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A GENETIC ALGORITHM OF OPTIMAL DESIGN OF BEAM AT RESTRICTED SAGGING

A genetic algorithm for solving the problem of optimal beam material distribution along length at a given restriction on maximum sagging value is suggested. A review of literature sources is conducted and it was shown that existing solutions cover partial cases only in which the position of the point with maximum sagging was defined previously. In the paper presented I-section beam with constant proportions is considered, i.e., beam width, caps, and web thickness are proportional to beam height in the current cross-section. A statically determined beam is being considered. The load applied to a beam can be arbitrary, including cases of non-symmetrical loads and differently oriented ones. The position of point(s) at which beam sagging is maximum are unknown at the beginning of optimization and are found in the process solution. The problem is solved in the linear definition. Beam mass was assumed to be an optimization criterion. The method of finite differences is used for beam sagging finding, i.e., for the solution of the differential equation of the bending beam with a variable cross-section. Discretization allows transforming the problem of design into the problem of beam height determination at a system of reference points. At this stage, found values of beam height must satisfy restrictions on reference point displacements. The suggested technique allows controlling beam displacement quite flexibly because restrictions on point displacement are considered separately and do not depend on each other. The suggested objective function is the linear superposition of beam mass and the possible penalty in case of beam maximum sagging over exceeding predefined values. The application of a genetic algorithm allows getting sets of beam thicknesses those, which guaranty reaching the minimum of the objective function. The model problem is solved. It is shown that the suggested algorithm allows effectively solves problems of optimal design of beams with restrictions on the maximum sagging value. The suggested approach can be developed for strength restrictions, statically undetermined structures, etc.

Keywords: *conditional optimization; method of finite differences; genetic algorithm.*

1. Introduction

There are many papers devoted to structural elements optimization. Optimization process can be concluded both in finding optimal parameters for definite structural parameters only [1-3] and in solving problem of structural optimization and finding function of optimal material distribution through structure volume. Beams are one of the most widespread structural elements and many papers are devoted to their optimal design [4]. This fact is stipulated by following: beam is relatively simple 1D-object. Generally, a problem of optimization is in finding such material distribution along beam length, which ensures minimal structural mass at some restrictions. At solving of such kind of problems definite parameters of beam cross-section are assumed to be frozen, for example, beam width. In this case a problem can be transformed to optimal beam height distribution along beam length. One of the most wide-spread restrictions applied is strength condition fulfilling (or condition of equal strength) of a structure. Other known restrictions are restriction on given point translation (displacement) [5-7], integral restriction of compliance [8], restrictions on free oscillations frequency [9], different types of buckling etc. If one analyzes behavior of cantilevered beam loaded with bending moments of the same sign it is possible to make conclusion that maximum sagging can be observed at free edge of a beam. The fact that location of a point which corresponds of maximum sagging is knows before analysis makes problem to be simpler and allows to formulate variation problem in classical form. However, generally predict coordinate of maximum deflection is not possible. This is stipulated by following: beam translations are derived differential equation with variable coefficients and displacements of a structure depend on material distribution along beam length, and also on location of applied load and boundary conditions.

In the paper [10] one of the authors suggested solving of beam optimization problem based on idea of beam discretization and consequent finding of beam optimal heights according to system of reference points disposed along beam length. Application of the method of finite differences allowed to transform optimization problem to the problem of non-linear separable programming, which in its turn can be transformed to the

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problem of linear programming by means of linearization. But unfortunately, described approach can't be generalized and developed for statically undetermined beams and for non-linear bended beams.

Therefore, the current paper more universal approach is developed. It is based on principally another ideas and can be spread for statically undetermined beams, another types of restrictions and equations of beams bending. Genetic algorithm is the basement of the method suggested.

Application of genetic algorithms for solution problems of topological optimization of structures is quite popular [4]. The reason of this relative simplicity and efficiency of genetic algorithms [11]. Large amount of papers deal with optimization of laminated composites structure because genetic algorithm operates with discrete sets of parameters [12]. In such cases, each element of sets of parameters describes characteristics of correspondent layer. However, there are some modifications of the method, which allow optimizing geometrical parameters distribution along structure length. For such cases calculation of displacements and stresses inside structure at given set of geometrical parameters is conducted, for example, by finite elements method [13-15], that can lead to calculation slowing.

This paper suggests approach for beam design based on discretization of a structure by length and determination of beam heights at the system of reference points. Displacements of beam with variable cross-section are calculated by conventional method of finite differences, which possesses high calculation rate.

Aim of the paper is solution of beam optimization task and development of the structures optimization method which can be generalized and applied for wide class of novel problems.

2. Problem formulation

Let's consider double-edge supported beam with variable cross-section and length L. Equation of beam bending has following view

$$EI(x)\frac{d^2w(x)}{dx^2}=M(x), \qquad (1)$$

where $EI(x)$ - beam bending rigidity; $w(x)$ - transversal displacement; $M(x)$ - bending moment in correspondent section.

Boundary conditions

$$w(0)=w(L)=0, \qquad (2)$$

Let's assume that beam cross-section keeps its proportions along entire beam length. In this case beam bending rigidity is the fourth order function of some linear dimension of beam section, for example, its height

$$EI(x)=EK_I h^4(x),$$

where K_I - coefficient depending of bean cross-section shape; $h(x)$ – beam height at section with coordinate x.

Beam cross-section in this case depends on beam height powered by two:

$$S(x)=K_S\cdot h^2(x),$$

where K_S - is coefficient, which equals to ratio of beam cross-section area to area of square with side length $h(x)$.

Volume of beam can be determined as

$$V=\int_0^L S(x)dx=K_s\int_0^L h^2(x)dx\,. \qquad (3)$$

If the density of beam material is averaged through current section then mass of a beam is proportional to beam volume and equals to product of beam volume multiplied by density.

Beam loaded with bending moment assigned as function of beam length $M(x)$. Therefore, beam is deformed under bending moment. Transversal beam displacements (sagging) have following restrictions

$$d_0\le w(x)\le d_1,\ \ x\in(0;L)\,. \qquad (4)$$

Here we assume that $d_0\le 0$ and $d_1\ge 0$ because beam side edges are unmovable (2). If $|d_0|=|d_1|$ then condition (4) can be written as $|w(x)|\le d_0$.

Thus, one has to find dependence $h(x)$, which guarantees minimal structural volume (i.e. and mass) (3) at restrictions on displacements (4). In their turn, displacements are described by equation (1).

3. Solution composing

To solve the problem let's compose algorithm based on simultaneous application of the method of finite differences for direct problem solving (composing solution of sagging equation (1)) and genetic methods of optimization to find beam height at reference points.

Beam was divided by system of nodes with numeration 0 to N. Division increment – $\delta = \frac{L}{N}$. Equation (1) in the form of difference has the view

$$w_{i-1} - 2w_i + w_{i+1} = \frac{\delta^2}{K_I}\frac{M_i}{h_i^4}, \tag{5}$$

where i - node number; M_i and h_i - correspondent bending moment and beam height at node узле i.

Beam heights h_i at each node are considered to be given and known.

It leads form boundary conditions (2) then

$$w_0 = w_N = 0.$$

Considering this condition system (5) in matrix form can be written as

$$\mathbf{AW} = \mathbf{Q}, \tag{6}$$

$$\mathbf{A} = \begin{pmatrix} -2 & 1 & & & & \\ 1 & -2 & 1 & & & \\ & 1 & -2 & 1 & & \\ & & \cdots & \cdots & \cdots & \\ & & & 1 & -2 & 1 \\ & & & & 1 & -2 \end{pmatrix}, \quad \mathbf{Q} = \frac{\delta^2}{K_I}\begin{pmatrix} \frac{M_1}{h_1^4} \\ \frac{M_2}{h_2^4} \\ \cdots \\ \frac{M_{N-1}}{h_{N-1}^4} \end{pmatrix},$$

$$\mathbf{W} = \begin{pmatrix} w_1 \\ w_2 \\ \cdots \\ w_{N-1} \end{pmatrix}.$$

Restrictions (4) can be written as

$$d_0 \le w_i \le d_1, \quad i = 1, 2, \ldots, N-1, \tag{7}$$

I.e. restrictions are applied to translation of each node. If discretization id quite frequent, elastic line of beam is smooth and doesn't contain slope breaks then replacement of restriction (4) with restriction (7) doesn't influence significantly to precision of problem solving.

Integral (3) can be found numerically with application quadratic formulas of trapezia. Considering that beam has hinged edges supported where mending moments are equal to zero (then $h_0 = h_N = 0$), one can get relationship

$$V = \delta \cdot K_s \left(h_1^2 + h_2^2 + h_3^2 + \ldots + h_{N-1}^2\right). \tag{8}$$

Application of genetic algorithm for finding optimal set of beam heights h_i at nodes $i = 1, 2, \ldots, N-1$ requires introduction of so-called fitness-function. Reaching of this function minimum guarantees minimal volume of beam (8) and fulfilling of restrictions (7). The member $\delta \cdot K_s$ in (8) is constant value. Therefore it can be neglected because its value doesn't influence of set of optimal heights h_i. This set we can consider as vector $\vec{h}$. Thus, fitness-function can be selected in following form:

$$\Phi(\vec{h}) = h_1^2 + h_2^2 + \ldots + h_{N-1}^2 + \\ + \begin{cases} 0, & w_{max} \le d_1 \\ Z \cdot (w_{max} - d_1)^2, & w_{max} > d_1 \end{cases} + \\ + \begin{cases} 0, & w_{min} \ge d_0 \\ Z \cdot (w_{min} - d_0)^2, & w_{min} < d_0 \end{cases} \tag{9}$$

Here $w_{max} = \max_{i=1..N-1}\{w_i\}$, $w_{min} = \min_{i=1..N-1}\{w_i\}$ - maximum and minimal translations (displacements) of reference points and minimal translations (displacements) of beam reference points (nodes) at given vector $\vec{h}$; Z – definite large number selected during algorithm adjusting. To calculate values of fitness-function one has to find beam deflections, which correspond to given set of heights $\vec{h}$ by means of system (6) solving.

I.e. if displacements don't overexceed restrictions then fitness-function is proportional to structure mass. But if maximum displacements elevate restrictions special penalties are involved for analysis. Such penalties are proportional to squared over exceeding of maximum displacement by given restrictions. Penalty functions are selected as quadratic to guarantee smoothness of function $\Phi(\vec{h})$ at minimal point. Such approach allows making algorithm convergence to be better. Minimum can be reached at condition of minimal structural length and doesn't overexceed restrictions by displacements. If mass is decreased more then displacements increase and penalty functions are "turned on" increasing function value.

Following algorithm is suggested for problem solving:

1. Original population of vectors $\vec{h}^{(j)}$, where $j = 1, \ldots, n$, (n - quantity of individuals in population) is created. Individuals can differ by average height, for example.

2. By data of sets with height $\vec{h}^{(j)}$ correspondent values of $\Phi_j = \Phi\left(\vec{h}^{(j)}\right)$ can be found using (8).

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3. *Selection.* Ranging of vectors $\vec{h}^{(j)}$ presented in population in accordance with correspondent values of fitness-function Φ_j. Select from population $2k$ (where $2k < n$) elements $\vec{h}^{(j)}$. Probability of getting into sample can depend either on number in ranged list or on Φ_j values in such way to guarantee getting to the sample the best sets of height $\vec{h}^{(j)}$ in population to ensure less values of fitness-function.

4. *Selection of parents.* Arbitrary select from sample k elements of the same «sex» (selection without returning), and select arbitrary pair to each of them from the rest k elements. Thus, k pairs of «parents» are selected.

5. *Mating.* Compose mask for each pair, i.e. vector which contains approximately the same quantity of figures zero and one and their total quantity equals to $N-1$. By this mask new code for derivative of current pair is composed. If a digit in mask is equal to zero (false) we take correspondent element from one of parents. If a digit is equal to one (true) then we take element from second parent. The result of this operation is creation of k derivative.

6. *Mutation.* In suggested version of algorithm mutation can happen at previous stage of mating.. I.e. mutations occur between individuals, which belong to the group of k derivatives. Let's assume that mutation happens in definite small amount of random elements $h_i^{(j)}$ per each of k derivatives. Mutation is in changing values of beam heights at several reference points of individual $\vec{h}^{(j)}$ on random value with zero mathematical expectation. The value of random deviation can be described, for example, by Gauss distribution with zero mathematical expectation. In this case probability of beam height increasing and decreasing at a node are the same.

7. After embedding changes to genome code all derivatives are return to population, volume of which is increased from n to $n+k$ individuals.

8. *Death.* Depopulation of individuals in population can be organized by two ways: a) arbitrary, with probability which is proportional to values of fitness-function («worse» code higher depopulation); b) after reaching definite age by individual (quantity of new generations in population).

9. Checking of stop criterion. If criterion is not reached (for example, given quantity of generation cycles N_g) – returning back to the item 3.

10. Selection of best individuals to separate population. If the population is filled with best individual – algorithm stops. Otherwise – returning back to the item 1.

11. Averaging of heights values at reference points by results of calculations. I.e. final population of the best individuals (which of them are produced after corresponding cycle of evolution) has to be averaged.

The goal of the last item of algorithm is in smoothening of some deviations, which are nevertheless presented in process of evolution algorithm realization.

4. Numerical realization

To illustrate operation of suggested algorithm let's consider single-span hinged supported beam loaded with concentrated force F, Fig 1.

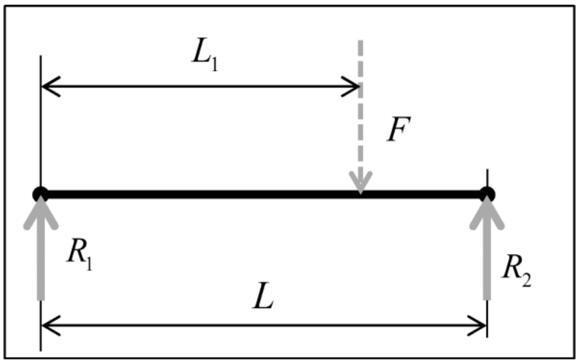

Fig. 1. Scheme of beam loading

Reactions in supports

$$R_1 = \frac{L-L_1}{L}F,\ R_2 = \frac{L_1 F}{L}.$$

Bending moments in beam

$$M(x) = \begin{cases} -R_2(L-x),\ L_1 < x < L; \\ F_1(L_1 - x) - R_2(L-x),\ 0 < x < L_1; \end{cases}$$

Let $L = 2$ m, $L_1 = \frac{5}{6}L$, $F = 9800$ N. In this case maximum bending moment but modulus equals $|M_{max}| = 2722,2$ N·m.

Select I-section caps two times less than its height and cap thickness is 0.1 of I-section height. Then $K_I = 0.0905$, $K_S = 0.16494$. Elasticity modulus equals to $E = 200$ GPa that corresponds to steel.

At first try to solve problem of bending of beam with constant cross-section. Cross-section of beam is I-section with constant height $h_0 = 50$ mm along beam length. Calculations have shown that maximum by modulus beam deflection appear in the first load case and reach value $W_{max} = 3.432$ mm.

Fig. 2 shows diagrams of beam elastic line and bending moments. Diagrams are drawn in dimensionless mode as ratio of displacement to maximum sagging (continuous line) and ratio of bending moments to maximum moment (dash line) mentioned above.

Then maximum sagging of beam with constant cross-section is used as restriction for optimization of beam height variable along beam length. Thus, we assign in restriction (4) $d_0 = -3.432$ mm and $d_1 = 3.432$ mm. Therefore, one can find beam height variable along beam length, which guarantees minimal mass of a structure and has the same maximum displacements as in a beam with constant cross-section.

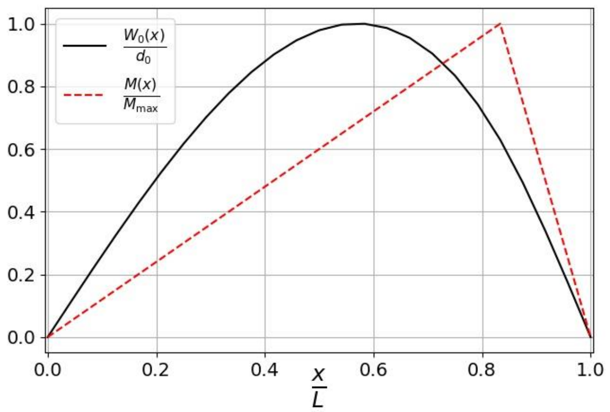

Fig. 2. Bending moment and sagging of beam with constant cross-sections

Following parameters were used at implementation of suggested algorithm:

– quantity of beam division intervals $N = 24$;

– quantity of individuals in population $n = 50$;

– quantity of pairs at the stage of mating $k = 8$;

– maximum portion of mutated genomes in mutating individual 0.2;

– ultimate age of individual after which individual removes from population, is equal to 6 cycles of generation (even if exact individual was not engaged to generation cycles);

– deviation of beam height at mutation is described Gauss distribution with zero mathematical expectation and mean-square deviation $\sigma = 0.2$ mm;

– quantity of generation cycles $N_g = 5000$;

– quantity of such cycles and volume of best selected individuals is equal to 40.

Fig. 3 shows diagram of beam height changing along beam length got for three cycles of problem solving, therefore, only some three solutions from the best final population from all best solutions.

Beam height is shown on Fig. 3 in dimensionless mode as ratio to height of beam with constant height $h_0 = 50$ mm. It can be seen that maximum beam height corresponds to neighborhood of the point where bending moment is maximal.

Moreover, solutions suggested differ significantly from each other due to random “interference” of mutation. To compensate influence of such random deviations the sample of 40 individuals was created. Each of them represents optimal solution found in process of genetic algorithm realization. Such technique allowed to determine average beam height value at each reference point. Result of such estimation is shown on Fig. 4.

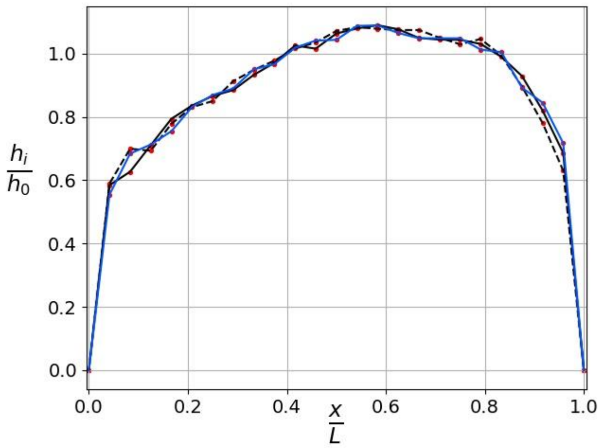

Fig. 3. Beam height at nodes

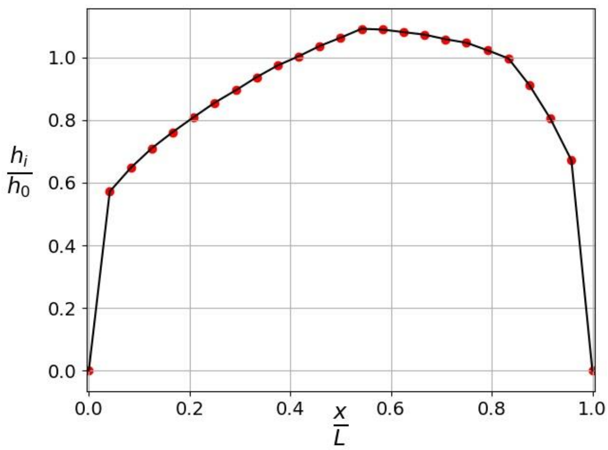

Fig. 4. Height of optimal beam at nodes

It can be seen that diagram has two slope breaks – at the point of maximum bending moment and at the point of concentrated force application. Result coincides with solution which got by another way [10] that proves correctness of operation of suggested algorithm.

Mass of the beam with constant cross-section

$$m_0 = V_0\rho = K_S h_0^2 L\rho = 9.26289 \text{ kg}.$$

Mass of beam with variable cross-section along beam length and having the same maximal displacement like beam with constant cross-section

$$m = V\rho = K_S\delta\left(h_1^2 + ... + h_{N-1}^2\right)\rho = 7.660458 \text{ kg}.$$

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Thus, developed optimal structure has mass 17.3% less than original one.

Fig. 5 shows diagrams of displacement for beam with constant height (W_0) and displacements for optimal beam (W^*).

It is clear that maximum deflections of beams are the same, but sagging of beam with variable cross-section along majority of total length are more than sagging of beam with constant cross-section.

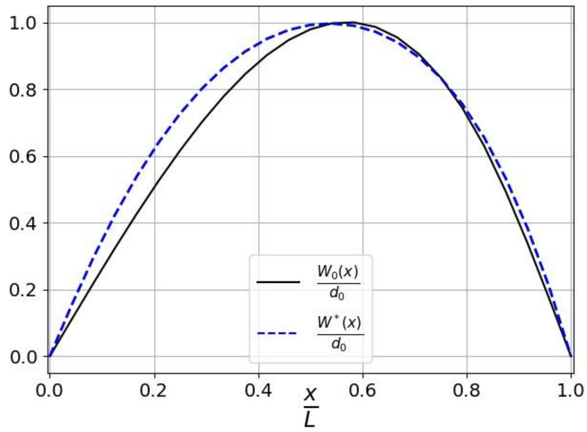

Fig. 5. Sagging of optimal beam and beam with constant cross-section

Sets of beam heights got after each cycle of genetic algorithm differ from each other due random mutations (Fig. 3), so the is a sense to estimate scattering of calculated values. Formally all reference points are equivalent let's consider deviations of heights from their mean values at all reference points by means of creation general array of deviations over all 40 individuals of final population. Diagram of deviation frequencies is shown on Fig. 6.

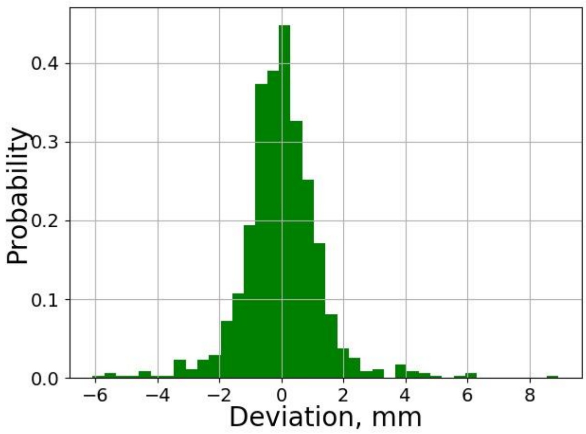

Fig. 6. Diagram of deviation frequencies

Mean-square deviation of this sample is approximately equal to 1.2 mm. Obviously, another realization of the algorithm will give slightly another diagrams of deviation frequencies and mean-square deviation. But calculation have shown that difference in obtained results is negligible.

Strictly speaking, scattering of beam heights at nodes near mean value is stipulated by influence of random mutations and mating. To estimate influence of algorithm parameters on final results let's change some parameters of algorithm. Try to increase duration of algorithm genetic selection from $N_g = 5000$ generations to $N_g = 10000$ generations. At the same time try to reduce mean-square deviation of mutations from $\sigma = 0.2$ mm to $\sigma = 0.15$ mm. All other parameters will keep the same. Correspondent frequency diagram is shown on Fig. 7.

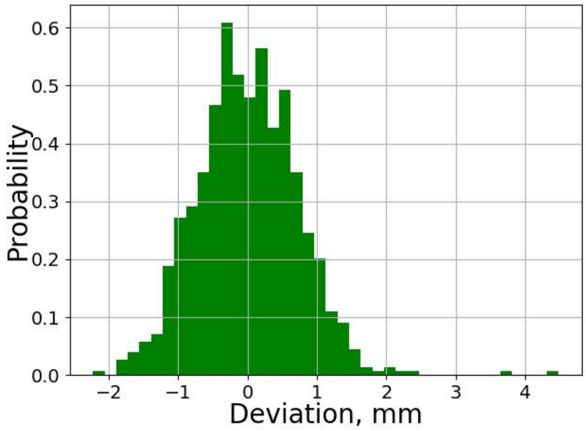

Fig. 7. Diagram of deviation frequencies

In this case mean-square deviation for given sample is reduced up to 0.7 mm (it was expectable). Heights distribution visually doesn't differ from one shown on Fig. 4.

5. Conclusions

Suggested in the paper genetic algorithm allows to find solution of beam height optimal distribution along beam length at given restrictions on maximum displacements. Moreover, in comparison with majority of known analogous problems the position of point with maximum beam deflection is unknown before analysis.

Algorithm shows good stability and gives similar results in wide range of algorithm parameters (volume of population, quantity of cycles, portion of mutative genomes etc). Since solving of direct problems about beam sagging is separated from optimization problem application of genetic algorithm allows quite simply to add restrictions on structural strength, buckling, apply such approach to statically undetermined structures, curved rods, non-linear beams sagging etc.

Further researches could be directed to algorithm perfection. For example, it is believable to be prospective to have significant portions of mutations and their mean-square deviation at beginning iterations. Then simultaneously with increasing quantity of generations in algorithm to reduce values of these parameters. This will allow from one hand to find quite quickly solutions close to optimal at beginning stages of iterations, from another hand to have minimal scattering of beam heights close to optimal at final stages of iterations

Moreover, suggested algorithm is planned to be used for solution of adhesive joints optimization [16, 17]. In this problem the law of two joining articles thickness variation along joint length will be objective function. Restrictions can be taken from strength condition but not the from rigidity like in current paper.

Contributions of authors: formulation of the problem – **S. Kurennov**; development of genetic algorithm – **S. Kurennov**, **K. Barakhov**; program realization of algorithm – **K. Barakhov**, **V. Stepanenko**; analysis of model problem, analysis results processing – **I. Taranenko**, **V. Stepanenko**; text of preliminary version of the paper **S. Kurennov**; editing and postediting – **K. Barakhov**, **I. Taranenko**. All authors have read and agreed to the published version of the manuscript.

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ГЕНЕТИЧНИЙ АЛГОРИТМ ОПТИМАЛЬНОГО ПРОЕКТУВАННЯ БАЛКИ ЗА НАЯВНОСТІ ОБМЕЖЕНЬ НА ПЕРЕМІЩЕННЯ

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Запропоновано генетичний алгоритм розв’язання задачі оптимального розподілу матеріалу по довжині балки за наявності обмеження на максимальну величину її вигину. Проведено огляд літератури, і показано, що відомі розв’язки обмежуються лише окремими випадками, для яких розташування точки максимального вигину відомо заздалегідь. У представленій роботі розглядається балка двотаврового перерізу постійних пропорцій, тобто ширина балки, товщини полиць і стінок балки пропорційні її висоті в даному перерізі. Розглядається статично визначні балка. Навантаження, що діє на балку, може бути довільним, у тому числі несиметричним і різноспрямованим. Розташування точок (або точки), в яких вигини балки максимальні – заздалегідь невідомо і знаходиться в процесі розв’язання задачі. Задача розглянута в лінійній постановці. В якості критерію оптимізації прийнято масу балки. Для знаходження вигинів балки, тобто для розв’язання диференціального рівняння вигину балки змінного перерізу використовується метод кінцевих різниць. Дискретизація дозволяє звести задачу проектування до задачі знаходження потрібних висот балки в системі вузлових точок. При цьому шуканий розв'язок має задовольняти системі обмежень на переміщення в вузлових точках. Оскільки обмеження на переміщення кожної вузлової точки розглядаються окремо і незалежно одне від одного, запропонована методика дозволяє гнучко керувати обмеженнями на переміщення балки. Запропоновано цільову функцію, яка є лінійною суперпозицією маси балки та можливого штрафу за перевищення максимального вигину заданого в умові. При роботі генетичного алгоритму з популяції існуючих наборів товщин балок відбираються ті, які забезпечують досягнення цільовою функцією мінімуму. Розв'язано модельну задачу, і показано, що запропонований алгоритм дозволяє ефективно розв'язувати задачі оптимального проектування балок за наявності обмежень на максимальну величину вигину. Запропонований підхід може бути розвинуто на наявність обмежень по міцності, статично невизначених конструкцій тощо.

Ключові слова: умовна оптимізація; метод кінцевих різниць; генетичний алгоритм.

ГЕНЕТИЧЕСКИЙ АЛГОРИТМ ОПТИМАЛЬНОГО ПРОЕКТИРОВАНИЯ БАЛКИ ПРИ НАЛИЧИИ ОГРАНИЧЕНИЙ НА ПЕРЕМЕЩЕНИЯ

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Предложен генетический алгоритм решения задачи оптимального распределения материала по длине балки при наличии ограничения на максимальную величину ее прогиба. Проведен обзор литературы, и показано, что известные решения ограничиваются лишь частными случаями, для которых положение точки максимального прогиба известно заранее. В представленной работе рассматривается балка двутаврового сечения постоянных пропорций, т.е. ширина балки, толщины полок и стенок балки пропорциональны ее высоте в данном сечении. Рассматривается статически определимая балка. Нагрузка, действующая на балку, может быть произвольной, в том числе несимметричной и разнонаправленной. Положения точек (или точки), в которых прогибы балки максимальны – заранее неизвестны и находятся в процессе решения задачи. Задача рассмотрена в линейной постановке. В качестве критерия оптимизации принята масса балки. Для нахождения прогибов балки, т.е. для решения дифференциального уравнения изгиба балки переменного сечения используется метод конечных разностей. Дискретизация позволяет свести задачу проектирования к задаче нахождения потребных высот балки в системе узловых точек. При этом искомое решение должно удовлетворять системе ограничений на перемещения в узловых точках. Поскольку ограничения на перемещения каждой узловой точки рассматриваются отдельно и независимо друг от друга, предложенная методика позволяет гибко управлять ограничениями на перемещения балки. Предложена целевая функция, которая представляет собой линейную суперпозицию массы балки и возможного штрафа за превышение максимального прогиба заданного в условии. При работе генетического алгоритма из популяции существующих наборов толщин балок отбираются те, которые обеспечивают достижение целевой функцией минимума. Решена модельная задача, и показано, что предложенный алгоритм позволяет эффективно решать задачи оптимального проектирования балок при наличии ограничений на максимальную величину прогиба. Предложенный подход может быть развит на наличие ограничений по прочности, на статически неопределимые конструкции, и т.д.

Ключевые слова: условная оптимизация; метод конечных разностей; генетический алгоритм.

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