2}h$	$y_c = \frac{h(2b+a)}{3(a+b)}$; $z_c = \frac{h(b+2a)}{3(a+b)}$	$I_y = \frac{h^3(b^2 + 4ab + a^2)}{36(a+b)}$; $I_{y_1} = \frac{h^3(b+3a)}{12}$; $I_{y_2} = \frac{h^3(3b+a)}{12}$	For upper fibers $W_y = \frac{h^2}{12} \times$ $\times \frac{(b^2 + 4ab + a^2)}{(2b+a)}$; For lower fibers $W_y = \frac{h^2}{12} \times$ $\times \frac{(b^2 + 4ab + a^2)}{(b+2a)}$	$i_y = \frac{h}{6(a+b)} \times$ $\times \sqrt{2(b^2 + 4ab + a^2)}$

Table Appx. 5.1 (continued)

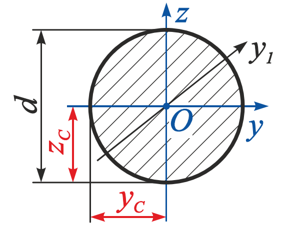
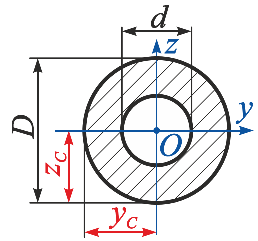
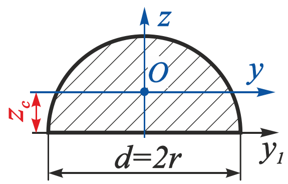
A cross-section shape	A cross-section area	A location of gravity center	Inertia moments	Resistance moments	Inertia radii
Circle  y and z are principal central axes	$F=\dfrac{\pi d^2}{4}$	$y_c=z_c=\dfrac{d}{2}$	$I_y=I_z=I_{y_1}=\dfrac{\pi d^4}{64}=\dfrac{\pi r^4}{4}$; $I_{yz}=0$; $I_{\rho_O}=I_y+I_z=\dfrac{\pi d^4}{32}=\dfrac{\pi r^4}{2}$	$W_y=W_z=W_{y_1}=$ $=\dfrac{\pi d^3}{32}=\dfrac{\pi r^3}{4}$; $W_{\rho_O}=\dfrac{\pi d^3}{16}=\dfrac{\pi r^3}{2}$	$i_y=i_z=i_{y_1}=$ $=\dfrac{d}{4}=\dfrac{r}{2}$
Ring  y and z are principal central axes	$F=\dfrac{\pi D^2}{4}\times$ $\times(1-\alpha^2)$ $\alpha=\dfrac{d}{D}$	$y_c=z_c=\dfrac{D}{2}$	$I_y=I_z=\dfrac{\pi D^4}{64}-\dfrac{\pi d^4}{64}=$ $=\dfrac{\pi D^4}{64}(1-\alpha^4)$; $I_{yz}=0$; $I_{\rho_O}=I_y+I_z=\dfrac{\pi D^4}{32}(1-\alpha^4)$	$W_y=W_z=$ $=\dfrac{\pi D^3}{32}(1-\alpha^4)$; $W_{\rho_O}=\dfrac{\pi d^3}{16}=$ $=\dfrac{\pi D^3}{16}(1-\alpha^4)$	$i_y=i_z=$ $=\dfrac{1}{4}\sqrt{D^2+d^2}=$ $=\dfrac{D}{4}\sqrt{1+\alpha^2}$
Semicircle  y and z are principal central axes	$F=\dfrac{\pi d^2}{8}=$ $=\dfrac{\pi r^2}{2}$	$y_c=\dfrac{d}{2}=r$; $z_c=\dfrac{2}{3}\cdot\dfrac{d}{\pi}=\dfrac{4}{3}\cdot\dfrac{r}{\pi}$	$I_y=\dfrac{d^4}{16}\left(\dfrac{\pi}{8}-\dfrac{8}{9\pi}\right)$; $I_{y_1}=I_z=\dfrac{\pi d^4}{128}=\dfrac{\pi r^4}{8}$; $I_{yz}=I_{y_1z}=0$	For upper fibers $W_y\approx 0{,}0239d^3$; For lower fibers $W_y\approx 0{,}0324d^3$; $W_z=\dfrac{\pi d^3}{64}=\dfrac{\pi r^3}{8}$	$i_y\approx 0{,}132d$; $i_z=\dfrac{d}{4}$

Table Appx. 5.1 (continued)

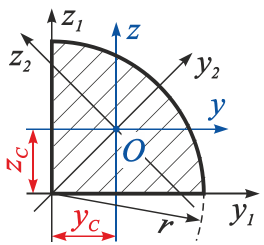
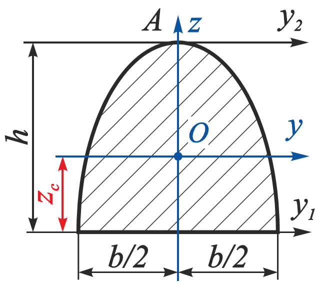
A cross-section shape	A cross-section area	A location of gravity center	Inertia moments	Resistance moments	Inertia radii
Quartercircle  y_2 and z_2 are principal central axes;	$F = \dfrac{\pi r^2}{4}$	$y_c = z_c = \dfrac{4r}{3\pi}$	$I_{y_2\,max} \approx 0{,}0714r^4$; $I_{z_2\,min} \approx 0{,}0384r^4$; $I_y = I_z \approx 0{,}0549r^4$; $I_{yz} = -0{,}0165r^4$; $I_{y_1} = I_{z_1} = \dfrac{\pi r^4}{16} \approx 0{,}196r^4$; $I_{y_1 z_1} = \dfrac{r^4}{8}$	For upper and right-handed fibers $W_y = W_z \approx 0{,}923r^3$; For lower and left-handed fibers $W_y = W_z \approx 1{,}245r^3$	$i_{y\,max} \approx 0{,}302r$; $i_{z\,max} \approx 0{,}221r$
Parabolic segment  y and z are principal central axes; **A** – parabola vertex	$F = \dfrac{2}{3}bh$	$z_c = \dfrac{2}{5}h$	$I_y = \dfrac{8}{175}bh^3$; $I_{y_1} = \dfrac{16}{105}bh^3$; $I_{y_2} = \dfrac{2}{7}bh^3$; $I_z = \dfrac{1}{30}bh^3$	For upper fibers $W_y = \dfrac{8}{105}bh^2$; For lower fibers $W_y = \dfrac{4}{35}bh^2$; $W_z = \dfrac{1}{15}bh^2$	$i_y = \dfrac{2}{5}h\sqrt{\dfrac{3}{7}}$; $i_z = \dfrac{b}{2\sqrt{5}}$

Table Appx. 5.1 (continued)

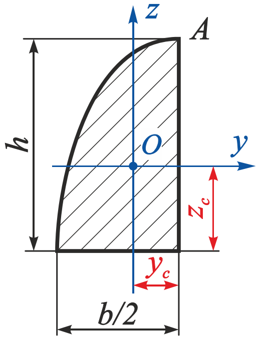
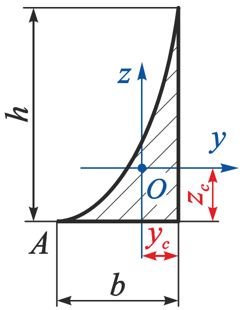
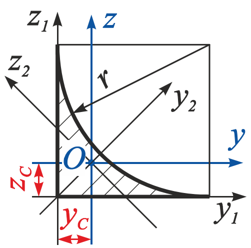
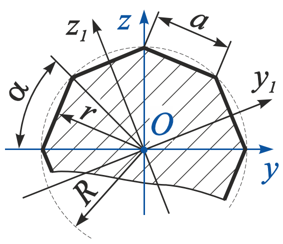
A cross-section shape	**A cross-section area**	**A location of gravity center**	**Inertia moments**	**Resistance moments**	**Inertia radii**
Parabolic semisegment  y and z are central axes; ***A*** – parabola vertex	$F = \frac{bh}{3}$	$y_c = \frac{3}{16}b;$ $z_c = \frac{2}{5}h$	$I_y = \frac{4}{175}bh^3;$ $I_z = \frac{19}{3840}hb^3$	For lower and right-handed fibers $W_{y_{min}} = \frac{2}{35}bh^2;$ $W_{z_{min}} = \frac{19}{48}hb^2$	$i_y = \frac{2}{5}h\sqrt{\frac{3}{7}}$
Parabolic triangle  y_2 and z_2 are principal central axes; ***A*** – parabola vertex	$F = \frac{bh}{3}$	$y_c = \frac{1}{4}b;$ $z_c = \frac{3}{10}h$	$I_y = \frac{1}{21}bh^3;$ $I_z = \frac{1}{5}hb^3$	For lower and right-handed fibers $W_{y_{min}} = \frac{10}{63}bh^2;$ $W_{z_{min}} = \frac{4}{5}hb^2$	$i_y = h\sqrt{\frac{1}{7}}$

Table Appx.5.1 (concluded)

A cross-section shape	A cross-section area	A location of gravity center	Inertia moments	Resistance moments	Inertia radii
Circular triangle  y_2 and z_2 are principal central axes	$F = 0{,}215r^2$	$y_c = z_c = 0{,}223r$	$I_y = I_z = 0{,}00755r^4$; $I_{y_1} = I_{z_1} = 0{,}0181r^4$; $I_{y_2} = 0{,}003r^4$; $I_{z_2} = 0{,}0121r^4$	$W_{z_2\,min} = 0{,}0097r^3$	$i_{z_2\,min} = 0{,}187r$
Regular polygon with n sides  y, y_1, z, and z_1 are principal central axes	$F = \frac{1}{4}na^2 \times$ $\times\, ctg\,\alpha =$ $= nr^2\,tg\,\alpha =$ $= \frac{nar}{2}$	$R = \frac{2}{2\,sin\,\alpha}$; $r = \frac{2}{2\,tg\,\alpha}$	$I_y = I_z = I_{y_1} = I_{z_1} =$ $= \frac{nar}{48}(6R^2 - a^2) =$ $= \frac{nar}{96}(12r^2 + a^2)$	—	$i_y = i_z =$ $= \sqrt{\frac{12r^2 + a^2}{48}}$; $i_{y_1} = i_{z_1} =$ $= \sqrt{\frac{6R^2 - a^2}{24}}$

Note. Inertia radius $i = \sqrt{I/F}$.

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ГЕОМЕТРИЧНІ ХАРАКТЕРИСТИКИ ПЛОСКИХ ПЕРЕРІЗІВ

(Англійською мовою)

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