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USE OF MATHEMATICAL MODELING FOR SOLVING TASKS OF OPTIMAL CONTROL OF AN ELECTRIC DRIVE

The object of study in this article is the control processes of electromechanical systems with an electric drive. The subject matter is a mathematical model in the problem of optimal control of the following electric drive. This article is devoted to the mathematical modeling of the control processes of electromechanical systems with an electric drive. In this paper, we use the approximation of vector-valued functions of one variable by splines of the first degree to solve the problem of rotation of the motor shaft in electromechanical system. As you know, it is not always possible to find exact solutions in optimal control problems. In this regard, there is a need to obtain approximate solutions for optimal control problems. Therefore, an urgent task is the development of new methods for the approximate solution to the problems of optimal control of the electric drive, which provide higher approximation accuracy. The following results are presented: Graphs of phase coordinates and optimal control of approximation by splines of the first order are obtained. The results obtained were compared with the exact solution. The given examples illustrate the high accuracy of approximate solutions to the control problem obtained using the proposed method. Conclusions. The scientific novelty of the results obtained lies in the fact that a method is proposed for an approximate solution to the problem of optimal control of a servo drive using spline functions, in which unknown control parameters are found simultaneously with unknown parameters of phase coordinates, which gives the best approximation to the exact solution.

Keywords: *functional; phase coordinates; control of the electric drive; first-degree interpolation splines; first-degree approximation splines; system of linearly independent functions; minimization of energy consumption for control.*

Introduction

The use of computer equipment in the form of industrial controllers in the control systems of automated electric drives shows that standard methods for numerical simulation of such systems do not provide sufficiently reliable results. So the work [1] proposes a modeling technique which takes into account the specifics of construction of modern electric drive. The described method allows one to simulate the dynamics of discrete-continuous systems as accurately as possible.

In work [2] obtained physical, mathematical and graphical models of translational and angular movements of a two-wheeled experimental sample. The obtained models differ from the known ones by taking into account the dynamic properties of sensors and electric drives, as well as relationships of movements. In article [3] developed the method of optimization by integral criterion. In this paper developed approaches to the formation of control algorithms for translational and angular movements of a non-stationary automatic control object.

The work [4] investigated the approximation of class and class functions by splines of the first degree. In particular, it was proved that the approximation of class functions by splines of the first order normally allows obtaining approximations with the optimal order of accuracy on this class. In works [5-6], a new method of solving the Cauchy problem for a system of linear ordinary differential equations was proposed and studied under the condition that the right-hand parts were approximated by splines of the first degree from the condition of the minimum of the corresponding residual.

In this article, the proposed method was applied for an approximate solution of the optimal control problem for minimizing energy costs when the motor shaft rotates through a given angle in a given time.

Unfortunately, it is not always possible to find exact solutions in optimal control problems. In this connection, there is a need to obtain approximate solutions to optimal control problems. Therefore, the scientific task is urgent – the development of new methods for the approximate solution of the problems of optimal control of the electric drive, which provide a higher approximation accuracy.

A particularly urgent control problem at the current stage of the development of a controlled electric drive is the need to improve the accuracy of the tracking electromechanical systems, robotic systems, highprecision control systems for the drive of radar antennas, support-rotating devices of optoelectronic systems for monitoring moving objects in air, land and sea spaces.

Solving this problem is possible within the framework of control theory using mathematical modeling of electric drive control processes and computational methods.

1. Formulation of the problem of controlling an electric drive using a system of ordinary differential equations

The motion of the object is described by a system of linear differential equations

$$\frac{dx_i}{dt} = \sum_{k=1}^{n} a_{ik}x_k + \sum_{l=1}^{r} b_{il}u_l + f_i(t),\ i=\overline{1,n},\ t\in[0,T] \quad (1)$$

or in matrix form

$$\frac{dx}{dt} = Ax + Bu + f,$$

where $x = x(t) - n$ is the dimensional vector of coordinates of the state of the object; $u = u(t) - r$ dimensional control vector; A_{n*n} and B_{n*r} coefficient matrices.

It is necessary to find the $u(t)$ - control and the trajectory corresponding to it $x(t)$, on which the functional reaches a minimum

$$J(x,u) = \int_0^T \left(\sum_{i=1}^{n} q_i x_i^2 + \sum_{j=1}^{r} r_j u_j^2 \right) dt \to \min_{u\in\Omega}, \quad (2)$$

where $q_i \ge 0$, $r_j > 0$ are given weighting factors. At the same time, the boundary conditions must be satisfied

$$x_i(0) = x_{i,0};\ x_i(T) = x_{i,1},\quad i=\overline{1,n}.$$

There are no restrictions on the coordinates of the object state vector $x(t)$ and the control vector $u(t)$. Entering a variable $\tau = \frac{t}{T}$, $0 \le \tau \le 1$, $x_i(t) = x_i(T\cdot\tau) = \tilde{x}_i(\tau)$ is a task (1) – (2) takes the form

$$\frac{d\tilde{x}_i}{d\tau} = \sum_{k=1}^{n} \tilde{a}_{ik}\tilde{x}_k + \sum_{l=1}^{r} \tilde{b}_{il}\tilde{u}_l + \tilde{f}_i(\tau),\quad i=\overline{1,n};$$

$$\tilde{x}_i(0) = x_{i,0};\ \tilde{x}_i(1) = x_{i,1},\ i=\overline{1,n};$$

$$J(\tilde{x},\tilde{u}) = \int_0^1 \left(\sum_{i=1}^{n} \tilde{q}_i \tilde{x}_i^2 + \sum_{j=1}^{r} \tilde{r}_j \tilde{u}_j^2 \right) d\tau.$$

Expressions for $\tilde{a}_{ik}, \tilde{b}_{il}, \tilde{f}_i(\tau)$ are given in the work. Integrating both parts of the differential equations along the segment $[0,t]$, we get

$$\tilde{x}_i(t) = x_{i,0} + \sum_{k=1}^{n} \tilde{a}_{ik} \int_0^t \tilde{x}_k(\tau)d\tau + \sum_{l=1}^{r} b_{il} \int_0^t \tilde{u}_l(\tau)d\tau + $$
$$+ \int_0^t \tilde{f}_i(\tau)d\tau,\quad i=\overline{1,n}.$$

We will approximate the phase coordinates $\tilde{x}_i(t),\ i=\overline{1,n}$ and coordinates of the control $\tilde{u}_j(\tau),\ j=\overline{1,r}$ vector by splines of the first order:

$$\tilde{x}_i(\tau) = \sum_{p=1}^{M_1} c_{i,p} h_p(\tau);\ \tilde{u}_j(\tau) = \sum_{q=1}^{M_2} d_{j,q} h_q(\tau),$$

where $h(\tau), h_p(\tau), h_q(\tau)$ – piecewise linear functions and

$$h(\tau) = \frac{1}{2}\left(|\tau-1| - 2|\tau| + |\tau+1|\right);$$

$$h_p(\tau) = h(M_1\tau - p);\ h_q(\tau) = h(M_2\tau - q).$$

The coefficients

$$c_{i,p}\left(i=\overline{1,n},\ p=\overline{1,M_1}\right),$$

$$d_{j,q}\left(j=\overline{1,r},\ q=\overline{1,M_2}\right)$$

according to the method of least squares can be found in integral form by minimizing the functional

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$$J(\tilde{x},\tilde{u}) = \gamma_1 \int_0^1 \left[\sum_{i=1}^{n} \left(\tilde{x}_i'(t) - \right.\right.$$

$$\left.\left. - \sum_{k=1}^{n} \tilde{a}_{ik} \tilde{x}_k(t) - \sum_{l=1}^{r} \tilde{b}_{il} \tilde{u}_l(t) - \tilde{f}_i(t) \right)^2 \right] dt +$$

$$+\gamma_2 \int_0^1 \left[\sum_{i=1}^{n} \left(\tilde{x}_i(t) - x_{i,0} - \sum_{k=1}^{n} \int_0^t \tilde{a}_{ik}(\tau) \tilde{x}_k(\tau) d\tau - \right.\right.$$

$$\left.\left. - \sum_{\ell=1}^{r} \int_0^t \tilde{b}_{il}(\tau) \tilde{u}_l(\tau) d\tau - \int_0^t \tilde{f}_i(\tau) d\tau \right)^2 \right] dt +$$

$$+\gamma_3 \int_0^1 \left(\sum_{i=1}^{n} \tilde{q}_i \tilde{x}_i^2 - \sum_{j=1}^{r} \tilde{r}_j \tilde{u}_j^2 \right) dt,$$

where $\gamma_1 > 0,\ \gamma_2 \ge 0,\ \gamma_3 > 0$ – some parameters, $\gamma_1 + \gamma_2 + \gamma_3 = 1$.

2. Examples of application of the proposed method for solving optimal control problems

Example 1. Consider a one-dimensional stationary linear system of automatic regulation, which corresponds to the differential equation

$$\frac{dx}{dt} = x + u + f(t);$$

it is considered that the object of regulation is under the influence of disturbance, where is time.

We find the optimal control and the corresponding state of the system under the initial condition. We define the quadratic functional as the criterion characterizing the quality of the control process

$$J(x,u) = \int_0^1 \left(x^2 + u^2 \right) dt \to \min_{x,u}.$$

On fig. 1. graphically shows the results of comparing the approximate values found by the proposed method with the exact ones

$$x(t) = \frac{1}{2}\left(3e^{-\sqrt{2}t} - t - 1 \right),$$

$$u(t) = \frac{1}{2}\left(-3\left(\sqrt{2}+1\right) e^{-\sqrt{2}t} - t \right).$$

Example 2. Applying the described method, we determine the software optimal process, which minimizes the dissipated energy - the time integral of the square of the control action. Consider a second-order system consisting of two serially connected ideal integrating links:

$$J(u) = \int_0^T u^2 dt \to \min_{u \in \Omega}; \qquad (3)$$

$$\Omega: \begin{cases} \dot{x}_1(t) = x_2(t); \\ \dot{x}_2(t) = u(t); \end{cases}$$

$$x_i(0) = x_{i,0};\ x_i(T) = x_{i,1},\ (i = 1,2);\ u(t) \in \Omega,$$

which is a fixed-end and fixed-time task.

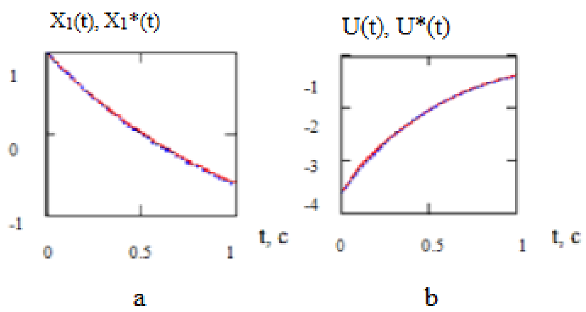

Figure 1. Comparison of dependence of:

a) phase coordinate with the exact one $X^*(t)$;

b) optimal control with precise $U^*(t)$:

$X_1(t)$, $U(t)$ – solid line, $X_1^*(t)$, $U^*(t)$ – dotted line

The considered problem can be given the following physical interpretation: it is necessary to turn the motor shaft to a given angle in the current time *T* with minimal energy consumption for control, which is characterized by functional (3).

We are looking for unknown functions in the form of splines

$$\tilde{x}_i(t) = \sum_{k=1}^{n} S_{i,k}(t),\ i = 1,2;\ \ \tilde{u}(t) = \sum_{k=1}^{n} S_{3,k}(t);$$

$$S_{j,k}(t) = C_{j,k} \frac{t - t_{k+1}}{t_k - t_{k+1}} + C_{j,k+1} \frac{t - t_k}{t_{k+1} - t_k},$$

$$t_k \le t \le t_{k+1},\ \ j = \overline{1,3},$$

$$S_{j,k}(t) = 0,\ \ t \le t_k \vee t > t_{k+1}.$$

We can find constants $C_{j,k}\ \left(j = \overline{1,3};\ k = \overline{1,n}\right)$ by minimizing the functional

$$J_1(\tilde{x},\tilde{u}) = \gamma_1 \int_0^1 \left[\left(\frac{1}{T}\tilde{x}_1'(t) - \tilde{x}_2(t) \right)^2 dt + \right.$$

$$\left. + \left(\frac{1}{T}\tilde{x}_2'(t) - \tilde{u}(t) \right)^2 \right] dt +$$

$$+ \gamma_2 \int_0^1 \left[\left(\tilde{x}_1(t) - x_{1,0} - T\int_0^t \tilde{x}_2(\tau)d\tau \right)^2 dt + \right.$$

$$\left. + \left(\tilde{x}_2(t) - x_{2,0} - T\int_0^t \tilde{u}(\tau)d\tau \right)^2 \right] dt + \gamma_3 T \int_0^1 \tilde{u}^2(t)dt.$$

Numerical calculations were carried out with the following of initial conditions:

$$T = 5\text{с},\ x_{1,0} = 0,\ x_{2,0} = 10\,\text{рад}/\text{с},$$
$$x_{1,1} = 50\,\text{рад},\quad x_{2,1} = 0.$$

The time segment was divided into 10 parts: $t_k = 0{,}1(k-1),\ \left(k = \overline{1, N+1}\right);\ N = 20.$

Note that in this example, the problem of software control is solved. The technical implementation of this software control is shown in fig. 2. An input signal $u^*(t)$ generated by a software clock mechanism (microcontroller) is applied to the input of an open system consisting of two integrators connected in series. At the moment t=0, the clock starts and, in accordance with its progress, the signal $u^*(t)$ changes as shown in fig. 3, a.

The results of numerical calculations are shown in fig. 3, which shows a comparison of the dependence of the optimal control $U^*(t)$ (Fig. 3, a) and the corresponding phase coordinates $X_1^*(t)$ and $X_2^*(t)$ (Fig. 3, b and Fig. 3, c) with the exact ones: $X_1(t), X_2(t), U(t)$ – solid line; $X_1^*(t)$, $X_2^*(t)$, $U^*(t)$ – dashed line.

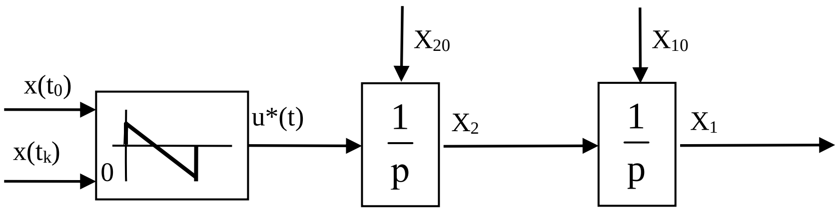

Figure 2. Technical implementation of optimal software control

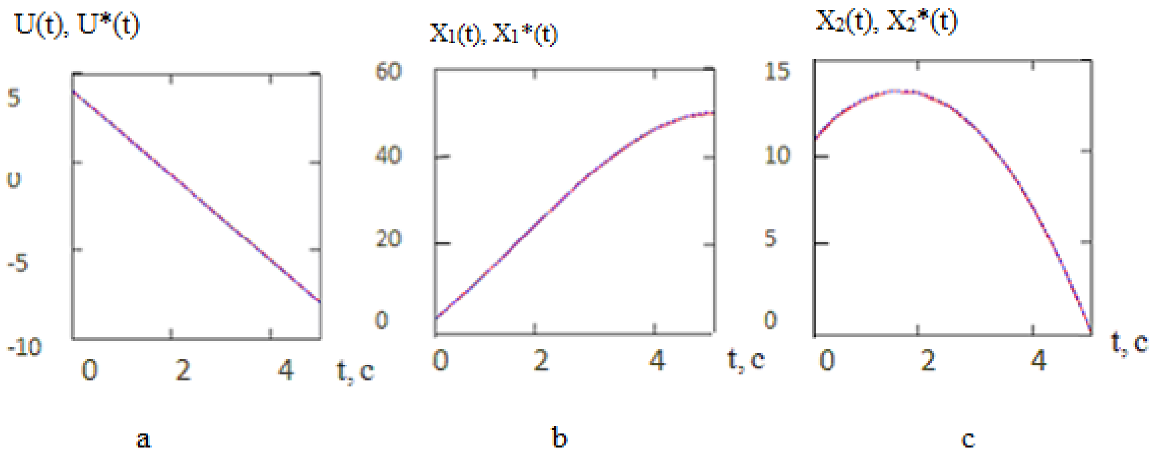

Figure 3. Comparison of dependencies:
a – optimal control $U(t)$ with accurate $U^*(t)$;
b – phase coordinate $X_1(t)$ (angle of rotation of the motor shaft) with accurate $X_1^*(t)$;
c – phase coordinate $X_2(t)$ (angular velocity) with exact $X_2^*(t)$

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The maximum relative error $\left(\varepsilon = \left| \frac{X_1(t) - X_1^*(t)}{X_1(t)} \right| \right)$ for does not exceed 0.04. Similar accuracy was obtained for $U^*(t)$ and $X_2*(t)$.

Conclusions

Analysis of the results of the computational experiment conducted on the basis of the created program package in the MATCAD system allows us to draw the following conclusions:

1. If the right-hand parts of the system of ordinary differential equations are polynomials of the variable, then to find the approximate solution of the system with constant coefficients, the choice of basis functions gives exponential accuracy.

2. If the basis functions are chosen differently for different components $y_1(x),...,y_n(x)$, then it is possible to obtain an exact solution for the right-hand parts, which are some approximation to the given right-hand parts.

3. Examples illustrate the high accuracy of approximate solutions obtained by the proposed method. The given examples show that the accuracy of the approximation depends on the properties of the solution: it is greater if the solution is monotonic on a segment compared to a solution whose behavior on this segment changes from monotonically increasing to monotonically decreasing and vice versa.

4. The research carried out in this work allows to increase the accuracy of calculations and reduce the computational load, without requiring an increase in the power of the hardware component of the computer system. The proposed method makes it possible to find a solution to the problem of optimal control of the electric drive, which is the best approximation to the exact solution in the norm $W_2^1[0,1]$. The approximation error is 0.04.

In the future, there is a possibility of generalizing the proposed method for solving the Cauchy problem for systems of nonlinear differential equations.

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ВИКОРИСТАННЯ МАТЕМАТИЧНОГО МОДЕЛЮВАННЯ ДЛЯ РОЗВ’ЯЗАННІ ЗАДАЧІ ОПТИМАЛЬНОГО УПРАВЛІННЯ ЕЛЕКТРОПРИВОДОМ

Г. А. Мірошниченко

Об’єктом дослідження в статті є процеси керування електромеханічними системами з електроприводом. У якості п**редмета** вивчення виступає математична модель в задачі оптимального керування електроприводом. **Метою** є дослідження обчислювального методу розв’язання задачі оптимального керування електроприводом. У даній роботі використано апроксимацію вектор-функцій однієї змінної сплайнами першого ступеня для розв’язування задачі обертання вала двигуна системи. Як відомо, в задачах оптимального керування не завжди можна знайти точні розв’язки. У зв'язку з цим виникає необхідність отримання наближених розв'язків задач оптимального керування. Тому актуальною **задачею** є розробка нових методів наближеного розв’язання задач оптимального керування електроприводом, які забезпечують більш високу точність наближення. Використовуваними **методами** **є:** загальні методи функціонального аналізу (побудувати функціонал, мінімум якого дозволяє отримати наближений розв'язок задачі оптимального керування електроприводом); методи обчислювальної математики (при виборі формул, що апроксимують фазові координати та керування з урахуванням початкових умов); метод знаходження найкращого наближення функції однієї змінної сплайнами 1-го порядку (при виборі системи точних розв'язків однорідної задачі Коші). Надані наступні **результати:** отримані графіки фазових координат та оптимального управління апроксимацією сплайнами першого порядку. Проведено порівняльний аналіз одержаних результатів з точним розв'язком задачі управління. Наведені приклади ілюструють достатньо високу точність наближених розв’язків задачі управління, одержаних запропонованим методом. **Висновки**. Наукова новизна отриманих результатів полягає в тому, що запропоновано метод наближеного розв’язання задачі оптимального керування слідкуючим електроприводом за допомогою сплайн-функцій, у якому невідомі параметри керування знаходяться одночасно з невідомими параметрами фазових координат, що дає найкраще наближення до точного розв'язку.

Ключові слова: функціонал; фазові координати; задача керування електроприводом; інтерполяційні сплайни першого степеня; апроксимаційні сплайни першого порядку; система лінійно-незалежних функцій; мінімізація витрат енергії на керування.

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