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## THEORETICAL ANALYSIS OF THE FORMING PROCESS OF CLOSED DIE FORGING WITH FLASH AND OPTIMIZATION DESIGN METHOD OF FLASH GUTTER

The subject of this article is the forming process of closed die forging with flashes and the optimal design of the flash gutter dimensions. The goal is to conduct an in-depth theoretical analysis of the forming process of closed die forging with flash and to research the optimal design of the flash gutter dimension, aiming to improve the scientificity and efficiency of the forging process, guide the optimization of process parameters in actual production, extend the service life of dies, reduce material waste, and enhance product quality and production efficiency. The tasks were to establish an accurate mathematical model for closed die forging with flash to analyze metal plastic deformation and stress distribution, to conduct in-depth research on key factors such as stress distribution, position of the neutral plane, and flash gutter design during the forming process, and to validate the mathematical model through finite element simulation using DEFORM-2D software. The methods used include theoretical analysis, mathematical modeling, and finite element simulation validation. The theoretical analysis provides a foundation for understanding the forming process and stress distribution, whereas the mathematical model allows for quantitative analysis. Validation of the finite element simulation provides a means to test and refine the theoretical analysis and mathematical model. The following results were obtained: the existing mathematical model underestimates the height of the main deformation zone, which results in an unreasonable flash gutter design. After verification and correction, the error in the optimized mathematical model did not exceed 10 %, and the flash amount was significantly reduced. Additionally, a stress analysis of difficult-to-fill cavity positions revealed that the entrance radius of the cavity significantly affects the final filling. Conclusions. The scientific novelty of the results obtained is as follows: A mathematical model of closed die forging with flash was established to analyze the metal plastic deformation and stress distribution, providing theoretical support for die design optimization, forging quality improvement, cost reduction, and productivity improvement. Using the Deform-2D finite element simulation, the optimal design criterion for the flash was refined, resulting in a substantial reduction in the flash amount and material savings.

**Keywords:** closed die forging with flash; theoretical analysis; finite element simulation; stress distribution; mathematical model; flash gutter.

### 1. Introduction

The theoretical analysis of closed die forging with flash holds significant importance in enhancing the scientificity and economy of the forging process. It serves as a guide for optimizing process parameters in actual production, thereby improving the service life of dies, minimizing material waste, and elevating product quality and production efficiency [1]. During the deformation process of closed die forging with flash, the rational design of flash gutter is crucial for ensuring forging quality, boosting production efficiency, and reducing material waste. Proper flash gutter dimension ensures that the forging completely fills the dies cavity during the forging process, preventing defects and thus enhancing the quality and precision of the forging. Additionally, by optimizing flash gutter dimension, deformation work can be minimized, reducing energy consumption and enhancing production efficiency [2].

Wang et al. [3] conducted a theoretical analysis of the deformation process in closed die forging with flash and proposed an optimized design criterion for flash. Sleenckx [4] in his review on flash design rules in closed die forgings, reviewed and discussed 16 different empirical formulas for flash design found in existing literature. He evaluated their accuracy by comparing them with the actual flash dimensions of 10 existing axially symmetric industrial forgings. Sheikhabaee [5] utilized 2D finite element simulations and response surface methodology to obtain the optimal dimensions for the die and preform, including the width and thickness of the flash gutter and the amount of flash at each major cross-section. Despite the abundance of theoretical analyses and empirical formulas for flash design in closed die forging with flash, none of these have good universality. Therefore, this article adopts a research approach that combines mathematical modeling with finite element simulation validation, enhancing the universality and accuracy of the



mathematical models, optimizing the flash design criterion, and providing a reliable theoretical basis for practical production.

## 2. Mathematical model of closed die forging with flash forming process

The primary purpose of establishing a mathematical model for closed die forging with flash is to investigate and analyze the plastic deformation behavior of metals during the forging process, as well as the distribution of stress. By constructing a mathematical model, we can gain a deeper understanding of the flow patterns and stress-strain states of metals during closed die forging with flash. This, in turn, provides theoretical foundations and technical support for optimizing dies design, improving forging quality, reducing production costs, and enhancing production efficiency.

The deformation process in closed die forging with flash consists of three main stages: upsetting, cavity filling, and final forging. The upsetting stage begins when the top die comes into contact with the end surface of the billet and continues until the lateral surface of the billet touches the side walls of the dies cavity. The cavity filling stage follows immediately after and continues until the dies cavity is completely filled. Finally, the final forging stage begins at the end of the cavity filling stage and continues until the top and bottom dies are fully engaged. The stress distribution calculation formula of the flat anvil upsetting model is as follows [6]:

$$\sigma_z = \left[ 1 + \frac{2\mu(R-x)}{h} \right] \sigma_s, \quad (1)$$

$$n_z = \frac{\sigma_z}{\sigma_s}, \quad (2)$$

where,  $\sigma_z$  – Stress perpendicular to the parting surface,

$\mu$  – Friction factor on the cavity wall,

$R$  – Radius of forgings,

$h$  – height of the billet,

$\sigma_s$  – Yield stress of the material,

$n_z$  – Stress distribution coefficient,

$x$  – the distance between the stress point and the central axis of the forgings.

When analyzing the deformation process of the metal within the flow neutral plane using the closed-die extrusion model, one can refer to the principles and formulas presented in reference [3]. Similarly, the deformation process of the metal outside the flow neutral plane can be analyzed by referring to the methods and conclusions in reference [6]. Since references [3] and [6] have conducted exhaustive theoretical analysis and experimental research on the relevant issues, this paper directly adopts the relevant conclusions and formulas.

### 2.1. Stress Distribution During the Upsetting stage

The forming types of upsetting can be classified into press-in upsetting, upsetting-type forming upsetting, and punching-type forming upsetting. This paper mainly focuses on the distribution of deformation forces during press-in upsetting.  $\mu_0$ –the friction factor at the flash gutter section.

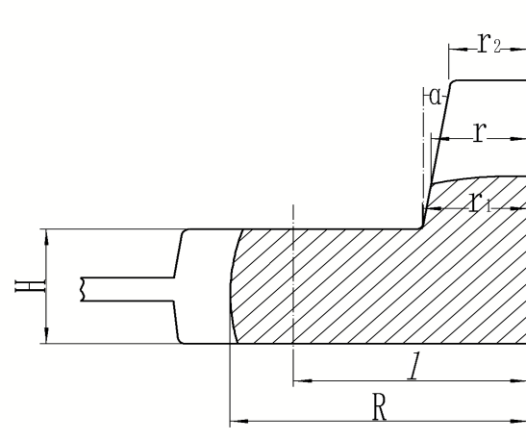


Fig. 1. Upsetting Stage

In the process of press-in upsetting, it can be divided into three stress zones, as follows:

$x \leq r$  :

$$n_z = 2(1 + \mu) \ln \frac{1}{r_2} + 2 \left( 1 + \frac{\mu_0}{\text{tg } \alpha} \right) \ln \frac{r_2}{x} + 2\mu_0 \frac{2x}{r_2}, \quad (3)$$

$r \leq x \leq l$  :

$$n_z = 2(1 + \mu) \ln \frac{1}{r_2} + 2 \left( 1 + \frac{\mu_0}{\text{tg } \alpha} \right) \ln \frac{r_2}{r} + 2\mu_0 + 2\mu \frac{x - r_2}{H}; \quad (4)$$

$l \leq x \leq R$  :

$$n_z = 2(1 + \mu) \ln \frac{r}{r_2} + 2 \left( 1 + \frac{\mu_0}{\text{tg } \alpha} \right) \ln \frac{r_2}{r} + 2\mu_0 + 2\mu \frac{x - r_2}{H}. \quad (5)$$

Regarding the determination of the flow neutral plane:

$$2(1 + \mu) \ln \frac{1}{r_2} + 2 \left( 1 + \frac{\mu_0}{\text{tg } \alpha} \right) \ln \frac{r_2}{r} + 2\mu_0 + 2\mu \frac{l - r_2}{H} = 1 + 2\mu \frac{R - l}{H}. \quad (6)$$

## 2.2. Distribution of Stress During cavity filling stage

During the forming process of the cavity filling stage, the forging can be analyzed by dividing it into three stress zones (considering a fillet of 2 mm for the most difficult-to-fill cavity). These zones are as follows:

$x \leq r_1$  :

$$n_z = 2(1 + \mu) \ln \frac{l}{r_1} + 2 \left( 1 + \frac{\mu_0}{\text{tg } \alpha} \right) \ln \frac{r_2}{r_1} + 1 + 1.5 \ln \frac{r_1}{2} + 2\mu_0 \frac{r_2 - x}{r_2}, \quad (7)$$

$r_1 \leq x \leq l$  :

$$n_z = 2(1 + \mu) \ln \frac{l}{r_2} + 2 \left( 1 + \frac{\mu_0}{\text{tg } \alpha} \right) \ln \frac{r_2}{r_1} + 1 + 1.5 \ln \frac{r_1}{2} + 2\mu + 2\mu \frac{x - r_2}{h}, \quad (8)$$

$l \leq x \leq R$  :

$$n_z = 1.5 + 2\mu \frac{b_0}{h_0} + \frac{R - x}{h}, \quad (9)$$

$R \leq x \leq R + b_0$  :

$$n_z = 1.5 + 2\mu \frac{R + b_0 - x}{h_0}. \quad (10)$$

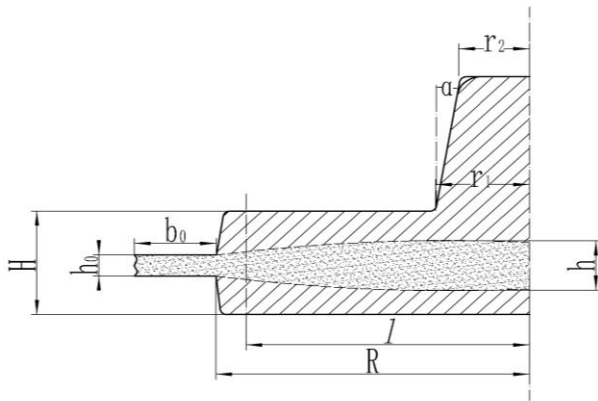


Fig. 2. Cavity filling stage

Regarding the determination of the flow neutral plane:

$$2(1 + \mu) \ln \frac{l}{r_2} + 2 \left( 1 + \frac{\mu_0}{\text{tg } \alpha} \right) \ln \frac{r_2}{r_1} + 1 + 1.5 \ln \frac{r_1}{2} + 2\mu \frac{R - l}{h} = 2\mu_0 \frac{b_0}{h_0} + \frac{R - l}{h}. \quad (11)$$

## 2.3. Distribution of Stress During the Final Forging Stage

During the forming process of the final forging stage, the neutral plane  $l = 0$  mm. Therefore, the forging

can be analyzed by dividing it into two stress zones, as follows:

$x \leq R$ :

$$n_z = 1 + \frac{2\mu_0 b_0}{r_1} + \frac{R - x}{2.9h_0}, \quad (12)$$

$R \leq x \leq R + b_0$ :

$$n_z = 1 + \frac{2\mu_0}{h_0} (R + b_0 - x). \quad (13)$$

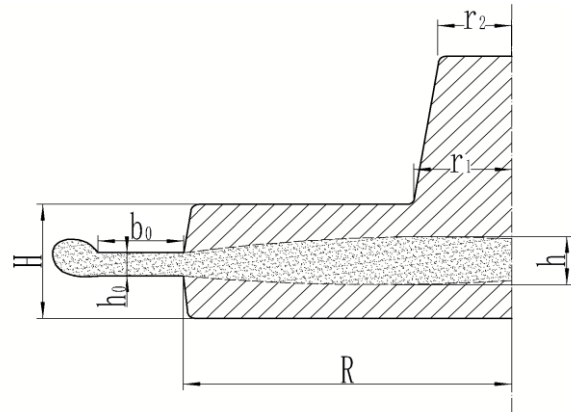


Fig. 3. Final forging stage

## 3. Theoretical design criteria for optimization of flash gutter

The height  $h$  and width  $b$  of the flash gutter in closed die forging with flash are critical parameters for the design of the final forging die. For decades, scholars in the field have exhausted both human and material resources in attempts to find methods for optimizing the design of flash gutter dimensions. However, they have not been successful, leading to the continued use of outdated empirical design methods [7].

In 1996, Wang et al. [3] proposed a theoretical optimization design method for the flash in closed die forging with flash, which established a correspondence between the shape and size of the forging and the flash at the end of the cavity filling process, as shown in Equation (14). The sum of the terms on the left side of the equation represents the stress required for the most difficult-to-fill part of the cavity to fill completely; the sum of the terms on the right side represents the stress generated by the flash resistance at the center of the most difficult-to-fill part of the cavity. By setting the width  $b$  of the flash gutter in the dies equal to  $b_0$  in Equation (14) and making the sum of the bridge height  $h$  and the compression amount  $\Delta H_3$  during the final forging stage equal to  $h_0$  in Equation (14), it can be ensured that the relationship expressed in Equation (14) holds true when the forging is completely filled.

$$2 \left( 1 + \frac{\mu_0}{\text{tg } \alpha} \right) \ln \frac{r_2}{r_1} - 0.8 + 1.5 \ln \frac{r_1}{2} + 2\mu \frac{R - 0.5r_2}{H} = 2\mu_0 \frac{b_0}{h_0} + \frac{R - 0.5r_2}{2.9h_0}. \quad (14)$$

After verification, it was found that Wang et al. [3] pointed out that the height evaluation of the deformation zone in the formula was not accurate enough, and the value of  $2.9h_0$  in Equation (14) should be corrected to  $5.8h_0$ . After correction, we get Equation (15):

$$2 \left( 1 + \frac{\mu_0}{\text{tg } \alpha} \right) \ln \frac{r_2}{r_1} - 0.8 + 1.5 \ln \frac{r_1}{2} + 2\mu \frac{R - 0.5r_2}{H} = 2\mu_0 \frac{b_0}{h_0} + \frac{R - 0.5r_2}{5.8h_0}. \quad (15)$$

#### 4. Finite Element Simulation Verification

DEFORM-2D is a high-precision finite element simulation tool specifically designed for analyzing complex metal forming processes. It boasts strong flexibility and a user-friendly graphical interface. It is widely used in the design and optimization of metal forming processes such as forging, extrusion, and drawing.

With the rapid development of computers and finite element simulation software, the use of simulation technology can be realized in any space and time under the displacement field, velocity field, strain field, stress field, temperature field and other field variables of the accurate analysis, the results obtained by the detailed and rich, to make up for the experimental and theoretical analysis of the shortcomings.

##### 4.1. FE Model

To save computation time, a 1/2 model is employed for the simulation, as shown in the Fig. 4. The billet

material is Ti-6Al-4V, the billet and dies temperature is both set at  $900^\circ\text{C}$ , the pressing speed is  $10\text{mm/s}$ , and the friction factor is 0.3. To better investigate the stress distribution, the point tracking function of Deform software is utilized, with 11 stress monitoring points set on the parting surface to facilitate the analysis of stress distribution.

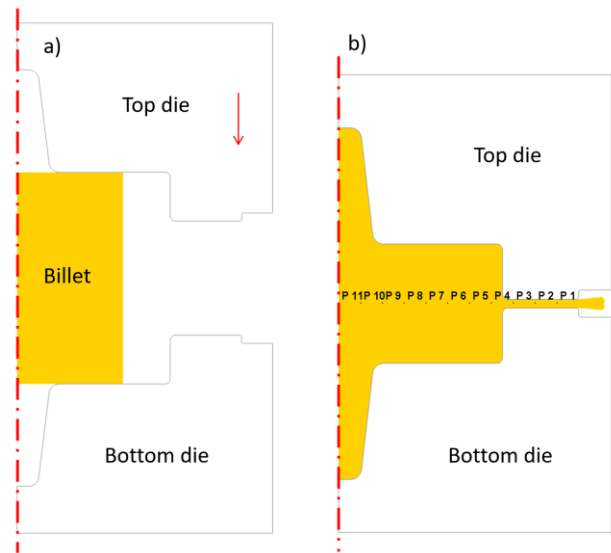


Fig. 4. 2D FE model: a – The model before forging, b – The model after forging

##### 4.2. Stress Distribution Analysis

Since the cavity filling stage has the greatest impact on the forming of forgings, the focus is on analyzing the stress distribution during this stage. The results show that the Y-axis stress exhibits a trend of first increasing and then decreasing along the axial direction of the forging towards the flash zones, as shown in the Fig. 5. The maximum stress point P4, occurs near the neutral plane, with a value of  $951\text{ MPa}$ .

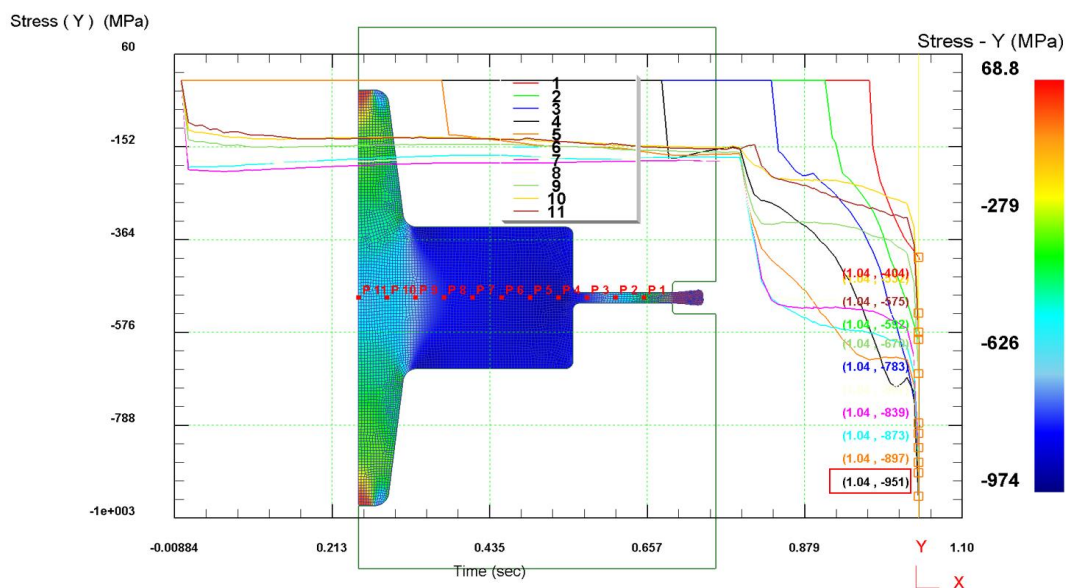


Fig. 5 Cavity filling stage @ stroke 10.4 mm

Through comparison, it is found that the maximum error between the value of mathematical model and the value of FE model is 11.6 %, occurring at point P11 on the central axis of the forging, as shown in the Tab. 1. The reason for this error is that Equation (7) neglects the influence of the entry radius at the most difficult-to-fill cavity. The errors for other stress analysis points in the main deformation zone are all within 10 %, and are basically higher than the simulation values. Therefore, this has certain guiding significance for actual production.

Table 1

Comparison of Y-Axis Stress

No.	Mathematical model /Mpa	FE model /Mpa	Error /%
P1	429	404	6.19
P2	559	592	-5.57
P3	756	783	-3.45
P4	1043	951	9.67
P5	976	897	8.67
P6	907	873	3.87
P7	838	839	-0.03
P8	770	807	-4.58
P9	701	670	4.63
P10	551	532	8.65
P11	661	575	11.30

### 4.3. Neutral Plane Analysis

The position of the neutral plane can be determined by Equation (11) as  $l=14.5\text{mm}$ . As shown in the Fig. 6, the actual neutral plane appears near point P4, i.e.,  $l\approx 14\text{mm}$ , with an error of 3.6 %. It is also found that the entry radius of the cavity that is most difficult-to-fill has a significant impact on cavity filling. When the entry radius is too small, the top surface of the rib is filled last, and in this case, the stress value is on the high side. When the entry radius is sufficiently large, the top corner of the rib is filled last, and the stress value is more reasonable at this time.

### 4.4. Analysis of the Main Deformation Zone During Cavity Filling Stage

Through the analysis of Y-axis strain, it is found that during the cavity filling stage, the height range of the main deformation zone is between 3.5...3.9 mm, as shown in the Fig. 7. This does **NOT** align with  $2.9h_0$  in the formula provided in reference. It should be corrected to  $5.8h_0$ .

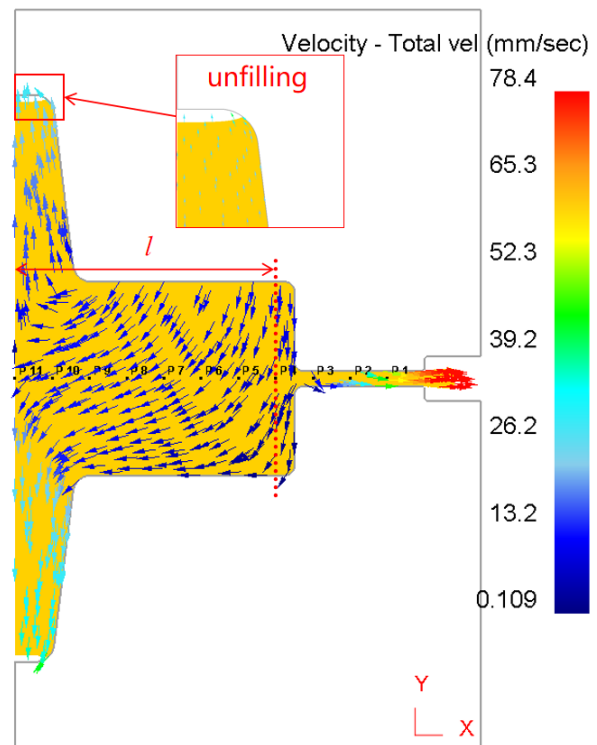


Fig. 6. Cavity filling stage @ stroke 10.3 mm

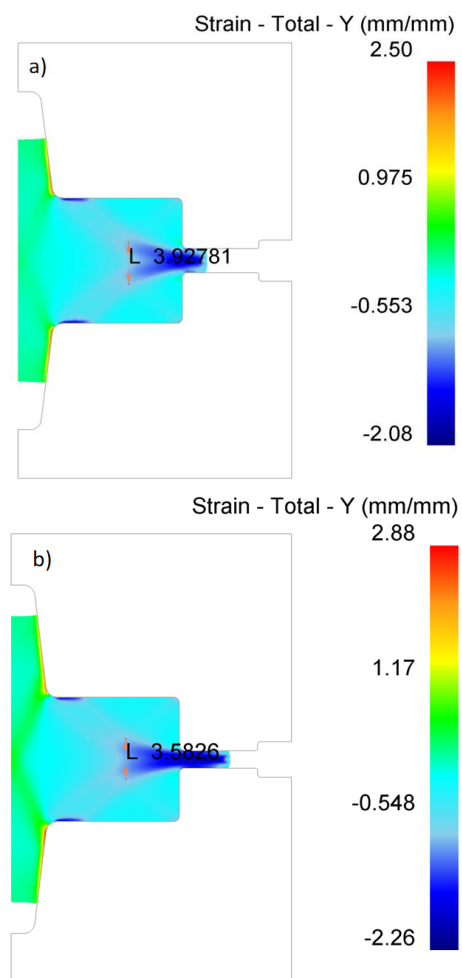


Fig. 7 Analysis of the main deformation zone: a – @ stroke 8.9 mm, b – @ stroke 9.6 mm



#### 4.4. Flash Analysis

The flash gutter height  $h_0=1.37$  mm was initially calculated using Equation (14). However, the simulation results revealed that this scheme resulted in excessive flash, as shown in the Fig. 8. Upon analysis, it was determined that the height of the main deformation zone utilized in Equation (14) was underestimated. Specifically, the height of the main deformation zone should be adjusted from  $2.9h_0$  to  $5.8h_0$  to derive the corrected Equation (15).

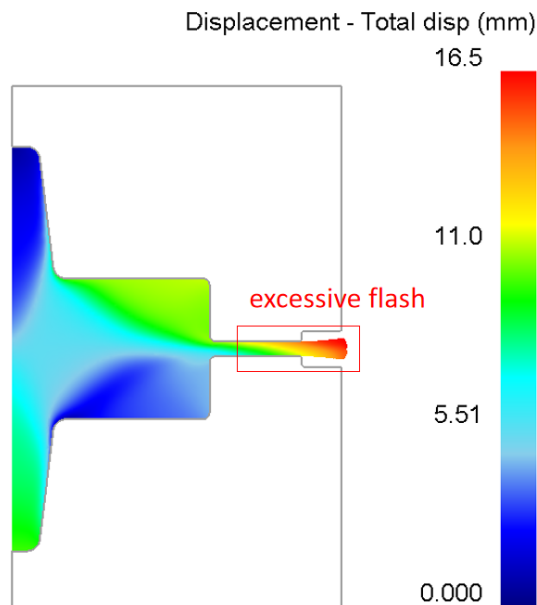


Fig. 8. Flash analysis after forging @  $h_0=1.37$  mm

Utilizing the revised formula (15), the flash gutter height was recalculated to be  $h_0=0.67$  mm. The rationality of the corrected approach was then verified through FE simulation, which demonstrated a significant reduction in the amount of flash produced, as shown in the Fig. 9. This adjustment highlights the importance of accurately assessing the dimensions of the main deformation zone in order to achieve optimal results in the forging process.

#### 5. Discussion

Through the above analysis, it is found that the existing mathematical model on the analysis of closed die forging with flash forming process has certain limitations and is not universal. When applied to different forgings, the existing mathematical model can be modified by combining finite element simulation technology. The corrected mathematical model can accurately predict the metal plastic deformation and stress distribution to guide the design of the die and improve productivity. Additionally the development of flash gutter optimization design criteria can effectively reduce the amount of flash and save materials.

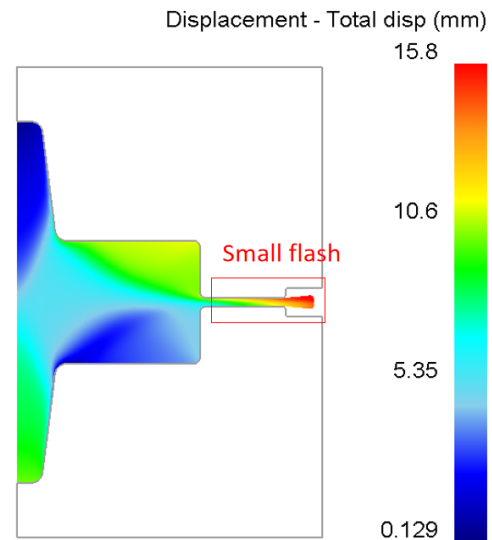


Fig. 9. Flash analysis after forging @  $h_0=0.67$  mm

#### 6. Conclusions

This article established an accurate mathematical model for closed die forging with flash to analyze metal plastic deformation and stress distribution, conducting in-depth research on key factors such as stress distribution, position of the neutral plane, and flash gutter design during the forming process. And enhancing the universality and accuracy of the mathematical model by using FE simulation method. The conclusion is as follows:

1. The errors of mathematical model in Y-axis stress calculations within the main deformation zone are confined to within 10%. Notably, the calculated stress values tend to be slightly higher than those obtained through simulation, offering valuable insights and guidance for practical applications in production.

2. The flash gutter design criterion in reference [7] has been revised, with the height of the main deformation zone in the formula corrected from  $2.9h_0$  to  $5.8h_0$ . The results show that the amount of flash is significantly reduced after the revision, effectively minimizing material waste.

3. Through analysis of the most difficult-to-fill zone of the cavity, it was found that the entry radius of the cavity has a significant impact on the stress distribution within  $x < r_1$ . When the entry radius is too small, the top surface of the rib is filled last, and in this case, the stress value is on the high side. When the entry radius is sufficiently large, the top corner of the rib is filled last, resulting in more reasonable stress values.

**Contributions of authors:** conceptualization, methodology – **Xiang Zhang**; development of model, software, verification – **Xiang Zhang**; analysis of results, visualization – **Xiang Zhang**; writing – original draft preparation, writing – review and editing – **Xiang Zhang**. formulation of tasks – **Volodymyr Borysevych**.

### Conflict of Interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, author ship or otherwise, that could affect the research and its results presented in this paper.

### Financing

This study was conducted without financial support.

### Data Availability

Data will be made available upon reasonable request.

### Use of Artificial Intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

All the authors have read and agreed to the published version of this manuscript.

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**ТЕОРЕТИЧНИЙ АНАЛІЗ ПРОЦЕСУ ФОРМУВАННЯ ЗАКРИТОГО ШТАМПУВАННЯ  
З ЗАЛИШКАМИ ТА МЕТОД ОПТИМІЗАЦІЇ ДИЗАЙНУ КАНАВКИ ДЛЯ ЗАЛИШКІВ***Сян Чжан, В. В. Борисевич*

Тема цієї статті - процес формування закритого штампування з залишками та оптимальний дизайн розміру канавки для залишків. Метою є проведення поглибленого теоретичного аналізу процесу формування закритого штампування з залишками та дослідження оптимального дизайну розміру канавки для залишків, з метою покращення науковості та економічності процесу штампування, керування оптимізацією параметрів процесу в реальному виробництві, продовження терміну служби матриць, зменшення відходів матеріалів та підвищення якості продукції та ефективності виробництва. Завданням є створення математичної моделі для закритого штампування з залишками для аналізу пластичної деформації металу та розподілу напруг, проведення поглибленого дослідження ключових факторів, таких як розподіл напруг, положення нейтральної площини та дизайн канавки для залишків під час процесу формування, а також валідація теоретичного аналізу за допомогою симуляції методом скінченних елементів за допомогою програмного забезпечення DEFORM-2D. Використані методи включають теоретичний аналіз, математичне моделювання та валідацію симуляції методом скінченних елементів. Теоретичний аналіз забезпечує основу для розуміння процесу формування та розподілу напруг, тоді як математична модель дозволяє провести кількісний аналіз. Валідація симуляції методом скінченних елементів надає засіб для перевірки та вдосконалення теоретичного аналізу та математичної моделі. Були отримані наступні результати: існуюча математична модель недооцінює висоту основної зони деформації, що призводить до нерозумного дизайну канавки для залишків. Після перевірки та корекції помилка оптимізованої математичної моделі не перевищує 10 %, а кількість залишків значно зменшується. Крім того, аналіз напруг у важкодоступних позиціях порожнин виявляє, що радіус входу порожнини має значний вплив на остаточне заповнення. Висновки. Наукова новизна отриманих результатів полягає в наступному: була встановлена математична модель закритого штампування з залишками для аналізу пластичної деформації металу та розподілу напруг, що забезпечує теоретичну підтримку для оптимізації дизайну матриць, покращення якості штампування, зменшення витрат та підвищення продуктивності. Інтегруючи симуляцію методом скінченних елементів Deform-2D, було уточнено критерії оптимального дизайну для залишків, що призвело до значного зменшення кількості залишків та економії матеріалів.

**Ключові слова:** закрите штампування з залишками; теоретичний аналіз; симуляція методом скінченних елементів; розподіл напруг; математична модель; канавка для залишків.

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