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PHYSICS
FOR PREPARATORY DEPARTMENT

Part II

Guidance Manual

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Подано другу частину стислого курсу фізики на базі старших класів для студентів підготовчого відділення з таких тем: «Молекулярна фізика», «Термодинаміка», «Електростатика» та «Постійний електричний струм». Навчальний посібник складається з 14 уроків, кожен з яких поєднує теоретичні викладки й пояснення, контрольні запитання й задачі.

Для англомовних слухачів підготовчого відділення університету.

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P78 Physics for Preparatory Department [Electronic resource]: guidance manual in 2 parts. Part II / O. V. Podshyvalova, A. M. Okhrimovskyy, O. I. Petrova. – Kharkiv : National Aerospace University «Kharkiv Aviation Institute», 2021. – 92 p.

Manual provides the second part of a concise course in physics based on a high-school level for the students of Preparatory department. This manual contains the topics: molecular physics, thermodynamics, electrostatics, and direct electric current. Manual consist of 14 lessons. Every lesson comprises theoretical statements and explanation, control questions and problems.

For English speaking students of university preparatory department.

Figs 16. Bibliogr.: 9 names

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CONTENTS

PREFACE

The purpose of classes at Preparatory department is to give foreign students the basic knowledge in the language of their further education that will assist their learning at university.

This manual is a logical continuation of the guidance “Physics for Preparatory Department: Part I” [1]. It offers the second part of a concise course in physics, based on the high school level, for ESL (English as a second language) students. This manual contains the topics: molecular physics, thermodynamics, electrostatics, and direct electric current. Every lesson comprises theoretical statements and explanation, so even the students who did not have high school physics will be able to understand the language of physics and learn the material. There are also control questions and problems in every lesson that will allow students to check their understanding and ability in applying theory to problems’ solution. At the end of the guidance, we offer self-training problem sets to recall Part I.

For better understanding and advance self-training, students are recommended to read textbooks [2–4]. The structure of the manual is similar to the structure of analogous publication in Ukrainian [5, 6] or Russian [7, 8].

We would like to thank our reviewers and colleagues at Physics department, especially Anatoliy Taran and Daniel Voronovich, for stimulating discussions about physics pedagogy and friendly criticism of our work.

We welcome suggestions and comments from our readers and wish our students great success in studying physics.

Lesson 17

KINETIC THEORY OF MATTER. IDEAL GAS

Kinetic theory of matter explains the macroscopic properties of substance by considering its molecular structure. Within this course a molecule can be defined as the smallest entity (unit) of a pure chemical substance that still keeps its chemical properties. The kinetic theory is based on following assumptions:

1. Any substance consists of a huge number of molecules.
2. The molecules are involved in a chaotic random motion.
3. The molecules interact with each other.

A system that satisfies these assumptions is often called a *thermodynamic system*. The simplest thermodynamic system to be analyzed with the kinetic theory is an ideal gas. The *ideal gas* model assumes that the molecules of the gas have a negligible size relatively to the distance between them and interact with each other during the elastic collisions only. So, the molecules of the ideal gas are considered as particles. It should be mentioned that air and other gases under the conditions closed to the normal pressure and room temperature can be treated as an ideal gas. Thus, it is reasonable to analyze the ideal gas model.

The state of an ideal gas of mass $\boldsymbol{m}$ is described by its pressure $\boldsymbol{P}$, volume $\boldsymbol{V}$, and temperature $\boldsymbol{T}$. These physical quantities therefore are called *state variables*.

We can represent the mass of the system $\boldsymbol{m}$ through the mass of one molecule $\boldsymbol{m_1}$ and the total number of identical molecules $\boldsymbol{N}$:

$$\boldsymbol{m = Nm_1}. \tag{17.1}$$

Experiments with various gases show that the product of gas pressure and its volume is proportional to the number of molecules and *absolute* (or *thermodynamic*) temperature:

$$\boldsymbol{PV = NkT}, \tag{17.2}$$

where $\boldsymbol{k = 1.380\ 649 \times 10^{-23}}$ **J/K** is the defining fundamental physical constant called *Boltzmann constant*. Check the 9th edition of the SI brochure [9] for more details. Equation (17.2) is known as *the ideal gas equation*.

On the other hand, one can derive the equation based on the kinetic theory assumptions:

$$PV = \frac{2}{3}N\langle E_K \rangle, \quad (17.3)$$

where $\langle E_K \rangle$ is the average kinetic energy of translational motion of gas molecules. This equation relates macroscopic properties, such as pressure and volume, to a microscopic one – the kinetic energy of random motion of all gas molecules.

Equating the right sides of equations (17.2) and (17.3), we can obtain an expression that clarifies the physical meaning of absolute temperature:

$$\langle E_K \rangle = \frac{3}{2}kT. \quad (17.4)$$

Thus, the last expression tells us that *absolute temperature is a measure of average kinetic energy of moleculs*. The SI units of absolute or thermodynamic temperature is the *kelvin*, symbol **K**.

In everyday life, most of people use the Celsius temperature scale. At the temperature $T_C = -273.15$ **ºC** molecules stop their motion and consequently their kinetic energy is equal to zero. So, at the absolute temperature scale this state corresponds to zero kelvin. The size of a unit on the absolute scale is chosen to be the same as that on the Celsius scale. Thus, to convert the temperature from the Celsius scale to the absolute one we can use the following equation:

$$T = T_C + 273.15.$$

In thermodynamic system, a macroscopic equivalent of the number of molecules is expressed through a physical quantity – *amount of substance*, symbol n. The SI unit of amount of substance is the *mole*, symbol **mol**. One mole contains exactly $6.022\ 140\ 76 \times 10^{23}$ elementary entities. This number is a fixed numerical value of another defining fundamental physical constant – the *Avogadro constant*: $N_A = 6.022\ 140\ 76 \times 10^{23}$ **mol**$^{-1}$. Thus, amount of substance can be written as

$$n = \frac{N}{N_A}. \quad (17.5)$$

The mass of a mole of a substance is called the *molar mass* and denoted M. The SI unit of molar mass is the kilogram per mole (**kg/mol**). We can represent the molar mass of a substance M from equation (17.1) through the mass of one molecule m_1 and the Avogadro constant N_A:

$$M = N_A m_1. \quad (17.6)$$

Taking into account equations (17.1), (17.5) and (17.6), the amount of substance can be represented as the following:

$$n = \frac{m}{M} . \qquad (17.7)$$

When a molecule consists of a single atom, the term *atomic mass* is often used instead of molar mass. The periodic table of elements is used to find out atomic mass. For a multi-atomic molecule, its molar mass is computed as the sum of the atomic masses of structural components.

Control questions

1. Write down the assumptions of the kinetic theory of matter.

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2. What is assumed in the ideal gas model?

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3. Give a list of state variables for a thermodynamic system.

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4. Give the definition of absolute temperature.

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5. Write down the ideal gas equation.

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6. Calculate the product of the Boltzmann and Avogadro constants.

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Problems

1. A nitrogen molecule N_2 moves with a speed $\boldsymbol{v}$ **= 430 m/s**. Find the momentum $\boldsymbol{p}$ of this molecule. The molar mass of molecular nitrogen is **$28 \cdot 10^{-3}$ kg/mol**.

(Answer: $\boldsymbol{p} = 2\cdot10^{-23}$ kg m/s)

2. Find the average kinetic energy of a monatomic gas molecule $\langle \boldsymbol{E_K} \rangle$ if the temperature of gas is **17 ºC**.
   (Answer: $\langle \boldsymbol{E_K} \rangle = 6\cdot10^{-21}$ J)

3. A room of volume $\boldsymbol{V}$ **= 80 m³** with air at pressure $\boldsymbol{P}$ **= 100 kPa** contains $\boldsymbol{N} = \mathbf{2\cdot10^{27}}$ molecules. What is the air temperature $\boldsymbol{T_C}$?
   (Answer: $\boldsymbol{T_C}$ = 17 ºC)

4. What is the total number $\boldsymbol{N}$ of molecules in **1 L** of water (H_2O) at room temperature and normal atmospheric pressure? The atomic mass of hydrogen is **1 g/mol** and the atomic mass of oxygen is **16 g/mol**.
   (Answer: $\boldsymbol{N} = 3.3\cdot10^{25}$)

5. The air pressure of inside a corked bottle is P_1 = **100 kPa** and its temperature is T_{C1} = **7 ºC**. It is known that the bottle uncorks itself when the gas pressure inside it is P_2 = **130 kPa**. To what temperature T_{C2} should the gas be heated to uncork the bottle ?
(Answer: T_{C2} = 91 ºC)

6. A mixture of helium He (M_1 = **$4 \cdot 10^{-3}$ kg/mol**) and molecular hydrogen H_2 (M_2 = **$2 \cdot 10^{-3}$ kg/mol**) is inside a balloon of volume **10 L**. The state variables of the mixture are the following: m = **1 g**, P_2 = **100 kPa**, T_C = **27 ºC**. Find out the number of molecules of each gas.
(Answer: $N_1 = 2 \cdot 10^{23}$, $N_2 = 0.5 \cdot 10^{23}$)

Lesson 18

THE IDEAL GAS EQUATION OF STATE. ISOPROCESSES IN GAS

Functional dependence between the state variables P, V, T *and* m *(or* n*) is called the equation of state.*

Taking into account equations (17.1), (17.2), (17.5) and (17.6), the equation of state for an ideal gas can be written as the following:

$$PV = \frac{m}{M}RT \text{ or } PV = nRT,$$

where $R = kN_A = 8.31$ J·K^{-1}·mol^{-1} is called the *ideal-gas constant*.

For a constant number of moles (or constant mass) of an ideal gas the product nR is constant, so the quantity $\frac{PV}{T}$ is also constant. Therefore

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = constant, \qquad (18.1)$$

where the subscripts **1** and **2** describe different states of the same mass of gas.

A thermodynamic process in which one of the state variables (P*,* V*, or* T*) remains constant is called an isoprocess.* Therefore, we will discuss three types of such processes.

1. *An isothermal process is a constant-temperature process* ($T = const$). For this process, equation (18.1) is reduced to:

$$P_1V_1 = P_2V_2.$$

For a constant mass of gas, the product of gas pressure and its volume remains constant if there is no change in gas temperature. The curve corresponding to this process in a PV-diagram (Fig. 18.1) is called an isotherm.

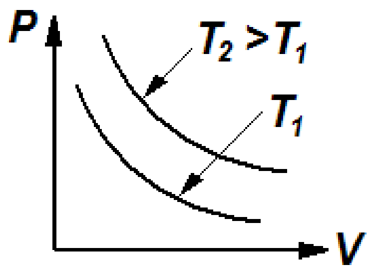

Fig. 18.1

2. *An isobaric process is a constant-pressure process* (**P = const**). For this process, equation (18.1) is reduced to:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}.$$

The last equation can be rewritten as

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}.$$

For a constant mass of gas, the gas volume is proportional to its temperature if the gas pressure remains constant. The plot of gas volume as a function of its temperature is a straight line (Fig. 18.2), called an isobar.

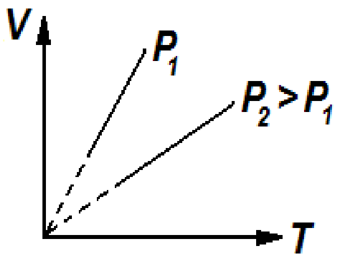

Fig.18.2

3. *An isochoric process is a constant-volume process* (**V = const**). For this process, equation (18.1) is reduced to:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}.$$

The last equation can be rewritten as

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}.$$

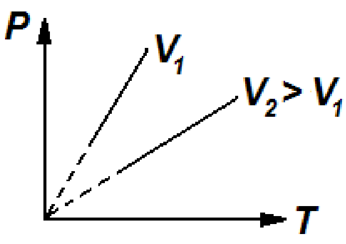

Fig. 18.3

For a constant mass of gas, gas pressure is proportional to its temperature if the gas volume remains constant. The plot of gas pressure as a function of its temperature is a straight line (Fig. 18.3), called an isochor.

Control questions

1. Write down the ideal gas equation of state.

2. What is called an isoprocess?

3. What process is called isothermal? Write down the corresponding law.

4. What process is called isobaric? Write down the corresponding law.

5. What process is called isochoric? Write down the corresponding law.

6. What is the constant in equation (18.1)?

Problems

1. A tank contains air, which can be treated as an ideal gas, at temperature $\boldsymbol{T_1} = \mathbf{27}$ **ºC** and pressure $\boldsymbol{P_1} = \mathbf{75}$ **kPa**. What is the pressure $\boldsymbol{P_2}$ of the air if it is heated to the temperature $\boldsymbol{T_2} = \mathbf{127}$ **ºC**? Assume that the volume of the tank is constant.
   (Answer: $\boldsymbol{P_2} = 100$ kPa)

2. A cylindrical tank with a tight-fitting movable piston contains air of volume $\boldsymbol{V_1}$ **= 5 L** at pressure $\boldsymbol{P_1}$ **= 1200 hPa**. The piston is slowly pulled out until the pressure of the air is decreases to $\boldsymbol{P_2}$ **= 100 kPa**. Find the new volume $\boldsymbol{V_2}$ occupied by the gas if the temperature remains constant.
(Answer: $\boldsymbol{V_2} = 6$ L)

3. In a cylindrical tank under a movable piston, the gas expands during the isobaric process from the volume $\boldsymbol{V_1}$ **= 3 L** to volume the $\boldsymbol{V_2}$ **= 4.5 L**. Find the final temperature of the gas $\boldsymbol{T_2}$ if its initial temperature is $\boldsymbol{T_1}$ **= 21 ^{o}C**. Express your answer in degrees Celsius.
(Answer: $\boldsymbol{T_2} = 168$ ^{o}C)

4. The gas inside a balloon of initial volume $\boldsymbol{V_1}$ **= 37.3 L** is always at the same pressure close to atmospheric one. What is the final volume of the balloon if it is cooled from the boiling point of water to the melting point of ice?
(Answer: $\boldsymbol{V_2} = 27.3$ L)

5. A **2-L** tank contains a quantity of hydrogen at **1.5 atm** and **17 °C**. The tank is closed and cooled until the pressure reduces to **0.75 atm**. What is the temperature T_2? Assume that the volume of the tank is constant.
(Answer: $T_2 = 145$ K)

6. The total lung volume for a typical teenager is **5 L**. A boy fills his lungs with air at an absolute pressure of **1 atm**. Then, holding his breath, he compresses his chest cavity, decreasing his lung volume to **4.75 L**. What is the pressure P_2 of the air in his lungs then? Assume that the gas temperature remains constant.
(Answer: $P_2 = 1.05$ atm)

Lesson 19

WORK IN THERMODYNAMICS. INTERNAL ENERGY. QUANTITY OF HEAT

Work done by expanding gas

Thermodynamic systems, which are able to expand, can perform work on their surroundings. It is easy to calculate work done by a gas expanding at constant pressure. We will derive it on the example of a gas inside the cylinder with a movable piston of area $\boldsymbol{A}$ (Fig. 19.1).

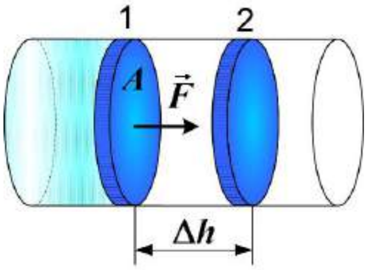

Fig. 19.1

The gas at pressure $\boldsymbol{P}$ exerts a force $\boldsymbol{F}$ on the piston. The magnitude of the force can be found from the definition of pressure: $\boldsymbol{F = PA}$. In the case of constant pressure, this force remains constant as well.

Let us assume that under the force $\boldsymbol{F}$ the piston has moved out through the displacement $\boldsymbol{\Delta h}$. Thus, the direction of the force and displacement are the same. By the definition of mechanical work, the work done by the gas over the piston can be calculated as:

$$\boldsymbol{W = F\,\Delta h}.$$

Then, the work done by the gas can be expressed in terms of the gas state variables:

$$\boldsymbol{W = PA\,\Delta h = P\,\Delta V = P(V_2 - V_1)}, \tag{19.1}$$

where $\boldsymbol{\Delta V = A\,\Delta h}$ is a change in the gas volume; $\boldsymbol{V_1}$ and $\boldsymbol{V_2}$ are the initial and final gas volumes, respectively.

Therefore, the work done by the expanding gas is positive. If external forces compress the gas, the change of its volume is negative. So the work done by the compressing gas is negative.

Internal energy

The molecules that a thermodynamic system consists of are involved in a chaotic random motion and interact with each other. Therefore, they possess the kinetic and potential energy. *The sum of the kinetic energies of all molecules of a system, and the sum of all the potential energies of interaction among the molecules is the internal energy of the system.* Usually, the internal energy of the system is denoted as $\boldsymbol{U}$.

Since the molecules of an ideal gas interact with each other only during the elastic collisions, the contribution of potential energy into the internal energy is equal to zero. Hence, the internal energy of an ideal gas $\boldsymbol{U}$ is the product of the number of molecules $\boldsymbol{N}$ and their average kinetic energy $\langle \boldsymbol{E_K} \rangle$. As the molecules of a monatomic ideal gas are only involved into translational motion, equation (17.4) can be used for calculating the *internal energy of a monatomic ideal gas*:

$$\boldsymbol{U} = \boldsymbol{N}\langle \boldsymbol{E_K} \rangle = \boldsymbol{N}\frac{\mathbf{3}}{\mathbf{2}}\boldsymbol{kT} = \frac{\boldsymbol{N}}{\boldsymbol{N_A}}\frac{\mathbf{3}}{\mathbf{2}}\boldsymbol{RT} = \boldsymbol{n}\frac{\mathbf{3}}{\mathbf{2}}\boldsymbol{RT} = \frac{\boldsymbol{m}}{\boldsymbol{M}}\frac{\mathbf{3}}{\mathbf{2}}\boldsymbol{RT}, \qquad (19.2)$$

where $\boldsymbol{k}$ is the Boltzmann constant, $\boldsymbol{T}$ is the gas temperature, $\boldsymbol{N_A}$ is the Avogadro constant, $\boldsymbol{R}$ is the ideal-gas constant, $\boldsymbol{n}$ is the amount of substance (number of moles), $\boldsymbol{m}$ is the gas mass, and $\boldsymbol{M}$ is the gas molar mass.

Expressions for the internal energy of polyatomic ideal and nonideal gases are beyond the scope of this course. However, it should be mentioned that the internal energy of any system rises with temperature.

Quantity of heat. Specific heat

From your everyday experience, you may know that two bodies, initially at different temperatures, reach the same temperature with time if they are in contact. That is due to the energy transfer between the bodies. In the result, the internal energy of a warmer body decreases by a certain amount and the internal energy of a colder one increases by the same amount of energy. Energy transfer from one body to another without performing work is called *heat transfer.* The corresponding amount of transferred energy, equal to the change of internal energy, is called a *quantity of heat* and denoted as $\boldsymbol{Q}$.

The SI unit of quantity of heat is the same as for energy and work – *joule* (**J**).

To change the body's temperature, it requires the amount of energy that depends on the temperature difference $\boldsymbol{\Delta T}$, the mass of the body $\boldsymbol{m}$, and the nature of the body's substance. *The specific heat of a substance* $\boldsymbol{c}$ *is a quantity of heat* $\boldsymbol{Q}$ *needed to increase the temperature of 1 kilogram of the substance by 1 kelvin*:

$$\boldsymbol{c = \frac{Q}{m\Delta T}}.$$

The SI unit of specific heat can be derived from the last equation and it is the *joule per kilogram-kelvin* (**J kg^{-1}·K^{-1}**).

From the specific heat definition, a quantity of heat required to change the temperature of a body can be calculated as the following:

$$\boldsymbol{Q = cm\Delta T = cm\,(T_2 - T_1)},$$

where $\boldsymbol{T_1}$ is the initial temperature of a body and $\boldsymbol{T_2}$ is its final temperature. The quantity of heat is positive if the body's temperature rises and it is negative if the body's temperature decreases.

Control questions

1. Write down the expression for work done by the gas in an isobaric process.

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2. In what case the work done by the gas is equal to zero? Explain.

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3. Write down the definition of internal energy of a thermodynamic system.

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4. Write down the expression for the internal energy of $\boldsymbol{n}$ moles of a monatomic ideal gas.

5. What is called a quantity of heat? What is its SI unit?

6. What is the specific heat of a substance?

Problems

1. The gas inside a balloon of initial volume $\boldsymbol{V_1} = \mathbf{27.3 \cdot 10^{-3}}$ **m³** remains at the atmospheric pressure $\boldsymbol{P} = \mathbf{1.0 \cdot 10^{5}}$ **Pa**. What is the work $\boldsymbol{W}$ done by the gas if the balloon is heated from the melting point of ice to the boiling point of water?
   (Answer: $\boldsymbol{W} = 100$ J)

2. The internal energy of **40 g** of helium gas ($\boldsymbol{M_{\mathrm{He}}} = \mathbf{4 \cdot 10^{-3}}$ **kg/mol**) increased by $\Delta \boldsymbol{U} = \mathbf{1662}$ **J**. What the gas final temperature $\boldsymbol{T_f}$ if it was initially at room temperature ($\boldsymbol{T_i} = \mathbf{21}$ **ºC**)?
   (Answer: $\boldsymbol{T_f} = 314$ K)

3. Calculate the equilibrium temperature $\boldsymbol{T}$ (in degrees Celsius) of the system if **0.40 L** of water at **20 ºC** is mixed with **0.60 L** of water at **70 ºC** in a thermally insulted container.
(Answer: $\boldsymbol{T}$ = 50 ºC)

4. In a cylindrical tank under a movable piston, gas is compressed during the isobaric process from the volume $\boldsymbol{V_1} = \mathbf{10}$ **L** to the volume $\boldsymbol{V_2} = \mathbf{6}$ **L**. Find the work $\boldsymbol{W}$ done by the gas if its pressure is $\boldsymbol{P} = \mathbf{1.05 \cdot 10^5}$ **Pa**.
(Answer: $\boldsymbol{W}$ = – 410 J)

5. An amount of argon gas inside the tank is equal to $\boldsymbol{n} = \mathbf{4}$ **mol**. What is the internal energy $\boldsymbol{U}$ of the gas if its temperature is equal to $\boldsymbol{T} = \mathbf{-23}$ **ºC**?
(Answer: $\boldsymbol{U}$ = 12.5 kJ)

6. To heat a piece of lead (Pb) of mass $\boldsymbol{m}$ = **100 g** from the temperature $\boldsymbol{T_1}$ = **15 ºC** to the temperature $\boldsymbol{T_2}$ = **35 ºC**, one should transfer to it $\boldsymbol{Q}$ = **260 J** amount of heat. Find the value of specific heat $\boldsymbol{c}$ for lead.
   (Answer: $\boldsymbol{c}$ = 130 J/(kg·K))

Lesson 20

THE FIRST LAW OF THERMODYNAMICS AND ITS APPLICATION

There are two ways in which energy can be transferred between the system (body) and its surroundings:

1) work done on the system W_{ext}, which means that the volume of the system and/or the shape of the system boundary changes due to the application of an external force;
2) heat transfer to/from the system Q, which always occurs if a temperature difference exists across its boundary.

Thus, both of them cause a change in the internal energy of the system ΔU, and in this case the energy conservation law can be written as follows:

$$\Delta U = W_{ext} + Q. \qquad (20.1)$$

According to Newton's third law, the force exerted by the system on the boundary with surroundings is equal to the external force in magnitude, but opposite in direction. Therefore, the work done by the system W is the negative work done on the system W_{ext}: $W = -W_{ext}$. Using this relation, equation (20.1) can be rewritten in the next form:

$$Q = \Delta U + W. \qquad (20.2)$$

The last equation is the *first law of thermodynamics*: *a quantity of heat delivered to the system is converted into the change of the system's internal energy and the work done by the system against its surroundings.*

We will apply the first law of thermodynamics for the isoprocesses in an ideal gas.

Isothermal process. As at such process the temperature remains constant, the temperature change is equal to zero: $\Delta T = 0$. Therefore, the change of the system's internal energy is equal to zero either: $\Delta U = 0$ (see equation (19.2)). Thus, relation (20.2) simplifies to the next form: $Q_T = W$. Consequently, all the heat delivered to the gas is converted into the work done by the gas.

Isochoric process. At this process there is no change in the gas volume, so $\Delta V = 0$. According to equation (19.1), the work done by gas is equal to zero. Therefore, relation (20.2) has the following form: $Q_V = \Delta U$.

Consequently, all the heat delivered to the gas is converted into a change in the internal energy of the gas.

Isobaric process is a process with constant pressure. In this case, the gas volume is proportional to its absolute temperature and the change of temperature causes the volume change and vice versa. Therefore, if heat is delivered, the gas performs the work and its internal energy changes. Thus, taking into account equation (19.1), relation (20.2) can be written as the following: $Q_P = \Delta U + P\Delta V$. Consequently, comparing with isochoric process, in the isobaric one a greater quantity of heat should be delivered to the gas to change its temperature by the same value.

We have considered the cases with no work done (isochoric process) and no internal energy change (isothermal process), but the process with no heat transfer between the system and its surroundings was not discussed.

Adiabatic process. If there is no heat transfer between the system and its surroundings ($Q = 0$), the system is treated as adiabatically isolated. Moreover, the corresponding thermodynamic process, in which the state variables (P, V, or T) change, is called an adiabatic process. Thus, equation (20.2) simplifies to the next form: $0 = \Delta U + W$ or $W = -\Delta U$. Consequently, the gas performs work only by reducing its internal energy. So, any adiabatic expansion of an ideal gas causes a decrease in its temperature.

Control questions

1. Write down the first law of thermodynamics.

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2. At what process is there no change in the internal energy of an ideal gas?

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3. At what process is the work done by the gas equal to zero?

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4. Compare the specific heat of a particular ideal gas under constant pressure and constant volume.

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5. What process is called adiabatic?

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6. Gas is compressed adiabatically. Does its temperature increase or decrease?

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Problems

1. While **17 kJ** of heat is delivering to the gas, it is performing **50 kJ** of work. What is the change in internal energy of the gas ΔU? Is the gas heating or cooling down? Explain.
(Answer: ΔU = – 33 kJ)

2. A monatomic gas, enclosed in a rigid tank (V = **const**), is heated from **20 ºC** to **50 ºC**. The mass density of the gas is $\rho = 1.5$ **kg·m^{-3}** and its specific heat in this process is $c_V = 3.12 \cdot 10^3$ **J·K^{-1}·kg^{-1}**. What is the change of gas pressure ΔP?
(Answer: ΔP = 93.6 kPa)

3. How much heat $\boldsymbol{Q}$ is supplied to helium (He) of mass $\boldsymbol{m}$ = **80 g** during the isobaric heating by $\boldsymbol{\Delta T}$ **= 10 K**? The molar mass of helium $\boldsymbol{M_{He}}$ **= 4·10^{-3} kg/mol**.
(Answer: Q = 4155 J)

4. During adiabatic expansion, **4** moles of argon gas performs the work of **4986 J**. What is the change in gas temperature $\boldsymbol{\Delta T}$? Has the gas heated or cooled down?
(Answer: ΔT = – 100 K)

5. A quantity of heat $\boldsymbol{Q}$ delivered to the gas during the isothermal process is equal to **37 kJ**. What is the change in internal energy of the gas $\boldsymbol{\Delta U}$? How much work $\boldsymbol{W}$ does the gas perform on its surrounding?
(Answer: ΔU = 0, W = 37 kJ)

6. A quantity of heat $\boldsymbol{Q}$ delivered to the gas during the isochoric process is equal to **60 MJ**. What are (a) the work done on the gas $\boldsymbol{W_{ext}}$ and (b) the change in its internal energy $\boldsymbol{\Delta U}$? Explain.
(Answer: (a) $\boldsymbol{W_{ext}} = 0$, (b) $\boldsymbol{\Delta U} = 60$ MJ)

7. The air inside a cylinder with a movable piston is at constant pressure of **1 atm**. When the gas compresses from the volume $\boldsymbol{V_1 = 5}$ **L** to the volume $\boldsymbol{V_2 = 3}$ **L**, its internal energy changes by **500 J**. Does the air absorb or liberate heat? If so, how much?
(Answer: $|\boldsymbol{Q}| = 0.7$ kJ)

8. During adiabatic compression, the temperature of **8 moles** of neon changes by $\boldsymbol{\Delta T = 50}$ **K**. Find out the work $\boldsymbol{W}$ done by the gas.
(Answer: $\boldsymbol{W} = 5$ kJ)

Lesson 21

HEAT ENGINE

According to the first law of thermodynamics, heat transferred to the system can be converted into the work done by the system. This idea is the heart of a device called a *heat engine*. A heat engine absorbs heat (from a source of heat at temperature T_H) and ejects a fraction of that energy by means of work. For practical purposes, a heat engine should operate in a cyclic process, a sequence of processes that in time brings the engine in the same state in which it started. But to have nonzero net work done by the engine over a cycle, it must return to the initial state being disconnected from the source of heat and connected to a heat sink at temperature T_L lower than T_H.

So, to provide the cyclic mode for a heat engine operation, the engine must have three elements:

1) a working substance, for example, gas in a cylinder under a movable piston;
2) a hot reservoir at higher temperature T_H;
3) a cold reservoir at lower temperature T_L.

Over a cycle, a quantity of heat Q_H is transferred to the working substance from the hot reservoir, the working substance performs some work W and the remaining quantity of heat Q_L is rejected to the cold reservoir (Fig. 21.1).

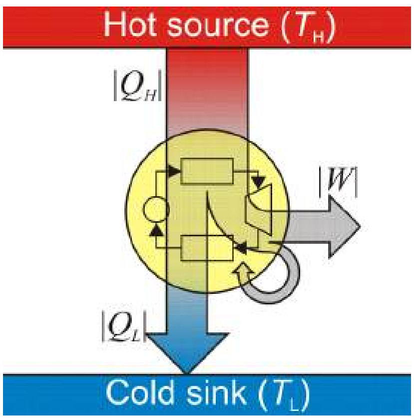

Fig. 21.1

To avoid any confusion with the sign, we will use the absolute values of the quantities of heat Q_H and Q_L. According to the energy conservation law, the work done by the engine can be calculated as the following:

$$W = |Q_H| - |Q_L|.$$

We define the thermal efficiency η of a heat engine as the ratio of work done over a cycle to the quantity of heat absorbed from the hot reservoir

$$\eta = \frac{W}{|Q_H|} = \frac{|Q_H| - |Q_L|}{|Q_H|}. \tag{21.1}$$

From this definition it is clear that the thermal efficiency must be less than one.

Engine based on the Carnot cycle

The most efficient heat engine is the one based on the Carnot cycle proposed by French physicist Sadi Carnot. Often it is called an ideal heat engine. The Carnot cycle consists of two isothermal and two adiabatic processes. A typical pressure-volume diagram of such cycle is shown in Fig. 21.2.

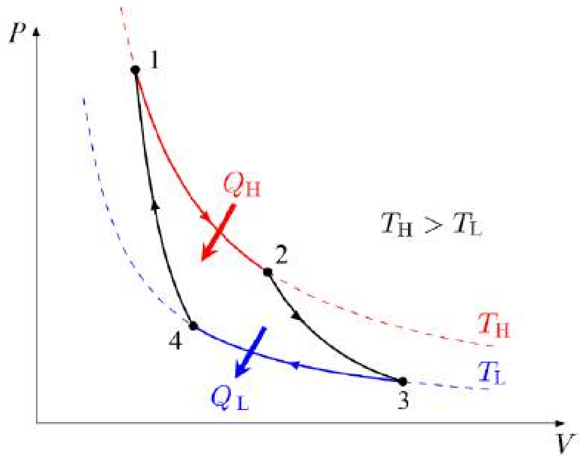

Fig. 21.2

Isothermal expansion of the engine’s working substance occurs between the states 1 and 2 while the substance absorbs the quantity of heat Q_H from the hot reservoir at temperature T_H. During this stage, the engine performs work on its surroundings equivalent to the absorbed heat. Then the working substance is disconnected from the hot source and adiabatically isolated. So, it continues to expand from state 2 to state 3 without heat exchange with surroundings. Thus, the working substance does work owing to its internal energy decrease. As a result, the substance temperature can be reduced to the temperature of cold sink T_L.

To return the working substance to its initial state, it should be compressed. The working substance undergoes isothermal compression between the states 3 and 4, when it is connected to the cold reservoir at temperature T_L and releases the quantity of heat Q_L. During this stage, surroundings of the engine perform work on the substance equivalent to the released heat. At the final stage, adiabatic compression of thermally insulated working substance occurs due to the work done by its surroundings until the engine achieves state 1.

It is possible to derive (but it is beyond our course) that for an ideal gas the quantities of heat Q_H and Q_L are proportional to the temperatures T_H and T_L, respectively. Proportionality coefficient is the same for both stages: 1-2 and 3-4. Thus, the thermal efficiency equation (21.1) for the Carnot cycle can be written as the following:

$$\eta_{Carnot} = \frac{T_H - T_L}{T_H}. \tag{21.2}$$

It should be mentioned that the Carnot cycle thermal efficiency has the greatest value among heat engines with the same temperatures of the hot and cold reservoirs.

Control questions

1. Specify the elements of a heat engine.

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2. Describe the energy transformation during a cycle of the heat engine performance.

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3. Provide the definition of the thermal efficiency of a heat engine.

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4. How many stages are in the Carnot cycle? What are they?

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5. At what stage (stages) does an engine based on the Carnot cycle perform work on its surroundings?

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6. Derive equation (21.2) assuming that $Q_H = bT_H$ and $Q_L = bT_L$.

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Problems

1. In a heat engine, a working substance absorbs **1 kJ** quantity of heat from the hot reservoir and performs **300 J** of work. Find the thermal efficiency of the heat engine η.
   (Answer: η = 30 %)

2. Over a cycle, the quantity of heat absorbed by a working substance of a heat engine is Q_H **= 60 kJ**. The thermal efficiency of the engine is η **= 25 %**. Find the work W done by this engine and the quantity of heat Q_L rejected to the cold reservoir.
   (Answer: W = 15 kJ, Q_L = 45 kJ)

3. Over a cycle, the quantity of heat rejected to the cold reservoir by a working substance of a heat engine is $\boldsymbol{Q_L} = \mathbf{240\ J}$. The thermal efficiency of the engine is $\boldsymbol{\eta} = \mathbf{20\ \%}$. Find the work $\boldsymbol{W}$ done by this engine and the quantity of heat absorbed from the hot reservoir $\boldsymbol{Q_H}$.
(Answer: $\boldsymbol{W} = 60$ J, $\boldsymbol{Q_H} = 300$ J)

4. Over a cycle, the quantity of heat absorbed by a working substance of a heat engine is **36 kJ** and rejected to the cold reservoir is **27 kJ**. Find the work $\boldsymbol{W}$ done by this engine and its thermal efficiency $\boldsymbol{\eta}$.
(Answer: $\boldsymbol{W} = 9$ kJ, $\boldsymbol{\eta} = 25\ \%$)

5. In a heat engine based on the Carnot cycle, a working substance absorbs heat from the hot reservoir at temperature **100 ºC**. The temperature of the cold sink is **0 ºC**. Find the thermal efficiency of the engine $\boldsymbol{\eta}$.
(Answer: $\boldsymbol{\eta} = 0.268$)

6. Over a cycle, the quantity of heat absorbed by an ideal heat engine is Q_H **= 8 kJ**. Temperatures of hot and cold reservoirs are T_H = **127 ºC** and T_L = **27 ºC**. Find the work W done by this engine and the quantity of heat released to the cold reservoir Q_L.
   (Answer: W = 2 kJ, Q_L = 6 kJ)

7. Over a cycle of an ideal heat engine, the quantity of heat rejected to the cold reservoir at temperature **256 K** is Q_L **= 360 J**. The thermal efficiency of the engine is η **= 20 %**. Find the work W done by this engine and the temperature T_H of hot reservoir.
   (Answer: W = 90 J, T_H = 320 K)

8. State 1 (see Fig. 21.2) of a heat engine using **0.5 moles** of ideal gas as the working substance is the following: V **= 8.31 L** and P = **6 atm**. Absorbed quantity of heat is **500 J** and rejected is **125 J** over the Carnot cycle. Find out the temperature T_H of hot reservoir, the thermal efficiency η of the engine and the temperature T_L of cold reservoir.
   (Answer: T_H = 1200 K, η = 75 %, T_L = 300 K)

Lesson 22

PHASE CHANGE. COMBUSTION

In previous lessons we discussed an ideal gas as the simplest model to analyze from a molecular viewpoint. For this model, the interactions between molecules are ignored. However, liquid and solid objects exist due to these interactions. Therefore, depending on the conditions, the substance can be in different states of matter: gaseous, liquid or solid. The term *phase* is generally used to describe a specific state of matter. A *phase change* is called a transition from one phase to another. There are two common types of phase changes: from solid to liquid (melting) and from liquid to gas (boiling). The phase change occurs with absorption or emission of heat and at a definite temperature for a given pressure.

It is evident that the quantity of heat $\boldsymbol{Q}$ required to change the phase is proportional to the substance's mass $\boldsymbol{m}$. Consequently, the ratio $\boldsymbol{Q/m}$, which is called *latent heat* and denoted as $\boldsymbol{L}$, describes an important thermal property of matter. Various substances need different quantity of heat per mass to change their phase. Therefore, the value of $\boldsymbol{L}$ for a substance depends on the type of the phase change and on the properties of the substance as well. The SI unit of latent heat is the *joule per kilogram* (**J/kg**).

From the definition of latent heat, we can derive the equation for amount of energy transferred to or from the substance at any type of phase change:

$$\boldsymbol{Q = \pm Lm}.$$

The sign depends on the absorption or emission of heat: the plus sign (heat absorption) is used when the substance melts or vaporizes; the minus sign (heat emission) is used when it freezes or condenses. For reversible processes like melting-freezing (fusion-solidification) or vaporization-condensation, the values of corresponding latent heat are the same.

Latent heats are specified for a particular type of phase change, for example, latent heat of fusion – $\boldsymbol{L_f}$ and latent heat of vaporization – $\boldsymbol{L_v}$.

It should be emphasized that the temperature of substance remains constant during the phase change process. For a particular process, this temperature is called melting temperature, boiling temperature and so on.

Combustion, which is a chemical reaction, is similar to a phase change in the way that due to it a definite quantity of heat is released. Therefore, the *heat of combustion* H_c is determined as the following:

$$H_c = \frac{Q}{m},$$

where Q is the quantity of heat released by burning fuel of mass m.

The energy value of food is determined in a similar way.

Control questions

1. What states of matter do you know?

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2. What is called a phase change?

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3. Write down the examples of phase changes.

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4. What is called the latent heat? What is its SI unit?

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5. Compare the latent heat of fusion and the latent heat of solidification for water.

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6. What is called the heat of combustion?

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Problems

1. How much heat Q is released by 3 **kg** of freezing water? The latent heat of fusion for water in solid state is $3.33 \cdot 10^5$ **J/kg**.
   (Answer: Q = 999 kJ)

2. A hailstone at 0**°C** falls through air with a uniform temperature of 0**°C** and lands on the pavement at the same temperature. From what initial height h should the hailstone fall to melt completely on impact? Assume that all energy is transferred to the hailstone and neglect air resistance.
   (Answer: h = 33.3 km)

3. How much heat Q is required to convert 10 **g** of ice at – 10 **°C** to water at melting temperature? The specific heat capacity of ice is $2.1 \cdot 10^3$ **J/(kg·K)**.
   (Answer: Q = 3.54 kJ)

4. What mass m of water at initial temperature of **20 °C** can be heated and evaporated by burning **100 kg** of coal? Assume there is no energy lost. The heat of combustion for coal is **15 MJ/kg**, the specific heat capacity of water is **4.2 kJ/(kg·K)** and its latent heat of vaporization is **2.26 MJ/kg**. (Answer: m = 575 kg)

5. A student, playing a computer game instead of studying, forgot about the kettle of water warming on the stove. When he remembered this and rushed into the kitchen, the last drop of water had just evaporated. Find out the amount Q of wasted electrical energy in **kW·h** if there was **1.6 L** of boiling water in the kettle.
(Answer: Q = 1 kW·h)

6. A group of skiers in a winter forest decided to have a five o'clock tea. How many hexamine fuel tablets N should be burnt to melt **1 kg** of snow at temperature **0 °C** and to heat it to the boiling temperature of water **100 °C**? Assume that **30 %** of heat is lost. The mass of one hexamine tablet is **6 g** and the heat of combustion for hexamine is **30 MJ/kg**.
(Answer: N = 6)

Lesson 23

ELECTRIC CHARGE. COULOMB'S LAW

According to modern views on the structure of matter, all atoms (and hence all molecules are formed by atoms) consist of interacting particles which have an intrinsic property, new for this course, – an *electric charge*. Electric charge or quantity of electricity is a scalar. Usually it is denoted by character q.

There are two types of electric charges: positive and negative. Elementary particles like electrons and protons carry an electric charge. According to the convention, introduced by Benjamin Franklin, the electron carries a negative charge, and the proton carries the same quantity of electricity, but a positive sign. Since an electron has no substructure, the magnitude of its charge is the smallest quantity of electricity that exists and is called the *elementary charge*, denoted as e. Any other charge is an integer multiple N of the elementary charge: $q = \pm Ne$, so the electric charge is a quantized quantity. At present, the elementary charge is the defining fundamental physical constant and is determined to be equal to $e = 1.602176634 \cdot 10^{-19}$ **A·s**. The SI unit of electric charge is the *coulomb* (**C**), 1 **C** = 1 **A·s**. For problems, we will use the approximate value of the elementary charge $e = 1.6 \cdot 10^{-19}$ **C**.

An atom is an electrically neutral system that consists of negatively charged electrons and a positive nucleus. The nucleus itself is composed of positively charged protons and electrically neutral neutrons. The proton's and neutron's masses are about the same and approximately two thousand times greater than the mass of an electron.

Most bodies are electrically neutral since the magnitude of a total positive charge of atomic nuclei is the same as the magnitude of a total negative charge of atomic electrons. The transfer of atomic electrons from one body to another results in the appearance of two charged bodies. Moreover, their charges are equal in magnitude, but have opposite signs. Therefore, a negative body charge means an excess of electrons while a positive body charge means a lack of electrons. Thus, *in an isolated system the algebraic sum of all electric charges is constant*. This statement is the *electric charge conservation law* that can be written as the following:

$$\sum_{i=1}^{n} q_i = const. \quad (23.1)$$

Charged bodies with the same sign of electric charge (like charges) repel each other, and bodies with opposite signs (unlike charges) attract each other. The interaction force between charged bodies is often called electric force.

Similar to the model of a body with a negligibly small size, used in mechanics, it is worth to introduce the model of a point charge: a charged body with a negligible size in comparison with the distance to other charged bodies. The electric forces exerted on each other by two point charges at rest are always along the line joining charges and obey Newton third law.

The experiments, for the first time carried out by a French physicist Charles Augustin de Coulomb, show that *the magnitude of the interaction force* $\boldsymbol{F}$ *between two point charges at rest in empty space is directly proportional to the product of the charges' magnitudes* $|\boldsymbol{q_1 q_2}|$ *and inversely proportional to the squared distance* $\boldsymbol{r}$ *between the charges*. This statement is called Coulomb's law and in mathematical form, it can be written as the following:

$$\boldsymbol{F = k_C \frac{|q_1 q_2|}{r^2}} \quad (23.2)$$

where $\boldsymbol{k_C}$ is a proportionality constant, often called the Coulomb’s constant. The value of $\boldsymbol{k_C}$ depends on the used system of units and in the SI it is $8.98755 \cdot 10^9$ N·m^2·C^{-2}. For problems we will use the approximate value $\boldsymbol{k_C = 9.0 \cdot 10^9}$ **N·m^2·C^{-2}**.

Control questions

1. What types of electric charges do you know?

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2. What is the SI unit of electric charge? Express it through the basic SI units.

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3. What is the value of the elementary charge?

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4. Provide the statement of the electric charge conservation law.

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5. Write down the Coulomb's law statement and its mathematical representation?

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6. Draw a sketch illustrating the electric forces between: a) two like point charges; b) two unlike point charges.

Problems

1. How many electrons should be added to or removed from the neutral sphere to charge it to the value: a) $\boldsymbol{q_1 = +4.8}$ **nC**; b) $\boldsymbol{q_2 = -0.32}$ **μC**. Specify the type of action (adding or removing).
(Answer: a) $3 \cdot 10^{10}$; b) $2 \cdot 10^{12}$)

2. Three small identical metal spheres initially have charges $\boldsymbol{q_1} = \mathbf{+\ 0.5\ \mu C}$, $\boldsymbol{q_2} = \mathbf{-\ 0.3\ \mu C}$ and $\boldsymbol{q_3} = \mathbf{-\ 0.8\ \mu C}$. After they are brought into contact, their charges redistribute equally between the spheres. Determine the magnitude $|\boldsymbol{q}|$ and the sign of the final charge on each sphere.
(Answer: $|\boldsymbol{q}| = 0.2$ μC)

3. Two small metal spheres have charges $\boldsymbol{q_1} = \mathbf{+\ 0.5\ \mu C}$ and $\boldsymbol{q_2} = \mathbf{-\ 0.3\ \mu C}$. The distance between the spheres' centers is $\boldsymbol{r} = \mathbf{30\ cm}$. Find the magnitude and direction of the interaction force $\boldsymbol{F}$ between them. Sketch a diagram.
(Answer: $\boldsymbol{F} = 15$ mN)

4. A very small charged sphere of mass $\boldsymbol{m} = \mathbf{10\ g}$ is hung by a thread. Another identical sphere of the same charge $\boldsymbol{q} = \mathbf{1\ \mu C}$ is placed below the first one. Find the distance $\boldsymbol{r}$ between the spheres so that there is no tension force in the thread. Treat the spheres as point charges.
(Answer: $\boldsymbol{r} = 0.3$ m)

5. While rubbing a plastic stick with a piece of fur, **20 billion** electrons transferred from one body to another. Determine the charges (including the sign) acquired by the stick and the piece of fur. Explain.
   (Answer: $q_{st} = -3.2$ nC; $q_{fur} = 3.2$ nC)

6. One of the simplest models of the hydrogen atom assumes that the electron is orbiting a proton along a circular path with a radius of $\mathbf{5 \cdot 10^{-11}}$ **m**. Estimate the speed v of the electron, taking into account the electric interaction between the particles. The mass of an electron is approximately equal to $\mathbf{9 \cdot 10^{-31}}$ **kg**.
   (Answer: $v \approx 2 \cdot 10^{6}$ m/s)

Lesson 24

ELECTRIC FIELD

In lesson 8 we stated two existing types of interaction described by contact and long-ranged forces. To describe a long-ranged interaction the concept of *field* is used. An example of field interaction is the gravitational force discussed in lesson 9.

As it is clear from equation (23.2), the electric force is a long-ranged one. Thus, for electric interaction it is convenient to introduce the concept of *electric field*. In other words, charged particles interact by means of electric fields. So, any charge modifies the property of the space around it or creates an electric field.

To analyze the electric field due to some charges at rest (*electrostatic field*), one can conduct a set of experiments with a test point charge. Such experiments show that at a particular point in space, the magnitude of the electric force experienced by a test point charge is always proportional to the magnitude of this charge. Therefore, the ratio of these magnitudes is an attribute of the particular point. However, the direction of the electric force is independent of the charge magnitude, but is reversed with a change in the test charge sign.

The *electric field* $\vec{\boldsymbol{E}}$ at a particular point in space is defined as the ratio of the electric force $\vec{\boldsymbol{F}}$ acting upon a small test charge $\boldsymbol{q_0}$ placed in this point to the charge value:

$$\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}}{\boldsymbol{q_0}} \ . \qquad (24.1)$$

The electric field is a vector quantity. Its *direction coincides with the direction of the electric force acting on the positive test charge*. A positive test charge experiences a repulsive force in the electric field created by a positive charge and an attractive force in the field created by a negative charge. So the electric field vector has direction from a positive charge and/or towards a negative charge.

The SI unit of electric field is the *newton per coulomb* (**N/C**).

Using Coulomb's law (see equation (23.2)), we can derive the expression for the magnitude (or strength) of the electric field created by a point charge $\boldsymbol{q}$ in empty space:

$$E = \frac{F}{q_0} = \frac{1}{q_0} k_C \frac{|q_0 q|}{r^2} = k_C \frac{|q|}{r^2}.$$

Electric field $\vec{E}$ *created by a system of* n *charges at a given point is equal to the vector sum of individual electric fields* $\vec{E}_i$, *created by every charge* q_i *at this point* (Fig. 24.1):

$$\vec{E} = \sum_{i=1}^{n} \vec{E}_i . \qquad (24.2)$$

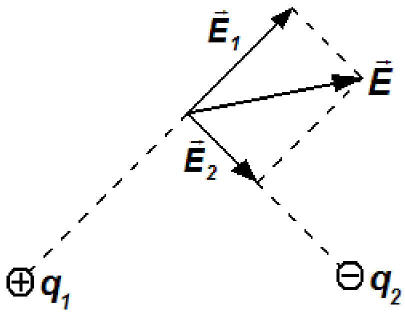

Fig. 24.1

The above statement and equation (24.2) constitute the *superposition principle of electric fields*. It should be mentioned that the superposition principle is a fundamental one and valid for any type of vector fields.

Electric field lines can graphically represent electric fields. We draw them so that the electric field vector $\vec{E}$ is tangent to the electric field line at each point. The direction of field lines is the same as the direction of the electric field vector. The electrostatic field lines start on positive and terminate on negative charges. The field lines for isolated positive and negative point charges are shown in Figs. 24.2, ***a*** and ***b***, respectively.

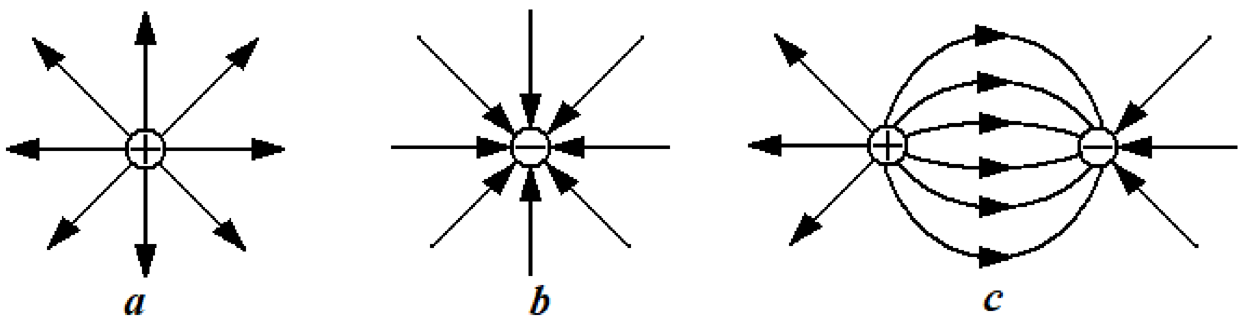

Fig. 24.2

Figure 24.2, ***c*** shows the pattern of field lines for the system of two unlike charges of the same magnitude.

If the electric field vector has the same magnitude and direction at any point of a certain region, then we call this field *uniform* in this region. We represent a uniform electric field by parallel and equidistant field lines. A uniform electric field is created by an infinite uniformly charged surface.

Control questions

1. Write down the definition of the electric field. What is its SI unit?

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2. What is the direction of the electric field vector?

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3. Derive the expression of the electric field strength created by a point charge.

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4. Describe the way to represent electric fields with electric field lines.

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5. Draw the field lines for the electrostatic field created by: a) an isolated positive point charge; b) an isolated negative point charge; c) the system of two unlike point charges of the same magnitude.

6. Derive the superposition principle of electric fields.

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Problems

1. The point charge of **1 μC** with mass of **10 g** is levitating above the unlikely charged surface. Calculate the magnitude of the electric field $\boldsymbol{E}$ created by the surface.
   (Answer: $\boldsymbol{E}$ = 100 kN/C)

2. The electric field created by a point charge $\boldsymbol{q}$ at a certain position A is equal to **100 N/C**. What should be the magnitude of the charge if the distance from the charge to A is **30 cm**?
   (Answer: $\boldsymbol{q}$ = 1 nC)

3. There are two point charges $\boldsymbol{q_+}$ **= +1 nC** and $\boldsymbol{q_-}$ **= – 1.5 nC** separated by a distance of **0.5 m** on the X-axis. Assuming that charge $\boldsymbol{q_+}$ is at the origin and $\boldsymbol{q_-}$ is on the positive part of the X-axis, find the magnitudes of electric field $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$ at the positions $\boldsymbol{x_1}$ **= – 0.1 m** and $\boldsymbol{x_2}$ **= + 0.1 m**.
   (Answer: $\boldsymbol{E_1}$ = 984.375 N/C; $\boldsymbol{E_2}$ = 862.5 N/C)

4. At some point in space, the horizontal and vertical components of the electric field are $\boldsymbol{E_x}$ = **– 1.2·10⁶ N/C** and $\boldsymbol{E_y}$ = **1.6·10⁶ N/C**. What is the magnitude of the electric force $\boldsymbol{F}$ experienced by a small particle with the charge $\boldsymbol{q}$ **= – 5 nC** placed at this point? What is the angle $\boldsymbol{\theta}$ between the force and the positive X-direction?
   (Answer: $\boldsymbol{F}$ = 0.01 N, $\boldsymbol{\theta}$ = 53.13º)

5. Two identical positive charges of **1 C** are placed at different vertices of the equilateral triangle with a side of **0.5 m**. Calculate the magnitude of the electric-field $\boldsymbol{E}$ at the third vertex of the triangle.
   (Answer: $\boldsymbol{E}$ = 62.35 kN/C)

6. Two point charges $\boldsymbol{q_1}$ **= 6 nC** and $\boldsymbol{q_2}$ **= – 16 nC** are at the distance $\boldsymbol{r}$ **= 5 cm** from each other. Find the magnitude of the electric field $\boldsymbol{E}$ at a point that is **3 cm** away from the positive charge and **4 cm** from the negative. Draw the diagram.
   (Answer: $\boldsymbol{E}$ = 108 kN/C)

Lesson 25

ELECTRIC POTENTIAL

In Lesson 13 we discussed gravitational and elastic forces as conservative. Another example of a conservative force is the electrostatic force. Therefore, the work done by electrostatic force on a charged body does not depend on the body's path, but depends only on the initial ($\boldsymbol{i}$) and final ($\boldsymbol{f}$) positions of the body.

Thus, the concept of potential energy, introduced for conservative forces, can be applied to the electrostatic force as well. It is worth to remind you the relationship between the work done by the conservative force $\boldsymbol{W_{if}}$ and the potential energy $\boldsymbol{U}$ of the corresponding interaction (see Lesson 13):

$$\boldsymbol{W_{if}} = \boldsymbol{U_i} - \boldsymbol{U_f}\,. \tag{25.1}$$

Since the electrostatic force experienced by a charge is proportional to its value (see equation (24.1)), the work done by this force on the charge to move it from the initial point to the final one has the same dependence on the charge. Hence, the electrostatic potential energy of the charge at a particular point should also be proportional to the charge.

The ratio of the electrostatic potential energy $\boldsymbol{U}$ *of a test charge at a point in an electric field to its value* $\boldsymbol{q_0}$ *is called the electric potential* $\boldsymbol{V}$ *at that point:*

$$\boldsymbol{V} = \frac{\boldsymbol{U}}{\boldsymbol{q_0}}. \tag{25.2}$$

The potential is a scalar quantity and can be both positive and negative. The SI unit of electric potential is the *volt* (**V**).

Combining equations (25.1) and (25.2), the work done by the electrostatic field on a point charge $\boldsymbol{q}$ to move it from the initial to final position can be written as the following:

$$\boldsymbol{W_{if}} = \boldsymbol{V_i} - \boldsymbol{V_f}, \tag{25.3}$$

where $\boldsymbol{V_i}$, $\boldsymbol{V_f}$, are potentials at the initial and final positions of the charge, respectively.

It is reasonable to assume that the electric potential at infinity is equal to zero, so the work done by the electrostatic field on a point charge q to move it from its initial position to infinity can be calculated as:

$$W_{i\infty} = q(V_i - 0) = qV_i,$$

where V_i is the potential at the initial position.

The electric potential due to a point charge q at distance r from the charge is the following:

$$V = k_C \frac{q}{r}. \tag{25.4}$$

This equation can be derived by using calculus, but it is beyond the scope of this course. It should be mentioned that the last expression is valid for electric potential at the point outside a uniformly charged sphere and on its surface, where r is the distance from the center of the sphere.

To calculate the electric potential of the electric field created by a system of n charges, we can apply the superposition principle. *The electric potential at a point of the field is equal to the algebraic sum of individual potentials, created by every charge at that point:*

$$V = \sum_{j=1}^{n} V_j.$$

For a uniform electrostatic field, there is the following relationship between its magnitude E and the potential difference $V_1 - V_2$:

$$V_1 - V_2 = Ed_{12},$$

where d_{12} is the distance between two points along the field line. It should be mentioned that V_1 is always greater than V_2. From the last equation we can obtain another SI unit of electric field, which is commonly used: [E] = **V/m**.

An equipotential surface is a surface every point of which has the same electric potential. Electric fields can be graphically represented not only by field lines, but by equipotential surfaces as well. It is easy to draw a pattern of equipotential surfaces from the pattern of field lines, using the rule: field lines are always perpendicular to equipotential surfaces.

Control questions

1. Is the electrostatic force conservative or nonconservative? What is the work done by the electric field on a point charge to move it along a closed path?

2. Provide the definition of electric potential. What is its SI unit?

3. What is the work done by the electrostatic field to move a point charge $\boldsymbol{q}$ from the position with potential $\boldsymbol{V_1}$ to the position with potential $\boldsymbol{V_2}$?

4. What is the electric potential due to a point charge $\boldsymbol{q}$ at any distance $\boldsymbol{r}$ from the charge? How to calculate the potential due to a system of charges?

5. What electric field is called uniform? How to calculate the potential difference in a uniform field $\boldsymbol{E}$?

6. Write down the definition of an equipotential surface. Sketch the pattern of equipotential surfaces of: a) a uniform field; b) the field created by a point charge.

Problems

1. Two identical positive charges of **1 μC** are placed at different vertices of an equilateral triangle with a side of **0.5 m**. Calculate the electric potential $\boldsymbol{V}$ at the third vertex of the triangle. What is the answer for two unlike charges (one positive and one negative) of the same magnitude?
   (Answer: $\boldsymbol{V}$ = 36 kV)

2. Find the potential $\boldsymbol{V}$ on the surface of the sphere if at distance $\boldsymbol{r_1}$ **= 50 cm** from its center the potential is $\boldsymbol{V_1}$ **= 400 V**, and at distance $\boldsymbol{L_2}$ **= 20 cm** from the sphere's surface the potential is $\boldsymbol{V_2}$ **= 800 V**.
   (Answer: $\boldsymbol{V}$ = 4 kV)

3. What work $\boldsymbol{W}$ is required to assemble the system of three positive point charges of **2 μC** each, located at the vertices of an equilateral triangle with a side of **0.3 m**.
   (Answer: $\boldsymbol{W}$ = 0.36 J)

4. Two negative point charges of magnitudes **6 nC** and **8 nC** are at a distance $\boldsymbol{r_1}$ **= 40 cm** from each other. Calculate the work $\boldsymbol{W}$ that should be done to decrease the distance between them to $\boldsymbol{r_2}$ **= 16 cm**?
   (Answer: $\boldsymbol{W}$ = – 1.62 μJ)

5. What is the speed $\boldsymbol{v}$ of an electron that has passed through the potential difference of $\boldsymbol{V_{12}} = \mathbf{200}$ **V** in the electric field? Use the following value for the electron's mass: $\boldsymbol{m_e} = \mathbf{9.1 \cdot 10^{-31}}$ **kg**.
(Answer: $\boldsymbol{v} = 8.4 \cdot 10^6$ m/s)

6. A uniform electric field is determined by a potential difference of $\mathbf{50}$ **kV** between equipotential surfaces spaced at a distance of $\mathbf{5}$ **cm**. How many electrons $\boldsymbol{N}$ should be added to a dust particle for it to experience an electric force of $\mathbf{160}$ **μN** in this field?
(Answer: $\boldsymbol{N} = 10^9$)

Lesson 26

CONDUCTORS AND CAPACITANCE. DIALECTICS

Some materials, called conductors, permit the easy motion of electric charge through them, while others, called dielectrics, do not. All metals are conductors. To support the electric field constant in time, all charges are assumed to be at rest. For them to be at rest, the electric field in the conductor must be equal to zero. Therefore, the potential difference in/on the conductor should be equal to zero as well. Consequently, the electric potential of every point of the conducting body is the same and equal to the potential of the conductor's surface. This potential is called the potential of the conductor.

The potential of the conductor is proportional to the electric charge stored on the conductor. The ability of a conductor to store an electric charge is described by the new concept of *electric capacitance* or simply *capacitance*. The capacitance $\boldsymbol{C}$ of an isolated conductor is determined as the ratio of the charge $\boldsymbol{q}$ stored on the conductor to its potential $\boldsymbol{V}$:

$$\boldsymbol{C = \frac{q}{V}}.$$

The SI unit of capacitance is the *farad* (**F**) that is equal to coulomb per volt: F=C/V. Practically, microfarad (**μF**) and picofarad (**pF**) are used:

$$\mathbf{1\ \mu F = 10^{-6}\ F, \quad 1\ pF = 10^{-6}\ \mu F = 10^{-12}\ F}.$$

The capacitance of a conductor depends on its sizes and shape. From equation (25.4) we can derive the capacitance for an isolated conducting sphere in empty space: $\boldsymbol{C = R/k_c}$, where $\boldsymbol{R}$ is the sphere's radius.

To improve the ability to store electric charge, two electrically separated conductors should be used. One of them can be positively charged, and the other gains an electric charge equal in magnitude but opposite in sign. This charging may be done by transferring electrons from one conductor to another. In this case the net charge of the two conductors remains zero, but each individual conductor is charged and can store this charge as long as it is required. It is evident that each of the conductors has its own potential: $\boldsymbol{V_+}$ or $\boldsymbol{V_-}$. As a result, there is a potential difference (or *voltage*) $\boldsymbol{V}$ between the conductors: $\boldsymbol{V = V_+ - V_-}$.

A capacitor is a system of two conducting plates separated by dielectric or vacuum. A capacitor with equal in magnitude but opposite in sign electric charges on its plates is called a charged capacitor. In the case of an ideal charged capacitor, the electric field is completely localized between its plates.

The capacitance $\boldsymbol{C}$ *of a capacitor is determined as the ratio of the charge magnitude* $\boldsymbol{q}$ *stored on one of its plates to the potential difference* $\boldsymbol{V}$ *across the capacitor*:

$$\boldsymbol{C = \frac{q}{V}}. \qquad (26.1)$$

A system of two parallel identical charged plates separated by the distance $\boldsymbol{d}$, much smaller than the square root of the plate's surface area $\boldsymbol{A}$ can be treated as an ideal capacitor, since its electric field is almost completely localized between the plates. Such a system is called a *parallel-plate capacitor* (Fig. 26.1).

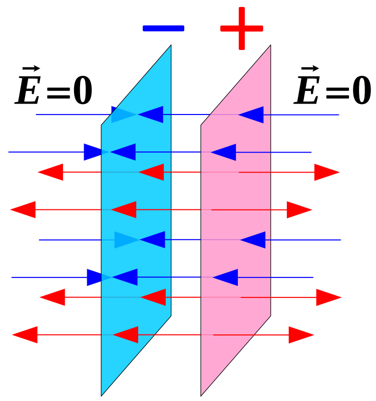

Fig. 26.1

The capacitance $\boldsymbol{C_0}$ of a parallel-plate capacitor with vacuum between its plates can be calculated by using the following:

$$\boldsymbol{C_0 = \frac{\varepsilon_0 A}{d}},$$

where ε_0 is often called the electric constant and its approximate value is $\varepsilon_0 = \mathbf{8.85 \cdot 10^{-12}}$ F·m^{-1}. The known for you constant $\boldsymbol{k_C}$ and the new ε_0 are related as $\mathbf{4\pi\varepsilon_0 \boldsymbol{k_C} = 1}$.

If a dielectric is placed between the plates of the capacitor, its capacitance $\boldsymbol{C}$ increases by some factor, which is called the *relative dielectric permittivity* of the substance and denoted by $\boldsymbol{\varepsilon_r}$. Thus, the capacitance of the capacitor with dielectric is calculated as the following:

$$C = C_0\varepsilon_r.$$

The value of ε_r depends on the dielectric's property and, by definition, is exactly **1** for vacuum. Since air is a rare substance, its relative dielectric permittivity is approximately equal to **1** as well.

The capacitance increases compared with a vacuum capacitor, since for exactly the same charge distribution, the electric field and potential difference in the dielectric reduces by the value of the relative dielectric permittivity.

Control questions

1. What is the potential difference between two arbitrary chosen points of a conductor in an electrostatic field? Explain.

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2. Provide the definition of the capacitance of an isolated conductor. What is its SI unit?

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3. Derive an expression for the capacitance of a conducting sphere.

4. What is called a capacitor? Definition the capacitance of a capacitor.

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5. Write down the equation for the capacitance of a parallel plate capacitor.

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6. How does the capacitance of a capacitor change if a dielectric is inserted between its plates? Explain.

Problems

1. A metal sphere of radius $\boldsymbol{R} = \mathbf{18\ mm}$ is charged to potential of $\boldsymbol{V} = \mathbf{150\ V}$. Find the capacitance of the sphere and the charge stored on it.
(Answer: $\boldsymbol{C} = 2$ pF, $\boldsymbol{q} = 0.3$ nC)

2. The area of the plates of a parallel-plate air capacitor is $\boldsymbol{A} = \mathbf{50\ cm^2}$, the charge of the capacitor is $\boldsymbol{q} = \mathbf{8.85\ nC}$, and the potential difference across it is $\boldsymbol{V} = \mathbf{200\ V}$. Calculate the distance $\boldsymbol{d}$ between the plates of the capacitor.
(Answer: $\boldsymbol{d} = 1$ cm)

3. In a parallel-plate air capacitor the distance between the plates is $\boldsymbol{d} = \mathbf{5\ cm}$. The capacitor is charged to $\boldsymbol{V} = \mathbf{200\ V}$ and disconnected from the source of voltage. What is the potential difference $\boldsymbol{V_1}$ across the capacitor if the plates are pulled apart to a distance $\boldsymbol{d_1} = \mathbf{10}$ cm?
(Answer: $\boldsymbol{V_1} = 400$ V)

4. The conducting body is charged to a potential of **200 V** and stores a charge of **0.6 nC**. Find the capacitance of the body. Calculate the radius $\boldsymbol{R}$ of the sphere with the same capacitance.
(Answer: $\boldsymbol{C}$ = **3** pF, $\boldsymbol{R}$ = **27** mm)

5. The area of the plates of a parallel-plate capacitor is $\boldsymbol{A}$ = **200 cm²**. The space between the plates is filled with a polystyrene slab (ε_r = **3.2**) of thickness $\boldsymbol{d}$ = **0.2 mm**. Find the capacitance $\boldsymbol{C}$ of the capacitor.
(Answer: $\boldsymbol{C} = 2.3 \cdot 10^{-9}$ F)

6. The capacitance of a parallel-plate air capacitor is $\boldsymbol{C}$ = **10 pF** and the potential difference across it is $\boldsymbol{V}$ = **50 V**. While the capacitor is connected to a source of voltage, it is immersed into distilled water (ε_r = **80**). What is the value of the new charge $\boldsymbol{q}$ on the capacitor?
(Answer: $\boldsymbol{q}$ = 40 nC)

Lesson 27

COMBINATION OF CAPACITORS. ENERGY OF A CHARGED CAPACITOR

When discussing the combination of capacitors, it is convenient to draw a schematic diagram of their connection. The capacitor symbol – two parallel lines of the same length – corresponds to the geometry of a parallel-plate capacitor.

Any combination of capacitors is able to store an electric charge. Thus, we can talk about its *equivalent capacitance*. This capacitance is calculated according to equation (26.1) as the ratio of the net charge stored within the capacitors' combination to the potential difference across this combination.

Capacitors in parallel

Let us consider a combination of $\boldsymbol{n}$ charged capacitors with capacitances $\boldsymbol{C_1}, \boldsymbol{C_2}, \ldots, \boldsymbol{C_n}$ connected in parallel (see Fig. 27.1 as an example for four capacitors). The top plates of these capacitors, connected by conductive wires, can be treated as one conductor. Therefore, they have the same potential. A similar conclusion is valid for bottom plates. Thus, all capacitors have the identical potential difference $\boldsymbol{V}$ across them, but the charges may be different: $\boldsymbol{q_1}, \boldsymbol{q_2}, \ldots, \boldsymbol{q_n}$.

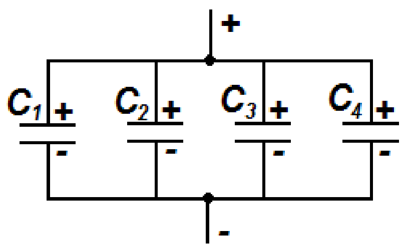

Fig. 27.1

According to the charge conservation law, the net charge $\boldsymbol{q}$ stored by such a system of capacitors is equal to the sum of the charges on each capacitor:

$$\boldsymbol{q = q_1 + q_2 + \ldots + q_n}\,.$$

The individual charges can be calculated using the definition of the capacitance (see equation (26.1)): $q_1 = C_1V$, $q_2 = C_2V$, . . . , $q_n = C_nV$. Then the net stored charge can be represented as the following:

$$q = C_1V + C_2V + \ldots + C_nV = (C_1 + C_2 + \ldots + C_n)V.$$

As a result, the expression for the equivalent capacitance of the combination of capacitors in parallel C_{par} is the next:

$$C_{par} = \frac{q}{V} = C_1 + C_2 + \ldots + C_n = \sum_{i=1}^{n} C_i .$$

Capacitors in series

Another combination of capacitors with capacitances $C_1, C_2, \ldots, C_n$ is the connection in series, when they are connected one after another. If we charge this initially uncharged combination, only the left plate of the leftmost capacitor would obtain some charge and the right plate of the rightmost capacitor would obtain a charge equal in magnitude but opposite in sign. Other plates form pairs of adjacent plates connected only to each other (see Fig. 27.2). These pairs must remain with zero net charge as an isolated system. Therefore, these plates can gain charge only due to the charge redistribution within an individual pair. Thus each plate has the same magnitude of charge q, but its sign is opposite for adjacent plates (see Fig. 27.2). This charge q is the net charge of the series combination of capacitors.

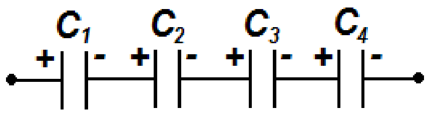

Fig. 27.2

We can determine the potential difference across each capacitor from equation (26.1):

$$V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, \ldots, V_n = \frac{q}{C_n}.$$

The potential difference across the capacitors' combination is equal to the sum of the potential differences across each capacitor:

$$V = V_1 + V_2 + \ldots + V_n = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}\right).$$

The ratio of the potential difference across the capacitors' combination to the net charge stored within the combination is the reciprocal of the equivalent capacitance $\boldsymbol{C_{ser}}$. Therefore, for a combination of capacitors in series we have the following expression:

$$\frac{1}{C_{ser}} = \frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n} = \sum_{i=1}^{n} \frac{1}{C_i}.$$

Energy stored in a charged capacitor

To charge a capacitor, that is, to add extra charge to the capacitor plate, we must overcome the repulsive force with the charge already stored on it. Thus, it is necessary to apply an external force to overcome this repulsion. The work done by an external force is equal to the energy of the electric field in a charged capacitor. When the capacitor discharges, this stored energy is brought back as the work done by the electric force.

The work $\boldsymbol{W}$ done by an external force to charge the capacitor up to some final value $\boldsymbol{q}$ and consequently to the potential difference $\boldsymbol{V}$ is equal to:

$$W = \frac{qV}{2} = \frac{CV^2}{2} = \frac{q^2}{2C}.$$

Taking into account that this work is equal to the stored electric field energy $\boldsymbol{U_E}$ and applying equation (26.1), the last relation can be written as the following:

$$U_E = \frac{qV}{2} = \frac{CV^2}{2} = \frac{q^2}{2C},$$

where $\boldsymbol{C}$ is the capacitance of the capacitor.

Control questions

1. Draw a schematic diagram of the combination of two capacitors in parallel. Compare the potential differences across each of the charged capacitors. Explain your answer.

2. Derive the expression of equivalent capacitance for the system of two capacitors in parallel.

3. Draw a schematic diagram of the combination of two capacitors in series. Compare the charges in each capacitor. Explain your answer.

4. Derive the expression of equivalent capacitance for the system of two capacitors in series.

5. Write down the expressions for calculating the energy stored by a charged capacitor.

6. You have three identical capacitors with capacitance C. Draw schematic diagrams of their combination to obtain the value of the equivalent capacitance equal to: a) $3C$; b) $C/3$; c) $2C/3$.

Problems

1. Three capacitors in series with capacitances $C_1 = 1$ **μF**, $C_2 = 2$ **μF** and $C_3 = 3$ **μF** are connected to the source of voltage with $V = 220$ **V**. Calculate the stored charge q and the potential differences V_1,, V_2 , V_3 across each capacitor.
   (Answer: $q = 1.2 \cdot 10^{-4}$ C; $V_1 = 120$ V; $V_2 = 60$ V; $V_3 = 40$ V)

2. Seven identical parallel-plate capacitors are connected in series. The area of capacitor plates is $A = 300$ **cm²** and the space between the plates is filled with a mica slab ($\varepsilon_r = 7$) of thickness $d = 0.15$ **mm**. Find the equivalent capacitance C of this combination.
   (Answer: $C = 0.177$ μF)

3. Three capacitors in parallel with capacitances $C_1 = 2$ **μF**, $C_2 = 4$ **μF** and $C_3 = 6$ **μF** are connected to the source of voltage with $V = 1$ **kV**. Find the charges q_1 q_2, q_3 in capacitors and the equivalent capacitance of this combination.
   (Answer: $q_1 = 2 \cdot 10^{-9}$ C; $q_2 = 4 \cdot 10^{-9}$ C, $q_3 = 6 \cdot 10^{-9}$ C; $C = 12$ pF)

4. Two capacitors in parallel are connected to the source of voltage with $\boldsymbol{V}$ = **2 kV**. Find the equivalent capacitance $\boldsymbol{C}$ of this combination if the charge stored in one of the capacitors is **4 nC** and the capacitance of another is **3 pF**.
   (Answer: $\boldsymbol{C}$ = 5 pF)

5. The area of the plates of a parallel-plate capacitor is **100 cm²**. The space between the plates is filled with a polystyrene slab ($\boldsymbol{\varepsilon_r}$ **= 3.2**) of thickness **0.1 mm**. Find the energy $\boldsymbol{U}$ stored by this capacitor if the potential difference across it is equal to **100 V**.
   (Answer: $\boldsymbol{U}$ = 11.5 μJ)

6. There are two capacitors with capacitances $\boldsymbol{C_1}$ **= 5 μF** and $\boldsymbol{C_2}$ **= 10 μF** and a source of voltage with **60 V**. Firstly, we connect to the source the combination of capacitors in series, then the same is done for combination in parallel. Calculate the energy $\boldsymbol{U}$ stored by the combination of these capacitors in both cases.
   (Answer: $\boldsymbol{U_{ser}}$ = 0.06 J; $\boldsymbol{U_{par}}$ = 0.27 J)

Lesson 28

CURRENT. RESISTANCE. ELECTROMOTIVE FORCE

In previous lessons we discussed the electrostatic field created by electric charges at rest. However, charging or discharging capacitors, we face the motion of charged particles or, in other words, the flow of electric charges.

A net flow of charged particles through the region is called *an electric current*. In electric circuits, these particles are electrons moving along a metal wire.

To describe the charge flow quantitatively, we use a physical quantity – *current*, denoted by character $\boldsymbol{I}$. The *current* $\boldsymbol{I}$ through the cross-sectional area of a conductor is defined as the ratio of the net charge $\boldsymbol{\Delta q}$ flowing through the area in a time interval $\boldsymbol{\Delta t}$ to this time interval:

$$\boldsymbol{I = \frac{\Delta q}{\Delta t}}\,. \tag{28.1}$$

The SI unit of current is the *ampere* (**A**), which is a basic SI unit. Ampere is related to the unit of electric charge, coulomb, as the following: A = C/s.

Electric current is a scalar quantity. However, the flow of charged particles has a direction along the conducting wire. According to the convention, the current has the same direction as the average velocity of positive chargers moving within the wire. A current that is constant in direction is called a direct current (DC).

Ohm's law

The electric current $\boldsymbol{I}$ through a conductor and the potential difference $\boldsymbol{V}$ across the conductor are proportional to each other:

$$\boldsymbol{V = IR}, \tag{28.2}$$

where proportionality coefficient $\boldsymbol{R}$ is called the *resistance* of the conductor. The SI unit of resistance is the *ohm* ($[\boldsymbol{R}] = \boldsymbol{\Omega}$) that is equal to volt per ampere. Equation (28.2) has its name in honor of the famous German physicist George Ohm. A conductor with resistance is called a *resistor*.

The resistance of a uniform conducting wire with length $\boldsymbol{l}$ and constant cross-sectional area $\boldsymbol{A}$ can be calculated as the following:

$$R = \rho \frac{l}{A},$$

where ρ is the resistivity of the wire's substance. The SI unit of the resistivity is the *ohm-meter* ($[\rho] = \Omega\cdot\mathrm{m}$).

Resistivity of a conducting substance depends on the temperature T:

$$\rho = \rho_0[1 + \alpha(T - T_0)],$$

where ρ_0 is the resistivity at a reference temperature T_0 (often taken as 273 K or 0 °C); α is the temperature coefficient of resistivity of this substance. Thus, the resistance depends on the size, geometry, temperature and properties of the conductor substance.

Complete circuit

To support a steady charge motion, i.e. the current, in a conductor, the latter must be part of a closed conducting loop with a source of energy (battery). Such a closed loop is called a *complete circuit*.

A simplified pictorial representation is used to study electric circuits. It is called a circuit diagram. Various circuit elements are represented in the diagram by circuit symbols. The circuit symbol for the capacitor we discussed in the previous lesson. Straight lines between the circuit symbols represent resistance-free connecting wires between the circuit elements. The circuit symbols for resistor (R) and battery ($\mathcal{E}$) are shown in Fig. 28.1. The negative terminal of the battery is at a lower potential and is represented in the symbol with a shorter but thicker line. A battery has an internal resistance denoted as r. Therefore, the resistance R of the resistor(s) connected to the battery is called the external resistance.

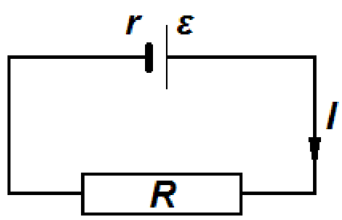

Fig. 28.1

The maximum possible potential difference that a battery can provide between its terminals is called the *electromotive force* or *emf* $\mathcal{E}$ of the battery.

The SI unit of emf is the same as of potential. Electromotive force $\mathcal{E}$ and internal resistance $\boldsymbol{r}$ are properties of the battery and do not depend on the current in the circuit.

The battery emf $\mathcal{E}$ in a complete circuit can be represented as the sum of the potential differences across the external and internal parts of the circuit:

$$\mathcal{E} = \boldsymbol{IR} + \boldsymbol{Ir}.$$

The product $\boldsymbol{IR}$ is usually called a *terminal voltage*. The electromotive force is equal to the terminal voltage if the internal resistance of the battery $\boldsymbol{r}$ is negligible compared to the external resistance $\boldsymbol{R}$ of circuit.

Control questions

1. What physical phenomenon is called an electric current? Define the electric current quantitatively. What is the SI unit of current?

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2. There is a conducting loop with a clockwise current in it. What is the direction of the electron's motion within this loop? Explain.

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3. Formulate Ohm's law. Define the physical meaning of resistance.

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4. What does resistance of a wire depend on?

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5. What is called the emf of a battery? What is the SI unit of emf?

6. What is the current in a complete circuit with emf $\mathcal{E}$, internal resistance $\boldsymbol{r}$ and external resistance $\boldsymbol{R}$?

Problems

1. What is the total number $\boldsymbol{N}$ of electrons passing through a cross-section of the conductor during **1.5 minute** if the current in the conductor is **1.6 A**?
(Answer: $\boldsymbol{N} = 9{\cdot}10^{20}$)

2. A current $\boldsymbol{I} = \mathbf{0.2}$ **A** flows through a nichrome conductor with a cross-sectional area $\boldsymbol{A} = \mathbf{0.5\ mm^2}$. The voltage across the conductor is $\boldsymbol{V} = \mathbf{1.6}$ **V**. Calculate the length $\boldsymbol{l}$ of the conductor. The resistivity of nichrome is $\boldsymbol{\rho} = \mathbf{1{\cdot}10^{-6}\ \Omega{\cdot}m}$.
(Answer: $\boldsymbol{l} = 4$ m)

3. During **0.5 minutes**, an electric charge of **45 C** passes through the conductor with a resistance of **5 Ω**. Find the voltage V across the ends of the conductor.
(Answer: V = 7.5 V)

4. Define the electric current I through a steel conductor with a length of **20 m** and a diameter of **0.8 mm**. The voltage across the conductor is **1.6 V** and the resistivity of copper is $\mathbf{1.0 \cdot 10^{-7}\ \Omega \cdot m}$.
(Answer: I = 0.4 A)

5. A rechargeable battery with emf $\mathcal{E}$ **= 1.2 V** and internal resistance r **= 0.5 Ω** is connected to a light bulb with resistance R **= 3.5 Ω**. Calculate the electric current I in the circuit.
(Answer: I = 0.3 A)

6. When we connect a resistor with resistance R_1 **= 5 Ω** to a battery, the voltage across the resistor is V_1 **= 10 V**. If we replace the first resistor by another one with resistance R_2 **= 12 Ω**, the voltage across it is V_2 **= 12 V**. Find out the emf $\mathcal{E}$ and the internal resistance r of the battery.
(Answer: $\mathcal{E}$ = 14 V; r = 2 **Ω**)

Lesson 29

DIRECT-CURRENT CIRCUITS

Electric circuits may include a different number of various circuit elements. In turn, circuit elements can be connected in many ways: in series, in parallel or their combination. Any combination of resistors is able to resist the electric charge motion through them. Therefore, we have to discuss the *equivalent resistance* of the resistors' combination. This resistance is calculated according to equation (28.2) as the ratio of the potential difference across the resistors' combination to the net current through it.

Series combination

Suppose a combination of $\boldsymbol{n}$ resistors with resistances $\boldsymbol{R_1, R_2, \ldots, R_n}$ connected in series, that means one after another (Fig. 29.1). An amount of charge passing through each resistor must be the same; otherwise, it would accumulate on the resistors or on the wire between them. Thus, the current $\boldsymbol{I}$ flowing through each resistor must be the same as well.

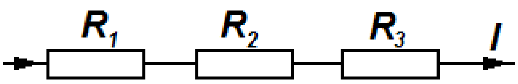


Fig. 29.1

We can determine the potential difference $\boldsymbol{V}$ across each resistor from equation (28.2): $\boldsymbol{V_1 = R_1 I, V_2 = R_2 I, \ldots, V_n = R_n I}$. The potential difference across the series combination of resistors is equal to the sum of potential differences across individual resistors:

$$\boldsymbol{V = V_1 + V_2 + \ldots + V_n}.$$

In terms of the net current through the combination $\boldsymbol{I}$ and the equivalent resistance $\boldsymbol{R_{ser}}$, according to equation (28.2), the last expression can be rewritten as the following:

$$\boldsymbol{IR_{ser} = IR_1 + IR_2 + \ldots + IR_n = I(R_1 + R_2 + \ldots + R_n)}.$$

Thus, the equivalent resistance $\boldsymbol{R_{ser}}$ for the combination of resistors in series is equal to the sum of resistances of individual resistors:

$$\boldsymbol{R_{ser} = R_1 + R_2 + \ldots + R_n = \sum_{i=1}^{n} R_i}. \tag{29.1}$$

From the last equation, one may conclude that the equivalent resistance of the combination of resistors in series is always greater than any individual resistance in the combination.

In case of the series combination of batteries, when unlike terminals connect to each other (Fig. 29.2), the equivalent emf of the combination is equal to the sum of the emf of individual batteries.

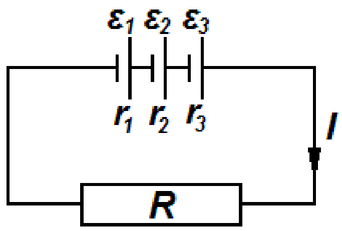


Fig. 29.2

The equivalent internal resistance of the series combination of batteries is calculated with equation (29.1).

A device called an *ammeter* is used to measure electric current. In the circuit diagram, the ammeter is indicated by the symbol **A** in a circle. To know the current in a particular part of the circuit, the ammeter should be connected in series. An ideal ammeter has zero internal resistance.

Parallel combination of resistors

Another combination of resistors with resistances $R_1, R_2, \ldots, R_n$ is the connection in parallel (see Fig. 29.3 as an example for three resistors). Conductive wires with zero resistance connect the left terminals of these resistors. Therefore, according to Ohm's, law they have the same potential at any point. A similar conclusion is valid for the right terminals. Thus, each resistor has the identical potential difference V across it, but the currents through them may be different: I_1 , $I_2, \ldots, I_n$.

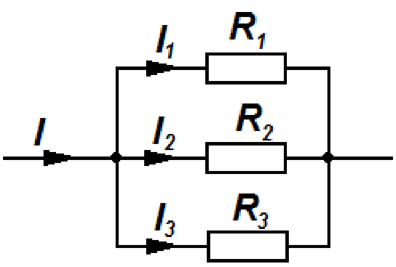


Fig. 29.3

The individual currents can be calculated using equation (28.2): $I_1 = V/R_1, I_2 = V/R_2, \ldots, I_n = V/R_n$. Based on the charge conservation law, the amount of charge passing through a parallel combination of resistors must be equal to the sum of the amount of charge passing through each resistor. So, the net current through the parallel combination of resistors can be represented as the sum of individual currents:

$$I = \frac{V}{R_{par}} = \frac{V}{R_1} + \frac{V}{R_2} + \ldots + \frac{V}{R_n} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}\right)V,$$

where R_{par} is the equivalent resistance of the resistors connected in parallel.

Consequently, the reciprocal of equivalent resistance is equal to the sum of the reciprocals of resistance for individual resistors:

$$\frac{1}{R_{par}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} = \sum_{i=1}^{n} \frac{1}{R_i}. \qquad (29.2)$$

From the last equation, one may conclude that the equivalent resistance of the combination of resistors in parallel is always lower than any individual resistance in the combination.

In case of a parallel combination of batteries with the same emf, when like terminals connect to each other (Fig. 29.4), the equivalent emf of the combination is equal to the individual emf.

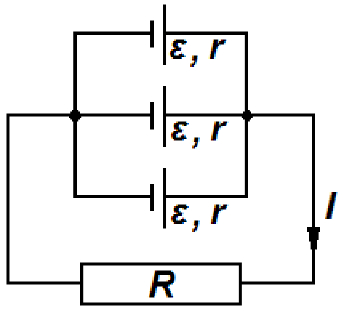


Fig. 29.4

The equivalent internal resistance of the parallel combination of batteries is calculated with equation (29.2).

A device called a *voltmeter* is used to measure potential difference (or voltage) between two points in the circuit. In the circuit diagram, the voltmeter is indicated by the symbol **V** in a circle. To know the voltage across a particular part of the circuit, the voltmeter should be connected in parallel to this part. An ideal voltmeter has infinite internal resistance.

Control questions

1. There are three identical resistors. Draw all possible diagrams of their combination.

2. What quantity is the same for each resistor in the series combination? Explain. What is the potential difference across the series combination of resistors?

3. What is the equivalent emf of a series combination of batteries? What is the equivalent internal resistance of this combination?

4. What quantity is the same for each resistor in the parallel combination? Explain. What is the net current through the parallel combination of resistors?

5. What is the equivalent emf of a parallel combination of identical batteries? What is the equivalent internal resistance of this combination?

6. If the resistance of the resistors in question 1 is equal to $\boldsymbol{R}$, determine the equivalent resistance for each diagram.

Problems

1. Three resistors with resistances $R_1 = 12\ \Omega$, $R_2 = 9\ \Omega$ and $R_3 = 3\ \Omega$ are connected in series. The voltage V across this combination is **120 V**. Find the current I in the circuit and the voltages $V_{1,2,3}$ across each resistor.
(Answer: $I = 5$ A; $V_1 = 60$ V; $V_2 = 45$ V; $V_3 = 15$ V)

2. A series combination of identical **8-Ω** light bulbs is designed to be plugged into a **220 V** socket. The maximum allowed current through a bulb is **0.5 A**. What minimum number N of the lightbulbs can be in this combination?
(Answer: $N = 55$)

3. The equivalent resistance of a parallel combination of two resistors is **1.2 kΩ**. What is the resistance R_2 of the second resistor if the first one has a resistance of **3 kΩ**?
(Answer: $R_2 = 2 \cdot 10^3\ \Omega$)

4. A ceiling lamp includes six light bulbs connected in parallel to a network of **220 V**. Each lightbulb has a resistance of **900 Ω**. Find the equivalent resistance $\boldsymbol{R}$ of the lamp and the net current $\boldsymbol{I}$ through the lamp switch.
(Answer: $\boldsymbol{R} = 150$ **Ω**; $\boldsymbol{I} = 1.47$ A)

5. A circuit consists of a battery with $\boldsymbol{\mathcal{E}} = \mathbf{12}$ **V**, $\boldsymbol{r} = \mathbf{1}$ **Ω** and a resistor with a resistance of **2 Ω**. What would be the change of the current in the circuit if the second identical battery is connected in parallel to the first one?
(Answer: 1.2 times increase)

6. Two resistors with resistances $\boldsymbol{R_1} = \mathbf{4}$ **Ω** and $\boldsymbol{R_2} = \mathbf{12}$ **Ω** connect in parallel to the series combination of four identical batteries ($\boldsymbol{\mathcal{E}} = \mathbf{1.5}$ **V**, $\boldsymbol{r} = \mathbf{0.5}$ **Ω**). Draw a diagram of the circuit and find the electric currents $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$ through each resistor.
(Answer: $\boldsymbol{I_1} = 0.9$ A; $\boldsymbol{I_2} = 0.3$ A)

Lesson 30

JOULE'S LAW

In Lesson 25 we discussed that the work $\boldsymbol{W}$ done by the electric field to move the amount of charge $\boldsymbol{\Delta q}$ through the circuit element with the potential difference $\boldsymbol{V}$ between its terminals (see equation (25.3)) is equal to $\boldsymbol{W = \Delta q V}$.

In this lesson we assume that the current in a circuit remains constant. The amount of charge $\boldsymbol{\Delta q}$, which passes through the conductor, can be represented from the definition of current $\boldsymbol{I}$ (see equation (28.1)): $\boldsymbol{\Delta q = I\Delta t}$.

Consequently, the work $\boldsymbol{W}$ done by the electric field can be written from the concept of current as well:

$$\boldsymbol{W = IV\Delta t}.$$

Since the current $\boldsymbol{I}$ through a conductor with resistance $\boldsymbol{R}$ is related to the potential difference $\boldsymbol{V}$ between its terminals (Ohm's law), the work $\boldsymbol{W}$ can be calculated as the following:

$$\boldsymbol{W = I^2 R\Delta t = \frac{V^2}{R}\Delta t}.$$

According to the energy conservation law, this work transformed into internal energy of the conductor with resistance, and this results in the conductor heating.

For practical purposes, the rate at which energy is delivered to a circuit element is of essential interest for us. Consequently, the concept of power can be applied (see Lesson 11). Dividing the last equation by $\boldsymbol{\Delta t}$, we can obtain an expression for the power $\boldsymbol{P}$ delivered by electric current $\boldsymbol{I}$ to a conductor with resistance $\boldsymbol{R}$:

$$\boldsymbol{P = I^2 R = \frac{V^2}{R}}. \tag{30.1}$$

Equation (30.1) is often called Joule's law and the corresponding energy is called Joule's heat.

We hope you remember that the SI unit of power is the *watt* (W = J/s). Taking into account equation (30.1), the unit watt can be represented as **W = V·A**. In everyday life, the power consumed by electrical devices is often marked in kilowatt: 1 kW = 10^3 W. By the way, energy companies prefer to measure electric energy in kilowatt-hours (kW·h) instead of the standard SI unit.

Control questions

1. Provide the definition of the current.

2. Write down the equation of the work done by the electric field to move the amount of charge $\boldsymbol{\Delta q}$ through the circuit element with the potential difference $\boldsymbol{V}$ between its terminals.

3. Formulate Ohm's law.

4. Provide the definition of power.

5. How to calculate the power delivered by electric current to a circuit element?

6. Express $\mathbf{1}$ kW·h in Megajoules.

Problems

1. A lamp is plugged into a **220 V** socket. A number of electrons passing through the filament of a light bulb during **1 min** is $\mathbf{6.82 \cdot 10^{19}}$. Define the power $\boldsymbol{P}$ of the lightbulb.
   (Answer: $\boldsymbol{P}$ = 40 W)

2. Two light bulbs connected in parallel are plugged to a network. The resistance of the first bulb is $\boldsymbol{R_1}$ **= 360 Ω** and of the second is $\boldsymbol{R_2}$ **=240 Ω**. Calculate the ratio of the powers $\boldsymbol{P_2/P_1}$ consumed by each light bulb. Which of two bulbs glows more brightly?
   (Answer: $\boldsymbol{P_2/P_1}$ = 1.5)

3. An electric kettle with a power of **2 kW** is plugged into a **220 V** socket. You want to boil **1.5 L** of water at temperature **20 °C** initially. What time $\boldsymbol{t}$ is it required to boil the water if the kettle's efficiency is **90 %**? The specific heat of water is **4.2 kJ/(kg·K)**. Water evaporation can be neglected.
   (Answer: $\boldsymbol{t}$ = 4 min 40 s)

4. A **60-W** light bulb is connected to a network with **240 V** voltage. Calculate the current $\boldsymbol{I}$ through the bulb and the resistance $\boldsymbol{R}$ of the bulb.
(Answer: $\boldsymbol{I}$ = 0.25 A, $\boldsymbol{R}$ = 480 **Ω**)

5. A resistor with **0.5 kΩ** resistance dissipates **80 W** when it is connected to a power supply. What is the potential difference $\boldsymbol{V}$ across the resistor and the current $\boldsymbol{I}$ through it?
(Answer: $\boldsymbol{V}$ = 200 V, I = 0.4 A)

6. There are two light bulbs and a power supply. If the bulbs are connected in parallel to the power supply, the first bulb dissipates the power $\boldsymbol{P_{1,par}}$ **= 60 W** and the second dissipates $\boldsymbol{P_{2,par}}$ **= 40 W**. What power does each bulb dissipate if they are connected in series to the power supply?
(Answer: $\boldsymbol{P_{1,ser}}$ = 9.6 W, $\boldsymbol{P_{2,ser}}$ = 14.4 W)

Self-training problem sets to recall Part I

Set 1

1. A helicopter flies **800 km** along the straight line, and then turns at the angle **90°** and flies **600 km** more. Find the path length s and the magnitude of the displacement $|\Delta\vec{r}|$ of the helicopter.

2. How much time t does a car need to increase its speed from υ_0 = **15 m/s** to υ = **25 m/s** if it is moving with constant acceleration a = **0.4 m/s²**?

3. Calculate the centripetal acceleration of a body on the equator due to the Earth rotation about its axis. The radius of Earth R_E is equal to **6400 km.** (*Hint*: duration of the earth's day is **24** hours).

4. What is the ratio n of the weights experienced by the same body on the Earth and the Moon? You will need the following data: the radius of the Moon is approximately **3.8** times less than the Earth's radius, and the mass of the Moon is **81** times less than the Earth's mass.

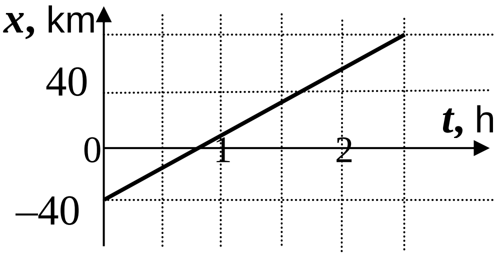
5. A tension force applied to a body of mass m = **40 kg** lifts it vertically upward at a height h = **15 m**. Find work W done by the force if the body moves with constant acceleration a = **2.2 m/s²**.

6. The work done by gravity force when an apple falls from 4 **m** height to the ground is equal to 8 **J**. Find the apple's mass m.

7. Find the sum R of two forces F_1 = **600 N** and F_2 = **800 N** acting on a body at the same point, if the angle between the vectors of forces F_1 and F_2 is α = **90°**.

Set 2

1. The graph of the point's position x as a function of time t is the following:



   From the graph, find the point's speed v and write down the equation of point's motion $x(t)$.

2. A body is dropped from rest from a height h = **180 m**. Find the time t of its falling and the speed v just before hitting the ground (g = **10 m/s²**).

3. A truck with mass m = **25 t** slows down to rest with constant acceleration a = **0.4 m/s²**. Find the magnitude of a net force F acting on the truck.

4. A vehicle of mass m = **15 t** moves with a speed of **18 km/h**. Find the momentum p of the vehicle.

5. A body of mass **4 kg**, initially at rest, is freely falling from some height. Find its kinetic energy K_E in a time of **2 s** after the beginning of motion.

6. A helicopter is moving upward with a constant speed of **4 m/s**. At a height of **75 m** a body falls out of it. What is the speed v of the body when it reaches the ground if we can neglect the air resistance?

7. What pressure P does a brick wall of height h = **10 m** exert on the ground if its average density is ρ_{brick} = $\mathbf{2 \cdot 10^3}$ **kg/m³**.

Set 3

1. A ball falls down on the floor from a height h_1 = **5 m** and then recoils to a height h_2 = **3 m**. Find the path length s and the magnitude of the displacement of the ball $|\Delta\vec{r}|$.

2. A train has a speed υ = **54 km/h** in a time t = **2,5 min** after the start. Find the acceleration a of the train.

3. A wheel of a diameter d = **40 cm** does N = **200** revolutions during the time t = **2 min**. Find the period of its rotation T, angular ω and linear υ speeds of a point on the rim of the wheel.

4. A vertical spring with constant of **75 N/m** is elongated by **2 cm** when a body is attached to it. What is the mass m of the body?

5. A body has moved through a distance of **15 m** along a horizontal surface under a force of **200 N**. The force is applied at an angle of **60º** to horizon. Calculate work W done by this force.

6. The constant of a spring is equal to **6 kN/m**. Calculate the elongation of initially nonstretched spring x if the work done to stretch it is **480 J**.

7. Three forces of an equal magnitude ($F_1 = F_2 = F_3 = F$) are applied to the same point of a solid body. Forces F_1 and F_3 form the angle **30°** with F_2. Define the magnitude of a resulting force R if these forces belong to the same plane.

Set 4

1. A car is moving during a time interval $t = 3$ **h** with a speed $v = 30$ **m/s**. Find out the path length s which the car has traveled during this time.

2. A body is freely falling from a height $h = 24.5$ **m**. Find the distance s passed by the body during the last second of its motion.

3. A body of mass $m = 4$ **kg**, falling in air (downward motion), at some instant has an acceleration $a = 9$ **m/s²**. Find a force of air resistance F_r, directed upward, if the gravity force F_g acting on the body is 4**0 N**.

4. A train of mass $m = 3000$ **t**, moving along a straight line, increases its speed from $v_1 = 18$ **km/h** to $v_2 = 54$ **km/h**. Calculate the magnitude of change of train's momentum Δp.

5. Calculate the kinetic energy K_E of a spaceship at its orbital motion with a velocity of **9 km/s**, if a mass of the ship is **8 t**?

6. A stone is thrown horizontally from a height $h = 1.05$ **m** above the earth surface. Initial speed of the stone is $v_0 = 10$ **m/s**. Find out its speed v when the stone hits the ground.

7. At what depth h is a hydrostatical water pressure **3** times greater than the normal atmospheric pressure if the density of water is $\rho_w = 1\cdot10^3$ **kg/m³** ?

Set 5

1. A body moves from the point with coordinates x_1 = **1 m**, y_1 = **3 m** to the point with coordinates x_2 = **5 m**, y_2 = **–2 m**. Make a graph; find the direction and magnitude of the displacement and its projections on the coordinate axes.

2. The speed of a train has decreased from υ_0 = **54 km/h** to υ = **36 km/h** during a time interval t = **10 s**. Derive an equation of the velocity υ as a function of time t and draw the velocity-time (υ-t) graph.

3. Linear speed υ_A of point A on the rim of a rotating wheel is **3** times greater than the linear speed υ_B of point B. Point B is **30 cm** closer to the wheel axis than point A. Find the radius R of the wheel.

4. A body of **70 N** weigh is at rest on an incline that is at angle **45°** to horizon. What is the magnitude of static friction force F_{fr}? Draw a diagram with all forces acting on a body.

5. A crane lifts a load of mass **1 t** with a constant speed of **20 m/min**. Lifting time is **60 s**. Find out work W done by the gravity force.

6. A body of mass **4 kg** falls from a height of **5 m**. Find out the work done by gravity force.

7. A body of mass **5 kg** is at rest on an incline at an angle of **60°** to horizon. Find a static friction force F acting on the body.

Set 6

1. A distance between points **A** and **B** is equal to **160 km**. From points **A** and **B**, two cars start moving toward each other simultaneously. Their speeds are: $\boldsymbol{v}_A$ = **50 km/h** and $\boldsymbol{v}_B$ = **30 km/h**. Find a) the time $\boldsymbol{t}$ when they meet; b) the distance $\boldsymbol{x}$ between the point of their meeting and point **A**.

2. A body is thrown vertically upward at an initial speed $\boldsymbol{v}_0$ = **40 m/s**. What maximum height $\boldsymbol{H}_{\mathbf{max}}$ will it reach? Find the total time $\boldsymbol{t}$ of the body's motion.

3. A body with mass $\boldsymbol{m}$ = **0.5 kg** is initially at rest. Then, under the action of a constant force $\boldsymbol{F}$ it moves through a distance $\boldsymbol{s}$ = **27 m** during the time $\boldsymbol{t}$ = **3 s**. Find the force $\boldsymbol{F}$.

4. Motion of a particle is described by the equation $\boldsymbol{x} = \mathbf{4 - 6}\boldsymbol{t} + \mathbf{3}\boldsymbol{t}^{\mathbf{2}}$ ($\boldsymbol{x}$ measured in meters and $\boldsymbol{t}$ – in seconds). Mass of a particle is **5 kg**. Find out the momentum $\boldsymbol{p}$ of the particle at the instant $\boldsymbol{t}$ = **3 s**.

5. The magnitude of a body's momentum $\boldsymbol{p}$ is **5 kg·m/s**, and its kinetic energy $\boldsymbol{K_E}$ is **12.5 J**. Find the mass $\boldsymbol{m}$ and speed $\boldsymbol{v}$ of the body.

6. At what height $\boldsymbol{h}$ was an apple if it hit the ground with a speed of **10 m/s**?

7. There is a mercury column of **5 cm** height in a high cylindrical vessel. The same volume of water is added to the vessel above the mercury. Find out the pressure $\boldsymbol{P}$ on the bottom of the vessel. The densities of water and mercury are $\boldsymbol{\rho_w} = \mathbf{1 \cdot 10^3}$ **kg/m³** and $\boldsymbol{\rho_m} = \mathbf{13.6 \cdot 10^3}$ **kg/m³**, respectively.

Set 7

1. Motion of a particle is defined by equations $x = 1 + t$ and $y = 4 + 2t$. Derive an equation of the path $y(x)$. Make a graph of the path.

2. Moving with acceleration $a = 0.5$ m/s² a sportsman has passed the distance $s = 100$ m during a time interval $t = 10$ s. What speed does he have at the beginning and the end of this distance?

3. Find an angular speed of the second hand on a clock.

4. A hockey puck of 0.25 kg is sliding along a horizontal icy surface. A kinetic friction coefficient is 0.04. Calculate the friction force F_{fr} acting on the puck.

5. An elevator of mass $m = 15$ t moves upward during a time interval $t = 5$ s with an acceleration $a = 0.4$ m/s². What work W does gravity force perform on the elevator?

6. A small sphere of a mass 200 g is hanging on a thread of 1 m length. Calculate the potential energy due to gravity U_g if the thread is deflected from vertical through an angle of 60°. We consider the position of equilibrium as the position with zero potential energy.

7. The length of an inclined rod **AB** in the supporting arm **ABC** is **1 m** and of a horizontal rod **BC** is **0.8 m**. A load of mass **15 kg** hangs at point **B** of the supporting arm. Define the force F_{AB} that squeezes rod **AB** and the force F_{BC} that stretches rod **BC**. Consider weightless rods.

Set 8

1. A bamboo grows with a speed of **0.005 mm/s**. By how many centimeters will it grow in **20 hours** (ΔL)?

2. A ball is projected horizontally from a height h = **5 m**. Its horizontal range is L = **9 m**. Find the initial v_0 and final v speed of the ball.

3. Under the force F = **4 mN** a ball moves through a distance s = **0.20 m** during the first second of its motion. Find the mass m of the ball.

4. A cannonball of a mass **30 kg** is shot with a speed of **70 m/s** from a cannon of mass M = **9 t**. Find out a speed of the gun u just after the shot.

5. A ball of mass **0.6 kg** is thrown horizontally with initial speed of **4 m/s** from some height. Find the total work done by the net force W_{tot} if ball's final speed is **6 m/s**.

6. A stone is thrown with a speed of **30 m/s** at an angle α = **30°** to horizon. Find out the maximum height h_{max} that the stone reached.

7. What will a height h of a liquid column at the normal atmospheric pressure be if the barometer is filled with water instead of mercury?

Set 9

1. A motor boat moving on the lake due northeast, passed the distance of **5 km**; and then moving due north, it passed **3 km** more. Find the boat's displacement and its magnitude. Assume that positive **X**-axis is due east and positive **Y**-axis is due north.

2. Acceleration of a car is $\boldsymbol{a}$ = **2 m/s²** and its initial speed is $\boldsymbol{v_0}$ = **3 m/s**. The car moves during a time interval $\boldsymbol{t}$ = **6 s**. What distance $\boldsymbol{s_6}$ has the car passed during the sixth second?

3. On the Big Ben tower clock the length of a second hand is **4.3 m**. What is the linear speed $\boldsymbol{v}$ of the hand's end?

4. Find out the acceleration due to gravity $\boldsymbol{g'}$ near the Mars' surface. The mass of Mars $\boldsymbol{M_M}$ = $\mathbf{6 \cdot 10^{23}}$ **kg**, the radius of Mars $\boldsymbol{R_M}$ = **3300 km**.

5. A car of mass $\boldsymbol{m}$ = **1 t** starts motion along a horizontal road and has passed a distance $\boldsymbol{s}$ = **20 m** during a time $\boldsymbol{t}$ = **2 s**. Find out power of an engine $\boldsymbol{P}$ if the car moves with constant acceleration (resistance to motion can be neglected).

6. Define potential energy $\boldsymbol{U_{el}}$ of a spring that was stretched by a force $\boldsymbol{F}$ = **3000 N** if its elongation is $\boldsymbol{x}$ = **40 mm**.

7. A uniform rod of length $\boldsymbol{L}$ = **0.5 m** and mass $\boldsymbol{m}$ = **2 kg** is attached at one end to a frictionless pivot and is free to rotate about the pivot in the vertical plane. The rod is released from rest in the horizontal position. What is the magnitude of an initial torque τ of the gravity force about an axis through the pivot?

Set 10

1. A subway escalator is at an angle $\alpha = 30°$ to horizon. A speed of the escalator is $\upsilon = 2$ **m/s**. To what height h does the escalator lift a man during a time interval $t = 0.4$ **min**?
2. A Red Cross airplane is flying horizontally with the speed $\upsilon = 252$ **km/h** at the height $h = 0.49$ **km**. A pilot drops a parcel with food for refugees' camp. How much time t is required for the parcel to reach the earth? At what distance L from the camp should a pilot drop a parcel to be picked up in the camp exactly?
3. A car of mass $m = 1.2$ **t** starts motion with constant acceleration and during the time interval $t = 20$ **s** moves through a distance $s = 100$ **m**. Find the car engine force F if a constant force F_r resisting to the car's motion is equal to 0.4 **kN**.
4. A man of a mass $M = 80$ **kg** stands on an ice and catches a ball of a mass $m = 0.4$ **kg**. A speed of the ball just before catching was $\upsilon = 25$ **m/s**. Find a speed V of the man with the ball in hands just after catching.
5. Calculate the momentum p of a load with kinetic energy $K_E = 25$ **J** if its mass is 8 **kg**.
6. An airplane of mass $2.5 \cdot 10^3$ **kg** is flying with a speed of 60 **m/s** at a height of 450 **m**. Then, to land the airplane a pilot turns off the engine. Determine the work W done by air resistance if the airplane reaches the ground with a speed of 20 **m/s**.
7. Areas of pistons in a hydraulic press are $A_1 = 1.5$ **cm²** and $A_2 = 300$ **cm²**. Work done to move a small piston through a distance $h_1 = 25$ **cm** is $W_1 = 125$ **J**. Find a force F_2 applied to the large piston and a height h_2 of its rising.

Set 11

1. A particle moves along a straight line according to equation $z = 6t^2$, where z is in meters and t is in seconds. Find the pass length s_{12} of the particle's motion for the time interval from $t_1 = 1$ **s** to $t_2 = 4$ **s**.

2. A truck covers **50 m** in **10 s** while smoothly slowing down to a final speed of **2 m/s**. Find the acceleration a and initial speed v_0 of the truck.

3. A car is moving with a speed $v = 36$ **km/h** along a circular road of radius $R = 50$ **m**. Find the value of the car's centripetal acceleration a_c.

4. Due to a force of **150 N** a spring is stretched by **3 cm**. Calculate the spring constant k.

5. A sportsman lifts a **600 N** weight by a height of **1.6 m** in a time of **4 s**. Determine his muscle power P.

6. The work W done by elastic force upon initially nonstretched spring is equal to **450 J**. Find out the spring constant k if an elongation is **15 cm**.

7. A uniform ladder weighing **60 N** rests against a frictionless wall. The ladder makes a **60°** angle with the horizontal. Find the horizontal F_H and vertical F_V forces the ground exerts on the base of the ladder.

Set 12

1. The distance between towns A and B along the river is **150 km**. A motor boat passes this distance with the stream of the river for **2 hour**. The same boat travels back (against the stream) in **2.5 hours**. Find out the speed of the river stream $\boldsymbol{\upsilon_r}$. (*Hint*: if the engine of the boat is turned off, the boat moves with a speed of the river stream.)

2. A body is thrown vertically upward from the ground at an initial speed $\boldsymbol{\upsilon_0}$. Show that the time of its downward motion is equal to the time of its upward motion.

3. A boy starts pulling a toy truck of mass $\boldsymbol{m} = \mathbf{0.6}$ **kg** with the force $\boldsymbol{F_p} = \mathbf{0.26}$ **N**. A constant resistant force $\boldsymbol{F_r}$, opposing the motion, is equal to **0.02 N**. Find a distance $\boldsymbol{s}$ the truck moves during the time $\boldsymbol{t} = \mathbf{3}$ **s**.

4. Two bodies with different masses move toward each other with the same speeds of **4 m/s**. After collision they begin to move together as one body with a speed of **2 m/s**. Find out the ratio of the bodies' masses $\boldsymbol{m_1/m_2}$.

5. The total work done by the net force $\boldsymbol{W_{tot}}$ over a body of mass **2 kg** is equal to **25 J**. Find the final speed $\boldsymbol{\upsilon}$ and momentum $\boldsymbol{p}$ of the body if it was initially at rest.

6. A body with mass of **400 g** is attached to the spring and able to move along a horizontal frictionless surface. What is the body's speed $\boldsymbol{\upsilon}$ when it passes an equilibrium position if the spring was stretched by **2 cm**? Spring constant is equal to **400 N/m**.

7. A large piston of a hydraulic automobile lift is **0.6 m** in diameter. What force $\boldsymbol{F_1}$ must be applied to the small piston of **3 cm** in diameter to lift a car with a mass of **1600 kg**?

APPENDIX

The seven defining constants of the SI base units

$\Delta\nu_{Cs}$	=	$9192631770\ \mathrm{s^{-1}}$	The ground state hyperfine structure transition frequency of the caesium-133 atom
c	=	$299792458\ \mathrm{m{\cdot}s^{-1}}$	The speed of light in vacuum
h	=	$6.62607015\times10^{-34}\ \mathrm{kg{\cdot}m^2{\cdot}s^{-1}}$	The Planck constant
e	=	$1.602176634\times10^{-19}\ \mathrm{A{\cdot}s}$	The elementary charge
k	=	$1.380649\times10^{-23}\ \mathrm{kg{\cdot}m^2{\cdot}K^{-1}{\cdot}s^{-2}}$	The Boltzmann constant
N_A	=	$6.02214076\times10^{23}\ \mathrm{mol^{-1}}$	The Avogadro constant
K_{cd}	=	$683\ \mathrm{cd{\cdot}sr{\cdot}s^3{\cdot}kg^{-1}{\cdot}m^{-2}}$	The luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz

Some properties of the Earth

L_E	=	$40\,000\,000$ m	Length of the Earth equator
R_E	=	$L_E/2\pi \approx 6.4\times10^6$ m	Average radius of the Earth
T_E	=	$86\,400$ s = 1 Day = 24 h	Period of Earth revolution about its own axis of rotation
M_E	=	6.0×10^{24} kg	Mass of the Earth
g	=	$9.80665\ \mathrm{m/s^2}$	Acceleration due to gravity near the surface of the Earth
R_O	=	1.5×10^{11} m	Average radius of the orbit of the Earth
T_O	=	365.25 Day = 3.15576×10^7 s	Period of the Earth's orbital motion, the solar year

MATHEMATICAL

Algebra

$$\boldsymbol{a^n \cdot a^m = a^{n+m};\qquad \frac{a^n}{a^m} = a^{n-m};\qquad a^0 = 1;\quad a^{-n} = \frac{1}{a^n};}$$

$$\boldsymbol{(ab)^n = a^n b^n;\qquad (a^n)^m = a^{nm};\quad \sqrt[m]{a^n} = a^{\frac{n}{m}};\qquad \sqrt[n]{a} = a^{\frac{1}{n}}}$$

$$\boldsymbol{(a \pm b)^2 = a^2 \pm 2ab + b^2;\qquad a^2 - b^2 = (a-b)(a+b)}$$

$$\boldsymbol{ax^2 + bx + c = 0;\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Trigonometry

$$\boldsymbol{\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)}$$

$$\boldsymbol{\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)}$$

$$\boldsymbol{\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta}$$

$$\boldsymbol{\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta}$$

$$\boldsymbol{\sin^2 x + \cos^2 x = 1}$$

Geometry

Pythagorean Theorem: $\boldsymbol{a^2 + b^2 = c^2, a}$, $\boldsymbol{b}$ are cathects, $\boldsymbol{c}$ – hypotenuse

Length of the circumference of radius ***r*:** $\boldsymbol{L_{Circ} = 2\pi r}$

Area enclosed by the circle of radius ***r*:** $\boldsymbol{A_{Circ} = \pi r^2}$

Area of a triangle with basis ***b*** and height ***h*:** $\boldsymbol{A_{Tr} = \frac{1}{2} bh}$

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