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SIMULATION OF ARMS DISTRIBUTION STRATEGIES BY COMBAT ZONES TO CREATE MILITARY PARITY OF FORCES

The problem of researching weapons distribution strategies to ensure effective combat operations in a military conflict zone is being formed and solved. The relevance of the study is related to the solution of the task of an operational-tactical nature in order to establish the military parity of forces in the conditions of the introduction of various equipment into the combat zone (CZ), which has different characteristics of combat capability from the set of warehouses and suppliers, with long supply chains. The problems that arise when establishing military parity of forces with the help of various weapons with different indicators of combat capability, the limited capabilities of suppliers, and the complications of supply logistics are analyzed. It is shown that asymmetric solutions, according to military equipment, are possible because of the use of technological innovations and high indicators of the combat effectiveness of weapons. It was concluded that the establishment of military parity of forces is possible thanks to the use of rational strategies for the distribution of weapons, which is an urgent problem in the conditions of modern hybrid warfare. Thus, this paper proposes a study of possible strategies for the distribution of weapons by CZ in conditions of threats of martial law of the country. The current study creates models that allow for the evaluation of possible arms distribution strategies, considering the diversity of military equipment, limited capabilities of suppliers, and risks associated with long logistics supply chains. Using set-theoretical analysis, methods of combinatorics, and enumeration theory, a systematic presentation of the distribution process of weapons is created. Possible strategies for the distribution of weapons by CZ to establish military parity of forces are formed and analyzed. Optimization models have been created for the rational distribution of military equipment under conditions of limited capabilities of manufacturers in terms of costs, production terms, and risks of arms supply. A simulation model is being developed to study the dynamics of the logistical process of distribution and supply of weapons in agent representation using the AnyLogic platform. Modeling allows you to analyze the impact of threats to the main indicators of the distribution and supply of weapons. An illustrative example of the distribution of weapons by CZ of a military conflict is presented. The scientific novelty of this research is related to the solution of the actual problem of the rational distribution of various military equipment by CZ in the conditions of modern hybrid warfare. The results of this study should be used for operational-tactical planning of the logistical process of distribution and supply of weapons to solve the problem of establishing military parity of forces in the CZ.

Keywords: *military parity of forces; various weapons; distribution of weapons; system model of distribution; optimization of distribution; supply logistics; simulation model; agent model.*

1. Introduction

Modern hybrid warfare is associated with innovations and new technological solutions that contribute to effective combat operations [1, 2]. The use of various types of military equipment, which is produced and supplied by various manufacturers, requires a rational distribution to create military parity of forces in the zone of military conflict [3, 4]. Therefore, a study is proposed in which the modeling of possible strategies for the rational distribution of military equipment by CZ is carried out, considering the combat capability of certain types of weapons.

1.1. Motivation

The variety of modern military equipment used in the military conflict zone leads to difficulties in creating military parity of forces in individual local CZ [5, 6]. However, the indicators of the combat capability of modern weapons (range, accuracy, zone of damage, etc.) allow, due to the superiority of quality over quantity, to create military parity of forces through asymmetry in the use of certain types of weapons (for example, the use of modern systems HIMARS, NASAMS, etc.) [7, 8]. Therefore, the relevance of the topic of the proposed publication, which analyzes and models possible strate-

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gies aimed at creating military parity of forces through the rational distribution of weapons by local CZ of a military conflict, is important.

1.2. State of the art

There are many problems that are associated with the creation of military parity of forces through the use and rational distribution of various weapons, but have partial solutions in the conditions of modern war, which are analyzed in publications [9, 10]:

1. Variety of weapons [11, 12]. Modern types of weapons have different indicators of combat capability, which complicates the distribution of military equipment by CZ [13, 14].

2. Establishing military parity of forces in different CZ has objective difficulties [15, 16]. These difficulties are related to the set of suppliers, limited capabilities of manufacturers and long supply chains of military equipment from warehouses [17, 18].

3. Distribution of weapons under wartime conditions [19, 20]. Strategies for the distribution of military equipment must be adapted to its diversity, multiple indicators of combat capability, and the military potential of the enemy in the military conflict zone under wartime conditions [21, 22].

4. Complexities of military transport logistics [23, 24]. For the distribution and supply of weapons to CZ, a heterogeneous transport environment is used, which in the conditions of threats of martial law is critical due to long supply chains [25, 26].

5. The modern nature of hybrid warfare requires the use of innovations and high-tech weapons, which makes it possible to establish military parity of forces due to asymmetric solutions [27].

6. Dynamics of changes in the nature of combat operations [28, 29]. This requires constant monitoring and analysis of the enemy's military potential, for the timely distribution and supply of weapons, which contributes to the creation of military parity of forces.

Thus, the manifestations of modern war in the form of a hybrid with the use of various weapons, different manufacturers and suppliers, and long logistics chains require the formation and analysis of a strategy for the rational distribution of military equipment by CZ to create military parity of forces.

1.3. Objectives and methodology

A contradiction arises between the requirements for creating military parity of forces in the zone of military conflict, in the conditions of military threats, and the lack of improvement of existing methods and a systematic analysis of problems related to the study of the strategy of distribution of weapons in the changing conditions of modern war. This contradiction is the object of research in the proposed publication, which presents methods and models for creating a methodology for the rational distribution of weapons in hybrid warfare.

The purpose of this study is to analyze and model possible strategies for the rational distribution of various weapons to ensure the creation of military parity of forces in CZ considering the enemy’s military potential. In accordance with the research objective, the following problems must be solved:

1. Create a systemic set-theoretic representation of the logistics of the distribution of military equipment across CZ during a military conflict.

2. Analyze possible strategies for the distribution of weapons, considering the military potential of the enemy.

3. Develop optimization models for the rational distribution of military equipment.

4. Develop an agent simulation model to study the logistics of arms distribution and supply under military threat conditions.

5. Present an illustrative example of the distribution of weapons by CZ.

Mathematical methods and models are used to study strategies for the distribution of weapons and military equipment: system analysis, enumeration theory, integer optimization, multivariate selection, and agent simulation modeling.

2. A systemic set-theoretic representation of the logistics of distribution of military equipment across combat zones of a military conflict

The presence of multiple types of weapons used in modern hybrid warfare, as well as the conduct of combat operations not in one but in a whole number of local military conflict zones, requires a set-theoretic representation for the decision on the distribution of weapons. Therefore, it is advisable to use combinatorics and enumeration theory methods to evaluate possible options for the distribution of military equipment by CZ [30]. The following stages were selected for evaluating set options for the distribution of weapons:

1. Assessment of the number of options for the distribution of weapons by CZ.

2. Formation of a set of distribution options for further analysis and optimization.

When enumerating the options for the distribution of military equipment, various tasks are possible:

1. Distribution of one type of weapon according to the local CZ.

In this case, the enumeration task is related to the use or non-use of the i-th CZ in the distribution of a specific j- th type of weapons, $i = \overline{1, M}$, where M is the number of CZ in a military conflict, $j = \overline{1, N}$, where N is the number of types of weapons.

Boolean variable $x_{ji} \in \{0,1\}$, where $x_{ji} = 1$, if i-th CZ is used to place the j-th type of weapon, $x_{ji} = 0$, in the other case. Then the number of distribution options:

$$K = 2^M.$$

Options can be formed using a binary counter. For example, M = 3, then K = 8 (the option 000 is rejected as fictitious). Options are presented in the following form:

1. 001,
2. 010,
3. 011,
4. 100,
5. 101,
6. 110,
7. 111,

where option 011, for example, means that the weapons are distributed in the second and third CZ, and in the first CZ – they aren't used.

2. Distribution of various types of weapons.

In this case, we will use an integer variable $x_{ji} \in \{0,1,2,3...\}$, where $x_{ji} = p$, means that the p-th weapon will be sent to the i-th CZ.

In this case, the number of weapons distribution options is equal to:

$$K = L^M,$$

where L is the number of types of weapons that will be used in a specific CZ.

The L-th counter can be used to form a set of distribution options. For example, where L = 3, is ternary counter. Then the set of options for the distribution of weapons: 32 = 9, and the options themselves have the form:

1. 11,
2. 12,
3. 13,
4. 21,
5. 22,
6. 23,
7. 31,
8. 32,
9. 33,

where 1 is weapons of the first type;

2 is weapons of the second type;

3 is weapons of the third type.

3. Consider a possible case in which the planning of the distribution of one type of weaponry is carried out in advance, and the finite set of weapons of mass destruction in the CZ has not yet been formed. In this case, we use the main results of enumeration theory [30]. By applying this theory, it is possible to count the number of options when mapping the set of weapons A into the set of CZ V. This formulation of the problem is equivalent to the problem of dividing the number A into V parts ($A \geq V, V = 1, 2, \ldots$).

4. Consider the case where the distribution of several types of weapons is planned when the finite set of CZ is not yet formed.

In this case:

$$A = \sum_{\mu=1}^{N} p_\mu ,$$

where p_μ is the number of μ-th type weapons;

N is the number of types of weapons.

Then, the sum of the symmetric groups of substitutions acts on a set of weapons of different types [30]:

$$H_A = S_{p_1} + S_{p_2} + \ldots + S_{p_N} .$$

After using the results of enumeration theory, we obtain the number of options [30]:

$$K = |H_V|^{-1} \sum_{h \in S_V} Z(H_A, \ldots, \sum_{j/i} jC_j, \ldots) =$$
$$= \frac{1}{V!} \sum_{h \in S_V} Z(S_{p_1} + S_{p_2} + \ldots + S_{p_N}; \ldots, \sum_{j/i} jC_j, \ldots).$$

5. Consider the case when it is necessary to distribute several types of weapons according to the already formed set of CZ (V is fixed with a specific value) in the zone of military conflict. In this case, the number of options [30]:

$$K = K_V - K_{V-1} =$$
$$= \frac{1}{V!} \sum_{h \in S_V} Z(S_{p_1} + S_{p_2} + \ldots + S_{p_N}; \ldots, \sum_{j/i} jC_j, \ldots) -$$
$$- \frac{1}{(V-1)!} \sum_{h \in S_{V-1}} Z(S_{p_1} + S_{p_2} + \ldots + S_{p_N}; \ldots, \sum_{j/i} jC_j, \ldots).$$

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3. Analysis of possible strategies for the distribution of weapons in combat zones

A complex military situation changes quickly during combat operations and requires the formation of possible strategies for the distribution of weapons by local CZ in a military conflict. Therefore, it is necessary to present and analyze the possibilities of the distribution strategy, considering the military potential of the enemy in the CZ (their number can be changed). We offer the following possible distribution strategies that can be used depending on the operational situation in CZ:

1. The existing military equipment in the CZ of the military conflict is sufficient to create a military parity of forces and conduct successful combat operations.

In this case:

$$n_{ji} \geq m_{ji},\ j = \overline{1, N},$$

where n_{ji} is the number of weapons of the j-th type in the i-th CZ;

m_{ji} is the number of weapons of the j-th type at the enemy in the i-th CZ;

N is the number of types of weapons used in the CZ.

In this case, the distribution of weapons is carried out taking into account the established plans for operational-tactical actions. The supply of weapons is necessary, first of all, for those CZ that are a priority for conducting combat operations, including those of an offensive nature.

These strategies are associated with the emergence of a shortage of weapons by individual types, which requires a rational distribution to create military parity of forces.

2. Not all types of weapons are available in sufficient quantity to establish military parity of forces.

In this case:

$$n_{jk} \geq m_{jk},\ n_{je} < m_{je},$$

where k is CZ in which the military balance of forces has been established by the j-th type of weapon;

e is the absence of military parity of forces in the j-th type of weapon.

At the same time, it is necessary to perform a rational distribution of weapons to create combat parity of forces, first of all, for those CZ in which violations of the nature of combat operations are possible (for example, the transition from offensive to defensive actions) due to an insufficient number of individual types of weapons.

3. Actual CZ, where there is a shortage of all types of weapons. In this case:

$$n_{jp} \geq m_{jp},\ n_{jq} < m_{jq},$$

where q refers to those CZ in which there is a shortage of all types of weapons; p refers to those CZ in which there is a shortage of several types of weapons.

In this case, it is necessary, as soon as possible, to perform the distribution of weapons, first of all, for the CZ, which are critical (a complete shortage of all types of weapons), since there is a possibility of changing the nature of the combat operations.

4. A strategy that uses combat capability indicators for distribution. In this case, at the expense of modern high-tech weapons and with increased indicators of combat capability (range, accuracy, zone of damage, etc.), it is possible to establish military parity of forces with a smaller number of weapons (advantage of quality over quantity). This is considered when distributing weapons by CZ.

5. A strategy based on an integral indicator of combat capability for all types of weapons in the zone of military conflict. In this case, the distribution of weapons is carried out taking into account the integral indicator of combat capability:

$$W = \sum_{j=1}^{N} w_j n_j,\ W = W_1 + W_2,\ W_1 \geq W_2,$$

$$W_1 = \sum_{j=1}^{N} w'_j n_j,\ W_2 = \sum_{j=1}^{N} w''_j m_j,$$

where W_1 refers to the integral indicator of combat capability, which considers all types of weapons in a military conflict;

W_2 refers to the integral indicator of the enemy's weapon;

n_j is the number of weapons of the j-th type in the zone of military conflict;

m_j is the number of weapons of the j-th type of the enemy's weapon.

In this case, successful combat operations are conducted at the expense of a greater number of existing weapons and the use of high-tech weapons with increased combat capability characteristics for all CZ of military conflict, where w_j is the combat capability indicator for the j-th type of weapon.

6. It is possible to use a mixed strategy in which strategies 2, 3, 4 are implemented.

4. Optimization models for the rational distribution of military equipment

The use of distribution strategies requires optimizing the choice of suppliers for each j-th type of weapon that will be sent to the CZ to create a military parity of forces. We will use the integer (boolean) programing method to create optimization models.

Given a boolean variable x_{jki}:

$$x_{jki} = \begin{cases} x_{jki} = 1, \text{ if weaspons of the j-th type} \\ \quad \text{will be supplied to the i-th CZ,} \\ \quad \text{k-th composition of suppliers;} \\ x_{jki} = 0, \text{ in another case.} \end{cases}$$

For the supply of weapons, it is necessary to take into account the limited capabilities of suppliers for the j-th type of weapons (n_{jk} is the amount of weapons of the j-th type that can be sent to the zone of military conflict by the k-th composition of suppliers).

Suppose that a possible group of suppliers can be formed using a binary counter:

$$l_j = 2^{p_j} - 1,$$

where p_j is the number of suppliers of the j-th type of weapon;

l_j is the number of options for forming the possible composition of suppliers for the j-th type of weapon.

When distributing weapons, it is necessary to consider the production time t'_{jk} and costs v'_{jk} for the j-th type of weapons for the k-th composition of suppliers, as well as the time t''_{jk} and costs v''_{jk} for the logistics of supplying weapons to the i-th CZ.

In a state of war, the country must consider military threats and the risks of their occurrence; therefore, we will use the following risks:

r'_{jk} is the risk of military threats associated with the production of the j-th type of weapons by the k-th possible composition of suppliers;

r''_{jki} is the risk in the logistics of supplying the j-th type of weapons by the k-th warehouse of suppliers for the i-th CZ.

We will create an optimization model for the rational distribution of weapons, considering time, costs, and risks.

If, as the objective function of the optimization model, the time required for the production and supply of weapons is used. Therefore, when distributing weapons, it is necessary:

$$\min T, T = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{l_j} (t'_{jk} + t''_{jki}) x_{jki},$$

where $x_{jki} = 1$, if the supply in the i-th zone uses the k-th warehouse of suppliers for the j-th type of weapons,0 is another case.

At the same time, to fulfill the requirements of strategy 2, 3, it is necessary that the number of weapons:

$$n_{ji} \geq m_{ji},$$

for all $j = \overline{1, N}$ and for all CZ, where m_{ji} is the number of weapons of the j-th type, which is present in the enemy's i-th CZ.

In addition, it is necessary to fulfill the limitation on costs V, which is related to the capabilities of suppliers:

$$V = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{l_j} (v'_{jk} + v''_{jki}) x_{jki}, V \leq \overline{V},$$

where $\overline{V}$ is the permissible cost.

It is also necessary to consider the military risks that are in the production and supply of weapons:

$$R = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{l_j} (r'_{jk} + r''_{jki}) x_{jki}, R \leq \overline{R},$$

where $\overline{R}$ is the permissible risk in the distribution of weapons.

In the presence of a high risk of the appearance of military threats, we will use risks as an objective function:

$$\min R, R = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{l_j} (r'_{jk} + r''_{jki}) x_{jki}.$$

In order to use 2, 3 strategies for the distribution of weapons, it is necessary to $n_{ji} \geq m_{ji}$ for all $j = \overline{1, N}$ and for all CZ $i = \overline{1, M}$, and to comply with time and cost constraints:

$$T \leq \overline{T}, T = \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{l_j} (t'_{jk} + t''_{jki}) x_{jki},$$

$$V \le \overline{V}, V = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{l_j}(v'_{jk} + v''_{jki})x_{jki}.$$

In order to use the 4 strategy of weapons distribution, which takes into account its combat capability, it is necessary that the combat capability:

$$n_{ji}w'_j \ge m_{ji}w''_j$$

for all types of weapons:

$j = \overline{1, N}$ and for all CZ $i = \overline{1, M}$,

where w'_j is combat capability of the j-th type of weapon;

w''_j is combat capability of the j-th type of enemy's weapon.

If the costs of production and supply of modern high-tech types of weapons with increased combat effectiveness are used as the objective function. Then, it is necessary:

$$\min V, V = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{l_j}(v'_{jk} + v''_{jki})x_{jki},$$

subject to compliance with the following restrictions:

$$T \le \overline{T}, T = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{l_j}(t'_{jk} + t''_{jki})x_{jki},$$

$$R \le \overline{R}, R = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{l_j}(r'_{jk} + r''_{jki})x_{jki}.$$

Thus, to use the 5 strategy, which has an integral combat capability indicator, it is necessary:

$$\sum_{j=1}^{N} w'_j n_j \ge \sum_{j=1}^{N} w''_j m_j.$$

If it is necessary to quickly create a parity of forces, considering asymmetry, using the values of the combat capability indicators of weapons in the zone of military conflict. Therefore, as an objective function, it is necessary to take time to produce and supply high-tech weapons:

$$\min T, T = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{l_j}(t'_{jk} + t''_{jki})x_{jki},$$

subject to the following restrictions:

$$V \le \overline{V}, V = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{l_j}(v'_{jk} + v''_{jki})x_{jki},$$

$$R \le \overline{R}, R = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{l_j}(r'_{jk} + r''_{jki})x_{jki}.$$

5. An agent simulation model

To assess in detail the results of the distribution of military equipment from the CZ, it is necessary to conduct simulations in the conflict zone, considering the dynamic process of distribution. For this purpose, an agent simulation model was created on the AnyLogic platform. These agents are in the model:

1. An agent that describes a heterogeneous transport environment (TE) for the supply of weapons to the CZ (transport routes, temporary storage sites, transshipment sites from one highway to another, etc.).
2. Agent of military threats (generator of threat occurrence).
3. An agent that describes vulnerabilities (triggered after the occurrence of threats in critical places of the transport network).
4. An agent that describes the damage (results in downtime in the transportation of weapons).
5. Agent of the route (trajectory of the route in the transport environment).
6. Agent of vehicles (the number of weapons is formed for the transportation of one batch of weapons).
7. Agent of supplier (arbitrary position in the transport network from which weapons are delivered to the CZ
8. Agent of CZ (the final position of the transport network for the supply of weapons).
9. Agent of statistics (formation statistical data and modeling results).
10. Agent of management (management of agents 2 – 9, considering the time scale and sequence of agents' actions).

In Fig. 1 the structural diagram of the agent model is represented.

Because of the simulation modeling, the following is formed:

- the trajectory of the route for the supply of military equipment, considering the time of supply and the number of weapons;

- movement delays associated with warehousing, temporary storage, and overloading are estimated;
- delays in supply due to the emergence of military threats;
- supply schedule.

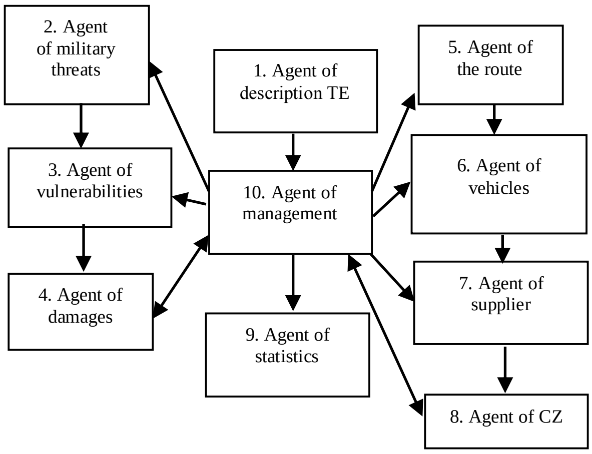

Fig. 1. The structural diagram of the agent model

The simulation was performed multiple times for statistical averaging of the results.

6. An illustrative example of the distribution of weapons in combat zones

To demonstrate the proposed approach to the distribution of weapons in a CZ, an illustrated example of the use of a distribution strategy for two CZ and three types of weapons used for combat operations is given. Through the conducted analysis, a set of possible suppliers for the first type of weapons was determined, as well as the number of these suppliers $n_1 = 3$. For the second type of weapon $n_2 = 2$, and for the third – $n_3=3$. Then the number of possible variants of the composition of the suppliers of weapons (see point 2): $K_1= 2^3-1 = 7$, $K_2 = 2^2-1 = 3$, $K_3 = 2^3-1 = 7$.

Further, for each possible k-th composition of the suppliers of weapons, it is necessary to evaluate the logistical possibilities of supply: quantity (n_{jk}), time (t_{jk}), costs (v_{jk}), and risks (r_{jk}), which are subsequently used to choose an option for the distribution of weapons.

Tables 1, 2, and 3 show options for the supply of weapons for the first, second, and third types of weapons, which were obtained using a binary counter (see point 2). In order to estimate logistics indicators in terms of time, costs and risk, averaging was carried out for different suppliers' warehouses to estimate their values per one unit of weapons: t_j is estimated in days, v_j is estimated in points (0 – 100), r_j is estimated in points (0 – 100).

Table 1

Variants in the composition of suppliers for the 1st type of weapons

No. warehouse	Composition of suppliers	n_{1k}
1	001	3
2	010	4
3	011	7
4	100	2
5	101	5
6	110	6
7	111	9

Table 2

Variants in the composition of suppliers for the 2nd type of weapons

No. warehouse	Composition of suppliers	n_{2k}
1	01	2
2	10	4
3	11	6

Table 3

Variants in the composition of suppliers for the 3rd type of weapons

No. warehouse	Composition of suppliers	n_{3k}
1	001	2
2	010	4
3	011	7
4	100	4
5	101	5
6	110	8
7	111	10

In the Table 4 presents the values of logistic indicators for one weapon unit for each type of weapon.

It is necessary to assess the possible military potential of the enemy to choose the composition of suppliers that will be used in the distribution of weapons in two CZ. For this purpose, while conducting reconnaissance operations (namely, with the help of UAVs), up-to-date data on the number of m_{ji} weapons of the j-th type in the i-th CZ were obtained (Table 5).

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Table 4

Logistics indicators of supply

No. type of weapon	t_j	v_j	r_j
1	3	15	6
2	2	10	4
3	4	20	8

Table 5

The number of enemy weapons

No.	m_{1i}	m_{2i}	m_{3i}
1	2	2	1
2	2	3	5

We will use for distribution strategy No. 2 (see point 3) for the distribution of weapons in two CZ, considering the following requirements:

$$n_{jki} \geq m_{ji},$$

where n_{jki} is the number of weapons of the j-th type supplied by the k-th warehouse of suppliers and directed to the i-th CZ.

The set of suppliers is analyzed and presented in Tables 1 – 3. The following sets of suppliers were chosen that satisfy the inequalities $n_{jk} \geq m_j$. For $m_1 = 4$, it will be $n_{15} = 5$, for $m_2 = 5$ it will be $n_{23} = 6$, and for $m_3 = 6$ it will be $m_{33} = 7$.

Further, to analyze options and form n_{jki} values, it is necessary to divide the n_{jk} value into two parts, which is equivalent to dividing the n_{jk} number into two components (two CZ). Based on the results of the enumeration theory (see point 2), the number of distribution options: $k_1 = 2$, $k_2 = 3$, $k_3 = 3$. The distribution options have the following form (the first number corresponds to the first CZ, and the second to the second CZ):

For the 1st type of weapon:

1. 14,
2. 23.

For the 2nd type of weapon:

1. 15,
2. 24,
3. 33.

For the 3rd type of weapon:

1. 16,
2. 25,
3. 34.

To choose distribution options considering the requirements of the second strategy, it is necessary that: $n_{jki} \geq m_{ji}$ for all j and i.

Therefore, the following options were chosen:

1. For the 1st type of weapons, there is a second distribution option:

$$n_{121} = 2,\ n_{122} = 3,\ n_{12} = n_{121}+n_{122} = 5,$$

where n_{12} is the number of weapons of the 1st type for the 2nd variant of distribution.

For the 2nd distribution option, the requirements of the 2nd distribution strategy are as follows:

$$n_{121} \geq m_{11},\ n_{122} \geq m_{12},$$

where $m_{11} = 2$, $m_{12} = 2$ (see Table 5), $m = m_{11}+m_{12} = 4$.

2. For the 2nd type of weapon, the 3rd distribution option is selected:

$$n_{231} = 3,\ n_{232} = 3,\ n_{23} = n_{231}+n_{232} = 6.$$

For the 3rd distribution option, the requirements of the 2nd distribution strategy are as follows:

$$n_{231} \geq m_{21},\ n_{232} \geq m_{22},$$

where $m_{21} = 2$, $m_{22} = 3$, $m_2 = m_{21}+m_{22} = 5$ (see Table 5).

3. For the 3rd type of weapon, the 2nd distribution option is selected:

$$n_{321} = 2,\ n_{322} = 5,\ n_{33} = n_{321}+n_{322} = 7.$$

For the 2nd distribution option, the requirements of the 2nd distribution strategy are used:

$$n_{321} \geq m_{31},\ n_{322} \geq m_{32},$$

where $m_{31} = 1$, $m_{32} = 5$, $m_3 = m_{31}+m_{32} = 6$ (see Table 5).

Considering the data in Tables 1 – 3, as well as the data in Table 4 and the distribution results for all three types of weapons in two CZ, Tables 6 and 7 were formed.

Table 6

The results of the distribution of weapons for the 1st CZ

No. type of weapon	Distribution option number	n_{j1}	t_{j1}	v_{j1}	r_{j1}
1	2	2	6	30	12
2	3	3	6	30	12
3	2	2	8	40	16

Table 7
The results of the distribution of weapons for the 2nd CZ

No. type of weapon	Distribution option number	n_{j2}	t_{j2}	v_{j2}	r_{j2}
1	2	3	9	45	18
2	3	3	6	30	12
3	2	5	20	100	40

The Tables present the values of the indicators of the quantity of weapons, time for delivery, and costs and risks of weapons delivery.

Discussion

The given example, which analyzes the distribution of weapons and military equipment by CZ in the conditions of a military conflict, demonstrates the effectiveness and practicality of the proposed approach in choosing strategies for the distribution of military equipment, assessing risks, and determining the time required for the timely delivery of weapons in conditions of martial law. The analysis of possible strategies for the distribution of military equipment allows the creation of conditions for the formation of military parity, considering the combat capability indicators of certain types of weapons, which affect the nature of hostilities and possible losses from the actions of the enemy.

Formal formulation in the form of combinatorics models, optimization and simulation modeling, usually, includes the following stages:

1. Analysis of a set of possible CZ in a military conflict.
2. Assessment of the availability of weapons and military equipment in the CZ.
3. Choosing a weapon distribution strategy from a set of possible ones to create military parity of forces.
4. Assessment and optimization of the main indicators related to the choice of a rational strategy for the distribution of weapons.
5. Modeling of logistic methods for supplying military equipment.

The proposed set of strategies is not final, which allows for the formation of new strategies for the distribution of weapons and military equipment, considering the characteristics of hybrid warfare. This is a new area of research and requires new approaches to the formation of military parity of forces.

Further research is planned to be related to innovations and technological aspects of combat weapons, which can change the nature and effectiveness of combat operations, as well as the conditions for the formation of military parity of forces.

Conclusions

The conducted research is related to the modeling of weapon distribution strategy to create military parity in the CZ. An analysis of existing problems related to the creation of military parity was conducted. On the basis of set-theoretical analysis, methods of combinatorics, and enumeration theory, a systematic presentation of logistical actions related to the distribution of weapons was created. During the formation of the distribution strategy, the variety of military equipment and the set of suppliers and supplies in the conditions of military threats are considered. Optimization models have been developed to estimate the costs, time, and risks of weapons distribution, which allow the analysis of the use of distribution strategies and their comparison with each other. An agent simulation model was created to study the dynamic process of the distribution and supply of weapons under the conditions of the emergence of military threats. An illustrative example of the distribution of weapons according to the proposed model is presented.

The proposed approach makes it possible to analyze strategies for the distribution of weapons in a military conflict zone, to form a set of suppliers considering their limited capabilities, and to develop plans for the supply of weapons.

Contribution of the authors: a systematic representation of the logistics process of distribution of weapons – **Oleg Fedorovich**; analysis of weapons – **Mikhail Lukhanin**; analysis of weapons distribution strategy – **Oleksandr Prokhorov**; creation of optimization models – **Oleg Slomchynskyi;** development of an agent simulation model – **Oleksii Hubka**; an illustrative example of the distribution of weapons – **Yuliia Leshchenko**.

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МОДЕЛЮВАННЯ СТРАТЕГІЙ РОЗПОДІЛУ ОЗБРОЄННЯ ЗА ЗОНАМИ БОЙОВИХ ДІЙ ДЛЯ СТВОРЕННЯ ВІЙСЬКОВОГО ПАРИТЕТУ СИЛ

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Формується та вирішується задача дослідження стратегій розподілу озброєння для забезпечення ефективних бойових дій в зоні воєнного конфлікту. Актуальність дослідження пов’язана з вирішенням завдання оперативно-тактичного характеру, для встановлення військового паритету сил, в умовах надходження різноманітної техніки в зону воєнного конфлікту, з різними характеристиками боєздатності від множини складів та постачальників, з довгими ланцюгами постачання. Аналізуються проблеми, які виникають при встановленні військового паритету сил за допомогою різноманітного озброєння з різними показниками боєздатності, обмеженими можливостями постачальників, ускладненням логістики постачання. Показано, що асиметричні рішення, за військовою технікою, можливі за рахунок використання технологічних інновацій та високих показників боєздатності зброї. Робиться висновок, що встановлення військового паритету сил можливо за рахунок використання раціональних стратегій розподілу озброєння, що є однією з актуальних проблем в умовах сучасної гібридної війни. Таким чином, у роботі пропонується дослідження можливих стратегій розподілу озброєнь за зонами бойових дій в умовах загроз воєнного стану в країні. Метою дослідження є створення моделей, які дозволяють оцінювати можливі стратегії розподілу озброєння, з урахуванням різноманіття військової техніки, обмежених можливостей постачальників та ризиків, пов’язаних з довгими логістичними ланцюгами постачання. Створюється системне представлення процесу розподілу озброєння за рахунок теоретико-множинного аналізу, з використанням методів комбінаторики та теорії перерахування. Формуються та аналізуються можливі стратегії розподілу озброєння за зонами бойових дій для встановлення військового паритету сил. Створюються оптимізаційні моделі для раціонального розподілу військової техніки в умовах обмежених можливостей виробників за витратами, часу виробництва та ризиками постачання озброєння. Розроблюється імітаційна модель для дослідження динаміки логістичного процесу розподілу та постачання озброєння, у агентному представленні, за допомогою платформи AnyLogic. Моделювання дозволяє аналізувати вплив загроз на основні показників розподілу та постачання озброєння. Наведено ілюстрований приклад розподілу озброєння за зонами бойових дій воєнного конфлікту. Наукова новизна дослідження пов’язана з вирішенням актуальної проблеми раціонального розподілу різноманітної військової тех-

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ніки за зонами бойових дій, в умовах сучасної гібридної війни. Результати дослідження доцільно використовувати для оперативно-тактичного планування логістичного процесу розподілу та постачання озброєння для вирішення завдання зі встановлення військового паритету сил в зоні воєнного конфлікту.

Ключові слова: військовий паритет сил; різноманітне озброєння; розподіл озброєння; системна модель розподілу; оптимізація розподілу; логістика постачання; імітаційна модель; агентна модель.

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