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MULTI-VALUED DECISION DIAGRAM IN IMPORTANCE ANALYSIS

The importance analysis allows reliability estimation depending on the system structure and its components states. Importance measures quantify the criticality of a particular component. Multi-State System (MSS) is mathematical model in reliability analysis that is used for description system with more than two levels of performance. Multiple-Valued Logic mathematical approach is used for MSS reliability analysis. New algorithms for calculation of the importance measures based on the methods of Logical Differential Calculus and Binary (Multi-Valued) Diagram are proposed taking into account features of MSSs. Benchmarking algorithms have been driven to the number of variables; number of driven nodes is foreseen to find all possible states. According to tests the two proposed algorithms are similar in point of view of computation complexity.

***Keywords**: Multi-State system, Multi-Valued Decision Diagram, Direct Partial Logic Derivative.*

Introduction

The principal problem in reliability engineering is analysis of system failure [1, 2]. Therefore the initial object for analysis is represented as system with two possible states as working and failure. Multi-State System (MSS) is mathematical model in reliability analysis that is used for description system with some (more than two) levels of performance (availability, reliability) [1, 3]. Examples of the MSS application in reliability engineering are in [1 – 4].

The MSS importance analysis is one of directions for estimation of MSS behavior [4 – 8]. Importance analysis is used for MSS reliability estimation depending on the system structure and its components states. The theoretical aspects of importance measures of MSS have been extensively investigated. Quantification of this is indicated by Importance Measure (IM). A.Lisniansky and G.Levitin [3] defined basic importance measures for MSS. They investigated basic IM for system with two performance level and multi-state components and their definitions by output performance measure have been considered in [3, 7]. Authors of paper [8] have been generalized this result for MSS and have been proposed new type of IM that is named as composite importance measures. New methods based on Logical Differential Calculus for importance analysis of MSS have been considered in paper [5, 9]. In papers [7 – 9] for MSS define IM such as Structural Importance (SI), Criticality Importance (CI), Birnbaum importance (BI), Component Dynamic Reliability Indices (CDRI) and Dynamic Integrated Reliability Indices (DIRI).

Structural Importance (SI) is concentrate on the topological structure of the system. There determines the proportion of working states of system in which the working of the *i*-th component makes the difference between system failure and working state.

Birnbaum importance (BI) of a given component is defined as the probability that such component is critical to MSS functioning. Next represents loss in the MSS when the *i*-th component was fails.

Component Dynamic Reliability Indices (CDRI) estimates the influence of the *i*-th component state change to MSS and is probability of MSS performance change depending on the *i*-th component state change.

Dynamic Integrated Reliability Indices (DIRI) is defined as the probability of MSS failure that was caused by the one of system component state breakdown.

The MSS performance level changes from zero to (*M*-1) and has *M* possible values. Each of *n* system components can be in one of mi (i = 1, …, *n*) possible states: from the complete failure (it is 0) to the perfect functioning (it is mi-1). A structure function is one of typical representations of MSS [2, 5] and is defined as:

$$\varphi(\mathbf{x}): \{0,\ldots,m_1-1\}\times\ldots\times\{0,\ldots,m_n-1\}\rightarrow\{0,\ldots,M-1\}, \quad (1)$$

where x_i is the i-th component; $\mathbf{x} = (x_1, \ldots, x_n)$ is vector of components states.

The structure function (1) represents system with two possible states (working and failure) if $m_i = m_j = M$ $(i \neq j;\ i, j = 1, \ldots, n)$.

Every system component states x_i is characterized by probability of the performance rate:

$$p_{i,s} = \Pr\{x_i = s_i\}, \quad s = 0, \ldots, m-1 . \quad (2)$$

In this paper we use the Multiple-Valued Logic (MVL) mathematical approach for MSS reliability analysis. The mathematical tools of MVL as Logical Differential Calculus for MSS quantification have been proposed in [10, 5]. The Logical Differential Calculus is mathematical tool that permits to analysis changes in function depending of changes of its variables. Therefore this tool can be used to evaluate influence of every system component state change. The principal disadvantage of the Logical Differential Calculus application in reliability analysis is increase of computational complexity depending on number of system component. In this case the Multi-Valued Decision Diagram (MDD) is used for initial function representation in MVL [11].

MDD is generalization of Binary Decision Diagram (BDD). BDD is an efficient method to manipulate the Boolean expression and, in most cases, BDDs use less memory to represent large Boolean expressions than representing them explicitly [12]. BDD is widely used in reliability analysis for Binary-State System analysis because a system is either in the operation state or in the fail state [12, 13]. MDD is natural extension for MSS analysis [14, 15]. But the MSS representation by MDD causes development of new algorithms for system reliability analysis. In papers [9, 16] algorithms for calculation of the IM are proposed. But simple adaptation of traditional calculation of the IM is used in these algorithms and advantage of MDD isn't exploited. We proposed new algorithms for IM calculation based on MSS representation by MDD. Logical Differential Calculus is used in these algorithms too.

1. Logical Differential Calculus

Calculation of IMs is based on different mathematical approaches and Logical Differential Calculus is one of them. Methods for MSS importance analysis have been proposed in [5, 14].

Logical Differential Calculus of MVL function includes different methods and algorithms for estimation of influence of variable/variables value change to the function value modification. Direct Partial Logic Derivatives (DPLD) are part of Logic Differential Calculus and can be used for analysis of dynamic properties of MVL function or MSS structure function.

These derivatives reflect the change in the value of the underlying function when the values of variables change [5].

DPLD with respect to variable x_i for MSS structure function (1) permits to analyse the system performance level change from j to $\tilde{j}$ when the i-th component state changes from s to $\tilde{s}$. This change is defined by the derivative:

$$\partial\varphi(j \to \tilde{j})/\partial x_i(s \to \tilde{s}) = \begin{cases} 1, & \text{if } \varphi(s_i, x) = j \text{ and } \varphi(\tilde{s}_i, x) = \tilde{j} \\ 0, & \text{other} \end{cases} \quad (3)$$

where $\varphi(a_i, x) = \varphi(x_1,\ldots, x_{i-1}, s, x_{i+1},\ldots, x_n)$; $\varphi(\tilde{s}_i, x) = \varphi(x_1,\ldots, x_{i-1}, \tilde{s}, x_{i+1},\ldots, x_n)$; a, $\tilde{s} \in \{0,\ldots,m_{i-1}\}$ and j, $\tilde{j} \in \{0,\ldots,M-1\}$.

The structure function (1) of coherent MSS has following assumptions [2]:

a) the structure function is monotone;

b) all components are independent and relevant to the system.

Therefore the DPLD (3) for the coherent MSS performance level reduction is defined as:

$$\partial\varphi(j \to j-1)/\partial x_i(s \to s-1) = \begin{cases} 1, & \text{if } \varphi(s_i, \boldsymbol{x}) = j \text{ and } \varphi((s-1)_i, \boldsymbol{x}) = j-1; \\ 0, & \text{other} \end{cases} \quad (4)$$

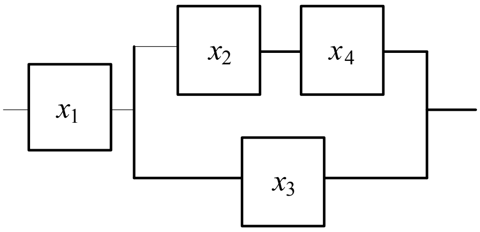
because the assumptions a) and b) cause gradual changes of the function value depending on the same variable change.

For example, consider the MSS of four component (n=4) in Fig. 1. The influence of the first variable failure to the MSS failure can be analyzed by DPLD $\partial\varphi(1\to0)/\partial x_1(1\to0)$. This derivative has eight nonzero values for variables vector $\mathbf{x}$ = (x_1 x_2 x_3 x_4): (1 1 0 1→0), (1 1 0 2→0), (1 0 1 0→0), (1 0 1 1→0), (1 0 1 2→0), (1 1 1 0→0), (1 1 1 1→0) and (1 1 1 2→0). Therefore the failure of the first component cause the system breakdown for working state of the second, the third and the fourth components or working state one of them. The system isn't functioning if the second and the third component are failed and failure of the fourth component hasn't influence to the system performance level change.

Therefore DPLD allows discovering the boundary system state for which change of the i-th system component state from s to s-1 causes modification of the MSS performance level from j to j-1.

2. Multi-Valued Decision Diagram

A MDD is a directed acyclic graph of MVL-function representation [11]. For the structure function (1) this graph has M sink nodes, labelled from 0 to (M-1), representing M corresponding constant from 0 to (M-1). Each non-sink node is labelled with a structure function variable x and has mi outgoing edges. In the MSS reliability analysis the sink node is interpreted as a system reliability state from 0 to (M 1) and non-sink node presents either a system component. Each non-sink node has mi edges (Fig.2) and the first (left) is labelled the “0” edge and agrees with component fail, and the mi-th last outgoing edge is labelled “mi -1” edge and presents the perfect operation state of system component.



The MSS structure function

$x_2\,x_3\,x_4$ \ x_1	0	1	2
0 0 0	0	0	0
0 0 1	0	0	0
0 0 2	0	0	0
1 0 0	0	0	0
1 0 1	0	1	1
1 0 2	0	1	1
0 1 0	0	1	1
0 1 1	0	1	1
0 1 2	0	1	1
1 1 0	0	1	1
1 1 1	0	1	2
1 1 2	0	2	2

Fig. 1. The MSS example for n = 4, M = 3 and **m** = (3, 2, 2, 3)

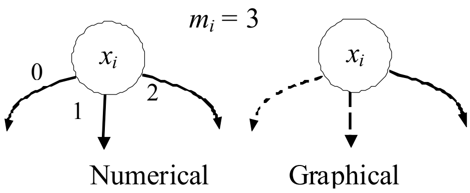


Fig. 2. Non-sink node example

The paths from top non-sink node to the zero-sink node are analysed for a MSS failure. The paths from top non-sink node to another sink node are considered for system repair by means of MDD. There is special software for MDD that allows calculating necessary measures [11]. In particular, the paths for system failure and system repair can be determined by it. For example, the MSS representation in Fig. 1 by MDD is in Fig. 3.

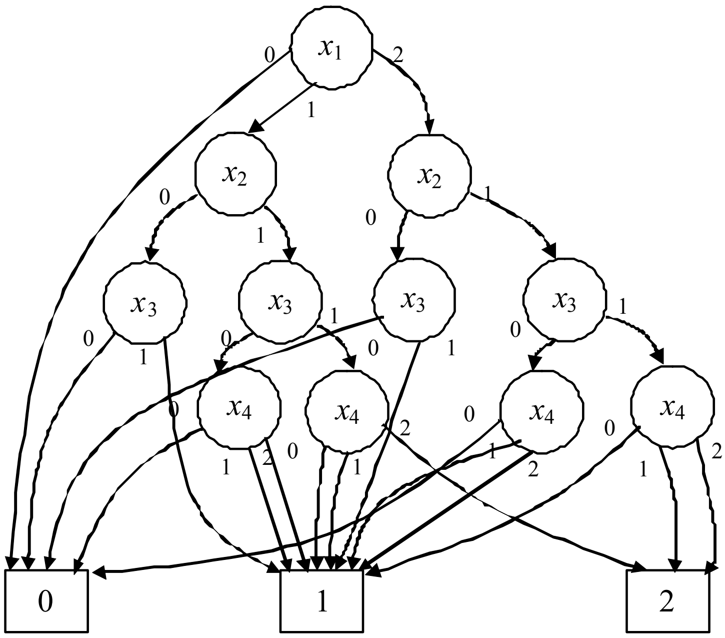


Fig. 3. MDD for structure function of the MSS in Fig. 1

3. MSS Importance Analysis

Consider definition of some IM in introduction in terms of DPLD. The SI takes into account the topological specifics of the system. It is used for analyzing such systems, which are in designs or we don't know the entire structure of the system. SI of the MSS for the i-th component state s is probability of this system performance level j decrement if the component state changes from s to s_i-1 depending on topological properties of system:

$$I_S(s_i \mid j) = \frac{\rho_i^{s,j}}{m^{n-1}} \qquad (5)$$

where $\rho_i^{s,j}$ is number of system states when the change component state from s to *s*-1 results the system performance level decrement and this number is calculated as numbers of nonzero values of DPLDs (4).

The modified SI represent of the *i*-th system component state change influence to MSS performance level decrement for boundary system state. In terms of DPLD (4) modified SI is determined as [5]:

$$I_{MS}(s_i \mid j) = \frac{\rho_i^{s,j}}{\rho_i^{(s,j)}} \quad (6)$$

where $\rho_i^{s,j}$ is defined in (5); $\rho_i^{(s_i,j)}$ is number of boundary system states when

$$\varphi(s_i, \mathbf{x}) = j$$

(it is computed by structure function of MSS (1)).

Birnbaum Importance (BI) of a given component is defined as the probability that such component is critical to MSS functioning [7, 8]. BI is probabilistic measure that can be interpreted as rate at which the MSS fails as the *i*-th system component state decreases:

$$I_B(s_i \mid j) = \Pr\left(\partial\varphi(j \to j-1) / \partial x_i(s \to s-1)\right), \quad (7)$$

BI measures (7) depend on the structure of the system and states of the other components, but is independent of the actual state of the *i*-th component.

CDRI indicates the influence of the *i*-th component state change to MSS performance level change [14]. This definition of CDRI is similar to definition of modified SI, but CDRIs for MSS failure take into consideration two probabilities: (*a*) the probability of MSS failure provided that the *i*-th component state is reduced and (*b*) the probability of inoperative component state:

$$I_{CDRI}(s_i \mid j) = I_{MS}(s_i \mid j) \cdot p_{i,s-1}, \quad (8)$$

where $I_{MS}(s_i \mid j)$ is the modified SI (6); $p_{i,s}$ is probability of component (2).

DIRI is the probability of MSS performance level decrement that caused by the one of system components state deterioration. DIRIs allow estimate probability of MSS failure caused by some system component (one of n):

$$I_{DIRI}(s \mid j) = \\ = \sum_{i=1}^{n} I_{CDRI}(s_i \mid j) \prod_{\substack{q=1 \\ q \neq i}}^{n} (1 - I_{CDRI}(s_q \mid j)). \quad (9)$$

The IM (5) – (9) are defined based on the DPLD (4). Therefore the algorithms for calculation of the DPLD by MDD are principal part for quantification of the MSS that is represented by MDD. There aren't established algorithms for calculation DPLD by the MDD. In paper [17] algorithm for calculation Boolean Partial Derivative by the BDD has been proposed only.

Furthermore, we consider the calculation of the state degradation of components due to the degradation state system. This change in state of a component or system will be called change from actual state to future state. Now is showed calculate this DPLD by MDD. The basic principle consists in searching the MDD and acquiring "appropriate routes". Appropriate route contains the actual (future) state of critical component for which calculate DPLD and leaf with the actual (future) state of the system. To determine these routes is suggested two possible algorithms, which permit a review of MDD. As a firs is presented an algorithm "from the root" on Fig. 4. This algorithm is initialized with root of MDD. Recursive searching is searched a MDD and identify the appropriate routes. If encounters at the recursion searching at some conflict with specified compute DPLD, in this route searching has stopped. This increases the efficiency of browsing the MDD, because are eliminated "irrelevant" routes that in this calculation is not important. Similarly is worked the second type of algorithm "from the leaf" presented on Fig. 5. As opposed to the first algorithm, that is required to initialize twice: first time with leaf of actual state of system and second time with leaf of future system state. The results of browsing MDD are suitable roads that are partial results and are presented on Fig. 5.

These roads still need to be compared with each other to find out what is already identifying DPLD calculation.

In comparing is also important to find routes that are different in the state of the whole value system that is a value in the actual and future state and in a state (actual, future) of component, whereby DPLD is calculated. On Fig. 6 are these routes presented a highlighted. In these routes the just get a state value of other components, what is the result of the calculation of DPLD.

Conclusion

In this paper MDD were considered to calculate the reliability indices by DPLD. The basic principles of MDD application in reliability analysis were presented. New algorithms for calculation of the DPLD for MSS structure function are proposed and tested based on Benchmark [18]. This benchmark has in the PLA – EXPRESSO format, which is used for Boolean functions. Therefore, the tests carried out on BSS constructed on the basic of the benchmarks. Benchmarking algorithms have been driven to the number of variables. Number of driven nodes is foreseen to find all possible states, which is possible calculate the DPLD. Tested BSS are shown in Fig. 7. According to tests the two proposed algorithms are similar in point of view of computation complexity.

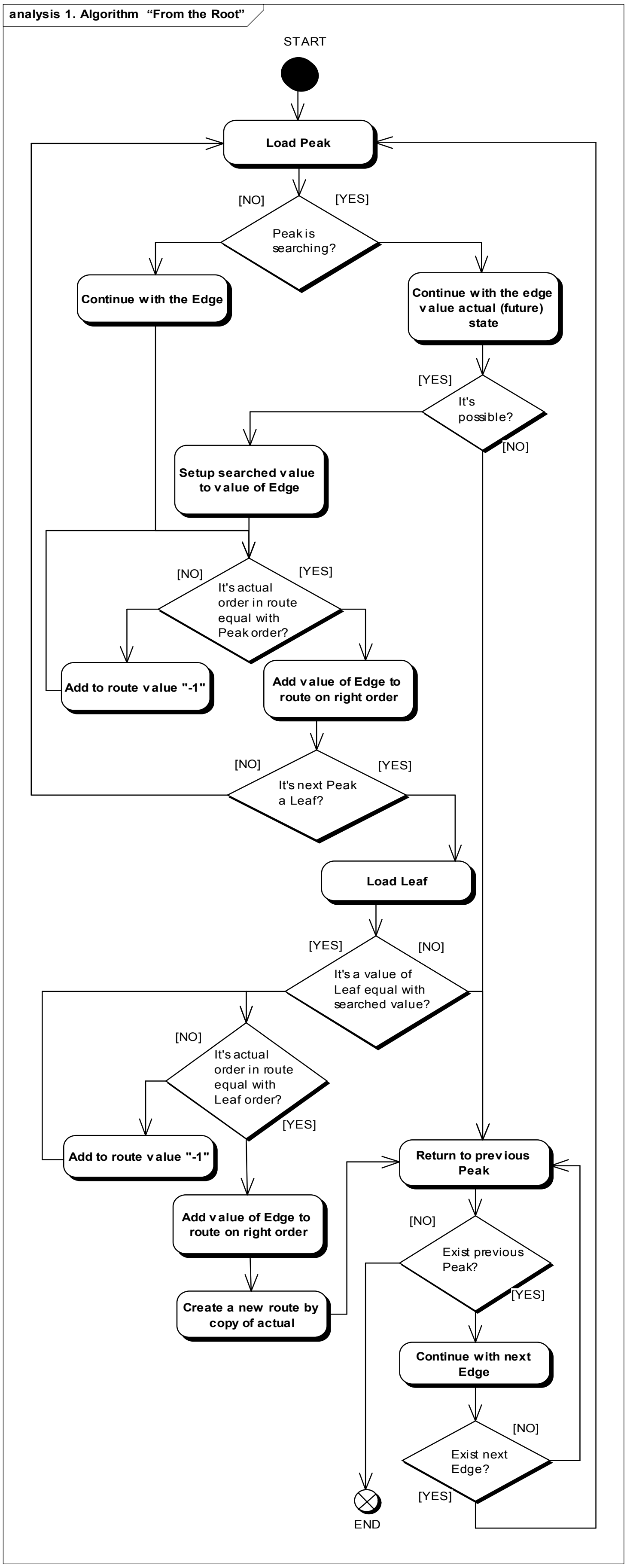


Figure 4. Algorithm 1 "from the root"

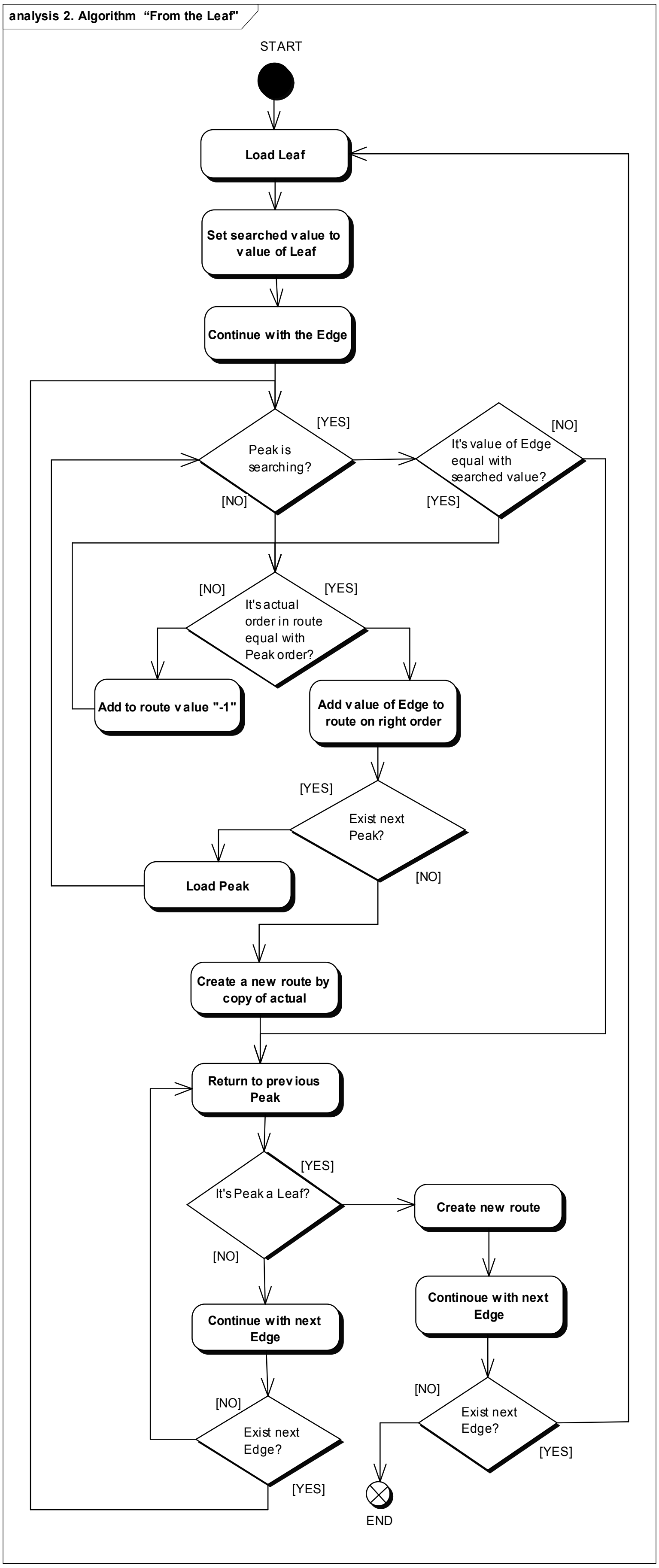


Figure 5. Algorithm 2 "from the leaf"

φ(**x**)	x_1	$\mathbf{x_2}$	x_3	x_4
1	1	**1**	0	1
1	1	**1**	0	2
1	1	**1**	1	0
1	1	**1**	1	1
1	2	**1**	0	1
1	2	**1**	0	2
1	2	**1**	1	0

φ(**x**)	x_1	$\mathbf{x_2}$	x_3	x_4
0	1	**0**	0	-1
0	2	**0**	0	-1

Figure 6. Suitable routed in table representation
with highlighted compared routes of the MSS in Fig. 1

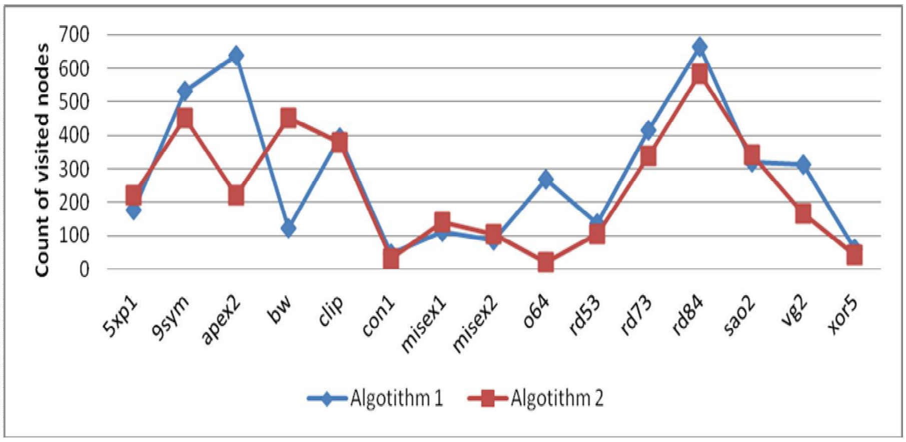


Figure 7. Algorithms comparison

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БАГАТОРІВНЕВІ ДІАГРАМИ РІШЕНЬ У АНАЛІЗІ ЗНАЧУЩОСТІ

О. Зайцева, В. Левашенко, Й. Костольни, В.С. Харченко

Аналіз значущості робить можливою оцінку надійності в залежності від структури системи та стану її елементів. Значущість визначає рівень критичності окремого елементу. Моделі систем з багаторівневою деградацією аналізуються. Для аналізу використовується багаторівнева математична логіка. Пропонуються нові алгоритми для визначення показників значущості, які базуються на методах логічного диференційного числення та бінарних (багаторівневих) діаграм. Порівняльний аналіз алгоритмів здійснювався по числу змінних; кількість керованих вузлів передбачено для знаходження всіх можливих станів. Підсумки дослідження показали, що два запропонованих алгоритму схожі з точки зору складності обчислень.

Ключові слова: системи з багаторівневою працездатністю, багаторівневі діаграми рішень, пряма часткова логічна похідна.

МНОГОУРОВНЕВЫЕ ДИАРАММЫ РЕШЕНИЙ ПРИ АНАЛИЗЕ ЗНАЧИМОСТИ

Е. Зайцева, В. Левашенко, Й. Костольны, В.С. Харченко

Анализ значимости делает возможной оценку надежности в зависимости от структуры системы и состояния её элементов. Значимость определяет уровень критичности отдельного элемента. Модели систем с многоуровневой деградацией анализируются. Многоуровневая математическая логика используется для анализа. Предлагаются новые алгоритмы для вычисления показателей значимости, которые базируются на методах логического дифференциального исчисления и бинарных (многоуровневых) диаграмм. Сравнительный анализ алгоритмов осуществлялся по числу переменных; количество управляемых узлов предусмотрено для нахождения всех возможных состояний. Итоги исследования показали, что два предложенных алгоритма схожи с точки зрения сложности вычислений.

Ключевые слова: системы с многоуровневой работоспособностью, многоуровневые диаграммы решений, прямая частная логическая производная.

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